Proceedings of the Second
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on the Peaceful Uses of Atomic Energy

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Volume 32
Controlled Fusion Devices

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PROCEEDINGS OF THE SECOND
UNITED NATIONS INTERNATIONAL CONFERENCE
ON THE PEACEFUL USES OF ATOMIC ENERGY

Volume 32
CONTROLLED FUSION DEVICES
More than 2,100 papers were submitted by the nations, the specialized agencies, and the International Atomic Energy Agency, which participated in the Second United Nations International Conference on the Peaceful Uses of Atomic Energy. The number of papers was thus about twice that involved in the First Conference. Provision was therefore made to hold five concurrent technical sessions in comparison with the three that were held in 1955. Even so, the percentage of orally presented papers was less in 1958 than in 1955.

In arranging the programme, the Conference Secretariat aimed at achieving a balance, allowing adequate time for presentation of as many papers as possible and, nevertheless, leaving time for discussion of the data presented. Three afternoons were left free of programme activities so that informal meetings and discussions among smaller groups could be arranged. No records of these informal meetings were made.

A scientific editorial team assembled by the United Nations checked and edited all of the material included in these volumes. This team consisted of: Mr. John H. Martens, Miss L. Ourom, Dr. Walter M. Barss, Dr. Lewis G. Bassett, Mr. K. R. E. Smith, Martha Gerrard, Mr. F. Hudswell, Betty Guttman, Dr. John H. Pomeroy, Mr. W. B. Woolen, Dr. K. S. Singwi, Mr. T. E. F. Carr, Dr. A. C. Kolb, Dr. A. H. S. Matterson, Mr. S. Peter Welgos, Dr. I. D. Rojanski and Dr. David Finkelstein.

The speedy publication of such a vast bulk of literature obviously presents considerable problems. The efforts of the editors have therefore been primarily directed towards scientific accuracy. Editing for style has of necessity been kept to a minimum, and this should be noted particularly in connection with the English translations of certain papers from French, Russian and Spanish.

The Governments of the Union of Soviet Socialist Republics and of Czechoslovakia provided English translations of the papers submitted by them. Similarly, the Government of Canada provided French-language versions of the Canadian papers selected for the French edition. Such assistance from Governments has helped greatly to speed publication.

The task of printing this very large collection of scientific information has been shared by printers in Canada, France, Switzerland, the United Kingdom and the United States of America.

The complete Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy are published in a 33-volume English-language edition as follows:

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Review of Controlled Thermonuclear Research at Los Alamos for mid 1958

By James L. Tuck*

At this first opportunity for open scientific discussions on this subject we look forward with eagerness to exchanging experiences with others and hearing of the courses taken towards the common goal in the various laboratories of the world.

Early Development

At this laboratory, the peculiarly intriguing nature of the problem and the admirable consequences of its solution have long been recognized. Lively discussions of such matters as plasma drift in a torus and its effect on achieving a laboratory thermonuclear reaction occurred between Fermi, Kerst, Landshoff, Teller, R. R. Wilson and the writer in 1946, and an unsuccessful search for neutrons from colliding Munroe jets of metal deuterides was made at about the same time, based on an earlier paper by Ulam and the writer. When the subject was reopened in 1951, the toroidal pinch (as proposed, with a superimposed $B_z$ field, by the writer in 1948) was selected as the confinement geometry for initial study and it passed through the usual vicissitudes and modifications—instability, linear dynamic pinches (Columbus), rf pinches, so called $B_z$ and wall stabilized pinches; while parallel speculations and experimental forays were made into cusped geometries (Picket Fence), spinning plasmas, shocks in axial and convergent geometries and magnetic mirrors. Our ideas and plans have undergone profound changes during the last seven years and are in process of undergoing another. As little as half a year ago, the chief obstacle to the achievement of a thermonuclear reaction via the stabilized pinch appeared to be contamination of the plasma by foreign atoms sputtered or evaporated off the walls. Doubtless this particular obstacle is still there, but in the meantime a crevasse has opened at our feet, in the form of our new experimental observations of high energy losses from the pinched plasma. If these turn out to be due to the newly predicted surface hydromagnetic instabilities, then we know how to overcome these and the outlook may be better than ever. If the losses turn out to be due to plasma oscillations, as is feared, then the outlook for the stabilized pinch as a possible reactor seems grave.

Present Requirements

The outstanding need in controlled thermonuclear research at present is for reliable quantitative observations on confined plasma, and the shortage of these can undoubtedly be blamed on the extreme mobility of plasma, the variability of gas discharges, and their sensitivity at high temperatures to impurities. It is only recently that primary measurements on the pinch effect have been made, of quality such that worthwhile derivations of other quantities from them could be made. At Los Alamos, the magnetic probe has unquestionably been the most effective measuring tool, yielding (from the pinch) plasma pressure, plasma electrical conductivity and, most recently, plasma mass. The mean temperature of the plasma, involving $(T_{\text{electron}} + T_{\text{ion}})$, has also been determined but not the much-sought-after individual components of it. Obviously, if ion temperatures could be made high enough, the trivial neutron-producing processes which bedevil us at present would also become trivial in yield, with the result that the thermonuclear yield could take its logical place as the ideal thermometer for ions.

In order to bring this into play, there has been a direct effort at Los Alamos to produce an identifiable thermonuclear reaction, without regard for reactor implications. Whether this has been achieved is still indeterminate: of the five experiments producing neutrons, Scylla looks probable as a thermonuclear source and is supported by a measurement of the neutron energy distribution, Columbus II neutrons have an energy anisotropy so small as to raise difficulties in interpretation by trivial processes, Columbus S-4 neutrons seem well correlated with a new instability and therefore appear suspect, as do the Perhapsatron S-4 neutrons on account of a much larger energy anisotropy. Ixion neutrons seem to be of two kinds, trivial ones associated with sheath breakdown and less determinate ones in a kind of tail of indefinite extent.

Obviously, the control of thermonuclear fusion depends on answers to problems in basic plasma physics. For example, are the high plasma losses observed in the pinch a characteristic of all confined
systems? Until such questions are answered, the technological problems of a fusion reactor are surely far away. What engineering design for a thermonuclear reactor would survive an increase in plasma diffusion rate by an order of magnitude?

As a matter of fact, some of the highly impulsive schemes—Columbus and Picket Fence—might survive such an increase but, for the time being, at Los Alamos we hope to steer clear of any large machine, keeping to a course of fairly basic plasma physics research in which practice and theory are intimately associated, around numerous modest-scale experiments.

In magnitude, our effort engages the activities of fifty-one people, including twenty-four physicists, the rest being engineers, technicians and secretariat. The staff is divided into two groups under the leaderships of Dr. K. Boyer and Dr. J. A. Phillips.

THEORETICAL

Stability

The theories of an infinite-conductivity pinch stabilized by an axial field—Kruskal and Tuck,1 Rosenbluth,2 and Colgate3 in the USA, Artsimovich4 and Shafarinov4 in the USSR, and Taylor4 in the UK—have become largely supplanted in our minds by the energy principle treatment7 by Suydam8 of the same geometry with diffuse boundaries between axial stabilizing field $B_z$ and confining field $B_r$. In this treatment the sign of $\delta W$, a function of the displacement, is the criterion for stability. The study was carried out by noting that the special character of the Euler-Lagrange equation of the variational principle permits one to evaluate $\delta W$ for a minimal displacement (i.e., a displacement which satisfies the Euler-Lagrange equation) for general configurations of plasma and field, without obtaining a detailed solution to this equation. If a minimum for $\delta W$ exists, such a procedure will find this minimum. However, certain configurations exist for which $\delta W$ is unbounded below (hence unstable) and it has not been shown that the above procedure will identify such cases. Thus a condition has been found which is necessary, but has not been shown sufficient for stability. It is

$$\frac{\delta W}{\delta B} \frac{d^2 B}{dr^2} \left( \frac{d}{dr} \frac{d}{d\varphi} \right)^2 > 0$$

where $\mu = B_0 r B_z$ and $\varphi = \text{plasma pressure}$. This imposes restrictions on the radial gradient of density in the pinch, such that it seems unlikely that a stable pinch with unidirectional $B_z$ and $B_r$ exists.

However, and here we differ from the older Rosenbluth criterion, stability is aided by an external-to-the-pinch reversed $B_z$ and $B_r$. Much importance is attached to this prediction, for although it adds complications to a thermonuclear reactor based on a pulsed $B_T$-wall stabilized, toroidal pinch (Pephapsatron) it revives hopes for a continuous rf pinch using cyclic $B_L$ and $B_R$ magnetic fields, to which we shall return in the conclusions. A computation of a stable configuration with thick boundaries and reversed $B_z$ has also been reported by Rosenbluth.9

A pinch current and plasma distribution has been found by Longmire9 (see also Bickerton15) which is stationary and in equilibrium with pressures and diffusion, assuming isothermal plasma and ignoring Joule heating. It is unlikely to be observed in the laboratory since it is unstable by the Suydam criterion above, but has didactic value for the light it throws on the compensation of the outward diffusion current $-\nabla W$ by the inward drift $E/B$.

Ixion Geometry

The axial magnetic field—radial electric field, spinning plasma geometry (as variously proposed by Lloyd Smith,16 Shipley,16 Luce,16 Baker16 and Gow17) with the addition of magnetic end mirrors, is known here as Ixion. A mathematical analysis of the motion predicts a strong enhancement of the mirror confinement. In such a device, collisions are by virtue of the Larmor energy since the rotation does not lead to collisions and, for ions created non-adiabatically in the system (as, for example, by ionization), the Larmor energy is approximately the same as the drift energy, $\frac{1}{2}mc^2(B/E)^2$. Thus, for example, with $E = 10$ esu/cm and $B = 2000$ gauss, the Larmor energy is 25 kev which is quite adequate for thermonuclear purposes. The current through such a device in equilibrium would of course ideally be zero, and the plasma confinement time diffusion-dominated. A plasma current does arise, however, by virtue of interaction between the rotating plasma and neutral atoms, either originally present or returning from the walls, as well as by virtue of a differential drift, between ions and electrons, due to the centrifugal force. In order, then, to achieve the long confinement time appropriate to a reactor, plasma drifting to the walls must be disposed of by some kind of diverter (cf. Stellarator) action and, to minimize the centrifugal drift, the radius must be large.

The initial formation of the rotating plasma also presents a problem as follows. The displacement current which flows in setting up the rotation has a magnitude corresponding to the enormous dielectric constant $4\pi\varepsilon_0 \mu_0 n/e^2$, i.e., of order of $10^8$ amp. This current, in flowing to the electrodes, would produce local space charges (sheaths) which would screen out the applied electric field from the bulk of the plasma unless arrangements are made for a ready supply of electrons and ions at the appropriate electrodes.

Runaway

Runaway is the term given to the production of an accelerated directed velocity in some components of the plasma by an applied electric field. A theory of immediate runaway in a fully ionized plasma of temperature $T$ in a strong electric field has been given by Dreicer.18 It turns out that the critical electric field $E_C$ required to produce such runaway is small and proportional to the ratio, density/temperature. For example, at particle density $n = 10^{18}$ cm$^{-3}$ and $\frac{\Delta T}{T} = 100$ ev, $E_C = 8$ volt/cm. The theory has now been extended to the more difficult case of weaker electric...
fields, where only part of, say, the electron distribution runs away immediately. These theories do not take into account the excitation of plasma oscillations by the runaway which, it seems superficially obvious, will reduce the runaway to some extent.

The pressure balance equation much used in the laboratory has also been modified by Dreicer as follows:

\[
\frac{B^2(r)}{8\pi} + \frac{P_{rr}(r) + \int_{R_e}^{r} \frac{B^2}{4\mu^2} dr + \int_{R_e}^{r} \frac{P_{\theta\theta} - P_{\phi\phi}}{r} dr}{8\pi} = \frac{B^2(R_e)}{8\pi} + \frac{P_{rr}(R_e)}{8\pi},
\]

where \( r, \theta, z \) are cylindrical coordinates, \( R_e \) is the radius of the outside wall, \( B \) is the magnetic field, \( B^2 = B_r^2 + B_\theta^2 + B_z^2 \), and \( P_{rr}, P_{\theta\theta}, P_{\phi\phi} \) are elements of the momentum flow tensor,

\[
P_{rr} = n k T + n m v^2 (B_0/B)^2,
\]

\[
P_{\theta\theta} = n k T.
\]

Here, \( v \) is the electron drift velocity parallel to \( B \); the mean square random thermal speeds are assumed to be equal in the \( \theta \) and \( v \) directions.

The new term,

\[
\int_{R_e}^{r} (n m v^2 \rho) (B_\theta/B)^2 dr,
\]

is the centrifugal force due to electron runaway. The question now arises, how much the very encouraging values for \( n k T \) deduced experimentally in this laboratory have been exaggerated by the neglect of this term. We shall return to this in the discussion.

Oscillations

The literature on plasma oscillations is voluminous, and we mention among the early contributors, the names of J. J. Thomson,29 Tonks and Langmuir,30 Landau,31 Bohm and Gross,32 and Vlasov.33 More recently, the subject of the excitation of plasma oscillations has been reported on by Akhiezer and Faynberg,35 Akhiezer and Polovin,36 Luchina,37,38 and very recently in the USA, Bene mann,39,40. These papers discuss the excitation of plasma vibrations by an electron beam or drift in an applied electric field, and they show that by a cooperative space charge interaction (without collisions) between ions and electrons, somewhat resembling bulk Helmholtz instability, the electron and ion oscillations can grow. Experimental confirmation of this process comes in an entirely different connection, namely the so-called double-beam traveling wave tube,33,34 in which amplification of space charge waves occurs by interaction between electron streams of differing velocities. (The identity of these two processes was pointed out by Bunemann.)

We now propose still another process for the excitation of plasma oscillations, depending on two-body interactions between ions and electrons as follows.84 For a plasma consisting of ions of density \( n_0 \) cm\(^{-3} \) and of electrons, both at temperature \( T \), an electric field \( E_a \) has induced a displacement of the electron velocity distributions in the \( x \) direction as a whole, so that the electric velocity distribution is symmetrical about a point \( v_0 \) (henceforward called \( v \)). The so-called dynamical friction force58 between the electron and ion distributions has been calculated on the above model by Dreicer:88

\[
\frac{\partial \psi}{\partial \xi} + \frac{\partial \psi}{\partial \xi} = \frac{\psi}{\eta} (E_a - E_b - E_c \psi(x))
\]

where \( E_a \) is the restoring space charge field related to the electron density \( n \) by:

\[
\frac{\partial E_a}{\partial \xi} = 4 \pi (n_0 - n)
\]

and the equation of continuity is

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n v) = 0.
\]

We discuss a small sinusoidal modulation in the drift velocity, which oscillates with the characteristic plasma frequency, \( \omega_p = (4 \pi n e^2/\mu)^{1/2} \), where \( \mu \) is the reduced electron mass.

The effect of the negative slope of \( \psi(x) \) is to introduce a negative damping, and we find the growth constant for small electron oscillations to be \( 4 \pi E_a x \mu v_0^3 \) where \( v_0 \) is drift velocity. A numerical example of the growth constant for the following values of the parameters: plasma density \( n = 10^{16} \) electron/cm\(^3 \), temperature \( T = 100000 \) ev, \( x = 2 e T / m v^3_0 = 2 \), \( v_0 = 10^9 \) cm/sec, \( E_a = 90 \) volt/cm, \( E_b = 19 \) volt/cm, \( \epsilon = e/300 \).
= 1.6 \times 10^{-18}, \text{ is } \omega_p = 7 \times 10^7. \text{ We see that the growth}
\text{ is such that large amplitudes could develop in a few micro seconds.}

For identification, we shall refer to this as the violin-string mechanism (the mechanism is closely
analogous to the setting into vibration of a string by the nonlinear friction of the bow).

For the large oscillations, we must refer to the limit
cycle of the nonlinear equation, the maximum down-
ward excursion of the velocity for \( z = 2 \) being
obtained by equalizing the two shaded areas, from
which we see that the amplitude can become large
enough to move the particles against the field in the
peaks.

The external manifestation of the process as de-
scribed might be a small increase in plasma resistance
(but not enough to bring it above the \( \frac{1}{2} \) power law
value) and an increase in the Maxwellization rate—cf.
Langmuir's paradox.36 These longitudinal oscillations
do not radiate but numerous cross-coupling pos-
sibilities for transition into radiative modes exist—as
for example via the fluctuating electron centrifugal
force in moving along curved magnetic field lines. For
the violin-string mechanism, the critical value of the \( z \)
parameter is 1, but it has not yet become clear how
large the fraction of the electrons having \( z > 1 \) must be
for growing waves to exist. The ratio \( z \) is, of course, a
critical parameter for the other mechanism also, for
which still larger growth rates have been predicted, so
there is some doubt which of these will be dominant
for the plasma oscillation phenomena concerned.

A theoretical analysis47 has also been made of the
economics and stability of the plasma confinement
process, variously called electrostatic39 or inertial40 in
which electrons are inwardly projected over the surface
of a sphere, the resultant turning point near the center
forming a space charge well for positive ions. It turns
out that a modest thermonuclear reaction might con-ceivably be maintained in a few mm\(^3\) in this way,
for experimental purposes, but no economic thermo-
nuclear reactor seems to be possible from this geo-
metry in the electronic form. However, an alternative
arrangement,40 with the ion and electron roles
reversed, and which technically we do not know how
to construct, looks quite promising.

**EXPERIMENTAL**

(The order is in historical sequence of development)

**Columbus II** 41

This is a high power linear pinch apparatus with the
following properties: tube—diameter, 10 cm; length, 30
cm; material, Mullite; condenser—capacity, 25 \times 0.8 \mu F;
voltage, up to 100 kv; peak current, 800 ka; time to
current peak, 2.2 \mu sec. The condensers (Fig. 2) are at
the periphery of a low inductance transmission line
connected to the discharge tube by a single
eight-plate subdivided vacuum spark gap.48 At 50 kv
tube voltage, the neutron pulses from this machine, of
duration \( \sim 1.5 \mu sec \), contain \( 3 \times 10^8 \) neutrons per pulse
for zero \( B_z \) and \( 2 \times 10^7 \) for 200 gauss \( B_z \).43 The yield at
large \( B_z \) has proved to be susceptible of improvement.

---

![Figure 2. Columbus II apparatus, schematic](1860.2)
by a conditioning treatment of the tube, together with a reduction of tube voltage from 50 to 40 kv. Nuclear emulsion measurements of the neutron energy distributions have been obtained up to 500 gauss $B_z$. Results of such measurements are shown in Fig. 3.

Neutron Yield

The neutron yield falls with increasing $B_z$ (Fig. 4), at first sharply and then more slowly. The neutron energy anisotropy also falls from the value (expressed as the energy of deuterons moving towards the cathode, assumed to react with stationary deuteron) of 57.5 kev at $B_z = 0$ to 7.2 kev at $B_z = 500$ gauss. Equivalent deuteron radial velocities appear from nuclear emulsion studies in other experiments to be lower than the velocities towards the cathode. If we assume that the primary neutron source is, in reality, monoenergetic and centered on the peak of the observed smooth neutron energy distribution, we can proceed to calculate the current of deuterons (having the appropriate energy from the observed center-of-mass velocity) incident on stationary deuterons at the compressed pinch density consistent with the observed yield. At low $B_z$, this gives a reasonable answer, e.g., at $B_z = 0$ and deuteron drift energy 57.5 kev, deuteron current $\sim 10^8$ amp. However, at high $B_z$, say 500 gauss, and deuteron drift energy 7 kev, the deuteron current is $6 \times 10^8$ amp: this is absurd, both on energetic grounds and also because it is larger than the total tube current $\sim 5 \times 10^8$ amp. (These currents are not dissimilar but it should be remarked that a fraction of the tube current—nearly all of it in a torus, where the net plasma momentum is zero—is carried by the electrons.) In speculating on the origin of these high $B_z$ neutrons, none of the usual instability characteristics are present, voltage signatures or sharp neutron peaks, and the discharge is conventionally stable in the $m = 0$ mode.

A simple explanation, in terms of a general thermonuclear reaction throughout the pinch, can likewise be excluded since it is not energetically possible for the whole plasma to have the required drift velocity. Some process is needed which would increase the deuteron-deuteron relative velocity in a small fraction of the plasma while maintaining in it the $4.1 \times 10^7$ cm/sec drift towards the cathode. At first sight, a Fermi-type mechanism, i.e., acceleration by reflections between moving magnetic discontinuities, might seem the most likely but in such a case more neutrons from deuterons moving away from the cathode should be observed. Ordered motion of the required kind can be imagined in a shock moving toward the cathode, as proposed by Phillips (the neutrons are known to be emitted uniformly along the pinch except in the vicinity of the anode). Another suggested process is a convection moving towards the anode and having in it an increased $B_z$. The conservation of flow through such a convection involves acceleration of the deuterons into the convection, accompanied by conversion of longitudinal motion into Larmor energy by the increased $B_z$ field, the required relative velocities being thus produced without calling on a collision process. Future experiments with the Columbus II apparatus involve a search for a relation between neutron pulse length and tube length, and a further attempt to apply the magnetic probe. The latter has so far proved too fragile, both mechanically and electrically.

Columbus S-4 44-46

This is a medium power linear pinch machine having a tube—diameter, 13 cm; length, 61 cm; material,
Figure 5. Columbus S-4 radial distribution of pressure and magnetic field strength $B_r$, $B_z$ and pressure (left) showing so-called hollow pressure distribution and (right) Suydam stability criterion for the foregoing: unstable between 2.5 and 5 cm

alumina (Mullite); condenser—capacity, 75 µf; voltage, 20 kv; peak current, 250 ka; time to current maximum, 6 µsec.

**Light Emission**

This apparatus has been used in a series of fundamental studies of the pinch, and from it have come most refined and reproducible observations. Important, in the achievement of cleanliness from impurities and reproducibility of operation, has been a conditioning of the tube by repeated discharges, with monitoring of the emitted impurity light and gas formation. In the clean state, the total visible light is <1% of that emitted from normal tubes, and no marked emission of gas into the pumping system is observed after a 250 ka discharge. Remaining changes in the performance from one discharge to the next were traced to fluctuation in pressure due to pumping instability, and were eliminated by a gas flow control servo operated from a Pirani gauge. In the final state, readings of $B_r$ and $B_z$ (except when instabilities are present) are reproducible to within a few percent.

Emission of H$_2$ light occurs in a brief flash, falling to zero for the main duration of the discharge. SII 4128 Å line intensity, used as an indication of wall impurities, does not appear until the second half cycle.

**Behavior of Current Sheath**

The magnetic probe measurements show (Fig. 5) that the currents flow in a well developed sheath in the initial stages (0–3 µsec), with a good $r^{-1}$ dependence for $B_r$, indicating negligible currents outside the sheath. From the equation for static hydromagnetic pressure balance,

$$\frac{B_r^2}{8\pi} + nkT = \frac{B_z^2}{8\pi} + \frac{r}{\tau} \int_{r_w}^{r} \frac{B_z}{r} dr,$$

we evaluate $nkT$. At 2.5 µsec, this is peaked at the radius of the current sheath and falls to zero on the axis. This hollow $nkT$ distribution has been predicted for some time to occur as a consequence of the accumulation of the swept-in gas at the sheath (snow plow effect) and its joule heating, but was never observed in earlier experiments with smaller diameter tubes. It turns out to have important consequences for the interpretation. Figure 5 also plots the two sides of the Suydam inequality from which we see that the pinch is unstable for radius >2.5 cm. After three or four microseconds, the azimuthal symmetry of the pinch is generally lost, in a manner depending on the magnitude of the initial $B_z$ stabilizing field. For low $B_z$, a helical deformation is found, such as would be predicted for $m=1$ instability. Higher $B_z$ fields effectively eliminate such gross motions but a "fluttering"
motion of the plasma boundary appears: correlation studies, of the signals from sets of closely spaced magnetic probes, indicate this motion to be turbulent in nature with related motion limited to regions of 1–2 cm extent. Figure 8 shows magnetic probe traces of the \( B_s \) signal as the fluttering boundary reaches the probe radius set at 4 cm, together with the neutron signal. This figure also gives the signal from a probe oriented to detect radial components of the magnetic field. Such probes detect the onset of instability sensitively. Neutrons are emitted from Columbus S-4 in a characteristic long pulse coinciding with fluttering and seen only when the clean state is achieved and fluttering is present. It seems reasonable to attribute the fluttering and the neutrons to the boundary layer instability of the kind predicted by Suydam.

![Figure 6. Columbus S-4 neutron emission coincident with fluttering of plasma boundary: Top \(- B_s \), showing onset of fluttering; middle \(- \text{neutron emission}; \) bottom \(- B_0 \) at 4 cm radius: \( B_s = 1000 \) gauss; trace speed, 3 \( \mu \)sec per division](image)

**Microwave Radiation**

Observations made with a microwave detector,\(^{47} \) at \( \lambda = 3 \) cm in the axial and radial directions show (1), an intense pulse of radiation in the first microsecond, (2) a quiet period 1 to 3 \( \mu \)sec and (3) a burst of radiation correlated in time with the fluttering. As the pressure balance calculations are extended to later times, the sheath which was well defined at 2.5 \( \mu \)sec, becomes intermixed at a discouraging rate so that, at 6 \( \mu \)sec, the distributions are found to correspond to \( J_0 \) and \( J_s \) current densities uniform across the tube.

Carrying the probe observations to the end of the first half current cycle resulted in the observation of entrapped currents in the pinch. Such currents have been reported before\(^{48, 49} \) and occur in a theory of imperfect sheath formation.\(^{50} \) At the time when the total tube current has reached zero, approximately one third of the original maximum current \( \sim 6 \times 10^4 \) amperes may still be flowing in the axial region, and back along a thin region adjacent to the wall. The phenomenon is, of course, due to the appreciable diffusion time for the internal currents to reach the exterior, together with the conductivity at the wall of some of the expanded gas in the pinch, which effectively screens the interior from the reversal in the applied voltage. Such reversed current distributions have some practical interest; a sufficient increase in the reversed current leads to a reversal of \( B_s \). Suydam’s criterion for stability can be met by a reversal in \( B_s \) and a stable pinch configuration involving reversed \( B_s \) has been deduced by Tayler.\(^{6} \)

**Conductivities**

Returning to the plots of \( B_s \) and \( B_0 \) versus radius—using \( \mathbf{V} \times \mathbf{B} = 4\pi \mathbf{j} \), we can derive \( \mathbf{B} \) and \( \mathbf{j} \). From three plots separated in time, using \( \mathbf{eV} \times \mathbf{E} = -\partial \mathbf{B}/\partial t \), we determine \( \mathbf{E} \), the electric field at all points. Knowing \( \mathbf{B} \) and \( \mathbf{E} \) at all points allows the evaluation of the parallel and perpendicular conductivities, \( \sigma_p \) and \( \sigma \perp \).

Figure 7 shows the results obtained for a discharge at 60 \( \mu \) deuterium pressure, 200 ka peak, \( B_s = 2000 \) gauss.

Highly peaked currents, \( J_0 \) and \( J_s \), at intermediate times indicate a well developed sheath. Note that \( E_g \) is small at the wall (it should extrapolate to zero at 7.2 cm for conservation of \( B_s \) flux). The parallel conductivity is seen to rise with time, reach a maximum (at \( t = 1.8 \mu \)sec and \( r = 3.5 \) cm) of 800 mho cm\(^{-1} \), and subsequently decline to a uniform lower value of \( \sim 150 \) mho cm\(^{-1} \). The perpendicular conductivity is not plotted in Fig. 7 as its value within the experimental error turns out to be zero, except at the same point in space and time as the maximum in \( \sigma_p \), where it reaches the value 20 mho cm\(^{-1} \).

**Plasma Inertia**

These parallel conductivities correspond to electron temperatures of 12 ev for 800 mho cm\(^{-1} \) and 5 ev for 150 mho cm\(^{-1} \). In the Fig. 7 plots of pressure versus radius, an anomalous plasma pressure is observed which alternates between the inside and outside of the current sheath. Suspecting that this was due to an inertia term, a careful plot of radius versus time revealed a small sinusoidal oscillation during the contraction (Fig. 8).

A calculation of the mass density in the sheath, using the accelerations measured from Fig. 8 and the anomalous part of the pressure plot, agrees, within the experimental error, with the total mass of contained gas that would be swept in by the sheath. Subsequently this was done more simply by noting that the small oscillation frequency about the equilibrium radius of a thin heavy shell of surface density, \( \rho_s \), under the particular external circuit conditions of \( B_s \) and \( J \) conserved, is

\[
\omega = \left( \frac{B^2}{4\pi\rho_s\rho_b} \right)^{1/2},
\]

where \( \rho_s = B_s^2 = B_0 \) is the magnetic field strength at the shell surface. An experiment to observe this oscillation frequency (Fig. 9) using a \( B_s \) probe on the axis, over a range of gas densities, confirms that the oscillation frequency varies as the square root of the gas pressure and that the \( B_s \) oscillation amplitude increases with gas pressure, as it should since the sheath velocity in Columbus S-4 is only slightly dependent on gas density at the selected operating condition.
parameters. The evaluation of $\rho_0$ in this way fills what has been a conspicuous gap in our knowledge of the pinch—namely, the completeness of the insweeping of the initial gas filling (since Columbus S-4 is apparently free from impurities during the first half current cycle).

**Temperatures**

This, together with the observed dependence of $\omega_2$ on initial gas filling density, leads to the further conclusion that no significant contribution to the deuterium gas filling occurs by desorption or removal of monatomic films of deuterium from the wall, or
detachment of the sheath from the wall, as has been feared. We next attempt to apply the knowledge of \( \rho_b \) obtained from the oscillation frequency, to the \( n k T \) in the sheath, deduced from the pressure balance equations, in order to extract \( T \). We take a peaked \( n k T \) distribution (Fig. 5) for example (see the discussion for the justification of this), and proceed to guess a density distribution in the sheath. The lowest temperature we can obtain throughout the sheath is by making it isothermal and matching the point density to the pressure. Dropping the inertia part of \( p \) in Fig. 7 and normalizing the resulting integral over the point density to the measured \( p_s \), we obtain the temperatures \( (T_e + T_i) \) of 6 ev at 1.4 \( \mu \)sec and 13 ev at 1.8 \( \mu \)sec. We cannot do this at 2.6 \( \mu \)sec since we have no check on \( p_B \), and, furthermore, the distribution is not hollow then. We can assume the peak in \( \rho \) contains all the initial filling gas, however; in which case, we obtain for \( (T_e + T_i) \) at 2.6 \( \mu \)sec the value 33 ev. Substitution in one of the expressions for the rate of equalization of temperature between electron and ion Maxwell distributions, shows, for particle density \( n \approx 10^{16} \text{ cm}^{-3} \) and \( T_e \approx 10 \text{ ev} \), that (a) if \( T_e > T_i \), there is insufficient time for \( T_i \) to approach \( T_e \), and (b) if \( T_e < T_i \) equalization will be close in a fraction of a microsecond. Accordingly, \( T_e > T_i > 0 \), and \( T_e \) lies between the values quoted above and half those values. The best agreement with data is for \( T_l = 0 \).

The first two temperatures assumed to be \( T_e \) are in excellent agreement with the parallel conductivities, and at 2.6 \( \mu \)sec, the conductivity goes down. We obviously suspect a breakdown of the confinement and we shall refer to these observations in the discussion.

Future experiments with Columbus S-4 might be (a) most urgently to attempt to make the system Suydam-stable, perhaps by using programmed \( B_z \) and \( B_e \); (b) a search for radiations in the plasma oscillation frequency region, i.e., \( 2 \times 10^{13} \text{ sec}^{-1} \), for the violin string mechanism, and \( 10^{14} \text{ sec}^{-1} \) for the Akhiezer \textit{et al.}–Bunemann mechanism.

Perhapsatron S-4

This toroidal wall-and-\( B_z \)-stabilized pinch apparatus, shown in Fig. 10, is the scaled-up successor of Perhapsatron S-3. The parameters are: torus diameter, 7 cm; major diameter, 35 cm; torus material, quartz; number of feed points, 2; condensers, \( 2 \times 225 \mu \text{F} \) at 20 kv; peak pinch current, \( \sim 520 \text{ ka} \); time to current maximum, 12.5 \( \mu \)sec. Figure 11 shows a characteristic oscillogram of voltage per turn, current, neutron intensity, and Si-4130 \( \lambda \) light intensity.

In general, we see that the current and voltage are more nearly \( \pi/2 \) out of phase, than for the S-3 machine, indicating that the current is limited by inductance rather than resistance as in the latter. The neutrons are emitted in the vicinity of the current maximum, and are more reproducible in intensity than has been previous experience. A mean neutron yield at 15 kv condenser voltage is \( 5 \times 10^4 \). The maximum yield observed is slightly in excess of \( 10^7 \) neutrons per pulse. The impurity light intensity is seen to rise rapidly during the neutron emission. The rise becomes earlier and steeper as the power (condenser voltage) is increased. Thus, impurities are probably an important factor in the performance of Perhapsatron S-4 at
times later than the maximum in the neutron intensity.

Magnetic Behavior

Magnetic probe measurements of $B_y$ and $B_z$ were made by a probe inserted along the radius at one point on the outside edge of the torus. The determination of $p$ from such records in toroidal geometry requires some explanation. From the $B_y$ and $B_z$ vs. time records, we map $B_y$ and $B_z$ over the plane of the minor axis as a function of time. Then $p$ in this plane is obtained from the equation

$$\frac{\rho + \frac{1}{8\pi} \left( B_y^2 + B_z^2 \right)}{x_1} = C - \frac{1}{4\pi} \int_{x_1}^{x_2} \left( \frac{2x B_y^2}{x^2 - a^2} + \frac{B_z^2}{x} \right) dx,$$

where $x$ is the radius from the major axis, $a$ is the radius where $B_y = 0$, and the constant $C$ is evaluated from some point where $p$ is known.

This derivation assumes only that the magnetic field is symmetrical about the plane of the minor axis circle and that there are no variations of field along the axis. The magnetic closed curves need not be coaxial with the minor axis, or with one another.

By analysing the probe data in this way, plots such as those shown in Fig. 12 may be obtained. In general, sheath detachment from the walls is imperfect. $B_z$ shows only very minor negative values at the outer edge so $m = 1$ spiral instabilities are not significantly present. The pressure distribution shows a hollow on the axis as late as 10 μsec, but with substantial flanks which reach the walls at all times. At later times, the pressure distribution becomes wild, and it seems likely that the pinch is moving $(a)$ from side to side in the plane of the mirror axis and $(b)$ above and below it. The latter motion invalidates the pressure balance equation assumptions so we cannot decompose the pressure distribution into its pressure and inertial components as was done with Columbus S-4.

The reproducibility of the data is insufficient to allow deduction of $a_y$ and of the pressure distributions—only one is zero on the axis, that at 3.75 μsec. We take this one and estimate $T$ from $n e T$ by equating the particles in the sheath to those that could be swept in from the original filling gas, and emerge with $(T_0 + T_1) \sim 130$ ev.

Since the voltage around the discharge is finite when the pinch current is passing through its maximum, we can estimate a gross resistance of the pinch, which turns out to be $2.8 \times 10^{-2}$ ohms. To obtain the conductivity of the plasma, we need to estimate the current path and, approximating this as a 45° spiral, we obtain the value $2.5 \times 10^3$ mho cm⁻¹, corresponding to a temperature of 5.5 ev.

Neutron Emission

The reproducible and relatively high neutron yield from Perhapsatron S-4, together with its small size, gives sufficient neutron intensity for a survey to be possible of the neutron emitting region. Using a lead-paaraffin collimator and small plastic scintillation detector, the resulting source distribution proves to be well centered on the minor axis of the torus, when viewed from two directions at right angles. Using an IBM 704 computer, the radial source primary distribution has been calculated from the experimental observations and the measured collimator resolving power. This gives the surprising result that the best fit to the data is for a source predominantly in a thin shell at the radius 1.3 cm. This is also the most probable position for the sheath from the magnetic probe data. The reliability of the data is such that a solid cylinder source cannot be excluded altogether for these measurements. Better statistics will easily resolve this point when the machine returns to the laboratory from the Geneva exhibit.

Figure 13 shows the neutron energy distributions from Perhapsatron S-4, from proton recoil counts in a cloud chamber neutron collimator system aimed respectively in the two directions tangent to the minor axis circle from a point in the plane of the torus.

The anisotropy corresponds to 10 kev deuterons, moving in the direction of gaining energy in the applied electric field, incident on stationary deuterons.

The distributions from the Perhapsatron are very little wider than the calibrations from a monoenergetic source, suggesting that the deuterons responsible are fairly monoenergetic. A small secondary peak 0.75 kev below the main peak of each distribution can be conveniently attributed to an inelastic scattering resonance by the iron core.

Microwave Emission

A superheterodyne microwave receiver $^{53}$ tuned to $\lambda = 3$ cm, when applied to the probe aperture, detected an extremely large signal some orders of magnitude outside the range of thermal radiation intensity appropriate to a black body radiation at kilovolt temperatures. The results are quite preliminary
but can obviously be ascribed to the plasma oscillation process mentioned above. Measurements in the electron plasma frequency region, $\lambda = 0.1$ mm, are in preparation.

**Effect of Crowbar**

By shortcircuiting the condenser supply at the moment of maximum current, the high current can, in principle, be extended, changing the time dependence of the current from a sinusoidal to an exponential decay. This procedure is known as crowbarring, a term borrowed from the Radiation Laboratory of the University of California. By substituting a very large capacitor or battery, whose potential is equal to the dissipative potential drop, $R_I$, the duration of the current maximum can be extended still further or indefinitely. The device is then known as an amplified or power crowbar. It has proved possible to apply the power crowbar to Perhapsatron S-4, using one 100 $\mu$F condenser connected, via ignitron, across each 7.5 $\mu$F, 20 kv condenser in the main supply. The power requirements of the pinch have proved to be so large that, in order to stay within the safe rating of the crowbar condenser (3.5 kv), the main condenser has had to be operated at a potential reduced from 14 kv to 7 kv. This reduces the neutron yield under normal operation to $\sim 2 \times 10^6$. The crowbar extends the duration of the current maximum so that approximately 28 $\mu$sec of essentially constant current operation are observed, throughout which neutron emission takes place, increasing the yield to $\sim 2 \times 10^8$ (Fig. 14). Magnetic probe measurements during the constant current period show an intermixed distribution which stays approximately constant until the end of the period. As a technical achievement, the crowbar is very satisfactory but the conclusions to be drawn from its behavior are disquieting. During the constant current phase, the current has the value 180 ka and $R_I$ has the value 5 kv, of which the resistive drop in the primary is negligible. (The oscillograms are somewhat complicated by the saturation of the iron core). Consequently it appears that the resistive drop in the pinch amounts to 5 kv at 180 ka.

A simple calculation of the conductivity, assuming only parallel conduction along the magnetic field

![Figure 12. Perhapsatron S-4 radial variation of B, Be and B](image1)

Conditions as in Fig. 11

![Figure 13. Perhapsatron S-4 neutron energy distributions, by cloud chamber](image2)

Conditions as in Fig. 11

lines, gives the same value as before, 250 mho-cm$^{-1}$, corresponding to 5.5 ev. The temperature by the resistivity is thus inconsistent with that derived from the pressure balance.

Further, the neutron emission and resistivity remain constant during the crowbar phase, during which period 27 kilo joules are deposited from the condensers, which would be sufficient to add 5.5 kev per electron to the plasma. The conclusion that the input is balanced by some dissipative process seems to be forced.

**Ixion**

The arrangement in which most of the Ixion experiments were conducted consisted of an axially symmetric magnetic mirror. The magnetic intensity was seven kilogauss in the median plane and rose to a maximum of 16 kilogauss at the throats of the mirrors. The radial electric field was approximately one kv/cm.

The diameter of the outer electrode was 24 cm and the distance between mirrors about 70 cm. The inner electrode was, on occasion, either a metallic rod 5 cm in diameter or a cylindrical region of injected plasma, roughly 3 cm in diameter. The gas was usually deuterium, admitted at a pressure of 1 $\mu$ Hg.

A typical oscillogram of the discharge current and voltage (Fig. 16) reveals that, for the first hundred microseconds, little current is drawn. Next a pulse of current is seen, which has a peak value of about 20 ka and lasts about 25 $\mu$sec, during which time the voltage drops about 50%. During the next 500 $\mu$sec the voltage decays exponentially to zero.
During the initial low current period there are indications of the presence of an anode sheath where most of the voltage drop occurs. The current pulse occurs when a rotating plasma is developed. Experimentally it appears that about one-fifth of the charge transferred to the device by the current pulse is associated with the polarization of the plasma caused by the tangential drift. The ion kinetic energy corresponding to the drift motion is about 30 ev as measured by the Doppler shift, assuming that the emitting atoms have acquired the plasma drift speed by collision or charge exchange. The subsequent exponential decay of the voltage is attributed to loss of plasma to the walls; the rate of decay is not inconsistent with the mechanism mentioned earlier.

If higher electric fields than 1 kv/cm are applied to the device at the time of breakdown, the discharge becomes completely dissipative, producing neutrons at the same time, and no lasting rotary drift is induced. This limiting electric field is much smaller than would be expected from an elementary analysis based on the equilibrium of a rotating plasma with the magnetic field. It appears that reasons for the present limitation on the voltage are rather to be found in the processes operating during the breakdown period, one of which is certainly the catastrophic evolution of gas from the electrode surfaces. It is planned to focus attention on these processes in the near future.

Columbus T-1

This is a long (6 m) linear pinch discharge tube of 15 cm diameter, having 1.6 cm thick aluminum walls divided into insulated sections 5 cm long. It was built primarily to test the Harwell (UK) group's concept of quasi stabilization of the pinch by conducting walls. The main interest in the results lies in the picture (Fig. 17) it gives of a particular mode of a spiral instability, at large amplitude in steady state motion or, alternatively, as a laboratory demonstration of a large amplitude transverse Alfvén wave. This complicated configuration was unravelled by a magnetic probe analysis in three dimensions.

Scylla

Scylla uses a rapidly rising axial magnetic field to ionize and compress a deuterium plasma in mirror geometry. Such arrangements have been proposed, with and without mirrors, as for instance by Post and Wilson in the USA and Terletski in the USSR, and studied experimentally in Jug, Collapse, and Totem Pole. The coil is energized from a bank of ten 0.88 μf condensers, and the inductance of the system is such that about half the 70–75 kv condenser voltage appears at the terminals of the single turn coil. The

Figure 14. Perhapsatron S-4, oscillograms in the crowbarred state. (From top to bottom: current, voltage, neutron intensity, SI 4128 Å light intensity)

Figure 15. Ixion apparatus

Figure 16. Ixion, oscillograms of current and voltage, also Doppler shift in Dγ
condensers are switched by individual 4-electrode spark gaps specially designed to have high precision in firing (Fig. 19). The tube is made of alumina (Mullite). Figure 20 is an oscillogram of the current together with the signal from the scintillator. The current is in the form of a damped oscillation of frequency 200 kc. The first sharp peak is due to X-rays and occurs at the voltage maximum and the three subsequent peaks due to neutrons are coincident with the current maxima. Neutrons do not appear in the first current half cycle, presumably on account of incomplete ionization; this hypothesis is supported by the observation that the first neutron peak passes to the third and fourth current maximum as the gas pressure is reduced. The maximum neutron emission (22 May) is \(2 \times 10^7\) per pulse.

The magnetic field can be shaped in a very simple way, by adjusting the internal cross section area surrounded by the coil, since, for the short times of the experiment, the magnetic flux through all sections is the same. The yield proves to be quite sensitive to the mirror ratio. For uniform fields, no neutrons are observed, and a maximum is found for \((\text{area of center})/(\text{area of neck}) \sim 1.1\). Higher mirror ratios, made by increasing the cross section at the center, reduce the yield, presumably by reducing the maximum magnetic field at the midpoint and consequently the compression. The neutron yield is sensitive to impurities and the base pressure achieved in the vacuum system, and after a long series of shots under the same condition, becomes quite reproducible.

![Figure 17. Columbus T-1, structure of large amplitude plasma perturbation](image1)

![Figure 18. Scylla, coil with measured magnetic field lines](image2)

![Figure 19. Scylla circuit](image3)
Figure 20. Scylla, oscillograms of coil current, neutron and gamma (X-ray) intensity, and general light intensity; timing marks at 2.7 µsec

For an initial D₂ pressure of 100µHg, thermonuclear neutron production would require a fully Maxwellized temperature of 1100 ev to give the observed yield and satisfy a pressure balance equation.

With this small source and appreciable neutron yield, the neutron and X-ray brightness is high and experiments yielding images of the neutron and X-ray emitting regions are a definite possibility. At present there seems no reason to doubt that a thermonuclear reaction is taking place, the difficulty at present being the usual one of not knowing whether the mean energy of the deuterons is high enough to give the observed yield, or whether some small group have an appropriately larger energy. The time of emission of the neutrons and the clear separation of X-rays and neutrons are all very favorable indications, but a decision must be deferred at least until the neutron energy distribution has been measured. The neutron emitting source has already been identified as close to the coil axis, and nuclear emulsions have been exposed. An important experiment with this apparatus will be to study the neutron emission while the current is sustained at its maximum by a crowbar procedure. Another experiment currently in progress is to determine, by a time-of-flight and momentum analysis, the nature and energy of the particles escaping from the mirrors at the time of maximum compression.

PAM

This is an experiment to study the acceleration of plasma, both for its own interest (and possible application to thermonuclear reaction studies by collision) and as a method of translocating plasma auxiliary to other machines. For example, Ixion receives its plasma by axial injection of plasma from such a device, while efforts are being made in Columbus T-3 to achieve pinches, at densities several orders of magnitude below those achieved hitherto, by the aid of such injection. The novel feature in these experiments is the immediate ionization, before dispersal, of a puff of cold gas injected into an evacuated tube which in most cases would be broken up into fragments or not ionized at all.

Figure 21. PAM apparatus and timing sequence: (above) plasma acceleration system, schematic; (below) typical timing sequence is provided with an axial magnetic field (Fig. 21). The initial ionization is either by a pinch, with axial expansion into the vacuum, or a tangential electric field from a one-turn coil connected to a high voltage condenser. Any neutral gas ahead of the expanding plasma is expected to be overtaken, ionized, and carried along, or left behind so that the plasma can be observed thereafter in vacuo.

The plasma has been successfully accelerated further by a traveling magnetic wave injected into a solenoidal transmission line. The velocity of the plasma is measured from the transit time between fast photomultipliers while the total momentum is measured by the classical ballistic pendulum. At the time of writing, the plasma velocity is \(5 \times 10^5\) cm/sec, plasma impulse 90 gm cm/sec. There seems no reason to doubt that the present techniques will permit the final velocity and mass accelerated to be increased by about a factor of ten.

Picket Fence

This and the following experiment are in a less developed state than the foregoing.

A test of the confinement properties of the cusped magnetic field configuration can very simply be made by using the fast Scylla condenser bank in combination with a suitable coil. Such a coil has been made, Fig. 22, which shows the measured magnetic field lines. Ultimately, when the technique of the crowbar has been extended to Scylla, it should be possible to maintain the field at its maximum strength and to observe the loss of plasma as a function of time. A direct comparison of the loss rates from mirror machine
versus Picket Fence geometries should thus be possible.

**Columbus T-2**

The range of densities over which good observations of the pinch effect have been made (taking into account the scaling laws for similar discharges) is very small. Indeed it seems that the densities studied have been dictated by considerations of obtaining breakdown. A properly chosen preionization system should allow, for example, the production of a pinch at, say, 1/100 of the usual particle density per cm length of the tube, which would bring down the pinch current by a factor of 10 or, alternatively, allow much higher temperatures to be confined at the same current. However, the most striking advantage of such an arrangement is expected to be the complete isolation of the wall from the initial bombardment; the pinch, in an arrangement such as this, being created in free space.

Figure 23 shows the large Columbus T-2 discharge tube, 90 cm in diameter and 3.6 m long in its new form. This has an all-metal wall consisting of a thin stainless steel bellows which conducts in parallel with the discharge.

Such an arrangement has been proposed by Thonemann. The pinch is formed by injecting a plasma column along the axial magnetic field lines: on bridging the two ends, it becomes the pinch. This apparatus has been operated only in an unsatisfactory form with subdivided conducting wall. The pinch is indeed formed in the predicted way, but the later phenomena are overshadowed by arcing between wall sections and arcing from wall to pinch. The new conducting wall may still develop arcing to the pinch, in which case, an insulating wall will be used.

**TECHNOLOGY**

**Magnetic Probe**

Our most serviceable tool for plasma physics has proved to be the magnetic probe. The technique is straightforward. A small coil (2 mm diameter, 20 turns) enclosed in an electrostatic shield, is mounted in the closed end of a tubular sheath of quartz or alumina, which projects radially into the discharge tube, and the coil is adjusted to the required radius and orientation. The integration of the resulting voltage signal is by an RC network (which has been preferred to the more sensitive electronic methods of integration on account of its great working range since the voltages developed by the coil can reach kilovolts). Care has to be taken that the frequency responses of all components of the system are adequate. Twenty megacycles is used for test. Doubts have been expressed as to the validity of magnetic field measurements made by a probe immersed in a plasma. For the Columbus S-4 measurements, the validity of the probe measurements in the presence of plasma have been established beyond all question as follows: the $B_z$ in this apparatus is produced by a low frequency condenser discharge through a solenoid wound on the exterior of the cylindrical return conductor of the discharge tube, reaching the tube interior by diffusion, and is measured in situ, in the absence of plasma, by standard laboratory techniques. The probe, whose integration time constant is too short to observe the slow rise of $B_z$, is calibrated ballistically using standard magnetic coils. From a set of probe measurements of $B_z$ vs. times at different radii, for a pinch under carefully maintained constant condition, the usual $B_z$ vs. $r$ curves

![Figure 22. Picket Fence, coil and measured magnetic field lines](image)

![Figure 23. Columbus T-2 apparatus](image)
are constructed for different times. The integral \( \int_0^{\infty} 2\pi B_z \, dr \), taken over such curves, should be constant provided that no flux leaked in through the outer conductor (conservation of \( B_z \) flux). For a set of such curves, taken with special care on a Columbus S-4 pinch which included plasma pressures up to \( 4 \times 10^6 \) dynes/cm\(^2\), the \( B_z \) was conserved to within 98% of its initial value. Note also the zero value of \( E_g \) at the wall in Fig. 7. Normal data shows 90–95% flux conservation.

Small probes, as described above, show remarkably little perturbation of the pinch so that, for example, a traverse along a diameter through a pinch column, from one side, gives a symmetrical record within the experimental error (provided that the pinch is stable). Obviously, some perturbation must result, however; in Perhapsatron S-4, a reduction of neutron intensity can be detected in the vicinity of a magnetic probe, the rest of the torus being unaffected. Unfortunately, as higher temperatures and longer confinement times are reached, the contamination and local cooling by the probe will inevitably become significant, and ultimately intolerable, and this useful technique will have to be abandoned.

Elaborate experiments have been made to find some other probe technique. Electron beams up to 400 kv proved quite useless for pinch currents > 10,000 amp. Highly energetic proton beams have frequently been proposed, but the measurement quality is likely to suffer greatly, and interpretations become very difficult as consequences of the complicated deflexion path through the pinch. A highly collimated microwave beam probe for the evaluation of electron density is immediately practical for many geometries, but not for the present high density pinches; these require an interferometer arrangement at say \( \lambda = 0.1 \) mm, which is certainly beyond microwave technology at the present time. Furthermore, there is some indication that the pinch itself may radiate more strongly than any contemplated generator at \( \lambda = 0.1 \) mm.

**Switch Technique**

Switch techniques play an important part in high power pulse discharge research. For discharges at medium voltages (up to 20 kv) and times to peak currents of as little as 6 \( \mu \)sec, specially developed ignitrons are used to connect many condensers simultaneously across the load. The ignitrons show consistency in triggering of approximately 0.1 \( \mu \)sec. As might be expected, the safe current carrying capacity per igniton depends on the duration of the current impulse. Characteristic maximum ratings for a half sine wave current pulse are: (GE 5550 igniton) 18,000 amperes peak, duration 60 \( \mu \)sec; 4000 amperes peak, duration 2 msec. A spoiled igniton has a low breakdown voltage. The spoiling process is believed to be associated with development of an arc spot on the metal walls of the tube. Installations of this kind are employed on Columbus S-4, Perhapsatron S-4, and Ixion and for \( B_z \) supplies universally. For extremely low-inductance high-voltage high-power applications, such as Columbus II and Scylla, ignitrons cannot be used. Two kinds of spark gaps have been developed.

**Vacuum spark**—The operation of Columbus II has depended on the success of a vacuum spark gap (visible in the upper half of Fig. 2) which was developed to meet the following requirements: trigger in less than
1 μsec, have an inductance <3 x 10⁻⁸ h and be able to hold off 70-80 kv, after having passed currents of 10⁴ amp. A simple vacuum gap readily holds off 70 kv before use but ceases to be able to do so after one discharge. Potential grading, using 8 diaphragms, solved this difficulty; capacitive subdivision of the potential between the diaphragms proved adequate for an applied potential rising to full value in 100 μsec. The choice of Teflon (polytetrafluoroethylene) for the insulators turns out to have been a happy discovery, for it has self-cleaning properties, after exposure to the metal vapor of the vacuum spark, so far unique to this material. In our experience, normal materials, e.g., porcelain, are quite impracticable for vacuum spark gaps, becoming coated by a metal film after a few discharges. Brass electrodes withstand the erosion of 5 x 10⁸ amp discharges well, the original Columbus II gap being still effective after many thousand discharges. The eroded surfaces have a highly polished appearance.

Four-electrode spark gap⁷⁰—The principle of automatic irradiation of a spark gap by photons from a subsidiary spark, in order to achieve precise breakdown characteristics, has been well described.⁷¹ Such an auxiliary gap adds a fourth electrode to the conventional three-electrode gaps (which, for precise operation, also depend on irradiation produced in a less direct manner.)

The principle has been extended to the well known 3-sphere gap as shown in Fig. 24. The central electrode is arranged by divider to float at a potential midway between the upper and lower electrodes. The incoming trigger pulse charges the center high impedance electrode via a small spark gap which irradiates the upper and lower gaps through the axial hole. When the center electrode reaches the breakdown potential of one gap, breakdown occurs, which immediately overvolts the second gap, completing the circuit. The not negligible advantage of this system is that, although the upper and lower electrodes are, of necessity, connected to systems of very low impedance, which would therefore require powerful trigger pulses to influence, the center electrode is in a high impedance system, capable of being triggered by a low power pulse.

An essential feature of this type of gap, to achieve reliably precise timing, is that the irradiation gap is adjusted in length to be small enough that its own unirradiated jitter never overlaps into the time when the center electrode potential reaches the breakdown point of one or the other of the main gaps. The jitter of these four-electrode spark gaps has not been measured but is known to be less than 10⁻⁶ sec for the following reasons: ten of the gaps are used in a parallel connection (see Fig. 19) in the Scylla apparatus, where the inter-condenser signal transit time is 0.03 μsec. The aggregate spread in firing must be less than this or some condensers will fire in anti-phase, with marked effects on the oscillograms (and serious strain on the condensers!). Such behavior is seen at 65 kv gap voltage but the gaps are completely reliable at 70 kv with no malfunction in several thousand operations. The upper voltage limit of the present design is about 110 kv, being a function of the selected gap spacing and insulator design.

**DISCUSSION**

**Pinch Temperatures**

Before we can discuss the measurements of the interior of the pinch, we must deal with the correction to the pressure balance equation due to runaway. This correction has the effect of making pressure seem larger than it really is, and we have no direct knowledge of the amount of runaway in the Columbus S-4 and Perhapsatron S-4 pinches. It turns out, fortunately, that we can demonstrate it to be negligible, in certain special cases. For the correction, as has been pointed out by Lovberg, is monotonic in r (centrifugal force is always outward). Consequently, for pressure distributions with p zero on the axis (so called hollow distributions), the runaway correction must be insignificant.

Let us consider the most precisely known of our pinches, the Columbus S-4 measurements at 1.4, 1.8 and 2.6 μsec shown in Fig. 7. When the corrections are made for the small radial oscillation of the pinch, the first two of these become hollow distributions and for the third, being such a small extrapolation from the first two, we shall consider the insweeping of the sheath to have continued and swept all the initial filling gas into the shaded area of Fig. 7.

We can derive a temperature Tp, from the sheath density ρs and the pressure, by the following:

\[ \omega_f = \left( \frac{B^2}{4\pi R_0 p_0} \right)^{1/2} \text{ gives } \rho_b, \]

\[ \frac{p}{\rho} \propto n T_p \]

Next we derive an electron temperature, T_e, from the conductivity:

\[ \sigma_i = 1.65 \times 10^{-7} \ln \Delta T^4, \]

where T is measured in ev. Then we make an energy balance calculation in the sheath, obtaining the rate of temperature rise:

\[ dT/d\tau \propto E \cdot j/n. \]

Table 1 shows the results obtained.

**Table 1. Comparison of Pinch Temperatures Calculated from Pressures and Conductivities**

<table>
<thead>
<tr>
<th>( \dot{E}) (eV)</th>
<th>mean ( E_{\text{ion}} ) (eV)</th>
<th>mean ( \rho_b ) (atoms/cm³)</th>
<th>mean ( \rho_{\text{electrical}} ) (mhos/cm)</th>
<th>( T_p ) (eV)</th>
<th>( T_e ) (eV)</th>
<th>( dT/d\tau ) ( \times 10^7 )</th>
<th>( \tau ) (μsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0 x 10⁵</td>
<td>1.6 x 10¹⁵</td>
<td>0.9 x 10¹⁵</td>
<td>1.6 x 10¹⁵</td>
<td>0.6</td>
<td>8</td>
<td>1.4</td>
<td>1.6</td>
</tr>
<tr>
<td>8.0 x 10⁵</td>
<td>3.8 x 10¹⁵</td>
<td>0.8 x 10¹⁵</td>
<td>8.0 x 10¹⁵</td>
<td>0.5</td>
<td>13</td>
<td>5.8</td>
<td>0.8</td>
</tr>
<tr>
<td>5.0 x 10⁶</td>
<td>4.4 x 10¹⁵</td>
<td>1.5 x 10¹⁵</td>
<td>50.0 x 10¹⁵</td>
<td>0.5</td>
<td>33</td>
<td>2.8</td>
<td>2.6</td>
</tr>
</tbody>
</table>

At 1.4 μsec and 1.8 μsec we see very fair consistency between the three quantities \( T_p \) and \( T_e \) and \( dT/d\tau \).
However, for the third time, $T_p$ is too high to be consistent with the conductivity and, although the heat input rate rises greatly, the conductivity temperature falls. Obviously some cooling process is required, starting between 1.8 and 2.6 $\mu$sec.

The later phases of Columbus S-4 and Perhapsatron S-4 show these inconsistencies to an increased degree. For example, in the crowbarred phase of the latter, the steady energy input rate is 900 megawatts, or 95 ev per electron per microsecond, yet no steady rise of pressure, neutron intensity or even impurity light intensity is observed. It seems urgently desirable to introduce the slit of an energy and momentum analyser into the discharge tube wall to perform a definitive experiment. [We are indebted to Colgate for experimental evidence for electron escape.]

Returning to the late time inconsistency between $T_p$ and $T_o$—we can reconcile this in essentially four ways:

1. increase particle density by transport of matter from the walls (i.e., retain the high pressure but bring $T_p$ down to $T_o$);
2. eliminate the high pressure by assuming a large runaway correction, again reducing $T_p$;
3. retain the high pressure but reduce $T_p$ by assuming that the pressure is due to some other form of energy density, e.g., turbulence; or
4. raise $T_o$ to $T_p$ by making some correction to the $f^\alpha$ power law conductivity formula.

Let us deal with these alternatives in order.

1. To bring consistency by means of extra atoms is rather inconsistent with the observed freedom from impurity radiation in both these experiments.

2. There is undoubtedly some runaway, for both experiments produce X-rays, but, to account for the high pressures by these alone requires, for example, a runaway of all the electrons in the system, at several hundred ev, which is both current-wise and energetically impossible.

3. It turns out that the turbulence hypothesis is a perfectly reasonable way of explaining how high pressures can be observed without the necessity for high temperatures. Consider the 10 $\mu$sec measurement in the Perhapsatron S-4 of Fig. 12. To account for the observed 2 atmos pressures would require an electron temperature in the original filling gas with, say, a fourfold compression, of $\sim 220$ ev. The conductivity is consistent with a temperature of 6 ev.

Let us assume that the pressure is given by $\frac{3}{2}nm\mathbf{\theta}^2$, where $\mathbf{\theta}$ is the root mean square speed in the turbulence, and that $T_p$ is in reality 6 ev. We now estimate the relaxation time for the decay of the turbulence into heat, and, for this, we require a mean eddy size, $\lambda$. From considerations of the size of the sheath and the resolving power of the probe, we shall take $\lambda = 0.5$ cm. Then the relaxation time $\tau$ (given by $4\pi\mathbf{\theta}^2\rho\lambda^3/\mathbf{v}$) turns out to be $\sim 1 \mu$sec for $\mathbf{v} = 9 \times 10^{10}$ cm/sec. If now the input energy primarily goes into turbulence, and escapes immediately as heat to the walls, the rate-determining process is the turbulence-heat decay rate. The instantaneous energy density in turbulence is given by energy input rate $\times \tau$. This turns out to be $\sim 4 \times 10^3$ erg/cm$^3$, in reasonable agreement with the observed $2 \times 10^3$ erg/cm$^3$.

4. For the last hypothesis, it could be that although good consistency was found using the $f^\alpha$ power law for the conductivity at early times, this law might break down as say plasma oscillations develop. Indeed, Bunemann has argued that this happens. In such a case, we retain $T_p$, and $T_o$ can be large, or even meaningless. We still, of course, have to keep the loss process which is not affected by the argument.

At present there is insufficient experimental evidence to distinguish between hypotheses 3 and 4.

**Loss Process**

Turning now to the prime cause of the loss process, two possibilities come immediately to mind, namely: (a) hydromagnetic surface instability of the Suydam type, and (b) plasma oscillations induced by the electric field. Both Columbus S-4 and Perhapsatron S-4 are Suydam unstable (see Fig. 5) so it seems likely that growing perturbations will be present. The loss process is seen to intervene at 2.6 $\mu$sec in Columbus S-4, before any fluttering has been detected but this could easily escape detection when just beginning. It is a straightforward experiment to check hypothesis (a): all that is required is to impose a Suydam stable configuration, and observe whether the conductivity or pressure rises. There are several ways of doing this, including the reversal of $B_z$ and $B_x$. So far, the preliminary experiments have involved reversed $B_z$ and have been ineffective in changing the conductivity but the $B_x$ programming equipment has been inadequate. The outcome of this experiment may be crucial for the future of the stabilized pinch as a reactor, so that it will receive much study.

As for hypothesis (b), the criterion for the excitation of plasma oscillations seems universally that $z$, the ratio of drift speed to thermal speed, be $> 1$. Looking into the measurements on Columbus S-4 and Perhapsatron S-4, for complete runaway of all the local electrons, the largest values of $z$ we can find, where $z = (m/e)(m/2kT)^{1/2}$, are $10^{-8}$ and 0.5 respectively. Since however the electric fields in these devices are appreciably lower than Dricer's values for total runaway, these values are probably strong underestimates, and we can be certain some fast electrons are present, since both apparatus produce X-rays.

Thus, about all that can be said, at present, is that conditions could easily be favorable for the excitation of plasma oscillations by runaway.

**Electromechanical Oscillations**

The electromechanical radial oscillations of the pinch reported in the section on Columbus S-4 have some interesting properties. For the case when the plasma is concentrated in a thin shell between the
central $B_z$ field and the external $B_\theta$ field, we have seen that the oscillation frequency is

$$\omega_r = \left(\frac{B^2}{4\pi\rho_0 R_0}\right)^{1/2}.$$  

The time $\tau$ for this configuration to become smeared out by diffusion of the currents is proportional to $2\pi \sigma R_0^2 / c^2$. Consequently, the number of oscillations of this type we would expect to see is $\omega_r \tau$, which is proportional to $R_0^2$. Thus, more oscillations should be obtained in large diameter discharge tubes.

After a time of order of magnitude $\tau$, the configuration will have changed to one resembling a solid cylinder composed of plasma and $B_z$ magnetic field, surrounded by $B_\theta$ magnetic field (in cylindrical geometry, the density rises more rapidly on the inside than on the outside of the shell). For this configuration, the oscillation frequency acquires a dependence on the plasma properties. Thus, for small oscillations, we have

$$\omega_r = 2.405((2B_z^2/8\pi) + \gamma_e \rho_e + \gamma_i \rho_i)/R_0^2 \rho^2,$$

where subscripts $e$ and $i$ refer to electron and ion properties taken separately and $\gamma$ is the ratio of specific heats. These are the usual bouncing oscillations, such as were noted by Bezbachenko et al. These oscillations might be used for plasma heating purposes, by exchange between the radial and longitudinal degrees of freedom, in the manner proposed by Spitzer and Schluter (magnetic pumping). Furthermore, $\gamma_e$ and $\gamma_i$ must change in a complicated but predictable way (in general from $5/3$ to $2$ as the temperature rises) from which some knowledge of $T$ may be extracted.

**Dynamically Stabilized Pinches**

It has frequently been suggested that pinches with radio frequency currents might have advantages in duration and stability over those with direct current, and rf pinches have been sporadically investigated experimentally. The frequencies have to be high, so that the duration of the free expansion at the surface of the pinch, which takes place near the zero of magnetic field there, is insufficient to allow the plasma to reach the wall. Magnetic fields of several thousand gauss at megacycle frequencies require large expenditures of power, and the resulting economics of such devices takes them out of the field of possible thermonuclear reactors. Incidentally, the same seems true of cavity confinement, although for experimental confinement not related to economic production of power, the rf cavity used in the strong focusing mode of Good shows much promise.

**Rotating Magnetic Fields**

Later, have come proposals for rotating magnetic field confinement where, since the confining field is never zero, the required frequencies are lower. However, calculations of the stability of plasma-rotating magnetic field boundaries have been discouraging. Nor is it hard to see why this has been so. Consider an infinitely conducting pinch confined by a magnetic field of constant magnitude, but whose direction at all points is along a tangent at the interfacial plane, rotating with constant angular velocity. Such a configuration passes from the pure pinch ($B_z$ in $\theta$ direction) to the pure axial field confinement cyclically, and the modes of instability vary cyclically between positive and negative values. Such systems are characterized by Mathieu equations whose solutions, except for restricted ranges of the parameters, are badly behaved, i.e., divergent in amplitude.

It now seems that this treatment has led to an error and an exaggeration of the degree of instability. Thin boundaries do not exploit the stabilizing properties of the magnetic shear to the fullest possible extent. An example of this is to be seen in the effect of $B_z$ external to the pinch on its hydromagnetic stability with thick and thin boundaries. As mentioned in the
theoretical section, with thin boundaries, external $B_z$ always diminishes the stability. For thick boundaries, a reversed $B_z$ can increase the stability.

For a rotating field, the magnetic fields take on a lamellar structure which is qualitatively most encouraging for stability. Solutions of Maxwell's equations for the penetration of a rotating magnetic field into a uniformly conducting cylinder (without dynamics) exhibiting this are shown in Fig. 25. (We are indebted to Riesenfeld for this solution.)

We see that for the currently observed pinch conductivities, $\sim 10^8$ mho cm$^{-3}$, frequencies in the range 10–250 kc give appreciable penetration.

The next steps, of considerably greater difficulty, are to put in the plasma dynamics and then examine the configurations for stability. It seems quite likely that a moderate degree of interlaminar instability could be tolerated in a constantly regenerated configuration of this kind but, at these low frequencies, surface instabilities would be fatal. Toroidal geometries for the excitation of pinches such as the above were discussed some years ago at Los Alamos but dropped for the reasons given above. Such geometries have a certain simplicity (Fig. 26).

For a simple verification of these speculations, efforts are being made to devise a ringing condenser system to provide, for a brief time, the large (>100 Mw) alternating current power required.

**Brute Strength Approach**

Suppose it turns out that the losses from the stabilized pinch are due to plasma oscillations? In such a case, since we have no idea how to prevent them, the stabilized pinch would seem unlikely to lead us to an economic thermonuclear reactor. Nevertheless, as an exercise, we can still in principle construct a pinch apparatus showing an energy profit. For this, we return to the old pre-stabilization Columbus X concept, of a single contraction pinch, whose size and density is such that the thermonuclear yield exceeds the energy input. That this must be possible in principle, we see as follows: consider a pinch which is formed arbitrarily rapidly, so that the losses are insignificant during the contraction. The disassembly time for a fixed optimum temperature is proportional to (compressed radius)/(sound velocity). Consequently the thermonuclear yield is given by

$$Y = \frac{A n^2 \nu^2 \rho \sigma \rho m (2kT)^4}{B^2}.$$

The investment cost, where all magnetic fields, losses, etc., are absorbed into the constant, is

$$I = B n^2 \frac{38 \alpha \rho kT}{2}.$$

The efficiency, $\epsilon = Y/I$, is proportional to $n \nu$; thus by making $n \nu$ large enough, $\epsilon$ becomes $>1$. To produce a reactor this way is somewhat dismaying. Even using tritium-deuterium, the pinch currents are $\sim 10^8$ amp, voltage gradients $\sim 10^8$ v/cm and the thermonuclear energy release takes the form of an explosion of power equivalent to $\sim 1$ ton of TNT per cm length of the pinch. Such impulsive confinement arguments can of course equally be applied to picket fence and spherically convergent shock systems, but the result is broadly the same. It would be an unpleasant form of reactor, of course, but hardly more so than the current proposals for thermonuclear power from H bombs.

**Plasma Oscillations**

While in this vein, we might speculate whether plasma oscillations might grow on the transverse current (in the direction $V \times B$) in the boundary of a confined plasma. Qualitatively, we see that a small electron charge accumulation in the surface of such a plasma boundary produces an electric field which leads to a ripple in the plasma surface, but does not relieve the accumulation of charge. Such a process was described by Alfvén.

Although it can be shown that the charge drift in a gradient of magnetic field is divergenceless; nevertheless, a situation might easily arise in the sheath where the ripple would grow as follows. Figure 27 shows a sheath having some trapped particles. The usual electric field $E_x$, arising from the difference in Larmor radii of ions and electrons in the direction normal to the interface, is present. If now $E_x/B_z$ increases outward away from the plasma, any small accumulation of electrons introduces a perturbation in the drift, which grows as it moves outward. The only self-consistent sheath analysis known to us is indeed unstable according to this.

It has been usual to take reassurance from the fact that charge can neutralize freely along the field lines. That this process is not always sufficient can be seen in the recent experimental observations of the instability of a hollow electron beam traveling along
an axial magnetic field. The process is precisely that described by Alfvén above, and the beam breaks up transversely to the magnetic field into parallel streams, in cross section resembling a vortex street.

Non-pinch Devices

So far we have discussed only the pinch. There remain several other configurations, notably Scylla, Ixion, Picket Fence, and the electrostatic inertial systems. We shall discuss these more briefly, since they have been studied less. We regard Scylla as an experiment to make and study thermonuclear reactions, and it makes a substantial number of neutrons. Before discussing these, we must have a criterion for a thermonuclear reaction.

The reason, of course, for the retention of the word thermonuclear in the title of our subject is that in order to produce an energy profit, we have to be able to allow many Coulomb collisions for each nuclear reaction. Thus the Maxwell distribution is an essential requirement. All the neutrons reported so far have almost certainly been produced by a small high energy component of a system whose average energy has been small. Such neutrons are irrelevant though not uninteresting. Next we consider a system that does have a high enough average energy to make neutrons but having all the particles with the same energy (δ-function distribution). We see that this is not enough either. We must demonstrate that it can be confined after it is Maxwellized. Thus the demonstration of a thermonuclear reaction might progress in the following steps:

1. the neutrons must be produced under conditions where no violation of pressure balance, thermonuclear yield rates, etc., has occurred;
2. the neutron energy distribution must exclude a high energy deuteron component as the neutron source, and anisotropy in the distribution must be accountable in terms of an overall plasma drift;
3. a strong but not conclusive proof of a Maxwell distribution would follow from neutron emission (even a rise of intensity) over many collision times (the loophole comes from the possibility of neutron emission from a cold deuteron plasma by photo-disintegration from Mev electrons); and
4. a direct measure of F, the plasma distribution function, which would surely be conclusive: in certain cases, notably in mirror geometries, it may even be practical.

The preliminary energy distributions of the Scylla neutrons indicate that Scylla is probably past stage 2 above. The present time confinement is about one collision time: the evidence for a thermonuclear reaction in Scylla is thus strong but not yet conclusive.

Ixion is an attractive geometry, especially as the "heating" process heats the ions selectively. It seems definite from the Doppler shift of the radiation that an ion drift and ion Larmor energy of about 30 ev has been achieved. At present, the standing current across the magnetic field is too large. For future experiments, an increase of apparatus radius is indicated, together possibly with initial ionization by condenser discharge and maintaining of the current by dc supply, in a strongly pumped system with diverters.

Picket Fence is interesting mainly for its strong inherent stability. Since its losses would seem always to be larger than those of the stable mirror machine, we regard it as a next-to-the-last-ditch reactor concept (Columbus X is the last ditch!) to be brought out if the mirror machine geometry is struck down by instabilities.

The electrostatic inertial system is still only a paper speculation.

CONCLUSION

A new surface instability of the $B_z$-wall stabilized pinch has been deduced theoretically. It would require reversed $B_z$ or $B_E$ to stabilize it.

Refined measurements have been made of the interior of pinched discharges. The $B_z$-wall stabilized pinch has been shown experimentally to have a large energy loss rate. The cause and mechanism for this is not yet known but believed to be due either to the new surface instability described above, or to plasma oscillations excited by electron runaway. If the former, the outlook for the pinch is good; the instability can be stabilized, and there is even increased promise for a continuously maintained dynamically confined pinch. If the latter, the outlook for the $B_z$-wall stabilized pinch as a reactor is poor.

Axial magnetic field-radial electron field systems are shown to have much interest theoretically, and it appears that the confinement properties of mirrors can be greatly improved. Experimentally, such a system has indicated qualitative agreement with theory, and an equivalent temperature of 30 ev.

A system for accelerating bursts of plasma into vacuum has been developed.

A rapidly rising axial magnetic field plasma compression system in mirror geometry is producing $>10^7$ neutrons per pulse, with a high probability of being thermonuclear at a temperature of over 1100 ev ($13 \times 10^6 \text{ oK}$).

ACKNOWLEDGEMENTS

The valuable advice and support of J. M. B. Kellogg in this work is gratefully acknowledged.

In addition to those mentioned in the text, it is a pleasure to acknowledge the generous cooperation of
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Neutron Production in Columbus II

By J. W. Mather and A. H. Williams*

Neutrons produced in a linear pinch discharge in deuterium gas have been reported by several investigators.1–6 These neutrons, in some instances, were reported to be produced in a narrow pulse (~0.1 μsec) and were also adversely affected by small values of applied axial magnetic field, \( B_z \). The processes generally agreed to be responsible for the observed phenomena are associated with some form of instability7 of the pinched discharge, since stabilizing fields of the proper magnitude reduced the yield to an immeasurably small value.

This paper is concerned with neutron production from a 40 kv, 500 ka linear pinch discharge (Columbus II device) in deuterium gas at 200 microns pressure, as a function of \( B_z \) field up to 4500 gauss. Typical voltage, current and neutron pulse traces as a function of time for an applied \( B_z \) of 4500 gauss are shown in Fig. 1.

A few general statements concerning the time of neutron emission and pulse duration can be made. The neutrons first appear within a few tenths of a microsecond after the first contraction for zero \( B_z \) field, with a duration of 0.5–0.8 μsec. The neutron pulse, in general, has a steep rising front, though the over-all shape varies from pulse to pulse. On the other hand, a \( B_z \) field causes the neutron pulse to be delayed by <0.6 μsec with respect to the first contraction, with a duration <1.5 μsec. The neutron intensity rises more gradually and a portion of the neutron pulse occurs at maximum gas current. These observations are not consistent with neutron production at a suddenly grown perturbation.

The neutron yield measured by a silver-activated Geiger counter, plotted against the \( B_z \) field is shown in Fig. 2a. It is seen that the yield dependence on \( B_z \) is fairly strong for values up to 100 gauss, with a further gradual fall-off up to 500 gauss. The value at 4500 gauss \((m = 1\) stable) is not as statistically reliable as those at lower \( B_z \) field; however, it is consistent with a simple extrapolation.

These increased neutron yields from Columbus II \((6 \times 10^8\) neutrons/pulse at zero \( B_z \)) have enabled nuclear emulsion studies of the neutron energy distribution to be extended to higher \( B_z \) fields than have hitherto been possible. The nuclear emulsion geometry for the following measurements has been described elsewhere.6 Since the sensitivity of the nuclear emulsion requires approximately \( 10^8 \) neutrons/cm², the exposures represent many discharges at the higher \( B_z \) fields. Histograms of the neutron energies from nuclear emulsions at the cathode are shown in Fig. 3 for applied \( B_z \) fields of 0, 100 and 500 gauss. The neutron energy corresponding to the peak of the neutron distribution is readily seen to decrease with increasing \( B_z \) field. The Cockcroft-Walton calibration of the nuclear emulsion is included (Fig. 3d) for comparison.

If the interpretation of the energy shift of the peak of the neutron distribution is made on the basis of axial accelerated deuterons colliding with a deuteron

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at rest, then the deuteron energy responsible for the neutron energy shift can be calculated. Fig. 2b shows the dependence of the energy shift, in terms of deuteron energy, as a function of axial magnetic field. If the neutrons are assumed to enter the nuclear emulsion at zero degrees to the axis, apart from a small angular spread, then it is possible to estimate the required deuteron current to account for the observed neutron yield. The result of this calculation is tabulated in Table 1, Column 6, along with the corresponding deuteron energy, Column 4. It is clearly seen that the deuteron currents become extremely large, larger than the actual machine current for the distribution of Fig. 3c. Figure 4 shows the neutron spectrum of Fig. 3b as a function of deuteron energy; it is shown that no peak in the $N_B$ vs. $E_D$ curve exists as in Fig. 3c, and thus it is impossible, on this basis, to associate a particular deuteron energy with the emission of the neutrons. By folding the D-D reaction cross section into Fig. 4, one obtains the numbers of deuterons responsible for the neutron yield as a function of deuteron energy. At small deuteron energies, extreme values of deuteron current would be predicted to explain the corresponding yield. Thus the data, analyzed on this basis, strongly discriminates against a simple D-D axial acceleration mechanism.

With the application of a $B_z$ field, the process of deuteron acceleration would naturally have been expected to occur along the spiral field lines of the pinch discharge; however, radial neutron measurements at 200 gauss $B_z$ reported earlier indicate that the maximum neutron energy is lower than those observed axially at the cathode. Thus, any proposed mechanism must explain the effect of small values of applied $B_z$ field on (1) the neutron yield and (2) the shift of the neutron energy distribution to smaller neutron energies.
**Table 1. Tabulation of Data Obtained from Neutron Energy Distributions**

<table>
<thead>
<tr>
<th>Ref., Fig. 1</th>
<th>$B_0$, gauss</th>
<th>Displacement of neutron energy peak from $2.45$, MeV</th>
<th>$E_D$, keV</th>
<th>Centre-of-mass velocity, cm/sec</th>
<th>Average neutron yield per burst</th>
<th>Deuteron current, amperes</th>
<th>Percentage of neutrons in energy interval 2.1-2.45 MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0</td>
<td>0.270</td>
<td>57.5</td>
<td>$1.1 \times 10^8$</td>
<td>$10^8$</td>
<td>$10^3$</td>
<td>8.5</td>
</tr>
<tr>
<td>b</td>
<td>100</td>
<td>0.190</td>
<td>12.8</td>
<td>$5.5 \times 10^7$</td>
<td>$3.2 \times 10^8$</td>
<td>$6 \times 10^4$</td>
<td>23.0</td>
</tr>
<tr>
<td>c</td>
<td>500</td>
<td>0.10</td>
<td>7.2</td>
<td>$4.1 \times 10^7$</td>
<td>$1.1 \times 10^7$</td>
<td>$6 \times 10^6$</td>
<td>29.6</td>
</tr>
<tr>
<td></td>
<td>4500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$2 \times 10^4$</td>
</tr>
</tbody>
</table>

* Deuteron energy calculated on basis of an axial deuteron colliding with a deuteron at rest.
* Deuteron current necessary to give the observed neutron yield at the deuteron energy $E_D$, pressure $p = 200 \mu$ Hg, pinch compression $(\rho_0/\rho_r) = 10$, target length 15 cm, pulse duration 1.5 msec, $E_D = (9880\rho_{Deuterons}^4 \exp(-4.189^{-1}))$.

The difficulties of interpretation are reduced if some mechanism can be found for increasing the relative deuteron velocity, retaining of course the estimated center-of-mass velocities toward the cathode. The possibility of a strong thermonuclear reaction in a small element of the pinch volume, streaming toward the cathode is not inconsistent with experiment. However, it can be shown from consideration of the energy available from the condenser bank that a thermonuclear reaction involving streaming of the entire pinch volume is excluded as a possibility.

**ACKNOWLEDGEMENT**

The authors wish to thank J. A. Phillips and J. L. Tuck for many helpful discussions concerning this experiment. It is a pleasure to acknowledge the scientific contribution of D. C. Hagerman in making this experiment possible. Further thanks are extended to L. Rosen for his cooperation in obtaining the neutron spectra. The efforts of Glen Livermore and Jerald Sherwood have been greatly appreciated in the operation of Columbus II.

**REFERENCES**

Field Configurations and Stability in a Linear Discharge

By L. C. Burkhardt and R. H. Lovberg*

A linear discharge apparatus, designated Columbus S-4, has been used in a series of studies of the nature of the stabilized pinch discharge. These studies have included observations of optical spectra and microwave emission, and field distribution measurements with magnetic probes. This paper will describe some recent magnetic probe experiments.

The Columbus S-4 apparatus consists of a porcelain (Mullite) discharge tube of 13 cm inside diameter and 61 cm inter-electrode spacing. Energy for the discharge is stored in a 75 μF, 20 kv capacitor bank whose inductance allows a current rise to 2.5 \times 10^5 amperes in 6 microseconds. An external solenoid provides a longitudinal stabilizing magnetic field (B_z), and a number of access ports make possible the insertion of probes for field mapping.

The magnetic probes which have been employed in the work to be described here are used in a conventional way. The terminals of the probe coil are connected through a passive integrating network to the input of an oscilloscope whose horizontal sweep is triggered by the signal from a discharge current transformer. Data thus displayed are recorded photographically.

CURRENT DISTRIBUTIONS

The procedure used in mapping the magnetic fields (B_x and B_z) and obtaining current distributions has been described in detail elsewhere.1,2 Briefly, the method is to record probe traces from a large number of discharges, moving the probe in radial position from one series of three or four discharges to the next, until a family of B_x(t) and B_z(t) curves is obtained which covers the interval from the tube axis to the wall. From these traces one may construct curves of B_x(r) and B_z(r) for separate times and, by suitable differentiation, obtain plots of j_x(r) and j_z(r). At positions and times when the acceleration of the plasma is small, it is possible to determine the plasma pressure (nkT) by equating the magnetic force on a unit volume to the pressure gradient; i.e., by setting j \times B = -Vp as discussed in the Appendix.

By use of the above procedure, we have obtained field and pressure distributions for a number of conditions of discharge current and stabilizing field.

Figures 1 and 2 are typical field and pressure curves for two times after initiation of the discharge. At 3 μsec, it is seen that the currents which determine the field configuration flow in a hollow cylindrical sheath approximately 1 cm thick with a radius of 2.5 cm. The plasma pressure is peaked at the current sheath position, indicating local joule heating. At 6 μsec, diffusion of the fields and thermal energy has taken place. For these distributions the system had the following parameters:

- Capacitor voltage = 14 kv
- Initial B_z field = 2000 gauss
- Deuterium pressure = 80 microns Hg
- Peak discharge current = 2.0 \times 10^5 a.

Under these conditions, the shot-to-shot reproducibility of probe traces is excellent up to a time of 4 microseconds. After this time, certain instabilities occur in the plasma, causing some scatter in the data as can be seen in Fig. 2. The nature of these instabilities will be discussed later.

Self-Trapping of Currents

The observation of probe signals out to times beyond a half-period of the energy storage system has revealed a significant phenomenon, namely, a tendency of the discharge to trap its own current in a closed circuit within the discharge tube. Specifically, it is observed that when the input current from the capacitor bank has oscillated back to zero at the end of a half-period, a current equal to about one-third of the peak discharge current still flows down the central region of the tube, returning along the inside wall. Figure 3 shows two sets of B_x probe traces taken at different radii, together with comparison traces of the total discharge current. While it can be seen that the net current inside a radius of 6.4 cm (the tube wall radius) goes through zero with the input current, the current inside 4.4 cm is still appreciable, and indeed, does not vanish until the total current has a large negative value.

A simple qualitative explanation of this effect may be ventured. If one supposes that a certain amount of neutral gas is left outside the current sheath as it undergoes its initial contraction,3 and that this gas is then ionized by radiations from the heated current channel, one concludes that the B_x flux external to
the main current is now imbedded in a conducting medium, and is not able to decay according to the demand of the external capacitor circuit; rather, its decay is determined by the characteristic penetration time constant of the plasma in which it is contained. The current requirement of the external circuit is met by the establishment of a negative current layer on the outside surface of the plasma. This phenomenon is, of course, no different from that encountered when a direct current flow through a thick conductor is interrupted. The current deep in the conductor continues to flow, and the requirement of a net current zero is met by the establishment of a negative skin current.

A simple order-of-magnitude calculation shows that this entrapment effect may be expected when the following inequality holds:

\[ \rho < \mu \rho g^2 / \tau_0, \]

where \( \rho \) = resistivity of plasma external to pinch, \( r \) = discharge tube radius, and \( \tau_0 \) = period of capacitor bank and connecting circuit. This is simply a statement of the requirement that the plasma time constant shall be greater than the period of the capacitor bank.

One concludes that this process may be a very efficient one for the containment of energy in large diameter tubes which are energized by fast banks.

**INSTABILITY STUDIES**

It has been widely predicted theoretically that the inclusion of a longitudinal field in a pinched discharge should remove the troublesome gross instabilities characteristic of a simple pinch.\(^4\)\(^-\)\(^9\) However, it is common experimental experience that even with the application of large longitudinal fields, a certain type of instability persists, as evidenced by the irregularity and irreproducibility of magnetic probe traces after a short time in the history of a discharge. Figure 4 is a typical set of superimposed probe signals, in which an abrupt departure of the traces from each other is seen to take place after 2.5 microseconds. In this particular case, a longitudinal field was employed which was theoretically adequate for the elimination of helical, kinking, and pinching-off instabilities.\(^4\)\(^-\)\(^9\) It should be remarked here that recent theoretical work of Suydam and Rosenbluth in the USA and Tayler in England has shown that the "\( B_z \) pinch" of finite sheath thickness may exhibit instabilities even though the requirements for stability of an infinitely thin current sheath have been met. In order to examine the possibility that the probe traces are scattered by small local instabilities rather than by gross pinch motions, an experiment was performed which examined a number of neighboring points on the pinch surface simultaneously.

A probe system was constructed in which four miniature magnetic loops were placed inside a ceramic tube of 5 mm od. The loops were spaced at 15 mm intervals, and all were oriented with their axes parallel to each other and perpendicular to the axis of their containing tube. This set of coils was inserted into the tube through one electrode so that all the coils were at the same distance from the discharge axis (3.8 cm), and all oriented to couple \( B_g \). In principle, then, if the pinch were to undergo a gross shift, it would move relative to all four coils at the same time, and the four traces should display nearly identical fluctuations. On the other hand, a very local surface instability might affect one coil strongly but not affect a more remote coil (up to 4.5 cm away). In the experiment, the equilibrium pinch radius was approximately 3 cm, so that the probe jacket was slightly outside the pinch.
Discharges were made at 1000 gauss $B_z$ (insufficient for gross stabilization) and at 2000 gauss $B_z$ which should give gross stability. Sets of traces obtained for these two conditions are shown in Fig. 5. It is plain that the whole column moves in the 1000-gauss case, whereas in the 2000-gauss case, the gross motions are hardly detectable. A study of nearly a hundred of these trace sets indicates that, for higher $B_z$, correlation of any given fluctuation is limited generally to adjacent pairs of loops, and less frequently to groups of three. Indications of gross motion of the column, as indicated by simultaneous fluctuation of all four loops, is quite rare.

Experiments have also been performed to search for correlated fluctuations on opposite sides of the discharge column. At 2000 gauss $B_z$, the result was entirely negative.

It is concluded that gross instabilities of the pinch have been eliminated by the application of a longitudinal magnetic field, but that local surface instabilities persist, and these are not greatly affected by the $B_z$ field.

ACKNOWLEDGEMENT

The authors wish to express appreciation to James A. Phillips and James L. Tuck for helpful discussions of these problems.

APPENDIX

The Effect of Inertial Force Terms in Pressure Balance Calculations

Calculations of plasma pressure in stabilized pinch systems have generally been based on the assumption that inertial forces on the plasma are negligible in comparison with the gradients of plasma and magnetic pressures. Under this assumption, one uses the expression

$$j \times B = \nabla \rho$$

(1)

together with experimental values of $B$ throughout the system to arrive at a distribution of $\rho$ over space and time. It is assumed here that $\rho$ is a scalar pressure.

Recently, some pressure distributions so derived have shown irregularities suggesting that inertial forces might indeed be important for the conditions being examined. Specifically, it was found that the plasma pressure at the axis of the tube, as calculated from Eq. (1), appeared to oscillate between large positive and negative values as compared with the pressure at the outside wall of the container. Since it seemed improbable that the plasma at the wall could ever attain the pressures necessary to keep the axial
pressure always positive, it was decided to examine the possibility of an inertial contribution.

Equation (1), modified to include inertial forces, becomes

\[ \mathbf{j} \times \mathbf{B} = \nabla p + \rho \mathbf{v} \frac{\partial \mathbf{v}}{\partial t} \]  

(2)

In the case of a cylindrical system which is symmetrical about its axis and in which there is no variation along the axis, we obtain

\[ \frac{1}{8\pi} \frac{\partial}{\partial r} \left( B_e^2 + B_s^2 \right) + \frac{1}{4\pi} \frac{B_e^2}{r} + \rho \frac{\partial \mathbf{v}}{\partial r} + F_t = 0 \]  

(3)
or, in integral form,

\[ \int_{r_1}^{r_2} \left[ \rho \mathbf{v} \right] d\mathbf{r} + \int_{r_1}^{r_2} \left[ \left( B_e^2 \right) / 4\pi r \right] + F_t \right] dr = 0. \]  

(4)

Here, \( F_t = \rho^\ast \).

Equation (4), with \( F_t \) assumed to be zero, was originally used to calculate pressure distributions \( p^\ast (r) \) at several times after initiation of a \( B_s \)-stabilized discharge. The results for three times are shown in Figs. 6 to 8, together with the measured magnetic field. Starting \( D_s \) pressure was 80 \( \mu \) Hg. In the original calculation, \( r_2 \) was taken to be the tube wall, and \( p^\ast (r_2) \) was assumed to be zero. As plotted, however, \( p^\ast \) in Fig. 6 is adjusted to be everywhere positive.

It is apparent from inspection of these plots that \( p^\ast \) is probably not a real pressure, since it swings from a high level at the outside wall to a high level at the axis in 0.3 microseconds, and 0.4 microseconds later has vanished at both axis and wall, leaving only a peak at the current sheath which separates the zones of high \( B_s \) and high \( B_e \). It is also unlikely that there are fluctuations in pressure associated with strong hydromagnetic waves, since the magnetic fields do not exhibit the corresponding swings which would be required.

However, the supposition that \( p^\ast \) contains inertial contributions, i.e., that

\[ p^\ast (r) = p (r) + \int_{r_1}^{r} F_t dr, \]

(5)

provides a simple explanation for the observed \( p^\ast \) configuration. An adequate model is one in which the mass and material pressure of the pinch are concentrated at the current sheath, i.e., form a thin cylindrical shell. Then, during an outward acceleration of this sheath the integral term in \( p^\ast \) will add a flat positive plateau between the sheath and the wall, since \( F_t \) exists only at the position of the gas; for inward acceleration, the plateau should appear inside the sheath. For zero acceleration, only the peak of pressure at the sheath radius should remain. To verify that these accelerations were actually occurring at the times of the plotted curves, a graph of \( r(t) \) for the current sheath was plotted by marking the position of the \( B_e (r) \) peak at a number of closely spaced times. This trajectory is shown in Fig. 9. It is easily seen that at 1.9, 2.2, and 2.6 microseconds, the accelerations have the required signs. A calculation of mass density in the sheath using accelerations derived from Fig. 9 and plateau heights from Figs. 6 and 7 agrees, within errors, with the assumption that the entire mass in the sheath is the gas which has been swept up, "snowplow" fashion, by the current layer in its inward motion.
FIELD CONFIGURATIONS AND LINEAR DISCHARGE STABILITY

Figure 10. Three superposed traces of $B_z(t)$ at $r=0$; sweep, 1 μsec/division
Initial $D_2$ pressures: a, 30 μ Hg; b, 60 μ Hg; c, 120 μ Hg; d, 400 μ Hg

It may easily be verified that the frequency, $\omega$, of small amplitude oscillations of the massive sheath about its equilibrium radius is given by

$$\omega^2 = \frac{B^2}{4\pi \rho_s}$$

(6)

where $B$ is either $B_\theta$ or $B_z$ at the sheath position (assuming negligible internal $nkT$), $\rho_s$ is the sheath surface density in g/cm$^2$, and $r$ is the equilibrium radius.

A series of runs was made to observe this "bouncing" frequency as a function of initial gas density, the $B_z$ at $r = 0$ being used as an indicator for the sheath oscillation frequency and amplitude. The results for $D_2$ pressures from 30 to 400 μ Hg are shown in Fig. 10. The oscillation periods are found to vary as the square root of initial gas pressure, as expected, and the amplitudes of the $B_z$ fluctuations increase with increasing sheath mass, again a predictable variation, since an increasing momentum is transferred to the sheath as it is made heavier. A noteworthy feature of the records in Fig. 10 is the increasing time of reproducibility with heavier sheaths. This may be tentatively explained as slower growth of instabilities because of greater mass.

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Low Voltage-Gradient Pinches in Metal-Walled Systems

By D. A. Baker, G. A. Sawyer and T. F. Stratton

Pinch experiments in metal walled tubes were initiated in order to check the concept of the eddy current confinement of unstable pinched discharges developed by the British controlled thermonuclear reaction group.

The insulating gaps implicit in metal walled tubes restrict voltage gradients to values low enough to prevent breakdown across the insulating gaps. The voltage gradients in the experiments described here were 1 to 8 v/cm. Axial magnetic fields were small, and the Rosenbluth stability criteria were not expected to be fulfilled.

EXPERIMENTS IN A 15 CM BORE LINEAR DISCHARGE TUBE

The first experiments were performed in a linear discharge tube 600 cm long with a 15 cm bore. The wall of the discharge tube was made up of 120 aluminum alloy rings spaced 2 mm apart by polyethylene rings. The rings were 5 cm long with 16 mm thick walls. An outer brass cylinder provided the return conductor for the coaxial discharge tube. Figure 1 is a drawing showing the electrical circuit and general features of the discharge tube.

The discharge was powered by a 3 kv, 200 kilojoule capacitor bank with ignitrón switches. A solenoidal winding provided axial magnetic fields (hereafter referred to as $B_z$ fields) up to 500 gauss. A trigger electrode imbedded in the cathode initiated the discharge successfully over a wide range of gas pressure and bank voltage. The tube was operated at voltage gradients of 1 to 5 v/cm, and pressures of 0.1 to 10 microns Hg of deuterium.

Currents in the tube were always resistance limited. Typically, the current rose to its maximum about 40 μsec after application of voltage to the tube, the rise time being determined by the effective $L/R$ period of the circuit. After reaching its maximum, the current decayed almost exponentially with a time constant of about $10^{-3}$ sec. Changes in the operating gas pressure had little effect on gas current throughout the entire range of 0.3 micron to 75 microns Hg within which the tube would break down. Without the spark plug trigger, breakdown could be achieved only over a narrow range of pressure, centered around 10 microns Hg, where $V/\rho d = 0.5$ v/cm micron Hg. The addition of a 200 gauss $B_z$ field raised the maximum current to twice its value with no $B_z$ field, but additional field above 200 gauss produced only a slight additional increase in current. The maximum current obtained in the discharge was $70 \times 10^6$ amp at 3 kv capacitor voltage and 500 gauss $B_z$.

Spectra of the discharge were dominated by lines originating from the aluminum and polyethylene in the walls of the discharge tube. Many of the impurity lines were stronger than the Balmer line $D_\beta$. Lines of $Al^2$, $O^2$, and $C^1$ were most prominent. $Al^2$ and $Al^1$ were faint; $O^2$ and $C^1$ were absent. There were many unidentified lines which were thought to be due to more highly excited impurity lines for which wavelength tables were not available. The degree of excitation of the impurity lines indicated a temperature of one to two electron volts. No lines of the tantalum anode or nickel trigger electrode in the cathode appeared in the spectra taken at the center of the discharge tube.

Time resolved spectra showed light from deuterium appearing as soon as current started to flow. After approximately 40 μsec impurity lines began to appear with the more easily excited lines appearing first. At about the time impurity lines began to appear, the deuterium light dropped to a low level. It was not shown whether the delay in appearance of impurities represented the time required to heat the walls enough to boil off wall materials, or signified the onset of instabilities which caused the discharge to strike the walls. The latter seemed more probable. Inspection of the tube after several weeks' operation showed many arc tracks on the metal walls and arcing between liners. Subsequent operation, after the surfaces had been remachined, revealed that restricting the electric field to 2 v/cm (10 v/gap) would allow indefinite running without arcs across liner gaps.

Moving-image camera (streak camera) photographs of the discharge were taken through a slit across the tube diameter. The camera swept the image of the full diameter slit across a film with the aid of a rotating mirror. The streak camera pictures exhibited the puzzling pattern of behavior reported by earlier workers. Such patterns are consistent with either a tightly pinched discharge wound into an irregular helix, or a large diameter pinch with rapidly moving large amplitude short wavelength disturbances.

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PINCH IN METAL-WALLED SYSTEMS

An attempt was made to distinguish between the two possibilities by a magnetic probe technique. The discharge was probed with a 160 turn, 3 mm diameter coil which was electrostatically shielded by a slotted stainless steel tube and protected from the discharge by a quartz envelope. The integrated probe signals showed a large oscillatory fine structure of about 10 μsec period corresponding to the turbulence observed on the streak camera pictures. Since the probe signals were irregular and were not reproducible from shot to shot a complete analysis of the magnetic fields would have required simultaneous measurements of $B_r$, $B_θ$, and $B_z$ at many positions in the tube. Such an experiment would have been formidable and, in addition, the presence of multiple probes in the tube would have introduced serious perturbations on the discharge.

Regular Oscillations of the Discharge

When low-current operation of the tube was investigated, however, it was noticed that the probe signals showed the irregular disturbance developing into a regular oscillation as the current fell below $10^4$ amperes. The oscillation persisted until the current fell to zero. If the discharge was operated at lower voltage so the current never exceeded $10^4$ amperes, the oscillation began without delay. Subsequent experiments led to a successful analysis of the current distribution in the regular oscillation phase of the discharge. The half-loop probes illustrated in Fig. 1 were used to study the general features of the disturbance. The integrated voltage from such loops gave a signal which was the excursion of $B_z$ passing through the loop. The signal was sinusoidal in character, with zero average. Two probes in the same plane but at right angles to one another gave sinusoidal signals with 90° phase difference, thus establishing that the disturbance in the discharge was rotating. The direction of rotation was determined from the loop leading in phase. The fact that the phase difference was 90° also showed that the disturbance was an $m = 1$ mode. Having established the mode and the direction of rotation, it remained to measure the $z$ dependence to see if the disturbance was a skipping rope or a helix. The sign and magnitude of the phase shift between two half loops placed along the $z$ direction showed the discharge to be a helix of pitch approximately equal to the tube diameter. The sense of the helix was such as to enhance the initial $B_z$ inside it, and it rotated so as to advance itself in the direction of the gas current.

Figure 2 illustrates some of the types of observations obtained for the helical oscillation. The bottom of the figure shows a streak camera picture of the tube with two views—from the side and from the bottom. After preliminary turbulence, the regular oscillations start, and examination reveals that the side view is 90° out of phase with the bottom view. The notch at...
the tube edge corresponds to an absence of current as shown by detailed analysis of the probe signals described below. The top half of the figure shows dual beam oscillograms of the half-loop signal and the $B_0$ signature from the small probe coil at 4 cm radius. Excellent time correlation between streak camera pictures and probe signals was obtained. Figure 3 is a sketch showing the helical disturbance on the discharge.

The reproducibility of the oscillation on the half-loop signals made it possible to achieve complete magnetic field plots with a magnetic probe, and to derive current density distributions for $j_z$, $j_\theta$, and $j_r$ from the field distributions. The magnetic probe coil for the appropriate field component was placed in the plane of the half-loop described above and the half-loop sinusoidal signal was used to fix the phase relationship of the magnetic probe signal with respect to the oscillation. The oscilloscope trace of Fig. 2 was typical of the data used. The fields $B_z$, $B_\theta$, and $B_r$ were obtained as functions of time for eight radii in the discharge tube. Because it was shown that the discharge was a regular rotating helix of known wavelength, one could transform data as functions of time into data as functions of $z$ and $\theta$ at a fixed instant of time. It was then possible to obtain the three space derivatives of the magnetic field components needed for calculating $\text{curl } B$ to yield the current density $j$.

Figure 4 shows profile maps of the $j_\theta$, $j_z$, and $j_r$ current density distributions, together with the appropriate models of the same data. The initial discharge conditions were 5 microns Hg and 100 gauss $B_z$; the current was 8 ka, the electric field 1.3 v/cm. These plots are the current distributions in a plane normal to the tube axis at an instant of time. At a later time, all three plots would be rotated clockwise. In addition, a plot of the current distribution further along the tube in the $z$ direction would be rotated with respect to the plot shown. The current $j_r$ is present to about half the amplitude of the other two and, because of the nature of the cylindrical co-ordinate system, is discontinuous at the origin where current crosses the middle. As a check, $\text{div } j$ was computed for each point of the plot and was zero within 20 per cent of the value of the largest individual term.

Spectroscopic measurements of Doppler shift were made to determine whether the plasma as a whole was rotating with the observed frequency. No Doppler shift was observed across the tube diameter and it was concluded that there was negligible rotation of the plasma. A line broadening of 0.25Å was observed, however. At the low densities used here, the broadening is largely due to Doppler effect and corresponds to a temperature of about 1 ev.

It is believed that the oscillation is a form of Alfvén wave, where the displacements are always perpendicu-
lar to the direction of propagation. Figure 5 shows the observed oscillation frequency as a function of the starting gas pressure. The pressure dependence can be represented quite well by a $p^{-1}$ law as expected for an Alfvén wave. Measurements of the wavelength, as a function of the same parameters, show that the product of frequency and wavelength is remarkably constant at $3 \times 10^6$ cm/sec, the speed of sound in deuterium at a temperature of 1 ev. The fact that the product of frequency and wavelength is very nearly equal to the speed of sound in the gas may eventually offer a clue to an explanation of the mode.

EXPERIMENTS IN A 15 CM BORE RACETRACK

Since it was feared that the electrodes in the linear discharge tube produced a serious perturbation of the discharge, a second pinch tube was constructed by assembling the aluminum ring sections of the previous experiment into a racetrack which was excited as an air-core transformer. The outer brass tube was the primary conductor and was driven by the capacitor bank. The plasma provided the secondary conductor. Voltage gradients, operating pressures, and axial magnetic fields were the same as those used on the linear tube.

The current-voltage oscilloscope traces at 2 kv bank voltage, 100 gauss $B_z$, and 1 micron Hg deuterium pressure (Fig. 6) show that the operation of this tube also was resistance limited. The voltage induced on a loop around the racetrack (top trace) and the current in the primary conductor (third from top) are typical of an oscillatory $LC$ circuit with small damping. The gas current (second from top), however, shows a resistive secondary. After the initial rise, the current drops rapidly as the induced voltage driving it drops until the voltage gradient is insufficient to maintain the discharge. Traces at higher $B_z$ show slightly more gas current and a current reversal nearly in phase with the induced voltage reversal. Circuit analysis indicates a plasma resistance of $40 \times 10^{-3}$ ohm, about the same as that found for the straight tube. The 8 $\mu$H inductance measured at initiation of the discharge is considerably higher than the 3-4 $\mu$H expected for a uniform rod of current nearly filling the tube and is consistent with a small diameter turbulent discharge. The largest secondary current in the racetrack was $40 \times 10^3$ amp.

Spectra of the racetrack indicated extensive impurities as in the linear tube. At currents below $10^4$ amp, a regular helical oscillation was observed similar to that observed in the linear tube, but its detailed structure was not determined. It has not been proved that the discharge current bent around the corners of the racetrack without passing into the metal walls of the tube. The lack of extensive arcing at the corners, however, indicated that most of the current was in the discharge, making it a truly electrodeless discharge.

Since the plasma characteristics of the electrodeless racetrack tube were substantially the same as those of the linear tube, it is believed that the operation of the linear discharge tube was not dominated by electrode effects.
Figure 8. Oscilloscope traces of current in the 90 cm bore discharge tube

The upper trace is the total current while the lower trace is the plasma current. Both traces have a sensitivity of $90 \times 10^3$ amperes per division and a time scale of 200 microseconds per division. The presence of large wall currents is indicated by the difference between the two traces. The dashed appearance of the traces is due to the dual trace switching preamplifier on the oscilloscope.

EXPERIMENTS IN A 90 CM BORE LINEAR DISCHARGE TUBE

A much larger linear discharge tube was constructed to examine the possible improvements afforded by a larger bore. The tube is 360 cm long with 90 cm bore (Fig. 7). The cylindrical tank has no insulating gaps but three insulating gaps are provided in each end plate to divide the voltage gradient, with the tank held at a voltage midway between cathode and anode by voltage divider resistors. The electrodes are hollow steel cylinders of 30 cm diameter and a guard ring around each electrode is held at a voltage midway between the electrode voltage and the tank voltage. The two end plates are identical except that the anode end is connected to the vacuum pump through the hollow electrode while the cathode contains a viewing window and trigger electrode to initiate the discharge. The anode is connected by copper straps to 16 insulated steel bars which provide a current return path back to the electrical feed point at the cathode end of the tube. A 3 kv, 300 kilojoule capacitor bank powers the discharge and provides voltage gradients up to 8 v/cm.

A magnetic winding against the tank inside the steel bars provides axial magnetic fields up to $2 \times 10^3$ gauss. The steel bars, in addition to serving as current return path, provide a low reluctance return path for the flux in the steel end plates and therefore help to keep the magnetic field constant near the ends of the tube. A window section at the center of the tube provides a view of the full tube diameter as well as ports for inserting magnetic probes.

The total currents drawn by the device were as high as $640 \times 10^3$ amp with a full voltage gradient of 8 v/cm. However, when the plasma current was measured using an internal current coil located at the center of the tube adjacent to the inner surface of the metal wall, it was found that with voltage gradients above 3 v/cm a large fraction of the total current was being carried by the walls. These wall currents increased both at low and high values of axial magnetic field, and became minimized at a field of approximately 500 gauss.

A set of oscilloscope traces showing both total and plasma currents are presented in Fig. 8. The difference of the two curves represents the wall current. It was observed that the wall current reversed during the latter portion of the current pulse although the voltage applied to the tube was still positive. This effect is attributed to the induced electric fields due to the changing azimuthal magnetic field surrounding the plasma current. When the plasma current is increasing, the sign of $\frac{dB}{dt}$ is such as to induce wall currents in the direction of the applied electric field. After the plasma current maximum, however, the induced field changes direction so that it opposes the applied field. Examination of the interior of the tube showed arc tracks only near the end insulators, showing that the wall currents result from arcing at the ends of the tube.

Streak camera pictures and magnetic probe signals indicate that the discharge is initiated as a fine thread, down the center of the tube, and immediately starts to oscillate back and forth across the tube with increasing amplitude. The turbulence of the discharge is clearly shown by the streak camera photographs (Fig. 9) which indicate that the discharge may strike.
the walls even at low voltage gradients where wall currents are not observed. An aluminum liner was inserted into the machine to increase the wall conductivity in an attempt to increase the eddy-current stabilization but the streak camera pictures showed no decrease of the turbulence in the discharge. A further attempt to stabilize the discharge was made by injecting plasma through the hollow cathode to initiate the discharge down the axis. Preliminary experiments with this modification show a slight delaying of the onset of wall currents as well as a 50 per cent increase in the voltage threshold for their appearance.

A stainless steel bellows liner is being installed in the discharge tube. The bellows will be electrically connected to the electrodes and will provide an even voltage gradient along the tube, thus eliminating the troublesome wall arcing.

ACKNOWLEDGEMENTS

Peter L. Scott, a summer student at Los Alamos in 1957, contributed greatly to the study of the helical oscillation described in the first section of this paper.

The authors wish to acknowledge the helpful advice and encouragement of Keith Boyer and James L. Tuck throughout the experiments described here. We also wish to acknowledge the invaluable technical assistance of T. M. Putnam, H. K. Jennings, J. W. Haley and A. T. Brousseau.

REFERENCES

All-Metal Discharge Tube Wall

By J. A. Phillips and J. L. Tuck*

For pinched discharges, we can identify four characteristic stages in an operation, namely: (1) plasma sheath formation at the wall (although it may be possible to devise systems, e.g., injected plasma, where this is absent); (2) heating of the pinched plasma; (3) thermonuclear reaction in pinched plasma; and (4) re-expansion to walls.

An overall requirement in producing a controlled thermonuclear reaction by these stages is that of achieving stable plasma confinement. Some success has been achieved in this direction as, for instance, in Perhapsatron S-3 and Columbus S-4. Following closely are two other problems, namely: (a) reduction of plasma contamination by wall materials during Stages 1 through 3; and (b) reduction of wall damage by the energy flux from the plasma in Stage 4.

In Problem a, material from the wall can be released by the following processes:

(a) Evaporation due to heating by: (1) ion and electron bombardment; (2) radiation; (3) eddy current heating; and (4) H+H recombination heating.

(b) Sputtering

(y) Chemical action, i.e., M+H→MH (gaseous)

(z) Desorption

In Problem b the same processes as above will be present, but the main consideration is that the surface shall remain unmelted.

At present we do not have enough data to evaluate the contributions of these various processes, although there is abundant spectral evidence of wall contaminations from both insulating and metal walls. There is also evidence that conditioning plays a part in pinched discharges presumably by Process z, the removal of pre-adsorbed layers. The exceptions, B₄C, diamond, may be poor from the standpoint of resistance to chemical reaction with atomic hydrogen. The position of Tungsten may be less outstanding than it appears when the considerations relating to the radiation enhancing power of high Z contaminants and the low resistance to sputtering are taken into account.

Clearly, metals may have marked advantages and the question arises—how to reconcile the conflict between the high electrical conductivity of the wall and the necessity for an axial electric field. For large, low-density-pinch torus machines, e.g., ZETA and Perhapsatron S-3, it seems perfectly practicable, using corrugated thin walls, to keep the wall currents to tolerable proportions. For machines with higher gradients (Perhapsatron S-5 etc.) the currents are too large. There are several ways, however, whereby the wall current can be reduced, e.g., (1) by high resistance, (2) by subdivision of the wall, and (3) by high inductance. A high electrical resistance may be inherent in the metal (unlikely) or the metal wall may be subdivided into a series of rings (washers) which are capacitively coupled, with the voltage between rings sufficiently small (~20 V) to prevent breakdown.

It appears that the most promising approach to limiting the current drain is by inductance. In this configuration, the wall is a continuous spiral as shown.

Table 1. Figure of Merit (Qₜ) for Selected Metals and Refractory Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Figure of Merit (Qₜ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diamond</td>
<td>~5000</td>
</tr>
<tr>
<td>Tungsten</td>
<td>1851</td>
</tr>
<tr>
<td>Boron Carbide B₄C</td>
<td>1443</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>1180</td>
</tr>
<tr>
<td>Graphite</td>
<td>961</td>
</tr>
<tr>
<td>Pure Copper</td>
<td>934</td>
</tr>
<tr>
<td>Tantalum</td>
<td>830</td>
</tr>
<tr>
<td>Beryllium</td>
<td>764</td>
</tr>
<tr>
<td>Silver</td>
<td>726</td>
</tr>
<tr>
<td>Nickel</td>
<td>672</td>
</tr>
<tr>
<td>Iron</td>
<td>600</td>
</tr>
<tr>
<td>Nickel Steel</td>
<td>550</td>
</tr>
<tr>
<td>Beryllia BeO</td>
<td>469</td>
</tr>
<tr>
<td>Silicon Carbide SiC</td>
<td>440</td>
</tr>
<tr>
<td>Beryllium Carbide</td>
<td>283</td>
</tr>
<tr>
<td>Zirconia</td>
<td>189</td>
</tr>
<tr>
<td>Alumina</td>
<td>183</td>
</tr>
</tbody>
</table>

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in Fig. 1. The cross section of the wire is so designed that the discharge cannot see through the turns of the winding. Insulation between turns on the side behind the discharge preventsshorting and with an electric field of 200 v/cm, a pitch of ten turns per cm gives a usable turn-to-turn voltage of 20 v. An advantage of the inductive current-limiting design is the automatic voltage gradient achieved, along the wall, which is relatively unaffected by local discharge drains.

Inherent in the spiral design is the longitudinal magnetic field produced by the current that does flow through the winding. If the sense of the pitch is so chosen that this magnetic field is in the opposite direction to that initially applied by an exterior winding, as is done in the usual stabilized pinch machines, the stability of the discharge may be enhanced.7

A continuous spiral metal wall has been tested in low current linear pinch machines by Dr. D. C. Hagerman at this laboratory. Using three turns per centimeter and voltage gradients of ~150 v/cm, good pinches have been formed as shown in Fig. 2. It has been noted that the metal spiral has been able to improve the performance of an insulated wall behind it. For example, no pinches are seen if a rubber wall is used, but if a spiral brass wall is inserted in front of the rubber good pinches are once again obtained. Experimentally, some difficulties have been experienced with this type of wall in linear pinch devices in which the electrostatic capacity between the spiral and the outer coaxial current return is too large. In such cases, because of the low velocity of the voltage wave down the spiral, large turn-to-turn voltages cause breakdown. It is expected that in toroidal geometry this effect can be eliminated.

If it turns out that metal walls can be used in high-power pinch devices, then, in addition to the advantages conferred by the heat conductivity discussed above, combinations of light elements can be considered which were previously unavailable.

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The Design and Performance of ZETA

By E. P. Butt,* R. Carruthers,* J. T. D. Mitchell,* R. S. Pease,* P. C. Thonemann,* M. A. Bird,† J. Blears† and E. R. Hartill†

ZETA is an experimental apparatus for studying the pinched ring discharge as a possible method of producing controlled thermonuclear power. The principle of this method is that the self-magnetic field of the discharge current isolates the plasma from the walls of the discharge tube. An advantage of this method, particularly as regards experimental studies, is that the electromagnetic fields required to heat and contain the plasma can be provided both readily and efficiently. An important obstacle is the inherent instabilities of a pinched current channel.

ZETA was designed to heat a plasma to about a million degrees, using a unidirectional current pulse lasting several milliseconds since the maintenance of temperature for a relatively long time is an essential feature of a power producing thermonuclear reactor.

The thermal energy of the charged particles per unit length, $2NkT$, in a simple equilibrium pinched discharge is related to the current $I$ by

$$I^2 = 2NkT,$$

where $N$ is the number of charged particles per unit length, $k$ is Boltzmann's constant and $T$ is the mean temperature. Experiments suggested that the minimum value of $N$ for which a high current discharge will form is about $10^{17}$ cm$^{-1}$. Consequently, currents of about $10^6$ amp are necessary to attain a temperature of about $10^6$ °K. The electric field and stored energy necessary to produce such current pulses could be predetermined only within wide limits; the design parameters were therefore based on the most pessimistic assumption, that of a fully unstable discharge with a predominantly resistive impedance at a frequency of about 200 c/sec. A large bore of tube, which was adopted to reduce this impedance, is essential in a power producing reactor to avoid excessive heat loading on the inner surface. It also produces a small ratio of the skin depth to radial dimensions for a given pulse duration.

To control instabilities, a metal torus was adopted and a toroidal coil was wound on the torus to generate a magnetic field $B_{m}$ parallel to the centre line of the discharge tube.

**DESIGN AND CONSTRUCTION**

**Electrical**

The basic circuit is shown in Fig. 1. Energy is stored in the capacitor bank $C$. By closing the switch $S_1$, the capacitors are discharged into the primary winding of the transformer $T$, thus generating a gas discharge in the torus. The transformer is iron-cored in order to use the capacitors as efficiently as possible. The design was based on the requirement that the gas current should exceed $5 \times 10^4$ amp for at least three milliseconds with a maximum resistive emf of 2 volt cm$^{-1}$ at current maximum. The bore of the torus was to be 1 metre.

To reduce the cost and volume of the capacitor bank, capacitors designed to operate with mainly unidirectional voltages were adopted, and the circuit elements were therefore designed to give critical damping. Under this circumstance, the current $I$ is given by

$$I = (V/L)\exp(-(Rt/2L),$$

with a peak value

$$I_{\text{max}} = (V/|e|C/L),$$

occurring at a time

$$\tau = (LC)^{1/2}.$$  

Here $V$ is the capacitor voltage, $L$ the circuit inductance, $t$ the time and $C$ the capacitance. The restriction of critical damping is relaxed when not operating at peak voltage, and also when the clamping circuit, discussed below, is in operation.

A detailed circuit is shown in Fig. 2. The main elements of this are described in the following sections.
The design and performance of Zeta

The Transformer

The cross section of the iron core threading the torus is determined by the product of the required electric field $E$ at the centre of the torus and the time for which it is applied. Using Eqs. (2), (3) and (4), the following expression is obtained:

$$D_1^2/D_2 = 1.09 \times 10^6 E/(S\Delta B)$$

where $D_1$=diameter of the core in cm; $D_2$=mean torus diameter in cm; $\tau=$pulse rise time in sec; $E=$electric field at peak current in volt cm$^{-1}$; $S=$space factor of the transformer core; and $\Delta B=$maximum change of core induction in gauss.

To minimize the size of the core, and hence also the length and the energy dissipation in the discharge, it is important to use material with the highest value of $\Delta B$. This is obtained in cold reduced steel where $\Delta B=32,000$ gauss when the core is biassed. Taking $E=2$ volt cm$^{-1}$, $S=0.6$, $\tau=1.25 \times 10^{-3}$ sec (equivalent to 3 m/sec width at half peak current), Eq. (5) yields $D_1^2/D_2=140$. Since the bore of torus required $D_2=D_1+100$ cm, the value of $D_2$ required is just over two metres. In the final design, it was found possible to accommodate slightly more iron; the core cross section is 24,000 cm$^2$, giving a core constant of 7 volt sec per turn. The mean path length of the gas discharge is 1160 cm.

The transformer, illustrated in Fig. 3, is built up from ring units made by winding steel strips 10 cm x 0.033 cm in the form of a clock spring with inner and outer diameters of 152 cm and 304 cm respectively. The units were annealed at 800°C after the winding. The interlaminar insulation is a magnesium silicate film formed during annealing. Each unit has an inner supporting mandrel and an outer retaining band made from 9.5 cm x 0.63 cm mild steel strip; the weight of each complete unit is just under five tons. These ring units are mounted in two adjacent groups of eighteen, forming two tunnels into which the torus is fitted. The units are insulated from one another by phenolic sheet. The end units have modified diameters to accommodate the curvature of the torus.

Each group of ring units carries 54 turns of rubber covered cable with a copper cross section 0.65 cm$^2$. Half these turns are terminated individually on one face of the core, and are the main primary winding; the other half are terminated on the opposite face and are used for biassing the core. These terminations allow the transformer ratio to be changed readily over the values 27, 13½, 9, 4½ and 3 to 1.

The core is biassed to 18,000 gauss by a current of 400 amp in the bias winding. This current is supplied continuously from a metal rectifier. The pulse current, due to the high voltage induced in the bias winding during the main pulse, is limited to a value equal to the dc bias current by a 54 mh iron cored choke.

The Capacitors

Energy may be stored as current in an inductance, as mechanical energy in a machine, as chemical energy in a battery or electrostatically in a capacitor. Consideration of these various possibilities established that capacitative energy storage had both technical and economical advantages, the greatest being the ease with which the rate of removal of energy may be varied over wide limits.

A study of the economics of capacitor manufacture and the working voltage of possible switches led to the adoption of 25 kv as the operating potential. With a discharge path length of about 1000 cm and an electric field of 2.0 volt cm$^{-1}$, the resistive emf is 2000 volts per turn. In a critically damped circuit, the peak resistive emf is $(2/e) \times$ (initial capacitor voltage). The best matching for maximum assumed discharge resistance is thus obtained with a 9:1 turns ratio. The design resistance of the discharge, transformed to the primary, is 1.6 ohms. The required capacitance is obtained from Eq. (4), $\tau=(LC)^{1/2}=2L/R=1/CR$. For $R=1.6$ ohm and $\tau=1.25$ m/sec, $C=1600 \mu F$. When this capacitance is charged to the full 25 kv, the stored energy is 500,000 joules. Flexibility to match the actual discharge impedance is provided by the variable turns ratio, and by the addition to the primary circuit of a pulse-shaping inductance with various tappings. The capacitor bank (Fig. 4) consists of 52 steel tank units each of 31 \mu F. Each tank contains 1920 units of conventional mineral-oil-impregnated paper and foil construction. Internal fuses are provided for the isolation of any units which become faulty. The capacitors are designed to withstand repeated voltage reversals, giving a peak to peak voltage swing of up to 25 kv.

To limit possible fault currents should a capacitor break down or a short develop, a graphite-loaded
Figure 3. Cut-away diagram of transformer

KEY.
1 CORE CLAMPING NON-MAGNETIC
2 CORE SUPPORTS NON-MAGNETIC
3 BIAS WINDING INTER-TURN CONNECTIONS
4 PULSE WINDING INTER-TURN CONNECTIONS
5 CORE RING
6 CORE RING UNIT INSULATION
7 PULSE SUPPLY BUSBARS
8 BIAS SUPPLY BUSBARS
ceramic-type resistor of 0.4 ohm is connected in series with each unit. Each resistor is capable of absorbing the whole 0.5 megajoule of stored energy.

The charging system is a dc supply fed through a transmitting type valve with a water cooled anode (E. 1872). This valve has a pure tungsten filament and is operated to give emission limited current. This provides a much steadier current loading on the dc supply than can be obtained with resistance-limited charging. The grid potential of the valve is pulsed on and off so that the valve also acts as a high voltage isolator between the dc supply and the capacitors.

The Switch

The capacitors are discharged into the transformer by a mechanical switch operated by compressed air.

To minimise electrical wear of the contacts, the switch was designed to have a short arcing time by using a double break contact system, by giving a closing velocity of 600 cm sec\(^{-1}\) to the moving parts, and by filling the contact chamber with air at ten atmospheres pressure. The switch is held closed by air pressure in the operating cylinder. The switch is opened either automatically, after a short time delay, or by a remote tripping circuit. The switch is not required to interrupt a large current, but the surge voltage, due to breaking a current of a few amperes, is limited by a 2 \(\mu\)F capacitor and 390 ohm resistor connected in series across the circuit.

The arcing time of this switch at 25 kv has been measured to be about 300 \(\mu\)sec. To prevent the current from rising to about 10 ka before the contacts are firmly closed, a saturable reactor (magnetic switch) is used in series with the mechanical switch. This saturable reactor was designed to limit the current to less than 100 amp for a hold-off time of 500 \(\mu\)sec with an impressed voltage of 25 kv, and to have a saturated inductance small compared with the total circuit inductance.

The parameters required for the design of a magnetic switch are: \(L_s\), the saturated inductance; \(t\), the hold-off time; and \(V\), the impressed voltage. Assume for the characteristics of the core and winding: \(\Delta B\), the flux swing to saturation; \(\Delta H\), the mmf change which results in \(\Delta B\); and \(I\), the current in the winding at end of time \(t\), i.e., the current producing \(\Delta H\). Then the following simple relations can be derived:

- \(L_s = V t \Delta H / (I \Delta B)\), which can be written \(I = V t \Delta H / (L_s \Delta B)\), and
- \(\text{volume of core required} = 4\pi 10^4 V^2 \Delta B L_s^{-1} (\Delta B)^{-2}\) cm\(^3\), where the units are \(V\) in volts, \(t\) in sec, \(L_s\) in henrys, \(\Delta B\) in gauss.

The requirements were satisfied by a biased 2\(\frac{1}{2}\) ton core of cold reduced silicon steel. The saturable reactor has a saturated inductance of 70 \(\mu\)H.

Circuit Protection Units

The remaining components are chiefly to protect the capacitors from an oscillatory discharge. Such a discharge occurs either under certain fault conditions, or when the secondary resistance is much lower than that assumed in the design.

The first such unit is the non-linear resistor, which has a high resistance at low currents. It was designed to limit the voltage reversal to 10\%, in case the secondary resistance were only \(\frac{1}{4}\) of the critical damping value, and to reduce the normal peak current by less than 10\%. The resistor, made of discs of “Metrosil,” has a design characteristic: \(V = 480/0.25\) volts \((I\) in amperes). Some calculated discharge transients with this resistor in circuit are shown in Fig. 5.

The second unit is the clamping circuit, designed specifically for a predominantly inductive secondary impedance. The circuit consists of two ignitrons (BK 178), with 0.08 ohm load-sharing resistors, connected in parallel across the transformer primary. The ignitrons are triggered when the transformer voltage falls to zero, and are held conducting by the holding anode. The circuit serves to trap most of the energy as inductive energy in the secondary circuit and has the effect of extending the duration of the current pulse.

Thirdly, the stored energy in the capacitors can be dissipated in a 2 ohm “dump” resistor. This can be connected manually, but is also connected by an igniton which is triggered in the event that the gas discharge fails to strike. Without this protection the transformer core saturates, and a low, inductive impedance is presented to the capacitors when they are fully charged.

Lastly, there is an \(RC\) circuit, connected across the transformer primary, to protect the non-linear resistor and the switch from voltage transients generated in the gas discharge.

Mechanical

The torus was designed to meet the following basic requirements: (a) it must be vacuum tight; (b) it must...
have the minimum number of electrical breaks to achieve maximum eddy current stability and for maximum exclusion of stray magnetic fields; and (c) its body must not form a short-circuit, and the potential differences at the gaps in it must not lead to breakdown and the formation of power arcs.

The chief problem was thus the conflict between requirements (b) and (c). On the one hand, experiments in 35 cm bore tubes showed that in the presence of a strong discharge, a single gap in hydrogen at $10^{-4}$ mm Hg broke down at about 100 volts; yet up to 2 kV/turn was to be applied to the ZETA torus. On the other hand, experiments with a 35 cm torus cut into several insulated sections revealed a significant decrease in eddy current stabilization and stray field exclusion. In the absence of ionization, a single gap in hydrogen at $10^{-4}$ mm Hg was observed to withstand 4 kV.

To meet requirement (b), the main torus body was made in two halves, of 2.54 cm thick aluminium; each half is mounted on a trolley for ease of assembly. The two main gaps where these halves join are inside the transformer tunnels. Electrical insulation at the main gaps is provided by bakelite and polythene gaskets, and the joints are made vacuum tight by neoprene gaskets. Each half was fabricated out of short sections of rolled aluminium plate welded together. In the centre of each half there is a square window block, containing ports and windows for observations. The four main diffusion pumps are connected to the bottoms of the window blocks.

A liner system was mounted inside the torus to prevent ionization from reaching the main gaps which would otherwise break down. The liners are insulated from one another by a strip of polytetrafluorethylene in "tongue and groove" joints, and are insulated from the main torus wall by mounting them on nylon bushes. There are 48 liners, giving a mean liner-to-liner potential of 40 volts at about 2000 volts per turn. The space between the liners and the main torus, which is about 3 cm in depth, is pumped by eight auxiliary diffusion pumps mounted outside the torus. A 0.6 cm bore water cooling pipe is welded into each liner, to provide the main cooling system for the torus. The construction of the torus and liners is illustrated in Figs. 6 and 7; the principal dimensions are summarized in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Principal Dimensions of the Torus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bore of torus</td>
</tr>
<tr>
<td>Thickness of torus</td>
</tr>
<tr>
<td>Bore of liners</td>
</tr>
<tr>
<td>Thickness of liners</td>
</tr>
<tr>
<td>Mean length of liners</td>
</tr>
<tr>
<td>Radius of curvature of quadrants</td>
</tr>
<tr>
<td>Length of straight sections:</td>
</tr>
<tr>
<td>at main gaps</td>
</tr>
<tr>
<td>in window blocks</td>
</tr>
<tr>
<td>Total mean circumference</td>
</tr>
<tr>
<td>Volume within liners</td>
</tr>
<tr>
<td>Approximate surface area facing the</td>
</tr>
<tr>
<td>discharge</td>
</tr>
</tbody>
</table>

Vacuum System

The total volume to be evacuated is about 12,000 litres. The working pressure of deuterium is between $10^{-4}$ and $10^{-8}$ mm Hg. The pumping system was designed to achieve a base pressure of $10^{-6}$ mm Hg in the presence of a leak rate of 3 litre-microns per second.

The leak rate requirement calls for a pumping speed of at least 3000 litres per second which is achieved by using two 14 in. diffusion pumps on each half torus.
The pumps are operated with DC 703 silicone fluid. Immediately above each pump is a refrigerated baffle (−15°C), and above this a large liquid nitrogen cooled trap with automatic level control. The torus–liner interspace is pumped by two 6 in. diffusion pumps mounted on each quadrant. All these pumps are backed by a rotary pump with gas ballast facilities. Each half of the torus is self-contained, except for a common backing line, and can be built up and fully tested whilst withdrawn from the transformer. At the normal working deuterium pressure of $\frac{1}{3} \mu$ Hg, the pumping speed was measured to be about 6000 l/sec.

Figure 8 shows a diagrammatic layout of the pumping system. Pumping lines (not shown) which by-pass the diffusion pumps are used for roughing out the torus. The majority of the vacuum valves are
electro-pneumatically operated, and protective circuits are used to shut the system down safely in the event of an excessive leak, or of pump or trap failure. Hydrogen or deuterium is admitted to the system through heated nickel tubes to ensure high purity. The gas flow to the torus is controlled by adjustment of the nickel temperature. Other gases are admitted through a motorized needle valve. Pressures are measured on calibrated ion gauges, connected by short 2.5 cm diameter pipes to the window blocks, and are accurate to within 10%.

Leak detection is normally carried out with a helium mass spectrometer leak detector; in normal operating conditions it is necessary to clear the torus of deuterium by running discharges in some other gas, to avoid detecting $D_2^+$ ions in the mass spectrometer. The leak rate normally achieved is between 1 and 5 litre-microns per second. Because of the large number (~300) of O-ring seals, it is not possible to bake the system, although heating to about 50°C was used on a number of occasions. The normal base pressure achieved was between 1 and $3 \times 10^{-6}$ mm Hg.

Both spectroscopic observations and mass-spectrometer analyses of samples of gas indicate that the proportion of impurities present during the discharge is not in general given by the simple ratio of base pressure to working pressure. This is because impurities removed from the walls play a major role. The number of atomic sites on the walls is about $10^{18}$ per cm length of discharge, so that if 10% of a monatomic layer is released into the discharge, the amount of impurities is equal to the amount of deuterium at normal operating pressure ($N \approx 10^{17}$ cm$^{-1}$). A striking feature of the apparatus is the clean-up which appears to take place during running.

**Control and Instrumentation**

The overall layout of ZETA is shown in Fig. 9. The discharge apparatus is in a chamber about 10 x 10 x 10 metres formed by ferro-concrete walls 1 metre thick for constructional and shielding purposes. This chamber is completely open on one side which is used for assembling the apparatus. There are large openings, about 1.5 m square, facing the window blocks of the torus on two other sides, from which observations are made in two laboratories. There is a similar opening on the fourth side, into the control room. There are also small portholes cut in the roof above the window blocks. The pulsed power and the main transformer bias supply are conveyed by coaxial transmission lines through the roof of the chamber.

The equipment for controlling, monitoring and recording the discharges is centralized in the control room. The main units here are: (a) a control desk, on which are mounted the chief manual controls, the timer, safety switches and monitoring oscilloscopes; (b) an eight-channel recording oscilloscope, on which selected transients from each discharge are photographed; (c) racks containing apparatus for controlling the stabilising magnetic field, the bias supplies,
the ionizing radiofrequency discharge, pressure gauges and fault monitoring equipment.

**Axial Magnetic Field**

The axial magnetic field is supplied by fifty 15-turn coils. Each coil is mounted between adjacent liner mountings. At the window blocks and main gaps, auxiliary windings were added to give improved uniformity of the field. The coil is wound from a flexible cable of 1.3 cm² copper cross section covered with varnished Terylene. It is suitable for continuous operation at currents up to 200 amp ($B_{\phi} = 160$ gauss). For higher magnetic fields the current is pulsed in synchronism with the main discharge pulse. Because of the time taken for flux to penetrate the torus wall, a current pulse duration of about 1 sec is necessary. Supplies are available to provide up to 1000 amp at 1000 v.

**Ionization of the gas**

The applied pulsed electric field is insufficient to cause a breakdown in hydrogen at a pressure of $10^{-4}$ mm Hg. The gas is therefore ionized by producing a radio frequency glow discharge inside the torus. A 3 Mc/s oscillator is connected capacitatively to two of the liners, giving an rf current of a few amperes. The oscillator is suppressed during the main current pulse to reduce interference in the various measuring circuits.

**Cyclic Operation**

The detailed switching sequence of the main discharge circuit is operated by a rotating drum timer. The drum rotates once every twelve seconds (in early experiments, ten seconds), and the operation of nine separate circuits can be adjusted to within 0.1 sec. The chief units switched from here are the charging circuit valve, the pulsed axial field and the mechanical switch. One of the circuits ensures that a complete cycle is always finished. Other circuits are used for operating such auxiliary items as discharge number registers and camera shutters.

By means of the drum timing, ZETA can be operated for many hours with only minor adjustments to the manual controls of gas pressure, axial field and condenser voltage. Operation ceases automatically under various fault conditions. Single shot operation can also be carried out.

Electronic pulses for operating equipment during the discharge are provided by a ten-channel pulse generator. Each channel has a separate delay circuit for presetting the time of the pulse to within about 5%, up to a total delay of 10 msec. The generator is initiated by pulses derived from voltages in the primary circuit of the transformer. These timing pulses operate oscilloscope time bases, the rf suppressor, the "dump" circuit, and experimental equipment such as image convertors and gating circuits for counting equipment.

**Monitoring and Measuring Equipment**

In such a high power circuit as ZETA, it is most desirable that the operator should be able to observe readily and reliably any unusual behaviour of the circuit before damage occurs. This requirement has
Figure 9. Over-all layout of ZETA
been met by installing two "Memotron" storage type cathode ray tubes, which retain the transients until they are deliberately erased. Any two of the eight waveforms being recorded on the eight-beam oscilloscope can be selected for presentation on the Memotron tubes.

The timing and sensitivity of the recording oscilloscope and the Memotrons are calibrated by standard pulses fed from a stabilized generator. The timing of these pulses is derived from a quartz oscillator, and the amplitude is determined by a gas-filled voltage reference tube.

The normal information fed into the 8-channel recording and monitoring system includes: (a) capacitor voltage, measured by a resistance potential divider; (b) primary circuit current, measured by current transformers placed both outside and within the clamping circuit; (c) secondary current, measured by integrating the potential given by a Rogowski coil placed around the discharge tube (Rogowski coils within the tube have also been used); and (d) voltage per turn, measured by a single turn around the transformer core and a resistance potential divider.

A large number of further signals derived from equipment such as neutron counters and spectroscopes can be fed into the 8-channel system.

To reduce interference from the electromagnetic fields set up by the discharge, separate earth circuits are provided for light current and heavy current circuits, and balanced twin cables and balanced input circuits are used on the oscilloscopes.

OPERATIONAL EXPERIENCE

After preliminary circuit tests using a stainless steel loop as the secondary, ZETA was first operated on August 12, 1957. The early experiments, carried out in hydrogen at low currents, served to check details of the circuit and instrumentation. In the next stage, conditions were found for which a high current discharge of interest to thermonuclear plasma studies could be obtained repeatedly. In particular, neutrons from D(d,n) reactions were detected.

The essential feature of such conditions is that the initial value, $B_{\phi 0}$, of the stabilizing field should be about 150 gauss, depending on the discharge current. The value of the stabilizing field for optimum neutron yield increases with current; and it appears that if the plasma traps all the $B_{\phi 0}$ flux, the pinch effect cannot produce a calculated enhancement of the initial flux density of more than 20 times. With larger fields the nuclear reaction rate decreases: with smaller fields there are many signs of serious instability, notably high resistance and widely fluctuating magnetic fields in the discharge. This is in accord with the stability criterion of Tayler.

ZETA has been operated for about six months, mostly under a limited range of conditions: Deuterium pressure $\frac{1}{5} \mu$ Hg, initial stabilizing field ($B_{\phi 0}$) 160 gauss, and currents in the range 140–180 ka, obtained with a 9:1 turns ratio. For most of the experiments the pulse shaping reactor was short circuited.

Operation of the machine at high current was limited to avoid excessive damage to the liner system. After prolonged running at high currents (>160 ka), the insulation between the liners deteriorated and liner-to-liner arcs became frequent, leading to insulation failures requiring a lengthy and expensive overhaul.

During the period August 1957–March 1958, there were about 150,000 discharges in ZETA, and four major overhauls of the liner system. The chief modifications to the apparatus were:

1. Replacement of an earlier form of liner-to-liner insulation (fibre glass tape) by P.T.F.E.
2. Improvements to the uniformity of the $B_{\phi 0}$ field at the main gaps and the window blocks.
3. The addition of the clamping circuit.

These modifications produced no major changes in the character of the discharge; indeed, a feature of ZETA is the consistency of performance in such respects as neutron yield and discharge resistance, from run to run, after overhauls and minor modification. Long periods of cyclic operation have been possible, the longest being 3500 discharges in 10 hours.

In this initial operating period, the reliability of the major components of the electrical circuit was satisfactory. Most trouble has been experienced with relatively standard items, such as air control valves in the main switch, failing after long periods of repetitive operation. Failures of the clamping ignitrons to hold off the main voltage have occurred, probably due to the absence of temperature control on the ignitrons. It has been our experience that the reduction of interference in control and instrumentation circuits is of the greatest importance for trouble-free operation and experimentation.

Electrical Characteristics

The equivalent circuit is shown in Fig. 10. Values of the various impedances actually used are in Table 2, where all quantities are referred to the secondary circuit on the standard 9:1 turns ratio. Analysis of this is simplified by ignoring current (which does not exceed 350 amp) in the shunt reactance of the primary winding, and by treating the effect of the non-linear resistor as simply reducing the capacitor voltage by about 4 kv. For fixed circuit elements, the maximum current $I_{\text{max}}$ and the time $t_{\text{max}}$ at which it occurs are then given by

$$I_{\text{max}} = (V_0 - 4000) \frac{\mu}{(C/L)^k} \exp(-\epsilon \cot^{-1} \epsilon); \quad (6)$$

$$t_{\text{max}} = (2L/R) \epsilon \cot^{-1} \epsilon; \quad (7)$$

where

$$\epsilon = \left[4L/R \frac{\mu}{C} - 1\right]^{-1},$$

$L$, $C$, and $R$ are the total inductance, capacitance and resistance of the circuit, $V_0$ is the actual voltage
initially on the capacitor bank, $n_t$ is the turns ratio, and time is measured from the point when the magnetic switch saturates. To the same degree of approximation, the current decays exponentially after the clamping circuit is triggered, with a time constant of $L^* / R^*$, where $L^*$ and $R^*$ are the total inductance and resistance of the clamped circuit.

Table 2. Fixed Circuit Parameters Referred to the Secondary on 9:1 Turns Ratio

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{sn}$—Magnetic switch inductance.</td>
<td>0.9 µH*</td>
</tr>
<tr>
<td>$L_f$ —Feeder inductance</td>
<td>0.21 µH</td>
</tr>
<tr>
<td>$L_L$ —Leakage inductance external to torus</td>
<td>2.17 µH</td>
</tr>
<tr>
<td>$L_t$ —Total external inductance</td>
<td>3.3 µH</td>
</tr>
<tr>
<td>$R_p$ —Primary resistance</td>
<td>1.25 x 10^-4 ohm</td>
</tr>
<tr>
<td>$R_m$ —Non-linear resistor</td>
<td>dropping a voltage $V_{m}=211$</td>
</tr>
<tr>
<td>$R_c$ —Clamping circuit resistance</td>
<td>2.5 x 10^-4 ohm</td>
</tr>
<tr>
<td>$L_c$ —Clamping circuit inductance</td>
<td>3 x 10^-2 µH</td>
</tr>
<tr>
<td>$V_{ob}$ —Arc voltage of ignitron</td>
<td>5V</td>
</tr>
<tr>
<td>$C$ —Capacity of condenser bank.</td>
<td>0.130 f</td>
</tr>
</tbody>
</table>

* fully saturated.

Figure 10. Equivalent circuit of ZETA

Figure 11 is a typical oscillogram obtained from a discharge under standard conditions—

Starting pressure: $1/2$ µ Hg deuterium plus 5% N₂;
Initial axial field: $B_{eo}=160$ gauss (on the centre line of the discharge tube).

Turns ratio: 9 to 1.

The gross characteristics of these oscillograms are those of a capacitor discharging into a predominantly inductive load, with the clamping circuit preventing oscillations. The rise time of 1.2 msec, and the maximum current of 140 ka are accounted for with $L=6.7$ µH and $R=5 x 10^{-3}$ ohm. This resistance is about a third of the critical damping value; the maximum current in the absence of any discharge resistance would be about 220 ka. The time constant after clamping is about 2 msec, rather more than is expected from values of $L^*$ and $R^*$ derived from the above values of $L$ and $R$, apparently because the effective resistance is less during the decay. The current does not decay exponentially to zero; there is a rapid rise of discharge impedance towards the end of the discharge, and abrupt current termination.

Values of peak current and of energy absorbed by the discharge, are shown as a function of capacitor voltage in Fig. 12. The time, $t_{max}$, to maximum current is about 1.2 msec throughout the range. Both this and the approximately linear increase of peak current with voltage in accord with Eq. (6) indicate only modest changes in $L$ and $R$ throughout the range, due partly to the relatively large value of fixed circuit inductance.

External Circuit Analysis

An example of the primary and secondary current waveforms is shown in Fig. 13. There is a relatively rapid rise of current during the initial 100 µsec up to the point A. Between 1.4 msec and 2.2 msec, the current is shared between the main capacitor circuit $I_1$ and the clamping circuit $I_2$. The corresponding voltage waveforms are shown in Fig. 14. Saturation of the magnetic switch is indicated by the rapid fall of $V_{SR}$ at the start of the pulse. For the first 100 µsec or so, much of the capacitor voltage appears across circuit elements external to the torus. The volts-per-turn at the surface of the torus, $V_T$, rises to over half the transformed capacitor voltage only after about 200 µsec, partly because of decreasing impedance in the external circuit, and partly because of increasing discharge impedance.

The observed volts-per-turn, $V_{ob}$, is the output of a single-turn loop around the iron core, roughly parallel to the inner circumference of the torus. This is, in effect, the voltage at a tapping point in the secondary leakage inductance $L_L$ external to the torus. Measurements of the inductance with the torus short circuited at the two main gaps yielded $L_L=2.2$ µH. The tapping point was found to be at 0.30 $L_L$. Consequently,

$$V_T = V_{ob} - 0.3 \frac{dL_L}{dT}. \quad (8)$$
The design and performance of ZETA

Measurements of the energy absorbed by the gas discharge are made by integrating the product \( V \cdot I \) with respect to time (Fig. 12).

The clamping igniton is fired at \( V_{ob} = 0 \), and from this point the discharge absorbs only energy stored in the leakage inductance \( L_L \). Energy dissipated in the clamping circuit is about 10% of the energy dissipated in the discharge after clamping. Energy remaining in the capacitors is dissipated in the non-linear resistor, and the capacitor voltage finally reaches the voltage across the clamp circuit which is about -300 volts. The relevant waveforms are shown in Fig. 15.

Discharge Characteristics

The inductance \( L_g \) and resistance \( R_g \) of the gas discharge are related to the voltage at the torus wall by

\[
V_T = I R_g + d(L_g I)/dt. \tag{9}
\]

The resistance \( R_g \) includes a term for eddy currents flowing in the torus walls and the liners, calculated to be \( 10^{-4} \) ohms. The inductance \( L_g \) corresponds to the magnetic energy generated within the torus.

Assuming an idealized model of the discharge, in which all currents flow on the skin of a pressureless plasma cylinder of radius \( r \), with the stabilizing field flux all trapped inside the plasma cylinder, the radius \( r \) of the channel is related to the current \( I \) and the bore, \( 2a \) of the torus by:

\[
a = \frac{2I}{aB_{so}} = \frac{I}{I'}, \tag{10}
\]

where \( I' \) is the current \( aB_{so}/2 \) at which the self-magnetic field \( B_{so} \) at the torus walls is equal to the stabilizing field \( B_{so} \).

The inductance per unit length of discharge corresponding to \( B_0 \) external to the plasma is thus \( 2 \log(I/I') \), and Eq. (9) becomes

\[
V_T = I R_g + 2 \lambda (dI/dt) [\log (I/I') + 1], \tag{11}
\]

where the additional term in the bracket corresponds to work done in compressing the axial field and \( \lambda \) is the total length of the discharge. The effective inductance \( L_B \) of the discharge for peak current calculations is given by \( \frac{1}{2} L_B |I_{max}|^2 = \text{total magnetic energy} \), that is:

\[
L_B = \lambda [1 - (I'/I)^2 + 2 \log (I/I')]. \tag{12}
\]

These equations are based on cylindrical geometry, and ignore the toroidal shape of the actual discharge. In so far as gas pressure alters the situation, it represents a correction to \( I' \); if the self-field has to support both \( B_0 \) and gas pressure, then \( I' \) will be larger than given above, and \( L_B \) will be less. In so far as the currents are distributed throughout the gas, \( I' \) and the radius of the discharge have to be redefined so that they represent appropriate averages. Taking the waveforms of Fig. 11, Eq. (12) requires \( L_B = 3.9 \) \( \mu \)H whereas the value obtained from the rise time and maximum current is \( 3.4 \) \( \mu \)H.

When \( I < I' \), the self-magnetic field cannot produce a pinched channel in which all the \( B_{so} \) flux is compressed. At the start of the pulse the inductance of the channel is determined by two quantities: (a) a small...
fixed inductance of 0.14 \mu H for the space between the liners and the torus walls; and (b) an inductance corresponding to the skin depth of the current. If the skin depth is large, so that current density is constant in a cylindrical approximation, the inductance is 0.6 \mu H; with a very small skin depth this inductance is negligible. In either case, the effective discharge inductance must be expected to be relatively small when \( I < I' \), and the rapid rise of current (Figs. 11 and 13) between zero and \( I = I' = 40 \) ka at point A supports this view.

The following quantities have been derived from observations and are shown in Table 3:

(a) \( \frac{V_0}{dI/dt} \) at the start of the pulse, which is the initial discharge inductance if \( dL/dt = \frac{dR}{dt} = 0 \);

(b) \( V_0/I \) at \( I_{\text{max}} \), which is the effective resistance at peak current—the true resistance only if \( dL/dt = 0 \);

(c) effective inductance at clamping from the \( L^* / R^* \) decay time, using estimate (b) for the resistance; and

(d) energy \( \frac{1}{2} V_0^2 L_{\text{max}} \) absorbed by the discharge between \( \frac{1}{2} I_{\text{max}} \) on the rise and \( \frac{1}{2} I_{\text{max}} \) on the decay of the current.

The values of effective resistance and inductance were checked in a number of cases by observing the variation of the decay time as a function of added inductance in the clamped circuit. This procedure yields independent estimates of \( L_{\text{g}} \) and \( R_{\text{g}} \) near current maximum, which agreed within about 30% of the estimates obtained from (b) and (c).

Table 3. Effective Characteristics of the Gas Circuit in Standard Conditions

<table>
<thead>
<tr>
<th>Capacitor voltage</th>
<th>Peak current, ka</th>
<th>Initial inductance, ( \mu H )</th>
<th>Effective resistance, ( \mu H )</th>
<th>Inductance at clamp, ( \mu H )</th>
<th>Energy absorption, k-Joules</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.5 keV</td>
<td>205</td>
<td>1.4 \pm 20%*</td>
<td>3.4 \pm 10%</td>
<td>4.9 \pm 10%</td>
<td>200</td>
</tr>
<tr>
<td>20.0 keV</td>
<td>180</td>
<td>1.6 \pm 20%*</td>
<td>3.0 \pm 10%</td>
<td>3.7 \pm 10%</td>
<td>143</td>
</tr>
<tr>
<td>18.5 keV</td>
<td>150</td>
<td>—</td>
<td>3.3 \pm 5%*</td>
<td>4.4 \pm 10%</td>
<td>91</td>
</tr>
<tr>
<td>16.5 keV</td>
<td>135</td>
<td>—</td>
<td>3.8 \pm 5%*</td>
<td>6 \pm 10%</td>
<td>59</td>
</tr>
</tbody>
</table>

* The limits shown are the spread of the measured values.

\( a,b,c,d \) Quantities defined under corresponding letters in text.

Variation of Conditions

The characteristics are largely independent of gas pressure over the range investigated (Fig. 16). Only a transitory low current discharge can be obtained when the deuterium pressure is reduced below about 0.06 \( \mu \) Hg (3 \times 10^{16} \) deuterium atoms per cm length). The apparent resistance of the discharge increases at higher pressures, and also if the discharge takes place in a heavier gas.

The characteristics are sensitive to magnetic stabilising field (Figs. 17 and 18); in particular the discharge becomes predominantly resistive with low or zero stabilising field. After cyclic operation for more than a few shots at a low field, the discharge often fails to strike.

The characteristics are sensitive to the cleanliness of the torus. The first discharges after exposure to the atmosphere are predominantly resistive, and are accompanied by a several-fold rise in pressure at each discharge. Optical spectrograms support these indications that the discharge releases large quantities of
gas (hydrogen, carbon, nitrogen and oxygen) from metal surfaces facing the plasma. The torus appears to clean up after about 1000 discharges in cyclic operation, depending on the degree of contamination. When “clean”, the resistive emf is about 500 volts per turn at current maximum, the volts-per-turn trace exhibits the striking transient peaks visible in Fig. 11, the neutron yield rises to values which are normally repeatable to within 25%, and a pressure drop of up to 50% is recorded by ion gauge readings at each discharge. Most observations after clean-up are repeatable over long periods of continuous operation, e.g. the current maximum is reproduced to within 5%. However, after a pause of even a few minutes some two or three discharges are required to re-establish given conditions.

The deuterium was mixed with 5% of nitrogen as estimated from ion gauge readings, which helped to limit the final voltage transient and also served as a tracer impurity for spectroscopic studies. This addition has little effect on the other electrical characteristics or on the neutron yield. Both this observation and the relative intensity of spectral lines suggest that, even after clean-up, a substantial fraction of the gas...
in the discharge is impurities of one sort or another. These could well be evolved at the arc-spots formed on the liner surfaces. The resistance is increased by inserting probes into the plasma and by reducing the uniformity of the stabilizing magnetic field.

Voltage Transients

The voltage transients, such as those visible in Fig. 11 are shown on an enlarged scale in Fig. 19. These transients are due to changes in current, generated by the dynamic behaviour of the plasma, flowing in the primary circuit inductances. They appear when the torus is clean and have a typical magnitude, near peak current, of 1–2 kV per turn, generated by a current change of between 0.05 and 0.1 I\text{max}.

The transients are not usually symmetrical. The current drops rapidly for about 10 μsec and recovers slowly during the next 90 μsec. Their frequency varies during the pulse with an average time between transients of about 100 μsec. This time is not markedly dependent on gas pressure or stabilizing magnetic field. The amplitude of the transients is much reduced after clamping.

If the transients are due to a sudden increase in resistance of the discharge, then the observed rate of fall of current, together with the estimated circuit inductance requires this increase to be about 0.1 ohm for about 10 μsec: if to an inductive effect, then the required inductance change is 0.3 to 0.6 μH.

These transients are correlated with X-ray production. In an analysis carried out for standard conditions, with I\text{max} = 180 kA, about 50% of the X-ray photons detected were emitted within 1 μsec of a voltage transient. There is also some evidence of correlation between neutron emission and the voltage transients.

Discharge Configurations

Magnetic Fields

The magnetic fields in limited regions of the plasma have been measured with search coils. The positions of the probe coils and the components measured are indicated in Fig. 20. Figure 21 shows plots of the magnetic field components obtained by averaging measurements over five pulses at the time of the peak current of 160 kA. The values of B_0 have been corrected for the initial field B_{0q}. The magnetic field direction varies continuously from an axial direction at the centre of the channel to an azimuthal direction at the outside over a distance of about 30 cm. The radius of the current channel, defined as the distance from the centre to the position of maximum B_q is about 25 cm. The axial field at the centre of the channel is about twelve times the initial value. This compression is somewhat smaller than the value of 19 calculated from the simple trapping analysis in cylindrical geometry. Figure 22 shows approximate current densities derived from the field measurements, using the expressions

\[ i_z = \frac{1}{r} \frac{d}{dr} (rB_\phi); \quad i_\phi = \frac{1}{l} \frac{d}{dr} (lB_\phi) \]

where \(r\) is distance from the estimated centre of the current channel, and \(l\) is the distance from the axis of the torus. The rise of \(i_z\) in the centre, and the predominance of \(i_\phi\) on the outside of the channel further emphasises the difference of the observed distribution from the simple skin-current model. In Fig. 23, the observed configuration is compared with those to be expected on the paramagnetic model, assuming constant temperature and cylindrical geometry, and on an assumed model in which the current vector is always parallel to the magnetic vector; that is, a


Figure 20. Magnetic field measurements
Probe positions and nominal components measured

Figure 21. Magnetic field components measured under standard conditions for: (a) probe position A; (b) probe position B

Figure 22. Current densities obtained from Figure 21, probe position A. The point \( r = 0 \) is the geometric centre of the tube

Figure 23. Comparison of observed and calculated current density ratios
A. Force free model, \( I_e/I_z = B_e/B_z \)
B. \( I_e/I_z \) from Figure 22
C. Paramagnetic model, \( I_e/I_z = B_e/B_z[2 + (B_z/B_e)^2]^{-1} \)

Figure 24. Magnetic field configuration as a function of time throughout the pulse
Probe position A. Standard conditions, 160 ka

force-free configuration. The actual configuration lies between the two. Pressure balance analyses suggest that the value of \( NkT \) that could be contained by the current channel is no more than one third the value given by the simple pinch equation.

The field configuration of Fig. 21 does not appear to arise as a consequence of the decay of skin currents. Values of \( B_e \) and \( B_z \) as a function of radius and time are shown in Fig. 24. The initial distribution of \( B_e \) indicates a uniform distribution of current throughout the channel, in agreement with the inferences drawn from the values of initial inductance. The compression of the current channel, which sets in between 100 and 200 \( \mu \)sec, appears to take place uniformly over the tube, and at no time is there evidence of a marked skin current. \( B_z \) is not corrected for \( B_{at} \) in Fig. 24.)

Measurements of both \( B_e \) and \( B_z \) with horizontal and vertical probes yield evidence of a complicated systematic drift of the current channel throughout the current pulse. Defining the centre of the current channel as the point where \( B_e = 0 \), results shown in Fig. 25 are obtained. The centre of the channel always moves outwards horizontally. When \( B_z \) and \( I_z \) are in the same direction (Curve A) this outward motion
appears to be rather greater than when they are opposed (not illustrated). There is also a vertical motion of the current channel. With a parallel arrangement of $B_z$ and $I_z$ the channel centre moves upwards (Curve B); with the normal antiparallel arrangement the motion is downwards (Curve C).

When $B_{z0}$ is very small or zero, there are rapid and violent fluctuations of the magnetic field, corresponding to the instabilities of the current channel. As $B_{z0}$ is increased, these fluctuations are reduced, and the fields rise and fall in step with the current. These rms fluctuations are illustrated in Fig. 26. Examples of the oscillograms are shown by Harding et al.\(^6\)

**Streak Pictures**

Evidence of a qualitative nature on the discharge configuration is yielded by streak pictures. In a heavy gas such as nitrogen, or before clean-up, streak pictures such as Figs. 27 and 28 are obtained. These show light coming mainly from a channel about 25 cm in radius. The channel appears to reach the wall of the tube only occasionally during the pulse. Luminous gas appears to be ejected from the walls at these times.

Such streak pictures have not been obtained in deuterium in "clean" conditions. Visible radiation from a pure hot deuterium gas is extremely weak, and light from impurity radiation predominates. An example of a quartz ultra-violet streak picture taken simultaneously with a visible light streak picture is shown in Fig. 29. Both pictures, though taken with different radiation, are remarkably similar. Both show a feature very characteristic of clean conditions: vertical bars of light which extend right across the tube. The time between bars is from 10 to 30 $\mu$s and they appear to come in bursts separated by about 100 $\mu$s. In many cases discontinuities in the light output can be correlated with the transients on the voltage waveforms.

The streak pictures show arc spots on metal surfaces in the field of view. Typically, these arc spots appear intermittently throughout the discharge pulse and several are seen per pulse.

**Plasma Parameters**

The state of the plasma is determined by the following parameters: electron density $n_e$; ion density $n_i$; electron temperature $T_e$; ion temperature $T_i$. These parameters must be presumed to vary throughout the discharge pulse and from one part of the plasma to another. At the present stage, it is only possible to report rough estimates near peak current under standard conditions.

**Electron Density**

The initial density of deuterium corresponds to an electron density of $0.9 \times 10^{13}$ cm$^{-3}$. To this must be added a correction for the electrons from multiply ionized impurities. At least, this must correspond to 5% of nitrogen ionized on the average about four times, so that $n_e$ is raised to $1.1 \times 10^{13}$ cm$^{-3}$. Supposing this to be compressed by a factor of $(I/I')^2$, or by the compression factor observed for $B_e$, densities obtained for the central region of the discharge are of the order of $10^{14}$ cm$^{-3}$ for currents of $140-180$ kA.
Transmission of 4 mm microwaves (critical electron density = $7 \times 10^{13}$ cm$^{-3}$) is only detected, for about 100 $\mu$s from the start of the current pulse at high currents. The densities at peak current under standard conditions exceed $7 \times 10^{13}$ cm$^{-3}$.

The compression calculations apply also to the deuterons: central densities of the order of $10^{14}$ cm$^{-3}$ are therefore to be expected, but there is no direct evidence on deuteron densities.

**Electron Temperature**

Estimates of the electron temperature have been made from microwave noise measurements and from the resistance of the discharge. The values obtained are summarized in Table 4. The estimates of $T_e$ made from the effective resistance at peak current are likely to be underestimates because current density and electric field are assumed to be uniform across the discharge, impurities are ignored, and the plasma is considered quiescent. The microwave measurements may give an underestimate of $T_e$ because a reflection coefficient of zero is assumed, and an overestimate in so far as the microwave noise is assumed to be all of thermal origin.

An upper limit is provided by the highest levels of ionization and excitation observed in the optical spectra. These are listed in Table 5. Only one of these lines, the CV triplet at 2278 Å requires appreciably more than 100 ev to excite it. The N$^\text{VI}$ and O$^\text{VII}$ iso-electronic lines with excitation potentials of 426 and 570 ev are not seen: they are at least five times weaker.

Calculations of collision excitation from the ground state$^{11}$ suggest that the intensity of the CV triplet is consistent with an electron temperature of less than $10^6$ °K if more than 1% of the ions are CV. This fact, together with the absence of other spectra requiring appreciably more than 100 ev to excite them, places an upper limit of about $10^6$ °K for the electron temperature. Precise estimates depend on more detailed knowledge of the atomic processes concerned, and particularly on the recombination time.

**Table 4. Estimated Electron Temperature $T_e$ in $^\circ K \times 10^{-5}$**

<table>
<thead>
<tr>
<th>Peak Current, ka</th>
<th>$T_e$ from 8 mm microwave observations</th>
<th>$T_e$ from resistance calculations$^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>60</td>
<td>$1.2 \pm 0.4$</td>
<td>0.8</td>
</tr>
<tr>
<td>84</td>
<td>$3.1 \pm 1.5$</td>
<td>0.9</td>
</tr>
<tr>
<td>110</td>
<td>$2.8 \pm 1$</td>
<td>1.0</td>
</tr>
<tr>
<td>130</td>
<td>$3.5 \pm 2$</td>
<td>1.0</td>
</tr>
<tr>
<td>150</td>
<td>$3.8 \pm 1$</td>
<td>1.1</td>
</tr>
<tr>
<td>180</td>
<td>$5.1 \pm 1$</td>
<td>1.2</td>
</tr>
<tr>
<td>205</td>
<td>--</td>
<td>$2.6$</td>
</tr>
</tbody>
</table>

$^*$ Resistance temperature estimates are based on formulae quoted by Spitzer$^{13}$ assuming (1) uniform current density over a channel 20 cm radius (2) uniform current density over a channel radius determined by $(I/I')$.

**Ion Temperature**

Doppler widths of impurity lines, interpreted as kinetic temperatures, yield values of $T_i$ of up to $7 \times 10^6$ °K. Results obtained are shown in Table 6. The experimental accuracy of individual measurements is about 25%. The results indicate the temperature increasing as pressure is decreased, and as $B_{20}$ is increased.$^6$ Temperatures measured throughout the discharge pulse do not vary widely, and suggest that
Table 5. Selected Vacuum UV Spectra from ZETA

Un-gated spectrograph; standard conditions; 175 ka

<table>
<thead>
<tr>
<th>Wavelength, Å</th>
<th>Identifications</th>
<th>Intensity, n.A</th>
<th>Energy to form ion, ev</th>
<th>Excitation potential, ev</th>
</tr>
</thead>
<tbody>
<tr>
<td>1215.31</td>
<td>D Lyα 1s 2S - 2p 2P&lt;sup&gt;0&lt;/sup&gt; (1)</td>
<td>8</td>
<td>—</td>
<td>10.1</td>
</tr>
<tr>
<td>949.48</td>
<td>D Lyα 1s 2S - 5p 2P&lt;sup&gt;0&lt;/sup&gt; (1)</td>
<td>1</td>
<td>—</td>
<td>13.0</td>
</tr>
<tr>
<td>1334-6</td>
<td>CII 2p 2p&lt;sup&gt;0&lt;/sup&gt; - 2p&lt;sup&gt;2&lt;/sup&gt; 3D (1)</td>
<td>6</td>
<td>11.2</td>
<td>9.3</td>
</tr>
<tr>
<td>903-5</td>
<td>CII 2p 2p&lt;sup&gt;0&lt;/sup&gt; - 2p&lt;sup&gt;2&lt;/sup&gt; 3P (1)</td>
<td>5</td>
<td>11.2</td>
<td>9.3</td>
</tr>
<tr>
<td>977.03</td>
<td>CH 2s 2S - 2p 1P&lt;sup&gt;0&lt;/sup&gt; (1)</td>
<td>4</td>
<td>24.3</td>
<td>12.6</td>
</tr>
<tr>
<td>1548-51</td>
<td>CIV 2s 2S - 2p 2P&lt;sup&gt;0&lt;/sup&gt; (1)</td>
<td>10</td>
<td>47.7</td>
<td>8.0</td>
</tr>
<tr>
<td>2270-8</td>
<td>CV 1s2s 2S - 1s2p 2P&lt;sup&gt;0&lt;/sup&gt; (2)</td>
<td>6</td>
<td>64.2</td>
<td>304</td>
</tr>
<tr>
<td>988-92</td>
<td>NII 2p 2p&lt;sup&gt;0&lt;/sup&gt; - 2p&lt;sup&gt;2&lt;/sup&gt; 3D (1)</td>
<td>2</td>
<td>29.5</td>
<td>12.5</td>
</tr>
<tr>
<td>684-7</td>
<td>NII 2p 2p&lt;sup&gt;0&lt;/sup&gt; - 2p&lt;sup&gt;2&lt;/sup&gt; 3P (1)</td>
<td>2</td>
<td>29.5</td>
<td>12.5</td>
</tr>
<tr>
<td>755.14</td>
<td>NIV 2s 2S - 2p 1P&lt;sup&gt;0&lt;/sup&gt; (1)</td>
<td>4</td>
<td>47.2</td>
<td>16.1</td>
</tr>
<tr>
<td>1238-43</td>
<td>NV 2s 2S - 2p 2P&lt;sup&gt;0&lt;/sup&gt; (1)</td>
<td>8</td>
<td>77.1</td>
<td>10.0</td>
</tr>
<tr>
<td>787-91</td>
<td>OIV 2p 2p&lt;sup&gt;0&lt;/sup&gt; - 2p&lt;sup&gt;2&lt;/sup&gt; 3D (1)</td>
<td>3</td>
<td>54.7</td>
<td>15.7</td>
</tr>
<tr>
<td>555-6</td>
<td>OIV 2p 2p&lt;sup&gt;0&lt;/sup&gt; - 2p&lt;sup&gt;2&lt;/sup&gt; 3P (1)</td>
<td>2</td>
<td>54.7</td>
<td>15.7</td>
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<tr>
<td>758-62</td>
<td>OIV 2p 2p&lt;sup&gt;0&lt;/sup&gt; - 2p&lt;sup&gt;2&lt;/sup&gt; 3P (1)</td>
<td>2</td>
<td>77.1</td>
<td>26.4</td>
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<tr>
<td>628.73</td>
<td>OIV 2s 2S - 2p 1P&lt;sup&gt;0&lt;/sup&gt; (1)</td>
<td>3</td>
<td>77.1</td>
<td>19.6</td>
</tr>
<tr>
<td>2781-9</td>
<td>OIV 3s 2S - 3p 2P&lt;sup&gt;0&lt;/sup&gt; (3)</td>
<td>4</td>
<td>77.1</td>
<td>72.4</td>
</tr>
<tr>
<td>1031-8</td>
<td>OIV 2s 2S - 2p 2P&lt;sup&gt;0&lt;/sup&gt; (1)</td>
<td>3</td>
<td>113.4</td>
<td>12.0</td>
</tr>
<tr>
<td>464-7</td>
<td>FIV 2p 2p&lt;sup&gt;0&lt;/sup&gt; - 2p&lt;sup&gt;2&lt;/sup&gt; 3P (4)</td>
<td>3</td>
<td>87.1</td>
<td>26.7</td>
</tr>
<tr>
<td>654-8</td>
<td>FIV 2p 2p&lt;sup&gt;0&lt;/sup&gt; - 2p&lt;sup&gt;2&lt;/sup&gt; 3P (4)</td>
<td>3</td>
<td>87.1</td>
<td>38.1</td>
</tr>
<tr>
<td>646.36</td>
<td>FIV 2p 2p&lt;sup&gt;0&lt;/sup&gt; - 2p&lt;sup&gt;2&lt;/sup&gt; 3P (5)</td>
<td>3</td>
<td>114.2</td>
<td>12.0</td>
</tr>
<tr>
<td>1639.00</td>
<td>Al IV 3s 1P&lt;sub&gt;1&lt;/sub&gt; - 3p 1D&lt;sub&gt;4&lt;/sub&gt; (6)</td>
<td>5</td>
<td>28.3</td>
<td>85.0</td>
</tr>
</tbody>
</table>

* Identification References


Intensity Number, n, is a qualitative estimate based on photographic plate density using a linear scale 0-10. This scale corresponds approximately to a variation in line intensity by a factor of 10<sup>4</sup>.

Figure 28. Drum camera streak pictures for helium. B<sub>20</sub> 160 gauss, 9:1 turns ratio, during clean-up, visible light, vertical window.
the measured values are not sensitive to the exact setting of the gate. The temperatures measured from O IV and N IV lines are some 50% lower than values obtained simultaneously from O III lines. The large scatter in the results appears to be associated with the cleanliness of the discharge.

Estimates of the deuteron temperature made from these measurements depend on establishing (a) the extent of equilibrium between the impurities and the deuterons, and (b) the magnitude of the mass motion. The lifetimes for ionization of O IV and N IV are about 10 and 12 µsec respectively, with \( T_e = 0.5 \times 10^6 \) °K and \( n_e = 10^{14} \text{cm}^{-3} \). The relaxation times for O IV and N IV collisions with deuterons are about 8 µsec and 15 µsec respectively, with a deuteron temperature of \( 10^6 \) °K, and an impurity temperature of \( 5 \times 10^6 \) °K. If the impurity is trapped in the gas at the start of the discharge, these times are quite short enough to establish equilibrium. The chief impurity lines used for Doppler width measurement have been observed mostly in the middle regions of the discharge tube. Figure 30 shows the intensity profile along the window together with temperatures obtained.

There is as yet no reliable direct evidence of the contribution of mass motion to the broadening of the spectral lines. The Doppler widths correspond to velocities of about \( 5 \times 10^6 \) cm sec\(^{-1}\). If the broadening were all due to the ions moving with the same velocity as the channel, then the deuterons would have an energy corresponding to about \( 0.7 \times 10^6 \) °K. Since there must be some degradation of the mass motion to thermal motion, as a consequence of viscous effects, shock waves and gyro-relaxation effects, the deuteron energy is likely to be intermediate between the values corresponding to \( 0.7 \times 10^6 \) and \( 5 \times 10^6 \) °K at currents between 140 and 200 ka.

### Energy Considerations

Comparison of the observed energies of the impurity ions with the estimated electron temperature yields evidence that these ions can achieve mean energies substantially greater than the mean energy of the electrons. For a deuteron plasma with \( n_e = n_i = 10^{14} \text{cm}^{-3} \) and \( T_e = 0.5 \times 10^6 \) °K, the relaxation time for deuteron–electron collisions is about 180 µsec, so the rate of energy transfer from the deuterons to the...
electrons is calculated to be about 10 watts \( cm^{-3} \), if the deuterons are maintained at \( 10^8^\circ K \). This corresponds to about \( 10^7 \) watts in the whole current channel.

The sudden falls of current, of about \( 10^4 \) amp at each voltage transient (Fig. 11), correspond to mechanical work being done on the plasma at the rate of about \( 3 \times 10^7 \) watts, if the transients are interpreted as being due to inductive changes. The duration of the transients is comparable with the deuteron-deuteron relaxation time, and longitudinal Alfvén waves are calculated\(^{13} \) to be damped mainly by viscous effects under the estimated conditions of the plasma. Consequently, it seems quite possible that the ions are maintained at a higher temperature than the electrons.

The thermal energy per unit length, \( \frac{2}{3} NkT \), of the gas contained by the pinch, must be less than that given by the simple pinch relation Eq. (1), because of the trapped stabilizing field. This relation can be rewritten

\[
\beta I^2 = 2NkT, \tag{13}
\]

where \( \beta \) is a factor which, according to the magnetic field measurements, is no greater than \( \frac{1}{3} \). Taking \( N = 2 \times 10^{17} \ cm^{-1} \) and \( \beta = \frac{1}{3} \), the mean temperature \( T \) is calculated to be \( 6 \times 10^5^\circ K \) at 100 ka and \( 2.4 \times 10^6^\circ K \) at 200 ka.

The maximum thermal energy trapped in the current channel is \( \frac{2}{3} \beta I^2 \). The power input is \( \Omega I^2 \) where \( \Omega \) is the effective resistance per unit length. If energy is lost as radiation at a rate \( P \), then the time \( \Delta t_1 \) taken to heat the plasma from zero temperature to a temperature at which the magnetic field can no longer contain it is given by:

\[
\Delta t_1 = \frac{3\beta}{4\Omega} \left[ 1 - \left( \frac{P}{\Omega I^2} \right) \right]^{-1}. \tag{14}
\]

This quantity, \( \Delta t_1 \), thus represents a crude “thermal stability time” which is only infinite if the power radiated is equal to the power input. At the currents so far used this condition cannot be satisfied by bremsstrahlung radiation alone.\(^{14} \) Preliminary measurements\(^6 \) indicate that, in clean conditions, \( P \) is substantially less than \( \Omega I^2 \). Hence, neglecting the radiated power, taking \( \beta = \frac{1}{3} \) and using the observed values of \( \Omega \), we obtain \( \Delta t_1 \approx 100 \mu sec \).

The time, \( \Delta t_1 \), may be compared with the “energy containment time” to be used in Lawson’s criterion\(^{15} \) for a measure of the efficiency of a thermonuclear device. This time is

\[
\Delta t_2 = \frac{3\beta}{4\Omega} \tag{15}
\]

which is equal to \( \Delta t_1 \) in the absence of radiation.

**SUMMARY**

ZETA is a fully engineered apparatus in which currents of up to 200 ka have been passed through gas...
in a torus. This torus has a bore of 100 cm and a mean circumference of 1160 cm. The current pulse has a duration at half height of about 2 msec. The current waveform corresponds to a capacitor discharging into an inductance and resistance in series; the resistance being about one third of the value for critical damping. The apparatus can run for long periods at a rate of one pulse every twelve seconds.

A basic condition for a thermal plasma in a gas discharge is that the drift velocity of the electrons shall be much less than their thermal velocity. In ZETA, where the average values are about 10^7 cm sec^{-1} and 5 x 10^8 cm sec^{-1} respectively, this condition is satisfied.

X-rays of 20 kev and upwards and D-D reactions due to non-thermal processes have been observed. If these processes are a result of runaway particles they account for only a fraction of the total current; i.e. 25 amp of runaway electrons and 1 amp of 17 kev deuterons.6, 6

Both inductance and magnetic search coil measurements show that the current channel is constricted into the centre of the discharge tube, and the stabilizing field is enhanced by a factor of about ten. The stabilizing field is not trapped by currents flowing only on the surface of the plasma and the observed distribution of magnetic fields has yet to be explained.

At peak current of 140–180 kA, the resistance of the discharge and the fluctuations of the magnetic field increase markedly as the stabilizing field is reduced from about 160 gauss. At very low fields the discharge often fails to strike in clean conditions.

Streak pictures taken with the normal stabilizing field indicate that the plasma is isolated from the walls for periods of up to 1 msec in nitrogen and in contaminated helium and deuterium. Under clean conditions, the magnetic field fluctuations, and the current and voltage transients, suggest that the plasma is by no means stationary.

Impurity ions in the channel have been observed spectroscopically to have energies up to 500 eV. These energies, which are much larger than the mean electron energy, may be accounted for by this motion of the plasma, both directly and as an ion heating mechanism.

The observed power input and the estimated maximum plasma energy (\(\frac{2}{\sqrt{\pi}}\)) suggest that the energy containment time \(\Delta t\) cannot exceed about 100 \(\mu\)sec. The efficiency of ZETA as a thermonuclear reactor is indicated by the Lawson product \(n_t \Delta t\), which is thus about \(10^{18} \text{ cm}^{-3} \text{ sec}\). This may be compared with the value of \(10^{18} \text{ cm}^{-3} \text{ sec}\) required for a power producing thermonuclear reactor (D-D).

**ACKNOWLEDGEMENTS**

We wish to acknowledge the continuous support and encouragement this work has received from Sir John Cockcroft, Dr. B. F. J. Schonland, Mr. D. W. Fry and Mr. C. H. Flursheim. Important contributions have been made by the authors of Reference 6, and by M. Rusbridge, W. Burton and J. Schofield. The experiments have been conducted with invaluable technical assistance from E. M. Jackson, H. Whittle, P. B. Clarke and A. B. Gillespie; and the work has profited from discussions with W. B. Thompson, R. J. Bickerton and our colleagues at the AEI Laboratories, Aldermaston.

**REFERENCES**

Theoretical Problems Suggested by ZETA

By W. B. Thompson,* S. F. Edwards,! J. Hubbard* and S. J. Roberts*

This paper is divided into three sections. The first section explores the consequences of the hypothesis that ZETA has produced a pinched discharge stabilized by a trapped axial field. In this configuration, stabilization is a result of the rapid change in magnetic field direction passing through the surface current sheet, and it is essential that the current be confined to a narrow layer. This in turn demands that the discharge be composed of a very hot, fully ionized gas, since otherwise the fields will mix rapidly and stability will be lost.

The second section considers what happens if the stability condition is violated. This need not result in the gas being lost to the wall, since one type of instability corresponds to a motion of the gas as a whole and the moving mass of gas may still be confined. In this process, the partial stability conferred by the partially trapped axial field and the image currents induced in the wall by the moving currents in the gas are both important. The mathematical problems associated with systems of this kind are most formidable, but an extremely simplified problem has been studied by W. B. Thompson and S. F. Edwards using analytic methods, and by S. J. Roberts and A. Curtis using computational techniques.

The third section discusses the transport coefficients, which have a vital role in the heating of the discharge and in preserving the stable configuration. There is not complete agreement on the values of these quantities, and in certain important circumstances new phenomena can be important. In these, the fluctuations in the electric field within the plasma are vital, and a method of analysis is developed in which the role of the fluctuating field is exhibited.

THE HEATING OF ZETA

In this section an attempt is made to give a theoretical description of the processes occurring in ZETA using only the simplest concepts of ionization, Joule heating and magnetic compression.

The history of the discharge will be considered in three phases. First, there is an initial stage of ionization and heating which lasts until the axial current is large enough to pull the discharge clear of the walls. This occurs when the axial current is equal to the axial field (180 gauss). In a tube of 50 cm radius this occurs at a current of 45,000 amp and occurs 150 μsec after the start of the pulse. Next, there is the contracting phase which lasts for approximately 2 msec with the fairly low electric fields used in ZETA. Finally, the contracted discharge is heated by ohmic dissipation, the currents widen and the azimuthal and axial magnetic fields mix until stability is lost.

Phase 1

Before the main voltage pulse is applied to ZETA, there is a small degree of ionization produced either by a rf discharge or by a direct current of about 2000 amp. In either case the initial ionization is about 0.1%, and since a typical starting pressure is 3/4 micron of deuterium (corresponding to about 5 × 10¹⁸ deuterium molecules or 10¹⁴ deuterons/cm³) we may expect an initial ion density of approximately 10¹⁰/cm³. This density, while low, can prevent the initial current from reaching the centre of the discharge. Initially, the current is limited by inertial effects and the magnetic field satisfies an equation of the form

\[ -\nabla B \sim 4\pi ne^2/mc^2 B. \]  

The initial penetration depth, \( l = \left( 3mc^2/4\pi ne^2 \right)^{1/4} \), is approximately 5.3 cm.

In the calculations associated with the first phase of the discharge, the internal electric field in the plasma has a most important role since it determines the energy which particles will acquire, and hence the relevant values of the atomic cross sections for elastic scattering, ionization, dissociation and excitation. In order to determine the electric field, it is necessary first to calculate the initial value of the current rise. This can be obtained from

\[ \frac{d}{dt}(\mathcal{L}I) + \mathcal{R}I = V \]

where \( \mathcal{L} \) and \( \mathcal{R} \) are the inductance and the resistance of the circuit and \( V \) is the applied voltage. This may be written in terms of the penetration depth \( l(t) \) as

\[ \frac{d}{dt} \left( \mathcal{L} + 2\mathcal{L} \frac{l(t)}{R} \right) + \frac{4\mathcal{L}}{R} \frac{dl}{dt} = V \]

where \( L \) and \( R \) are the length and radius of the torus...
and $\mathcal{L}$ is the inductance associated with the magnetic field outside the torus. The solution of Eq. (2) is

$$I = \frac{1}{L} \frac{1}{[1 + \alpha(t)]} \int [1 + \alpha(t)]^2 V dt$$

(3)

where $\alpha(t) = \frac{2I(t)}{L} \frac{L}{R}$

The electric field is obtained from

$$\frac{\partial E}{\partial x} = -\frac{1}{c} \frac{\partial H}{\partial t}$$

which, if we assume that initially $\partial E/\partial x$ can be replaced by $E/t$ and, since $\frac{\partial H}{\partial t} = \frac{2i\omega}{R}$, takes the form

$$E = -\frac{1}{c} \frac{2i\omega}{R} t$$

(4)

Since $L \approx 3 \mu H$ and $L \approx 1000 \text{ cm}$, we have $\alpha \approx \frac{3}{4} \frac{t}{R}$

and from Eq. (3) it is found that the initial current rise, $\partial I/\partial t$, at 2 kev applied emf is approximately $10^5$ amp/sec. The internal electric field, from Eq. (4), is then found to be approximately 0.21 $\text{v/cm}$.

Since the mean free path for electrons is approximately $10^5 \text{ cm}$, the energy of an electron on collision is about 210 ev. At this energy the cross sections for elastic scattering, $\sigma_0 \approx 0.153 \sigma_0$; excitation, $\sigma_{ex} \approx 0.51 \sigma_0$; and ionization $\sigma_1 \approx 0.68 \sigma_0$; where $\sigma_0 \approx 0.8 \times 10^{-14} \text{ cm}^2$. We assume here that the cross section for ionization of a deuterium molecule is twice that for the ionization of an atom and that the dissociation cross section is close to the excitation cross section.

The time needed to make the first collision is approximately $\frac{1}{4} \mu \text{sec}$, and ionization proceeds with an initial rate of $\partial n/\partial t \approx n_1/\tau$. Since $\tau = 1 \mu \text{sec}$, the ionization rises rapidly and we must discuss the electric field in a system with rapidly varying conductivity.

We must solve

$$\frac{\partial^2 H}{\partial s^2} = \frac{4\pi \sigma(t)}{c^2} \frac{\partial H}{\partial t}$$

(5)

By replacing the variable $t$ by $s = \int \frac{c^2}{4\pi \sigma(t)} dt$, Eq. (5) is reduced to the simple form

$$\frac{\partial^2 H}{\partial s^2} = \frac{\partial H}{\partial s}.$$  

(6)

Now, however, the boundary condition on the surface becomes a little elaborate, the solution being approximately

$$H(x, t) = H_0(t) \left( \frac{n_1}{e} \sqrt{s/\tau} \right)$$

where $H_0 = \left( \frac{1}{c^2} e^2 / \sigma_0 \right)^{1/2}$ is the penetration depth at time $t = \tau$, and is equal to 25 cm.

The field near the surface at any time is determined by the instantaneous value of the surface field, while the field deep within the plasma is chiefly that remaining from the early stages before the conductivity became large.

At the same time, the energy of particles can be estimated by solving

$$\frac{d}{dt} \left( \frac{1}{2} m v_i^2 \right) = \frac{1}{\tau_1} \left( \frac{1}{2} m v_i^2 \right) = \frac{1}{\tau} E,$$

$$\frac{d}{dt} \left( \frac{1}{2} m v_\perp^2 \right) = \frac{1}{\tau} v_\perp^2,$$

(8)

where $\tau$ is the fraction of the energy left, after a collision, which is available for the motion transverse to the field, $\tau_1$ is the collision time and $v_i$ and $v_\perp$ are the components of velocity perpendicular and parallel to the field. The angular parentheses indicate average values.

The average energy, $\bar{E} = \frac{1}{2} m \langle v_i^2 \rangle + \frac{1}{2} m \langle v_\perp^2 \rangle$, at time $t$ is, from Eq. (8),

$$\bar{E} \approx \frac{1}{2} \frac{e^2}{m} E^2 (x, \tau) \tau^2 e^{-\tau/t} \text{ near the centre}$$

$$\approx \frac{1}{2} \frac{e^2}{m} E^2 (x, \tau) \tau^2 (1 + f) \text{ near the edge}.$$  

(9)

When the relevant values are put in equations (7) and (9) we find that, by the time the gas is about 10% ionized, the ions gain an average of approximately 150 ev and the electrons, which make many inelastic collisions, gain approximately 80 ev. Even after the field is reduced, the particles can raise the degree of ionization to about 50%, while the ion temperature falls to 20 ev. This occurs in the first 60 $\mu \text{sec}$, after which the azimuthal field has risen to values comparable with the axial field, the depth of penetration has dropped, and direct acceleration of particles along magnetic field lines becomes relatively less important; however, the temperature continues to rise because of Joule heating so that

$$T = T_1 \left( 1 + \frac{H_0^2}{8\pi n_k T_1} \left( \frac{3 l_\text{p}}{4} \right)^{1/2} \right),$$

(10)

where $l_\text{p} = \left( 4 \frac{\pi \sigma_1}{e^2} \right)^{1/2}$ and $\sigma_1 = \sigma(T_1)$.

Since $H_0(0) = 180 \text{ gauss}$ and $T_1 \approx 15 \text{ ev}$, we find that, from Eq. (10), the final temperature is about 20 ev per particle and the final penetration is about 1.8 cm.

Phase 2

In the second phase, the discharge is contracted by the rising azimuthal field. Since the collapse time is about 1 $\mu \text{sec}$ and the velocity of a hydromagnetic wave is approximately $10^7 \text{ cm/sec}$, the collapse occurs with pressure balance maintained. Neglecting the gas pressure compared with the magnetic pressure we get the relationship

$$(R(r) \mathcal{L} = (2\pi L/RH_0) \int E(t) dt$$
and if we introduce an averaged electric field $\langle E \rangle$ and a length $R^*$, so that the external inductance can be written as $L \ln (R^*/R)$, we obtain an expression for the radius $r$ of the plasma column at a time $t$

$$\frac{R}{r} \ln \left( \frac{R^*}{r} \right) = \frac{1}{t_e} \int_0^t \langle E(t) \rangle \, dt \tag{11}$$

where $t_e = \frac{1}{4} R H_{e0} \langle \langle E \rangle \rangle$.

Since the voltage drops to zero in approximately 2 msec starting from 1 kev/m, the final conditions are radius $r_t \approx \frac{1}{3} R$ and $I_{\text{max}} \approx 150$ kA. At the same time, the temperature is increased, by adiabatic collapse, to about 80 ev.

Joule heating has been completely neglected in this calculation. It can be included if the density and temperature variations are taken as functions of time. We introduce the variables $\xi$ and $s$, defined by

$$\frac{1}{2} \frac{d^2}{ds} = \int_0^r \frac{\rho}{\rho_0} \, r \, dr \quad \text{and} \quad s = \int_0^r \frac{c^2}{4 \pi \sigma_1} \frac{\rho}{\rho_0} \, dt,$$

where $\rho_0$ is the initial density. We then find that $H_{4}/\rho$ and $H_{4}/\rho r$ satisfy simple differential equations in the variables $\xi$ and $s$, where, from Eq. (11), $s$ can be written as a function of $r$:

$$s = \frac{c^2}{4 \pi \sigma_1} \frac{t_e}{r} \ln \left( \frac{R^*}{r} \right) \tag{12}$$

The dissipation of magnetic energy during the compression can be calculated; it is sufficient to increase the gas energy by approximately 50 ev per particle. Although this almost doubles the gas temperature, it has a negligible effect on the dynamics, which is still dominated by the magnetic field effects.

Thus, at the end of the collapse we have an energy of about 130 ev per particle corresponding to a temperature of about 1.4 x 10^4 degrees K.

**Phase 3**

In the third phase, more heat is added by ohmic dissipation. If heat losses are negligible and heat conduction high we can find the increase in temperature from the expression

$$\left( \frac{T}{T_5} \right)^{3/2} = \left( \frac{T_5}{T_2} \right)^{3/2} + \frac{2}{5} = \left[ \frac{2}{t_5} \frac{3}{2} (1.5 + 1.62 b^9_2) \right]^{5/2} \tag{13}$$

where $t_5 = (4 \pi \sigma_3 / c^2) a^5_3$, $a_3 = \sigma(T_3)$, $\beta = 8 \pi \times [H_{4}(r_f)]^{-2}$ and $b_1 = H_{4}/H_{4}(r_f)$.

Since $b = 0.4$, $t_2 = 0.25$ sec and $b_1 \approx 1$, we find that in 2 msec $T$ is increased by $0.3 T_5$ during which time the field penetrates 1/10 the total radius.

**Radiation Energy Loss**

In the temperature estimates made so far, we have considered the radiation loss by un-ionized deuterium, but have made no allowance for the energy loss by bremsstrahlung and by radiation from any impurities present. The first of these is negligible, since it is not greater than approximately $2 \times 10^{-8}$ w/cc, corresponding to about 100 ev/electron-sec, but the second is more serious. The principal contaminating gases are oxygen and nitrogen, and possibly aluminium, and atoms of these gases are not fully ionized at temperatures around 100 ev. However, the oxygen and nitrogen are rapidly ionized in about 50 usec to the K shell where the excitation levels are too high to be easily excited at 100 ev. Thus the energy radiated by these atoms is radiated in competition with ionization and is not likely to be greater than about 400 v/atom for oxygen and 250 v/atom for nitrogen.

The total energy loss due to these impurities is approximately 6500 ev/electron, where $\phi$ is the concentration of oxygen and nitrogen. For aluminium the situation is rather different. Here the concentration of Al^X and Al^XI is approximately 0.4 and 0.25 of the total aluminium. These two ions possess low-lying energy levels, the strongest lines being of energy 37 ev and 22 ev. These levels have excitation times of the order of a few tens of microseconds and therefore the energy loss per electron is approximately $4 \times 10^{32}$ aluminium ev/msec. Hence, if the concentration of aluminium rises much above 1% it constitutes a serious source of energy loss.

Some estimate of the total radiation loss is obtained from the observation that variations in the air concentration below 10% do not seriously affect the temperature, but it is seriously altered by a 20% concentration. This suggests that the energy radiated is of order 150 ev/electron in the pulse. But if the energy gained from Joule heating is calculated, keeping the temperature constant at 100 ev, it is found to be approximately 180 ev/electron in the pulse and so compensates for that lost by radiation. Thus the actual temperature is probably in the region of $10^6$ degrees K. This disagrees with the Doppler measurements in O^Y and O^Y, but since these ions have very short lives, it is doubtful if they are in thermal equilibrium and their motion is probably the effect of the radial electric potential, $\phi$, which is related to the temperature by $\phi = (kT/\rho) \ln (\rho_{m}/\rho_0)$. It is easy to devise processes which give ions the necessary motion.

Another estimate of the temperature may be made by comparing the apparent resistance of the current channel, which is approximately $1.6 \times 10^{-3}$ ohm, with that estimated assuming an ionized gas at $10^8$ degrees with the current confined to a skin of 1.5 cm depth, which gives a resistance of approximately $0.8 \times 10^{-3}$ ohm. These are not in serious disagreement.

**Runaway Ions**

The next problem which must be considered is the neutron production, which far exceeds the thermonuclear flux at $10^8$ degrees, and also far exceeds the thermonuclear flux at the Doppler temperatures of 2 to $3 \times 10^8$ degrees; we must, therefore, expect that the neutrons do not have a thermonuclear origin. In devices where the gas pressure contributes little, and the pressure balance is between the axial and azimuthal fields, the electric current has a considerable component along the field lines and hence is capable of accelerating particles. This produces only low
instabilities do grow, the motion of the plasma de-
linear effects or by image currents. If such limited
not necessarily imply large losses to the walls, for the
in the discharge. These particles can gain energies of
about 20 kev in $\frac{1}{3}$ mesc and could produce a flux of
approximately $10^6$ neutrons/sec which is not far from
approximately $10^7$ fast particles
of the particles present, and only those which
are in the region of changing magnetic fields can run
away. However, we can easily find $10^4$ fast particles
in the discharge. These particles can gain energies of
about 20 kev in $\frac{1}{3}$ mesc and could produce a flux of
approximately $10^6$ neutrons/sec which is not far from
the observed flux. During the earlier phases of the
discharge, the electric field is larger and runaway ions
can appear even more easily; thus we can provide
enough runaway ions to produce the observed neutron flux.

PLASMA TURBULENCE

The picture of the plasma presented above is in
serious contradiction with experiment at one point. Magnetic fields have been measured by inserting
probing coils into the discharge, and it was found that,
although the axial field was trapped, the azimuthal field penetrated deep into the discharge. This suggested
a uniform volume current rather than a surface cur-
rent sheet. The motion of the gas during the first
phase has been neglected; but such motions should
occur, since at early times the conductivity is low
enough for the ionized gas to diffuse through the axial
field. However, this is possible only during the first
few microseconds, and it is difficult to see how the axial current density can exceed 5 amp/cm$^2$ before the
conductivity reaches a high enough value for the axial field to trap the gas. Compression would
amplify this, but the volume current remains a small
fraction of the total current.

Moreover, the apparent stability of the observed
field configuration is obscure. Tayler, 6 who has studied
the effect on stability of a finite current channel,
suggests that the observed fields should be unstable,
and we must consider the possibility that the dis-
charge in ZETA is not, in fact, stable. Instability does
not necessarily imply large losses to the walls, for the
growth of instabilities may be limited either by non-
linear effects or by image currents. If such limited
instabilities do grow, the motion of the plasma de-
depends on small random departures from an unstable
equilibrium configuration and might be expected to
show some of the characteristics of turbulence. If the
plasma is turbulent, the large field mixing is not sur-
prising, nor is a rapid gain in energy of the ions. To
proceed—in view of the random nature of the initial
conditions, we should seek statistical information
about the motion. To treat this problem fully would
require an understanding of hydromagnetic turbulence
which is lacking, and we have confined ourselves to one
simple model of the motion. If the plasma moves as a
whole, without altering its cross section ($m=1$ modes
only), then we can forget the hydrodynamic problems
and try to understand the statistical properties of the
motion of a wriggling current channel in a conducting
tube.

Statistical Treatment

The simplest treatment of this is to consider the
walls as dominant everywhere except near the axis,
which is equivalent to considering the discharge as
almost straight. We can then discuss the motion of an
axial current channel which is projected away from
the axis with a certain initial velocity. The channel
will then move in the potential field of the walls, the
potential when the channel is at any position being propor-
tional to the inductance at that point. If we
ask for the statistical properties of the system, where
the average energy is known, we can use the methods
of statistical mechanics and obtain, for the probability
of finding the current channel in a unit volume near $r$, the
expression

$$P(r)d\nu = \exp(-\int L(r)/kT)$$

where $kT$ is an average of the energy and $L(r) = \log(1-r^2/R^2)$ is the self inductance of the current
channel at a distance $r$ from the centre, compared with
that on the axis. This can be written as

$$P(r)d\nu \approx [(y+1)/(1-r^2/R^2)]\pi R^2d\nu$$

where $y$ is a constant not far from unity. It is obtained
by assessing the energy gained when the discharge is
subject to a perturbation of the form $a\sin kx$. The
force acting is

$$F = I^2abl\sin kx \ln(2/k\nu_0)$$

and it changes sign when the displacement $y$ is given by

$$y^2 = R^2[1-(kR)^{-3}\ln(2/k\nu_0)].$$

Hence the energy gained is

$$\frac{1}{2}I^2[bR^2 \{R^2 \ln(2/k\nu_0) - 1\} - \ln(k^2R^2 \ln(2/k\nu_0))].$$

This must now be averaged over the permitted spectrum to get

$$\gamma = \left(bR^2 \frac{1}{6} \ln \frac{2}{k\nu_0} - \frac{1}{2} \ln \left(bR^2 \frac{2}{k\nu_0}ight) \right)$$

where $k_0$ is an appropriate mean of $k$ which, if the
modes are independent, is just $k_{max}$, the maximum
possible value of $k$. 

Alternative Treatment

It is possible to give a more sophisticated treatment of this problem in which the shape of the curve is taken into account. The energy associated with the curved current channel depends upon the shape of the curve, hence if we write a partition function as \( \exp(\int \mathcal{A} \rho \, dt) \), \( \mathcal{A} \) must be dependent on the form of the curve \( \mathbf{r}(s) \) along which the current channel lies. The partition function is thus a function of the channel shape. In fact, the potential energy may be written as

\[
V(\mathbf{r}(s)) = \frac{H^2}{8} \rho \cdot \mathbf{r} \cdot \int ds \left( \left( \frac{d\mathbf{x}}{ds} \right)^2 + \left( \frac{d\mathbf{y}}{ds} \right)^2 + \left( \frac{d\mathbf{z}}{ds} \right)^2 \right)^{\frac{1}{2}} \]

where these terms represent the tension due to a trapped axial field, the self inductance in the absence of walls and a term representing the image effect of the walls. To treat this we obtain the Fourier transform of the curve \( \mathbf{r}(s) \) and expand \( V \), keeping the dominant terms. We write

\[
y(s) = \Sigma y_k \sin(kx), \quad z(s) = \Sigma z_k \cos(kx)\]

Then

\[
V(y_k) = -\Sigma_k [(aK_0(kr_0) - b)k^2 - c] (y_k^2 + z_k^2) + d\Sigma_k y_k y_{k'}^2 + z_k z_{k'}^2 + \ldots, \quad (21)
\]

where \( r_0 \) is the discharge radius, which must be finite to avoid singularity in the self inductance, and \( K_0 \) is a modified Bessel function of the second kind.

The functional distribution function takes the form

\[
P(y)dy = \exp(\Sigma_k F(k)(y_k^2 + z_k^2)) - d\Sigma_k y_k y_{k'}^2 + z_k z_{k'}^2 + \ldots) \Pi dy_k dy_{k'} \quad (22)
\]

The possibility of carrying out this analysis depends on the fact that, for large enough \( k \), \( b > aK_0(kr_0) \) while, for small enough \( k \) the non-linear wall terms represented by \( d \) will eventually dominate. Because of these effects, the potential, \( V \), is infinite and positive for all except a limited range of \( k \), implying the existence of an equilibrium helix for this field configuration which, incidentally, is rather artificial, in that in the model there is a trapped axial field but no external axial field. In order to discover the probability of finding the channel centre at fixed points \( Y(\mathbf{x}), Z(\mathbf{x}) \) we must evaluate the functional integrals

\[
\int \delta(Y-y) \delta(Z-z) P(y,z) \, dy \, dz \quad (23)
\]

but the evaluation of these is not straightforward, and the procedure adopted is to linearize the problem at the \( P \) stage. We introduce an approximation, \( V = V_0 \), and first insist that \( V_0 \) be chosen as a function of \( F \), the function whose mean is required, so that the correct value is given when averaged over \( F(V_0) \); i.e.,

\[
\langle J(0) \rangle = \int J \exp(-V_0) \, dy = \langle J \rangle = \int J \exp(-V) \, dy \exp(-V_0). \quad (24)
\]

If we write \( V = V_0 + (V - V_0) = V_0 + V' \), expand in powers of \( V' \) and retain only the linear term, we obtain an integral equation for \( V_0 \)

\[
\langle J(0) \rangle \langle V(0) \rangle = \langle JV(0) \rangle. \quad (25)
\]

When this integral equation is solved, using a quadratic form for \( V_0 \), it is found that the solution depends on the term in \( d \). If the sum \( y_k^2 \Sigma y_k^2 \Sigma y_k^2 \) greatly exceeds \( y_k^4 \), which is likely if many modes are excited, we can replace \( \Sigma y_k^2 \) by its mean, and obtain an expression for the radial distribution function in the form

\[
P(r) \sim \exp(-K r^2) \, R^2 \quad (26)
\]

where

\[
K = (8d/[N \chi T])(1 - (2N \chi T)^{-1}(8d/[N \chi T]) \int F(k) \, dk
\]

\[
= 2\langle \langle E \rangle^2 \rangle^{-1}(1 - \langle \langle E \rangle^2 \rangle^{-1} \times [\langle (I^2 \ln K_0(\rho) - H^2 r_0^2) \rangle R^2 + \langle I^2 \langle E \rangle^{-2} \rangle].
\]

Numerical Computation

To study the properties of the wriggling discharge, a computational experiment has been carried out. The force acting on the discharge over much of the range in \( k \) is proportional to the curvature and directed along the principal normal, i.e.,

\[
F(\mathbf{r}(s)) = C I^2 \partial^2 \mathbf{r} / \partial k^2 + F_{\text{wall}} \quad (27)
\]

The quantity, \( C \), in here is not actually a constant, but if an axial field is included it becomes negative for large curvatures. We can represent the effects of this by suppressing large curvatures when they occur. If we ask for the motion of an element of the current channel at \( \mathbf{r}(s, t) \) we may write

\[
\rho_0 \frac{d^2 r(s)}{dt^2} = C I^2 \frac{d^2 \mathbf{r}}{dt^2} + F_{\text{wall}}. \quad (28)
\]

The wall force can be written in a simple form

\[
\frac{1}{2} \left( \frac{1}{R - \rho} - \frac{1}{R + \rho} \right), \quad \text{where} \quad \rho = (0, y, z), \quad \text{per unit displacement} \quad \text{parallel to the wall, directed along the projection of the normal from the wall on to the plane normal to the current channel. These equations are extremely difficult to treat analytically, but are quite simple to handle numerically on a computer with a store large enough to carry the information needed to represent the position and velocity of the curve with sufficient accuracy to determine the curvature.}
\]

The calculation is complicated by the presence of a computational instability which in some circumstances can become completely dominant and swamp even the short wave length instability of the analysis. It can be recognized since it grows with the number of steps in the computation, while the physical instabilities grow with the lapsed time. Hence, by altering the magnitude of the computational step, it can be modified. To eliminate this mathematical singularity, and the physical singularities at high curvatures, a smoothing formula has been used to attenuate the
highest harmonics developing in \( r(s) \). When the smoothing formula is used, and a coarse enough integration procedure is used, a region is found where the results do not depend on the computational step, nor on the nature of the smoothing formula, and it seems reasonable that in this region the numerical solution does indeed represent a solution of the approximate equation of motion.

Using this procedure, the motion of a current channel of length \( L \) in a cylinder of radius \( \frac{L}{10} \), represented by either 300 or 100 points, has been followed for a time of between 5 and 15 times the characteristic instability growth time. The initial conditions were obtained by selecting small random numbers to represent amplitude and phase of the first four Fourier components of the initial velocity and position, and the smoothing formula was used to attenuate all harmonics above the 15th. Under these conditions, the curve did not develop sharp kinks, but remained fairly smooth. The results were obtained by (a) calculating the radial distribution function for the channel at each 10th time step, and (b) displaying the coordinates of the 300 or 100 points on a screen at each step, photographing these, and preparing a film showing the calculated motion. Some results for a typical case are shown in Figures 1 and 2.

The distribution function had not become completely steady by the end of the calculation, but had a peak, at about \( R/5 \) from the centre, which oscillated back and forth. This peak distribution function does not resemble the Gaussian form which was all we succeeded in obtaining from the analytical theory. This is because the Gaussian form appeared only if the energy was shared almost equally among many modes, while in our computation we probably have not yet produced a wide enough distribution of the energy.

**THE TRANSPORT COEFFICIENTS**

If the discharge is turbulent, the local fields in the plasma may be very different from those in thermal equilibrium. Since it is these local fields that determine the straggling of particle trajectories and hence transport properties, we should like to develop a method of discussing transport processes which displays the role of these microfields, and hence which can be generalized to cases other than thermal equilibrium. In approaching this problem, we have considered one particularly simple problem, the diffusion of electrons through a strong magnetic field, but have treated it in a way that displays the dependence on the microfield and handles the long-range component of the forces with particular care.

The magnetic field is considered so strong, the temperature so high and the density so low that the Larmor radius, \( \frac{\langle p \rangle}{\omega_B} \), is much less than the Debye length, \( \frac{\langle p \rangle}{\omega_p} \); i.e., that the gyromagnetic frequency, \( \omega_B/mc \), is much greater than the plasma frequency, \( \omega_p = (4\pi^2n/e^2)^{1/2} \), or \( nmc^2 < H^2/4\pi \). The great simplification introduced by this is that, to a reasonable approximation, the particles move adiabatically in the fluctuating microfields, i.e., \( v = (E \times H)/cH \). In order to obtain the diffusion, we need the mean value of \( d(28x^2)/dt \), the mean square straggle of the particle position in unit time. We obtain this by writing

\[
\dot{x} = v(x, t) = -cH \times E(x, t)/H^2 \tag{29}
\]

which can be written

\[
\dot{x} = -c/\Sigma H_{\mu\nu} \times H_{\nu\mu} \exp[c (\omega t - \mathbf{k} \cdot \mathbf{x})]. \tag{30}
\]

We make the approximation that in the exponential we can write \( x(t) = x_0 + v_t t \) where \( v_t \) is the component...
THEORETICAL PROBLEMS SUGGESTED BY ZETA

of particle motion along the magnetic field; then

$$\delta \delta = \frac{e}{H^2} \sum_{E_k, \omega} H \times E_{k, \omega} \frac{\exp[(i(\omega - \omega_0) t - i k \cdot x_0)]}{t(\omega - \omega_0)}$$  \hspace{1cm} (31)

Assuming that $E_{k, \omega}$ is parallel to $k$, i.e., that only Coulomb fields are important, we obtain, on squaring (31) and taking an ensemble average of the electric field distribution,

$$\langle \delta \delta \rangle = \frac{e^2}{H^2} \sum_{E_k, \omega} (H \times k)^2 \langle E_{k, \omega} \rangle \delta(\omega - \omega \cdot \nu)$$ \hspace{1cm} (32)

for large $t$. This reduces the problem to a calculation of $\langle E_{k, \omega} \rangle$, the electric field spectrum. Until this stage, the analysis does not depend on the particle distribution and is applicable to turbulent as well as to equilibrium states. However, we do not have any method for calculating high frequency components of $\langle E_{k, \omega} \rangle$ for a turbulent system, and at this stage have considered a system in thermal equilibrium. For this case we can invoke a generalized Nyquist theorem and write the fluctuating field spectrum as $\langle E_{k, \omega} \rangle = \kappa T R(\omega, k)$, where $R(\omega, k)$ is the admittance of the plasma, which in turn is given in terms of the complex dielectric constant $\varepsilon(\omega, k) + i \sigma(\omega, k)$ as

$$R(\omega, k) = \frac{4 \pi \sigma(\omega, k)}{\omega \varepsilon + \sigma}$$ \hspace{1cm} (33)

Now by calculating the response of the plasma (composed of non-interacting particles) to an applied field, we can obtain an expression for the dielectric constant. If the wave vector, $k$, is small and the damping is small, the field spectrum is dominated by peaks arising from the vanishing of $\varepsilon(\omega, k)$ and these peaks represent the contribution to the spectrum due to the collective motions of the plasma, i.e., the plasma oscillations. If the collision damping goes to zero, the limiting process must be carried out rather carefully at these resonances; a natural width is found, a result of the damping produced by those particles travelling along with the plasma waves. This represents the inverse of the generation of a wake of plasma oscillations by a simple fast particle, as described by Bohm and Pines. At very large wave numbers, this damping term is very large, and the fluctuating field is determined by the variation in particle density due to their free motion, hence reduces to the Holtzmark field, describing diffusion due to ordinary collision processes.

The value of the dielectric constant in the magnetic field, as calculated by Gertsenshtein, may be used to calculate the field spectrum, and this used to determine the diffusion coefficient. Preliminary calculations seem to show that the diffusion coefficient has the same form and is of approximately the same size as that obtained by standard methods in the limit of strong magnetic fields.

REFERENCES

Stable Plasma Column in a Longitudinal Magnetic Field

By I. N. Golovin, D. P. Ivanov, V. D. Kirillov, D. P. Petrov, K. A. Razumova and N. A. Yavlinsky*

The results of the experimental study of a pulse discharge between two electrodes in a straight tube in the presence of a longitudinal magnetic field are presented in this paper. The discharge starts approximately along the tube axis and gradually expands. When the strengths of the longitudinal magnetic field and the magnetic field of the current at the plasma boundary are related in certain definite ways the discharge column remains stable and does not reach the tube walls. For strong currents, when the field of the current is comparable to the longitudinal field, the plasma column fills the whole tube cross section and becomes unstable.

To obtain a high temperature in a discharge without electrodes it is essential in the first place to ensure the macroscopic stability of the plasma column. In the experiments described, therefore, we have studied pulsed deuterium discharges in a straight tube, with electrodes in the presence of a longitudinal magnetic field.

The maximum current varied from 3000 to 300,000 amp. The duration of the first half-cycle of the current ranged from 300 to 1800 μsec. The magnetic field in the same direction as the current could be adjusted from 0 to 27,000 gauss. The initial deuterium pressure in the experiments varied from 0.005 to 5 mm Hg.

The amplitude of the current through the gas and the longitudinal magnetic field could be varied independently in different experiments. Thus, for a wide range of absolute values of the current through the gas, the ratio of the longitudinal magnetic field $H_0$ to the magnetic field of the current $H_a$ could be varied from 1 to 10.

**EXPERIMENTAL EQUIPMENT**

Investigations of the stable plasma column were carried out with several units very similar in construction, size and characteristics of the feeding circuits. A schematic view of such a unit is shown in Fig. 1.

The discharge tube is a porcelain or glass tube of 18 to 23 cm diameter and 80 cm high. The faces of the tube are covered by porcelain plates on which copper or stainless steel electrodes are mounted. In the experiments described the following electrodes were used: flat electrodes 9 cm and 17 cm in diameter, and hemispherical electrodes 4 cm in diameter. The distance between the electrodes in all the experiments was ~70 cm, unless otherwise stated.

The discharge tube was surrounded by a 5 mm thick coaxial stainless steel return conductor whose stabilizing effect should be negligible. The longitudinal magnetic field in the discharge tube was excited by a coil through which a $C_2$ capacitor bank of 10,000

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* Original language: Russian.
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Figure 2. Oscillograms of the voltage between the electrodes and the discharge current for systems with various durations of the first half-period with a large $H_0/H_0$ ratio: $H_0 = 15,000$ gauss, $i_m = 20$ ka, $p = 0.2$ mm Hg

to 100,000 $\mu$F was discharged. The pulse current in the tube was produced by the discharge of the $C_1$ capacitor bank (800 to 6800 $\mu$F) through a spherical spark gap.

The $C_1$ and $C_2$ banks were made up of pulse capacitors of the IM–3–100 type each with a 100 $\mu$F capacity and a charge voltage of 3 kV.† The amplitude of the discharge current was adjusted from 3 to 300 ka and the duration of the first current half-wave from 300 to 1800 $\mu$sec by varying the capacitor bank voltage and by the insertion of an additional inductance $L$ into the circuit.

The $C_1$ and $C_2$ banks worked jointly in the following way. When the current in the coil reached a maximum, the synchronization circuit triggered the spark gap in the $C_1$ circuit, producing a discharge through the gas. The discharge circuit oscillation period was chosen to be much less than that of the circuit producing the longitudinal magnetic field. Thus, the discharge occurred in an almost constant magnetic field. The strength of the longitudinal magnetic field could be raised to 27,000 gauss.

The discharge was investigated in deuterium (at pressures $p$ from 0.005 to 5 mm Hg) and in argon. The tube was filled with deuterium through a palladium filter ensuring the appropriate purity of the gas. After the discharge the tube was evacuated to a pressure of $1-2 \times 10^{-6}$ mm Hg by an oil diffusion pump with a trap cooled with liquid nitrogen.

**PROCEDURE AND RESULTS**

For each of the experimental conditions chosen the following parameters were measured: (a) the total discharge current, (b) the voltage between the electrodes,
Figure 4. Duration of the "plateau" as a function of the longitudinal magnetic field intensity;  \( p = 0.2 \) mm Hg,  \( I_m = 185 \) kA  

\( \text{Magnetic field intensity, kiloersteds} \)

(c) the current in the concentric zones of the tube cross section and (d) the mean value of the longitudinal magnetic field in the tube. In addition, the plasma column was photographed by a high-speed camera, the discharge spectrum was also photographed and the time variation of the intensity of separate spectral lines was recorded.

The discharge current was measured by a Rogovsky belt, the voltage from it being integrated by an  \( RC \) circuit. A low-resistance divider, whose self-inductance was much less than its effective resistance, was connected between the electrodes, in parallel with the discharge column. In order to record the current, the voltage, and other characteristics of the discharge, a double-beam pulse oscilloscope of OK-17M type was employed.

Oscillograms of the voltage between the electrodes and of the discharge current are shown in Fig. 2. The discharge current pulse is similar in form to a sine curve. Its duration and magnitude vary scarcely at all with change in  \( H_0 \) and  \( p \). This is explained by the fact that the total impedance of the discharge circuit is much greater than the discharge resistance. The oscillograms of the voltage show that the inductive component of the discharge impedance is considerably less than its resistance.

Figure 3 shows that the voltage between the electrodes has two typical stages. At first it is approximately constant; we call this the "plateau" stage. Then the voltage rises and a very transient voltage is observed. The time prior to the beginning of the
Figure 7. Ratio of the current density in a given zone to the mean current density in the tube cross section; \( p = 0.2 \) mm Hg, \( l_m = 18 \) ka, at (a) \( H_0 = 5000 \) gauss and (b) \( H_0 = 15,000 \) gauss.

Figure 8. Ratio of the current density in a given zone to the mean current density; \( p = 0.2 \) mm Hg, \( l_m = 185 \) ka, \( H_0 = 8000 \) gauss.

Figure 9. Determination of the current distribution over the tube cross section by current densities measured in separate zones.

Figure 10. Time variation of plasma column "current" radius. Stainless steel electrodes ~ 9 cm in diameter; \( p = 0.2 \) mm Hg, \( l_m = 18 \) ka, duration of the first half-period ~ 1600 \( \mu \)sec: Curve 1, \( H_0 = 5000 \) gauss; curve 2, \( H_0 = 15,000 \) gauss.

The second stage ("plateau" duration) is inversely proportional to the rate of the current increase. This time rises as the longitudinal magnetic field increases (Fig. 4). At rather large values of \( H_0 \) the "plateau" is observed all the time the current flows through the gas (Fig. 2).

The voltage between the electrodes at the moment when the discharge current reaches its maximum increases linearly with the amplitude of the current and depends little on the initial pressure \( \phi \) and the magnetic field \( H_0 \). The size of the electrodes and the material of which they are made influence the magnitude of voltage between them.

Two procedures were used for studying the configuration of the plasma column: (1) the transverse distribution of the current in the plasma column was...
Figure 11(a). Time variation of plasma column "current" radius. Solid line, copper electrodes of 4 cm diameter, dashed line, stainless steel electrodes of 17 cm diameter; duration of the first half-period, 300 μsec measured with Rogovsky concentric belts, and (2) the plasma column was photographed by a high-speed camera. The Rogovsky belts placed in quartz tubes 8 mm in diameter were inserted inside the tube. The equal-sensitivity of the belts made it possible to balance one belt against another in order to measure directly the current in each of the three concentric zones between the belts. The arrangement and the sizes of the belts are given in Fig. 5.

Figure 6 presents oscillograms of the current in the central and near-wall tube zone. For the same zone these oscillograms have the same current and time scales. At the initial value of magnetic field ($H_0 = 27,000$ gauss) the current in the near-wall zone is very small; with the decrease in $H_0$ the current in the near-wall zone increases, but in the central part it drops. Figures 7 and 8 present the results of the analysis of such oscillograms.

When $H_0$ has the value 15,000 gauss, the density of the current in the near-wall zone is negligible, while the current density in the central zone is 15 times larger than the mean current density at the beginning of the process, and 7 or 8 times larger at the end of the process. When the intensity of the magnetic field is equal to 5000 gauss, the current density distribution becomes more regular (Fig. 7). With a still smaller ratio of $H_0$ to $H_f$, when the "plateau" of the voltage is sharply pronounced, the current distribution at the moment corresponding to the end of the "plateau" changes appreciably: the current in the central zone drops; at the extreme it increases sharply and there appear strong oscillations corresponding to the redistribution of the current over the tube cross section (Fig. 8).

Thus, the measurements of the current density distribution over the tube cross section have shown that under definite conditions a discharge may be realized in which the current through the near-wall zone is negligible ($j < 1$ amp/cm$^2$), i.e., the plasma column interacts weakly with the tube walls and exists as long as the current passes through the gas.

For such discharges the imaginary radius $a$ of the current column was plotted as a function of time. This "current" radius was defined as follows: a quadratic dependence of current density on the radius $j = A(1 - B r^2)$ was taken, and the constants $A$ and $B$ were calculated assuming that the entire current inside the internal and central zones coincided with the measured values. The current in the near-wall zone was neglected (Fig. 9).

Figures 10 and 11(a) show the dependence of "current" radius on time. The oscillograms corresponding to the first half-period of the current (300 or 1600 μsec) were treated. For all the operating conditions some increase in the "current" radius at the beginning of the process is typical. Then the radius remains almost invariable until the current vanishes.

The change of the luminous plasma column width with time was photographed simultaneously with the measuring of the current distribution. For this purpose a narrow cross slot was made in the middle of the coil producing the longitudinal magnetic field (Fig. 1). Such photographs for discharges with the first half-period equal to 300 or 1600 μsec were treated. For all the operating conditions some increase in the "current" radius at the beginning of the process is typical. Then the radius remains almost invariable until the current vanishes.

In the streak photographs of the plasma column luminous sinusoidal stripes attract the attention. The amplitude of oscillation of the stripes increases with the rise of the current in the discharge and decreases with the rise of the longitudinal magnetic field. During the first half-period their frequency decreases, which is much more visible in the photographs of a discharge in argon (two lower photographs on Fig. 11(b)). The explanation of these regular motions requires further investigation. However, it is possible to state that they are not associated with the distortion of the basic part of the current through the gas.
From the photographs of the discharge, it is possible to plot the dependence of the plasma column "luminous" radius on the longitudinal magnetic field and on the pressure.

The dependence of the plasma column "current" and "luminous" radii on the longitudinal magnetic field (Fig. 12) shows that the "luminous" radius exceeds almost twice the "current" radius (according to the above-described procedure of analysis), but the laws of their variation in time are similar. The curves plotted for various electrodes differ slightly. For electrodes with a 17 cm diameter there has been observed an increase of the "current" radius with the rise of the magnetic field. It should be noticed that the "current" radius for electrodes with a diameter of 4 cm cannot be less than 2 cm on account of the large size of the central measuring belt.

The radius of the plasma column increases as the longitudinal magnetic field decreases and as the discharge current or the distance between the electrodes increases. The curves in Fig. 13 show that the radius of the plasma column depends slightly on the pressure. The plasma column at the pressure of 5 mm Hg occupies the whole tube, and is unstable. The oscillograms show in this case a very transient voltage. It should be noticed that for such a discharge \( \omega_i^2 \tau_i^3 < 1 \), where \( \omega_i \) is the Larmor ion frequency and \( \tau_i \) is the ion–electron collision time.

The changes caused by the discharge in the distribution of the longitudinal magnetic field over the tube
the coil producing the longitudinal magnetic field were balanced against the measuring turns. The sensitivity of the compensating turns was chosen so that in the absence of a discharge, the signal from unit magnetic field linking both loops should be as small as possible. Since the external resistance of the magnetic coil circuit is small in comparison with the inductive impedance of the coil, the magnetic flux in it remains constant during the discharge, and hence the compensating turns cannot record the magnetic field alterations inside the discharge. The emf induced in the turns was integrated by the RC circuit.

For all discharges there was observed an increase in the longitudinal magnetic flux within the discharge.

The oscillograms of the magnetic flux alterations are presented in Fig. 14. The analysis of such oscillograms (Fig. 15) shows that the gas pressure in this case is considerably lower than the magnetic pressure.

In discharges separated from the walls, when \( H_0 \gg H_\phi \), the increase in the longitudinal magnetic field within the discharge is small in comparison with its nonexcited value. Therefore, the measurements under these conditions are only of a qualitative nature. They show that the equilibrium of the magnetic tensions is considerably disturbed only at small currents when even at low temperatures \((10-20 \text{ ev})\) the gas pressure becomes comparable with the magnetic field pressure.

A study of the nature of the spectrum in the visible region was carried out for various discharges throughout the entire current pulse; the time variation of separate lines of this spectrum was also studied.

The discharge spectrum was photographed by a ISP-51 spectrograph accommodating glass optics. The time variation of separate spectral lines was investigated by a UM-2 monochromator provided with a FEU-19 photomultiplier. To protect the photomultiplier from the dispersed magnetic field the monochromator with the photomultiplier is placed in a metal encasement. The signal received from the photomultiplier was amplified, and recorded by an oscilloscope.

The radiation was observed through a small hole through the discharge tube so that the plane of the loops was perpendicular to the tube axis. In order to increase the accuracy of measurement a method of compensation was employed. Compensating turns wound over the coil producing the longitudinal magnetic field were balanced against the measuring turns. The sensitivity of the compensating turns was chosen so that in the absence of a discharge, the signal from unit magnetic field linking both loops should be as small as possible. Since the external resistance of the magnetic coil circuit is small in comparison with the inductive impedance of the coil, the magnetic flux in it remains constant during the discharge, and hence the compensating turns cannot record the magnetic field alterations inside the discharge. The emf induced in the turns was integrated by the RC circuit.

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Figure 14. Examples of current oscillograms (upper trace) and measurement of magnetic flux within a discharge (lower trace). The upper oscillogram corresponds to a system with $p = 0.2 \text{ mm Hg}$, $I_m = 90 \text{ ka}$, $H_0 = 4500 \text{ gauss}$, and the lower with $p = 1 \text{ mm Hg}$, $I_m = 180 \text{ ka}$, $H_0 = 4500 \text{ gauss}$.

Figure 15. Time variation of the increment of the magnetic flux within the discharge measured experimentally and the value $\frac{n^2 H^2}{2 \pi}$ calculated for conditions corresponding to an increment of the magnetic flux that is at magnetic pressure balance. Conditions: $p = 0.2 \text{ mm Hg}$, $I_m = 1800 \text{ ka}$, $H_0 = 4500 \text{ gauss}$.

Figure 16. Oscillograms of the time variation of the $D_n$ intensity and the total discharge current for $p = 0.01 \text{ mm Hg}$, $I_m = 45 \text{ ka}$ and various magnetic fields. Oscillograms (c), (d) and (e) are obtained with greater amplification on the photomultiplier than (a) and (b). The field strengths $H_0$ are: (a) 0, (b) 5, (c) 5, (d) 10, (e) 15 kilogauss.

Figure 16 shows the oscillograms of the photomultiplier signals induced by the radiation of the Balmer line $D_n$ from the discharge. These oscillograms show a decrease in the intensity of the Balmer lines after the superposition of the magnetic field. This decrease is sharper for a larger field.

The oscillograms in Fig. 17 convincingly confirm the existence of a stable discharge. They express the character of the time variation of the radiation intensity of the silicon lines (4128 $\AA$ and 4130 $\AA$), and...
evidence a significant relaxation of the interaction of the discharge plasma with the tube walls when the longitudinal magnetic field is sufficiently large and the radius of the plasma column is smaller than the interior radius of the discharge tube. All the photomultiplier signal oscillograms presented are synchronized with the discharge current trace.

DISCUSSION OF RESULTS

The measurements of the current density distribution over the cross section of the discharge column and the streak photographs show that under experimental conditions the discharge starts near the tube axis. This fact is observed for all the values of the longitudinal magnetic field and very often for $H_0 = 0$. This is possibly associated with the configuration of the electric fields before the breakdown. Since the rate of the current increase is small and the plasma resistance at these discharge stages is high ($R \gg \omega L$), the discharge develops in the central region where the conductivity is much higher. The original discharge channel expands but does not reach the walls and remains stable when $H_0$ is large enough. Investigation of the dependence of the radius of the plasma column on the longitudinal magnetic field was described in Ref. 1.

The radius of the stable plasma column as shown in Ref. 2 must satisfy the condition

$$a \geq \frac{\lambda}{2\pi H_0}$$  

where $\lambda$ is the wavelength of excited instability, $H_0$ is the intensity of the field of the current at the discharge column boundary and $H_0$ is the intensity of the longitudinal magnetic field. Numerical comparison of the radii obtained experimentally and those calculated by means of the formula

$$a = \frac{l H_0}{\pi H_0}$$

shows that Eq. (1) is correct if $\lambda$ is taken as $2l$, where $l$ is the electrode separation—70 or 135 cm.

The data presented (Fig. 12) show that the decrease in the intensity of the longitudinal magnetic field if $H_0 < 15,000$ gauss leads only to an increase in the plasma column radius but not to instability. One of the possible explanations of such a discharge behaviour may lie in the following mechanism: after the start in the narrow discharge channel, the current density increases with time and may turn out to be considerably larger than the current density allowed by the stability conditions (1) for the discharge column radius. In this case instability is developed in the narrow column and its wavelengths will be the smaller, the poorer the fulfilment of condition (1). Macroscopic motions of the unstable column lead to a partial ionization of the neutral gas enveloping the plasma column and, consequently, to an expansion of the conducting region through which the current flows. Such a process will take place until the current density becomes stable, equal to $j$, the value $H_0$ on the boundary drops and conditions are created for the existence of a stable plasma column.

We do not have at our disposal sufficient data to evaluate the time of existence of the stable plasma column. In comparing the data presented in Fig. 12, one can see that the stable plasma column in experiments with a duration of the first half-cycle of 1600 $\mu$sec has a larger radius than in experiments with a duration of the first half-cycle of 300 $\mu$sec, although all the other conditions are the same. It may be assumed that diffusion phenomena become essential when the period is increased. However, it may be...
possible that the diffusion phenomena affect the process during a shorter period of time, and the stable column radius may be defined by the ratio of the diffusion rate to the charged particle drift.

In experiments with narrow electrodes, when the plasma column radius exceeds the radius of the electrode, the voltage across the gap increases and becomes transient. But this is not associated with instability of the discharge column, as no chaotic redistribution of the current density over the cross section is observed. This may be associated with the fact that in this case the electrons near the electrodes have to travel across the magnetic field.

The conductivity of the plasma column in the magnetic field is anisotropic. In such a plasma the current flows along a spiral line intensifying the magnetic field within the plasma column.

Calculations show that the increment of magnetic flux caused by the anisotropy of conductivity,

$$\Delta \phi = \frac{\pi a^2 H_s^2}{4H_0},$$

can balance only half of the pressure produced by the field $H_s$.

In experiments with currents of 100 ka or more there is always observed an equality between the magnetic pressures of the field of the current and of the increased longitudinal magnetic field within the plasma column.

If we assume that the plasma column expands, a "diamagnetic" effect, i.e., a magnetic field repulsion effect, must be observed. The expansion of the ionization region resulting from the mechanism described above does not require a repulsion of the field lines. On the contrary, the expansion of the ionization region might take place simultaneously with the constriction of the plasma column and with an increase of the magnetic field intensity within it, in order to satisfy the requirements of equality of the magnetic pressures.

Measurements of the voltage between the electrodes and of the current through the gas provide all the data for the evaluation of the plasma conductivity and the degree of ionization.

By excluding the inductance component from the measured value of the voltage between the electrodes, it is possible to determine the value of $E$, the intensity of the electric field in the plasma. The evaluation of the plasma conductivity in stable regimes shows that, at the moment of maximum current, in the central zone it reaches the value of $3 \times 10^{14}$ cgse, while in the internal zone it reaches only $1 \times 10^{14}$ cgse. At small values of $H_0/H_s$ the conductivity increases only until the discharge column touches the tube walls. After this moment the conductivity in the central zone decreases to values close to the mean value over the discharge cross section.

It follows from the evaluation of the energy transferred to the electrodes that the plasma temperature cannot exceed 50 ev. Therefore, if the conductivity is of the order of $5 \times 10^{14}$ cgse, the degree of gas ionization must exceed 20 per cent. This also agrees with the measurements of the $D\beta$ line intensity with time, which decreased with increase in the magnetic field, and had a pronounced maximum in the zero region of the current (Fig. 16).

**CONCLUSIONS**

The following conclusions may be made, based on the experimental material:

1. In straight gas discharge tubes when the distance between the electrodes exceeds the tube diameter appreciably and also in the absence of a coaxial damping conductor, the discharge column in a longitudinal magnetic field can be stable for more than a thousand microseconds.

2. After the breakdown, the region occupied by the discharge expands and the rate of this expansion increases with increase in the derivative of the current and decreases with increase in the intensity of the longitudinal magnetic field.

3. At large $H_0/H_s$ ratios, when the relation $H_0/H_s > \lambda/(2\pi a)$ is satisfied, the discharge column does not reach the walls and remains stable while the current flows.

4. If the condition in item 3 is not satisfied, the discharge fills the whole tube volume and the stability fails.

5. The ionization in the stable plasma column exceeds 20 per cent.

6. The longitudinal field within the plasma column increases up to the value sufficient to ensure equality in the magnetic pressures (neglecting the insignificant pressure of the plasma heated to 10–100 ev).

**ACKNOWLEDGEMENTS**

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**REFERENCES**


Investigations of the Stability and Heating of Plasmas in Toroidal Chambers

By G. G. Dolgov-Saveliev, D. P. Ivanov, V. S. Mukhovatov, K. A. Razumova, V. S. Strelkov, M. N. Shepelyev and N. A. Yavlinsky*

Electrodeless discharges can be employed to produce thermonuclear reactions providing that one is able to heat a long-lived stable plasma column to the required temperature.

Some possible stability conditions have been derived theoretically. In addition experimental studies of discharges in chambers containing electrodes and also of electrodeless discharges in toroidal chambers have been performed with the purpose of solving these problems.

It seems quite natural to attempt to heat the plasma by Joule heating as at first glance this possibility appears to be simplest and easiest, at any rate during the first stage of formation of a plasma possessing a high concentration of charged particles.

Some results of investigations of electrodeless discharges in deuterium are reported in the present paper. The experiments were carried out with tubes of various dimensions under conditions involving a great variety of discharge parameters.

EXPERIMENTAL ARRANGEMENTS

The experiments described in the present paper were performed with toroidal chambers of specifications given in Table 1. In arrangements 1, 4 and 5 the discharge tubes were made of a dielectric or stainless steel and surrounded by a copper case which damped the long-wave oscillations of the plasma column. In arrangements 2 and 3 the copper walls served simultaneously as a vacuum chamber and damping sheath. Alternating as well as quasi-constant magnetic fields could be used in chambers 1, 3, 4 and 5 inasmuch as along with transverse cuts each of these chambers had a longitudinal cut along the equator of the torus. In chamber 2 only a constant stabilizing magnetic field could be employed (chamber without a longitudinal cut).

The discharge was produced with the aid of a special coil encircling the torus. This coil will be called the exciting coil. In order to be able to change the initial values of the electric field \( E \) and the initial rate of increase of the current \( I \) over a broad range independently we did not employ steel cores such as those used in our earlier work. For the same purpose means were provided for connecting the capacitors in parallel and in series in various combinations. The voltage on the capacitors was 5 kv and their capacity was 150 \( \mu F \).

A photograph of one of the experimental arrangements is shown in Fig. 1. The basic electrical circuit shown in Fig. 2 was used for all chambers excluding chamber 2. In experiments with this tube condenser bank \( C_3 \) and the auxiliary equipment connected to it were replaced by a constant current source.

In the experiments described here as well as in the earlier work emission of material from the walls was of great importance. Streaming of tube wall vapor into the discharge led to large losses of energy which was expended in ionization of the vapor, in charge exchange and in other elementary processes. It has been found in addition that in chambers with dielectric walls static charges appear during the first stages of the discharge and these charges increase the loss of charged particles at the walls. The combination of these factors greatly impeded the interpretation of the results. For this reason we employed chambers with metallic walls instead of discharge chambers made of dielectrics (glass, quartz, porcelain). Intense arcs were observed in electrolytic copper chambers with two insulating sections. A result of this was that the conducting sheath screened the electric field pro-

<table>
<thead>
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<th>No.</th>
<th>Dimensions in cm</th>
<th>Material of chamber wall</th>
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<tbody>
<tr>
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<td>15</td>
<td>Quartz</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>Copper with two insulation sections</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
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</tr>
<tr>
<td>4</td>
<td>50</td>
<td>Chromium-nickel steel with two insulation sections</td>
</tr>
<tr>
<td>5</td>
<td>62.5</td>
<td>Chromium-nickel steel (without sections)</td>
</tr>
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Original language: Russian.
* Academy of Sciences of the USSR, Moscow.
† Developed under the guidance of M. M. Morozov and S. K. Medvedev.
producing the discharge current, and copper vapor and vapor of the insulating materials contaminated the deuterium plasma.

Chambers having insulating sections and made of heat-resistant chromium–nickel steel with a high electric resistivity were found to be more convenient. However, spectral measurements always indicated the presence of impurities attributable to the material of the insulating sections as well as to the material of the chamber walls.

In one of the latest types of experimental assemblies a closed chamber (5 in Table 1) made of chromium–nickel steel was employed. The wall thickness of this chamber was 0.22 cm and its total electrical conductivity was about 0.1 ohm. The thin-walled tube was surrounded by a 2 cm thick vacuum copper case. The inner surface of this sheath was separated by 1.5 cm from the inner tube. A schematic diagram showing the construction of the apparatus is presented in Fig. 3. In the gas discharge tube used in this arrangement a uniform electric field existed along the discharge axis during the process; moreover the appearance of arcs was excluded and the vacuum conditions were better. The absence of qualitative measurements does not permit one as yet to estimate the degree of contamination of the deuterium plasma by the wall materials.

METHOD OF MEASUREMENT

The following measurements were performed in the present experiments.

1. Voltage measurements.—The voltage per turn induced in a loop located in the equatorial plane of the torus or the voltage from a divider connecting the parts of the conducting sheath of the chamber was recorded with the aid of an oscillograph.

2. Discharge current measurements.—The emf induced in a Rogovsky belt by the magnetic field of the current was fed to an integrator consisting of a resistance $R$ and capacitor $C$. In chambers with non-conducting walls the belt encompassed the torus tube. In the metallic wall chambers the discharge current was measured by two belts, an outer one which encircled the torus tube and an inner one located within the chamber. In experiments with the closed metallic chamber, the current in the gas was determined by subtraction of the current in the metal from the total current. The former was recalculated without account of the inductive resistance of the chamber and was assumed to be proportional to the voltage per turn corresponding to the total current. In some cases the subtraction was performed with the help of a special circuit. The error in determining the current in the gas was not large inasmuch as in the experiments described here the ohmic resistance of the metallic chamber was much larger than the inductive resistance.

3. Magnetic field measurements.—The magnetic field of the current $H_T$ and the longitudinal magnetic field $H_B$ were measured with coils of $0.4 \times 0.6$ cm dimensions. The coils were mounted in quartz or porcelain tubes of about 0.6 cm external diameter. After integration by the $RC$ network the emf of the coil was recorded on an oscillograph.

4. Spectrographic observations.—The time variation of the intensity of the spectral lines of the discharge was recorded with an ISP 51 spectrograph with glass optics. The photographic film on which the time variation of the spectral line intensities was recorded was mounted on a revolving drum. The integral discharge spectrum in the visible and ultraviolet regions was measured with a quartz spectrograph.

5. Plasma probing with ultra-high-frequency radio waves.—To determine the electron concentration in the discharge the plasma was probed with ultra-high-frequency radio waves. Pulsed generators producing waves with $\lambda = 0.4$ cm and $\lambda = 0.23$ cm were employed for this purpose. The emitting and receiving horn were arranged as shown in Fig. 4. The ultra-high-frequency oscillations were modulated by a low frequency (30 kc/sec) and shaped into packets of 3-msec duration. After detection, the signals accepted by the horn were fed to the oscillograph. The cross sections...
Figure 3. Schematic diagram of experimental arrangement: a, coil for excitation of a vortex electric field; b, copper shield for reduction of scattered fields; c, longitudinal magnetic field coil; d, copper stabilizing coil; e, thin-walled vacuum chamber made of a high-resistance alloy; f and g, nipples for evacuation of the chamber and forechamber; h, observation window (for photography, ultra-high-frequency probing, spectral measurements, adjustment of measuring belts and magnetic probes).

of the waveguide channel, and the emitting and receiving horns, were made circular in order to ensure transmission of the oscillations through the plasma which was crossed by helical lines of force of the magnetic field.

6. Photography of the discharge glow was done with a high-speed photographic recorder. The photographs give the time variation of the glow intensity of the gas discharge in the plane of the observation window (see Fig. 3).

MEASUREMENTS

1. Stray Field

Our experimental arrangement was essentially an air transformer whose secondary coil consisted of a gas loop. In systems of this type the stray field $H_s$ of the primary coil reaches very high values. The stabilizing, conducting 2-cm-thick sheath reduced the stray fields considerably. The sheath had four sections—two with insulating slugs and two constructive sections with metallic slugs. Stray fields permeated these sections. Moreover, the current flowing in the closed metallic sheath which served as the discharge chamber also produced a vertical magnetic field component.

In order to reduce the magnitude of this vertical component a copper shield was interposed in the discharge chamber between the exciting coil and the torus. The vertical component of the field produced by the current flowing in the chamber could be somewhat reduced by arranging the copper stabilizing sheath near the thin-walled case of the chamber and by fixing the latter in an eccentric position with respect to the conducting surfaces.

Curves characterizing the maximal field strengths $H_s$ along the toroidal axis before and after assembly of the shield are shown in Fig. 5. It should be noted that with increase of the reference frequency of the circuit the penetration of scattered fields into the shield decreases. Since under the conditions of our experiments the role of the active resistance of the plasma loop was always important, the peak of the magnetic field strength $H_s$ was shifted by a certain angle with respect to the peak of the current.

Decrease of the magnitude of the scattered fields led
to an improvement in the conditions for formation of a discharge. Thus with a charge voltage of 15 kv a discharge could be induced in deuterium at a pressure of 10⁻⁵ mm Hg without a high-frequency source for production of initial ionization. Moreover, destruction of the plasma column after decrease of the current in it proceeded at a slower rate. A purely ohmic discharge process was observed when the stray field strength was large. In this case the voltage per turn and current in the gas passed through zero simultaneously. Under the same conditions the presence of the shield led to the result that the current was somewhat delayed (by 100-150 μsec) with respect to the voltage.

2. Discharges in Strong Magnetic Fields

On the basis of the data reported in Ref. 4 one may presume that heating of a plasma produced in a strong magnetic field \( H_t < H_0 \), where \( H_0 \) is the longitudinal magnetic field strength and \( H_t \) is the discharge current magnetic field strength) is limited by two factors—by the strong interaction with the chamber walls and by loss of hydrodynamic stability. Under conditions when the plasma fills the whole cross section of the chamber, instability can be inferred from indirect observations. It would seem to be profitable to investigate the possibility of formation, in a strong magnetic field, of a plasma column which interacts weakly with the walls. With this aim in mind the following investigations were carried out in a toroidal chamber made of quartz: (1) Elucidation of the conditions under which the discharge current and longitudinal magnetic field vary synchronously. (2) Investigation of conditions under which the frequency of variation of the longitudinal magnetic field exceeds by several times the reference frequency of the discharge current. Measurements under conditions of a synchronously increasing field were carried out at a peak longitudinal magnetic field strength up to 5000 gauss and discharge currents up to 25 ka. Oscillograms of the voltage per turn, of the discharge current and of the longitudinal magnetic field strength are presented in Fig. 6. Beneath the oscillograms a streak photograph of the discharge is presented, which was taken simultaneously through the side and top slits in the copper stabilizing sheath. The time scales of the streak photograph and oscillogram are identical. The discharge current in this experiment attained 18 ka and the longitudinal magnetic field 3.5 kilogauss. The distribution of the field strength \( H_t \) at successive periods of time as measured with the aid of magnetic probes is shown in Fig. 7. It follows from these data that the discharge begins to develop in the center of the chamber (as observed in experiments in the absence of a longitudinal field) and subsequently spreads completely over the cross section of the chamber.

Measurements of the current distribution over the chamber cross section carried out with a small coil (magnetic probe) indicated that although the current axis was displaced towards the inner wall of the chamber, some contraction of the discharge filament did occur. After the current attained its peak value the distribution of the current density over the cross section became more uniform.

In experiments involving a rapidly increasing field and the same chamber the duration of the first quarter of the discharge cycle was 145 μsec and the frequency of variation of the longitudinal magnetic field was 15,000 c/sec. In Fig. 8 we present oscillograms of (1) the discharge current, (2) variation of the current in the coil producing the longitudinal magnetic field, (3) voltage per turn of the chamber, and (4) magnetic field strength in the center of the chamber measured with a magnetic probe; and also (5) a streak photograph of the discharge. After 20 μsec the current attains a value of about 5 ka. At this moment the longitudinal magnetic field is turned on. As a result, the rate of increase of the current approximately doubles. After 30 μsec the current in the longitudinal field coil reaches its peak value. However, the discharge current continues to increase and reaches 22 ka. When the current in the longitudinal field coil vanishes, the longitudinal magnetic field frozen in the plasma persists there for 6-8 μsec. The strength of the longitudinal field along the chamber axis remains approximately equal to that of the magnetic field of the current at the boundary of the plasma column.

A comparison of the streak photograph with the oscillograms reveals that the column contracted by 4 to 5 times; simultaneously the voltage increased and
3. Quasi-Constant Magnetic Fields

With some of the experimental arrangements studies were made of gas discharges in deuterium in a quasi-stationary longitudinal magnetic field produced by discharging condensers through a coil wound on a torus. The small toroidal chamber (4 in Table 1) was made of heat-resistant chromium-nickel steel 0.08 cm thick. The two halves of the torus were connected by two porcelain rings. The ring surfaces facing the inner part of the chamber were protected by shields made of the same steel. The copper stabilizing shields, which simultaneously serve as a frame for the longitudinal magnetic field coil, had eight transverse slits and one slit in the equatorial plane of the torus. In this arrangement the duration of the first quarter of the voltage cycle and the equal duration of the first half-period of the current cycle was about 500 µsec. The dependence of the peak discharge current on the magnetic field for various charge potentials is shown in Fig. 9. Some oscillograms of the current for a given charge voltage and various values of the magnetic field are presented in Fig. 10.

Streak photographs of the plasma filament (see the photographs in Fig. 11) measured at a charge voltage of 25 kv and an initial deuterium pressure of $10^{-9}$ mm Hg give some idea of the nature of the discharge. From 20 to 30 µsec after breakdown of the gas a narrow glowing channel, which broadened with increase
of the current, appeared on the background of a weak diffuse glow in the chamber. This luminous region pulsed and had protuberances which reached the chamber walls. With increase in the initial magnetic field the luminous channel became broader and the bursts became more regular although their brightness decreased. With further increase in the longitudinal magnetic field the plasma began to fill up the whole chamber volume. The inhomogeneity and disorder of the glow impedes interpretation of the photographs under these conditions.

More detailed investigations of the constricted plasma column were made at various durations of the process in a chamber whose tube diameter was 47 cm and torus diameter 125 cm. Oscillograms of the total current in the secondary circuit and of the voltage per turn are presented in Fig. 12 for frequencies of 220 c/sec and 750 c/sec. The calculated values of the current in the gas are given on the oscillograms. It should be noted that for a frequency of 770 c/sec the peak current on the second half-wave exceeds that on the first half-wave. Under the conditions we have used, the peak discharge current continued to depend linearly on the electric field. The dependence of the peak discharge current on the magnetic field strength is linear and for this chamber does not differ from the dependence presented in Fig. 8 for the chamber with a tube diameter of 26 cm.

Streak photographs of the discharge are shown in Fig. 13. These photographs were taken when the duration of the first quarter of the voltage period was about 1 μsec.

The glowing discharge column in the center of the chamber had a uniform and fibrous structure. The wavelike shape indicates that the column as a whole oscillates in a vertical plane without touching the chamber walls. Throughout the process a glow of the gas is observed. At certain moments the brightness of the glow increases.

Of undoubted interest is a photograph of the column which refers to the second half-period of the current. Regularly spaced helical luminous bands are wound around a central core which oscillates with a frequency smaller than that during the first half-period. The amount of light at the time is much smaller than that during the first half-period of the current.

Comparison of streak photographs made for various values of the longitudinal magnetic field shows that under our conditions (presence of stray magnetic fields $H_z$ of about 100 gauss along the toroidal axis) stability is impaired with decrease of the longitudinal magnetic field and the duration of the column is reduced. The lower streak photograph in Fig. 13 corresponds to $H_0 = 1300$ gauss and gives some idea about the course of the process during the first and part of the second half-period of the current. This photograph differs from the previous one ($H_0 = 900$...
Figure 11. Streak photographs of plasma filament. The chamber boundaries are designated by white lines.

Figure 12. Oscillograms of the total current in the secondary circuit, of the current in the gas and of the voltage per turn for $U = 15$ kV, $H_0 = 400$ gauss, $p_0 = 3 \times 10^{-3}$ mm Hg; 1, current in chamber sheath (gas current is zero) and voltage per turn; 2, total current, gas current and voltage per turn; 3, current in chamber sheath (gas current is zero) and voltage per turn; 4, total current, gas current and voltage per turn.
in that the discharge column is shifted downward relative to the chamber axis during the first half-period and shifted upward during the second half-period. The plasma column depicted on this photograph at the moment the current attained its peak value has a smaller diameter. Finally during the second half-period of the current the amplitude of the column oscillations is greater than that of the oscillations during the first current half-period.

From the streak photograph one can derive the time variation of the radius of the discharge column. However, various measurements and in particular those reported in Ref. 3 indicate that the current distribution over the cross section may be inferred from the light measurements. The "current" radius, i.e., the current distribution over the chamber cross section derived from mean weighted quantities, is always smaller than the radius determined from the streak photographs. The dependence of the conductivity of the discharge column on the longitudinal magnetic field is presented in Fig. 14. The conductivity was computed from the discharge current and from the electric field strength at the moment when the latter attains its maximum. Curve 1 refers to a chamber with a tube diameter of 26 cm and curve 2 to a chamber of 47 cm diameter. Since the difference between these curves is smaller than the experimental errors, it may be concluded that in the experiments described in this paper the conductivity depends weakly on the chamber dimensions. If one takes into account that the "light" radius used in the calculations is greater than the "current" radius and also that the current density along the discharge axis is very considerable, the maximal plasma conductivity may be estimated to equal $\sigma = 10^{15}$ cgs.

The time of appearance of spectral lines emitted by the wall material can be roughly determined from the streak photographs. For illustration a streak photograph of the spectrum is presented in Fig. 15. Examination of phototgraphs made under various conditions shows that the impurity lines appear after the lines emitted by deuterium. At a reference frequency of 770 c/sec the lines due to the wall material appeared after 100 $\mu$sec when the current peak was 150 ka and after 200-220 $\mu$sec when the current peak was 85 ka. For a frequency of 220 c/sec and $\mu = 110$ the Fe, Cr and Ni lines appeared after 80-100 $\mu$sec.

The spectra photographed with the quartz spectrograph contain lines which persisted during the entire time the current flowed through the gas. These spectra are characterized by the deuterium lines, by the presence of lines emitted by multiply ionized impurity atoms (O, N) and by the presence of intense Fe, Cr and Ni lines.

Radiofrequency oscillations of a frequency of 75,000 and 130,000 Mc/sec were used to determine the concentration of the charged particles in the plasma.

Ultra-high-frequency oscillations of 0.23 cm wavelength (130,000 Mc/sec) were completely reflected by the plasma formed in a torus at an initial deuterium
pressure of $10^{-3}$ mm Hg. The amplitude of the recorded signal was much smaller at lower initial pressures. When $p_0$ was reduced still further the recorded signal did not differ from that passing through a neutral gas. Some oscillograms referring to the passage of ultra-high-frequency oscillations through a plasma (1) and reflection of the oscillations (2) are shown in Fig. 16. The second trace is a record of the discharge current. The experimental data available to us are not sufficient, however, to permit us to draw any unambiguous conclusions. The information derived from streak photographs of the discharges indicates that from 40 to 60% of the magnetic field is captured during the contraction process. If one assumes that in the initial stage of the discharge an equal fraction of particles is captured, this would mean that ionization in the discharge column is almost complete. If a larger number of particles were captured by the field the degree of ionization would be correspondingly lower.

It should be mentioned that even when the oscillations are completely reflected during the first half-period of the current a signal transmitted by the plasma will almost always be recorded during the second half-period. Evidently impurities emitted from the walls not only cool the plasma but also lead to a decrease of the concentration of the charged particles.

CONCLUSIONS

The experimental data presented above permit one to draw the following conclusions:

1. The plasma compressed by the current detaches from the walls and forms a column. Capture of the magnetic field is not complete and in our experiments does not exceed 60% of the initial field.

2. The observed macroscopic oscillations of the
plasma column do not permit one to consider that it is fully stable; there is even less reason to suppose that it does not interact with the walls.

3. The conductivity of the plasma column observed in the present experiments does not exceed $10^{16}$ cgse, which corresponds to an electron temperature of 15–30 ev. Evidently higher plasma conductivities were also not achieved in other arrangements of a similar type.

4. The energy transferred to the chamber walls by particles and radiation during the existence of the plasma column should be measured and the effect of impurities on heating of the deuterium plasma should be determined.

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REFERENCES

Experimental Studies of the Pinch Phenomenon

By R. Aymar, C. Etievant, P. Hubert, A. Samain, B. Taquet and A. Torossian

The purpose of the work described in this paper was to investigate the production and stabilization of the pinch (striction) phenomenon so as to apply it to the creation of controlled thermonuclear reactions.

These experiments began in 1956, thanks to facilities made available to us by Electricité de France, at its testing centre of Fontenay. First, the experiments were concerned with rapid rectilinear discharges, then they went on to ring-shaped discharges.

The first paper about this work was presented at the Third International Conference on Ionization Phenomena in Gases. Even though few data have been obtained since that date, we believe that it is worthwhile to describe the course of the work and to discuss the progress achieved.

LINEAR DISCHARGE EXPERIMENTS

Discharges Without a Magnetic Field

A first series of experiments designed to reproduce the phenomena described by the Russian experiments was initiated with the aid of a bank of 60 µf, 35 kv condensers, which discharged in a Pyrex chamber 28 cm in diameter and 90 cm in length, with a period of some 20 µsec. The parasitic self-inductance of the circuit is equal to \( L_e = 60 \text{ mH} \).

After some efforts to improve the purity of the deuterium and to eliminate various causes of gas losses, we observed the production of neutrons (some \( 10^6 \) to \( 10^7 \) at each discharge) and the creation of hard X-rays. The X-rays appeared in coincidence with the beginning of the discharge and the pinch occurrence.

A spectrographic study of the light emitted warranted the conclusion that most of the light radiation is produced in the terminal phases of the discharge, when the gas is contaminated by the materials evaporated from the walls.

An over-all study indicates that the widening of Balmer lines, which happens under those conditions, is not incompatible with the supposition of 100% ionization during the discharge.

Magnetically Stabilized Discharges

This second series of experiments has been conducted in order to verify the theory of stabilization of the pinch as it is now described by various authors. Experimental work on the same subject by Bez-bachenko and his group has already given very interesting results. However, the disposition described by the latter authors does not provide complete stabilization conditions, because of the lack of a conductive envelope about the chamber.

Apparatus

A schematic view of the apparatus is shown in Fig. 1. The equipment is the same as for the first experiments, but this time the Pyrex chamber is one meter long. It is surrounded by a copper jacket 0.4 mm in thickness, which provides for the return current. This jacket, which is shaped to match the outside of the glass cylinder very closely, is provided with an observation slit 7 cm high in its central portion. Current flow at this gap is made possible by a “squirrel cage” of wires soldered to the edges of the slit.

A substantially uniform magnetic field is created, parallel to the axis, by a winding which is also split into two sections to permit observation. Voltage at the terminals of the coil is supplied 0.5 sec before the discharge, so that the magnetic field has time to penetrate into the chamber despite the copper jacket. The longitudinal field obtained is between 0 and 1500 gauss.

The experiments are carried out with deuterium. The filling pressure is somewhere between \( 10^{-4} \) and \( 10^{-3} \) mm of Hg for a partial pressure, due to impurities, of less than \( 10^{-5} \) mm Hg. The initial voltage applied to the chamber is between 20 and 35 kv.

The means provided for the observation of the phenomena are, simply:

(a) the recording, as a function of time, of the voltage applied to the electrodes and of the intensity of current; and

(b) ultra-rapid cinematography (one picture per µsec) of the phenomena which take place in a section at right angles to the axis of the chamber. This section, 3 cm in height, is in the vicinity of the center of symmetry.

Preliminary Results

Oscillograms

The oscillograms give voltage \( E \) at the terminals of the chamber and discharge current \( i \). Figure 2 shows some oscillograms which have been recorded. The
currents achieved are some $2 \times 10^6$ amp. The phenomenon lasts about 10 µsec.

Let us note that, if $L_e$ is the external self-inductance of the discharge circuit and $V$ the voltage at the terminals of the capacitors, we can write:

$$V - E = L_e \frac{di}{dt}$$  \hspace{1cm} (1)

On the other hand, if $R$ is the resistance and $L$ the self-inductance of the discharge, we shall have:

$$E = Ri + L \frac{di}{dt} + i \frac{dL}{dt}$$  \hspace{1cm} (2)

We can assume that the electron temperature $T_e$ is such that $2nkT_e \sim \beta^2$, where $n$ is the number of electrons per centimeter of discharge, and $k$ is Boltzmann's constant.

At a time 1 µsec after the discharge begins, the resistivity in the plasma is some $3 \times 10^{-6}$ ohm cm. Using this resistivity, we find a penetration of field $B_0$ into the plasma of about 0.5 cm. The resistance of the discharge is of the order of $10^{-3}$ ohm.

Allowing for the order of magnitude of $E$ and $i$, we can then write, 1 µsec after the beginning of the discharge:

$$E = \frac{dL}{dt} = L \frac{di}{dt} + i \frac{dL}{dt}$$  \hspace{1cm} (3)

On the other hand, if we assume that the plasma column has a radius $r$ which is well defined and that the currents are superficial, we have

$$L = 2 \times 10^{-7} \log \left( \frac{r_0}{r} \right) \text{ henry},$$  \hspace{1cm} (4)

in which $r_0$ is the radius of the metal jacket.

Under those conditions, we can compute a "radius" $r$ of the column, as a function of time, according to the following equation:

$$2 \times 10^{-7} \log \left( \frac{r_0}{r} \right) = \left( \frac{1}{\mu} \right) \int_0^t \frac{E dt}{V}$$

$$= -L_e + \left( \frac{1}{\mu} \right) \int_0^t \frac{V dt}{V}$$  \hspace{1cm} (5)

Assuming that we make a gross error during integration for the first microsecond, the relative error which can be made on $\log \left( \frac{r_0}{r} \right)$ after 3 µsec does not exceed $\frac{1}{2}$.

The real radius is larger than the computed value. Figure 3 gives voltages $E$ and $V$, as well as current $i$ and the computed radius $r$, as functions of time, for three values of applied longitudinal magnetic field $B_{z0}$.

The motion of the column wall changes direction when $dL/dt = 0$; i.e., when $E = (L/L_e) (V - E)$. Since $L \gg L_e$, this is when $E \simeq V$ and $di/dt = 0$. The corresponding points are marked $\alpha$ and $\beta'$ in Fig. 3.

Under our experimental conditions, viz., $10^{-3} < P < 10^{-1}$ mm Hg and $20 < V_0 < 35$ kv, the radii $r$, as computed, are some 2 cm for $B_{z0} = 0$.

We verified that (in e.m.u) the dimensionless quantity

$$H = (2mr)^{-1} \int_0^t \frac{\rho dt}{\rho}$$  \hspace{1cm} (6)

in which $\rho$ is the mass of the plasma per unit of length, has a value of approximately unity.

Moving Picture Films

The cinematograph pictures are difficult to interpret, for we do not always know what the camera sees. Figure 4 is a reproduction of selected films for different values of $B_{z0}$. The pictures are taken with a
special ultra high-speed motion picture camera (Beckman and Whitley, Model 189).

On Film 1 \((B_0 = 0)\) we see the pinch \((0 \alpha)\). The minimum radius of the column is 2 cm. The rate of contraction is 3.5 cm/\(\mu\)sec. The plasma column then appears to explode according to a mode of revolution having a longitudinal wavelength of 2 cm. The distortions increase \(e\) fold in some 0.6 \(\mu\)sec.

The theory on the instability of revolution deformation\(^7\) \((m = 0)\) and the wavelength \(\lambda\) of a plasma column without a longitudinal magnetic field, as confined by field \(B_0\), leads to the following result:

\[
T(\alpha) \approx 2 c^{-1} \text{ for } \alpha \sim \gamma
\]

where \(T\) is the time constant for the amplification of the distortion, \(\gamma\) the radius of the column, \(c^2 = B_0^2/2\rho\) with \(\rho\) the density of the plasma. We find, in our case, \(T = 2 \times 10^{-7}\) sec.

Film 2 \((B_0 = 400\ \text{gauss})\) gives a radius \(r_a\) of 2.5 cm and a contraction rate of 5.5 cm/\(\mu\)sec. The explosion noted on Film 1 seems to have disappeared. The constriction is maintained for 4 \(\mu\)sec after the first pinch. Field \(B_0\), computed at the time of maximum striction, assuming total conservation of the flux in the plasma, is of the order of 12,000 gauss. Field \(B_0\) is some 16,000 gauss. Let us remember that the stabilization of mode \(m = 0\) is obtained for \(B_0/B_\gamma \leq \sqrt{2}\).

Film 3 \((B_0 = 1100\ \text{gauss})\) gives a radius \(r_a\) of 4.5 cm. The column keeps a regular appearance for 5 \(\mu\)sec, at least, following the first pinch.

The radii read on photographic Films 2 and 3 were shown on Figs. 3b and 3c. Agreement is good in this case.

It can be seen that the motion picture method may be of service in the study of the phenomena which accompany the pinch. However, in the present condition of our technical knowledge, the very low pressure discharges, which are the most interesting, are unfortunately not bright enough to lend themselves to the method.

RING DISCHARGE EXPERIMENTS

The building of a ring discharge device was decided upon when it became obvious that long-lasting stabilization could be hoped for, and that such stabilization was incompatible with the existence of electrodes, the evaporation of which may contaminate the discharge.

The principle used was intended to combine the stabilizing action of a longitudinal magnetic field with that of a close-fitting metal jacket around the discharge tube. We endeavored to retain the peculiar advantages inherent in various experiments already described in the literature,\(^8\)-\(^10\) none of which had all the elements indispensable for perfect stabilization.

Equipment

Figure 5 shows that the equipment consists of: (a) a hermetically closed torus-shaped envelope, made of Pyrex, with the diameters \(\phi_1 = 78\ \text{cm}\) and \(\phi_2 = 8\ \text{cm}\); (b) a metal jacket made up of copper braid wound in two crossed layers; (c) an inductive field winding, the turns of which are parallel to the
The principal observations which can be made are the following:

1.—In the absence of a stabilizing field, the current goes through a maximum after about 10 \( \mu \)sec, then drops to a sort of plateau before falling to zero again (see Fig. 9).

2.—The voltage oscillograms always show disorganized fluctuations of low amplitude, the pseudo-period of which is about 1\( \mu \) sec, showing that the plasma is not devoid of agitation.

3.—A great many oscillograms show, at some moments, some irregularities of greater amplitude which appear simultaneously in the current and in the voltage. These irregularities appear as a damped oscillation, the period of which is about 10 megacycles, for a duration of approximately 2\( \mu \) sec. Very likely, we are dealing with oscillations of the plasma; however, their exact nature is not yet clear.

**Spectroscopic Observations**

The light emitted by the discharge is analyzed with an optical (visible region) spectrograph, the image of which is spread out by a rotating mirror device.

The inductive field is created by the discharge of the bank of capacitors mentioned above, while the power needed for the stabilizing field \( B_z \) is supplied by a dc generator.

The essential characteristics of the assembly are: (i) an acceptable degree of coupling, despite the lack of a magnetic circuit; (ii) a reduction of the leakage inductance; (iii) matching of the inductive circuit to the source of current; and (iv) reinforcement of the stabilizing action of the metal jacket by connecting the six sections of the field coil in parallel.

**Experimental Findings**

The results consisted essentially of electrical, spectroscopic and X-ray measurements. All the experiments we shall mention here were carried out on discharges in deuterium.

**Electrical Observations**

We obtained oscillographic tracings of voltage \( V \) (number of volts per turn) and of current \( I \) carried by the discharge. The relevant signals were supplied by suitably arranged coils.

Figure 6 shows a typical example of oscillograms made under these conditions while Figs. 7 and 8 show how the maximum current varies during the discharge, in terms of the pressure and value of the stabilizing magnetic field.
Toroidal discharges in deuterium

Figure 7. Maximum discharge current vs. gas pressure

Toroidal discharges in deuterium

Figure 8. Maximum discharge current vs. $B_{22}$

Toroidal discharges in deuterium

Figure 9. Current oscillogram for discharge in deuterium

Figure 10 shows a measurement of the blackening, under those conditions, of the Hβ Balmer line and on a singly-ionized silicon line (5041–5056 Å).

It will be noted that the line Hβ appears at the beginning of a discharge, disappears for a while, and then shows up again accompanied by impurity lines. The presence of the magnetic field has the effect of retarding the moment of the return, which clearly shows its stabilizing role.

X-ray Observations

Intense X-radiation was detected under some conditions. Maximum emission took place with a magnetic field of 500 gauss and a filling pressure of $5 \times 10^{-2}$ mm of mercury. During the experiment, the electric field generally was 20 v/cm. An absorption measurement enabled us to estimate a maximum energy of approximately 150 kev for the spectrum of the emitted photons.

The recording of the signal given by a sodium iodide scintillator showed that the X-ray pulses are produced 4 μsec after the start of the discharge and last about 2 μsec. This duration is greater if the charging voltage of the condensers is lower.

The production of these X-rays is accounted for by the acceleration of electrons which go around the ring about 30 times. Certainly, we are dealing with a mechanism which is different from that at play during the development of instabilities in linear discharges; for the various parameters, such as the electric field, the magnetic field and the moment of appearance, are not at all the same.

CONCLUSIONS

The results obtained during our experiments show that it is possible to improve the stability of a pinched discharge. This conclusion is in agreement with the results published a short time ago by other laboratories. We hoped to continue research along these lines with devices which are being made, in which some technical improvements will be included.
ADDENDUM

Initial X-ray Production from a Stabilized Ring Discharge

Experiments have continued, using a new torus which differs from the first model only in having an electrolytically deposited copper shell, about 2 mm thick, on the Pyrex tube. The shell has two windows through which the X-rays emerge, but in any case the 5 mm glass wall absorbs the softer X-rays.

**Total X-ray Intensity**

Table 1 shows the results of measurements made with an argon-filled pressurized ionization chamber for discharges in deuterium under various conditions of gas pressure and magnetic field. The observed diminution of X-ray intensity at low pressures may be due to difficulty in initiating the discharge. The emission becomes very irregular at high values of the magnetic field. The X-ray emission for discharges in argon is only slightly less than for discharges in deuterium.

**X-ray Energy**

The effective energy of the X-rays has been deduced from absorption measurements made by placing copper absorbers in a collimated beam detected by a sodium iodide crystal. For discharges in deuterium at a pressure of $4 \times 10^{-2}$ mm Hg, the effective energies, $V_e$, under selected operating conditions are:

<table>
<thead>
<tr>
<th>$B_z$, gauss</th>
<th>800</th>
<th>800</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$, v/cm</td>
<td>20</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>$V_e$, kev</td>
<td>187</td>
<td>110</td>
<td>117</td>
</tr>
</tbody>
</table>

It is recognized that these measurements do not exclude the possibility of a complex spectrum having a small high-energy tail.

**Time of Emission**

Figure 11 shows typical oscillograms of X-ray emission and discharge current. The X-rays appear always at the beginning of the discharge and have not been observed at subsequent alternations of the current. There does not seem to be any relation between the emission of X-rays and the inflection in the discharge current curve marking the beginning of the pinch.

The lack of a preionization device had the unexpected effect of prolonging the X-ray signal from 2 to 7 µsec.

The sum of the observations forces us to the conclusion that emission takes place while the gas is still cold and incompletely ionized.
Figure 13. Design of torus with composite liner

- 1, main vacuum pumps
- 2, auxiliary pumps
- 3, iron yokes
- 4, ports
- 5, primary windings
- 6, stabilizing field windings
- 7, outer wall
- 8, thin stainless steel liner
- 9, cooled lapped copper segments
Effect of Local Magnetic Perturbations

To investigate the mechanism of X-ray production, the effect of a local perturbation of the stabilizing magnetic field has been studied, since such a perturbation ought to prevent electrons from being accelerated during multiple passages around the discharge ring. This hypothesis is confirmed by the following three experiments.

(a) For a perturbing field, $B_p$, perpendicular to the plane of the torus the X-rays disappear for $B_p \approx B_z$.

(b) A magnetic mirror, produced by means of a supplementary field coil, will also suppress X-ray emission. Table 2 gives the values of $B_M$, the maximum strength of the supplementary field, and $a \equiv (B_M + B_2)/B_2$, the Mirror Ratio, at which X-ray emission is suppressed for selected values of $B_z$ and other operating parameters.

(c) An inverse or negative mirror, produced by reversing the current in the supplementary field coil, suppresses X-ray emission for $B_M \approx B_z$.

These experiments justify the hope that the harmful effects of those electrons which become decoupled from the pinched discharge may be suppressed by the judicious use of magnetic mirrors.

Large Aluminium Torus

An aluminium torus 2 m in diameter is being constructed for the further study of stabilized ring pinch discharges. Its principal characteristics are as follows:

- Major diameter: 2 m
- Minor diameter: 30 cm
- Wall thickness: 1 cm
- Stabilizing field: 0-2000 gauss
- Induced electric field: 0-7 V/cm
- No. of magnetic circuits: 16
- Iron cross section: 3200 cm²

Three variants of interior fittings are provided:

(a) Interlocking aluminium liners, insulated so as to float electrically;

(b) A corrugated stainless steel liner 0.4 mm thick, acting as a potential divider because of its ohmic resistance (the annular space between the liner and the exterior wall evacuated separately from the discharge chamber); and

(c) A very thin stainless steel vacuum-tight liner protected from the discharge by 88 copper rings for which it serves as a potential divider.

Figure 12 is a photograph of the apparatus in its present state (version a) while Fig. 13 illustrates the design of version c.

REFERENCES

2. C. Breton et al., ibid., p. 163.
High Intensity Discharges in Deuterium in a Metal Wall Torus

By J. Andreoletti, C. Breton, J. Charon, P. Hubert, P. Jourdan and G. Vendrayes*

In two recent papers1, 2 we described the production and investigation of high intensity discharges in a toroidal chamber, the walls of which were made of Pyrex, externally covered by metal shielding. It seemed worthwhile to us to repeat these experiments in a metal walled chamber as, in fact, theoretical studies3 have shown that the stabilizing role of such a conductive jacket, with respect to the over-all changes in the shape of the plasma column, are all the more efficient as the radius of this envelope is closer to the initial radius of the discharge. On the other hand, the experiments of the Harwell team4 showed that a pinched discharge can be initiated under such conditions. An important advantage of a metal walled chamber is found in its mechanical behavior, which raises far fewer problems than do other types of chambers. The metal chosen is aluminium, largely because of its low atomic number.

Objectives

The research was undertaken, first, to investigate problems having to do with very high current discharges (over 50,000 amp.) in an aluminium torus, particularly:

(a) initiation of discharge—longitudinal magnetic field \(B_z\), pre-ionization, number of breaks, etc.;

(b) insertion of insulating sleeves and connections in the torus section, the behavior of which must be good mechanically, electrically and under a vacuum.

(c) Electric coupling of primary circuit and plasma.

Secondly, it was desired to study the discharges themselves by various techniques, particularly: ultrarapid moving pictures, spectrography, magnetic probes, microwaves, neutron scintillators, etc.

The purpose of this paper is to describe the experimental setup in its present condition of completion and to outline some of the preliminary results obtained.

EXPERIMENTAL ARRANGEMENTS

The discharge chamber (which is called Equator I) is an aluminium torus with a mean major diameter of 80 cm and a minor inside diameter of 7 cm. The aluminium wall thickness is 4 mm.

In order to avoid any turn short-circuit effect, the torus is made of two sections (Fig. 1), the two halves connected by two insulating sleeves or connections. Various lengths of insulating breaks, ranging from 2 to 50 mm, have been used. The insulating materials investigated to date are Pyrex and polytetrafluoroethylene. It is proposed to use quartz, ceramic (such as mullite) and alumina-coated aluminium sleeves.

An evacuating tube having a cross section of 3 cm is connected to the vacuum equipment which includes an oil diffusion pump (separated from the chamber by a metal valve with an indium gasket and two liquid nitrogen traps) and a vane type pump. Deuterium filling is by the same channel as that used for pumping.

The periphery of the torus carries four sampling ports, at which it is possible to insert magnetic or electrical probes or arrange a number of suitable devices for investigation, such as spectrographs or scintillators.

Around the torus are 36 coils designed to create the longitudinal magnetic field \(B_z\). These coils are made up of six layers of enamelled wire 2 mm in diameter. For 0.5 sec at the time of a discharge, they can be connected to a dc rotary generator with a nominal output of 1 Mw. The current through the coils can be adjusted by means of a resistor bridge. The longitudinal magnetic field \(B_z\), which may be created in the torus chambers by this arrangement, can thus be varied from 0 to 5000 gauss.

The discharge makes up the secondary of a transformer, the primary of which is made up of a certain number of turns of inductive loops arranged about the torus. Allowing for the fact that investigation of the insulating sleeves requires frequent disassembly of the torus and of its electric circuits, the primary coils, until now, have been made up simply of circular turns arranged in the plane of symmetry of the torus, as close as possible to it as shown in Fig. 1. This primary coil is made of copper wire, 2 mm in diameter, under a
Magnetic field windings
Primary inductive winding
Testing ports
To the capacitors

Figure 1. Equator I, schematic

polythene covering. Most discharges took place using twenty inductive turns of wire.

The primary-plasma coupling can be improved by means of magnetic circuits wound about both the primary and the torus. In this fashion, coupling takes place in part through the air and in part through the iron. The results given here were obtained with a makeshift circuit having a low maximum induction and substantial leakage in the air gap. We propose to improve the coupling in the near future by eliminating the air gap and using oriented grain sheet, 0.35 mm thick, with a maximum induction of 16,000 gauss.

The wiring diagram of the setup is shown in Fig. 2. The bank of condensers consists of 30 elements, of 2 μf each, which can be charged to a maximum dc voltage of 50 kv, for a total of 75 kilojoules, maximum energy, available for each discharge.

The condensers discharge in the primary of the transformer through a sphere type spark gap. Discharge is controlled by means of a voltage pulse applied to an intermediate plate located between the two spheres of the gap.

The primary current is oscillating. This creates mechanical stresses in the dielectric of the capacitors. In order to avoid their deteriorating too rapidly, it is possible to short-circuit the primary by means of a mercury igniton, when the voltage at the terminals of the primary circuit first passes through zero. Such an arrangement was used for all discharges in which the capacitor charging voltage exceeded 30 kv.

In order to facilitate the initiating of the discharge in the gas, the latter is made slightly conductive by means of a high-frequency discharge, which is applied for a few seconds before the pulse is supplied by the capacitors. The high-frequency generator has a power of 2 kw and a frequency of 27 Mc. Both inductive and capacitive high-frequency discharges have been tried. The most efficacious method appears to be to apply the high-frequency voltage between the two insulated metal shells which make up the aluminium torus. This hf discharge is interrupted within 5 μsec after the discharge pulse.

Figure 3 shows the general arrangement. A floor was arranged over the capacitor bank to carry the torus and its accessories.

The high-voltage units are enclosed in a wire-netting cage, preventing access to the experimental setup while tests are being run. The equipment needed for the investigation of discharges is arranged about the torus. Note, in particular, the spectrograph (in the background) which is sighted on the discharge through one of the ports described above. The image of the spectrum so obtained can be formed, by means of an optical bench, on a horizontal slit. A time analysis of the light passing through this slit can be made by means of a camera with a horizontal-axis rotary mirror.

An ultra-rapid Beckmann camera which can give 25 photographs at 1-μsec intervals (close-up, on the left) can also give 25 motion-picture exposures of the plasma column during discharge. We are currently working on the development of liquid and plastic scintillators, as well as on BF₃ counters, for the detection of the fusion neutrons which may be emitted during the discharges.

A thin section of the aluminium torus serves as a window for the detection of X-rays by means of an ionization chamber or an NaI scintillator.

The capacitor charging devices and the observation oscillograph are outside the high-voltage enclosure. Once the capacitors have been charged to the required voltage, all operations are automatically sequenced by means of a camshaft and electronic relays. A coincidence device, in particular, provides means of bringing the discharge about when the mirrors of the two rotary-mirror cameras are in a suitable position.
PRELIMINARY FINDINGS

The parameters, for the range investigated, are:
- Pseudo-period: 700–2000 μsec
- Electric field, $E_z$: 1–5 V/cm
- Pressure: $10^{-3}$–1 mm Hg of deuterium
- Initial magnetic field, $B_z$: 0–4400 gauss
- Current: 0–104 kA

**Gap Problems**

Although this investigation is still in a very early stage, results to date seem to indicate that:

1. discharge initiation is made easier by an increase in the number of gaps;
2. Pyrex sleeves are out of the question, because they break after a few discharges;
3. the polytetrafluoroethylene joints, in contrast, are particularly rugged (some of them, used for more than 150 discharges with currents of more than 50,000 amp, showed no trace of deterioration when dismantled. The creeping or de-gassing problems having to do with these joints or connectors seemed to be less important than might first have been thought); and
4. during the discharge, a heavy arc passes between the edges of the gap; there is an actual transfer of metal from one edge to the other. Preliminary observations seem to indicate that the current in the aluminium shell, during the discharge, may amount to a full 7 to 10% of the current in the gas. Eliminating this arc appears likely to be the main problem in producing discharges in metal chambers.

**Initiating the Discharge**

The initiation of the discharge is greatly facilitated by the presence of a longitudinal magnetic field, even a very weak one ($\approx 50$ gauss). For the initiation of discharges at low pressures ($10^{-3}$ mm Hg) we are studying, together with hf ionization, the possibility of making the gas slightly conducting by means of a stream of electrons from an electron gun.

**Current and Electromotive Force**

The intensity of the discharge is measured by means of a current transformer which surrounds the torus (Rogowsky belt). A single measurement gives both the derivative, $dl_z/dt$, of the current, and the current, $I_2$, itself—the current being deduced from $dl_z/dt$ by means of an electronic integrator.

The electromotive force, $U$, is measured by taking the voltage at the terminals of a circular coil tangent to the external jacket of the torus along its large mean diameter.

Figure 5 is a photographic reproduction of oscillograms for the current, $I_2$, its derivative, $dl_z/dt$, and the voltage, $U$. The horizontal calibrating lines show 180 kA for the current and 600 V for the voltage. The horizontal time axis shows a calibrating signal every 250 μsec on all three oscillograms; in other words, 3 msec for the whole of the sweep. Filling pressure here is $1.2 \times 10^{-2}$ mm Hg of deuterium, and the longitudinal magnetic field, at the start, is 120 gauss.

Figure 6 shows oscillograms for $I_1$ (primary current), $I_2$, $dl_z/dt$ and $U$ with a total scanning time of only 1000 μsec. The ignitron, here, has short-circuited the battery of capacitors at the time when the voltage dropped to zero at the terminals of the primary inductor. Current $I_1$ continues to drop exponentially in the circuit made up of the ignitron and a series resistance, 0.5 ohm, and the inductance of the primary circuit. The calibration, on $I_1$, is 8250 amp between the two horizontal lines and, for $I_2$, 66,000 amp per square. Calibrations on $dl_z/dt$ and $U$ are the same as in Fig. 5.

These oscillograms are remarkably reproducible, which seems to indicate good discharge stability.
Influence of Magnetic Field $B_z$

Figures 7 and 8 show the variation in the maximum current ($I_{2 \text{max}}$) of the first half-cycle of the secondary current as a function of the magnetic field, for filling pressures of $1.2 \times 10^{-2}$ and $2.5 \times 10^{-3}$ mm Hg, and charging voltages of 30 kv. The graphs also show the plasma resistance which corresponds to current $I_{2 \text{max}}$.

For the pressure of $2.5 \times 10^{-3}$ mm, it has not been possible to start the discharge at zero longitudinal magnetic field.

At a pressure of $1.2 \times 10^{-2}$ mm Hg, we note a sudden variation in the current for a magnetic field variation between zero and 50 gauss. Some slight optimum of current $I_2$ would appear to have been noted for a magnetic field of about 3000 gauss.

Effect of the Capacitor Charging Voltage

Figure 9 gives the maximum current of the first half-cycle of $I_2$ for a capacitor charging voltage between 25 and 40 kv, i.e., for a stored energy ranging from 19 to 48 kilojoules, and the plasma resistance corresponding to this maximum current for the same variation of the charging voltage.

It will be noted that $I_{2 \text{max}}$ increases substantially as the charging voltage, i.e., as the square root of the stored energy, and that the resistance, $R_2$, of the plasma decreases with the charging voltage. The electromotive force, $U = R_2 I_2$, corresponding to $I_{2 \text{max}}$ varies rather little with the charging voltage.

Effect of Filling Pressure

In the presence of a strong magnetic field, $B_z$, the maximum current, $I_{2 \text{max}}$, of the first half-cycle varies but little with the initial filling pressure.

The results shown in Fig. 10 were obtained with a charging voltage of 30 kv and a magnetic field, $B_z$, of 1000 gauss. Other investigations are under way for initial magnetic fields $B_z$ much weaker than those.

Discharge Spectra

Figure 11 shows some spectra of the discharge, photographed at a distance of approximately 40 cm from a gap area, for a pressure of $1.2 \times 10^{-2}$ mm Hg and a charging voltage of 30 kv, as well as for various magnetic fields.

There is a very marked widening, due to the Stark effect, of the $\beta$ and $\gamma$ lines of the Balmer series of deuterium. Computations are under way to evaluate the electronic density corresponding to these line spreads.

Beside each spectrum of Fig. 11 is shown the initial magnetic field $B_{zo}$. The widening of line $H_\beta$ was measured with a microdensitometer. Figure 12 shows the variation of this width, measured at half-height, for various magnetic fields $B_z$. It will be noted that maximal widening of the $\beta$ and $\gamma$ lines is found for a magnetic field $B_{zo}$ between 300 and 400 gauss.

Temperatures

The present status of the research does not permit a good evaluation of temperature. However, it can be indicated that crude application of the formula...
I^2 = 4 NkT would give \( T = 1.3 \times 10^6 \) °K for the tests made with \( B_z = 0, V = 30 \) kv, \( p = 1.2 \times 10^{-2} \) mm Hg and \( I = 48.5 \) ka.

Some measurements are now being planned for the purpose of evaluating the discharge temperature regardless of magnetic field \( B_z \).

CONCLUSION

The preliminary studies already conducted with Equator I indicate that it is possible, in an Aluminium torus, to achieve very high current discharges in deuterium. The appearance of the current and voltage curves is not incompatible with good stability of the discharge so obtained. The currents already recorded do not invalidate the assumption that a high temperature has been achieved.

The main problem to be solved is the arcing between the edges of the insulation break. It is important to see what fraction of the current is removed from the discharge in this fashion, and to determine to what extent the contamination this causes contributes to the cooling of the plasma.

ACKNOWLEDGEMENTS

We particularly wish to thank Messieurs Givelet and Dupuy, of the Ateliers de Mécanique de Saclay, who greatly helped us in building the torus and the ancillary equipment in as short a time as possible.

The major part of this research was carried out at the testing laboratories of the Electricité de France, at Fontenay-Aux-Roses. We wish here to convey our thanks for the valuable help of the E.D.F. in those experiments, and particularly to thank Messieurs Nasse, Arribes, Anres and Gary, whose constant help has been most welcome, as well as all the technicians whose enthusiasm and devotion to duty enabled us to surmount a great many difficulties.

ADDITIONAL AUTHORS

Hydromagnetic Waves

A spectrograph with a rotating mirror was set up at the observation port equidistant from the two gaps in the metal torus (Fig. 1). The resulting time-resolved spectrograms (Fig. 13) show a succession of bands in the continuous background, generally occurring at intervals. A microdensitometer has been used to determine the length of the intervals. Such a phenomenon could be explained by the existence of transverse hydromagnetic waves excited by the formation of arcs at the insulating gaps and propagated along the magnetic field lines in the plasma.

Table 1. Comparison of Observed and Theoretical Plasma Wave Velocities

<table>
<thead>
<tr>
<th>Discharge number</th>
<th>Alfvén speed, ( v_0 ) cm/sec</th>
<th>Observed speeds cm/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>395</td>
<td>3</td>
<td>2.3, 2.3</td>
</tr>
<tr>
<td>397</td>
<td>3</td>
<td>2.3, 2.3, 2.3</td>
</tr>
<tr>
<td>399</td>
<td>3</td>
<td>3.1</td>
</tr>
<tr>
<td>440</td>
<td>5.4</td>
<td>4.6, 6, 11, 11</td>
</tr>
<tr>
<td>441</td>
<td>4.5</td>
<td>5, 5, 5, 5, 5.6</td>
</tr>
</tbody>
</table>

To check this hypothesis, the observed velocities have been compared with the Alfvén speed, \( v_0 \), for the non-constricted discharge: \( v_0 = B_{20}(4\pi\rho_0)^{1/2} \), where \( B_{20} \) and \( \rho_0 \) are the longitudinal field strengths and plasma density at the beginning of the discharge. General agreement is shown by the results given in Table 1.

In Discharge 440 the observed speed increased appreciably during an increase in the plasma current. Letting \( S \) equal the cross section of the plasma column, one can write

\[
\rho S = \rho_0 S_0, \quad B_z S \leq B_{20} S_0
\]

whence

\[
a_0/a = \left( S_0/S \right) \geq v/v_0
\]

where \( a_0 \) and \( a \) are the initial and pinched radii, and \( v \) is the Alfvén speed for the pinched discharge in a field \( B_z \). The data for Discharge 440 indicates a linear compression \( a_0/a \geq 2.4 \).

Additional authors for the addendum were P. Anres and H. Arribes, Electricité de France, Centre d’Essais de Fontenay-aux-Roses.
DISCHARGES IN A METAL TORUS

Figure 13. Time-resolved spectrogram of Equator I discharge and microdensitometer trace of continuous background

Figure 14. Steatite coupling sleeve for Equator С torus
1. torus shell; 2. sleeve; 3. teflon seal; 4. conducting ring for preionization of the gas

Equator С

The new copper torus, Equator С, has the same dimensions as Equator I. Its principal distinctions are an internal coating of 0.3 mm of enamel and a Steatite insulating sleeve of the form shown in Fig. 14. This design shields the vacuum seal from the discharge and provides a long path length for possible arcs between the ends of the metal shells. Both indium and polytetrafluoroethylene (teflon) have been used for sealing the joint. With this arrangement it was shown that the current flowed entirely in the plasma and not at all in the metallic shell.

This experimental apparatus permits the use of magnetic circuits premagnetized to saturation before the discharge. Under these conditions, the product of the average induced emf, $\bar{E}$, and the characteristic time, $\tau$, before saturation is $\int_0^\tau \! E \, dt = \bar{E} \tau = 2.5 \times 10^8$ maxwells. The primary circuit is coaxial with the discharge tube and provides 31 conductors. The maximum electric field in the plasma can be varied between 2.5 and 5 v/cm by varying the condenser potential from 20 to 40 kv. The discharge characteristics depend upon whether or not the iron saturates and upon the strength of the magnetic field. Figure 15 shows typical current and voltage oscillograms for magnetic fields of 100 and 1000 gauss. For the lower field the gas discharge current, $I_2$, rises more rapidly at first but falls off very quickly.

Conclusion

The Equator С apparatus, with its enamelled copper torus and Steatite insulating sleeves that shield the edges at the gaps from the discharge, appears to give a satisfactory solution to the major problem that arose in the operation of Equator I, viz., arc suppression.

REFERENCES

Joint General Atomic–TAERF Fusion Program

By D. W. Kerst*

The thermonuclear work being undertaken at General Atomic rests strongly on the results of several theoretical studies by Dr. Marshall N. Rosenbluth concerning the stability criteria and the magnetic-field diffusion mechanism of heating for toroidal pinch discharges. Several features of this theoretical work are described in two papers prepared for this Conference.1, 2

The stability requirements call for not only an initial axial magnetic field but also a reversed axial magnetic field, which may have to be applied immediately after the discharge is formed, since it has been shown that otherwise surface instabilities will result.

The magnetic energy stored within and around the discharge is high immediately after formation of the discharge, when the central axial magnetic field and the external \( \theta \) magnetic field are still separated. As time goes on, however, the diffusion of these fields through the plasma produces a state of less stored magnetic energy and consequently of higher gas temperature as a result of the ohmic heating produced in the diffusion process. This automatic form of heating during the disassembly of the plasma structure is sufficient to bring the gas to a temperature for a thermonuclear reaction, and the parameters of the torus which are suggested by this theoretical treatment are shown in Table 1. The various quantities can be scaled to a different-sized torus, which may be desirable for practical reasons. Recently, plasmas in which the current layers are not of infinitesimal thickness have been under theoretical study.

The experimental work has consisted of several parts: the study of charge exchange in hydrogen ionic and atomic collisions, the study of some linear pinch discharge systems with high stabilizing axial magnetic fields, developments on a small scale for a large toroidal geometry, and experiments with various diagnostic methods, including electrical, optical, and shock-tube methods.

The experiments on atomic collisions have consisted of measurements of cross sections for the ionization, the excitation of Lyman-alpha radiation, and elastic scattering for the case of electron bombardment. In addition, charge-exchange cross sections between deuterons and deuterium atoms have been measured. Figures 1–4 show these results. In Fig. 4, notice that the calculations of Dalgarno and Yadav, using a perturbed stationary-state approximation (Curve E), are close to the experimental results which show a very large cross section for charge exchange.3, 4

The behavior of a linear discharge with a large axial stabilizing magnetic field was examined by small probe coils to determine the local magnetic field, and also by optical means. In the case of a rather small diameter tube (4.4 cm id) with axial magnetic fields of 15,000 to 20,000 gauss it was observed that the measurements of the field within the plasma could be continued for the duration of the discharge, since the measurements were fairly reproducible, whereas with small fields, such as 8000 gauss of axial magnetic field, the measurements were reproducible only for the first 2.5 \( \mu \text{sec} \) of the discharge. In addition, with the high axial magnetic fields, the axial discharge current began to develop in the center of the discharge tube almost immediately. The earliest observations made were about 0.1 \( \mu \text{sec} \) after initiation of the discharge—before wall impurities could get in. The current density throughout the tube had essentially the same distribution, with its maximum in the center of the tube at all times, but the magnitude of the current increased as the discharge developed (Fig. 5).

Table 1. Possible Characteristics of a Diffusion-Limited, Self-heated D-T Reactor

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
<th>Scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major radius of torus</td>
<td>30 cm (arbitrary)</td>
<td>( r_0 )</td>
</tr>
<tr>
<td>Minor radius of torus</td>
<td>6 cm</td>
<td>( r_0 )</td>
</tr>
<tr>
<td>Initial plasma radius (( r_0 ))</td>
<td>1.5 cm</td>
<td>( r_0 )</td>
</tr>
<tr>
<td>Current</td>
<td>3 \times 10^6 amp</td>
<td>( (r_0)^0 )</td>
</tr>
<tr>
<td>Pressure at wall</td>
<td>400 atmos</td>
<td>( r_0^{-2} )</td>
</tr>
<tr>
<td>Initial pinched density (( \rho_0 ))</td>
<td>1.3 \times 10^7/cm³</td>
<td>( r_0^{-5} )</td>
</tr>
<tr>
<td>Burning temperature (( T_b ))</td>
<td>6 kev</td>
<td>( (r_0)^0 )</td>
</tr>
<tr>
<td>Disassembly time</td>
<td>0.15 sec</td>
<td>( r_0^3 )</td>
</tr>
<tr>
<td>Total magnetic energy</td>
<td>4 \times 10^9 joules</td>
<td>( r_0 )</td>
</tr>
<tr>
<td>Losses (copper torus)</td>
<td>2.5 \times 10^7 joules</td>
<td>( r_0 )</td>
</tr>
<tr>
<td>Energy produced</td>
<td>3 \times 10^9 joules</td>
<td>( r_0 )</td>
</tr>
<tr>
<td>Temperature rise of copper</td>
<td>500°C</td>
<td>( r_0^{-2} )</td>
</tr>
<tr>
<td>Efficiency of burning</td>
<td>10%</td>
<td>( (r_0)^0 )</td>
</tr>
</tbody>
</table>

* John Jay Hopkins Laboratory for Pure and Applied Science, General Atomic Division of General Dynamics Corporation, San Diego, California. Research on controlled thermonuclear reactions is a joint program carried out by General Atomic and the Texas Atomic Energy Research Foundation.
With a somewhat larger tube, having a diameter of 6.8 cm, similar behavior was observed; but, in addition, a transient current sheath was superimposed on the stationary current distribution. This current sheath existed for less than 1 µsec at 2.5 µsec after the start of the discharge. Observations at lower axial magnetic fields were in general agreement with those made in other laboratories, but sharp current sheaths were not always found. Conductivity estimates and pressures derived from the magnetic field distributions in these small discharge tubes indicated temperatures of the order of 10 electron volts.

The optical observations showed a strong rising continuum of radiation occurring at around 5 µsec in the case of high axial magnetic fields, and occurring earlier in time for the case of lower magnetic fields. Much of this radiation is the result of impurities.

Further experiments are to be made with this continuum and with the use of the Doppler broadening in an attempt to attain an accurate determination of the temperature of the plasma. In order to calibrate the method at the high densities used in our discharges, where collision broadening is important, pure plasmas generated in an electrically driven shock tube are under study since the temperature in the shock front is calculable.

An analyser for the neutral particles emerging from the plasma is being constructed. This will determine the species of neutral atoms, their energy, and their time of emergence.
Figure 5. Axial and azimuthal fields and currents in the 4.4 cm diameter discharge tube
The main concern of the experimental group at present is with establishing a discharge which is free from impurities. This program involves the testing of materials and of methods of initiating the discharge with the current concentrated in the center. Multiple magnetic probes are being used to understand better the spatial distribution of some of the variations in the discharge (Fig. 6), and we are turning to toroidal geometries because of the violent erosion of the electrodes when we attempt to use currents and fields of interesting magnitudes. This toroidal geometry is composed of what might be called six small linear tubes bent into a torus and connected end to end, with the full voltage attainable in linear discharge systems applied to each of the six sectors. This is done with a flux-shielding method which consists essentially of six air-core pulse transformers linking the discharge tube and producing nowhere more than one-sixth of the total voltage around the whole discharge tube.

REFERENCES

Experimental Pinch Stabilization with Large Axial Magnetic Field


Aluminum oxide discharge tubes of 4.4 cm and 6.8 cm inside diameter and 30 cm length have been used with a capacitor bank of 5000 to 10,000 joules to examine the character of deuterium discharges with high values of axial stabilizing magnetic fields (see Fig. 1). Forty-eight 7.5 μF capacitors, each with its own type-5550 ignitron and coaxial cable combination having a total inductance of 0.42 μH, were connected to a parallel-plate header at one end of the tube. The total inductance in series with the tube was 0.016 μH and the inductance of the discharge usually was 0.055 μH. Generally, the 3 mm quartz sheath for the magnetic probe would break if the condensers were charged to much more than 10,000 V. Consequently, most of the measurements described here were made with 8000 V and fewer capacitors. Currents were 100,000 to 400,000 amp with an initial pressure of 150 μ of deuterium, and the initial stabilizing axial magnetic fields were from 1000 to 20,000 gauss.

The solenoid, which generates the axial stabilizing magnetic field, and single-turn pick-up loops indicated that there were local high voltages, between turns of the solenoid, which caused sparking and destructive effects, especially when the axial field was small. An additional thin closed, or tightly lapped, shield was necessary to prevent the escape of the high-frequency flux which was responsible for this external sparking (Fig. 1).

It is characteristic of the discharge in the 4.4 cm tube that, with an axial field of 7.25 kilogauss which should give stability at our current if a current sheath were formed and compressed to 0.5 of the initial radius, there is irreproducibility of the magnetic field, Bz, detected by a probe on successive discharges. Only during the first 2 μsec do the data reproduce; thereafter great deviations are seen which have the largest fractional variation near the center of the tube. A clear change in the reproducibility of the data occurs if the stabilizing field, Bz, is increased to 14.5 or 20 kilogauss. At these fields, the data are much more
Figure 2. Radial variation of magnetic field and current density in discharge in a 4.4 cm tube.
Figure 3. Radial variation of pressure in a discharge

reproducible on successive discharges and a small wandering of the magnetic center of the discharge takes place in the same way during each discharge. In addition, with this large $B_z$, conducting plasma appears throughout the tube with its maximum axially directed current density in the center of the tube. This is the case at all times measured—namely, from 0.3 to 10 $\mu$sec, a little past the time of peak currents. This characteristic of the discharge is shown in Fig. 2. The absence of an inwardly moving current sheath is very different from the usual observations at smaller $B_z$, which do not show the peak current density in the middle. Tests were made with the larger, 6.8 cm, tube to search for the current sheath under conditions similar to those shown by Burkhardt, Lovberg, and Phillips, and fair agreement with these authors was obtained. However, the current sheath is not very sharply defined. At high axial field this large tube also gave the peak axial current density at the tube center at all times observed.

Spectroscopic observations have been made by sighting longitudinally through slots in the electrodes. When the axial magnetic field was 7.25 kilogauss, the onset of the continuous spectrum, presumably due to wall impurities, occurred before the peak discharge current was reached. At 14.5 kilogauss initial $B_z$, the onset of the continuum was postponed until a time after the peak current had been passed.

The pressures derived from the field distributions (given in Fig. 3) show the pressure at 1.0 $\mu$sec and the pressure at 5 $\mu$sec, near the time of maximum current. Each curve has an unknown but different integration constant which must be added. It is thus characteristic of the discharge that the peak axial current density is in the center, but that the peak pressure is in a sheath around the center, intermediate between the somewhat compressed central $B_z$ and the $B_e$ resulting from the discharge.

REFERENCES
Toroidal Discharges in Deuterium with External Magnetic Field

By Kai Siegbahn and Per Ohlin*

Some experimental work will be described which concerns discharges in straight tubes and furthermore two different ways of obtaining a constricted ring discharge in deuterium. In one type of experiment a constant external magnetic toroidal field is applied and an \( H_E E_z \) pinch is formed by discharging a condenser bank with a stored energy of 48 kj. In a second type of experiment a very large bank of condensers is used, giving a stored energy of about 1 Mj, which produces a variable toroidal magnetic field with a time constant of up to 3 msec, giving simultaneously two different kinds of plasma con- striction, namely \( H_E E_z \) and \( H_Z E_Q \) pinches.

Fusion experiments in deuterium gas discharges using the pinch effect have been undertaken in Uppsala during the last two years. In some previous communications we have described some experiments concerning heavy current discharges in straight tubes, the currents being of the order of 400 ka. In those experiments a low-inductance condenser bank (\( V_{\text{max}} = 50 \text{ kv; } C = 60 \mu F; \text{ } L_{\text{short circuit}} = 6 \mu \text{H} \)) was coaxially coupled to a discharge tube 30 cm in diameter and 60 cm long (Figs. 4 and 5). The tube was made of porcelain and coaxially surrounded by an aluminium cylinder, carrying the return current, and also by a solenoid (\( B_{\text{max}} = 750 \text{ gauss} \) ) for stabilizing the plasma (Fig. 1). It was found that fairly stable conditions could be achieved during the short discharge time (\( T \sim 15 \mu \text{sec} \)). Under these conditions it was possible to observe 8 subsequent pinches (Fig. 2) or, in one case, 6 pinches (Fig. 3). In some discharges there was evidence for pinches occurring also in several subsequent half-periods following the first half-cycle.

Figure 4 shows the current and voltage fluctuations during the formation of the pinches. Usually these are accompanied by "moderate" bursts of \( \sim 10^5-10^6 \) neutrons per pulse. Figure 5 shows a typical example of the development of a fast "sausage" type of instability followed by a burst of more than \( 10^6 \) neutrons. We have never found these fast transients when there was an external magnetic field. We conclude, therefore, that this type of instability is effectively eliminated by a longitudinal magnetic field.

When the neutron yield in the fusion process was studied as a function of, e.g., the applied longitudinal magnetic field, it was found that, once the sausage instability neutrons were eliminated at fairly low fields, the neutron yield was not affected by increasing the magnetic field (Fig. 6). This weak dependence on the magnetic field does not agree with several previous findings, which show an extremely strong dependence. In some reported investigations, all neutrons have completely disappeared for fields below 100 gauss. In our case, however, at 750 gauss the neutron yield was still about 50% of its value at small fields. Our results seem to be consistent with the Aldermaston results.

We have also observed a rather infrequent phenomenon which is not yet understood, partly because of its rareness. Figure 7 is an oscilloscope trace of some relatively slow oscillations which, although they may later be found to have some trivial cause, may also be due to a plasma oscillation—probably of the ionic type because of the low frequency (\( \sim 1.4 \text{ Mc} \)). It is interesting to note that the oscillation is almost undamped.

In the early spring of 1958 two new experimental devices were constructed, both having discharge tubes of toroidal geometry. The smaller of these devices (Fig. 8) consists of an aluminium torus (\( \Phi_1 = 8 \text{ cm; } \Phi_2 = 60 \text{ cm} \)) surrounded by magnet coils

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* Institute of Physics, Uppsala.
Figure 2. dI/dt oscillogram showing six successive pinches, \( p = 0.009 \text{ mm Hg}; B_z = 750 \text{ gauss}; \text{Sweep: } 1 \mu \text{sec/cm}; V_0 = 23 \text{ kV} \)

Figure 3. dI/dt oscillogram showing six successive pinches, \( p = 0.01 \text{ mm Hg}; B_z = 300 \text{ Gauss} \)

Figure 4. Neutron yield as a function of externally applied magnetic field

Figure 5. Neutron yield as a function of externally applied magnetic field
and split into two sections with viewing windows in the insulating connections. The primary in the current transformer is a coaxial metal tube connected to the condenser bank \((C = 1640 \mu\text{f}; V_{\text{max}} = 7.5 \text{ kv})\) via a triggered spark gap. A high-frequency transmitter connected to the discharge tube forms the pre-ionizing unit. By means of the applied magnetic field, one can achieve a confinement of the plasma extending over a quarter of a cycle, i.e., about 70 \(\mu\text{sec}\) according to Fig. 9. The characteristics of the oscillogram are similar to those obtained with, e.g., Sceptre III, showing a drop in resistance when the plasma is leaving the wall and a corresponding increase in resistance when it is touching it again at the end of the confinement period. With this device it is possible to achieve maximum currents of 200 ka.

The second device is much bigger than the previous one and is supplied with a condenser bank giving a stored energy of up to 1 Mj \((C = 8200 \mu\text{f}, V_{\text{max}} = 15 \text{ kv})\). The condensers are arranged in an open circle around the discharge unit. Figure 10 shows how the 960 condensers are coupled in 40 separate groups, each consisting of 24 condensers. They are rated at 7.5 kv max, under the present conditions, but the voltage of the bank is raised to 15 kv by means of a series connection within each group. This way of coupling also allows the condenser cases to be kept at ground potential, with the high tension poles symmetrical to earth. Each group is connected to the discharge device by means of a coaxial cable, a separate spark gap being provided to each cable.

As the condenser bank is subdivided for safety reasons, the division must be maintained also within the discharge device, which can be regarded as a core-free transformer with two sets of primary windings and the gas as a secondary. One primary gives an induced voltage in the direction of the axis of the torus body, here called \(H_z-E_z\) field; the other gives a magnetic field in this direction, here called \(H_x-E_x\) field. Figure 11 shows the primary windings in a wooden frame and also the three-electrode spark gap arrangement.

Close to the torus body there are thus 40 coils tightly packed together to give an \(H_x-E_x\) field inside the torus. These coils consist of 4 separate copper windings, cast in araldite to a wedgelike shape to fit around the torus. Each winding has 6 turns and by series or shunt connection of the windings—6, 12 or 24 turns—different time constants can be chosen for the discharge. A maximum magnetic field of about 40,000 gauss can be obtained in the \(H_z\) field. The coils have been tested at more than twice this value without breaking. Outside the \(H_x-E_x\) coils there are 80 turns to give a field perpendicular to the first one: an \(E_y-H_y\) field. There are thus two turns for each group of condensers, which can be connected in series or in parallel at will.

Normally both primaries should be connected to each other and to the condenser bank in such a way that they follow each other in phase during the discharge, i.e., have the same frequency. This restricts
the ways of coupling to 17, but if different frequencies are allowed for the two fields even further possibilities exist. The primary current varies, of course, with the coupling. It can be as high as 2000 ka for one of the couplings. With both kinds of fields fed from the condenser bank, the peak primary current for the $H_x-E_z$ field can reach a maximum of 1000 ka and for the $H_y-E_z$ field a maximum of 1600 ka. The frequency for the different couplings varies between about 350 and 2600 sec$^{-1}$.

On account of the very large current, the mechanical strength of the entire apparatus becomes of utmost importance. The solution chosen is to press both kinds of primary windings in two heavy blocks of wood which, in their turn, are held together by means of strong bolts. The $H_y-E_z$ windings are held in place directly on the wooden blocks by means of grooves but the $H_x-E_y$ coils are only pressed together between the blocks which, for this purpose, are machined to small tolerances.

There is a spacing, between the coils, which gives access to the torus body for vacuum pumping and for observing the discharge through a window.

Of essential importance for getting consistent results are good preionization of the gas and reliable, simultaneous triggering of the 40 spark gaps. The former is done by means of a capacitive coupling of the gas to a damped high frequency oscillation, initiated by the discharge of a condenser bank. The spark gaps consist of three carbon electrodes. Two main electrodes are completely separated by a third electrode which, on triggering, is given a high tension oscillation from a condenser discharge.

The toroid discharge tube itself has the external dimensions: $\phi_1 = 28$ cm; $\phi_2 = 130$ cm. The tube material is either porcelain or metal. In the former case the tube is surrounded by a metal sheet.

The whole experimental arrangement was built to provide a flexible research tool for the study of different features of high temperature plasma.
Record of Proceedings of Session A-7

Controlled Fusion Devices, Part I

THURSDAY MORNING, 4 SEPTEMBER 1958

Chairman: Sir George P. Thomson (UK)
Vice-Chairman: Mr. O. Kofoed-Hansen (Denmark)
Scientific Secretaries: Messrs. T. Coor and C. Sanchez del Rio

PROGRAMME

P/1860 Review of controlled thermonuclear research at Los Alamos for mid 1958.............. J. L. Tuck et al.
(Presented by J. L. Tuck.)

DISCUSSION

P/1519 The design and performance of ZETA.............................................. E. P. Butt et al.
(Presented by R. S. Pease.)

P/2 Theoretical problems associated with ZETA......................... W. B. Thompson et al.
(Presented by R. J. Tayler.)

DISCUSSION

P/2226 Stable plasma column in a longitudinal magnetic field ................. I. N. Golovin et al.
(Presented by I. N. Golovin.)

P/2527 Investigations of the stability and heating of plasma in toroidal chambers.............................................. G. G. Dolgov-Saveliev et al.
(Presented by E. I. Dobrokhotov.)

DISCUSSION

P/1181 Experimental studies of the pinch phenomenon.............................. R. Aymar et al.
(Presented by P. Hubert.)

P/1062 Joint General Atomic-TAERF fusion program .............................. D. W. Kerst.

P/147 Toroidal discharges in deuterium with external magnetic field........ K. Siegbahn and P. Ohlin
(Presented by K. Siegbahn.)

DISCUSSION

DISCUSSION OF P/1860

Mr. J. Kistemaker (Netherlands): Mr. Tuck stated that the radial electrical field strength in Ixion is maintained between the wall and the central electrode. First, can anything be said about the central electrode's being positive or negative with respect to the wall, or how much the current-voltage characteristics change, if at all? Secondly, how well does the central plasma column behave as an electrode?

Mr. J. L. Tuck (USA): These are highly pertinent questions. First of all, the effect of changing the polarity was described theoretically. Experimentally, there undoubtedly appears a difference in polarity, but I cannot remember which way it goes.

Secondly, the central plasma column apparently works quite well and does, indeed, impose this radial electric field with longitudinal electric field.

Mr. M. A. Leontovich (USSR): Mr. Tuck, in what
way do you deduce the absence of impurities from the walls for the whole half period, 12 \(\mu\)sec?

Mr. Tuck (USA): The experiment consisted of monitoring the performance of the device, impurity-wise, as the machine was run systematically and frequently. It was found that there was a steady decrease in the appearance of impurity lines. Therefore, it was decided to continue this process until it stopped improving. At the end of so many thousand discharges, the impurity lines were zero in the first half cycle. The current was quite large, about 200,000 amp. The wall was of alumina, \(\text{Al}_2\text{O}_3\), which is apparently able to stand the taking off from it of a one hundred thousand ampere sheath without contributing impurities. This, I would imagine, is only temporary because more powerful pinches ultimately would have to damage the walls.

Mr. P. Hubert (France): Mr. Tuck, in what form is the energy transferred to the wall in the stabilized pinch?

Mr. Tuck (USA): We know the energy input and, we think, the pressure and temperature of the pinch. We therefore, by inference, deduce that energy must be deposited on the wall. We do not know the nature of the transfer of energy from the pinch to the wall, but arithmetical considerations show that it is very easy to dispose of the very large energy input by an occasional particle getting to the walls. It does not take very many particles to dispose of these rather large energy inputs like 900 Mw. There are also workers in other laboratories of the United States who do have information about the nature of this.

Mr. Y. B. Fainberg (Ukrainian SSR): Mr. Tuck, is there any experimental evidence for the existence of a second cause of instability—plasma oscillation?

Mr. Tuck (USA): I would like to talk about the theory first. There is a great body of work, especially in Russia, as a matter of fact, on plasma oscillations. Quite recently, in America, Bumemann of Stamford very wisely pointed out the possibilities of extremely rapid growth that these oscillations have. There is also a new mechanism being proposed for the excitation of plasma vibrations called the "violin string" mechanism. It is not as fast producing as what I call the Akhiezer-Bunemann process but, on the other hand, is very much there.

The criterion for their excitation is that the longitudinal velocity of the electrons should be greater than the thermal velocity itself. If you look at these pinches and make these calculations, you will find that they are just about marginal. The ratio of mean longitudinal drift speed to thermal speed is one-half, and one is the critical point: under these conditions, no measurements can be trusted to better than one-half. With regard to any evidence for seeing these plasma vibrations—there is undoubtedly the emission of very intense radiation from these devices. For instance, microwave crystals at the ends of waveguides are burned out by the intensity of radiation. However, there is a difficulty, and we are developing a combined microwave-infrared detection technique. The wave length which is expected for the plasma oscillations in free space is 1 mm, a very awkward length.

DISCUSSION OF P/1519 AND P/2

Mr. M. Hoyaux (Belgium): Messrs. Tuck and Pease, can the discrepancy between the power figures you have given for the maintenance of the stabilized pinch current be explained by the presence in ZETA and the absence in Columbus S-4 of substantial amounts of impurities?

Mr. R. S. Pease (UK): I think this is possible, but I do not think that the power is lost by radiation from impurities in what we call the clean discharge of ZETA. I do not know what the discrepancy is due to, but I think we might look at the scaling laws to see whether they give a clue to the mechanism involved.

Mr. P. Hubert (France): Mr. Taylor, the electric field transients in ZETA are probably greater than the voltage transients measured directly by the voltage loop. Have you included such effect in calculating the production rate of runaway deuterons?

Mr. R. J. Taylor (UK): I am fairly certain that such an effect has not been included. Dr. Thompson considers that he can produce the right number of neutrons from a simple mean electric field. If the other effect is as important, we might have the problem of wondering why we do not see more neutrons than in fact we actually see.

DISCUSSION OF P/2226 AND P/2527

Mr. G. N. Harding (UK): Mr. Golovin, what experiments were made to determine the perturbing effect of the Rogowski belts on the discharge?

Mr. I. N. Golovin (USSR): Experiments have been conducted both with and without the Rogowski belt. The Rogowski belt had no appreciable effect on the light pictures or the volt-amper oscillograms.

Mr. J. L. Tuck (USA): Mr. Golovin, why does the temperature of the plasma, as indicated by its conductivity, remain so low for the cases of long duration (1000 \(\mu\)sec), high \(B_0\) (15,000 gauss) pinches?

Mr. Golovin (USSR): The temperature was not measured directly but calculated from the conductivity: in the formula, allowance is made at the temperature of the plasma, as indicated by its conductivity, remain so low for the cases of long duration (1000 \(\mu\)sec), high \(B_0\) (15,000 gauss) pinches.

Mr. A. E. Robson (UK): Mr. Dobrokhотов said that arcing cannot occur with a stainless steel torus lining. Has he considered arcing due to potentials between plasma and wall, and can the impurity concentration in the discharge be accounted for otherwise than by arcing?
Mr. Dobrokhotov (USSR): The authors are thinking only of an arc which can be set up in the gap between the conducting parts of the torus. The characteristic traces of an arc were not observed on the inside of the torus on dismantling the torus chamber. The presence, in the discharge, of impurities from the material of the wall is, in fact, explained by the contact of the discharge with some parts of the wall.

Mr. P. Hubert (France): Mr. Dobrokhotov, the oscillograms which you have shown, concerning the discharges in the quartz torus, show irregularities identical to those which we have observed in similar conditions. How do you explain this? I mean the irregularities which look like voltage sparks followed by high frequency oscillations down to the discharged state. In our experiments we wondered whether it was a failure of the apparatus or the excitation of plasma waves.

Mr. Dobrokhotov (USSR): Not enough experimental data have yet been collected to enable us to point to a definite mechanism accounting for this irregularity.

Mr. R. F. Post (USA): Mr. Dobrokhotov, have you measured the ratio of thermal to turbulent motion of the impurity atoms?

Mr. Dobrokhotov (USSR): These measurements have not yet been carried out.

DISCUSSION OF P/1181, P/1062 AND P/147

Mr. Fitch (UK): Mr. Siegbahn, how soon do you get down to a bank inductance of 0.006 \( \mu \)h? It is difficult to reconcile the voltage oscillations in the pinch with the \( \frac{d}{dt} \) peaks that you show.

Mr. K. Siegbahn (Sweden): This self-inductance was at short circuit, so it is higher when the discharge tube is there.

Consideration of additional questions was deferred until an informal meeting in the afternoon.
# Session A-9

**CONTROLLED FUSION DEVICES, PART II**

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A Summary of the Berkeley and Livermore Pinch Programs

By Stirling A. Colgate*

The containment of a plasma by a magnetic field can take either of two rather distinct forms. In one case the plasma causes only a small perturbation in a static vacuum magnetic field. In the other case currents flowing within the plasma may be said to generate or bound the magnetic field, thus creating configurations that are radically different from the vacuum field configurations. The pinch program is concerned with phenomena of the latter kind.

Since pinch-type devices are not limited to vacuum magnetic fields, a much greater degree of freedom exists in designing useful configurations. For instance, plasma stability is often improved to a marked extent by sharply defined or even oscillatory spatial magnetic field distributions, which are hard to achieve in a vacuum. It is almost redundant to point out that pinch-type configurations are suited to the containment of plasma energy densities that are comparable to the energy density of the containing magnetic field. This is an important feature in the economics of any potential thermonuclear reactor.

Because pinch magnetic fields are dependent on plasma current, they must disappear as the pinched plasma escapes from containment by instability mechanisms or by diffusion. At low plasma temperatures the plasma diffusion rate is rapid. Consequently the containment times thus far achieved even in stable plasma configurations have been short, meaning some tens of microseconds in devices of 10-cm size. In order to progress toward practical thermonuclear devices, the principal objective must be to prolong containment times by improving the electrical conductivity of the plasma. Those pinch configurations which are grossly unstable are, of course, unsuitable for practical thermonuclear work. Therefore our purely dynamic experiments are conducted only to study basic shock heating and instability mechanisms.

As the plasma escapes from containment, and the pinch magnetic field collapses, the plasma acquires something like the energy density of the magnetic field. In the ideal sense this corresponds to heating, but from a more realistic standpoint it has the more disastrous consequence of loss of containment. Apparently part of this energy is lost in the form of high-energy particles striking the wall and part in the form of impurity radiation. Neutron emission has been observed in most of the experiments, but the relevance of small neutron yields to bulk plasma temperature is not obvious. Therefore our basic evaluation of progress in pinch-type experiments is the reduction of the dissipation rate of the magnetic fields. This is because dissipation destroys containment, and containment is the most fundamental goal of the magnetic bottle. If you have long-time containment, you can achieve high temperature by heating at your leisure, but if you have heating with no containment, then you have neither containment nor high temperature.

My present pessimistic viewpoint is that most of the pinch devices that depend upon high current density within the plasma are beset with an enhanced dissipation rate which is disastrous to pinch containment. This dissipation is derived either from an electron plasma current instability (analogous to the wriggling of a fire hose with high-velocity water flowing through it) or from hydromagnetic turbulence. Both have been predicted in theory and observed in experiment.

**STABILIZED PINCH**

(Ferguson and Furth)

One of the principal efforts is directed towards understanding and utilizing the stabilized pinch effect for the containment of a thermonuclear plasma (Fig. 1). Stability can be achieved by entrapment of an axial magnetic field \( H_s \) in the pinch column, provided a number of secondary conditions are met. The pinch radius must be kept larger than one-fifth the radius of the return-conductor shell, and the plasma pressure must be low compared to \( H_s/\sqrt{\pi} \). The region containing the hot plasma and the \( H_s \) field must be sharply bounded from that containing the pinch field \( H_p \).

These theoretical stability criteria of Rosenbluth have been tested in linear pinch experiments. Under improper conditions, in particular for substantial \( H_s \) appearing in the external \( H_p \) field region, the helical instability mode was induced. The magnetic field behavior in time and space was studied by means of

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* University of California Radiation Laboratory, Livermore, California. This summary covers the work of O. A. Anderson, W. R. Baker, A. Bratenahl, W. B. Kunkel and R. V. Pyle at the Berkeley Laboratory and of J. P. Ferguson, H. P. Furth and R. E. Wright at the Livermore Laboratory.
magnetic probes. The occurrence of instabilities was observed synchronously with probes and with optical equipment. The sign of the helical instability was shown to depend on the direction that $H_z$ has in the $H_0$ field region. The interdiffusion of the $H_z$ and $H_0$ regions was found to proceed at a rate derived from pressure balance analysis of the magnetic probe data. These results correspond closely to calculated equilibrium temperatures for plasma in thermal contact with cold electrodes. The region external to the pinch proper (the $H_0$ field region) was found to contain not a vacuum but a pressureless plasma of good conductivity. It follows that sharpness of boundary between the $H_z$ and $H_0$ field regions is contingent on rapid diminution of $H_z$ flux external to the pinch as the plasma pulls away from the tube wall. This can be accomplished electrically by introducing a cancelling $H_z$ flux external to the pinch as the plasma pulls away from the tube wall. $H_z$ distributions which oscillate in space have also been established by oscillating $H_z$ at the tube wall in the time during pinch compression.

To achieve higher temperatures and so impede resistive interdiffusion of the $H_z$ and $H_0$ field regions, experiments are now being conducted in toroidal geometry (Fig. 2). The pinch current is generated by induction, and the drainage of heat into electrodes is eliminated. Preliminary experiments performed at low power level have allowed approximate reproduction of linear pinch results. As $H_z$ flux expands by diffusion into the $H_0$ field region, its energy content is diminished and the plasma is heated correspondingly. A pinch surface layer several diffusion depths thick is theoretically given a third of the characteristic temperature $T_c = H_z^2/8\pi nk$, provided radiative losses are small. How much of the pinch can be heated depends on the diffusion depth tolerable from the stability point of view. In actual experiments with a Gamma torus, with a $10^4$ joule, 30 kev, 300 ka bank (Fig. 3) and a ceramic toroidal pinch of 10 cm minor diameter (Fig. 4), we have found little evidence that the plasma temperature exceeds that attained in the linear pinch work. The electrical conductivity corresponds to about 10 ev, even though more than 5000 ev per particle is dissipated resistively in the first quarter-cycle of the current. Neutron yields corresponding to about 300 to 500 ev ion temperature are observed, but these appear to be due to nonthermonuclear mechanisms of ion acceleration. We observe very great sensitivity of neutron yields to the steepness of the trapped $H_z$.
distribution in the pinch. Plasma heat-loss rates of the observed magnitude are hard to explain except by heavy element impurity radiation, or by some equally disastrous mechanism. During the first quarter-cycle time of our toroidal pinch we have excluded by time-resolved vacuum ultraviolet measurements the possibility of impurity radiation loss. We have discovered instead a catastrophic loss of runaway 3 to 5 kev electrons. The flux of runaway electrons striking the wall, as measured by the soft X-ray flux, is comparable in magnitude to the pinch current of 250 ka. The implication of the non-adiabatic behavior of these electrons, namely that they move freely across the magnetic lines of force of the pinch, is that they represent a disastrous energy-loss mechanism. We suspect either a current-induced instability or hydromagnetic instability that drives plasma turbulence, resulting in the nonadiabatic electron behavior.

It seems likely that the high electrical resistivities that have been observed should be explained in terms of this effect, rather than in terms of a very low electron temperature. If so, one cannot anticipate an improvement at higher power levels and consequently the so-called stabilized pinch may turn out to be unsuitable as a useful magnetic bottle.

SHEET PINCH DEVICES
(Anderson, Baker, Kunkel and Pyle)

A pinch can be stabilized by means other than the entrapment of an axial magnetic field. It is predicted theoretically that a current-carrying plasma sheet of infinite extent possesses positive stability for some perturbation modes and at least neutral stability for others. This stability is approached in sheet pinch devices of modest size. Both the flat sheet and the cylindrical, edgeless configuration (the latter having been given the name “Triax”) are being studied. The Triax pinch device is shown in Fig. 5.

An obvious disadvantage of the sheet-like plasma is the relatively small compression ratio that is obtainable at a given current, as compared to an unstabilized filamentary pinch. This feature is, however, also characteristic of ordinary pinches stabilized by longitudinal magnetic fields. The principal advantage of the Triax pinch over the stabilized linear pinch is the absence of magnetic field lines intersecting metal electrodes. Experiments with thermocouples in the electrodes and variation of effective tube length by means of inserted floating “dummy” electrodes indeed both point to low heat transfer to the electrodes, in agreement with theoretical predictions.

Determination of the current density distribution as a function of time, using magnetic probes in low power level discharges ($T \approx 10$ ev, starting density 400–1000 $\mu$ of $D_2$, mean compression factor 5, peak current $\approx 6 \times 10^5$ amp, total energy supply $\approx 10^4$ joules, 50 $\mu$, 20 kv), clearly shows well-reproducible, repeated oscillations of the plasma sheet thickness (Fig. 6). These results are in full agreement with inductance calculations made from the tube voltage. Under suitable conditions, up to ten oscillations can be
discerned (lasting for about 3 μsec) and there is no sign of hydromagnetic instability. In fact, time-resolved spectroscopic observations in the visible part of the spectrum indicate full ionization of deuterium after the second pinch and low recombination until the second current reversal. The observed resistance of the plasma is in agreement with that calculated using the temperature and the mean compression deduced from probe measurements.

In the attempts to obtain a temperature sufficiently high for thermonuclear reactions, lower initial gas densities (down to 75 μ of D₂) and higher currents (up to I_{max} ≈ 1.5 × 10⁶ amp) are being used. In this case slow starter currents of about 4 × 10⁴ amp peak and about 10 μsec duration are needed to ionize the gas fully and preheat it, presumably to above 10-ev temperature, without causing appreciable compression. Sudden pinching of this plasma by the main discharge current is capable of producing several hundred ev temperature after thermalization. Again, several bounces of the plasma sheet thickness are observed, smaller and more rapid now than at low level. After these bounces a burst of neutrons (of the order of 10⁴ to 10⁵ per discharge) lasting for a few tenths of a μsec is detected. A thermonuclear explanation would require an ion temperature of 300–400 ev. Simultaneously a “bump” in the tube voltage, 2–3 kV high, is seen indicating a distinct temporary increase in tube impedance. The exact cause of this increase in tube impedance has not yet been ascertained and hence the nature of the neutron production is still uncertain. Direct acceleration of deuterons along the plasma seems to be ruled out because subdivision by insertion of the dummy electrodes did not appreciably reduce the neutron production. Emission of visible impurity light, predominantly from silicon ions, sets in during or shortly after the voltage bump. The observed resistance of the discharge points to an electron temperature of probably not more than 50 ev, yet ten times this energy per particle is introduced and so again we are beset with a dissipation rate that is not yet understood.

**HOMOPOLAR**

(Anderson, Baker, Bratenahl, Furth and Kunkel)

Long-time containment of a plasma is usually associated with quasi-static configurations where the plasma is at rest in the laboratory frame. However, containment of a long-lived thermonuclear plasma can theoretically be achieved in systems in which the plasma is in rapid rotation.

Of the many interesting rotational plasma configurations, the homopolar geometry has thus far received most attention (Fig. 7). Here a radial current and an axial magnetic field are used to set plasma into azimuthal drift motion. A transient radial current flows during the acceleration stage, and this can serve incidentally to pinch the plasma axially away from insulating end plates. The axial magnetic field employed in practice has been of the mirror machine type. The centrifugal force associated with plasma rotation tends to hold the ions away from the axis, and there-
fore traps them in the region of bulging field. Thus axial containment is achieved at first by the azimuthal pinch field and later by the centrifugal trapping effect. Rotational kinetic energy can be converted to plasma heating by viscosity or turbulent mixing, and so a measurement of the rotational decay time determines the containment achieved.

The experimental program to date has been directed exclusively to documenting the physics of rotating plasma using primarily argon. Plasma angular momentum has been measured unambiguously as a function of time, by short-circuiting the machine at various stages of its cycle and observing the charge drawn out. Ion kinetic energies above 500 ev in argon and 50 ev in deuterium have thus been demonstrated. Doppler-shift measurements indicate, if anything, still larger energies. The presence of very high centrifugal plasma pressure has been shown by the deformation of the starting magnetic field. For 5-μsec plasma-acceleration times, substantial plasma rotation has been observed up to 200 μsec later. Several large homopolars, having thermonuclear potentialities, have been completed (Fig. 8). An important incidental use of the homopolar machine is as a fast-discharge capacitor. Dielectric constants of $10^6$ to $10^7$ have been produced with ease, and output-current rates of rise of $5 \times 10^{11}$ amp/sec have been generated.

**Figure 8.** Homopolar with nonuniform field; also short-circuit signals of current to and from homopolar showing conservation of charge and hence rotational angular momentum for 160 μsec.
One very critical test of the division of energy between ions and electrons in a shock is the observation of a thermonuclear yield. For this and other obvious reasons we are attempting to observe a transient but provable thermonuclear neutron yield from a partially stabilized dynamic pinch (Fig. 9).

It is well known that the standard dynamic pinch is subject to sausage (or $m = 0$) instabilities which grow nonlinearly and effect the rapid self-destruction of this pinch configuration. The study of thermonuclear reactions in the dynamic pinch is made doubly difficult because the copious nonthermonuclear yield from $m = 0$ instability mechanisms masks the true yield, and also because the rapid deterioration of the pinch geometry prevents further heating by adiabatic compression following the hydrodynamic stage. If the $m = 0$ instability is inhibited by the use of an axial magnetic field, then the helical (or $m = 1$) mode dominates, but the time scale of instability growth to destruction is extended. A disadvantage of included axial magnetic field is that it weakens the hydrodynamic ion interaction and serves to energize the plasma electrons instead. For this reason, as well as for reasons of optimizing plasma versus magnetic field energy content, the stabilizing axial field is applied to the pinch column in the form of an encasing sheath. The feasibility of this "screw-dynamic pinch" configuration was deduced from stabilized pinch experiments proving the space outside the pinch column to be not vacuum but conductive pressureless plasma. The configuration is generated simply by imparting a slight helical pitch to the pinch tube return conductor. The resultant field distributions have been derived analytically and verified experimentally in some detail. For high compressions, the magnetic energy storage in the screw-dynamic pinch approaches that in the standard dynamic pinch, or half that of the dynamic pinch with 50% internal stabilizing field pressure. Preliminary experiments conducted with a 15,000 joule, 70 kv, 250 ka supply have shown the expected hydrodynamics and instability pattern. Neutron pulses of about $10^5$ were recorded coincident with the $m = 1$ instability and correlated in magnitude with the violence of this instability rather than the applied voltage. The neutron yield was unaffected by the presence of substantial fractions of helium, but could be totally suppressed at 300 microns starting pressure. The mechanism of production is not understood, but it is thought that Fermi-type acceleration of fast deuterons rather than bulk heating of the plasma might account for the yield observed. A bank of $10^6$ joule energy storage and 300 kv is being applied to the study of a larger screw-dynamic device (Figs. 10 and 11).
THE PINCH EFFECT†

In a plasma-containing device of the pinch type the magnetic field is generated chiefly by currents flowing in the plasma itself. The pinch magnetic field tends to impart energy to the plasma by means of Joule heating and compression, and by various instability mechanisms. In these processes the plasma can be expected to acquire an energy density something like that of the magnetic field itself.

As magnetic field energy is translated into plasma energy the plasma becomes more difficult to confine, and eventually it is lost altogether. One of the chief problems in pinch work is to keep the plasma from acquiring energy too fast, and so being released prematurely from its confinement.

In the ordinary dynamic pinch the magnetic field energy goes very quickly into the development of catastrophic instabilities. Such a device is clearly unsuitable as a practical fusion reactor, because it does not provide sufficient reaction time for the plasma. We have therefore been interested in producing pinches that are stabilized by an included longitudinal magnetic field. If present theories prove correct such a machine can serve as a highly economical type of fusion reactor.

Stabilizing the Pinch

The fastest growing instability of the ordinary dynamic pinch is the \( m = 0 \) or sausage-type instability. That a moderate amount of longitudinal magnetic field will stabilize the pinch column against this instability has been recognized by many researchers. A calculation of instability growth rates was made in 1953 by Kruskal and Tuck, who found that in the presence of a longitudinal field the \( m = 1 \) or corkscrew instabilities of long wave length tend to predominate. Linear pinch stabilization by means of a longitudinal field plus external conducting shell was analyzed in the magnetohydrodynamic approximation by both Kruskal and Rosenbluth as early as 1954. Essentially the same analysis has also been made by Shafranov and Tayler. The preconditions for absolute pinch stability were apparently not recognized until, at the suggestion of Colgate, the well-known calculations by Rosenbluth were carried out in 1956. It then became clear that pinch stability depends critically on the absence of longitudinal magnetic field from the region external to the pinch. If this condition is strictly met the pinch is stable up to radial compressions of 5:1 for null plasma pressure, and up to compressions of 2.5:1 for a plasma pressure equal to half the pinch pressure. If, however, the longitudinal field strength external to the pinch is as much as half of the pinch field strength at the plasma surface, then the pinch becomes unstable at 1.8:1 compression, even for null pressure.

Some early experimental work and intuitive interpretation anticipates the results of the Rosenbluth theory, while other classical experiments seem to have fallen into the unstable region of operation. The \( m = 1 \) instability was first detected by us experimentally, using the technique illustrated in Fig. 1. A loop surrounding the glass pinch tube measures the amount of \( H_z \) flux that has been pulled into the tube during pinch formation. The ratio of the inside diameters of the metal and glass tubes is denoted by \( \delta \). Some integrated signals are displayed in Fig. 2. In the stable regime (\( \delta = 1.1 \)), one sees a well-defined increase of flux inside the tube, corresponding in magnitude to the \( H_z \) flux initially located between the glass and metal walls. In the unstable regime (\( \delta = 1.3 \)), the signal starts out exactly as before, but suddenly becomes anomalously large. The \( H_z \) flux between the glass and metal walls has been reversed. This is exactly what is expected to happen when the \( m = 1 \) instability appears (cf. Fig. 1). One also expects an asymmetric \( H_z \) distribution to appear at the same time. This effect is measured by \( H_z \) flux loops connected at diametrically opposite points just outside the glass tube. The integrated signals are displayed in Fig. 3. An image converter picture taken at the time of maximum difference signal in the \( H_z \) loops is shown in Fig. 4.

Note that pinch stability is obviously much worse for \( \delta = 1.3 \) than for \( \delta = 1.1 \), other things being the same. The ratio \( \delta \) is a measure of the \( H_z \) flux left outside the pinch column when a pinch is formed. The traces of Fig. 2 therefore demonstrate the predicted sensitivity of pinch stability to external \( H_z \) field. By means of many similar measurements, the basic outlines of the Rosenbluth theory have been

† First portion of the paper, originally submitted under the title The Stabilised Pinch.

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confirmed. Since this work was done mostly in tubes of small size (3 in. id), the duration of the discharge was not sufficient to prove conclusively the absence of slowly growing instability modes. For reasons of electrical resistivity, the duration of pinch discharges scales as the square of the tube radius; on the other hand, instability growth times scale directly as the radius. A very large stabilized pinch therefore allows a much more convincing check on the stability theory. This is one of the reasons why the advent of ZETA has been of such importance in stabilized pinch development.

The Rosenbluth stability theory treats the extreme case where the pinch current is a surface current on an infinitely conductive plasma column. The actual experiments, of course, are done on pinches where the current has some volume distribution that may be more or less peaked on the pinch surface as discussed below. The stability theory for volume current distributions is being investigated by Tayler, Rosenbluth, Suydam and others. It is found that if sharply defined magnetic field distributions and surface currents are replaced by "diffuse" volume distributions and broad current-carrying regions the effect is definitely harmful to stability. Rosenbluth has already treated the case of non-scalar plasma pressure. This analysis has been extended by Watson, Kaufman and Chandrasekhar. The main result is that the stability regions outlined by the simple hydromagnetic analysis do not generally impose sufficient conditions for absolute pinch stability when the plasma pressure is anisotropic. Calculations of stability in toroidal geometry are being carried out by Mittleman and Newcomb.

Stability measurements on toroidal pinches have yielded results rather similar to those obtained in linear geometry. Figure 5 shows $H_z$ and $H_\theta$ measurements made by probes situated at the glass wall of a 4 in. minor diameter 6:1 torus (without iron core). At low compression ratios the discharge is apparently
A TOROIDAL STABILIZED PINCH

Figure 5. Magnetic field measurements at the wall of a toroidal stabilized pinch, for various values of initial $H_z$.

Table: Magnetic field measurements

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<thead>
<tr>
<th>$H_z$</th>
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Figure 6. Simultaneous probe and image-converter data on a toroidal stabilized pinch.

$H_2$ initial is 1000 gauss, maximum $H_9$ at the wall is 4000 gauss, initial deuterium pressure is 200 $\mu$m.

The timing of the image-converted picture is indicated by a marker on the probe signal.

Stable, but at high compressions the $H_z$ field becomes negative outside the pinch column. One suspects an $m = 1$ instability, and this is verified by the simultaneous probe and image-converter measurements shown in Fig. 6.

A characteristic feature of the toroidal pinch is that instability wave lengths must be integral factors of the torus circumference. This is especially important at very low pinch currents (below the Kruskal limit), where all instability wave lengths are greater than the circumference. Just above the Kruskal limit, the pinch tends to pass through an unstable regime, because the $H_z$ field outside the pinch column is, in general, still large. For $\delta = 1.3$, the region of pinch stability is re-entered after a 1.06:1 compression; for $\delta = 1.1$, the required compression is only 1.006:1. These figures are of some relevance to low-level pinch pre-heating.

Plasma Containment and Heating in the Stabilized Pinch

Since the plasma of a stabilized pinch has finite electrical conductivity, both the plasma and the $H_z$ field will tend to diffuse out in the course of time. As a result, the energy content of the magnetic field configuration is diminished, and the plasma is heated. The electrical skin depth in the plasma is a good measure of diffusion distance as a function of time. If finite thermal conductivity is allowed for, but radiation losses are neglected, it is found\(^{11}\) that the mean plasma temperature over a skin depth in from the pinch surface is about

$$T_p = \frac{1}{3}T_e = \frac{H^2}{2\pi n k},$$

where $H$ is the magnetic field amplitude and $n$ the particle density in the undisturbed region of the pinch. $T_e$ is the maximum temperature compatible with pressure balance at density $n$.

As the $H_z$ and $H_9$ fields interdiffuse, the stability of the pinch diminishes (cf. the delayed instability signals of Fig. 2). On the same time scale on which pinch stability is lost, the plasma of the pinch is heated. In making a practical thermonuclear machine of the pinch type, one must therefore arrange for the plasma to gain energy fast enough to overcome radiation losses.
Magnetic probe measurements made on linear pinches (Fig. 7) point to magnetic field distributions differing to a considerable extent from the ideal stabilized pinch configuration. At the same time, the conductivity of the plasma appears fairly good. One obtains temperatures of 20 to 50 ev, depending on whether the conductivity is inferred from the diffusion rate of $H_9$ or $H_e$. Slightly higher temperatures can be arrived at by a pressure-balance calculation (Fig. 8) based on magnetic probe measurements. This method, however, is open to some question because the magnetic field inside the glass envelope of the probe need not be identical with the field outside. A maximum temperature of 50 ev is expected from an analysis of heat conduction loss to the electrodes. A 50 ev temperature corresponds to about $\frac{1}{2}T_e$. The density of these pinches is generally such (initial $D_2$ pressure, 170 micron) that the ion and electron temperatures must remain closely the same.

**Flux Trapping Outside the Pinch**

In view of the fairly respectable electrical conductivities observed, we were at first somewhat puzzled by the lack of sharpness of the $H_e$ distributions measured by our magnetic probes. As has been remarked earlier, the sharpness of the $H_e$ distribution is a matter of considerable practical concern, because of its relevance to pinch stability. Part of the diffuse-ness of measured distributions is certainly due to plasma resistivity, and part can be blamed on the distortions inherent in probe measurements. There is, however, a more basic reason for broad regions of mixed $H_e$ and $H_9$. It can be shown both theoretically and experimentally that the space between the pinch column and the pinch tube wall is not a vacuum, but a good electrical conductor.

No experimentalist who has contended with occlusion problems is likely to doubt that the external particle density is at least $10^{-8}$ times the initial density. For an initial electron density in excess of $10^{16}$ per cm$^3$, the external "pressureless plasma" will have a density in excess of $10^{18}$ electrons/cm$^3$. The electron drift velocity at as high a current density as 1000 amp/cm$^2$ corresponds to less than 100 ev energy for each electron, so that carrier density is no limitation. Accordingly, the electrical conductivity of the external plasma is determined principally by its temperature. If the pinch is diffusion-heated, the dilute external plasma should tend to become even hotter than the dense plasma inside the pinch proper, but impurity radiation may be the controlling factor here. At any rate, the temperature and conductivity inside and outside of the pinch can be expected to be of comparable magnitudes.

This external plasma conductivity will in general be sufficiently high to entrap any $H_e$ flux external to the pinch proper and prevent it from distributing itself uniformly in the tube. The exact shape of the $H_e$ distribution is determined by the variation of the
boundary value $H_z$ at the tube wall as a function of compression. If no voltage is applied to the $H_z$-generating coil during pinch formation (or if the pinch tube is surrounded by a conducting shell impermeable to $H_z$ flux), then the variation in time of $H_z$ at the tube wall will depend purely on the quantity $\delta$, which is the ratio of the glass tube id to the id of the $H_z$ coil or conducting shell. The region between the inner surfaces of the glass and of the $H_z$ coil acts as a reservoir of $H_z$ flux, which is gradually depleted during pinch compression. In the plane-parallel approximation, the resultant $H_z$ distribution in space is simply given by

$$H_z = H_{zo} \exp \left[\frac{(x-s)/s}{e}\right] \quad 0 \leq x \leq s$$

$$\frac{s}{x} \leq x$$

where $e = \delta - 1$. The region $0 \leq x \leq s$ is the low-density conducting-plasma region, and the region $s \leq x$ constitutes the "pinch proper". This result has been derived in the infinite conductivity limit. Note that, even in this limit, arbitrarily "mushy" $H_z$ distributions can occur, provided $\delta$ is sufficiently large.

Machine calculations have been carried out to obtain the magnetic field distribution created in cylindrical geometry, for various values of $\delta$. This is important in order to interpret the magnetic probe measurements made inside the pinch tube, and for a number of other purposes. For instance, an estimate of pinch radius from pinch current will be inaccurate if based on the assumption of vacuum outside the main pinch column. One needs to allow for the quantity $\delta$ peculiar to the pinch tube, in order to get a more precise value. The effect of conductivity external to the pinch can be neglected only for $\delta$ very close to unity.

Some $H_z$ distributions obtained experimentally for various values of $\epsilon$ are shown in Fig. 9. An even more striking proof of conductivity external to the pinch is afforded by Fig. 10. In this case, $H_z$ at the tube wall was oscillated rapidly during the time of pinch formation, thus generating corresponding oscillations of trapped $H_z$ field in space. This result implies that rf methods of pinch stabilization and heating are probably impractical. The technique of trapping arbitrary $H_z$ distribution in the low-density plasma outside the pinch proper is finding some useful applications in stabilized and partly stabilized pinch work.

In a 4 in. id toroidal pinch called "Gamma" (Fig. 11) we are using a fast $H_z$-reversing bank to obtain sharply defined $H_z$ distributions. By driving the $H_z$ field at the wall strongly negative, one can create an $m = 1$ instability in such a direction as to reduce $H_z$ at the center of the tube. This is opposite to the sense of the corkscrew that forms when $H_z$ external has the same sign as $H_z$ internal. One of the most striking results obtained with Gamma is that the electrical conductivity remains at low values (indicative of 10 ev temperature) even when $T_o$ exceeds 3000 ev, at currents above 200 ka and initial deuterium densities of 10$^{15}$ cm$^{-3}$. Very similar results have been obtained in other experiments. Indeed, the quantity (current decay time)/(tube radius)$^2 \approx 10^{-6}$ sec/cm$^2$ is roughly a constant for all present machines. There is some temptation to conclude that perhaps
the classical theory of electrical conductivity is incorrect. Possibly the plasma temperature is really rather high, but certain bounded instability processes take place that enhance the resistivity effect due to particle collisions and so simulate low plasma temperature.\(^\text{16}\) While this pattern of thought is somewhat plausible, there is thus far no convincing proof that high plasma temperatures do in fact occur, and there is considerable evidence to the contrary.

**RADIATION FROM A TOROIDAL STABILIZED PINCH\(^\ddagger\)**

The electrical circuit of the Gamma toroidal pinch tube is shown in Fig. 12. The pinch current is driven by two low-inductance 10 kv, \(2.5 \times 10^4\) joule capacitor banks connected in series. A slow-rising current is fed into the \(H_z\) field windings through an isolating inductance, and can be canceled or even reversed by an auxiliary bank in a time shorter than the quarter cycle of the pinch current. Figure 13 shows some typical traces of circuit behavior for a program that slightly reverses the current in the \(H_z\) field windings, i.e., that makes the average \(H_z\) at the tube wall slightly negative.

### Neutron Production

Early in the course of experimentation with Gamma, neutrons were observed coming from the discharge,\(^\text{17}\) and were identified by moderating them in paraffin and noting the characteristic decay of the neutron capture gamma-ray signal (Fig. 14). A time-resolved measurement (Fig. 15) shows that the neutrons are emitted near current maximum. Neutron production as a function of starting pressure is also given. The yield was found to be maximum at about \(10 \mu\), the lowest pressure that permits good entrap-ment of the \(H_z\) flux in the pinch. The neutron production was extremely sensitive to the \(H_z\) field program (Fig. 15), the maximum yield corresponding to the most peaked distribution of current density in the pinch column. Figure 16 (a) shows the neutron yield as a function of the initial \(H_z\), the optimum \(H_z\) program being used throughout. The addition of 10% of air under otherwise optimal conditions diminished the yield by about 50%. The neutron production could also be quenched progressively by inserting a \(\frac{1}{2}\) in. lucite rod into the discharge. As is shown in Fig. 16 (b), the effect was far more marked with the neutron detector near the probe than with the detector at the opposite side of the torus.

The interpretation of the neutron yields has been somewhat problematical. Presumably, the mechanism of neutron production in the “stabilized” pinch differs

\(^\ddagger\) Title of the second portion of the paper, submitted as a supplement to the first.
from that discovered in the ordinary dynamic pinch, \( ^{18} \) because there is no evidence of catastrophic instability at the time of emission. The present mechanism is also far less sensitive to impurities, and has a different dependence on \( H_z \) field. A high degree of mutual consistency has been evident among the bodies of data gathered by various groups on toroidal stabilized pinches ranging in size from 2.5 to 50 cm minor radius. \(^{8,13,14} \) The neutron data are nearly identical with each other and with the data given above, provided one scales the particle density inversely as the square of the tube radius, and the pinch duration directly as the square of the radius. \(^{17} \) One is tempted to suppose that the same mechanism is responsible for neutron production in all these machines. As to the question, whether the plasmas are thermonuclear, this cannot be decided on the basis of neutron emission as long as the yields remain at such low levels. The present neutron data are, at best, informative concerning a small number of energetic deuterons, and have little logical relevance to the mean energy of the deuteron population, other than to impose an upper limit.

The plasma temperature can be estimated somewhat more directly on the basis of bulk plasma phenomena such as conductivity, apparent plasma pressure, and various spectroscopic effects. The pressure balance method and the analysis of the Doppler broadening of impurity lines tend to yield temperatures in the thermonuclear range. These data, however, are like the neutron data in that they only set an upper limit to plasma temperature. Doppler broadening can be accounted for in terms of plasma turbulence; and large apparent plasma pressures may be due, for instance, to turbulence, run-away electrons, or unexpected concentrations of particle density. Plasma conductivity measurements, which set a lower limit to electron temperature, consistently indicate values from 50 ev downward. Measurements on ionization rates give somewhat similar results. Since at such low electron temperatures the ion-electron thermalization time is a small fraction of the pinch duration, these values are, of course, incompatible with thermonuclear neutron production. Recently this dilemma has lost some of its force, thanks to the discovery on ZETA that the observed neutrons are generated by energetic accelerated deuterons. \(^{19} \) Nonetheless, there are still many points that remain unclear. For instance, it is difficult to see how any simple deuteron run-away mechanism is compatible with the rod insertion experiment (Fig. 16). Neutron
emission would be expected to drop equally all around the torus when an obstacle to run-away is introduced.

Energy Balance

Leaving aside the neutron question, there is a second and more basic problem in interpreting the toroidal pinch data. A great deal more energy seems to go into the discharge than can readily be accounted for. In typical operation of Gamma pinch (Fig. 13), one may start with an energy of $5 \times 10^4$ joules ($10.5$ kv) in the main pinch banks and $1000$ joules ($4000$ gauss) in the initial $H_z$ field. By the time the maximum current of $264$ ka is reached, there are $8200$ joules left in the main pinch banks. The stray magnetic energy outside the pinch tube amounts to $9800$ joules, if an iron transformer core is used. The magnetic fields inside the tube (Fig. 17) account for $16,000$ joules, which leaves about $17,000$ joules, or a third of the starting energy, unaccounted for. The "resistive" voltage drop around the pinch tube at current maximum is $8.5$ kv, corresponding to an energy dissipation of $2300$ joules per $\mu$sec. It is consistent to conclude that the $17,000$ joules which are missing after the first quarter-cycle of the pinch current have been dissipated in the plasma. For a starting deuterium pressure of $20 \mu$, this figure corresponds to $4$ kev per initial particle. Evidently, no such large amount of energy is present in the plasma at current maximum. Even by the most optimistic interpretation, the observed neutron yields "indicate" ion temperatures of only $300$ ev. The pressure balance results of Fig. 17 need special interpretation for the reasons given earlier, and one also notes that the absence of an outward shift of the zero of $H_z$ is inconsistent with pressure balance theory in a torus. It is probably legitimate, however, to use the pressure balance method in setting an upper limit to the plasma energy content of about $700$ ev per initial particle. On a theoretical basis it had been expected that the toroidal stabilized pinch would prove difficult to heat, but relatively free from loss mechanisms. Instead, it now appears that energy is transferred to the plasma at an extremely high rate, and is lost again almost as quickly.

Loss Mechanisms

Visible and UV Radiation

Of the potential loss mechanisms, most are ruled out simply by the magnitude of the observed effect. It is possible to explain the loss in terms of radiation from partly stripped impurity ions, if contaminations of the order of one oxygen ion per deuteron are assumed. This possibility has been investigated experimentally in some detail. Figure 18 shows the emitted visible light (most of which consists of the $4128 \AA$ line of Si™). The light flux during the first $10 \mu$sec is less than a thousandth of the total flux. By means of SWR ultraviolet sensitive film, exposed with and without a soft glass filter, it has been determined that about four-fifths of the radiated energy is in the form of ultraviolet, and that the total energy radiated amounts to $(3.0 \pm 1.5) \times 10^4$ joules. In order to obtain a time-resolved signal, a photo cell with a heated platinum cathode was
A TOROIDAL STABILIZED PINCH

DIVERTED LINES

PYREX GLASS ROD
ACTS AS PHOSPHOR & LIGHT PIPE

Figure 19. X-ray probe and diverter coil experiment

attached to the pinch tube, and kept evacuated by differential pumping. The clean platinum-surfaced photo cell has been shown by Weissler\textsuperscript{21} to have a uniform efficiency of approximately 5\% for wavelengths from 200 to 1000 Å. Figure 18 shows that a rapid onset of the ultraviolet signal occurs only at very high gas pressure. At low pressure, about 6\% of the ultraviolet flux is emitted before current maximum. The estimate of total radiated energy agrees with that from the SWR film exposure, if one assumes a mean quantum energy of 10 ev.

**X-rays**

It is true, then, that much of the input energy of the pinch eventually turns to radiation; but during the first quarter-cycle of the current this process seems to play a secondary role. An alternative loss mechanism which would be expected to occur preferentially during the first quarter-cycle is electron run-away. It is well known that, with poor pre-ionization, the toroidal pinch can produce initial bursts of X-rays up to several Mev. A spatial analysis of hard X-ray dosage has been made with Victoreen ionization pencils. A persistent pattern was observed,\textsuperscript{§} showing sharp peaks of intensity near the two ports of the torus. The pattern could be altered by introducing artificial perturbations of the $H_z$ field. When a strong pre-ionizing current is used on Gamma, the hard X-ray signal outside the torus shell disappears. On inserting an X-ray probe with a 1-mil beryllium window (Fig. 19) inside the tube, a large X-ray flux is, however, still observed (Fig. 20). By varying the material and thickness of the foil window, the mean X-ray energy at pinch initiation has been set at about 1.5 kev (the addition of 0.34 mil of Al attenuates the signal by a factor of 0.4). It was also found that the X-rays become continuously softer during the first one-eighth cycle of the current. Whether a soft X-ray flux persists during the second one-eighth cycle could not be determined with present probes which cut off below 1 kev. The energy content of the X-ray flux above 1 kev is within an order of magnitude of what is required to explain the energy loss from the plasma in terms of run-away electrons striking the wall. Figure 20 shows the dependence of yield on initial deuterium pressure. The signal was found independent of probe orientation parallel or anti-parallel to the $H_z$ lines, and there was no signal unless the probe could "see" the outer half of the torus shell. It is concluded that the observed X-rays are provided by electrons striking the outer half of the shell, rather than by striking the probe itself. With a diverter coil wrapped about the probe port and with the probe retracted into the port, it was possible to produce a large signal whenever the run-away electrons were steered into the port by the diverter field. With the diverter field reversed or at low strength compared to the initial $H_z$, there was no signal. An inspection of the inside of the tube has shown a blackening and glazing of the porcelain at two points on the outer half of the shell, near the tube ports, where inhomogeneities in the $H_z$ field function as natural diverters. (To produce such a glazed spot it is estimated that more than a gram of material must be heated to 1000°C.)

**Discussion**

Since the natural geometric inhomogeneities of the $H_z$ field are rather weak, it is difficult to see how large

\[ X \text{ RAY \ YIELD} \]

![Graph](image)

\[ \text{PRESSURE= MICRONS} \]

![Graph](image)

\[ \text{X RAY \ YIELD} \]

![Graph](image)

\[ \text{WALL-OPERTURBATION POSITIVE X} = \text{NEGATIVE} \]

![Graph](image)

\[ \text{X RAY \ YIELD} \]

![Graph](image)

\[ \text{WALL-OPERTURBATION POSITIVE} \]

![Graph](image)

§ The Spatial Asymmetry of the Hard X-ray Pattern was First Observed with X-ray Film at Los Alamos, by J. Tuck et al.
fluxes of run-away electrons can cross directly from the pinch surface to the wall. One is forced to conclude that there exist either substantial fluctuations of the magnetic field over an electron Larmor orbit, or else, rather large transverse electric field components. The requisite magnitude of $E_1$ is given by $E_1/H \sim v/2mc$, where $v$ is the run-away velocity. In the case of Gamma, $E_1$ should amount to some thousands of volts per cm. The magnitude of the variation in $H$ over a Larmor radius that is required to cause electrons to escape freely along non-adiabatic trajectories is given by $\Delta H \sim \pm H/\pi$. It is easy to see that an average pressure contribution of a few tenths $H/\pi m$ will result. The excitation of large amplitude transverse Alfvén waves by run-away electrons, and the consequent non-adiabatic behavior of electron trajectories, have been investigated at length by the stellarator group. Experimentally, the presence of large magnetic field and plasma fluctuations in the so-called "stabilized" pinch has been demonstrated variously with Langmuir probes, magnetic pickup loops, and microwave detectors. Especially elegant experiments in the last two categories have been performed at Los Alamos by Lovberg et al. To what extent the experimental observations can be accounted for in terms of bounded hydromagnetic instabilities, run-away-driven Alfvén waves or plasma oscillations excited by run-aways remains to be seen. 

Whatever theory emerges must incidentally account for the extremely large plasma resistivities that have been observed in toroidal pinches. When the starting pressure in Gamma is made comparable to pressures used in typical linear pinch work (above 100 $\mu$), the electrical conductivity can be made to reach comparable values, corresponding to about 30 ev. At starting pressures of 20 $\mu$, where much higher temperatures might be expected, the conductivity is actually diminished, and corresponds to as little as 10 ev. One possible explanation is that the plasma temperature is actually as low as 10 ev. A simple calculation shows, however, that for 20-\(\mu\) starting density even a 10-ev electron will tend to run away in the observed 50-volt/cm electric field. Unimpeded, such an electron will reach 100 kev in a few tenths of a \(\mu\)sec. Since the mean energy of the X-rays observed under conditions of good pre-ionization is quite low, it follows that electrons are either prevented from accelerating freely, or else that they escape to the tube wall after very little acceleration. It is useful to note that the problem of explaining either process in terms of fluctuating fields is precisely identical with the problem of accounting for the plasma resistivity. If the pinch current can be shown to be carried by run-away electrons that eventually escape to the tube wall, it also follows automatically that the "resistive" energy input into the pinch plasma is carried to the wall.

One practical conclusion regarding Gamma pinch can be drawn quite independently of any theory of field fluctuations or any particular opinion about plasma temperature. The plasma resistivity is evidently between 20 and 100 times as great as the highest resistivity that is tolerable from the point of view of a useful thermonuclear reactor, namely the classical resistivity of a 200-ev plasma. Even if the ion temperature can eventually be brought into the 100-kev range by raising the input power, as long as the electrical conductivity requirement is not met, the plasma containment time in a pinch of practical size will be too short to allow economical operation. On the basis of present experiments it is unlikely that a simple increase of input power density vs. particle density above present levels will improve the plasma conductivity even slightly. It would seem, therefore, that the main emphasis of stabilized pinch research, should at this point belong not to the attainment of high plasma temperatures, but to the understanding of energy dissipation phenomena in the plasma.

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A Partly Stabilized Dynamic Pinch

By S. A. Colgate, J. P. Ferguson, H. P. Furth and R. E. Wright*

The dynamic pinch effect can be used to heat the ions of a plasma at an extremely high rate. Initially, all the particles of the pinched plasma acquire the same radial velocity, which means that only the ions acquire a large directed kinetic energy. Following the first dynamic bounce, the ion kinetic energy is translated into ion temperature. If the pinch column can maintain itself intact while the pinch current continues to rise, then the ion temperature is further enhanced by adiabatic compression.

Early dynamic pinch experiments at first encouraged the idea that thermonuclear ion temperatures had been reached by the expected processes. It turned out, however, that the observed neutron yields were attributable to the presence of a small component of highly energetic deuterons in an otherwise relatively cold plasma. Strong evidence was found that the observed neutron abundances, of a few times $10^8$ per shot, were associated with deuteron acceleration by the $m = 0$ or sausage type instability of the pinch. An incidental discovery was that a small amount of longitudinal magnetic field trapped inside the pinch column would reduce the neutron yield several orders of magnitude, presumably by impeding the $m = 0$ instability.

More recent experiments have been designed to make use of this discovery, both to improve pinch stability and to inhibit the non-thermonuclear neutron yields. Considering how welcome these yields are, of course, very significant in experimental obstacles to the observation of a thermonuclear plasma.

The stabilizing effects of a longitudinal magnetic field, applied either inside or outside the pinch column, have been analyzed by a number of authors. Generally speaking, it is found that, for $H_s$ comparable to the pinch field $H_0$, the instability most likely to predominate is of the $m = 1$ or corkscrew type. The corkscrew instability is free from one of the most unpleasant features of the sausage instability, namely that its growth becomes nonlinear once its amplitude exceeds a fifth of the pinch radius $r_0$. The presence of the corkscrew instability begins to impair plasma containment seriously only when its amplitude exceeds its wavelength, which is typically about $2\pi r_0$.

The inclusion of a longitudinal magnetic field in the pinch column raises an issue that may be serious. As long as the radially accelerated ions of the dynamic pinch can interact only with each other, it is clear that the initial kinetic energy will be translated almost exclusively into ion temperature. If a longitudinal magnetic field is present inside the plasma, then the plasma electrons will receive their share of the initial kinetic energy, and they may even be heated preferentially.

We have therefore been interested in the possibility of stabilizing the dynamic pinch by means of an $H_s$ field external to the pinch column. This possibility has long been recognized in principle but has received little attention in practice because of the apparent waste of magnetic energy in the required magnetic field configuration. For instance, if one assumes the pinch configuration of Fig. 1, where $H_s = H_0$ on the surface of a 10:1 pinch, and there is vacuum outside the pinch column, then the stored magnetic energy amounts to $54.2$ on a scale where the product of the plasma pressure and volume is taken as unity. By way of comparison, the stored magnetic energy in an ordinary 10:1 pinch is only $4.6$. The magnetic energy of a 10:1 pinch with internal $H_s$ equal in pressure to the plasma pressure amounts to $10.2$. These numbers are, of course, very significant in experimental practice. Only a limited amount of electrical energy enters the pinch tube in the time interval up to the first dynamic bounce, and one cannot afford to waste this energy in stray inductance.

Our present interest in dynamic pinch stabilization by an external $H_s$ field was kindled by the observation that the region outside the pinch column generally contains, not a vacuum, but rather a dilute plasma of good conductivity. As a result, the $H_s$ external to the pinch column proper does not distribute itself as first anticipated, but actually assumes a much more favorable distribution.

THE SCREW-DYNAMIC PINCH DISTRIBUTION

A convenient way to apply an $H_s$ and $H_0$ field simultaneously to a plasma is to impart a helical twist to the return conductor of the pinch tube. Such a "Screw-Dynamic" pinch device can be made, for instance, with strips of copper braid which are
insulated from each other. Instead of the usual azimuthal pinch driving field, a helical driving field is now generated. The pitch of the helical driving field can be determined as follows.

Let us assume a pressureless plasma of infinite conductivity outside the pinch column proper. Then the pressure balance equation becomes

$$H_z \frac{dH_z}{dr} + H_0 \frac{d(rH_0)}{dr} = 0 \quad (1)$$

It is easy to show that the fluxes $\phi_2$ and $\phi_0$ between a given plasma point and the pinch axis must be conserved. Therefore $\phi_0$ has a fixed functional dependence on $\phi_2$:

$$\phi_0 = \phi_0(\phi_2) \quad (2)$$

Differentiating Eq. (2) and substituting from Eqs. (3) and (4), one obtains

$$H_0 = \mu \tau H_0 \quad (5)$$

where $\mu = \frac{d\phi_0}{d\phi_2}$ is a function of $\phi_2$ only, and therefore is a constant at any given plasma point. Note, however, that $\mu$ is not in general a constant in the laboratory system. The distribution of $H_z$ and $H_0$ in space is defined by Eqs. (1) and (5), plus a complete history of $H_z$ variation at the tube wall as a function of $H_0$ variation at the tube wall, starting at the time of pinch initiation.

If the return conductor of the pinch tube is a helix, then the wall condition on $H_z$ can be written simply as

$$H_0 = \mu_0 H_0 \quad (6)$$

where the radius of the pinch tube has been taken as unity and $\mu_0$ is a constant. Since each point of the dilute plasma outside the pinch column proper has at some time coincided in space with the laboratory point $r = 1$, it follows that $\mu = \mu_0$ for all plasma points. Therefore, $\mu$ is a constant in the laboratory system as well as in the plasma system. This peculiar property of the Screw-Dynamic pinch distribution greatly facilitates mathematical analysis.

Substituting Eq. (5) into Eq. (1) and solving, we obtain

$$H_z = MH_{z0} \quad (7)$$

$$H_0 = r \mu MH_{z0} \quad (8)$$

where $M = \frac{1 + \mu}{1 + \gamma^2}$, $r_0$ is the pinch radius and $H_{z0}$ is the value of $H_z$ at $r = r_0$. These distributions have been plotted in Fig. 2, for the case $r_0 = 0.1, \mu = 10$.

The magnetic energy density distribution is given by

$$u_m = M u_m_0 \quad (9)$$

and the total magnetic energy can be written

$$U_m = U_m_0 \gamma G_z(r_0, \gamma) \quad (10)$$

where

$$G_z = \left[\frac{r_0^2(1 + \gamma^2)}{\gamma^2}\right] \ln\left[\frac{1 + \gamma^2/r_0^2}{1 + \gamma^2}\right] \quad (11)$$

and

$$\gamma = \frac{r_0}{\mu_0} = \frac{H_0/H_{z0}}. \quad (12)$$

For $\gamma \to 0$, Eq. (11) reduces to $G_z \approx 1 - r_0^2$.

The function $G_z$ has been plotted in Fig. 3 for several values of $r_0$. For purposes of comparison, one can derive an analogous function for the ordinary dynamic pinch:

$$G_0 = 2r_0^2 \ln(1/r_0) \quad (13)$$

For the case $r_0 = 0.1, \gamma = 1$, we then have $G_z = 0.078$ and $G_0 = 0.046$. Thus there is only 1.7 times as much stored magnetic energy in the Screw-Dynamic pinch as there is in the standard pinch, given the same plasma pressure $u_m_0$ and pinch radius $r_0 = 0.1$. As one goes to high pinch compressions, the situation becomes even more favorable.

**EXPERIMENTAL RESULTS ON A SMALL MODEL**

The theory of the Screw-Dynamic pinch has been checked in a 4-in. diameter 8-in. long pinch tube, powered by a 15,000-joule capacitor bank. The bank consists of twenty 7.5-µF capacitors connected, 5 in series and 4 in parallel, so as to form an effective
capacity of 6 μ. The capacitors are separated by individual air spark gaps, which are triggered by simultaneous over-voltaging. The range of satisfactory operation is from 35 to 70 kv total voltage. The maximum current attained is about 250,000 amp.

The return conductor of the pinch tube consists of braid strips having a variable pitch, but usually slanted by one part in 10, so that \( \mu = 10 \) (Fig. 4). The resultant \( H_z \) distribution has been measured by means of a magnetic probe. Some typical \( H_z \) signals are shown in Fig. 5(a), (b) and (c). Values of \( H_z \) obtained for 100 μ pressure of \( D_2 \) and 60-kv bank voltage at the time of first dynamic bounce are plotted in Fig. 6. Evidently, the agreement with theory is satisfactory.

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The detailed variation in time of the \( H_z \) signals is easy to interpret. There is essentially no signal until the shock front sweeps by the probe, which requires about 0.7 μsec in the case of the probe at \( r = 0.1 \). Then there is a damped oscillation, indicative of an oscillation in pinch radius. Finally, after about 1.5 μsec a catastrophic instability seems to occur. The \( H_z \) signal of Fig. 5(a) (taken at the tube wall), corresponds to the familiar reverse-field signal obtained from the ordinary stabilized pinch when the corkscrew instability develops.\(^{10}\)

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The discharge was monitored by means of a \( \frac{dI}{dt} \) loop, various external flux loops (see Fig. 4), and a voltage divider. One of the most interesting signals was the one for \( \frac{d\phi_z}{dt} \) outside the pinch tube, as measured by a loop surrounding the return conductor. When the pinch current rises inside the tube, the simultaneously generated \( H_z \) field must return on the outside. The \( \phi_z \) loop measures this signal, plus any additional signal brought about by pinch asymmetry.

Evidently there is a marked fluctuation in the expected direction at the time of the corkscrew instability (Fig. 5). We have also used a \( \phi_z \) loop, which can receive no signal at all as long as the pinch column maintains axial symmetry. A typical \( \frac{d\phi_z}{dt} \) signal is shown in Fig. 5(b). One should perhaps note here that the observation of the instability component in the \( \phi_z \) and \( \phi_h \) signal demonstrates that the electrical conductivity inside the pinch tube is limited—otherwise no such signal could get out. One can also see (Fig. 5) that the external signal is delayed by a few tenths of a μsec, relative to the onset of the instability.

If deuterium is used in the discharge, then at precisely the time when the instability signal first becomes apparent on the \( \phi_z \) loop, we observe a burst of neutrons in the range \( 10^4-10^5 \) (Fig. 7). No very distinctive or consistent features seem to occur at this time on either the \( \frac{dI}{dt} \) or the voltage-divider signal. The neutron yield is greatest for 50 to 100 μ starting pressure, and is virtually unaffected by small amounts of impurity, such as 5% helium. If the tube is filled with hydrogen, following operation in deuterium, there is a small residual yield.

The magnitude of the neutron yield is fairly erratic from shot to shot, varying by factors 3 to 10. Most of the data were taken at 60 kv bank voltage, since raising the voltage to 70 and even 80 kv did not increase the neutron yield substantially. At 50 kv the yields became smaller and more inconsistent. There have been no cases observed of large yields where the accompanying \( \phi_z \) instability signal has not been extremely marked. However, there have been numerous cases of marked \( \phi_z \) signals where the neutron yield has been small. In a few per cent of the cases, especially at low densities, there are multiple neutron bursts (Fig. 7(b)). At starting pressures of 200 μ the neutron yield is greatly reduced and at 300 μ neutrons

\[ \text{Figure 3. Total magnetic energy storage in the field of the screw-dynamic and of the ordinary dynamic pinch} \]

\[ \text{Figure 4. The screw-dynamic pinch and associated flux-measuring loops} \]
PARTIALLY STABILIZED PINCH

H₂ AT WALL
3000 gauss/div

H₂ AT r = 0.2
15,000 gauss/div

H₂ AT r = 1
15,000 gauss/div

\[ \frac{d\phi_z}{dt} \text{ ext.} \]

\[ \frac{d\phi_z}{dt} \text{ ext.} \]

\[ \frac{d\phi_z}{dt} \text{ ext.} \]

\[ \frac{d\phi_z}{dt} \text{ ext.} \]

\[ \frac{1}{\sqrt{2}} \mu \text{sec/Div}, 100 \mu D_{2}, 60 \text{ kv} \]

Figure 5. \( H_z \) measurements at various pinch tube radii, also \( \frac{d\phi_z}{dt} \) and \( \frac{d\phi}{dt} \) measurements outside the pinch tube are never observed. At these high densities the instability signal is also very much diminished. All in all, one can say that there seems to be considerable interrelation between the instability process and the mechanism of neutron production, but it is difficult to prove a causal relationship.

In looking over some of our older data\(^1\) on neutron production in the presence of a trapped \( H_z \) field, we find substantial similarity to the present results. The neutrons appear at the same time and are roughly of the same number under comparable conditions. Several other experiments have been reported recently\(^3\), which all exhibit a certain basic consistency.

One can say with assurance that the observed neutrons must be different in origin from the old \( m = 0 \) neutrons, simply because the \( m = 0 \) acceleration mechanism cannot persist in the presence of a substantial \( H_z \) field. It is also significant that the large neutron yields associated with the \( m = 0 \) mode were extremely sensitive to impurity contamination. A percent or so of a heavy gas would reduce the yield by orders of magnitude. In the Screw-Dynamic experiment and the other similar experiments we seem to be left with a hardly residual neutron yield which is not highly sensitive either to impurities or to the amount of \( H_z \) field present. A contamination of 5% helium or air is barely sufficient to reduce the neutron emission noticeably.

It is probably idle at the present stage to attempt to deduce whether the residual neutron yields are of thermonuclear origin. Neutron measurements are directly informative only concerning those deuterons in the multi-kev region which participate actively in the fusion process. Whether these few deuterons
constitute the Maxwell tail of a plasma having several hundred ev temperature or whether they are accelerated directly by plasma turbulence, is a question that must be decided by measurements on the bulk of the plasma. An exception occurs, of course, if the yields are so obviously spurious as to inhibit all further speculation concerning a thermonuclear origin.

In view of the close association of our neutron yields with the corkscrew instability, it seems highly likely that a Fermi-type process of ion acceleration is at work. Our probe measurements show that at instability time there is a good deal of extremely rapid plasma bulk motion with which energetic deuterons could thermalize. Some crude calculations confirm this possibility. Conditions are particularly suited to the Fermi mechanism in the low density region just outside the pinch proper, where the Alfvén velocity is high and ion mean free paths are long.

Some of the questions that puzzle us at the moment will perhaps be resolved during a forthcoming experiment with a larger Screw-Dynamic device in the $10^4$–$10^5$ joule range. It is also hoped that much more pronounced high temperature plasma phenomena will be achieved in the larger experiment.

REFERENCES


Collapse—The Shock Heating of a Plasma

By S. A. Colgate and R. E. Wright

There have been numerous independent suggestions to use high speed shocks to heat deuterium gas to thermonuclear temperature (E. Teller, R. R. Wilson, H. Grad, W. Marshall), and extensive experimental work in this field is being carried on by, e.g., Kolb and Janes. Our own work in this field has been directed towards a fundamental understanding of the shock process, in the limit of no particle collision, to find out if, within this limit, the ion heating following the passage of the shock is large enough to give rise to a thermonuclear reaction.

The reason for the emphasis on the limit of no particle collision (within the dimensions of the system) is that for practical magnetic field strengths sufficient to contain plasma at a thermonuclear temperature of say 10 kev, the density of plasma is so low that the collision mean free path becomes many times greater than the dimensions of the system. Under these conditions it is generally agreed that the dominant length in the shock transition process is determined by the cyclotron radius of the ions or electrons in the magnetic field, and that the forces of charge separation dominate the process of momentum change of ions.

The more exact theories treat the case where the magnetic field energy density \( H^2/8\pi \) behind the shock dominates the partial pressure of plasma oscillatory or thermal motion. In the case where magnetic field pressure is dominant, the oscillatory or thermal energy density behind the shock resides primarily in the ions.

One of us has discussed the probable consequences of extremely strong shocks (in the limit of no particle collision), for the case where the magnetic pressure behind the shock does not dominate plasma energy density. It is tentatively concluded that in this limit the energy density will reside in electron plasma oscillations which relax into an electron temperature before the ions are heated. It is hoped that the experimental program will eventually test these theories.

In shock-heating a body of plasma, the basic procedure is to apply a large pressure that rises in a time short compared to the transit time of sound across a plasma dimension. A large fraction of the energy input into a shock will produce irreversible heating, and this large and sudden heating is the objective. The geometry used is constructed so that the plasma is in the region of the mirror—or axial field—and the shock is created by the sudden rise of an axial magnetic field. If the plasma acts as a perfect conductor during the time scale of dynamic phenomena, then the magnetic pressure outside, \( H^2/8\pi \), becomes the magnetic piston. In the initial experiments, conducted two years ago, this sudden rise of the magnetic field was applied to the cold, neutral gas with the hope that the induced electric field would electrically break down and ionize the gas to a plasma in a time short enough to form a shock. These experiments were done in two sizes; in a 4-in.-diameter system in which the shock magnetic field rose to 3000 gauss in 2 \( \mu \)sec, and in a very small coil (tiny collapse) 2-cm diameter with a magnetic field rising to 10,000 gauss in 0.05 \( \mu \)sec. In general, for both experiments, the process of ionization did not occur fast enough to give a clearly defined shock in a perfectly conducting plasma. Consequently, an experiment was started in which the three processes (1) ionization, (2) current boundary layer formation, and (3) shock were to be separately accomplished in an optimum time sequence.

It is desirable to use the minimum gas or plasma density possible so that the magnetic field strength needed to form a strong shock is minimum. A P. I. G. discharge achieves of the order of a few percent ionization at an initial pressure of 10 microns deuterium. Then the process of “slow collapse” is designed to increase the ionization to approximately 100%, push the plasma slightly away from the outside glass wall, and at the same time form a current boundary layer thin compared to the radius.

**EXPERIMENT**

Slow collapse is produced by an alternating magnetic field created by the discharge of four 7.5 mf capacitors at 10 kv by ignitrons through the single turn coils (Fig. 1). The magnitude of the field is such that \( H^2/8\pi \) \( (H \approx 2000 \text{ gauss}) \) is larger than the plasma pressure at 20-ev temperature and the frequency is such that \( \frac{1}{4} \) cycle time \( (3 \mu\text{sec}) \) corresponds to roughly a 20-ev ion transit time across the system. Before ionization is complete, the induced electric field of the changing magnetic flux induces large currents in the partially ionized plasma, thereby further heating and
ionizing it. When the electrical conductivity becomes large enough, or the fraction ionized becomes large enough, the plasma is more strongly coupled to the oscillating field and a 10- to 20-ev weak shock will be formed. Externally this appears as an alternating expansion and compression of the plasma at a velocity corresponding to a 20-ev ion. This temperature shock is optimum for ionization because the collision cross section and ionization cross section are large. In addition, the temperature of the plasma is still low enough so that the heat flux to the glass wall does not vaporize the wall surface. The plasma rapidly becomes sufficiently conducting so that there is a major phase lag between the magnetic fields inside and outside. This, coupled with the information that the plasma is moving away from the glass walls, is the appropriate condition for the application of the fast shock. Internal magnetic and electric field probe signals and image converter pictures verify this concept of the condition immediately before the application of the strong shock.

Figure 1 shows the geometry of the experiment. The vacuum envelope is 4-in. pyrex pipe. The bias field coils and slow collapse coils give an axial mirror-type field. The single-turn fast collapse coil is mounted in the center in order to maximize the shock strength for the limited magnetic field energy available. Consequently, it is not in mirror geometry and there is no containment after the shock is applied; the plasma is merely squirted out at the ends. This geometry is adequate to study the first passage of the shock itself, but later experiments are envisaged with subsequent shock containment.

The slow collapse heats and ionizes the gas sufficiently rapidly so that by the beginning of the fourth current maximum of the slow collapse cycle the plasma is sufficiently ionized and heated to be a "perfect" conductor, and a current boundary layer is formed away from the glass wall. The fast collapse shock is applied shortly after this.

The bias magnetic field is small (~200 gauss) applied 300 µsec before slow collapse. It permits the P.I.G. ionization to start 20 µsec before slow collapse and serves also to vary the amount of field trapped inside the plasma during slow collapse.
by the value of the trapped field and plasma pressure. The fast shock is applied at approximately the time of the second picture when the plasma is just beginning to leave the glass wall.

Figure 4 shows the oscilloscope traces that describe the effect of the applied shock created by the spark gap firing of a single 0.25-mf, 80-kv condenser (shorted ringing frequency of 2.5 megacycles). Frame (A) shows an overlay of two traces taken with the magnetic probe inside the glass envelope, axially centered under the fast collapse coil and at one-half radius position. The upper trace, V, was taken with no plasma (vacuum) and shows a typical crowbarred magnetic field rising from a negative value of 800 gauss to a positive value of 1500 gauss in 0.3 \mu sec. The lower, P, trace in Frame (A) is the same signal except that a plasma has been added, with all the necessary pre-conditioning. The first feature that is evident is that no signal occurs until after a delay of approximately 0.2 \mu sec. The shock has to travel something like 2.5 cm before reaching the probe and so this is interpreted as a minimum speed of approximately 10^7 cm/sec. The next feature is that the initial probe signal is negative rather than in the positive direction of the applied fast field. The negative signal is what is expected in the case of a shock, because the compression behind the shock of a negative field is independent of the properties of the applied field, and depends only upon the piston pressure \( H^2/\sigma_r \). The shock compression of a negative initial trapped magnetic field should show an increase of the negative field—exactly as observed. Following the passage of shock, it is expected that the subsequent radial flow of the plasma will uncover the probe and expose it to the positive applied shock field. The probe indicates the positive field after 0.4 \mu sec. The bottom two traces, Frames (C) and (D), show the same phenomena of the shock on the internal magnetic probe (plasma and vacuum) at a much slower sweep speed in order to illustrate the combined effects of the slow and fast shocks. The relative time of the shock and shock arrival cannot be compared exactly because of the jitter in shock initiation. In the previous upper left trace this jitter was observed to be small, and was negated by triggering the oscilloscope from the applied shock field.

DISCUSSION

If the plasma surface acts like a perfect conductor and moves like a piston, then it should react back on the initial circuit; that is, it should behave as a variable inductance that is changing within the time of the primary frequency. Most of the inductance of the fast collapse circuit is located in the single-turn coil around the glass envelope. If a shorted turn of copper were placed just inside this coil, the inductance would be reduced and the current (for a given voltage on the condenser) would rise faster and higher than without the shorted turn. If now the shorted turn of copper moved radially inward just after the field had been applied, the inductance would be increased, work would be done on the shorted turn equal to the magnetic pressure times change in volume, and the current would decrease by an amount corresponding to the conversion of magnetic energy to mechanical energy of the work done on the changing inductance. Exactly this behavior of the plasma is documented in Frame (B) of Fig. 4. The external magnetic field, or current to the fast collapse coil is shown with and without plasma. The slower rising curve (the fields are measured negative) that ends up at a larger magnitude corresponds to the vacuum applied shock. When now the plasma is introduced so that it is at maximum radius at the time of the shock, then the current rises more rapidly at first, indicating that the plasma behaves like a shorted turn inside the coil, but then, at 0.2 \mu sec, the current crosses over the vacuum field case and ends up at a lower magnitude. This implies that the current-carrying plasma has moved radially inward away from the glass walls, and that this motion has been reflected back on the primary circuit as a changing inductance. The work done is the thermal and kinetic energy given to the plasma. To calibrate the effective radius at which the initial current was flowing in the plasma, actual copper rings of various sizes were placed inside the glass and then the reactive effect observed by the different rates of rise of initial current. Of course, in the case of a static copper shorting ring the current rose to and remained at a higher value than in the vacuum field case. In this fashion it was found empirically that the initial effective plasma current flowed within 0.5 cm of the glass walls. The inductive interpretation of the circuit assumes that the resistive penetration is small, as indeed it must be to explain the observed large phase shifts at the very much lower frequency of the slow collapse.

We have, in addition, attempted to verify the charge separation structure of the shock front by the use of dual Langmuir probes. We have succeeded in applying the technique, so far, solely to the case of the weak shock of the slow collapse cycle.

The shock front should be dominated by charge separation provided the collision mean free path is very long compared to the electron Larmor radius. For the slow collapse cycle:

\[
H \geq 1000 \text{ gauss}
\]

Electron energy = 10 ev
Electron Larmor radius = 0.006 cm
Collision mean free path = 0.15 cm (D$_2$ at 10 μHg; temp. 10 ev)

Since the energy the ions receive in falling through the integral of the space charge separation field is about equal to the kinetic energy of their fluid flow behind the shock, we should expect a voltage signal normal to the shock equal to the ion kinetic energy in electron volts in the case of a shock that is thin compared to the probe spacing. This signal is observed.

At a radial distance two-thirds of the tube radius, all three orthogonal components of the electric field are present. These components were measured by a dual-wire probe (see Fig. 1) oriented variously in the three directions, radial, axial, and chord. The oscillating magnetic field of the slow collapse generates successive shock waves that travel radially toward the axis and reflect. There is also an axial component due to the mirror effect of two separate collapse coils. After a few successive current oscillations, the shock builds up in strength because the resultant heating and ionization of the gas make it a better conductor and consequently more strongly coupled to the magnetic field. These shocks reach a maximum at the fourth current maximum (second condenser cycle), and so all probe signals were studied at this period.

The electric signal from the probes is always measured in parallel with a circuit impedance which must be large compared to the plasma impedance, or else a time-varying correction must be applied. It was determined experimentally that the signals were essentially the same whether they were measured across a 100-ohm or 100,000-ohm impedance, so that in practice they were fed through two 50-ohm cables to opposite oscilloscope plates.

Figure 5 shows a set of chord and radial signals, each with an overlay of the same signal with the probe rotated through 180 degrees. The self-consistency of the signal, even under inversion of the probe, indicates a reproducible phenomenon solely associated with probe orientation within the plasma.

The electric field inside the plasma should be the sum of the velocity induced field $\mathbf{v} \times \mathbf{H}$ and the charge separation field $e\int n_e - n_i dx$. If the fluid flow is strictly radial in the region of the dual probe, then the chord signal should measure the $\mathbf{v} \times \mathbf{H}$ field and the radial signal the charge separation field. A magnetic probe coil measurement of $d\Phi/dt$ is also taken at the position of the Langmuir dual probe. To a first approximation the $d\Phi/dt$ signal and the chord dual probe signal should be the same since the plasma velocity is a measure of the rate of change of magnetic flux. It can be seen in Fig. 5 that these signals are approximately the same, thus verifying the interpretation qualitatively. The chord signals give a velocity of $2 \times 10^6$ cm/sec, in agreement with measurements of the progress of the interface. The radial signal is...
displaced somewhat toward earlier times, which is in agreement with the concept that the charge separation gives rise to the forces that cause the plasma motion. The polarity of the radial signal is independent of magnetic field direction and is predominantly of one sign (whereas the chord signal reverses sign). The polarity of the radial signal on the first peak of each cycle is in agreement with a radially inward acceleration, i.e., the negative electron charge is ahead of the positive ion charge. Subsequent reflected shocks cause a reversal of this potential, but since the outgoing shock is never as strong as the ingoing one, the negative reversals are weaker. The magnitude of the radial signal is in good agreement with an expected 20-ev ion kinetic energy.

Figure 5 also shows a polar plot of the chord-radius signal as a function of angle, taken at a given time. The sine curve result is an indication of the reproducibility and true plasma origin of the signals.

On the right, Fig. 5 shows the axial electric field measurements. Because of the mirror configuration of the collapse coils, there is an axial component to the shock, but at the midpoint between the mirrors, this component should vanish, giving zero electric field. A sequence of measurements as a function of position between the mirror coils shows that the axial electric field does go through a null and reversal.

The electric field and magnetic field measurements are in good agreement with present shock theory, and it is anticipated that future experiments will tend toward much higher shock strengths in a containment geometry—either mirror or stabilized pinch.

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Sheet Pinch Devices


It is predicted theoretically that a current-carrying plasma sheet of infinite extent possesses positive stability for some perturbation modes and at least neutral stability for others. Consequently three types of sheet-like discharges are being studied at Berkeley. The first of these, which has been given the name "Triax", consists of a cylindrical plasma sleeve contained between two coaxial conducting cylinders as shown in Fig. 1. A theoretical analysis of the stability of the cylindrical sheet plasma predicts the existence of a "sausage-mode" instability which is, however, expected to grow more slowly than in the case of the unstabilized linear pinch (by the ratio of the radial dimensions). The second pinch device employs a disk-shaped discharge with radial current guided between flat metal plates, this configuration being identical to that of the flat hydromagnetic capacitor without external magnetic field. A significant feature of these configurations is the absence of a plasma edge, i.e., there are no regions of sharply curved magnetic field lines anywhere in these discharges. The importance of this fact for stability is not yet fully investigated theoretically. As a third configuration a rectangular, flat pinch tube has been constructed, and the behavior of a flat plasma sheet with edges is being studied experimentally.

An obvious disadvantage of the sheet-like plasma is the relatively smaller compression ratio that is obtainable at a-given current as compared to an unstabilized pinch. This is, however, also characteristic of pinches stabilized by longitudinal magnetic fields. The principal advantage of the Triax pinch over the stabilized linear pinch is the absence of a plasma edge, i.e., there are no regions of sharply curved magnetic field lines anywhere in these discharges. The importance of this fact for stability is not yet fully investigated theoretically. As a third configuration a rectangular, flat pinch tube has been constructed, and the behavior of a flat plasma sheet with edges is being studied experimentally.

The voltage across the tube and the total current through the tube are measured by means of a voltage divider and a flux loop in one of the connecting cables. These signals are displayed on an oscilloscope and photographed. Spectroscopic observations, both with and without time resolution, have been made and give some information as to the impurities present, time of appearance of impurity and deuterium lines, and the profiles of deuterium lines broadened by the Stark effect. The profiles were obtained by firing successive shots at different settings of a 0.5-meter grating monochromator. Under low-level conditions, there was sufficient intensity and reproducibility to yield very satisfactory profiles.

The magnetic field distribution inside the discharge tube was determined by means of conventional probe techniques. For convenience of assembly the probe tubing, which is used both as insulator and vacuum jacket. Larger sizes are being planned. The inner conductor has been of 5-, 2.5-, and 1.25-cm diameter. In earlier work at low power levels, Pyrex insulators instead of quartz were used. Other materials are being contemplated, and one tube with exposed metal walls is under construction.

The 1 µ, 30 kv capacitors used can supply peak currents of 30,000 amp each. Low-level runs employ 45 of these capacitors in parallel, while for high power levels a bank of 100 µ is used. For other purposes, such as starting current and switch triggering, smaller groups of from 4 to 24 capacitors have been used. All connections and leads consist of RG 8/U or RG 9/U coaxial cables.

Switching has previously been accomplished by means of pressurized spark gaps filled with air and fired by a sudden release of pressure or, if programmed switching is desired, triggered by a shock wave from an auxiliary, small spark gap incorporated in the switch assembly. Recently a triggered, low pressure, high level switch has been developed.

The gas, usually 99.7%, pure deuterium, flows continuously through the system to avoid accumulation of volatile impurities, and pressures of from 50 to 1000 microns have been used.

DIAGNOSTIC EQUIPMENT AND TECHNIQUES

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PRINCIPAL EQUIPMENT AND TECHNIQUES

We have used tubes of 1 meter and 50 cm length, the diameter of the outer return conductor being about 10 cm. This size is determined by the available quartz tubing, which is used both as insulator and vacuum jacket. Larger sizes are being planned. The inner conductor has been of 5-, 2.5-, and 1.25-cm diameter. In earlier work at low power levels, Pyrex insulators instead of quartz were used. Other materials are being contemplated, and one tube with exposed metal walls is under construction.

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was introduced through the grounded end electrode, and was enclosed in a quartz tube extending to the mid-point of the Triax discharge. No evidence was found that this arrangement affected the discharge measurably.

Two large liquid-scintillator tanks were used to detect neutrons from the discharge, one being operated close to and one several meters from the Triax. Both the initial proton-recoil pulse and the individual neutron-capture pulses were recorded. The over-all detection efficiency was 5% when the detector was close to the discharge tube.

Various other auxiliary tests have been performed from time to time. An electrostatic probe was used to determine whether an electrode sheath existed (none could be found) and to estimate the resistive voltage drop along the center of the plasma.

Several floating electrodes were introduced into the path of the discharge to determine the effect of electrodes on the voltage across the tube and on the neutron production. Subdivision of the pinch by these floating electrodes had little effect on the behavior of the pinch.

Some measurements were made on the heat delivered per discharge to an end electrode, using a fast responding thermocouple. These measurements verified roughly the theoretical predictions as to the low heat transfer.

TRIAX PINCH MEASUREMENT

In general, the measurements fell into two main subdivisions according to the value of $\chi$, the ratio of maximum magnetic pressure to initial gas pressure. In the low-power work, for which well-behaved performance was observed without preionization only when the initial pressure was above 200 $\mu$, $\chi$ was always less than $5 \times 10^4$. In high-power work ($\chi > 10^5$), when the initial pressure was relatively low and particularly when higher currents were used, a starting current was required for satisfactory results.

Low-Level Observations ($\chi < 5 \times 10^4$)

Typical oscillograms of various signals obtained with a Triax are shown in Fig. 2. This tube, with Pyrex walls and a 5 cm diam. inner conductor, was filled to 475 $\mu$ initial pressure; a 45-$\mu$f bank of condensers charged to 20 kv was discharged through it. Each trace in Fig. 2 consists of three superimposed traces from three separate discharges to show the reproducibility from shot to shot.

The first trace (Fig. 2a) shows the tube voltage. The oscillations are due to variations in the plasma thickness, because the total current is changing only slowly. At the time of a pinch (maximum compression), the inductance and its negative second time-derivative are a maximum and the voltage is decreasing at its maximum rate. Eight rebounds from the pinch are discernible in Fig. 2a. The oscillations are smoothly damped and show no signs of instability. The time to reach the first pinch agrees well with the value calculated under the assumption that all the gas is swept into the plasma sheet.

The second signal (Fig. 2b) is a measure of $dI/dt$, and of course shows oscillations that are nearly the mirror image of the voltage oscillations. The time integral of this signal is proportional to the total discharge current, and is shown in Fig. 2c. The lack of structure indicates that the total inductance of the
The circuit is not greatly affected by the oscillations inside the pinch tube.

The value of the voltage across the tube when $dI/dt = 0$ is a measure of the resistive drop because at this time $dL/dt$ is small. Under the assumption that the current flows in a channel about 5 mm wide (cf. Fig. 4b), an electron temperature can be calculated by use of the conductivity-temperature relation given by Spitzer. In this particular low-level case, this temperature is about 15 ev.

Figure 2d was obtained from a flat loop wedged between the outer conductor and the outer Pyrex insulator. This signal shows the rate of change of current density flowing along the outer conductor. Comparison of Fig. 2d with Fig. 2b shows that the local current density on the outer conductor is not proportional to the total current. If cylindrical symmetry is assumed, this is presumably due chiefly to a mode of oscillation in which the radial position of the plasma rather than the thickness varies. Such oscillation is expected whenever the plasma sheet is not formed initially in its equilibrium position.

Note also that the frequency of the slower radial mode (seen only in Fig. 2d) is not as reproducible from shot to shot as that of the thickness mode, indicating that these radial oscillations tend to get out of phase as time goes on. Simple theoretical considerations predict a different dependence of the two frequencies on variations in experimental conditions such as initial pressure. The lower damping of the radial mode is also in accord with theory.

The interpretations given here have been checked by detailed magnetic-probe studies and also by spectroscopic observations, to be discussed later.

**Magnetic Probe Studies**

Probe studies in the low-power Triax pinch have shown fairly good reproducibility up to about 2.5 μsec, depending on various factors such as gas pressure and voltage. Figure 3 shows a three-dimensional view of the current density as a function of both radial position and time, extending over the first 1.5 μsec of the discharge. The nature of the pinch and the existence of a rather large current penetration is apparent from this figure. The width at half-maximum of the current channel at the time of the first pinch is apparently 1/3 of the original thickness before breakdown. The oscillations in thickness are more easily seen in Fig. 4b where the width of the current channel (determined by the positions of half-maximum current-density values) is plotted against time. This width is closely correlated with the effective inductance in the discharge, and comparison with the observed tube voltage bears out the interpretation given above. A pressure balance calculation, using the measured mean compression together with the assumption that the plasma has the same temperature everywhere, leads to estimated temperatures of about 4 ev at the time of the first pinch and 15 ev at the current peak.

**Spectroscopic Study**

Spectrograms (without time resolution) of a low-energy Triax showed a line spectrum with no continuum. The principal impurities were silicon and oxygen from the insulators, carbon from gaskets, and copper from electrodes, as expected. A time-resolved profile of the deuterium line $D_0$ was obtained with a monochromator and photoelectric detector. In Fig. 4 the top trace is the tube voltage against time, the second curve shows the thickness of the plasma sheet as deduced from magnetic-probe measurements described earlier, and the third and fourth traces are typical of a large number of curves of light intensity vs. time taken at various wavelengths in the neighborhood of $\lambda 4860$ (the latter being the wavelength of the center of the profile of $D_0$). The fourth trace shows that the profile of $D_0$ is very much broadened by inter-atomic Stark effect at the times of the first and second pinches. An application of the Holtsmark
theory\(^6\) to the profile at the time of the first pinch
(0.8 \(\mu\)sec) indicates that while the density of ions is not
uniform throughout the plasma there is a layer in
which the density reaches some thirteen times the
density of atoms in the gas before the discharge begins.
This result is in reasonable agreement with the con-
cclusions from the probe studies. By the time of the
third pinch, the deuterium atoms in the dense parts
of the plasma have become completely ionized and
cease to radiate. The remaining light, visible in the
trace at \(\lambda 4860\) at 1.5 \(\mu\)sec and later, must come from
neutral atoms at or near the walls where the density
is low, as shown by the narrow profile at these late
times.

Figure 5 shows at slower sweep speed the voltage,
current and intensity of the line \(D_\beta, \lambda 4860\). The burst
of light at the time of the first zero in the current indi-
cates that at this time the plasma spreads to the walls
and thereafter again forms into a pinched sheet as the
current builds up. After the second zero, the current
remains at too low a value to hold the plasma away
from the wall, and there follows a prolonged emission
of \(D_\beta\) because of recombination at the walls.

The Triax Pinch at High Level \((\chi > 10^5)\)

Operating a Triax tube at high level and a pressure
of approximately 100 microns leads to well-behaved
pinches only when the main discharge is preceded by
a low-energy starter discharge which ionizes and pre-
heats the gas. This predischarge is estimated to give
the plasma a temperature of about 15 volts, the current
being such that the magnetic pressure approximately
balances the particle pressure against the walls.

Because of the contamination problems with Pyrex,
quartz insulators were used in these high-level runs.
Switching apparatus was developed which allowed
firing the starter current \((16 \mu\) charged to 20 kv, pro-
ducing 40,000 amp peak in 12 \(\mu\)sec\) for a predetermined
length of time and then firing the main current. Nor-
mal operation of the tube shows no impurities intro-
duced by the starter current. The effect of the starter
current on pinch formation at 140-\(\mu\) pressure in a
50-cm tube having inner and outer conductor radii of
5 and 10 cm, respectively, is shown in Fig. 6. The
upper trace (taken without starter) does not show the
marked pinch dynamics, while the next (with starter)
shows good pinches. Figure 6c shows the current trace
with a proton-recoil signal from the neutron detector
superimposed on the current. Peak current was about
1.3 \(\times 10^4\) amp, and 2 \(\times 10^4\) neutrons/pulse were ob-
erved. Note the time coincidence of neutron produc-
tion with the pronounced bump on the voltage curve,
which is preceded by a series of pinch oscillations of
decreasing amplitude. Such a bump is of course sug-

gestive of instabilities. Several arguments, described
below, militate against interpretation of this voltage
bump in terms of known instabilities, but a determined

search for such instabilities is continuing. Furthe-
more, the thermonuclear origin of the neutrons has not
been ruled out.

Interpretations

(a) If the neutrons are assumed to be ther-

monuclear, their number is consistent with the tem-

peratures possible from known heating mechanisms in

the tube (resistive heating, shock heating, and adia-

batic compression, which could have produced a

temperature of about 300 volts). This consistency was

lacking in the much larger yields from the linear

pinch.

(b) Neutron production has been shown to be

roughly uniform throughout the length of the tube,

thus ruling out some proposed nonthermonuclear pro-

cesses for neutron production. The low neutron

production precludes the measurement of neutron

energy by nuclear-emulsion techniques, as was done

with the simple dynamic pinch,\(^7\) but a high-pressure

cloud chamber is being readied which may make this

measurement possible.
(c) Addition of a longitudinal magnetic field of 200 gauss shifts the neutron production to a later time, but does not seem to affect the number of neutrons appreciably.

(f) Because instabilities in the Triax might be expected to be enhanced if the wall separation were increased, another tube having an inner conductor of 2.5-cm diameter was constructed. The high-level characteristics of this tube are shown in Fig. 7. The voltage bump is not much larger than in Fig. 6.

Various characteristics of the tube mentioned under (f) above are shown in Fig. 7, taken at 75-micron initial pressure and with the main bank of 100 μF charged to 20 kV. Starter current was used as before. The new voltage curve, Fig. 7a, is considerably different from that in Fig. 6b and is interpreted as being due to a superposition of the two modes of oscillation of the plasma mentioned previously.

The neutron yield for this tube, shown in Fig. 7b superimposed on the current trace, was higher than for the previous tube; i.e., $2 \times 10^9$ per pulse. The production begins near the end of the bump on the voltage curve.

The last three traces in Fig. 7 show the behavior of the light emitted from the tube. The most interesting feature is the sudden appearance of impurities, represented by Si$^{++}$ (λ4552) just at the end of the neutron production.

Experiments with streaking cameras are under way and radiation in the vacuum ultraviolet will be studied, in an effort to understand more fully the mechanism of the Triax discharge. These experiments, coupled with a determination of neutron energy, will help to establish whether the neutron production is compatible with a thermonuclear process.

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Study and Use of Rotating Plasma

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This paper is intended as a preliminary report on the Homopolar device (axial magnetic field with radial electric field), which is the rotating plasma configuration most thoroughly investigated thus far. A study of this configuration for aerodynamic purposes was reported in 1950 by H. C. Early et al., and a more complete report was issued in 1952. Some ion sources having the same basic geometry were reported in 1955 by L. P. Smith and J. Luce. The hydromagnetic Homopolar machine was conceived by William R. Baker and Oscar A. Anderson in 1956, as a device for the containment and heating of a thermonuclear plasma. More recently, there have been significant contributions to this technique by Gow and Wilcox, and by Boyer, Hammel, Longmire, Nagle, Ribe, and Riesenfeld.

THEORY

The analysis presented in this paper applies principally to the “ideal” Homopolar, that is, the configuration free from electrode-sheath drops and other disturbing but remediable phenomena. The experiments described here have been carried out mostly under high-density (pinch-type) conditions which favor the creation of a totally rotating plasma. As models of the Homopolar grow larger, it will become possible to use the present pinch-type technique of plasma formation at much lower density, just as in the case of the large toroidal stabilized pinch.

The Rotational Equilibrium State

Whenever currents flow in plasmas across magnetic field lines, as for example in most pinch-type configurations, the resultant Lorentz force must be balanced by static or inertial plasma pressures. Where such pressure is lacking (e.g., in the low-β stabilized pinch), the current flows along field lines or ceases to flow. In the Homopolar, Fig. 1, a transient radial current is drawn while the plasma is accelerated to its equilibrium rotation. In equilibrium, the driving electric field vanishes in the plasma rest frame, and the radial current ceases. The equilibrium velocity of rotation is

\[ v_0 = \frac{{[E_0 + (\rho v_0^2/e)]}}{H}. \]

The second term, a centrifugal effect, depends on \( e/m \) and, although small, results in different drift velocities for electrons and ions. As a result, an azimuthal current flows and the associated Lorentz force just balances the centrifugal pressure of the plasma. If the plasma conductivity is finite, there is also an azimuthal electric field resulting in a slow outward drift of plasma—a diffusive process impelled, so to speak, by centrifugal pressure.

A sudden application of electric field to the Homopolar would produce trochoidal ion orbits as in Fig. 2(B). In practice, the applied voltage drops sharply at the instant of ionization and then builds up again as the plasma rotation develops. The effective electric field in the plasma in general rises slowly compared with an ion Larmor period. The ion orbits therefore are likely to be of the form shown in Fig. 2(A). Although the direction of drift is the same for particles of either sign, the mean radial displacements of ions and electrons are in opposite directions. The effect is similar to the displacement phenomenon in a dielectric. The associated dielectric constant, \( K \), may be evaluated by ascribing the drift energy density of the plasma to the electric field energy density. Neglecting the second term of Eq. (1), we have

\[ \frac{KE^2}{8\pi} = \frac{E^2}{8\pi} + \frac{1}{2}\rho v_0^2 = \frac{E^2}{8\pi} \left(1 + \frac{4\pi \rho c^2}{H^2}\right), \]

where \( \rho \) is the mass density of the plasma.

If the plasma in a rotational device may be regarded as a dielectric, then the machine itself must resemble a capacitor. The Homopolar experiment is essentially equivalent to connecting an uncharged capacitor, the Homopolar, to a charged capacitor, the source condenser bank, through an intermediate resistance and inductance (Fig. 3). In general there results a damped oscillation with the current and voltage approximately 90° out of phase, and equilibrium is reached with the charge so distributed that the voltages on the two capacitors are equal (Fig. 4). The current ceases at

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equilibrium except for a trickle through the leakage resistance, $R_L$, representing viscous drag and heating.

The kinetic energy of the rotating plasma is the analogue of stored energy in a capacitor $\frac{1}{2}CV^2$. Another constant of the motion, the angular momentum $P_\theta$, is the analogue of the charge, $Q = CV$. The torque impulse due to the current through the Homopolar must in the ideal case equal the angular momentum imparted to the plasma disc. It is easily shown that, neglecting losses, we have

$$P_\theta = \int H \, r \, dr \, J \, dt = \Phi Q / 2 \pi,$$

(4)

where $\Phi$ is total magnetic flux.

The capacitor analogy may be carried to its logical extreme with the addition of a "crowbar" shorting switch to the circuit of Fig. 3.

If the crowbar switch is closed at a time when there is voltage across the Homopolar, there must result a surge current through the Homopolar and crowbar (Fig. 5). There can be little doubt that the charge represented by the outpouring of the current gives a measure of the angular momentum stored in the rotating plasma, since the crowbar current is opposite in direction to the charging current. The voltage and current oscillations after crowbarring represent hydro-magnetic oscillations of the plasma similar to the motion of a torsion pendulum.

**Plasma Containment by Centrifugal Force**

In an ordinary mirror machine, only those ions are contained that have a sufficiently large ratio of azimuthal to axial velocity. Other ions are lost through an "escape cone" in velocity space. Particles are continually diffusing into this cone by collisions. At higher temperatures, diffusion—and hence loss rate—decreases while thermonuclear yield increases on account of the behavior of the appropriate cross sections. Much higher operating temperatures are therefore required in a practical machine in order to control the loss rate. The escape cone is a serious inconvenience.

If one applies a radial electric field to a mirror machine, there are at first components of $E$ parallel to $H$, but since in general we have

$$E^2 / 8 \pi \ll nkT,$$

(5)

the plasma will readjust itself so that $E$ becomes orthogonal to $H$. Once this has happened, it turns out that all first-order effects in $E$, tending either to contain or to expel particles, cancel out. The second-order effect in $E$ permits containment of all particles up to a certain kinetic energy. In other words, for sufficiently large $E$ the escape cone can be truncated arbitrarily far out in velocity space.

This proposition is seen to be plausible when one notes that the azimuthal drift in the electric field produces a centrifugal force tending to keep particles from approaching the axis of drift. Since passage through the mirror involves approach to the machine axis, there is thus an effective mirror repulsion which is independent of particle sign and, like the centrifugal force, increases with the square of $E$.

The idea of plasma containment in a mirror machine by means of the "gravitational" effect was first proposed by Baker and Anderson in 1956. A rigorous mathematical analysis was recently undertaken by Nagle, Ribe, and Boyer leading to the elegant theory of Longmire. An elementary derivation giving the same results in the limit of weak mirror ratio, $R_m$, was made by H. P. Furth. It is found that the maximum axial kinetic energy that can be contained is

$$W_\perp = W_\perp (R_m - 1) + \frac{1}{2} m v^2 (R_m - 1) / R_m,$$

(6)
in which $W_\parallel$ and $W_\perp$ are the energies of thermal motion parallel and perpendicular respectively to $H$. Note that the enhanced containment, second term of Eq. (6), depends on the magnitude of the rotational kinetic energy of a particle, and also on $R_m$. Unfortunately, very large values of $R_m$ appear to be of little use in achieving a high degree of enhanced containment. It should be said, however, that the analysis leading to Eq. (6) is valid only in the limit where the mirror field varies slowly over the particle trajectory (adiabatic limit).

It should also be noted that the containment effect is much smaller for electrons than for ions, given a common velocity of rotation $v_0$, but the consequences need not be serious. The loss of electrons through the escape cone sets up an axial electrostatic potential equal to the mean energy of escaping electrons. This potential will be weak if the electron temperature is kept well below the ion temperature, as it should be in a practical machine. A very serious practical consideration is that, if the rotational kinetic energy of the ions is merely made equal to the mean ion thermal energy, then the more energetic ions escape at about the same rate as from an ordinary mirror machine. It is necessary therefore to set the rotational energy equal to the energy of the most energetic ions one desires to have participate significantly in the fusion process. For example, for $R_m \sim 2$, $E = 4$ kv/cm, $H = 4$ kilogauss, and ion density a few times $10^{13}$, one would have containment up to 10 kev. This is satisfactory for an experimental thermonuclear plasma device but not for an economic power-producing machine. For much higher energies, the over-all voltages required in large machines may present difficulties.

**Plasma Heating**

If a thermonuclear kinetic energy density can be imparted to a rotating plasma, one may hope in a general way that some of this will be converted into disordered motion. Since the Homopolar, unlike the ordinary mirror machine, has no simple “escape cone” in velocity space, there is no objection in principle to particle collisions. Various diffusive processes of
thermalization are therefore conceivable. The situation is similar to that in the stabilized pinch. In both cases a long-lived equilibrium state can readily be created, and one looks next for processes that will convert stored nonthermal energy density into thermal energy density at a suitable rate.

The plasma of the Homopolar, like the plasma of the stabilized pinch, is heated in the process of outward diffusion. This is just the ordinary Joule heating process. Since the velocity distribution in the rotating plasma tends to go as $1/r$, there is also viscous drag between plasma layers. The associated heating effect is comparable to Joule heating. Probably there can also be some turbulent mixing of plasma layers, resulting in additional thermalization.

While the charging current of the Homopolar is drawn, there results a pinch-type compression in the axial direction. It can be shown that this effect is always sufficient to produce a moderate pinch away from the wall, but insufficient to compress the plasma appreciably, provided the energy density of the plasma rotation is comparable to that of the axial magnetic field.

Aside from these built-in heating mechanisms, there are also various artificial means one can employ. For example, superimposing a rapidly oscillating component on the radial electric field can set up a standing hydromagnetic half-wave in the axial direction. The wave propagation velocity is the Alfvén speed,

$$v_A = \frac{cK^{-1}}{4\pi},$$

and amounts to only $3 \times 10^7$ cm/sec for a typical dielectric constant of $K = 10^4$. The viscous drag brought into play by the associated velocity gradients along field lines is large and should result in a rapid conversion of directed kinetic energy into heat energy. Experience with deuterium pinch devices has shown that $\frac{\eta_0 r_0^2}{4} \approx 2 \times 10^{16}$ cm$^{-1}$ for optimum performance, where $r_0$ is the tube radius and $\eta_0$ the initial particle density. It is an interesting coincidence that an Alfvén half-wave length $\lambda/2$ at the deuteron cyclotron frequency is just such as to give $\frac{\eta_0 r_0^2}{4} = 2 \times 10^{16}$ cm$^{-1}$.

As an alternative to impressing an oscillatory electric field, one can simply perturb the cylindrical symmetry of the Homopolar. Once a certain level of thermal and rotational energy has been established it can also be enhanced by radial compression as in a mirror machine.

### EXPERIMENTAL RESULTS ON HOMOPOLAR I

Homopolar I is illustrated in Fig. 1. The discharge space is bounded by a central electrode 3 in. in diameter, an outer electrode 9 1/2 in. in diameter, and a pair of Vicor glass plates 1/2 in. apart that lie against the pole faces of an electromagnet. The glass serves to separate the plasma discharge current from the return current that flows over the copper-plated pole faces. The discharge current is supplied by a condenser bank, usually 48 $\mu$F. This source is connected to the outer electrode through 48 coaxial cables like the one shown in the figure. Deuterium, helium, neon, argon, and iodine vapor have been employed in these hydromagnetic experiments.

At the time of breakdown, the plasma is pinched away from the glass. The radial pinch current, typically of the order of 50 kA, produces a torque as it flows across the axial magnetic field, and the torque accelerates the plasma into the rotational state. The pinch phase is accompanied by a rapid fall of the applied voltage, which subsequently rises again during the acceleration phase. The high voltage then decays slowly during the rotation state because of losses and internal heating. The current shows a rise during the pinch and early acceleration phase, but then falls again to a very low value as the equilibrium rotation state is reached.

Under certain circumstances the voltage and current have an oscillatory character but in most of the experiments the discharge is approximately critically damped. The phase of the voltage and current are always such as to demonstrate the capacitors-like behavior of the Homopolar. Accordingly, the Homopolar is represented in Fig. 3 as the capacitor $C_H$, and the loss mechanism (which includes internal heating) by the shunt resistance $R_L$.

Typical oscillatory behavior is shown in Fig. 4. The upper pair of traces, with magnetic field switched off, show voltage and current in phase. In this case $C_H$ in Fig. 3 should be replaced by a short circuit. The period of oscillation is $T_s = 2\pi [(L_s + L_H)C_s]^{1/2}$. The lower traces, with 13 kilogauss magnetic field show the $\sim 80^\circ$ phase shift between voltage and current. As the gas pressure is reduced, the period, which is given by

$$T_{814} = 2\pi [(L_s + L_H)C_s/C_s + C_a)],$$

is also reduced, in quantitative agreement with the reduction of the dielectric constant $K$, Eq. (3), of $C_H$. It will further be noticed that the equilibrium
It is quite clear that the Homopolar presents interesting possibilities as a practical capacitor. Rates of change of current of \( \sim 5 \times 10^{11} \) amp/sec, up to peak currents of 300 ka, are readily obtainable in Homopolar I. The low inductance, \(<0.001 \mu\text{H}\), and high dielectric constant, \(>10^6\), result in a very compact system of electrical energy storage. Further considerations of the plasma Homopolar as a hydromagnetic capacitor will be published elsewhere.\(^7\)

Besides showing the comparatively long lifetime of the rotational state, the crowbar has provided a useful diagnostic technique to determine the dielectric constant under variations of pressure and magnetic field. Since the vacuum capacity of the Homopolar is 0.75 \(\mu\text{F}\), \(K\) is given by

\[
C = 0.75 \times 10^{-12} K = V^{-1}\int i dt = Q/V,
\]

where \(V\) is voltage at time of crowbar and the integral is taken over the first quarter cycle of the crowbar current. It was possible to vary \(K\) between the limits of \(10^6\) to \(10^8\) and, over this variation, the ratio of the observed dielectric constant to that calculated from Eq. (3) falls in the limits

\[
1 < \frac{K_{\text{obs}}}{K_{\text{th}}} < 2.
\]

Both the nature of the observed limits on the variation of \(K_{\text{obs}}/K_{\text{th}}\) and the behavior of \(C\) with time provide diagnostic information. The lower limit of the variation is taken to mean that essentially all the initial gas has become a rotating plasma. The upper limit can be understood in a qualitative way by comparison with an idealized case. It has already been mentioned that when the centrifugal plasma pressure becomes comparable with the magnetic field energy density, the plasma leans heavily on the magnetic field lines, bending them outward, increasing the mirror ratio. In the idealized case the conductivity is taken to be infinite so that in the absence of diffusion across magnetic field lines, plasma and field distort together. It is easy to show that the capacity is modified as follows:

\[
C_H = C_{\text{HO}} \frac{2 \ln(r_2/r_1)}{(r_2^3 - r_1^3)} \int_{r_1}^{r_2} \frac{\psi r \, dr}{\int_{r_1}^{r_a} \psi(r) \, dr},
\]

where \(H = \psi H_0\) is the distended magnetic field, \(r_1\), \(r_2\) are the inner and outer radii of the Homopolar,
and \( C_{H0} \) is the unmodified capacity. If all the magnetic field is crowded against the outer electrode, then \( C_H = 2.45 C_{H0} \). This upper limit is compatible with the experimental values of \( K_{obs}/K_{th} \). If the finite plasma conductivity is taken into account, then the calculation becomes much more complicated. One obvious conclusion is that when plasma is actually lost by resistive diffusion out of the system, then \( C_H \) will be diminished. A reduction in \( C_H \) after long times has been observed under conditions of large \( E/H \), although under most conditions \( C_H \) remains essentially constant during the lifetime of the rotational state. Above 10 kv, Homopolar I appears to break down across the glass insulators.

Figure 6 shows an analysis of the data of Fig. 5(A). Curve A gives the charge \( Q \) of Eq. (4) at the various crowbar times. Curve D is the true charging current obtained from A by differentiating the curve. It is seen to be less than the observed input current, Curve B, and starts somewhat later. Evidently excess current flows through the device without increasing angular momentum of the plasma, and thus represents current flowing in \( R_H \) of Fig. 3. This current is associated with viscosity of the initially cold gas and with viscous drag on the walls. From Eq. (1) one obtains \( v_\theta \sim 3 \times 10^8 \text{ cm/sec}, \) with a maximum of \( 6 \times 10^8 \text{ cm/sec}. \) The corresponding argon energies are 180 ev and 660 ev. \( C_H \) remained constant at 5.7 \( \mu \text{F} \) for the time of observation; in this case \( K_{obs}/K_{th} = 1.0. \)

A distinction should be made between long-lived rotational effects and actual containment. The containment of the plasma in the rotating state depends on the notion of trapping by centrifugal force in a trough of magnetic field, but actual containment must depend on whether the plasma is in fact leaning on the field, or whether it is leaning on the walls. Evidence that the plasma is indeed leaning on the field is shown in a typical magnetic probe or spin loop signal, the third trace of Fig. 7. This signal was obtained by a probe coil located between the glass plate and the pole piece.

The coil is oriented to detect the enhancement of the radial component \( H_r \) produced by the rotating plasma. The second term in Eq. (1) for the velocity \( v_\theta \) results in an azimuthal current density

\[
\jmath_\theta = \rho v_\theta^2/\mu H \sim E^2/\mu H^2.
\]

This current is the source of \( H_r \), the centrifugally produced distortion of the magnetic field. As proof that the component \( H_r \) is produced in the manner stated, it is observed that \( H_r \) changes sign with reversal of \( H \) but not with reversal of \( E \). It is gratifying to see that \( H_r \) has the same general lifetime as the rotating state. One concludes that the plasma is leaning on the field lines for the duration of the rotating state.

For final proof that the plasma is in a rotating state, the Doppler shift of spectral lines may be observed. Provision was made for spectrographic observations along a line of sight passing within \( \frac{1}{2} \text{ in.} \) of the inner surface of the outer electrode. By photographing the Homopolar spectrum with the magnetic field first in one direction, then in the other, one can observe shifts in the lines due to the Doppler effect. The shifts are symmetrical about reference lines with no field. Figure 8 shows the shift of \( 3888 \text{ Å} \) He and some impurity lines under the conditions 8 kv, 17 kilogauss, 600 \( \mu \text{ Hg} \) of He gas for normal and reversed direction of the magnetic field. The lower comparison spectrum is for He and Hg. The shift corresponds to a velocity of \( \sim 3 \times 10^6 \text{ cm/sec}. \) It is expected that higher velocities would be found near the central electrode.

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Radiation Pressure Confinement, the Shock Pinch and Feasibility of Fusion Propulsion

By Milton U. Clauser and E. S. Weibel*

This report describes the research work on two approaches to the control of the fusion reaction. First, the containment of a plasma in a cavity by means of a radiation field is analyzed. Secondly, the work on nonadiabatic or shock heating is described. In addition, the feasibility of thermonuclear rocket propulsion is examined.

CONFINEMENT OF A PLASMA BY RADIATION PRESSURE

An approach which has received little attention earlier is based on the fact that reflected electromagnetic radiation exerts a pressure on the reflector. A plasma is an excellent reflector up to the plasma frequency, so that it can be subjected to radiation pressure.1

The configuration that looks most promising is based on a toroidal cavity. A filament of plasma forming a toroidal core (Fig. 1) runs along the center line of the cavity. The arrangement thus resembles a coaxial cable bent into a circle and closed into itself, the cavity walls being the outer conductor and the plasma forming the center conductor. The cavity is excited in the lowest transverse magnetic mode at its cutoff frequency. The radiation consists of a purely longitudinal electric field and a purely transverse magnetic field. The electromagnetic energy travels radially and is reflected back and forth between the walls and the plasma. With sufficient intensity, the radiation pressure at the surface of the plasma balances its pressure and maintains it away from the walls.

The fields and the plasma partly penetrate each other. The following is a brief account of the analysis which determines the electron and ion densities and the fields as a function of radius. The particles are supposed to be under the influence of the average fields which they modify by their presence. It is convenient to start from the Lagrangian for the motion of a charged particle:

\[ L = \frac{1}{2} m (\dot{r}^2 + (\dot{\varphi})^2 + \dot{z}^2) + \varepsilon \dot{A} z - \phi \]  

where \( r, \varphi, z \) are its cylindrical coordinates

\[
A_z = -\frac{1}{\alpha} E_0(r) \sin \omega t, \quad \phi(r)
\]

and

\[
\dot{z} = \frac{p_z}{m} + \frac{eE_0(r)}{m\omega} \sin \omega t, \quad \dot{\varphi} = \text{const.}
\]

The radial equation of motion, after eliminating \( \varphi \) and \( z \), is

\[
m\ddot{r} = \frac{\varepsilon E_0^2(r)}{mr^3} \frac{d}{dr} \left( \frac{\varepsilon E_0^2(r)}{2m\omega^2} \right) \sin^2 \omega t
\]

\[
\quad - \frac{e\dot{E}_0(r)}{m\omega} \sin \omega t - e \frac{d\phi(r)}{dr}.
\]

It can be shown that, for sufficiently rapid oscillations, only the time average of the right hand side is important:

\[
m\ddot{r} = \frac{\varepsilon E_0^2(r)}{mr^3} \frac{d}{dr} \left( \frac{\varepsilon E_0^2(r)}{4m\omega^2} + \phi(r) \right)
\]

The particle thus moves as if it were in a potential well \( \phi(r) \), in addition to the potential \( \phi(r) \) which is created by charge separation. This potential provides a force attracting the particle to the center regardless of its sign. It is this term which represents the radiation pressure.

Without going through the details of justification, Boltzmann's law shall be used to determine the distribution of particles under the influence of this potential

\[
n_j(r) = n_0 \exp \left[ -\frac{\varepsilon E_0^2(r) - e\phi(r)}{4m\omega^2} T - \frac{\varepsilon \phi(r)}{kT} \right]
\]

This holds for ions \( j = 1 \) as well as electrons \( j = 2 \) if the proper values for the mass and charge are substituted. It is worth mentioning that an analysis based on a continuous two-fluid model of the plasma leads to exactly the same expressions for \( n(r) \).

To obtain a self-consistent solution, one must require that Maxwell's equations hold with the charge and current densities, calculated from Eq. (5):

\[
\phi'' + \frac{1}{r} \phi' + e(n_1 - n_2) = 0,
\]
where $\varepsilon(n_1 - n_2)$ is the charge density. The current density, which is represented by the last two terms of Eq. (7), has been obtained from Eq. (2), by averaging $\varepsilon(n_1 - n_2)$ over all momenta, $p_n$, which are assumed to be randomly distributed.

Equations (5), (6) and (7) can be solved for $n(r)$, $E_0$; $\phi$. To obtain a general picture of the behavior, one can neglect the charge separation by assuming in (7), $n_1 = n_2 = n$ and using for $n$ the geometric mean of $n_1$ and $n_2$ according to Eq. (5):

$$n(r) = n_0 \exp \left[ - \frac{\varepsilon^2 E_0^2(r)}{8 M \omega^2 kT} \right]$$

with $\frac{1}{M} = \frac{1}{m_1} + \frac{1}{m_2}$. (8)

This leads to a single nonlinear equation for $E_0(r)$:

$$E_0(r) + \frac{1}{r} E_0'(r) + \left( \frac{\varepsilon^2 n_0}{M} \exp \left[ - \frac{\varepsilon^2 E_0^2(r)}{8 M \omega^2 kT} \right] \right) E_0(r) = 0.$$ (9)

A typical solution is shown in Figure 2. In solving this equation, $n$ is set equal to zero for $r > b$, where $b$ designates the radius at which $E$ assumes its maximum and $n$ its minimum. In the cases of interest, $n_{\text{min}}$ is so small that the discontinuity is not noticeable.

Physically this means that plasma has been removed from $r > b$ which slightly upsets the equilibrium: particles will escape at a rate equal to $n_{\text{min}}$ times the thermal velocity. One can then determine the half life of the containment.

$$\tau = \left( \frac{m}{kT} \right)^{\frac{1}{2}} \frac{a^3}{b} \exp \left[ - \frac{\varepsilon^2 E_0^2(b)}{8 M \omega^2 kT} \right]$$ (10)

where $a$ is the plasma radius shown in the figure. This time can be exceedingly long; hence, there is practically no diffusion of plasma out of confinement.

Multiplying Eq. (7) with $(1/\omega)(dE_0/dr) = B_0$ and integrating it once, one obtains

$$\left[ 2 n h T + \frac{1}{4} (E_0^2 + B_0^2) \right]_{n_1}^{n_2} + \int_{n_1}^{n_2} \frac{B_0^2}{r} dr = 0.$$ (11)

The last term in this equation is a reminder that the electromagnetic forces cannot be described exactly by a hydrostatic pressure. The gas pressure that can be balanced is larger than the electric and magnetic "pressure" $\frac{1}{2}(E_0^2 + B_0^2)$; the factor $\frac{1}{4}$, instead of $\frac{1}{2}$, is due to the time averaging of $\sin^2 \theta$. 

To sustain the radiation in the torus, radio-frequency power must be supplied to cover the loss of energy to the cavity walls. This loss is large, because of the skin effect. In an economical reactor, the power generated by fusion must exceed the loss. This leads to an estimate for the size and power level of such a reactor. Both turn out to be exceedingly large. It is hoped, however, that ordinary magnetic confinement and this type of radiation pressure confinement can be combined so that the bulk of the required pressure is provided by magnetostatic fields while a super-imposed oscillating field eliminates drifts and stabilizes the plasma.

**NONADIABATIC, OR SHOCK HEATING OF A PLASMA**

In the containment schemes which utilize the pinch effect, two forms of plasma heating have been used: adiabatic compression and ohmic heating. When the pinch is stabilized by an internal field, adiabatic compression loses importance and ohmic heating becomes increasingly difficult above $10^8$ K because of increasing conductivity. Nonadiabatic heating due to finite velocity of the plasma boundary is a further possibility. If the plasma behaves as a fluid, then for large velocities of the boundary a shock wave will form. The energy transferred to the plasma ions is approximately $m U_0^2$ where $m$ is the ion mass and $U_0$ is the boundary velocity. If the mean free path is large compared to the plasma dimensions, the energy transferred is $2mU_0^2$ which is the same order of magnitude for our purposes. In either case, all of the plasma will converge toward the center of the column and be contained long enough for the ion motions to be thermalized. Hence it is of only academic interest as to whether or not an identifiable shock wave will develop.

**The Shock Pinch**

If the plasma is to be heated nonadiabatically by the motion of the boundary, the boundary must move with a velocity comparable to the ion velocity corresponding to the temperature to be attained. The average thermal velocity of deuterons at their so-called ignition temperature of $4 \times 10^8$ K is $2.2 \times 10^6$ m/sec, which is $\frac{1}{10}$ the speed of light. If the energy is to come from an electrical storage system, it must be initially contained in a volume no greater than $(140)^3$ to $(140)^3$ times the volume of the discharge tube.

Analysis of a specific configuration will help to illustrate the nature of the problem. For simplicity, let us consider a linear pinch tube and examine the limiting conditions imposed by not being able to neglect the ion velocities compared to the speed of light (not relativistic considerations).

A cylindrical capacitor with circular plates is located concentrically with the discharge tube as shown in Fig. 3.

**Order-of-Magnitude Considerations**

When the switch, which is shown schematically, is closed, a current flows through the plasma and the magnetic field starts to drive the plasma toward the center of the tube. At the same time, an electromagnetic wave propagates radially outward toward the radius $r_2$ where it is reflected back toward the center. The total energy in the capacitor requires a time to reach the discharge tube of approximately $2\pi c / U_p$. If the average velocity of the magnetic piston is $U_p$, 

$$E_0 + \frac{1}{r} E_0' + \left( \frac{\varepsilon^2 n_0}{M} \exp \left[ - \frac{\varepsilon^2 E_0^2(r)}{8 M \omega^2 kT} \right] \right) E_0 = 0,$$ (7)
then it travels in to some fraction of the tube radius in a time $t_e$ given by $r_1/U_p$. Since the energy transfer will occur in the same period of time, $2r_1/\epsilon \sim r_1/U_p$. If the ions bounce off the piston with twice its velocity, then, for a temperature of $4 \times 10^6 \ K$, $U_p = 1.1 \times 10^6 \ m/sec$. If the dielectric constant is unity, $r_2/r_1$ is of the order of 140. Thus the electrical energy which goes into the plasma must be contained in a radius which is less than 140 times the discharge tube radius.

![Figure 1](image1.png)

If we know the maximum density at which the electrical energy can be stored, the plasma density can be determined. Let us assume that one-quarter of the energy goes into the plasma:

$$\frac{3}{2} nk T \pi r_1^2 h = \frac{1}{2} E^2 \pi r_2^2 h,$$  \hspace{1cm} (12)

where $n$ is the particle density, $\epsilon$ is the dielectric constant, and $E$ is the electric field. The average ion velocity is $U = (3kT/m)^{1/2}$ and the ion density is given by

$$n \sim \frac{\epsilon E^2}{12kT} \frac{c^3}{U^3}$$ \hspace{1cm} (13)

Since $c^3$ varies inversely as $\epsilon$, the attainable density is independent of the dielectric constant. If a dielectric such as mylar, with a dielectric strength of $E = 10^8 \ \text{v/m}$, is chosen:

$$n \sim 2.5 \times 10^{22} \ \text{deuterons/m}^3$$

or

$$2.5 \times 10^{18} \ \text{deuterons/cm}^3.$$

This is an encouragingly high density for a plasma at this temperature even though it represents an upper limit unless better dielectrics are available. It can deteriorate rapidly if additional inductance slows down the discharge and puts more of the energy into the magnetic field.

### Equation of Motion

A number of more nearly exact solutions have been obtained for the shock heating. The simplest uses the free particle model assumed in the previous paragraph. The piston pressure required for the acceleration is $P = \rho U_p U = 2\rho U_p U^2$ where $\rho$ is the gas density. If we neglect the internal magnetic field, which will be needed for stabilized containment, then the plasma pressure must be balanced by the external magnetic field:

$$P = B E^2/2\mu,$$ \hspace{1cm} (14)

where $B_0 = \mu I/2\pi r$ and $I$ is the plasma current.

The solution of Maxwell’s equations for the cavity with a constant diameter cylindrical conductor shows that most of the energy is in the fundamental mode and is equivalent to a lumped parameter circuit with:

$$L = \frac{\mu_0 h}{2\pi} \ln \frac{r_2}{r_1} \quad \text{and} \quad C = \epsilon \pi r_2^2 / h.$$

This formulation should still apply when the diameter of the plasma conductor is changing. The circuit equation, including the variable inductance, can be put in the form:

$$\frac{d}{dr} \left[ \xi \ln \frac{2}{\xi} \frac{d}{dr} \right] + \xi^2 + N - 1 = 0$$ \hspace{1cm} (15)

where

$$\xi = r/r_1, \quad \xi_2 = r_2/r_1, \quad \tau = c/r_2 \delta,$$

$$N = \left( \xi_2^2 \xi_2 \right)(\epsilon E^2/\rho c^3),$$

Solutions for $\xi$ and a dimensionless current $\xi = I r_2 / CV nc$ are plotted in Figs. 4 and 5. From these it can be
seen that a critical condition occurs when \( N = 1 \).
For the assumptions that have been made, all of the electrical energy will be transferred to the plasma when \( N = 1 \). Upon close examination, it turns out that much of the energy is transferred when the piston is near the axis and at a time when the assumptions about the piston not overrunning particles more than once and not crossing the axis have been violated.

Somewhat more realistically, it can be shown that approximately one-quarter of the energy may be transferred to the plasma.

Then

\[
N = \frac{1}{2} \left( \frac{eE^2}{\rho c^2} \right)^{1/2} \left( \frac{r_2}{r_1} \right)^2 = 1
\]

and

\[
\frac{1}{2} \pi r_2^2 \delta eE^2 = \frac{1}{2} \pi r_1^2 \delta p U_p
\]

From these equations we obtain \( 2r_2/c = r_1/U_p \) which is the same relation as was obtained from the order-of-magnitude considerations. Therefore, the estimate of the plasma density is similarly valid.

**Experimental Apparatus to Test**

**The Linear Shock Pinch**

The analysis gives a clue as to where care had to be taken in the design of the experimental apparatus used to verify that a plasma can be shock heated to the desired temperature. The electric field in the capacitor must be as high as possible and the inductance of the switch must be as small as possible or the plasma density will have to be so low that it will be difficult to initiate the discharge.

The electrodes in the linear pinch are detrimental because of the contamination which they inevitably put into the plasma and because of the thermal quenching of the plasma in contact with the cold metal. The toroidal configuration will eliminate these difficulties. For the conventional pinch with relatively slow current rise time, the toroidal discharge can be induced by magnetically linking the torus with a primary coil. For the shock pinch with very fast rise
times, a different arrangement is necessary. Fig. 8 is a schematic of one possible configuration. The primary discharge between adjacent condenser plates is through coaxial cylindrical conductors. The switches are a part of the conductors. All of the switches will be fired simultaneously, or at least within a time which is short compared to one ringing cycle.

The magnetic coupling between the plasma torus and displacement currents in the capacitors, along with the real currents in the cylindrical conductors, will induce a current in the plasma. The equations which describe the performance of this configuration are essentially the same as for the linear pinch except that some of the parameters are slightly modified.

Stabilized Containment after Shock Heating

Theoretical studies\(^6,7,8\) show that a heated plasma can be stably contained in a pinch discharge if a longitudinal \(B_z\) field is trapped in the plasma and a concentric conducting wall is used. Experiments verify that such a stable confinement is possible.\(^9\) This method of containment imposes certain restrictions. Since the \(B_z\) field must be strong enough to stabilize the short wavelength perturbations and since the sum of the plasma pressure and the magnetic pressure due to \(B_z\) must be equal to the pinching magnetic pressure during containment, the analysis indicates that only a limited volume compression is usable. In the practicable situation, this amounts to not more than a three-to-one change in radius or ten-to-one change in volume.

APPLICATION OF THERMONUCLEAR POWER TO ROCKET PROPULSION

The high density of nuclear energy in comparison with chemical energy makes it particularly attractive for use in rocket propulsion.\(^10\) Fission energy has already received attention in this regard, but it has an inherent difficulty in that the reaction products do not make a good propellant. No one seems to have devised a way to transfer the energy from the fission products to a suitable propellant, such as hydrogen, without the interposition of solid material and without the consequent severe limitation in temperature and specific impulse.

In contrast, the reaction products of the fusion reaction make an excellent propellant. A further difficulty looms, however, because some of the reaction power is in a troublesome form. For the D-T reaction at a temperature of \(4 \times 10^8\) °K, about 20% of the power is in charged particles, about 0.6% is in bremsstrahlung radiation power and the rest is in the neutrons. In a missile, any power which escapes from the reaction zone and manifests itself as heat in the

With a ten-to-one volume change, adiabatic heating is negligible but shock heating is not so limited, for it depends more on the rate of change than on the actual change in volume. Imagine that the magnetic piston has the ideal motion shown in Fig. 9. During the interval \(t_1 < t < t_e\) the magnetic piston drives the plasma from the wall into a radius \(r_0\) with a velocity sufficient to heat the plasma as described earlier. The energy put into compressing the \(B_z\) field has been neglected, since this energy is less than the energy put into the plasma by the ratio of the final to the initial volume. As the piston approaches the radius at which the plasma is to be contained, its motion is stopped. This can be accomplished by tailoring the shape of the capacitor plates. By suitable cutouts in plate area at appropriate radii, the current through the discharge can be adjusted to the proper function of time which will slow the piston as its radius approaches \(r_c\).

After the piston has been brought to rest, it should be possible to hold the reacting plasma in a state of stabilized containment for a period long enough to regain the energy required to bring it up to temperature and to produce some useful power. The method of regaining the heating energy and producing useful power will determine what further needs to be done to the plasma. For this reason we next turn to an interesting application.
structure must somehow be dissipated. Although the neutrons contain a large part of the energy, their small cross section will largely insure that they escape from the region without collision. The radiation is in a part of the spectrum where it may be difficult to make the reaction chamber walls transparent to it. Even though it is a small percentage, it will be almost impossible to reradiate it from the missile; hence, attention must be given to disposing of about 0.6% of the total power or about 3% of the charged particle power.

If an additional fluid is used as a coolant to exhaust the unwanted heat overboard, then for maximum propulsive efficiency it should be mixed with the charged particles and the mixture exhausted with the highest possible velocity. To illustrate, let us assume that lithium is the coolant and that all of the radiation power must be dissipated. In Fig. 10 is shown a section of the reaction container. Lithium is circulated through the wall and upon boiling is injected into the chamber. Let us adopt the following notation:

\[ P_+ = \text{power in charged particles} \]
\[ P_T = \text{radiation power} \]
\[ H_p = \text{enthalpy of lithium through boiling} \]
\[ C_p = \text{specific heat of lithium vapor} \]
\[ T_s = \text{boiling point of lithium and the maximum structural temperature} \]
\[ T_p = \text{temperature of lithium and } \text{He}^+ \text{ mixture} \]
\[ M_p = \text{lithium mass flow rate} \]

The radiation power transferred to the lithium is:

\[ P_T = 0.03P_+ - H_pM_p \]

After the lithium vapor enters the chamber, it mixes with the He\(^+\) and the mixture comes to a temperature \( T_p \):

\[ T_p - T_s = \frac{P_+}{C_pM_p} - \frac{H_p}{0.03C_p} = 2.7 \times 10^5 \text{°K}. \]

This temperature corresponds to a specific impulse of 3200 sec which is more than ten times that obtainable from chemical rockets.

The minimum amount of power required for magnetic containment of the reacting plasma is determined by the ohmic heating due to the current in the plasma.

To evaluate the containment power, let us make the following assumptions:

(a) the rocket motor produces \( 10^4 \) kg of thrust, which corresponds to \( P_+ = 1.7 \times 10^9 \text{w} \);
(b) the D-T plasma density is \( 10^{22} \) particles/m\(^3\) at \( 4 \times 10^6 \text{°K} \);
(c) the stabilizing magnetic field pressure is 0.9 times the containing magnetic field pressure; and
(d) the plasma is in a torus whose major radius is ten times the cross-section radius \( r \).

The plasma current for containment is:

\[ I^2 = \frac{4\pi^2B_o^2}{\mu^2} = \frac{8\pi^2B_o^2}{\mu} \left( \frac{1}{1-0.9} \right) \]

The resistance\(^{11}\) of the plasma of length \( l \) is \( R = 4 \times 10^{-14}l/\mu^2 \).

The minimum containment power, which is due to the ohmic dissipation of the pinching current in the plasma, can be calculated to be \( 1.1 \times 10^9 \text{w} \), which is less than 1% of the charged particle power. It should be possible to convert this much of the charged particle power directly by reaction of the plasma on the magnetic field.

Another requirement for power is to produce the stabilizing field. In this case, it is a compromise between the power required and the weight of the coil. If the power is set equal to the containment power, then the coil weight is about one-third of the thrust.

**CONCLUSION**

For a thermonuclear power rocket motor of about 20,000 lb thrust, only about 1% of the charged particle power would need to be converted to electricity and the rest could remain in the form of heat. Since there are few applications with such a favorable ratio, the fusion rocket may be the most appropriate initial application of thermonuclear power. The capacitors need not be heavy and the stabilizing field coil is of reasonable weight, so the total rocket weight may not prove prohibitive.

**ADDENDUM**

Stabilization of a Magnetically Confined Plasma by an Alternating Magnetic Field

The results of a new approach to the stabilization of a plasma suspended in a magnetic field are described in the following.

At equilibrium, the plasma occupies a cylindrical volume of radius \( a \), its interior being field free. The confining pressure is exerted by a magnetic field, \( B_o \), of the form

\[ B_{or} = 0 \]
\[ B_{0a} = F_0\phi/a \]
\[ B_{0a} = 0 \]

Such a field can be produced by a system of conductors wound on a coaxial cylinder of radius \( b \). Apart from serving as the sources for the field (A.1), these conductors must also provide a boundary for the magnetic fields that result from deformations of the plasma surface. The field amplitudes shall be:

\[ (a) F_0 = \sqrt{2}F_1 \cos (2\pi f_t) \]
\[ G_0 = \text{const.} \]
\[ (b) F_0 = \sqrt{2}F_1 \cos (2\pi f_1) \]
\[ G_0 = \sqrt{2}G_1 \sin (2\pi f_1) \]

with either \( f_1 = f_2 \) or \( 2f_1 > f_2 \). In case (a) it is assumed that the plasma is perfectly conducting; in case (b) it is only necessary to assume that the penetration depth (skin depth) of the field is negligible.

\(^{11}\) By E. S. Weibel.
compared to the radius of the plasma column: 
\[ f_2 \gg 1/\alpha^2 \] 
where \( \alpha \) is the conductivity of the plasma. Any of the frequencies, \( f, f_1, f_2 \) shall lie well within the range

\[ \frac{1}{2b} \gg f \gg \frac{v}{2a} \]  \hspace{1cm} (A.3)

where \( v \) designates the sound speed in the plasma. The first inequality allows the field to be treated as quasi-stationary: the second inequality requires the field oscillations to be so rapid that, during one cycle, the deformation of the plasma cannot grow beyond a small fraction of its diameter. Therefore, only time-averaged values of the magnetic pressure will be taken into account. Thus the equilibrium pressure exerted by the field on the plasma is

\[ p_0 = \frac{1}{2} (G_x^2 + F_1^2), \]  \hspace{1cm} case (a) \( \alpha = 0 \),

\[ p_0 = \frac{1}{2} (G_x^2 + F_1^2), \]  \hspace{1cm} case (b) \( \alpha = 1 \).

Assume that the plasma surface suffers a small deformation \( r = a + \delta a(\varphi, z) \). As a result, an excess plasma pressure, \( \delta p_\mathrm{P} \), appears at the surface. At the same time the magnetic field will be modified to \( B = B_0 + \delta B \) and the magnetic pressure changes by \( \delta p_\mathrm{B} \).

All perturbations are assumed to be small and will be linearized in \( \delta a \). It is therefore possible to make use of the Fourier transforms of these perturbations. The individual Fourier components depend on the coordinates \( \varphi, z \) and on the time only through the factor, \( \exp(i\omega t - k h \varphi - i\omega t) \), where \( n \) is an integer while \( k \) is a continuous real wave number. A characteristic equation (A.6) below will determine a discrete set of eigenvalues for \( \omega \).

Within the plasma, the displacement \( w(\varphi, z) \) and the excess pressure \( \delta p(\varphi, z) \) satisfy the acoustic equations

\[ \nabla^2(\delta p) - \frac{1}{v^2} \frac{\partial^2(\delta p)}{\partial t^2} = 0 \]

\[ \frac{\partial^2 w}{\partial t^2} = \gamma p_0 \nabla(\delta p) \]

where \( v \) denotes the sound speed and \( \gamma \) the ratio of the specific heats. This set of equations determines \( \delta p \) at the plasma surface as a function of \( \delta a \):

\[ \delta p_\mathrm{P} = \gamma p_0 \frac{(\omega a/ v)^2}{1 - (\omega a/ v)^2} \frac{1}{I_n(ka)} \frac{n(ka) \delta a}{a} \]  \hspace{1cm} (A.4)

where \( h^2 = (\omega/v)^2 - \lambda^2 \).

The deformations of the magnetic field are represented by a gradient of a scalar potential: \( \delta B = \nabla(\delta \psi) \), \( \nabla^2(\delta \psi) = 0 \), which is possible because of (A.3). The change in magnetic pressure becomes

\[ \delta p_\mathrm{B} = X_n(\delta \psi)(\delta \psi_0), \]  \hspace{1cm} (A.5)

where

\[ X_n(h) = -\frac{\gamma p_0}{\nu n^2} \frac{1}{I_n(ka)} \frac{1}{J_n(ka)} \frac{1}{I_n(ka)} \frac{1}{J_n(ka)} \]  \hspace{1cm} (A.6)

Performing the averaging, \( X_n(h) \) becomes

\[ X_n(h) = -\frac{\gamma p_0}{\nu n^2} \frac{1}{I_n(ka)} \frac{1}{J_n(ka)} \frac{1}{I_n(ka)} \frac{1}{J_n(ka)} \]  \hspace{1cm} (A.5)

It can be shown that \( X_n(h) > 0 \) if

\[ F_1^2 < \frac{2}{(\alpha/v)^2 - 1} G_x^2. \]

The characteristic equation is obtained by equating the plasma pressure (A.4) and the magnetic pressure (A.5) at the surface of the plasma:

\[ \gamma p_0 \frac{(\omega a/ v)^2}{1 - (\omega a/ v)^2} \frac{1}{I_n(ka)} \frac{n(ka) \delta a}{a} = X_n(h). \]  \hspace{1cm} (A.6)

It is possible to show that only real values of \( \omega \) can be solutions of this equation. Thus any disturbance of the plasma surface will cause only oscillations which do not grow in amplitude. Hence this plasma-field configuration is stable.

A physical picture of how stability is achieved shall be given. To this end various simple confining fields which are special cases (A.1) shall be compared.

Consider first the case, \( F_0 = \text{const.}, G_0 = 0 \). The instability of this situation is well known and directly traceable to the \( 1/r \) dependence of \( B \). It occurs for static confinement whenever the center of curvature of the field lines lies within the plasma.

In the case \( F_0 = 0, G_0 = \text{const.} \), the field lines are straight and parallel to the \( x \)-axis, and the plasma turns out to be neutrally stable.

As a third case, let \( F_0 = \text{const.} \) and \( G_0 = \text{const.} \). Now the magnetic field lines are helical. Deformations in the form of spiral grooves, having the same pitch as the field lines tend to grow, since they do not deform the field lines to first order. This situation, then, is unstable.

In the cases under consideration which are described by equations (A.2a) and (A.2b) the field lines are still helical but of rapidly alternating pitch. The change of pitch was chosen to be so rapid (A.3) that a helical deformation which might start to grow at a given instant is wiped out in the next by the field lines which cross the grooves at an increasing angle. However, the stabilizing field must not be made too strong, otherwise its own inherent instability, discussed in the first case, dominates the situation. This is the reason for the condition (A.4).
REFERENCES

Review of Controlled Thermonuclear Research at A.E.I.
Research Laboratory

By T. E. Allibone,* D. R. Chick,* G. P. Thomson† and A. A. Ware*

At temperatures of several hundred million degrees, in deuterium, the energy generated by nuclear reactions will exceed the radiation loss. This fact suggested to one of us (G.P.T.), in 1946, that useful power might be generated from such reactions, provided that the loss of energy to the walls could be sufficiently reduced by magnetic fields. Experiments were begun at Imperial College, London, in February 1947, to produce very large currents in a low-pressure gas discharge. The containment of the gas was to be achieved by the pinch effect and a toroidal discharge tube was used to eliminate end losses. Currents in the range $1 \times 10^4$ to $2 \times 10^4$ amp were induced in the gas by discharging a condenser of maximum energy 400 joules through a primary winding coupled with the gas. These experiments were the first to demonstrate a marked pinch effect in a gas discharge, although inertial effects caused the contracting discharge to overswing and expand again, followed by further contractions and expansions.

In order to achieve temperatures of several million degrees and demonstrate a thermonuclear reaction it was clear from the Bennett relation, $2NkT = I^2$, that currents of the order $10^5$ amp were required. $(N$ is the total number of particles per unit length of the discharge, $k$ Boltzmann's constant, $T$ the temperature and $I$ the current.) When, however, the condenser energy was increased in 1950 to 18,000 joules, serious crazing and evaporation of the small Pyrex glass discharge tubes occurred and it became necessary to develop superior materials to contain the large currents.

At this juncture thermonuclear research was made secret by the Government and, because of the undesirability of conducting secret research at a university, the staff and apparatus were moved to the Associated Electrical Industries Research Laboratory, Aldermaston, in 1951. In the development of suitable discharge tubes, quartz and porcelain were tried, but it soon became apparent that only metals could withstand the heat pulse involved. With metal tubes, a new problem was encountered; arc breakdown occurred across the gap which must be left in the tube to avoid a short-circuit. This arc breakdown has been the main problem in the development of metal discharge tubes. In addition, as the research was extended to currents of longer duration the fundamental problem of the discharge instability was encountered. Most of the research at A.E.I. has been directed towards solving these two main problems, namely the arc breakdown and the discharge instability.

DEVELOPMENT OF METAL DISCHARGE TUBES

The quartz and porcelain discharge tubes used had tube diameters 3-5 cm and torus diameters 20-30 cm. With pulsed oscillatory currents of amplitude about $3 \times 10^4$ amp and frequency 20 kc/s, the quartz tube was found to vapourise appreciably after 5 microseconds and the porcelain after about 20 microseconds. A simple calculation showed that these times were consistent with most of the heat generated in the gas being passed rapidly to the tube walls and the walls vapourising when their surfaces approached boiling point. The time for which a given surface can withstand a heat flux of $q$ calories cm$^{-2}$ sec$^{-1}$ is given by:

$$t = \frac{X}{f}$$

where $\lambda = \pi \theta_0 K \rho s / 4$, and $\theta_0$ is the boiling temperature, $K$ the thermal conductivity, $\rho$ the density and $s$ the specific heat. On this basis, the most suitable materials for the discharge wall are those having high values of the quantity $\lambda$. Unfortunately, most electrically insulating materials have a low value of $\lambda$ because of their poor thermal conductivity. The only exceptions are a few oxides and, in particular, beryllia, but these are very difficult to fabricate. The other materials having high values of $\lambda$ are electrical conductors and of these the metals with high $\lambda$ are the most suitable.

The first metal tube was made in 1951 of aluminium and had two gaps. In preliminary experiments satisfactory discharges of several thousand amperes were obtained with about 100 volts per turn. At higher currents and voltages serious arc breakdown occurred across the gaps, causing a short-circuit of the discharge and melting the metal at the gaps. The first step taken to prevent arc breakdown was to increase the number of gaps to reduce the gap voltage. A series of multi-gap tubes was developed, culminating in 1955 in a large...
64-gap torus with tube diameter 30 cm and torus diameter 100 cm, in which currents up to $8 \times 10^4$ amp were produced (see Fig. 1). At currents much less than the maximum and an applied voltage corresponding to only 10 volts per gap, appreciable arc breakdown still occurred at some of the gaps. The discharge was very unstable and wriggle velocities up to $10^7$ cm sec$^{-1}$ were observed. The instability caused serious bombardment of the tube wall and may well have caused local enhancement of the gap voltages, but, when a high temperature plasma is adjacent to a metal surface, an arc can form independently of applied voltages.

Since 1955 a considerable amount of work has been done to understand the nature of arc breakdown in order to find ways of preventing it. Investigations have been made of the role of the surface condition in glow-to-arc transitions in the hope of finding a suitable material or surface treatment with which transitions did not occur. Success has been achieved in greatly reducing the amount of arc breakdown on small test electrodes immersed in the plasma of a 1000-amp hydrogen ring discharge. The electrodes were “conditioned” by allowing arc discharges to form on their surfaces under controlled conditions, a process similar in some respects to that widely used in the manufacture of certain types of electronic tubes. With all the metals tried the probability of a ring discharge resulting in arc formation, which was initially unity, was found to decrease with the number of discharges. For example, in the case of copper and up to 4000 volts applied between test electrodes of 10 sq. cm area, the probability could be reduced from near unity to about 0.01 in some 100 flashes. Careful chemical cleaning or baking in vacuo reduced the number of arcs required to condition a surface but no treatment was found to obviate the need for ‘conditioning’.

Various metals have been compared in these tests and, in particular, copper was found to arc less than aluminium. Because of this and the fact that copper has a larger value of $\lambda$, copper liners were used in Sceptre III to cover the aluminium near the gaps. This enabled over 40,000 discharges with currents from 50,000 to 200,000 amp to be passed without arc damage seriously interrupting research. Similar operations without the copper liners lead to serious damage of the aluminium after only several hundred discharges.

Considerable progress has been made in understanding the dynamic behaviour of the arc spots in the presence of a high-current discharge. The arc spots undergo retrograde motion due to the self-magnetic field of the discharge. This effect causes the arcs to travel along the discharge tube and enter the gaps, and it is in the confined space of the gaps that serious arc damage occurs. Most of the damage is on the anode side of a gap. Further details of this work are given in a separate paper.

**Study of the Discharge in a Metal Torus**

Kruskal and Schwarzschild showed theoretically that a self-constricted discharge is inherently unstable. There are many modes of instability but probably the most important is that known as the “wriggle” instability which causes any slight kinks in the discharge filament to grow in amplitude. It was first observed in the United Kingdom in 1953. If such an instability develops in a metal tube the eddy currents induced in the walls produce magnetic fields which tend to prevent the wriggle developing, and it was originally hoped that this effect would control the instability. However, in experiments with a glass torus, surrounded by a metal sheath, high speed photography showed that stability was retained for current values only slightly in excess of the currents giving rise to instability in the absence of a metal sheath. At high currents the discharge appeared to wriggle over the whole tube. A simple analysis shows that eddy current forces will not balance the instability forces arising from short wave instabilities until the wriggle approaches very near the wall.

As so little was known of the properties and, in particular, the wavelength of the wriggling discharge in a metal tube, it was agreed with AERE that a study should be made of the phenomenon, especially as the decision had been taken to construct ZETA at AERE, in which some instability was expected to occur. The discharge tube used initially was a four-gap aluminium torus with tube bore 30 cm and mean diameter 105 cm. Coupling between the primary and the gas was aided by a four-ton iron core and the primary was wound on the core to reduce its stray magnetic field in the discharge regions. The condenser bank, which was discharged through an eight turn primary by a spark-gap switch, had a maximum energy storage of 66,000 joules. In practice, however, the energies used were limited to about 6000 joules because, with greater energy inputs, arcing occurred at all the gaps, resulting in a short-circuit of the discharge. The current pulse time was about 1 millisecond. A rotating mirror camera...
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Figure 2. Racetrack torus with "Pepper-Pot" section.
Internal bore of tubing, 12 in.

was used to photograph the discharge as seen through a slot running across the discharge tube, and by viewing the discharge from two directions at right angles, a two dimensional, time-resolved record of the discharge was obtained. Current and voltage oscillograms were also recorded and the apparatus provided time correlation between the streak photographs and the electrical waveforms.

In order to obtain information regarding the third dimension, parallel to the tube axis, and in particular to determine the wriggle wavelength, further experiments were carried out with two straight sections, 60 cm long, inserted in the torus and making it race-track in shape. The sections were square in cross section and one, known as the "Pepper-Pot", had an array of holes perforated in two of its sides as shown in Fig. 2. The holes were 1.3 cm in diameter and were spaced 2.5 cm apart in both directions. They were drilled in such a way that their axes converged on a focus at the position of the camera lens. Photographs were taken first with an image converter camera giving single-frame microsecond exposures, and secondly with a Courtney-Pratt lenticular plate camera giving multiple-frame photography with exposure and framing times both 10 microseconds.

Very little success has been achieved in photographing hydrogen discharges because of the very rapid discharge movement and the low light level involved. Because of this most of the work has been done with argon discharges. The currents studied have been in the range 5000-20,000 amp and the main results can be summarised as follows. At the low electric fields used (0.3 to 1.0 v cm⁻¹), the discharge forms as a constricted filament near the centre of the tube and no pinch effect oscillations occur. The discharge has no kinks at this stage but these soon develop and the discharge filament becomes approximately sinusoidal in shape. The sine wave grows in amplitude with time and in addition moves along the tube with a velocity of the same order of magnitude as the rate of growth in amplitude. Figure 3 shows two sample image converter photographs of the racetrack section taken during the development of the instability, and the graphs in Fig. 4 give the magnitude of the wriggle growth velocity in the radial direction and its variation with the applied electric field for different argon pressures. In addition to the development of the instability, the width of the discharge channel is increasing during this period, and measurements of current and gas resistance suggest that the gas kinetic pressure is increasing more rapidly than the confining magnetic pressure.

For a given set of experimental conditions the time of initial onset and rate of growth of instability were found to repeat accurately from one pulse to another. The wriggle wavelength was about 40 cm in all cases...
and only the rate of growth varied with pressure and current. In other words it was the same mode of instability appearing first each time and, for fixed conditions, the time of onset and position of the instability did not vary. These results suggest that the particular mode observed is due to departures from perfect toroidal geometry in the torus rather than to fundamental properties of the discharge.

The wriggle amplitude continues to grow until the discharge is apparently touching the tube wall. At the lower currents studied, the discharge thereafter wriggles in a confused and random manner throughout the remainder of the pulse and other modes of instability may well be present. At currents greater than about $1.2 \times 10^4$ amp, and after the discharge has reached the walls, streak photographs show bars of light extending across the tube width which last about 10 microseconds and appear at random intervals. At still higher currents (greater than about $1.5 \times 10^4$ amp) this "Barring" effect occurs for only a short period in the pulse and is then followed by an abrupt and permanent reduction in the light emitted by the gas, although the oscillograms show a marked simultaneous increase in current. It is believed that the effect is due to short-circuiting arcs having formed at all the gaps at this instant. All these effects—the growth of wrigging, the barring, the reduction in light intensity and the sudden increase of current—are shown in Fig. 5 which shows simultaneous streak photograph and current oscillogram records for an argon discharge.

Prior to the short-circuiting arcs occurring at all the gaps, local intermittent arcing can take place at some gaps independently of others, and the barring phenomenon (which will be referred to later, since it appears on all records of hydrogen discharges in ZETA and Sceptre) is believed to be associated with this arcing. An arc produces a local burst of gas and metal vapour which will lead to a region of high gas pressure moving along the tube.

Study of the Discharge in Applied Magnetic Fields

Before the observation of the discharge instability in 1953, a few experiments had been carried out in which toroidal magnetic fields were applied parallel to the discharge. These fields were intended to maintain the containment of charged particles during the periods around the current zeros in the case of oscillatory discharge currents. Apart from a reduction in the constriction of the discharge and a lowering of gas resistance no marked effects were observed. Since 1953 many experiments employing magnetic fields have been performed with a view to stabilizing the discharge channel.

Some of the earlier experiments with steady magnetic fields have been described already. In particular, a toroidal magnetic field, $B_\phi$ applied to currents of a few thousand amperes in a glass torus was found to destroy the pinch effect. This occurred even when the magnetic field was applied after the discharge had constricted. In many cases the photographs showed sharp-edged light and dark regions travelling across the discharge tube. The velocity of these bands appeared independent of magnetic field and had the magnitude to be expected for an acoustic type magnetohydrodynamic wave.

In other work with an alternating toroidal magnetic field, Miles confirmed the results of Bickerton of AERE that circulating currents induced in the $\phi$ direction suppressed wriggling. It was found, however, that the discharge sat near the outer wall of the torus for part of a half cycle of the alternating magnetic field and at other times near the inner wall. In an analysis of this type of discharge, Liley showed that such an effect could be explained by taking account of the gradient of $B_\phi$ and the phase angle between $B_\phi$ and the $\phi$ currents in the gas. This work was not pursued,
however, since the results showed that very large reactive powers were required in order to produce toroidal magnetic fields capable of controlling large gas currents.

In 1956 Bickerton showed that a stable discharge could be obtained with a steady toroidal magnetic field when combined with a metal torus. Experiments, therefore, were carried out with a steady magnetic field applied to the discharge in the four sector aluminium torus described in the previous section. In this work, $B_\phi$ was again found to increase the gas current, the increase being as much as five times in the case of low pressure hydrogen. The current range studied was from 5000 amp to about 30,000 amp. With argon, the streak photographs were similar to those with glass tubes. The amount of constriction of the discharge when it first becomes visible is progressively reduced by increasing $B_\phi$, and in all cases the transverse wave motion has again been observed at this point. The discharge subsequently expands and fills the whole tube. With hydrogen at the lower currents, photographs show a diffuse discharge filling the whole tube with some diagonal streaks, suggesting a helical instability. At the higher currents these diagonal streaks are observed only at the beginning of the pulse; at later times the streak photographs have a turbulent appearance with intermittent bars similar to those already mentioned. The effect of $B_\phi$ is to increase the number of bars and make them more marked in appearance.

In 1957 the “Pepper-Pot” racetrack sections were fitted to this torus and, after the experiments without magnetic field, coils were added to extend the study to discharges with $B_\phi$. It was at this time that the first results were obtained with ZETA at AERE and these indicated: (i) that at high currents in a metal torus with $B_\phi$, a stable discharge with trapped field was obtained and (ii) that the amount of arcing in the presence of such a discharge was very much reduced, compared with similar currents in the absence of $B_\phi$. These results gave encouragement to studying higher currents in the racetrack torus, and it was found that in spite of the tube’s having only eight gaps, large gas currents could be obtained in the presence of $B_\phi$ without arcs short-circuiting the discharge. Currents of $10^9$ amp were obtained with the full condenser energy.

With argon, the appearance of the discharge is markedly different under these conditions (Fig. 6). The discharge forms initially over the whole tube, contracts to a minimum diameter and then expands again, and over this time the discharge has a more clearly defined edge. After the discharge has expanded to the walls the streak photographs become confused for a time and then seem to indicate a sharp-edged constricted discharge with bars superimposed on it. The two phenomena are not independent since marked sideways perturbations occur in the sharp-edged channel at the time of each bar. These excursions of the channel, however, are limited in amplitude and the channel does not touch the tube wall. The constriction of the channel is less the higher the initial gas pressure and the higher $B_\phi$.

With hydrogen the photographs in most cases show only the bars, but in a few cases a sharp-edged core has been observed. With colour films this core is blue, whereas the bars are mainly red. Some streak photography was carried out with hydrogen, using a line of holes along the length of the “Pepper-Pot” and this showed that the bars were travelling along the tube. The direction of motion was that of the electron drift velocity in the discharge for most of the pulse, but changed to the opposite direction towards the end of the pulse. The velocity appeared independent of $B_\phi$ and gas current and was always in the range $10^6$ to $2 \times 10^6$ cm sec$^{-1}$.

Following the work of Ramsden at AERE, measurements were made of the ion temperature from the Doppler broadening of oxygen ($O^+$) impurity lines, since these were from the most highly ionised atoms observed in the discharge. Voigt profiles were used to correct the line contours for instrumental broadening and, by the middle of October 1957, broadening corresponding to temperatures more than $10^8$ °K were recorded.

**EXPERIMENTS WITH SCEPTRE**

Following Bickerton’s observation of stability in a metal tube with $B_\phi$, the work of Bickerton, Liley and others led to the understanding of the part played by trapped magnetic fields in such discharges. A detailed analysis by Tayler gave the conditions necessary for stability which are now well known. As a result several experiments were proposed at the end of 1956, at AERE and AEI, all incorporating this method of stabilisation but with different heating mechanisms. Liley suggested that adequate heating of the gas could

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‡ Sceptre: Stabilised Controlled (E) Pinch Thermonuclear Reaction Experiment.
be achieved by ohmic heating due to both the $\phi$ and $\phi$ gas currents, aided by the adiabatic compression of the gas during contraction.

Two small tubes (Sceptre I and Sceptre II) based on this idea were in construction in 1957 when the large racetrack torus yielded the promising results described above. It was decided to rebuild this large torus without the racetrack sections and its primary was adapted to produce higher currents. This apparatus was renamed Sceptre III.

Figure 7 shows a photograph and Fig. 8 a plan diagram of Sceptre III. The torus is made of aluminium tubing with a bore of 30 cm and a wall thickness of 1.2 cm. The eight porcelain insulators at the gaps are protected by eight pairs of interlaced copper liners, and double vacuum gaskets made from indium wire are used throughout. Quartz-covered slits—one in the vertical plane and one horizontal—allow observation of the discharge. The primary has been placed near the torus to reduce the leakage inductance which is now an important factor because of the low impedance of a stabilised discharge. It consists of four separate windings, of eight turns each, and is wound as uniformly as possible over the surface of the torus so that the local magnetic fields of the primary current cancel in the discharge space.

The toroidal magnetic field is produced by 102 layer-wound coils placed around the torus perimeter beneath the primary, and at pumping and viewing ports the field is maintained constant by compensating coils. The maximum $B_\phi$ is 1000 gauss. The condenser bank with maximum energy 66,000 joules and maximum voltage 30 kv is discharged through the primary by means of a spark gap switch.

The gas used so far with Sceptre III has been almost exclusively deuterium and all the results given below refer to this gas. The pressure range has been $4 \times 10^{-4}$ to $4 \times 10^{-3}$ mm Hg. Work has been done with the primary connected to give transformer turns ratios of 8:1, 16:1 and 24:1.

### Experimental Results

#### Conditioning Effect

The behaviour of Sceptre III shows a conditioning effect. Following a period with no discharges, and particularly after the apparatus has been dismantled and re-assembled, the first few pulses show a high gas resistance,§ with a high level of radiation from the impurities oxygen and nitrogen, and no neutron emission is detected. With further discharges the impurity level of oxygen and nitrogen and the gas resistance decrease, though the copper and aluminium impurity increases. The degree of ionisation of the oxygen atoms increases and the lines become broader. The current waveform changes in shape and, after a time, exhibit the characteristic changes of slope referred to below. Simultaneously, neutrons are detected and, with further discharges, their number increases. Lastly, the voltage observed across the gaps in the torus shows more fluctuations under the neutron producing conditions.

After a period of repeated discharges the various parameters reach steady values and the results listed below refer to this "conditioned" state.

#### Current and Voltage Oscillograms

In Fig. 9, peak gas current is plotted as a function of magnetic field for several condenser voltages with the eight turn primary and for one voltage with the sixteen turn primary. The length of the current pulse is proportional to the number of primary turns and the decrease in peak current with the number of turns is

§ In some cases the first two or three discharges showed very high currents which were believed to be due to short-circuit arcs occurring at all the gaps.
due to the increased damping that results. There is a slow increase of current with increase in pressure.

Figure 10 shows sample oscillograms for voltage, current and rate of change of current. The current waveforms have a characteristic shape. When the current reaches a certain critical value there is a fairly abrupt change in $\frac{dl}{dt}$ and a corresponding change when the current returns to this value. The value of this current corresponds to a self-magnetic field $B_0$ at the discharge tube wall equal to the applied magnetic field $B_0$. The waveforms are consistent with the gas having a kinetic pressure low compared with the magnetic pressure and with at least some of the discharge plasma filling the whole tube until the critical current is reached, then constricting and trapping most of the $B_0$ flux, and finally expanding to the walls when the current returns to this value. This behaviour is to be expected since, with trapped $B_0$, there is an outward pressure of $\frac{B_0^2}{8\pi}$ which must be exceeded by $\frac{B_0^2}{8\pi R}$ before pinching can occur.

A considerable amount of information can be obtained from simple inductance measurements. First, before the discharge leaves the wall, and assuming the discharge is stable, the leakage inductance of the gas is given by:

$$L_1 = 4\pi R \ln \left( \frac{d}{a} + \frac{\gamma}{4} \right)$$  \hspace{1cm} (2)

where $R$ is the major radius of the torus, $2a$ is the width of the discharge and $d$ is the distance of the primary turns from the centre of the discharge. The factor $\gamma$, associated with the magnetic field within the discharge, will be zero for an infinitely thin skin current and will rise to unity for a uniform current density across the discharge. Measurement of $L_1$ will yield therefore a measure of $\gamma$ since the other components can be calculated. Unfortunately, the method is rather inaccurate since the $\gamma$ component is only a small fraction of the total inductance.

Secondly, when the current exceeds the critical value at which the discharge leaves the wall, assuming $B_0$ to be completely trapped and the kinetic pressure to be small compared with the $B_0$ magnetic pressure, the rate of change of inductance leads to an effective increase, of $4\pi R$, in the inductance of the discharge. This follows from (a) the circuit equation,

$$L_1 \frac{dI}{dt} + I \frac{dL_1}{dt} + R_g I = V,$$  \hspace{1cm} (3)

where $I$ is the measured gas current, $R_g$ the "effective" gas resistance and $V$ the volts per turn, and (b) from the discharge pressure balance relationship,

$$B_0 = \frac{2I}{a} = B_0 = \alpha \frac{b^2}{a^2} B_{00},$$  \hspace{1cm} (4)

where $2b$ is the diameter of the discharge tube, $B_{00}$ the applied field and $\alpha$ the fraction of $B_0$ flux trapped, and where $\alpha$ is assumed near unity.

This yields

$$\frac{1}{I} \frac{dI}{dt} = -\frac{1}{a} \frac{da}{dt},$$

and hence

$$\frac{dL_1}{dt} + \frac{dL_1}{da} \frac{da}{dt} = \frac{4\pi R}{I} \frac{dI}{dt}.$$  \hspace{1cm} (5)

Equation (3) then becomes

$$L_2 \frac{dI}{dt} + R_g I = V,$$  \hspace{1cm} (6)

where

$$L_2 = L_1 + 4\pi R.$$  \hspace{1cm} (7)
A measurement of the change of \(dl/dt\) at the critical current yields the change of effective inductance. If the measured increase in inductance is \(\beta 4\pi R\), then \(\beta\) should equal unity if the above assumptions are correct.

Lastly, from Eqs. (2) and (4) we have

\[
L_3 = 4\pi R \left[ \ln \left( \frac{2Ic}{B_0} \right) - \ln \left( \frac{1}{2} \right) + \beta \right]. \tag{8}
\]

\(L_3\) has been measured at peak current from the ratio \((dV/dI)/(d2I/dt)\) (on the assumption \(dR/dt \approx 0\)) and, since all other quantities are known in Eq. (8), a value can be obtained for \(\alpha\).

Table 1 shows a sample set of experimental results obtained for \(V/I\), at peak current, and the quantities \(\alpha\), \(\beta\) and \(\gamma\) for the eight turn primary. For this set,

<table>
<thead>
<tr>
<th>(B_0) (gauss)</th>
<th>(V/I) (ohms)</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\gamma)</th>
<th>(\delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.022</td>
<td>0.58</td>
<td>0.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>0.013</td>
<td>0.82</td>
<td>1.1</td>
<td>0.68</td>
<td>0.58</td>
</tr>
<tr>
<td>600</td>
<td>0.009</td>
<td>1.2</td>
<td>0.56</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>0.005</td>
<td>0.85</td>
<td>1.2</td>
<td>0.53</td>
<td>0.39</td>
</tr>
<tr>
<td>1000</td>
<td>0.006</td>
<td>1.30</td>
<td>0.8</td>
<td>0.32</td>
<td>0.17</td>
</tr>
</tbody>
</table>

The kinetic pressure is known to be appreciably less than the magnetic pressure and the simple results just derived can be used. For kinetic pressures comparable with the magnetic pressures, as with the sixteen turn primary, a more exact analysis must be applied. The values of \(\alpha\) indicate that the discharge is trapping most of the \(B_0\) magnetic field. The values of \(\beta\) are near unity and confirm the assumptions made in the above analysis. The values of \(\gamma\) are the least accurate of the three since they involve measuring a small increment in the inductance. The values of skin depth corresponding to these values of \(\gamma\) are shown in the sixth column of the table as fractions of the discharge tube radius. The inductance and hence \(\gamma\) have not been measured at the beginning of the first half cycle because the gas resistance is unknown and likely to be large at this point. The high value of the ratio \(V/dI/dt\) before the discharge leaves the wall indicates either a high gas resistance or high inductance.

The measurements of \(V/I\) at peak \(I\) cannot be directly taken to be a measure of gas resistance. This is because resistance associated with eddy currents in the torus walls and arcs must be accounted for in the ratio \(V/I\), while any component of \(dI/dt\) which is not in phase with the current will also be included in this ratio.

Streak Photography

The predominant feature of the streak photographs is the "bars" which are observed in all cases with deuterium and hydrogen discharges. During the "conditioning" of the discharge a sharp-edged core is also observed which is blue in colour, whereas the bars are mainly red. With the decrease in impurity content which results from 'conditioning', this core becomes invisible. Its continued presence is indicated by the more sensitive detection of the light from highly ionised impurity atoms described below.

Spectroscopic Measurements

The spatial and temporal variation of the intensity of impurities lines has been studied by combining a photomultiplier with a monochromator. This work indicates that the oxygen \(O\) light comes mainly from a narrow core at the centre of the discharge, with a diameter from \(\frac{1}{4}\) to \(\frac{3}{4}\) of the tube diameter. On the other hand, \(O\) has minima at the centre of the discharge and is otherwise fairly uniform across the tube.

At a condenser voltage of 25 kv and a pressure of \(1.4 \times 10^{-3}\) mm Hg, the intensity distributions have been obtained for different \(B_0\). At 250 gauss the waveforms exhibit considerable fluctuations, suggesting some instability of this central core, and in this particular case the \(O\) intensity is a maximum at a radius of about 4 cm and is less at the centre. At higher fields the waveforms show only small fluctuations and the intensity has a sharp maximum in the centre. The oscillograms indicate that the \(O\) light appears shortly after the change of slope on the current waveforms and has a maximum intensity at about peak currents; at all times it comes mainly from the central core of the discharge.

Measurements have also been made of the Doppler broadening of the \(O\) lines using a Hilger medium quartz spectrograph. In one case the line profile was checked to be Gaussian down to \(\frac{1}{2}\) of the peak intensity. The shape is consistent with the oxygen ions having a Maxwellian distribution of velocities and the temperature indicated by the line broadening. It
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Figure 12. Neutron yield as a function of axial magnetic field at 25 kv condenser voltage, 16:1 turns ratio, and various deuterium pressures (1μ = 10⁻⁴ mm Hg)

is not proof of the temperature, however, since a random instability could have velocities which, when averaged over time, give a Maxwellian distribution.

On the assumption that the broadening is wholly due to thermal motion, the ion “temperatures” have been calculated. The line breadth measurements come from photographic exposures for the whole duration of the current pulse and for many discharges. The “temperatures” therefore are to some extent average values, but the time variation of O⁷⁺ intensity weights the average in favour of temperature at peak currents.

Figure 11 shows a graph of ion “temperature” against applied magnetic field $B_φ$. With the eight-turn primary the ion “temperatures” were found to decrease with increasing condenser voltage over the range studied (11–20 kv) but with sixteen turns it increases with increasing voltage.

Nuclear Measurements

Neutrons and protons from the D(d,n) and D(d,p) reactions respectively have been observed from the discharge, and X-rays of energies up to 300 kev are also present, mainly in an intense pulse at the beginning of the current cycle. The neutrons appear shortly after the change of slope on the current waveform and are emitted in a continuous manner throughout the rest of the current pulse and also part of the way into the second half cycle. Figure 12 shows examples of the variation of neutron yield with time; in each case it is for one flash of the discharge. The neutron yield is seen to continue in each case beyond the current zero into the second half cycle of the current. Some delayed counts are expected to be due to capture gamma rays in the scintillation counter, but the number of observed counts at the current zero and afterwards is too great to be explained in this way. There is a real neutron emission at these times.

Nuclear plates, shielded by thin metal foils to reduce fogging by light and very soft X-ray, have been exposed to the discharge: proton tracks, of length corresponding to an energy of 3 Mev, have been observed. The energy spectra of the protons observed at 30°, 90° and 150° to the direction of current flow in the discharge have been measured. Significant differences in the mean proton energies have been observed. (A provisional value for the difference between the energies at 30° and 150° is 0.17 ± 0.04 Mev.) These indicate that at least some of the protons are due to nuclear reactions involving deuterons accelerated preferentially in the direction of current flow. The statistical accuracy of the results is not yet high enough to exclude the possibility that some of the protons may be of thermonuclear origin. The proton energy measurements confirm that these arise from the D(d,p) reaction and since their yield is in agreement with the measured neutron yield, the origin of the neutrons is also confirmed as being the D(d,n) reaction. From the direction of the proton tracks it can be deduced that these originate near the centre of the torus rather than from the walls.

Four indium activation counters have been positioned around the outer walls of the torus at roughly 90° intervals, and the induced activities measured. These were constant within the statistical accuracy of the results, which is 10%, indicating that the neutron yield did not vary appreciably round the torus.

Figure 13. Time resolved neutron yields for 25 kv and 30 kv condenser voltages
SCEPTRE THEORY

Before drawing conclusions from the experimental results, it is of interest to summarize the theory of ohmic and adiabatic heating, as developed by Liley, since this has served as a yardstick with which to compare experimental results. In this theory it has been necessary to make many simplifying assumptions and these can be listed as follows:

1. complete trapping of the applied magnetic field up to the time of peak current;
2. the kinetic pressure of the gas is small compared with the magnetic pressure;
3. the skin depth is a constant fraction of the discharge radius;
4. the exchange time, for energy transfer between electrons and ions, is short compared with the current pulse, so that their respective temperatures are always equal;
5. the temperature is constant across the discharge;
6. the rate of change of current, after the discharge leaves the wall and up to the time of peak current, is constant; and
7. all heat losses are negligible.

The basic equations are then:

\[ 2\pi R \frac{d}{dt} \left( \frac{3}{2} NkT \right) = R_0 I_0^2 + R_0 I_0^2 - 4\pi nkT \frac{d\phi}{dt} \]  

\[ \frac{B_0^2}{8\pi} + nkT = \frac{B_0^2}{8\pi} \]  

\[ B_0 = \frac{b^2}{2\pi} B_0 \]  

\[ R_0 I_0 + L_1 \frac{dI_0}{dt} + I_0 \frac{dL_1}{dt} + \frac{1}{\nu C} \int I_0 dt = 0; \]

where \( N \) is the total number of electrons and ions per unit length of the discharge tube, \( I_0 \) the total current circulating in the \( \theta \) direction, \( R_0 \) the corresponding gas resistance, \( n \) the number of electrons and ions per unit volume, and \( \nu \) the number of primary turns. Equation (12) is used only to give the period of the current pulse. The leakage inductance used was that for Sceptre where \( \phi b^4 E_0 \) is equal to 1.5.

With the various assumptions listed above, the temperature reached at peak current is found to be relatively independent of compression over the range \( 1.2 < b/a < 2.5 \) and is given in degrees Kelvin by

\[ T = 3 \times 10^7 \beta (\gamma' \beta b^4 E_0)^{-\frac{1}{3}} \]

where \( J \) is the energy in joules per centimetre of the discharge tube length, \( \beta \) is the initial pressure in microns of mercury, \( E_0 \) is the initial applied electric field in volts per cm, \( \gamma' \) is a factor proportional to skin depth and is unity when the skin depth is half the tube radius. It has been taken as unity in the present calculation. The theory indicates that the temperature should rise fairly rapidly after the discharge leaves the wall and then more slowly up to peak current.

Conclusions

It is possible to calculate the deuterium temperature which would be necessary to produce the observed neutrons by thermonuclear reactions. Examples of these “temperatures” are shown in Fig. 11, where they are compared with the corresponding ion “temperature” from Doppler broadening.

When the first results were obtained with Sceptre III, the good agreement between the Doppler broadening “temperatures”, neutron “temperatures” and theoretical “temperatures” was strong evidence that the true temperatures were those indicated and that the theory indicated the method of heating. The more recent experimental results, given above, and further theoretical considerations have brought to light several serious discrepancies which cast doubt on these conclusions. The main discrepancies can be summarized as follows.

1. Although no electron temperatures have been measured yet, what evidence there is indicates an electron temperature less than the measured ion “temperatures”. The weakness of the \( \gamma \) lines relative to \( \gamma' \) suggests that the electron temperature is not greater than 10\(^{6} \) K. The apparent high gas resistance also suggests an electron temperature less than this value.

2. There is a marked discrepancy in many cases between the ion “temperature” and the “temperature” deduced from the neutron yield, and the nuclear plate experiments indicate that at least some of the protons, and consequently some of the neutrons, are of non-thermonuclear origin.

3. In some cases, if all the gas which is initially present in the discharge is at the temperature indicated, the kinetic pressure would exceed the magnetic pressure \( B_0^2/8\pi \) and the discharge should not be constricted. The current waveforms, however, give evidence that the discharge is constricted.

The following conclusions can be drawn from the results.

First, it is reasonable to conclude that both the Doppler broadening and neutron yields give upper limits for the true deuterium temperature. From this it follows that in some cases all of the neutrons...
observed are produced by non-thermonuclear processes. For example, in Fig. 11 at 25 kv and 600 gauss, the Doppler broadening indicates a temperature of $1 \times 10^6 \, ^\circ K$ at which the thermonuclear reaction rate is undetectable, but nevertheless a large yield was obtained. Second, the evidence indicates an electron temperature appreciably greater than $10^7 \, ^\circ K$ but less than $10^8 \, ^\circ K$. The deuterium ion temperature may be obtained. Second, the evidence indicates an electron temperature appreciably greater than $10^7 \, ^\circ K$ but less than $10^8 \, ^\circ K$.

Finally, the inductive and capacitive energy remaining at peak current, as estimated from the oscillograms, shows that over half of the initial condenser energy has been dissipated in the gas by that time. Since for an initial pressure of $1.4 \times 10^{-3} \, \text{mm Hg}$ this energy is sufficient to raise all the original particles to the temperature $3 \times 10^6 \, ^\circ K$, it must be concluded that most of the energy is being lost from the discharge or shared with impurities.

Despite these many uncertainties, however, there is one outstanding achievement indicated by the results, and particularly by the study of $O^+$ light, and this is that a stable discharge was achieved for several hundred microseconds.

ACKNOWLEDGEMENTS

The authors wish to point out that this paper describes the work of a team of scientists and engineers whom it was their pleasure to direct. The nuclear physics measurements were made by members of the Nuclear Physics Section of this Laboratory.

The authors would also like to acknowledge the close co-operation with the Atomic Energy Research Establishment, Harwell, and, since 1956, the stimulus of exchange of information with the United States. Most of the work was carried out under a contract with the Atomic Energy Authority.

Mr. Ware presented Paper P/3, above, at the Conference and added the following remarks:

With reference to the spectroscopic study of light from highly ionised oxygen, Fig. 14 shows intensity profiles for the $O^+$ light at different times during the discharge pulse. There are some high frequency fluctuations on the intensity oscillograms, but the light in the outer regions of the tube is always small compared with the intensity in the centre of the tube. This suggests that gross instabilities have been overcome for the duration of the pulse.

Measurements of Doppler broadening have recently been made, viewing the discharge in the tangential direction (parallel to $\phi$) as well as the radial direction.

The line broadening was found to be the same for both directions. The results also indicated a shift of the broadened line when viewed tangentially. The $O^+$ particles have a directed velocity parallel to the positive gas current from $10^6$ to $2 \times 10^6 \, \text{cm/sec}$.

Magnetic measurements have been made with a small probe inserted into the discharge. Figure 15 shows typical $B_\phi$ and $B_\rho$ profiles for peak current and the conditions shown. The $B_\phi$ profile shows that the discharge has an edge outside which there is either zero or a small negative current density. The $B_\rho$ profile shows that the applied magnetic field is trapped by the discharge, and in addition extra $B_\phi$ is produced within the discharge so that $B_\phi$ outside the discharge.

### Table 2. Proton Energy Measurements in Sceptre

<table>
<thead>
<tr>
<th>$B_\phi$ field, gauss</th>
<th>Peak gas current, ka</th>
<th>Deuterium pressure, $\mu$</th>
<th>Nominal angle of observation</th>
<th>Number of proton tracks</th>
<th>Relative mean proton energy, Mev</th>
<th>Energy shift $E_{204} - E_{135}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>100</td>
<td>1.8</td>
<td>45°</td>
<td>56</td>
<td>3.07 ± 0.03</td>
<td>0.16 ± 0.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>90°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>135°</td>
<td>40</td>
<td>2.91 ± 0.03</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>140</td>
<td>1.4</td>
<td>45°</td>
<td>204</td>
<td>3.07 ± 0.02</td>
<td>0.23 ± 0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>90°</td>
<td>42</td>
<td>3.00 ± 0.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>135°</td>
<td>145</td>
<td>2.84 ± 0.02</td>
<td></td>
</tr>
</tbody>
</table>
Figure 14. OY light intensity distributions at various times after initiation of the discharge

Figure 15. Magnetic field distribution at peak current

is negative. The oscillograms show substantial fluctuations.

With reference to the measurement of proton energies, Fig. 16 shows histograms of the proton energies for $B_0$ fields of 500 and 1000 gauss. The discharge conditions and detailed results are tabulated in Table 2. In this work the protons studied were those emitted at angles of 45°, 90° and 135° with respect to the positive gas current.
The Stellarator Concept

By L. Spitzer Jr.*

The confinement of a very hot ionized gas under laboratory conditions would appear to require the use of a strong magnetic field. While an enormous variety of magnetic configurations is possible, two relatively simple geometries may be set apart at the outset, both involving infinite cylinders with axial symmetry. In the first of these, the magnetic field is produced by an axial current flowing through the gas. This configuration is the so-called "pinch effect", which has been extensively discussed in the recent literature. In the second simple geometry, the magnetic field is parallel to the axis, and is produced by external currents, flowing in solenoidal windings encircling the plasma. Such a straight cylinder, with an externally produced field, forms the basis of the "pyrotron" proposed by R. Post.

The stellarator, like the pyrotron, utilizes an external magnetic field, produced by coils encircling a tube containing the heated gas. However, instead of a finite cylindrical tube, the stellarator employs a tube bent into a configuration topologically like a torus, without ends. Such a tube will be referred to as "toroidal". Relative to the pyrotron, this configuration has the advantage, in principle, of permitting more complete confinement, since end losses are eliminated. Relative to the pinch discharge, the stellarator offers the advantage, again in principle, of permitting equilibrium in a steady state. Both these advantages might be of importance in a controlled thermonuclear reactor. The present paper outlines the basic concepts involved in the confinement and heating of a gas in a stellarator.

One basic complexity in the stellarator results from the fact that the simple torus, in which the magnetic lines of force are circles centered at the axis of symmetry, does not permit equilibrium confinement of a plasma in a straightforward manner. Microscopically this result follows at once from the particle drifts associated with the inhomogeneity of the magnetic field. These drifts, first pointed out by Gunn,2 and analyzed in detail by Alfvén,8 produce motions perpendicular both to \( \mathbf{B} \) and to \( \nabla B \); these motions are in opposite directions for electrons and positive ions. The resultant separation of charges produces electric fields which sweep the ionized gas towards the wall. Macroscopically, this same result is obtained from the fluid equations discussed below.

Basically, the confinement scheme in the stellarator consists of modifying the magnetic field so that a single line of force, followed indefinitely, generates not a single circle but rather an entire toroidal surface, called a "magnetic surface". The tube enclosing the gas is, ideally, one of these surfaces, enclosing an entire family of such magnetic surfaces. It is shown below that such surfaces can be produced, to a high approximation. A later section discusses the confinement of a plasma in a system characterized by magnetic surfaces. Heating of the gas is treated in another section; the techniques considered involve heating to intermediate temperatures (about \( 10^6 \) °K) by currents flowing parallel to the confining magnetic field, and subsequent heating to very high temperatures by means of a pulsating magnetic field, with a wide variety of possible pulsation frequencies. The present paper constitutes an introduction to the series of papers from Project Matterhorn, printed in the Proceedings of the 1958 Geneva Conference.

**ROTATIONAL TRANSFORM AND MAGNETIC SURFACES**

In this section we consider certain properties of a stellarator magnetic field which do not depend directly on the presence of a plasma in the system. In particular, we are interested in how far a line of force in a stellarator tube may be followed before it intersects the tube wall. If we could show in all rigour that only one magnetic surface passed through each point within the tube, then no line of force would ever intersect the tube wall. While this result is not exactly true, it is apparently true to a high order of approximation. The demonstration of this result depends on a certain abstract property of the stellarator magnetic field, called a "rotational transform".

We first discuss the properties of such transforms, and postpone until later a discussion of how they are produced.

**Properties of Rotational Transforms**

Let us pass a plane through the stellarator tube at some point. For convenience in representation, we shall take the plane to be perpendicular to the magnetic field, \( \mathbf{B} \), in the central region of the tube, although this restriction is not required. We assume

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* Project Matterhorn, Princeton University, Princeton, New Jersey.
that the magnetic field is non-zero at all points in the cross-sectional plane. Then through any point, such as \( P_1 \), for example, a line of force will pass. This line of force may now be followed, in the direction of the magnetic field, along the stellarator tube, until it has completed one circuit of the toroidal tube and intersects the cross-sectional plane at a point \( P_2 \), as shown in Fig. 1. In the ideal torus, \( P_2 \) coincides with \( P_1 \) and every line of force is closed after one circuit. If the degeneracy of the torus is removed, \( P_2 \) no longer coincides with \( P_1 \).

One might raise some question as to whether any of the points \( P_2 \) lie inside the tube wall. It is not difficult to construct a toroidal tube with a solenoidal winding such that most of the lines of force stay within the tube for at least one circuit. The interesting question is rather which lines intersect the tube wall after many circuits, and we may safely assume that for most points \( P_1, P_2 \) will exist and will lie inside the tube wall. If we exclude, for the moment, those areas for which \( P_2 \) does not lie within the tube wall, then for every point \( P_1 \), we have a point \( P_2 \); the transformation of the set of points \( P_1 \) into the set of points \( P_2 \) is called a “magnetic transform” of the cross-sectional plane.

A magnetic transform of this type is characterized by an important property. Let us take the density of points \( P_1 \) to be proportional to the component of \( \mathbf{B} \) normal to the cross-sectional plane. Since the plane is transformed into itself by the magnetic transform, the density of these points must be the same function of position before the transform as after. A transform with this property has been called “measure preserving” by Kruskal.

Let us now assume that the magnetic transform is also primarily rotational. This condition states that at least the outer portions of the plane all rotate in the same direction in a single transformation. As we shall see subsequently, there are several ways of achieving this result in a toroidal system. According to Kruskal, it follows from this assumption and from the Brouwer fixed-point theorem that there must be at least one point in the plane which is transformed into itself. In some types of stellarator, the magnetic transform involves only small deformations of the plane, in addition to a general rotation. In such systems there will be only one point that transforms into itself, and only one line of force that is closed after a single circuit around the stellarator. This line is called the “magnetic axis”.

A second basic result on measure-preserving rotational transforms, also established by Kruskal, is that any point, other than one of the fixed points, when followed through successive transformations, will not move far from a single closed curve. This result is illustrated in Fig. 1, where the points \( P_3, P_4 \) generated by successive magnetic transforms of the point \( P_1 \), all lie close to a single closed curve. Thus a single line of force, after many circuits around the tube, generates a magnetic surface.

More precisely, let us introduce coordinates \( r, \theta \) in the cross-sectional plane depicted in Fig. 1; \( r \) may be measured from the magnetic axis, denoted by the point \( O \). The value of \( \Delta \theta \) between \( P_1 \) and \( P_2 \) is denoted by \( \iota \), and is called the “rotational transform angle”. Let \( \theta = 0 \) at \( P_1 \), and let us assume that \( \theta \) equals \( 2\pi \) for the point \( P_2 \). The distance \( \Delta r \) from \( P_1 \) to \( P_2 \) is called the “deviation from closure” of the point \( n \). Evidently \( \Delta r \) measures how far the line of force has strayed from a closed curve. Kruskal has shown that \( \Delta r \) decreases more rapidly than any power of \( 1/n \). Hence one may surmise that \( \Delta r \) varies as about \( \exp(-Kn) \), where \( K \) is some dimensionless constant.

The physical reason for this result can best be understood in the special case when the normal component of the magnetic field is constant over the cross-section plane. The analysis of more general systems may be reduced mathematically to a consideration of this special situation. In this case, the density of points in the plane must remain constant in successive transformations. Let us now draw, in the cross-section plane, a closed curve connecting point 1 and its successive transformed points as smoothly as possible. Since the H-transform now preserves areas, the total area enclosed within this curve must remain constant in successive transformations. Hence all points on the curve cannot move inwards with successive transformations. If some move in, others must move out.

In the special case in which the \( \theta \) coordinate of every point returns to its original value after \( n \) transforms, it is possible for some points on the curve, together with all their transformed points, to move steadily in, while the points between move steadily out. Thus the closed curve develops wrinkles in successive transformations; this rate of wrinkling decreases very rapidly with increasing \( n \). In the more general case in which the \( \theta \) coordinate of a point never returns exactly to its initial value (to within a multiple of \( 2\pi \)), one would expect a further averaging out of these radial motions to occur. Even if the transform angle, \( \iota \), is not small, the deviation of a line of force from a magnetic surface should be small; if necessary, a group of transforms, which give an \( \iota \) very nearly a multiple of \( 2\pi \), can be taken as the basic transform, and Kruskal’s theorem used to demonstrate that a line of force never departs very far from a single closed surface. We
shall therefore assume in the following discussions that each line of force does in fact generate a single magnetic surface, and that a single line, if followed sufficiently far, comes arbitrarily close to any point on this surface.

Methods for Producing a Rotational Transform

To produce a rotational transform in a vacuum field it suffices to twist a torus out of a single plane. Virtually any such distortion will remove the degeneracy of the ideal torus and produce a rotational transform.

The simplest such system is the figure-eight, historically the first geometry proposed\(6,7\) for a stellarator. The topography is indicated in Fig. 2. The 180° curving end sections LM and KN are in planes each tilted at an angle, \(\alpha\), to the parallel planes in which the reverse-curvature sections LK and MN are placed. Figure 3 indicates that a rotational transform is present. This figure represents cross-sectional planes at K, L, M and N, all as seen from one end of the device. The point O represents the magnetic axis, while \(P_1\) and \(P_2\) denote the successive intersections of a single line of force with a cross-sectional plane at K. Intersections of this line of force with the other three planes are denoted by crosses. The solid lines represent the path followed by the magnetic axis. From K to L and from M to N there occur simple translations of the cross-sectional plane, while from L to M and from N to K, a rotation about an axis, inclined at an angle \(\sigma\) to the vertical, is involved.

Evidently, the line of force which passes through \(P_1\) in plane K, and is then followed through one circuit, through planes L, M and N, intersects plane K again in a point \(P_2\), rotated by a rotational transform angle \(\iota\). Examination of the figure shows that for this geometry

\[
\iota = 4\alpha
\]  

(1)

Moreover, \(\iota\) is independent both of distance, \(r\), from the magnetic axis, and of angle, \(\theta\). In an actual system, mutual interference between the stray fields of the curving sections LM and MN will modify these results slightly, but the general features remain unchanged.

A rotational transform angle may be produced in a variety of other ways. When a plasma current is flowing around the simple torus, a rotational transform appears, despite its absence in the vacuum field. If steady-state confinement is envisaged, however, a rotational transform must be present in the vacuum field. The most important alternative method for producing such a rotational transform is the use of a transverse magnetic field, whose direction rotates with distance along the magnetic axis. We shall follow a line of force and show that a transform angle appears.

Let us consider an infinite cylinder, with coordinates \(r\), \(\theta\), and \(z\). We consider the \(r\) and \(\theta\) coordinates of a single line of force as \(z\) increases. The coordinates of points along such a line of force are related by the differential expression

\[
\frac{dz}{dr} = \frac{r d\theta}{B_z} = \frac{B_r}{B_\theta}
\]  

(2)

Suppose now that \(B_r\) and \(B_\theta\) are produced by 2\(l\) wires, wound helically, on the outside of the cylinder, so that currents flow in opposite directions in adjacent wires, and with a pitch \(2\pi/l\). If we denote the tube radius by \(r_0\), and if \(kr_0\) is small compared to unity, then for small \(r/r_0\) we have, from the appropriate solutions of Laplace's equation

\[
B_r = Ar^{l-1} \sin(l\theta - hz)
\]  

(3)

\[
B_\theta = Ar^{l-1} \cos(l\theta - hz)
\]  

(4)

where \(A\) is a constant characterizing the strength of the transverse field. There is also a component of \(B_\theta\) associated with the current in the helical wires, but its magnitude is less than \(B_r\) by a factor \(kr\). We assume that \(B_{\theta 0}\), the component of \(B_\theta\) produced by a separate solenoidal winding, is the dominant axial field.

Let us follow a line of force whose coordinates are \(r_0\) and \(\theta_0\) in the absence of the transverse field. Equation (2) may now be integrated by means of a power-series expansion in \(A\), the coefficient in Eqs. (3) and (4). To first order in \(A\), \(r = r_0\) and \(\theta = \theta_0\) vary as the cosine and sine, respectively, of \(l\theta - hz\); to this order, the line of force is a helix, and its intersection with a plane moving along the \(z\) direction is a circle. Solving to second order in \(A\), we must take into account that for \(l\) equal to 2 or more, \(B_\theta\) is larger on the outside of the circle \(r_\theta > r_0\), where \(B_r\) is positive, than on the inside \(r_\theta < r_0\), where \(B_r\) is negative. As a result, the positive values of \(d\theta/dz\) in Eq. (2) more than offset the negative ones, and \(\theta\) increases systematically with increasing \(z\). A detailed integration by Johnson and Oberman\(8\) shows that
$\omega$, the increase of $\theta$ in one period $2\pi/h$ of the helical field, is given by

$$
\omega = \frac{\pi A\alpha^2}{kBz^2} \left[ 2(l-1) + h^2 \beta^2 + O(h^4) \right].
$$

The term in $(h^2)$ is included to give results for $l$ equal to unity; in this case a rotational transform arises from the variation of $B_z$ with $r$. The configuration for which $l$ is unity, with a helical magnetic axis, and its use for confining a plasma were proposed by Koenig, who first studied the use of helical fields in connection with the stellarator. For small $\alpha r$, an appreciable $\alpha$ is more readily obtained with transverse fields of higher multiplicity for which the transverse field vanishes at the magnetic axis. The properties of such fields, and of the magnetic surfaces associated with them, have been extensively studied by the Matterhorn theoretical group, under E. Frieman.

In the experimental program at Matterhorn, described in the subsequent papers, rotational transforms have been produced both with the figure-eight geometry and with transverse fields, with $l$ equal to 3. In either case, the existence of a rotational transform is readily confirmed experimentally by observation of a narrow electron beam, which follows the line of force. As pointed out subsequently, the chief advantage of the multipolar transverse field over the figure eight is the greater hydromagnetic stability which, in theory, it should yield.

**CONFINEMENT**

The objective of confinement theory is to demonstrate that the number of particles striking the tube wall is negligibly small. An exact proof would presumably require a detailed solution of the Boltzmann equation, together with the field equations. For the complex magnetic configuration of the stellarator this would be a difficult task indeed. An approximate treatment will be followed here.

First we shall use the macroscopic equations for the plasma, based on a number of simplifying assumptions. With these equations, together with the field equations, we can show that an equilibrium situation is possible, in which the plasma is macroscopically confined. Since particles moving with some particular velocity might conceivably escape, even though the plasma as a whole were confined, we shall next consider the trajectories of single particles, in the electric and magnetic fields determined from the macroscopic equations. These two approximate methods, taken together, indicate nearly perfect confinement, if collision and cooperative phenomena are ignored. The hydromagnetic stability of these equilibria has been extensively analyzed by Frieman, Krukal and their collaborators. The results of this analysis, summarized briefly below, indicate stable equilibrium under certain conditions. All these results, taken together, encourage the belief that magnetic confinement in a stellarator may be adequate for a controlled thermonuclear reactor.
momentum, and hence the macroscopic velocity must be vanishingly small. A fuller analysis of effects associated with macroscopic velocities would be desirable.

On the basis of these assumptions, the equations of equilibrium become, in emu.

\[ \mathbf{j} \times \mathbf{B} = \nabla \rho \]
\[ \nabla \times \mathbf{B} = 4\pi \mathbf{j} \]
\[ \nabla \cdot \mathbf{B} = 0. \]

The generalized Ohm’s Law determines the electric field, in terms of the pressure gradient for the positive ions, while Poisson’s Law then gives the charge density. Since neither of these quantities is of particular significance in the present analysis these equations may be omitted.

On the basis of these equations it was shown several years ago that no simple equilibrium is possible in a torus if the lines of force are assumed to be circles centered at the axis of symmetry. We introduce cylindrical coordinates \( r, \phi, \) and \( z, \) with \( z \) taken along the axis of symmetry of the torus; we assume that only \( B_\phi \) differs from zero, and that all quantities are independent of \( \phi. \) If we now take the curl of Eq. (6), eliminating \( \mathbf{j} \) by means of Eq. (7), we obtain

\[ \frac{1}{R} \frac{\partial B_\phi}{\partial z} = 0. \]  

Thus, for equilibrium, either \( R \) must be infinite or the magnetic field (and pressure) must be independent of \( z. \) Equation (9) for toroidal fields \( (B_\theta = B_z = 0) \) corresponds to a theorem by Ferraro who also take diffusion into account. Here we follow an earlier and simpler treatment, and demonstrate the existence of solutions to Eqs. (6)–(8) by the simple artifice of showing how to construct such solutions. We assume that \( \rho \) is a small quantity, and obtain a solution by successive iteration. The zero order solution is taken as the vacuum solution, with \( j_0 \) the current in the external coils, and \( B_0 \) the vacuum field, with its magnetic surfaces. The solution of order \( n \) is then defined by the equations

\[ j_n \times B_{n-1} = \nabla \rho_n \]
\[ \nabla \times B_n = 4\pi j_n \]
\[ \nabla \cdot B_n = 0. \]

Evidently, \( \rho_n \) must be assumed constant on the magnetic surface obtained in the previous iteration, but is otherwise arbitrary. It may generally be assumed that \( \rho_n \) is in each case a monotonically decreasing function of distance from the magnetic axis. If the solution converges, it must evidently yield a solution of Eqs. (6)–(8).

In principle, this iteration scheme is straightforward, but in practice the algebra is cumbersome. It turns out that the chief obstacle to convergence is the distortion of the magnetic surfaces by the magnetic fields associated with the plasma current \( j_n \), along the magnetic field. Even though this current density is low, the currents must travel an appreciable distance, and even the weak magnetic field associated with these currents may distort the magnetic surfaces out of all recognition.

The approximate condition for convergence in a simple figure-eight device may be derived in the simplest manner. The transverse current \( j_\perp \) is evidently about equal to \( \rho / B_\theta R, \) where \( B_\theta \) is the axial vacuum field and \( R \) is the distance of the tube wall, or plasma boundary, from the magnetic axis. The divergence of \( j_\perp \) results from the inverse proportionality, between \( B_\theta \) and \( R, \) and is about equal in magnitude to \( \rho / B_\theta R \), where we take \( R \) to be the radius of curvature of the magnetic axis. The divergence of \( j_\perp \) is also equal to this quantity, and over a tube length equal to \( R, \) \( j_\perp \) will build up to about \( \rho / B_\theta R. \) Hence \( j_\| \) is of the same order of magnitude as \( j_\perp. \) Over a tube cross-section, \( j_\| \) will have opposite directions on opposite sides. The magnetic field on the axis, due to this plasma current, will be of the order of \( 2\pi \rho j_\| / B_\theta \). The new magnetic axis will therefore be inclined at an angle \( 2\pi \rho / B_\theta \) relative to its direction in vacuo. The condition for negligible distortion of the
magnetic surfaces is that the deviation of the magnetic axis be small compared to \( r \). If the currents \( j_j \) flow along half the axial length, \( L \), of the machine, on the average, before cancellation, this condition yields

\[ \beta \ll 8r/R \]  

(13)

where \( \beta \) is defined by

\[ \beta = 8\pi\rho/B_0^2 \]  

(14)

and \( \rho \) is evaluated on the magnetic axis. A more precise discussion,\(^1\) taking into account the detailed variation of \( j_j \) over the cross-section, yields a coefficient \( 16\pi/\pi \) instead of \( 8 \) in (13); this computation assumes that \( L \) much exceeds \( 2\pi R \) and that the transform angle \( 4\pi \) (see Fig. 3) is small. If inequality (13) is not satisfied, the method of iteration fails, and it is not known what type of solution, if any, may exist.

In an infinite cylinder, values of \( \beta \) as great as unity might be envisaged. In the figure-eight stellarator, of the type shown in Fig. 2, \( r/L \) can scarcely exceed 0.02, and \( \beta \) must therefore be small compared to 0.1. If the rotational transform is produced by transverse fields, however, the transform angle, \( \psi \), for the device this situation, and hence equilibrium values of \( \beta \), in \( r/R \), that the upper limit on \( \beta \) is proportional to \( 2\pi \), in principle. It is readily shown that the upper limit on \( \beta \) is proportional to \( \delta r/\pi R \), in this situation, and hence equilibrium values of \( \beta \) substantially greater than 0.1 should be possible, although at the cost of somewhat greater overall axial length.

The question of the hydromagnetic stability of such configurations has been extensively studied by the Matterhorn theoretical group, under E. Frieman. Basic concepts and methods of analysis have been published by Bernstein, Frieman, Kruskal and Kulsrud,\(^2\) with application to the stellarator in the paper by Johnson, Oberman, Kulsrud and Frieman.\(^3\) Because of the importance of this work, the results will be summarized briefly here.

Instabilities tend to be most marked if the lines of force can interchange positions with the least possible bending. In the case of an axially symmetric field, if \( B_\psi \) vanishes everywhere, the lines of force can interchange positions without bending, and if bulges are present in the field the plasma is unstable. However, if a \( B_\psi \) component is present, so that lines of force are helices about the cylinder axis, and if \( B_\psi/\pi r \) increases with \( r \) so that the pitch of the helices decreases with increasing \( r \), then the outer and inner lines of force cannot be interchanged without appreciable bending, and the plasma tends to be stable. In the same way, if the transform angle, \( \psi \), varies with \( r \) in a stellarator, the outer lines of force are topologically different from the inner ones, and the plasma is stable against all hydromagnetic disturbances, provided that \( \beta \) is less than some critical value, \( \beta_\psi \). Computations of \( \beta_\psi \) for a cylinder, with helical transverse fields, with \( l = 3 \), added to an axial confining field, indicate\(^4\) that values of \( \beta_\psi \) as great as 0.1 could be obtained if the approximate theory could be trusted somewhat beyond its range of validity. There is some reason to believe that corrections for finite radius of gyration may increase \( \beta_\psi \) by a factor of about 2, although the theory is still very incomplete in this respect. An experimental test of this theory has not yet been obtained, although the corresponding theory applied to kink instability in the stellarator (see below) is apparently in close agreement with the observations. The maximum value of \( \beta \) for which a stellarator plasma is stable can probably best be determined by experiment.

Single Particles

In the absence of collisions, confinement of single particles will be shown to follow quite generally from the existence of a rotational transform and from the asymptotic behavior of a gyrating particle in a strong magnetic field. It has been shown by Kruskal\(^2\) that the magnetic moment, \( \mu \) (about equal to \( m\nu/2 \)), of a gyrating particle is constant to all orders of \( \alpha k \), where \( k \) is the gyration radius and \( \alpha \) is about \( \sqrt{\nu/2 \pi} \).

Constancy of \( \mu \) to first order in \( \alpha k \) had previously been demonstrated by Alfvén,\(^5\) and to second order by Hellwig.\(^6\) Similarly, Kruskal has also shown\(^7\) that the motion of the guiding center is independent of the phase of gyration to all orders of \( \alpha k \). Kruskal’s theory does not yield either a simple definition of \( \mu \), to all orders in \( \alpha k \), nor yet a simple definition of the guiding center, but shows that such definitions must exist and how, in principle, to construct them. In consequence of these results, we may assume that for each particle the total energy, \( W \), is not the only integral of motion, but there exists also a second integral, the magnetic moment, \( \mu \).

Thanks to these results, it may now be shown that successive intersections of particles with a particular cross-section plane, similar to that discussed above, produce a transformation similar in its properties to the magnetic transform generated by successive intersections of a line of force with the same plane. We restrict consideration to particles within ranges \( dW \) and \( d\mu \) centered at some energy, \( W \), and some magnetic moment, \( \mu \), and let the density in phase space, within these narrow ranges of \( W \) and \( \mu \), be constant everywhere. Within an interval of time \( \Delta t \), the guiding centers of these particles will intersect the cross-sectional plane at a number of points, \( P_1 \).

The density of such points will be a known function of position in the plane, depending on the magnetic field \( B \), and the electric potential \( \Phi \), through the two integrals of motion.

Each particle whose guiding center has intersected the plane at a point \( P_1 \) will ultimately cross the plane again, at a point \( P_2 \). Normally, if a particle passes through a point, the three components of its velocity, \( w \), are arbitrary, and its subsequent trajectory is not determined. In the present case, the two integrals of motion determine \( w_\perp \) and \( w_\parallel \), and the third velocity component, the phase of gyration, has no effect on the motion. Hence to each point \( P_1 \), there corresponds one and only one point \( P_2 \). Moreover, the particles which have produced all the points \( P_1 \) in the time \( \Delta t \) will all
produce points $P_2$ within the same time interval, and hence the density of intersection points in the transformed plane will be the same function of position as in the original plane. Thus this "particle transform" is measure-preserving in the same sense as is the magnetic transform discussed earlier.

From the same arguments as before it follows that free particles are confined in a stellarator to a very high approximation, provided that the particle transform is primarily rotational. Such a transform will be assured for particles whose velocity is mostly parallel to $B$ ($w_\parallel > w_\perp$), so that no reflection can occur from regions of relatively high field. The rotational magnetic transform guarantees a rotational particle transform for such particles. Qualitative experimental confirmation of this prediction is obtained from observations of runaway electrons reported by the Matterhorn experimental group$^{10}$ under M. Gottlieb. Electrons travelling at speeds near the velocity of light are observed to make about $5 \times 10^6$ circuits around a stellarator, during the ten milliseconds or so after the applied voltage is reduced to zero, but the confining field is still moderately high.

For particles which are trapped between two regions of relatively high field, or which are moving at a relatively very slow rate along the magnetic field, further arguments must be invoked to guarantee a primarily rotational particle transform. Two separate mechanisms are important. Firstly, the diamagnetic effect of the plasma produces a radial gradient of the axial field, and this inhomogeneity produces a drift of guiding centers about the magnetic axis. Secondly, a radial electric field will produce a similar rotation. Such an electric field is required by the assumption that the macroscopic velocity vanish, since only a radial electric field can cancel out, in a steady state, the velocity associated with a radial pressure gradient. It has been shown by Spitzer$^{26}, 84$ that such a radial field arises naturally when the gas is ionized and heated. A more detailed analysis of these effects$^{27}$ indicates that these two effects produce adequate confinement for most particles with relatively low $v_\parallel$.

One important exception should be noted. For some particles, the different effects producing rotation may cancel out, leaving only the unidirectional drift produced by the curvature of the field. The seriousness of this effect is reduced by two factors. As distance from the magnetic axis changes, the different mechanisms producing rotation will change in different ways, and the cancellation will disappear. Moreover, the cancellation will be exact only for a particular particle energy, $W$, and a particular magnetic moment, $\mu$; collisions will change these quantities, and restrict the time during which a unidirectional drift will occur. These effects have been discussed elsewhere$^5, 27$; the analysis, while admittedly approximate and incomplete, indicates that this process is not of great importance, although it may increase the diffusion rate somewhat above the value given by electron-ion collisions.
increases so rapidly with increasing energy that electrons which are in the tail of the Maxwellian distribution and which are sufficiently energetic to excite and ionize atoms may gain very appreciable energies in one free path.

Before any experiments had been carried out on ohmic heating, it was pointed out by Kruskal\(^\text{22}\) that discharges in the stellarator should be subject to kink instability. This hydromagnetic instability was predicted for heating currents greater than the critical current that will reduce the transform angle to 0 (or increase it to \(2\pi\)). This critical current is now generally known as the “Kruskal limit”.

Extensive observations of ohmic heating have been carried out by the Matterhorn experimental group, under M. Gottlieb, and are reported in several subsequent experimental papers.\(^{16-13}\) The data indicate clearly that nearly complete ionization is attained, with electron and ion temperatures in the neighborhood of \(5 \times 10^6\) °K. The occurrence of the predicted kink instability, at currents above the Kruskal limit, is fully verified experimentally. However, the detailed predictions of the ohmic heating theory are not substantiated, presumably because of the non-Maxwellian distribution of electron velocities. In support of this hypothesis, intense X-rays from runaway electrons are observed, with energies up to \(10^8\) ev.

These data emphasize the very great importance of impurities from the walls streaming into the discharge. In the early observations, the carbon and oxygen ions presumably outnumbered the helium ions during the later stages of the discharge, and sharply reduced the electron temperature. With the use of ultra-high vacuum techniques, resulting in base pressures below \(10^{-9}\) mm of Hg and relatively clean surfaces, the efflux of wall impurities has been reduced by more than an order of magnitude.

Another method of reducing the impurity level has been the use of a divertor. This device was proposed relatively early\(^4\) to take away from the discharge the particles nearest the wall and to avert bombardment of the discharge tube by charged particles. In the divertor, an outer shell of flux is diverted or bent away from the main discharge into a large auxiliary chamber. Any impurities produced by wall bombardment in the divertor chamber return relatively slowly into the main discharge tube. A schematic diagram of a divertor is shown in Fig. 4. The theory of this device, together with observations on its effectiveness in reducing the impurity level, without an ultra-high vacuum, is reported in the subsequent paper by Burnett, Grove, Palladino, Stix and Wakefield.\(^8\) Apparently the divertor reduces the ratio of impurity ions to helium ions by a factor of about 1/5.

The most important new observational result that has emerged from these ohmic heating studies is the evidence on cooperative phenomena. During ohmic heating, the plasma is anything but quiescent. Runaway particles start abruptly to hit the tube wall, producing X-rays, sometimes in short bursts, at times dependent on the magnitude of the confining field. The plasma in the stellarator is not well confined by the magnetic field, and reaches the wall in about \(10^{-4}\) seconds when voltage is applied around the stellarator. After ohmic heating, a current of runaway electrons, amounting to some 10 amp/cm\(^2\) may persist for several milliseconds after the voltage is turned off, and then abruptly disappear, producing a burst of X-rays and additional ionization and excitation in the plasma.

The detailed study of these phenomena should increase our understanding of plasma physics and enable one to predict how an ionized gas might behave in a full-scale thermonuclear reactor.

**Magnetic Pumping**

Pulsation of an axial confining field produces an oscillating electrical field, encircling the tube axis. This electric field can increase the energy of gyrating charged particles. If the pulsation frequency is much less than cyclotron frequency, however, this increase in energy is computed most simply from the magnetic moment, \(\mu\), which, according to the results by Kruskal\(^{24}\) should be very accurately constant. Under these conditions, the increase in kinetic energy of motion, transverse to the field, is given by the usual formula for adiabatic compression of a gas, provided \(\gamma\) is set equal to 2, corresponding to the presence of two degrees of freedom. Instead of thinking about the electric fields induced by the pulsating magnetic field, we may think of the external lines of force as constituting a piston, and the pulsation of the external field as providing a pumping action.

Evidently if the plasma were entirely adiabatic, or isothermal, the work done on the gas during the compression would just equal the work done on the piston during expansion. No heating would result. To obtain net heating in a pumping cycle there must be a phase lag between temperature and density. Such a phase...
lag may be produced in a variety of different ways, and hence there are many frequencies at which magnetic pumping can be effective.

One mechanism for producing such a phase lag is the effect of collisions in exchanging energy between motions parallel and perpendicular to the lines of force. This effect was analyzed early\(^\text{33}\) at Project Matterhorn, as a possible substitute for ohmic heating, but was dropped because it was not an effective method for completing the ionization of a gas. More recently, this mechanism has been analyzed by Berger and Newcomb,\(^\text{36}\) and, independently, by Schlüter,\(^\text{38}\) with identical results. The analysis at Princeton is given in the subsequent paper by the Matterhorn theoretical group.\(^\text{38}\) It is clear that magnetic pumping at the positive ion collision frequency can, in principle, heat a fully ionized gas to very high temperatures; however, the rate of heating falls off as \(T^{-4}\) with increasing temperature, an inconvenient drop.

Another method of producing the desired phase lag is to pump in a short section of tube, with a pulsation period about equal to the time required for a positive ion to travel through the pumping section. In this situation the temperature lags because of loss of heat out the ends. If the mean free path is short compared to the length of the pumping section, magnetic pumping at this frequency produces acoustic waves, which travel along the magnetic field. For long mean free paths, the particles may be treated as free, and the energy is effectively thermalized by fine-scale mixing. The analysis\(^\text{34}\) of this “transit-time heating”, reported in a subsequent Matterhorn paper,\(^\text{36}\) shows that this method should be an effective means for heating a plasma to very high temperatures, particularly since the rate of energy input increases as \(T^{-6}\) with increasing temperature, if the frequency is optimized at each temperature.

Experimental verification of heating by magnetic pumping has not yet been possible at Project Matterhorn, since the radio-frequency power available was insufficient to balance plasma losses, due to inadequate confinement and too high an impurity level.

**Ion-cyclotron Resonance Heating**

Pulsation of the confining field at the cyclotron frequency of the positive ions should give very rapid heating at very low ion densities. The effect is most conveniently understood not as a macroscopic pumping but as a microscopic resonance between the oscillating electric field and the gyration of the positive ions. However, this type of heating produces separation of charges, and at appreciable plasma densities the resultant electrostatic fields prevent any appreciable heating in this way. It was pointed out by Stix that use of two adjacent heating sections, with identical pulsation frequencies, but differing in phase by \(180^\circ\), would make it possible for electrons to cancel out this separation of charges by flowing back and forth along the lines of force, and thus permit heating at the ion cyclotron frequency even at substantial plasma densities.

The axisymmetric free oscillations of a cylindrical plasma column, in a strong axial field, were analyzed in detail by Stix,\(^\text{37}\) who found that indeed for sufficiently short wave lengths a plasma resonance existed close to the ion cyclotron frequency. Later analyses both by Stix\(^\text{33}\) and by Kulmrad and Lenard\(^\text{36}\) have considered the input of energy into the gas both at the exact ion cyclotron frequency and at the adjacent plasma resonance frequency. It appears that a substantial amount of energy can be fed into the gas at the plasma resonance frequency, and that thermalization of the energy can readily be achieved in a small system. At low \(\beta\) (low ratio of material pressure to magnetic pressure in the vacuum field) this technique offers the great advantage, in principle, over magnetic pumping that the coupling between the external circuits and the plasma is very much better. For a large system, at moderate \(\beta\), it is not obvious from theory alone which system would be most effective.

Detailed observational results on plasma heating at frequencies near ion-cyclotron resonance are reported in the associated experimental paper by Stix.\(^\text{14}\) Measures on the external circuits at low power indicate that, in fact, resonant loading is observed at frequencies at and below the cyclotron frequencies both for hydrogen and helium. The energy fed into the gas exceeds the energy dissipated in the external circuit, a prerequisite for efficient heating. Measures at high power appear to be generally consistent with expectations, although direct evidence of plasma heating has not, as yet, been obtained.

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In presenting Paper P2170, above, at the Conference, Mr. Spitzer summarized Papers P358, 359, 362, 363, 364 and 2170, from this Session, and P357, 1875 and 1876 from Session A-5 (Vol. 31) under the title "Survey of the Stellarator Program", as follows.†

The controlled thermonuclear program at Princeton University has concentrated on plasma confinement in an endless toroidal tube, in which a strong magnetic field is produced by external coils. Such a system we have called a stellarator. This paper reviews very briefly the theoretical foundations of the stellarator program, and then summarizes the observational results which our experimental group has obtained.

Equilibrium

Our theoretical work has been chiefly concentrated on three main topics—first, the equilibrium of a hot plasma within a stellarator; second, the stability of this equilibrium; third, the initial ionization and heating of a gas to establish this equilibrium. Our work on each of these three subjects will be summarized briefly here. Early studies of equilibrium showed that in the ideal torus, where the lines of force are circles centered at the axis of symmetry, equilibrium plasma confinement was not possible, in any simple way. This result follows at once from the well known particle drifts across an inhomogeneous field. These drifts produce a separation of electrical charge. The resultant electric field is not neutralized by currents, since an electric field transverse to a magnetic field produces no current in a steady state, and this electric field sweeps the plasma to the wall. This catastrophe can be averted, in principle, if the degeneracy of the torus is removed, and if the field lines are effectively twisted, and no longer close on themselves after one circuit around the tube. The situation is shown in Fig. 1, which portrays a cross-section of the stellarator tube, perpendicular to the magnetic axes. Point P1 represents an intersection of some line of force with this plane. The same line of force returns on itself after one circuit around the tube. The other lines rotate about the central regions of the tube. One line of force, the magnetic axis, returns on itself after one circuit around the tube. The other lines rotate about the magnetic axis in successive circuits; the angle of rotation in a single circuit—that is, the angle between the magnetic axis in successive circuits; the angle of rotation in a single circuit—that is, the angle between the magnetic axis and the rotational transform angle.

One might wonder whether successive intersections of a line of force with this cross-section plane might spiral away from the magnetic axis, finally leaving the tube altogether. Fortunately this does not happen. Kulsrud has shown that the successive intersection points in this cross-section plane, produced by a single line of magnetic force, generate a closed curve to a very high approximation. Thus inside the stellarator tube a single line of force generates a closed toroidal surface, which we call a magnetic surface. This family of nested magnetic surfaces forms the basis of confinement in the stellarator.

The time available permits only a very brief summary of how a rotational transform angle can be produced. Evidently a current along the magnetic lines

† Figures 1-4 are in the preceding text.
of force will cause such an effect. However, our goal in the stellarator is to produce confinement in a steady state and to achieve this goal we must produce a rotational transform in the vacuum field. We have found two ways of doing this. In the first, the vacuum tube and the surrounding solenoidal windings are twisted out of a common plane into a form resembling a figure eight, for example. The rotational transform angle is then simply the integral of the torsion around the magnetic axis. In the second method, the tube remains in one plane and additional helical windings are added outside the tube. Four or six groups of conductors are used with the current directed oppositely in adjacent groups. Either system can produce a rotational transform angle as great as 180°. These two systems have been fully illustrated in the U.S. fusion exhibit, and will not be described further here.

The equilibrium of a plasma in a magnetic field possessing magnetic surfaces has been analyzed in some detail by the Matterhorn theoretical group, under the supervision of Edward Frieman. The results of these studies are, in brief, that a steady equilibrium is possible. The separation of electric charges, which destroys equilibrium in the simple torus, is now no problem, since any electric charges accumulating on one side of the tube can flow around to the other side by simply moving along the lines of magnetic force. The properties of the equilibrium state have been shown by Kruskal and Kulsrud to be related directly to the natural invariant functions of the problem—the total mass enclosed within a magnetic surface, and the rotational transform angle on this surface—as functions of the total enclosed flux through a cross-sectional plane. In addition, a study of the trajectories of single particles indicates that these should be confined within a stellarator to a very high approximation, provided, of course, that the plasma remains quiescent; that is, provided that cooperative phenomena do not destroy the confinement.

Stability

Since we may conclude that a plasma confined in a stellarator should possess a steady equilibrium state, we now pass on to the second area for investigation, the stability of such equilibria. This has been a major field of investigation by the Matterhorn theoretical group. A general method has been developed by Bernstein, Frieman, Kruskal and Kulsrud for computing whether a plasma is unstable for any hydrodynamic disturbances. This method is based on a computation of $\delta W$, the change in energy for any arbitrary perturbation; the computation is based on the fluid equations, with the assumption that the macroscopic velocity vanishes in the equilibrium state. Convenient methods have been developed for finding the perturbation which gives the least $\delta W$ in any system. If this least $\delta W$ is negative, the system is unstable. This method has been applied to the stellarator by Johnson, Oberman, Kulsrud and Frieman. The results indicate that the figure-eight stellarator tends to be generally unstable, but that the helical windings give a system that is completely stable hydromagnetically, provided that $\beta$, the ratio of material to magnetic pressure, is not too great.

The stability produced by these windings results from the fact that the rotational transform angle increases markedly with distance from the magnetic axis. In any hydromagnetic instability, within a toroidal tube, some lines of force move toward the wall, while others move inwards. The system is most unstable if the lines of force can interchange positions with very little bending. When the rotational transform angle varies with distance from the magnetic axis, different magnetic surfaces are topologically different; considerable bending of the lines is required if one line is to move inwards and another, outwards.

Our detailed stability calculations have all been based on the fluid equations, which are not always realistic in a hot, fully ionized gas. However, Kulsrud has shown that the more complicated single-particle analyses of Kruskal and Oberman, and of Rosenbluth, give essentially the same stability criterion as does the simple $\delta W$ formulation. We conclude that in principle the stellarator, with helical windings, is completely stable against hydromagnetic disturbances, for $\beta$ less than a certain critical value. This upper limit on $\beta$ is somewhat uncertain, since the theory has only been carried through to first order in $\beta$, and since corrections for finite Larmor radius may be important. This critical value of $\beta$ probably lies between 0.1 and 0.4.

Heating

The various methods used for heating plasma in stellarators have also been analyzed in considerable detail. Two general methods have so far been considered, both based on the use of an electric field to ionize and heat the gas initially present in the tube. First, the electric field has been assumed parallel to the magnetic field. In this situation, where an electric field around the stellarator is induced by transformer action, current flows along the lines of force, and the ohmic losses heat the gas. This method is called ohmic heating. Second, the electric field has been assumed perpendicular to the magnetic field. Since external electrostatic fields tend to be shielded by the plasma, we have restricted ourselves to induced electric fields produced by the pulsation of the confining magnetic field in one section of the stellarator. Since the lines of force move in and out, compressing and rarefying the gas as the magnetic field changes, this method is called magnetic pumping. A variant of this method, proposed by Stix of Matterhorn, and described in a previous session, utilizes a magnetic field oscillating at about the cyclotron frequency of the positive ions.

The analysis of ohmic heating in the stellarator is simpler than in the pinched discharge because there are no dynamical effects to worry about; the magnetic field produced by the ohmic heating current is small compared to the confining field, and the lines of force

‡ In the U.N. National Scientific Exhibition.
do not move much. However, the physical processes occurring are many. Detailed numerical computations on the rates of ionization and heating have been carried through on an electronic computer by Berger, Bernstein, Frieman and Kulsrud for helium. These analyses assume perfect magnetic confinement and a Maxwellian velocity distribution for the electrons. While these two assumptions are not entirely precise, and while the computations take into account only the main excitation and ionization processes, it is believed that the results should provide a reasonably accurate prediction of ohmic heating effectiveness, provided that other phenomena, such as hydromagnetic instabilities, electrostatic oscillations and other types of cooperative activity do not carry plasma and energy to the wall too rapidly.

When this method of heating was first being analyzed at Matterhorn, Kruskal pointed out that the ohmic heating current would produce a kink instability, similar to that observed in the pinched discharge, for a current density greater than a certain limiting value, which we now call the Kruskal limit. In this simplest mode of hydromagnetic instability, all the lines of force within the current channel move uniformly toward the wall. Fortunately the Kruskal limit is rather high, exceeding some 200 amp/cm² in our devices, and currents well below this limit can achieve effective heating at temperatures below a million degrees. In addition to the kink instability, the theory also predicts higher modes, in which the perturbation varies over the plasma cross-section.

Heating of a fully ionized gas to very high temperatures can, in principle, be achieved by magnetic pumping, provided, as usual, that unforeseen cooperative effects do not introduce excessive losses to the wall. The theory indicates that the positive ions are heated most effectively if the period of magnetic field pulsation is about equal to one of the two natural periods of these ions. One of these periods is the time required for Coulomb collisions to produce a deflection of 90°.

Heating at this frequency has been discussed independently by Schliiter. The second natural period is the time required for a positive ion, moving at thermal velocity, to travel through the heating section, in which the magnetic field is pulsating. This interval is called the transit time, and has the practically convenient property that it changes relatively slowly as the temperature increases. The detailed analyses carried out by our theoretical group indicate that magnetic pumping at the transit-time frequency should be an effective way, in principle, to heat a gas to thermonuclear temperatures. Unfortunately, the rate of heating by this technique is relatively slow, and losses from the plasma must be kept very low if very high temperatures are to be reached by this means. Heating at plasma resonance, near the ion cyclotron frequency, is a promising method, especially since Stix has shown that the resultant rate of heating may be relatively rapid.

**Results**

After this brief review of theory, we may now pass on to observational results. To test these ideas, a half-dozen small research stellarators, with auxiliary equipment, have been designed and built at Princeton by engineering groups under Norman Mather and Robert Mills. These devices, referred to as our B series, have discharge tubes some 5 to 10 cm in diameter, and between 250 and 600 cm in axial length. Solenoidal coils surrounding each tube produce axial confining fields up to 50,000 gauss; operation is generally at fields between 10,000 and 40,000 gauss. Some of these devices utilize a figure-eight shape to obtain a rotational transform; others employ the helical windings.

Figure 5 indicates the normal sequence of events with one of these devices. The solenoidal coils are energized from a capacitor bank of \( \sim 10^4 \) joule, whose terminals are shorted when the voltage approaches zero. Near the peak of the field, breakdown is obtained with a radio-frequency electric field, typically at 250 kc, and ohmic heating is produced with a pulse of 30 to 300 v applied from another capacitor bank.

A schematic diagram of the tube, showing the various experimental techniques we have employed in these stellarators, appears in Fig. 6. The solenoidal coils are omitted from this diagram. The rf breakdown voltage is applied across an insulating section in the stainless steel discharge tube. The ohmic heating voltage is applied to the primary winding of the ohmic heating transformer. The plasma current is measured by its magnetic field, in the usual way, and electron densities are determined from the phase shift of 8 mm and 4 mm electromagnetic waves traveling across the discharge. We have made extensive use of spectroscopic methods, and have measured the X-rays emitted when energetic electrons strike the inner edge of a tungsten aperture limiter, a circular disc with a hole centered at the magnetic axis. The most striking result obtained with this technique is the observation of X-rays, with energies of several hundred kilovolts, for an interval of some ten milliseconds after

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*Figure 5. Time sequence of fields in B research stellarators*  
The axial confining field is produced by external solenoid coils; the voltage is applied around the toroidal tube.
the ohmic heating pulse. Evidently, energetic single particles are confined for more than a hundred thousand circuits around the stellarator, an indication of very good confinement for single particles.

Figure 7 shows typical results obtained with an ohmic heating discharge in helium; the initial pressure, \(3 \times 10^{-4}\) mm Hg, confining field, 27 kilogauss, and applied electric field, 0.067 v/cm. It is readily shown from the high ratio of He\(^{11}\) to He\(^{1}\) light, at about two-thirds of a millisecond after the voltage is applied, that the ratio of ions to neutral atoms at this time is very high, probably greater than a hundred to one. At about this time the electron density starts to drop. This decay of the electron density during ohmic heating, which we have now measured in considerable detail, is called "pump-out". A short time after maximum density, the current peak is reached, and the current starts downwards. Meanwhile substantial amounts of impurity light appear in the discharge.

**Analysis**

Extensive data of this type have been obtained and analyzed by the Matterhorn experimental group, under the supervision of Melvin Gottlieb. The primary problem in which we have been interested is the general area of cooperative phenomena, including all types of plasma activity such as oscillations, instabilities and turbulence. It is generally acknowledged that if such phenomena did not occur and the plasma were quiescent, a thermonuclear reactor would certainly be possible, at least in principle. This basic investigation of plasma physics forms the central part of our program. A secondary problem is the investigation of impurities, which stream off the walls into the discharge. Since purity of the gas is of the greatest importance in a thermonuclear reactor, the investigation of this area is of considerable practical importance.

We are working in three areas of plasma physics. One, which we think we understand, is the existence of a limiting current, which corresponds closely with the current predicted by Kruskal for the onset of kink instability. Another is the rate at which ohmic heating proceeds, and the associated problem of energy balance. A third is pump-out, the disappearance of plasma from the discharge. Finally, a brief discussion will be included of impurities, specially on the practical problem of how these may be reduced in abundance.

**Kink Instability**

Let us first, then, look at the observational evidence for kink instability, for currents greater than the Kruskal limit. In Fig. 8, taken from a paper by Kruskal, Johnson, Gottlieb and Goldman, is shown the maximum current reached in a helium discharge, as a function of the ohmic heating field applied. For voltages only slightly greater than are needed for breakdown, the current is observed to rise to a maximum value that is nearly independent of initial pressure, in the range between \(10^{18}\) and \(10^{14}\) He atoms per cm\(^3\). This maximum current in a single pulse rises only...
If this observed limiting current is plotted against the confining field, we find a closely linear relationship, as shown in Fig. 9. This result agrees with theoretical predictions. The solid circles give results for the current in one direction, the open circles for reverse current. The ratio of the two slopes is 1.19, in very close agreement with the theoretical value 1.22 obtained from the Kruskal theory for the geometry of this particular machine. While it is not clear just what contortions the plasma performs when the Kruskal limit is exceeded, we are fairly confident that the hydromagnetic kink instability does in fact appear under the conditions predicted theoretically. The higher modes of instability, also predicted by the theory, have not as yet been observed.

**Ohmic Heating**

We next look at the comparison between the ohmic heating theory and the observed rate at which the discharge develops. Figure 10, taken from a paper by Coor, Cunningham, Ellis, Heald and Kranz, shows this comparison for a heating field of 0.11 v/cm, and an initial helium pressure of $8 \times 10^{-4}$ mm Hg. During the first 250 $\mu$sec, the current, the electron temperature found from the resistivity, and the intensity of HeII light behave about as expected, but at later times these quantities level off for more than a millisecond, as compared with a continuing rise predicted by theory. Evidently some loss mechanism becomes important after about 250 $\mu$sec and prevents the further heating of the plasma. The nature of this loss mechanism is still obscure.

**Pump-out**

We pass on now to the third topic, about which we are more ignorant—the disappearance of electrons from the discharge, or pump-out. It is clear that the electrons and positive ions in the gas are somehow reaching the wall and sticking there. The effect appears to be identical in hydrogen and deuterium. Extensive measures of pump-out rates in hydrogen discharges have been made by Gorman, Ellis and Goldberg. Over the limited range of densities measured, the observed decay appears to be roughly exponential; the decay rate in reciprocal seconds is plotted in Fig. 11 as a function of initial pressure, for a confining field of 30 kilogauss. The decay rate does not seem to vary slowly with further increase in the pulsed heating field. Moreover, when the current is at or above the Kruskal limit the voltage is observed to fluctuate rapidly and X-rays disappear; evidently the plasma develops violent activity under these conditions.

If this observed limiting current is plotted against the confining field, we find a closely linear relationship, as shown in Fig. 9. This result agrees with theoretical predictions. The solid circles give results for the current in one direction, the open circles for reverse current. The ratio of the two slopes is 1.19, in very close agreement with the theoretical value 1.22 obtained from the Kruskal theory for the geometry of
appreciably as the ohmic heating field is changed, although the electron temperature (as determined from the measured resistivity) changes by a factor of two as this electric field varies. It is apparent from the figure that the observed decay rate decreases linearly with increasing initial pressure. This result is very plausible if the number of neutral atoms returning to the discharge, per atom reaching the wall, varies linearly with the initial pressure. Since the number of adsorbed hydrogen molecules on the stainless steel wall might well be proportional to the initial pressure, this behavior is entirely reasonable.

If we extrapolate this line back to zero initial pressure, and assume that a hydrogen atom sticks to a metal wall, in the absence of adsorbed hydrogen atoms to be knocked off, we obtain the intrinsic rate at which ions leave the discharge. In Fig. 12 these decay rates, extrapolated to zero pressure, are plotted against the confining field, on a logarithmic scale. Evidently these intrinsic electron decay rates, under the conditions of these particular measures, vary about as the inverse square root of the confining field. We have no theoretical explanation of this dependence. These measures have not as yet been carried out in a theoretically stable system, with helical windings; possibly some mode of hydromagnetic instability is responsible. At any rate, pump-out is evidently a very important phenomenon in a stellarator, and its explanation provides a fascinating problem in basic physics.

Impurities

We turn now to the fourth major topic, the impurity level in a stellarator. With the millisecond time scale and small radius of our devices, the inflow of impurities can be relatively high, especially as a result of the intense particle bombardment of the tube walls produced by pump-out.

Several steps have been taken to keep down the impurity level. One very effective method is to clean the walls, by baking for several hours at 450°C. This treatment reduces the base pressure to about 10^{-10} mm Hg, and does indeed reduce the impurity level by about two orders of magnitude as compared with unbaked systems. To obtain such high vacua in systems with volumes of some ten liters and with many observation ports and gasketed joints, has required the development of a substantial technology by our vacuum group under Don Grove. Prolonged pulsing with ohmic heating reduces the impurity level but has not yet proved to be an adequate substitute for prolonged baking.

Another successful technique for reducing the impurity level involves use of a divertor. In the divertor, an outer shell of magnetic flux is bent away from the discharge tube into a separate chamber. Particles diffusing toward the wall enter the divertor and strike the wall far from the main discharge. Any impurities released in the divertor do not readily find their way back into the discharge.

Figure 13 shows a quarter cross-section of a divertor used on an unbaked stellarator designed by Stix. The divertor is rotationally symmetric about the horizontal axis shown; the vertical axis is a plane of symmetry. The thin curves represent magnetic lines of force. Devices of this type are readily designed on a magnetic analogue board devised by Wakefield.

A series of spectroscopic measurements was made by Burnett, Grove, Palladino and Stix with and without divertor. When the divertor was not activated, that is, when the negative coil was not energized, a limiter disc with a hole in the center was used to confine the discharge to the same central region of the tube as with the divertor. Figure 14 shows a comparison of the intensities of the O' line, with and without divertor. The curves represent mean intensities observed photoelectrically as a function of time after the beginning of the ohmic heating pulse. If the divertor was used in place of the limiter, the intensity of this impurity line

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**Figure 12.** Decay rate of electron density extrapolated to zero initial pressure as a function of confining field.

**Figure 13.** Quarter-section of experimental divertor. The horizontal axis is an axis of cylindrical symmetry; the vertical axis is a plane of symmetry.
Figure 14. Observed intensity of OV line with and without the divertor. The axial confining field is 20,000 gauss, the applied electric field, 0.24 v/cm, the initial helium pressure, $1 \times 10^{-4}$ mm Hg.

Figure 15. Observed Doppler width of OV line with and without the divertor. Experimental conditions are as in Fig. 11.

was reduced to about a tenth of its former value. Figure 15 shows that the kinetic temperature, obtained from the Doppler width of this same line, is materially increased by use of the divertor. The measurements are not yet sufficiently complete to separate effects of temperature from those of abundance, nor to unravel effects produced by radial variations of both temperature and impurity level. However, they support the belief that the divertor produces a marked reduction in the impurity level, with a large consequent increase in temperature.

**Conclusion**

To conclude, our theoretical work has shown that (1) an equilibrium state for a confined quiescent plasma can exist in a stellarator; (2) this equilibrium is hydro-magnetically stable; and (3) heating to thermonuclear temperatures should be possible by ohmic heating, followed by magnetic pumping, provided the plasma is sufficiently quiescent. The observational research has shown that confinement of high energy electrons is excellent, and that ohmic heating produces a hot, virtually fully ionized plasma. However, the gas is definitely not quiescent. The onset of the gross hydro-magnetic kink instability, predicted by Kruskal, has been fully confirmed observationally. In addition, the evidence on pump-out indicates that there may be other types of activity occurring that we do not as yet understand. The study of these effects will form a challenging and major part of our program during the next few years. A secondary objective of our program is the further reduction of the impurity level. If pump-out can be controlled, and the impurity level kept sufficiently low, we might hope to achieve much higher temperatures by use of magnetic pumping, an objective which has not yet been possible with the present high loss rate from the plasma.

These experimental objectives should be materially aided by the construction of the C Stellarator, a research tool considerably larger than our B devices. With a tube diameter of 20 cm, a field of 80,000 gauss pulsed for times up to one second, and extensive facilities for diagnostics and for heating, this device should be a powerful tool for analyzing the complex phenomena occurring in hot plasma.
On the Ionization and Ohmic Heating of a Helium Plasma

By J. M. Berger,* I. B. Bernstein,† E. A. Frieman† and R. M. Kulsrud†

One useful method of heating a plasma confined by a magnetic field is to impose a constant electric field parallel to the magnetic field.1, 2 Due to the finite resistivity of the plasma, there will be a conversion of electric energy into heat, some of which appears as internal energy of the plasma. In this paper we determine what value of this electric field is needed to ionize and heat the plasma in a reasonably short time.

The plasma is imagined to be at a low temperature and partially ionized when the constant electric field is first imposed. The Joule heat cannot all go to raise the kinetic energy of the ions and electrons since some of the energy is lost in ionization of neutral atoms by electron impact, while some is lost to the plasma through excitation of the neutral atoms which then radiate their energy out of the system. Also, at higher temperatures, bremsstrahlung radiation due to free-free transitions of the electrons in the coulomb fields of the ions becomes a significant loss mechanism. Further, the remaining Joule energy is communicated directly to the electrons which move more readily in the electric field, and is only subsequently given to the ions by electron-ion collisions. Finally, the rate of Joule heating depends on the conductivity of the plasma, which varies with temperature. The number of electrons is continually changing because of the ionization mechanism already mentioned.

During the development of the discharge, the electron distribution function is not expected to be completely Maxwellian since the collision cross section responsible for the approach to equilibrium is a rapidly decreasing function of energy. The distortion is most severe for the energetic particles, those above a certain critical energy. They are essentially unaffected by collisions and gain energy continually from the electric field, acquiring the loop voltage on each transit around the discharge tube. They can be said to “run away” from the body of the electron distribution. The critical energy is roughly that at which an electron gains from the electric field, in one mean free path, an energy equal to its (thermal) kinetic energy.

If the critical energy is several times the mean thermal energy it is legitimate to assume that the distribution function is Maxwellian for the purpose of computing the conductivity. Also, if the critical energy is much larger than the energy of those electrons contributing most to excitation and ionization, it is legitimate to assume a Maxwell distribution in calculating the average rates at which these processes occur.

However, there must always be some electrons which have run away and are circulating at high energies. If there are enough of these they can well alter the nature of the discharge; for instance, by giving rise to a skin effect, or perhaps collective motions. Throughout the present work such runaways and their associated effects will be consistently ignored.

It will be found that the criteria for achieving a Maxwell distribution are not well satisfied for the densities and electric fields for which the calculations were made, and one therefore should resort to the Boltzmann equation.

One might expect a further distortion of the distribution from Maxwellian to exist in the depletion of electrons from the tail in the inelastic processes of ionization and excitation. However, one finds that in the cases treated, the tail is resupplied by elastic collisions at such a rate that the distortion due to this effect is not serious. However, the assumption of a Maxwell distribution greatly simplifies the calculations and yields results which are qualitatively correct.

If the above assumptions are satisfied, the discharge in helium develops as follows. When the electric field is applied, the current is at first limited by inductive effects which initially are much larger than the resistive effects. After a short time, the inductive effects become negligible compared to the resistive effects, and the current is then given closely by Ohm's law. The electron temperature rises until the electrons have sufficient energy to excite and ionize the neutral atoms, at which point the temperature stays nearly constant until almost all the atoms are ionized and the energy sink they provide is removed. The temperature then rises again until second ionization and excitation of the singly ionized atoms set in. It pauses again, until second ionization is complete.

The current then starts to rise again since the only remaining energy loss, the bremsstrahlung loss, varies as $T^3$ while the power input goes as $T$. As the current continues to rise, the temperature increases, yielding an increase in conductivity which leads to a skin effect. This current is thus confined to a thin annular region on the discharge boundary. This results in the heating,
Figure 1. Densities $n_e$ and $n_+$ and temperatures $T_e$ and $T^+$ vs. time.

Figure 2. Current, $j$, and light intensities ($\lambda 4921$ and $4686$ Å) vs. time.

finally, of only the discharge skin. This effect, however, does not enter significantly for the range of temperatures considered in the calculations.

The ion temperature, which is the same for singly and doubly ionized helium because of rapid collisions, lags far behind the electron temperature throughout the entire process, since the transport of energy from electrons to ions is relatively inefficient compared to the rate at which the electrons are heated. Further, the rate of this transport decreases as the electron temperature rises (as $T_e^{-4}$) so that ohmic heating of ions becomes inefficient when the electrons are heated to temperatures of more than several hundred volts. The ions cannot be expected to reach temperatures of more than 100 volts. Consequently, one cannot reach thermonuclear ion temperatures with ohmic heating alone. A more serious limiting effect is the existence of a hydromagnetic instability associated with large currents, as predicted by Kruskal.\textsuperscript{9} When the current exceeds a critical value, depending on the size of the confining magnetic field, the instability sets in and the assumptions of this calculation become highly suspect.

**EQUATIONS**

For simplicity, we consider the gas to be at uniform density, contained in a long cylinder, and in a homogeneous externally maintained axial magnetic field. The electric field is also, of course, in the axial direction, as is the current which it produces.

The equations advanced to describe the development, in time, of the discharge are the following. Let $j$ denote the axial current density, $S$ the cross sectional area of the discharge, $\epsilon$ the electric field strength, $l$ the length of the discharge, and $\sigma$ the electrical conductivity. Then the equation relating $j$ and $\epsilon$ is

\[ \frac{dS}{dt} = \left( \frac{3}{2} nekT + \frac{3}{2} n+n+1 \right) \frac{d\epsilon}{dt} \left( \frac{2\pi m\epsilon}{m_+} \right)^{1/2} \frac{m_0}{m_+} \left( \frac{m_0}{m_+} \right)^{1/2} \ln \left[ \frac{2\pi m\epsilon}{m_+} \left( \frac{m_0}{m_+} \right)^{1/2} \left( \frac{m_0}{m_+} \right)^{1/2} \ln \left[ \frac{12\pi m_0}{\sigma} \left( \frac{m_0}{m_+} \right)^{1/2} \left( \frac{m_0}{m_+} \right)^{1/2} \right] \right] \]

where, in Gaussian units,

\[ \frac{1}{\sigma} = \left( 0.582 + \frac{1}{10\epsilon} \right) \frac{\pi}{kT} \frac{Z_{eff}^2}{kT} \]

and the effective atomic number $Z_{eff}$ is given by

\[ Z_{eff} = \frac{Z}{n/n + n_+ + (Z_{eff} - Z)} = 1 + \frac{n_+ + n_{++}}{n_+ + n_{++}} \]

Eqs. (2) and (3) are obtained, for the mixture of positive ions here considered, from Spitzer\textsuperscript{4} by linear interpolation between the integral values of the atomic number for which he presents the results.

The equation of electron energy balance is,

\[ \frac{dE}{dt} = \frac{1}{\sigma} \left( \frac{2\pi m\epsilon}{m_+} \right)^{1/2} \frac{m_0}{m_+} \left( \frac{m_0}{m_+} \right)^{1/2} \ln \left[ \frac{12\pi m_0}{\sigma} \left( \frac{m_0}{m_+} \right)^{1/2} \left( \frac{m_0}{m_+} \right)^{1/2} \right] \]

The quantities $n_e$, $n_0$, $n_+$, $n_{++}$ and $N$ are respectively the number densities of electrons, neutral helium, singly ionized helium, doubly ionized helium, and helium nuclei, the last of these being constant in time; $\hbar$ is Boltzmann's constant, $T_e$ the electron temperature; $T_+$ is the temperature of the $He^+$ ions which is taken to be the same as $T_++$, the temperature of the He++ ions. The left-hand side of Eq. (4) represents the rate per unit volume at which energy is fed into the plasma by the current. The first term on the right expresses the rate at which the internal energy of the electrons increases, the second term the rate at which energy is expended in exciting and ionizing neutral helium, and the third term the rate at which energy is
OHMIC HEATING OF A PLASMA

expended in exciting and ionizing He+. The quantities $E_i$ and $E_i^+$ stand for the energy required for a particular excitation, while the $v_i$ and $v_i^+$ stand for the product of the associated collision cross sections and the electron speed, averaged over the Maxwell distribution of the electrons. The expressions for $\Sigma v_i E_i$ and $\Sigma v_i^+ E_i^+$ used in the computations were the following analytic fits to results obtained employing experimental cross sections:

$$v_{ion} \Sigma v_i E_i \ldots = a + b(kT_e) + A - B(kT_e) + C(kT_e)^2 + D(kT_e)^3 + E(kT_e)^4 \ldots$$

The constants for the various fits are given in Table 1.

### Table 1. Coefficients for Calculation of $v$, $\Sigma v E$

<table>
<thead>
<tr>
<th>Cross section</th>
<th>$E_i v_i$</th>
<th>$E_i^+ v_i^+$</th>
<th>$v_{ion}$</th>
<th>$v_{ion}^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>$1.17248 \times 10^{-18}$</td>
<td>$7.84995 \times 10^{-21}$</td>
<td>$3.89217 \times 10^{-10}$</td>
<td>$4.53767 \times 10^{-11}$</td>
</tr>
<tr>
<td>c</td>
<td>$1.4740 \times 10^{-14}$</td>
<td>$3.07369 \times 10^{-8}$</td>
<td>$1.21018 \times 10^{-4}$</td>
<td>$1.7800 \times 10^{-5}$</td>
</tr>
<tr>
<td>A</td>
<td>$9.80288 \times 10^{-10}$</td>
<td>$1.43009 \times 10^{-10}$</td>
<td>$3.57046 \times 10^{-9}$</td>
<td>$9.07731 \times 10^{-10}$</td>
</tr>
<tr>
<td>B</td>
<td>$1.10027 \times 10^{-18}$</td>
<td>$7.36446 \times 10^{-21}$</td>
<td>$3.77845 \times 10^{-10}$</td>
<td>$4.1765 \times 10^{-11}$</td>
</tr>
<tr>
<td>C</td>
<td>$3.12718 \times 10^{-18}$</td>
<td>$1.63000 \times 10^{-24}$</td>
<td>$1.15091 \times 10^{-11}$</td>
<td>$8.1165 \times 10^{-13}$</td>
</tr>
<tr>
<td>D</td>
<td>$8.95259 \times 10^{-3}$</td>
<td>$2.37593 \times 10^{-3}$</td>
<td>$6.01543 \times 10^{-3}$</td>
<td>$1.44940 \times 10^{-3}$</td>
</tr>
<tr>
<td>E</td>
<td>$2.31606 \times 10^{-2}$</td>
<td>$6.00339 \times 10^{-6}$</td>
<td>$5.74003 \times 10^{-6}$</td>
<td>$6.49858 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

where, in addition, there appear separately the constants for the ionization rates. The units have been so chosen that the results are in cgs units if $kT_e$ is expressed in electron volts.

The third and fourth terms on the right-hand side of Eq. (4) represent respectively the rates at which He$^+$ and He$^{++}$ gain energy per unit volume via collisions with electrons, while the last term gives the energy loss per unit volume via bremsstrahlung.

In addition to the preceding equations, there are those expressing the ion energy balance, and the various number and charge conservation laws, viz:

$$\frac{d}{dt} \left( \frac{3}{2} n_+ (n_+ + n_{++}) kT_e \right) = n_0 n_+ k(T_e - T_0) \left( \frac{2m_e T_e^4}{m_+} \right)^{\frac{3}{2}} \ln \left( \frac{12\pi n_e (\frac{k T_e}{4\pi n_e e^2})^{\frac{1}{2}}} {2\pi n_e^{\frac{3}{2}} (\frac{k T_e}{4\pi n_e e^2})} \right) + n_0 n_{++} k(T_e - T_0) \left( \frac{2m_e T_e^4}{m_+} \right)^{\frac{3}{2}} \ln \left( \frac{6\pi n_e (\frac{k T_e}{4\pi n_e e^2})^{\frac{1}{2}}} {3\pi n_e^{\frac{3}{2}} (\frac{k T_e}{4\pi n_e e^2})} \right) - n_0 n_+ \frac{3}{2} (kT_e - T_0) Q \left( \frac{4k (T_{++} + T_0)}{m_+} \right)^{\frac{1}{2}}$$

The last term on the right-hand side of Eq. (6) represents the effects of charge exchange collisions. In such an encounter it is assumed that the “fast” ion is converted to an ion at the temperature, $T_0$, of the neutral atoms. $T_0$ is taken to be the temperature of the external walls and is constant in time. The charge transfer cross section, $Q$, was computed to be $5 \times 10^{-16}$ cm$^2$.

Experimental results on ohmic heating in the B-1 Stellarator are reported in Reference 7.

### Experimental Results

The equations were integrated on the N.Y.U. Univac for a variety of electric fields and densities. The results are given in Figs. 1 and 2 for an electric field of 0.11 volts/cm, which corresponds to 50 $\text{v}$ around the stellarator, and an initial neutral density of $2.4 \times 10^{13}$ helium atoms/cm$^3$. The levelling off of the electron temperature during the main part of the first ionization phase is prominent, and further there is a detectable decrease in the rate of temperature rise during the second ionization phase. It is clear that the ion temperature lags seriously behind the electron temperature. Note that the current density follows the electron temperature quite well except during the initial stages of the discharge. The intensity curves for the neutral helium line 4921 $\text{A}$ and the singly ionized
helium line $\lambda$4686 Å were also calculated and are presented in Fig. 2. For similar cases, in which the integration was carried out to longer times than in this case, it was found that the ion temperature reached 100 v in approximately four milliseconds. At this time the electron temperature was approximately 1,000 v. As mentioned previously, the conditions necessary for the electron distribution to be considered Maxwellian for the purpose of the calculation were not well satisfied. The critical energy $E_{\text{crit}}$ in electron volts, for electrons to run away, is given by

$$E_{\text{crit}} \approx 4.5 \times 10^{-18} n_e/\varepsilon$$  \hspace{1cm} (11)$$

if $\varepsilon$ is in volts/cm and $n_e$ in cm$^{-3}$. For full ionization, in the case considered, $E_{\text{crit}}$ is approximately 80 v. Since the electrons which contribute most to the conductivity are those whose energies are several times thermal energy, one cannot expect the conductivity to be given accurately for $T_e$ much above 10 v. Further, the electrons responsible for first ionization and excitation lie in the range 30–40 v and therefore the calculated rates for these processes are not accurate. For the case of second ionization the situation is worse in this regard but it should be noted that this does not affect the course of the discharge seriously. A detailed comparison of the calculated and experimental results is carried out in Reference 7.

ACKNOWLEDGEMENT

The authors would like to thank Lyman Spitzer Jr. for much advice, encouragement and constructive criticism in connection with this problem.

REFERENCES

Experiments on the Ohmic Heating and Confinement of Plasma in a Stellarator

By T. Coor,* S. P. Cunningham,† R. A. Ellis,* M. A. Heald* and A. Z. Kranz*

In 1951 Spitzer proposed the stellarator as a device for confining plasma and heating it to thermonuclear temperatures. Since that time, a series of devices has been constructed and operated at Princeton University for experimental research in such systems. This paper summarizes experimental results in the fundamental areas of heating and confinement. Accompanying papers present evidence for the importance of cooperative processes in the plasmas of these devices.2, 3

STELLARATOR PRINCIPLES

Confinement

Confinement of plasma in a stellarator is provided solely by externally produced magnetic fields. As described in detail elsewhere, the magnetic configuration is that of a torus with rotational transform.4 This geometry gives plasma confinement of a higher order than that given by a simple toroidal field. It does so by virtue of providing a path, along magnetic lines of force, by which currents may easily flow to neutralize charge accumulations produced by particle drifts in the general toroidal field. Two methods have been used to give rotational transform to the Princeton devices. The first, illustrated in Fig. 1, distorts the torus into the form of a "figure eight", while the second employs currents in auxiliary helical conductors on the surface of a regular toroidal tube. In both cases the general "axial" field is provided by current flowing in copper coils surrounding the tubes. In this and in the accompanying papers,2, 3 the results presented were obtained on equipment employing the figure-eight geometry only.

Even with the rotational transform, plasma confinement will not be perfect. Collisions between unlike particles in the plasma will always give rise to a diffusion across lines of force. In a quiescent plasma in which cooperative oscillations are negligible, this classical collision diffusion will be the dominant process. If instabilities and other cooperative phenomena are present, particles may reach the wall even more rapidly than in a quiescent plasma.

Following Spitzer it can be shown that, assuming classical collision diffusion, the mean confinement time, \( \tau_0 \), for helium is given by

\[
\tau_0 \sim \frac{(20B^2T^2R^2)/n}{\text{sec}}, \quad (1)
\]

where \( B \) is the confining field, \( T \) the temperature, \( R \) the plasma radius, and \( n \) the particle density. Inserting typical values for our devices, \( B = 3 \times 10^4 \) gauss, \( T = 10^5 \text{ °K} \), \( R = 1 \text{ cm} \), \( n = 10^{13} \text{ cm}^{-3} \), we find \( \tau_0 \sim 0.6 \text{ sec} \). Therefore, if virtually quiescent conditions can be found, confinement times of the order of a half second might be expected.

Bohm,7 on the other hand, has suggested that some sort of turbulence with randomly varying electrostatic fields exists even in "quiescent" plasmas and gives for the average macroscopic diffusion velocity

\[
v_\perp \approx \frac{4 \times 10^{18}}{nB} \nabla \phi \text{ cm/sec}, \quad (2)
\]

where \( \phi \) is the plasma pressure. This gives for a mean confinement time

\[
\tau_0 \sim \frac{(0.002BR^2)/T}{\text{sec}}. \quad (3)
\]

For helium under the conditions assumed above, this would give a confinement time of 600 \( \mu \text{sec} \). Even though Bohm and his collaborators adduced experimental evidence in support of this rate of diffusion, Simon and Neidigh8 at Oak Ridge have refuted their result and claim that there is good evidence that Bohm diffusion does not exist, at least in a "quiescent" plasma. Nevertheless it is reasonable to assume that fluctuating electric fields associated with instabilities and other cooperative phenomena can, under some conditions, cause enhanced diffusion across magnetic lines of force.

Heating

Ionization and preliminary heating of plasma confined in stellarator geometry is accomplished by making the closed tube of plasma the secondary of a transformer, as shown in Fig. 2. Flux changes in the transformer iron produce an axial electric field in the plasma which accelerates the electrons and to a much lesser extent the ions. The energy thus given to the particles is randomized and used in completing the ionization of the gas and "heating" the resulting plasma. It is to be emphasized that this ohmic heating
Detailed calculations have been made by Berger et al.,\textsuperscript{10} on the ohmic heating of hydrogen and helium in stellarators. These calculations, in which the fractional ionization and electron and ion temperatures are followed, allow for the important atomic and single-particle processes that might take place in the discharge but assume that the energy distributions stay Maxwellian. Some doubt is cast on the applicability of these results by the fact that, in practice, the applied electric heating fields are large enough to distort the electron energy distribution away from Maxwellian. In particular, the group of runaway electrons (electrons which are gaining energy from the field faster than they lose energy by collisions) which arise early in the heating pulse are capable of more rapid ionization of the neutral gas than the main body of thermal electrons. At the same time, the runaway may be capable of exciting some cooperative mechanism that will transfer energy to the ions at a more rapid rate than the simple electron-ion Coulomb collisions assumed in the Berger model.

To make an exact calculation of the progress of a discharge, in which the electron energy distribution is allowed to be non-Maxwellian and in which cooperative processes are included, is of course impossible at the present time, since the nature of many of the processes is unknown. The best we can hope for in a comparison of the experimental results with heating theory is an indication of how important a part the unknown processes are playing in the heating.

EXPERIMENTAL APPARATUS

This and the accompanying papers\textsuperscript{3, 4} are concerned principally with results obtained in the operation of the B-1 stellarator. This machine uses “figure-eight” geometry and ohmic heating. A schematic diagram of the B-1 device with its associated diagnostic instrumentation is shown in Fig. 3. The discharge tube is fabricated from stainless steel, is 5 cm in diameter, and 450 cm in axial length. A short ceramic (alumina) section is welded into the otherwise conducting discharge tube at one point. This ceramic break pre-
vents the discharge tube from short-circuiting the heating transformer and provides a convenient point for applying a 250-kilocycle radio-frequency pulse for initial breakdown.

Pulsed magnetic confining fields up to 30 kilogauss are used in the B-1 device. Energy storage for this field is obtained from a capacitor bank of \(10^6\) joules. Switching is performed by ignitrons of standard type.

Subsequent to the initial breakdown pulse, ionization is completed and the plasma is heated by means of a unidirectional axial electric field produced by two laminated iron transformer cores. The duration of this field is limited by saturation of the iron (\(\sim 0.1\) v-sec). Under normal operation the ohmic heating voltage waveform approximates a rectangular pulse; this is termed "constant-voltage" operation. Because of the Kruskal hydromagnetic instability which occurs at a definite critical current, it is sometimes useful to insert resistance in the primary circuit of the ohmic heating supply, thereby giving approximately 'constant-current' operation at a point below the critical value. The ohmic heating field can be removed quickly at a pre-set time by means of a 'crowbar' circuit, shown in Fig. 2, consisting of an ignitron switch to short-circuit the primary of the heating-field transformer.

In order to minimize direct wall bombardment by the plasma, and control the size of the discharge column, an aperture limiter was introduced in many experiments. The limiter is a wolfram mask with 3.2 cm diameter hole centred on the magnetic axis of the stellarator. The limiter has incidental uses in connection with diagnostic instrumentation, providing a reference potential for Langmuir probe measurements, and a localized target with high atomic number for the production of X-rays by fast electrons losing confinement from the discharge.

The instantaneous heating field is inferred from the voltage measured across the ceramic break in the stainless steel discharge tube. The plasma current is measured by means of a toroidally-wound pickup loop encircling the tube, the output of which is electronically integrated. These and other diagnostic signals are observed on multiple-channel synchronized oscilloscopes.

It is not possible within the scope of this paper to discuss in detail or justify the many types of instrumentation which have been used to analyze the progress of discharges under various conditions. Electron temperature is inferred from the plasma conductivity derived from observed heating voltage, plasma current, and aperture; from relative strengths of related singlet and triplet lines in the neutral helium spectrum; and from the amplitude of an induced signal coupled by the plasma between toroidally-wound transmitter and receiver coils, operating at a frequency low enough that skin effect can be neglected. Ion temperature is inferred from Doppler broadening of spectral lines of the working gas and of highly ionized impurities. Confinement effects are inferred from the maximum energy of X-rays produced by fast electrons; from time variations in electron density, derived from phase-shift of a transmitted millimeter microwave beam; and from Langmuir probes located outside the active discharge aperture. Instability processes of various types and wall effects are inferred from spectral lines of impurity atoms; from streak photography; and from emission of X-rays and high-level non-thermal microwave noise. Langmuir (electric) and magnetic probes have not been found useful in the active region of the discharge. Because of the rotational transform, an obstacle such as a probe intercepts in principle all plasma outside the magnetic surface adjacent to the inner tip of the obstacle, thus becoming an aperture limiter. While probe studies were nevertheless possible in early devices in which high impurity levels existed because of wall outgassing, it has been found that the high plasma temperatures and abundant runaway electrons of our baked devices consistently destroy the probe in one or two pulses.

The results of early experimentation led to the conclusion that the behavior of discharges in devices of this type was completely dominated by outgassing of the walls if conventional vacuum systems were used. Therefore, since 1956 all devices have been constructed so as to be fully bakable to 450°C. With baking, a base pressure of the order of \(2 \times 10^{-10}\) mm Hg has been achieved, and the influx of impurities has been reduced by about two orders of magnitude. In spite of these efforts, material from the wall is still a significant factor in the behavior of these devices under most operating conditions. During operation, spectroscopically pure gas is introduced continuously through a controlled leak to maintain an operating pressure between \(10^{-4}\) and \(10^{-3}\) mm Hg. While the behavior of hydrogen and deuterium discharges has been studied in some detail, more extensive work has been done with helium, because of better characteristics for optical spectroscopic analysis and greater freedom from complex molecular and chemical phenomena.

The time scale and sequence of operation is shown in Fig. 4. The magnetic confining field is essentially constant during the interesting portions of the dis-
amplitude, duration, and timing of the radio-frequency breakdown pulse and the shape of the heating field waveform (constant-current or constant-voltage operation, and termination by crowbar). The following discussion refers to helium discharges unless otherwise stated.

The Current Rise

Upon application of the ohmic-heating field, the plasma current and electron density rise rapidly. Representative cases of current and voltage behavior for constant-voltage operation are shown in Fig. 5. Figure 6 shows the current rise for several values of heating fields. At high heating fields, the current reaches the Kruskal instability limiting current in 100 μsec or less. The limiting current for the B-1 charge. The timing of all operations is controlled by pre-set counters driven by a 100-ke/sec master oscillator. Operation is limited to about one pulse per minute by capacitor bank charging and heat dissipation of the field coils.

**EXPERIMENTAL RESULTS**

In this section the major experimental observations at various stages of the discharge are summarized. The principal variables are the value of confining field (10 to 30 k-gauss), the ohmic-heating field (0.04 to 0.5 v/cm), and the initial gas pressure (10⁻⁴ to 10⁻³ mm Hg). Other parameters are the

![Figure 5. Representative constant-voltage ohmic-heating discharges in helium](image)

Confining field, 27 k-gauss; pressure 5×10⁻⁴ mm Hg

![Figure 6. Development of current in helium discharges for various heating fields](image)

Confining field, 27 k-gauss; pressure, 5×10⁻⁴ mm Hg
device with a 4.1 cm diameter geometrical aperture is about 2200 amp at a confining field of 27 k-gauss. At moderate fields the current typically rises to a value (< 500 amp) well below the Kruskal limit, remains at this plateau value for as much as several milliseconds, and then rises rapidly to the Kruskal limit. In general, the second rise always proceeds to the Kruskal limit although at low heating fields (Fig. 6, case A) the field ends before a second rise is evident. The plateau currents are essentially independent of confining field, and are shown as a function of heating field and pressure in Fig. 7. The plateau is accompanied by X-ray production and intense non-thermal microwave noise generation.

Typical spectroscopic data obtained by Photomultiplier techniques are shown in Fig. 8. The neutral helium line peaks early in the current rise and then falls slowly. The slowness of fall may result from the influx of cold gas from the external portions of the vacuum system, in particular the observation port, and perhaps also from wall outgassing. Ionized helium light peaks slightly later during the current rise and then falls to low intensity. Spectroscopic evidence indicates first ionization in excess of 95%, and probably a high degree of second ionization. Spectral lines of impurity atoms, such as C\textsuperscript{4+} 4647 Å and O\textsuperscript{4+} 4415 Å, emerge well before the Kruskal instability current is reached. Preliminary observations of Doppler broadening at the time of the current plateaus of Fig. 6 (this feature is barely visible at about 70% of maximum current in Fig. 8) indicate ion temperatures far in excess of what would be expected from ohmic heating alone. These measurements, of great potential significance, must be carried much farther before dependable conclusions can be drawn.

In Fig. 8 the average electron density, inferred from 8.6 mm micro-wave phase-shift data, begins to fall before the Kruskal critical current is reached, indicating imperfect confinement of the plasma.

A comparison of theoretical calculations\textsuperscript{18} with the observed time dependence of plasma current, electron temperature, and ionic (4686 Å) He\textsuperscript{XII} light is shown in Fig. 9 for a typical case where two current plateaus are observed. The temperature was inferred from plasma conductivity data. The agreement between theory and experiment is good in the early phase of the discharge. The theory predicts the existence of the first current plateau, and approximately predicts the observed value of current and electron temperature. However, the duration of the first current plateau is considerably longer than theoretically predicted; this effect is presumably caused by the influx of cold gas into the discharge region. Furthermore, some He\textsuperscript{XII} is observed much earlier than theory predicts, at a time when first ionization of helium is not complete.

Operation at the Kruskal Limiting Current\textsuperscript{2}

When the current reaches the Kruskal kink-instability critical value, both plasma current and voltage show violent fluctuations, as indicated in Figs. 5 and 8; the average current then decreases somewhat with time. No X-rays are produced at this time, indicating that single-particle confinement is too short for runaway electrons to reach high energies.
Depending on the outcome of the competition between the driving of ions into the walls and the resulting intense outgassing of the walls, the charged particle density either falls, as in Fig. 8, or remains high. The time constant for decay of electron density in Fig. 8 is about 300 µsec. Spectral lines of impurities (principally carbon, oxygen, and hydrogen) are very intense, particularly in the case where outgassing dominates and the electron density remains high. Although no quantitative measure of the abundance of impurity ions in the discharge exists, it is clear that they are a major factor even under ultra-high vacuum conditions.

**Operation at "Constant Current"**

By placing a relatively high resistance in series with the heating-field power supply, it is possible to limit the plasma current to a value well below the kink limit. Typical current and voltage behavior is shown in Fig. 10. Under these conditions the charged particles are again rapidly lost from the discharge. Figure 11 shows the variation in electron density during these constant current discharges.

A brief X-ray burst appears at the time of the current rise. As the discharge progresses and the charged particle density falls, there is at first little and then increasingly intense X-ray production. It is probable that a significant part of the current in the final stages of the discharge is carried by runaway electrons. In the usual case, as the ohmic-heating transformer cores saturate, the current falls slowly, often in a series of small steps accompanied by X-ray bursts. Occasionally, the current and electron density both drop abruptly, with a consequent sudden rise in the heating voltage. This effect, illustrated in Fig. 12, is presumed to be some type of plasma instability, perhaps associated with runaway electrons.

At sufficiently high initial gas pressures the wall emission rate is high enough to maintain the charged-particle density in the discharge. To study wall emission effects, data similar to Fig. 11 were taken as a function of pressure. Electron density loss rates, tangents to observed decay curves at $5 \times 10^{12}$ electrons/cm$^3$, are shown in Fig. 13 for hydrogen. An extrapolation to zero pressure suggests a confinement time of the order of 100 µsec in the absence of wall feedback effects.

The current and voltage data of Fig. 10 can be used to compute plasma resistivity if an effective plasma diameter is assumed. This diameter can be estimated from the Kruskal instability current or from geometrical considerations, these being in satisfactory agreement.$^2$ For the Model B-1 stellarator the effective diameter without wolfram limiter is 3.6 cm with an estimated accuracy of 15%. One infers the electron temperature, using the theoretical resistivity relationship which assumes Maxwellian electron distributions,$^4$

$$\eta = K (\ln \Lambda) T_e^{-\frac{5}{2}}$$

the electron kinetic temperature $T_e$ is in degrees Kelvin; $\ln \Lambda$ is a slowly varying function of tempera-
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ture and density of the order of ten in our experiments; the constant \( K \) is \( 6.5 \times 10^3 \) ohm cm for hydrogen, \( 11.1 \times 10^3 \) for doubly-ionized helium). For the data of Fig. 10 the resistance is relatively constant at about 5 milliohms during the latter two-thirds of the discharge, corresponding to a temperature of about 90 electron-volts. This computation of temperature is subject to question because a significant fraction of the total current may be carried by runaway electrons, and indeed the "thermal" electrons may not have a Maxwellian velocity distribution. Furthermore, the actual current channel may be broken up or distorted by cooperative processes. Low-frequency ac conductivity measurements cannot be made during heating because fluctuations in the heating current produce too high a noise level.

Electron temperature measurements based on the relative intensities of corresponding singlet and triplet lines in the helium spectrum are inaccurate because of poor knowledge of the excitation cross-sections, the non-Maxwellian distribution, and low experimental light intensities. However, such measurements give better than order-of-magnitude agreement with resistivity data.

Ion temperatures, from Doppler line widths, are difficult to measure during heating because of the low light intensity. Under typical conditions, \( \text{He}^{11} \)

4686 Å appears cold (\(< 5 \text{ ev}\)) in the later stages of the discharge when one might expect the temperature to be a maximum. The possibility of mass motion, due to cooperative processes, and the possibility that cold gas continually entering the discharge may contribute disproportionately to the radiating ions limit the validity of these observations. Light observations are normally made transverse to the stellarator axis, an unfavorable geometry for observing volume rather than surface radiation.

Discharge Behavior after Heating

Many discharges terminate when the charged-particle density falls so low that the electron energies can no longer be moderated and all particles run away. If wall emission has been high enough that substantial density is left when the heating field ends, the density falls rapidly at first and then more slowly, with a time constant of the order of a millisecond. To retain a significant density at the heating pulse end, the field can be removed by crowbar before much gas has been lost and before the walls are heavily bombarded. Typical data are shown in Fig. 14. Again the density drops quickly at first and then more slowly. Decay time constants of the order of 6 msec have been observed under these conditions. However, low-frequency conductivity measurements made at this time indicate a completely cold plasma. A very large

![Figure 13. Electron density loss rates during heating as a function of operating pressure. Heating current, 1500 amp; confining field, 27 k-gauss.](image)

![Figure 14. Average electron density following crowbar termination of heating field. Heating field (0.2 \text{ v/cm}) applied 0.75 msec prior to crowbar; helium, 5 \times 10^{-4} \text{ mm Hg.}](image)

![Figure 15. Stepwise decay of plasma current following removal of heating field by crowbar 1.8 msec after start of oscilloscope trace. Concave curvature of baseline is instrumental; current reaches zero 12.6 msec after start of trace. Heating field, 0.3 \text{ v/cm; confining field, 27 k-gauss; helium, 4 \times 10^{-4} \text{ mm Hg.}](image)
prolonged burst of X-rays is observed some 10 msec after the discharge, when the confining field is between one-half and one-third of its peak value.

A very striking phenomenon, characteristic of the crowbarred heating-field, is that the current does not fall abruptly, but rather decays slowly in a series of abrupt "steps" and plateaus. This effect is shown in Fig. 15. The steps are accompanied by X-rays and microwave noise. For some conditions the electron density is observed to increase suddenly, by as much as a factor of two, at the time of a current step. Since the after-current would appear to consist principally of runaway electrons, this period of the discharge, like the rise of current, may be controlled largely by associated plasma instabilities. These effects are considered in greater detail in the accompanying paper.  

Comparison of Hydrogen and Helium  

The behavior of the B-1 device operated with the two atomic species is qualitatively similar. The rate of decay of the plasma density during ohmic heating is from two to five times greater in hydrogen than in helium, presumably because of the greater binding of hydrogen on the walls. The plateaus on the rise of the current are not observed in hydrogen. A further difference occurs during the radio-frequency breakdown discharge. Hydrogen is removed very rapidly during the discharge, to the extent that the ohmic heating discharge is inhibited for several milliseconds afterwards. Helium, on the other hand, shows negligible net loss of gas during the breakdown discharge. This difference may result from dissociative recombination and charge exchange processes in the feeble-ionized radio-frequency discharge. No significant differences between hydrogen and deuterium have been detected.

CONCLUSIONS  

Although many details of the operation of stellarators are yet to be explained in full, most general features of the observations described above appear to be understood. One of the most clear-cut results obtained in the operation of these devices is the effectiveness of the figure-eight geometry in providing single-particle confinement. Runaway electrons persist for many milliseconds after the heating pulse terminates, as evidenced by the late burst of X-rays which occurs when the confining field has diminished markedly. This persistence is clear evidence that the rotational transform of the stellarator produces confinement. These electrons make about $5 \times 10^6$ circuits of the loop during a time when there is negligible axial current along the discharge tube to produce any rotational transform, a phenomenon which would not be possible in a simple torus, where the drifts are of the order of one gyration radius per revolution.

The most significant measurement of plasma confinement appears to be the confinement time during ohmic heating in hydrogen, obtained from Fig. 13. The result, approximately 120 $\mu$sec at 27 k-gauss, is much shorter than the classical collision diffusion time and is nearer the value predicted by Bohm's equation, a result which may be fortuitous. The confining field dependence is not yet known. We have no knowledge of the process that is responsible for this rate of plasma disappearance from our devices. The runaway electrons or non-Maxwellian particle distributions that exist during ohmic heating may produce plasma oscillations which give rise to enhanced diffusion across lines of force. Also there exists the possibility of interchange instability or higher modes of the kink instability of Kruskal. Evaluations of the ratio of material pressure to magnetic energy density, a parameter which appears in the theory of the interchange type of hydromagnetic instability,  

yield values of the order of $10^{-5}$ for discharges under various conditions. Theoretical estimates of the critical value of this parameter for the B-1 device are not available because of the complex geometry. While the data are not inconsistent with the occurrence of an interchange type of instability, its presence cannot be assumed without some more direct evidence.

The observations on the plasma decay after the ohmic heating is terminated unfortunately give no good evidence as to the confinement of a "quiescent" plasma. The rapid fall in electron temperature after the end of the ohmic pulse is clear evidence that enough impurities have entered the plasma to catalyze recombination. However, the runaway electrons which continue to exist in the plasma during this period have enough energy content to keep the ionization process going and maintain the plasma in spite of recombination. There is some difficulty in explaining the rate of ionization, but since the transverse velocities of runaway electrons may also be large, the runaway electron flux effective in ionization could be considerably greater than that detected by axial current measurements.

The existence of runaway electrons and non-Maxwellian particle distributions makes detailed comparison with the simple theory of ohmic heating unrewarding, but the general behavior of the ionization and heating process is about as predicted. It has been shown to be possible with relatively low electric fields to ionize a gas almost completely and to heat it to maximum temperatures of the order of 100 electron-volts in spite of cooperative processes that limit the confinement time during heating and in spite of an appreciable influx of impurities from the walls.

ACKNOWLEDGEMENTS  

We wish to acknowledge the assistance of R. G. Tuckfield and H. J. Winthrop in this work.

REFERENCES  

14. See Ref. 5 above.
Runaway Electrons and Cooperative Phenomena in B-1 Stellarator Discharges

By W. Bernstein, F. F. Chen, M. A. Heald and A. Z. Kranz*

An accompanying paper has described the general characteristics of discharges in B-model stellarators with the figure-8 geometry. From the data presented, it is clear that the ohmic heating process in these machines is not entirely understood. In the belief that they would provide a useful diagnostic tool, a detailed investigation of the runaway electrons in helium discharges was begun. In addition, it was thought that the characteristics of the discharge itself might be influenced by the presence of runaway electrons.

Runaway electrons will arise when there exist an electric field and a density such that some electrons, those faster than a certain critical velocity, will gain more energy from the field than they lose by collisions. Since the collision cross section decreases with energy, these electrons then follow the lines of force around the stellarator and continue to gain energy as long as a longitudinal electric field exists. Because of the rotational transform, the maximum energy to which these electrons may be accelerated is determined by the drift across the lines of force in one of the 180° curves of the stellarator. For the B-1 device, this limiting energy is about 3 Mev at 30 kilogauss.

Direct observation of the runaway electrons themselves is exceedingly difficult, so the X-rays produced when they strike the walls have been observed. Under certain conditions X-ray emission is accompanied by microwave generation at power levels much greater than expected from thermal radiation.

This paper will describe measurements of the energy and intensity of the X-rays as functions of time during the stellarator discharges. These include X-rays appearing during the early stages of the ohmic heating pulse, and those occurring after abrupt termination of the heating field. It will be apparent that the X-ray emission pattern cannot be explained on a single particle model and that cooperative phenomena must be present.

INSTRUMENTATION

An aperture limiter, consisting of a piece of 0.05 cm thick tungsten with a 3.2 cm diameter hole centered on the magnetic axis, was placed perpendicular to this axis. This provided a known cross section for the charge and served as a high-Z target which all the runaway electrons leaving the discharge region must strike. The X-rays produced at the aperture limiter were observed at 90° to the electron beam, through a glass window with a transmission of about 50% for 30 kev X-rays.

The X-rays were detected using standard scintillation counter techniques. A Dumont 6292 photomultiplier tube and a NaI(Tl) crystal, 5 cm in diameter and 5 cm thick, served as the detector. It was placed about 5.5 m from the limiters and was shielded by 7.5 cm of lead on all sides. Collimators of various diameters were inserted in front of the crystal to adjust the counting rate, and absorbers were used to remove the low energy portion of the spectrum. The cathode follower output of the detector was fed to a non-overload linear amplifier, and the individual X-ray pulses were displayed on an oscilloscope screen.

Energy calibrations were made with radioactive sources of known energy (Cs and Co). X-ray energies were measured from the calibrated linear relationship between vertical deflection amplitude and X-ray energy.

Some coincidence measurements were made using two detectors aimed at the limiter; each was shielded by 7.5 cm of lead on all sides. The resolving time of the coincidence circuit was 0.1 μsec; accidental coincidences were evaluated by the insertion of a 0.4 μsec delay line in one channel. Tests with radioactive sources were made to insure that the resolving time was independent of the counting rate and the presence of the delay line. The resolving time was determined both by measuring random coincidences from two intense, uncorrelated radioactive sources, and by a measurement using two pulses which could be delayed with respect to each other.

Observations of microwave noise were made with a superheterodyne receiver at about 35,000 Mc. The 30 Mc intermediate-frequency amplifier bandwidth was 8 Mc; no image rejection was used. The sensitivity was sufficiently low that the receiver was unsaturated.
by the intense signal levels observed. Exploratory work was done using waveguide crystal-mount video detectors which were sensitive over broad frequency bands centered at 9,000, 35,000 and 70,000 Mc.

NORMAL DISCHARGE MEASUREMENTS

The behavior of the current as a function of the ohmic heating field at a pressure of $5 \times 10^{-4}$ mm Hg is shown in Fig. 6 of the accompanying paper. The two distinct modes of current rise typical of low and high heating fields will be called Mode A and Mode B, respectively. At low heating fields (0.06 v/cm) the current rises to an initial plateau of about 500 amp and after approximately 3 msec increases to 2200 amp (at 27 kilogauss), the Kruskal limiting (kink instability) current for a 4.1 cm diameter geometrical aperture; this is Mode A. With increasing heating field, the amplitude of this first current plateau increases, whereas its duration decreases. At heating fields higher than 0.1 v/cm, this plateau disappears, and the current rises smoothly to the Kruskal limiting value; this is Mode B. At higher pressures, higher heating fields are required to obtain similar discharge characteristics.

Typical current and X-ray emission patterns for a high heating field (Mode B) are illustrated in Fig. 1. It can be seen that X-rays are emitted during the smooth current rise to the Kruskal limiting current; in the low voltage case (Mode A), they are emitted during the first current plateau. They are not emitted while the current is at the Kruskal limiting value but again appear in the later stages of the discharge when the current decreases. This section will consider those X-rays emitted in the early stages of the discharge.

Energy Measurements

The energies of the X-rays were measured to determine whether the maximum energies were consistent with the ohmic heating fields applied. Since the X-rays observed were produced by thick target bombardment, it was felt that little significant information could be obtained with regard to the electron energy distribution; only the maximum energies observed had significance. The counting rates were reduced by collimation and absorption. Only those X-ray pulses which were undistorted and clearly separated from adjoining ones were used to obtain the energy distribution. Figure 2 shows a histogram of the number of X-rays versus energy for an applied heating field of 0.12 v/cm, a confining field of 26.5 kilogauss, and a pressure of $4 \times 10^{-4}$ mm Hg; the X-rays were counted during the interval between 240 and 260 $\mu$sec after the heating field was applied. At this time the current (Mode B) had risen smoothly to a point just below its maximum value. The peaked distribution resulted from the absorbers used to discriminate against the low energy end of the X-ray distribution. If the measured heating field is integrated over 260 $\mu$sec, the maximum possible energy of a single electron is calculated to be 420 kev, in agreement with the observed maximum energy. It is thus probable that some electrons were confined and accelerated from the time the heating field was first applied.

Figure 3 shows a similar histogram obtained under the same conditions, for a lower heating field, 0.068 v/cm (Mode A). These X-rays were emitted at the end of the first current plateau, between 840 and 860 $\mu$sec after the heating field was first applied. The maximum predicted energy is 1.35 Mev; however, considerably higher energies were observed. Since the counting rates were quite low, and only undistorted X-ray pulses were analysed, the observed intensity of high amplitude pulses cannot be attributed to random pile-up. If correlated emission of X-rays occurred,
caused by a group of runaway electrons striking the limiter in a time interval short compared with the 0.2 μsec rise time of the detector, the detector would not be able to distinguish the individual events but would sum them to give a spuriously high amplitude.

To test for this correlated emission of X-rays, the coincidence arrangement described previously was used. The ratio of real to accidental coincidences, using the delay line insertion technique, was found to be 2.9 ± 0.2. Thus, correlated emission of X-rays from the limiter, in time intervals less than 0.1 μsec, the resolving time of the coincidence circuit, does occur; this is evidence that discrete groups of runaway electrons strike the limiter. However, the possibility of unexpectedly high electron energies, perhaps due to in-phase acceleration by local electric fields, cannot be ruled out. Further experiments are required to clarify this point. Both correlated emission and anomalously energetic electrons are indicative of the occurrence of cooperative phenomena.

**Time Measurement**

The emission of X-rays during the rise of the current begins rather abruptly. It appears, for two reasons, that the abrupt start is due to some sort of internal disturbance in the discharge which suddenly begins to carry runaway electrons to the limiter. First, the abrupt start cannot be caused by electrons first reaching the detector threshold; the latter is about 30 kev, whereas energies over 100 kev are possible at the beginning of emission if acceleration began when the heating field was first applied. Moreover, as the magnetic field is increased with the heating field kept constant, the emission begins later, when the energies are presumably higher; hence it is not a matter of detector threshold. Second, the abrupt start cannot be caused by electrons first reaching too high an energy to be confined by the magnetic field, because electrons over 2 Mev have been observed to be confined by a 26 kilogauss field. At the time of emission the electrons have had time to be accelerated to at most 300 kev; only a small fraction of the runaway electrons near the edge of the discharge can lose confinement at that energy, and at any rate one would expect the onset of emission to be more gradual.

To see whether the X-ray emission is related to a current-dependent instability, the value of plasma current at the onset of X-ray emission was plotted. It is shown in Fig. 4 as a function of heating field, for several values of confining field, at a pressure of 1.5 × 10⁻³ mm Hg. Figure 5 shows similar plots, under the same operating conditions, for the plasma current at the end of the X-ray emission period. It can be seen that the behavior of the two sets of data is very similar. For both cases at the higher heating fields (Mode B), these currents, plotted as functions of heating field, appear constant at values which depend on con-
fining field. At the lower heating fields (Mode A), the behavior is entirely different; there is now a marked dependence of these currents on heating field, but little dependence on confining field. The X-ray emission starts during and ceases at the end of the first current plateau; thus there is no significant difference in the magnitude of the currents at the beginning and end of X-ray emission. The first current plateau magnitude seems to be independent of confining field.

In this stage of the discharge, intense microwave generation, in the frequency range 10,000–70,000 Mc, has been observed. To date, detailed measurements have been restricted to the 35,000 ± 2,000 Mc range. The power levels of the received noise are of the order of 0.1 watt. The microwave generation generally occurs during the time of X-ray emission but may also occur considerably earlier, when the mean electron density first approaches the value (1.5 x 10^{13} cm^{-3}) at which a 35,000 Mc beam is no longer transmitted unimpeded by the plasma because of plasma resonance. Under certain conditions no microwave generation is observed during X-ray emission, possibly because the frequency of the generation has shifted entirely out of the response band of the receiver.

In Fig. 6, plasma currents corresponding to the beginning and end of X-ray emission (Mode B), taken from the flat regions of the preceding figures, are plotted against confining field for two pressures. A family of curves through the origin of coordinates is obtained; some unexplained pressure dependence is observed only for the currents at the end of X-ray emission. The magnitudes of the currents shown at the end of X-ray emission (neglecting pressure) are in good agreement with the magnitudes of the Kruskal limiting currents calculated^5 for an assumed mean aperture of 7 cm^2, as shown. From these data, it appears that there is a relationship between the X-ray pattern and the Kruskal instability. Electrons are not expected to be confined and accelerated to sufficiently high energies to produce observable X-rays during the time the Kruskal instability dominates the discharge; the existing runaway electrons probably are not confined after the onset of the instability. Thus X-ray emission is not expected for the duration of the instability, in agreement with the experimental observations.

There are, however, several significant arguments against a causal relationship between a Kruskal instability and the X-ray emission pattern. At the low heating fields, the currents observed at the beginning and end of X-ray emission are far below those predicted, from the theory,^6 for this instability to develop. In fact, the absence of any confining field dependence in the observed currents is a strong refutation of the existence of any hydromagnetic instability. The absence of a relationship between the Kruskal instability and the X-ray pattern is less clear at high heating fields. If one assumes that the instability causes the start of X-ray emission, then it is difficult to explain the long duration of emission (50–200 µsec). If the Kruskal instability ended the X-ray emission, a pronounced increase in the intensity of the emitted radiation would be expected at the end of the emission period, since all the remaining runaway electrons would lose confinement at that time. However, the X-ray intensity shows no such peak, as shown in Fig. 7. (These data are not corrected for the transmission through the absorbers used.) This time distribution implies that all the runaway electrons are removed from the discharge before the Kruskal limiting current is attained; perhaps this is a necessary condition for reaching the Kruskal limit. The microwave generation pattern occurs at both the high and low heating fields and suggests that the same type of process perhaps occurs in both cases.
CROWBAR DISCHARGE MEASUREMENTS

Significant information about the relation of runaway electrons to plasma characteristics may be obtained by the abrupt removal (crowbar) of the heating field during the discharge. In particular, both fast heating (constant voltage) and slow heating (constant current) cases have been examined using this technique. In many respects the results are similar.

Typical X-ray, voltage, and current patterns obtained when the voltage is crowbarred just at the beginning of X-ray emission are compared with normal patterns in Fig. 8; the operating conditions are high constant-voltage heating field (0.28 V/cm), high confining field (26.5 kilogauss), and a pressure of $1 \times 10^{-3}$ mm Hg. In the normal case, the current rises smoothly to the Kruskal limiting value; the voltage is essentially constant. The burst of X-rays begins at about 60 /usec after the heating field is first applied and terminates when the current reaches the Kruskal limiting value.

In the crowbarred case, the current rises normally until the time of crowbar (60 /usec), and then decays slowly to a low residual value below 500 amp. The measured voltage is normal until the time of crowbar; the slow decay thereafter is the $L(dI/dt)$ voltage produced by the current decay. The crowbarred X-ray pattern shows the beginning of X-ray emission (again at 60 /usec); here emission continues for several milliseconds. If the heating field is crowbarred after the current has reached the Kruskal limiting value, the current decays rapidly to zero; no later X-rays are emitted. In general, the later the crowbar time during the initial X-ray emission period, the lower the total X-ray intensity afterwards; the microwave generation ceases at approximately the time of crowbar.

It is apparent that many more X-rays were emitted after the heating field was crowbarred than were emitted in toto in the normal case. Using low counting rates and fast oscilloscope sweep speeds, a comparison was made of the number of X-rays emitted during the entire emission time of a normal pulse, and the number emitted during the time interval 80 to 530 /usec after crowbar; the heating field was crowbarred 60 /usec after it was applied. The ratios were obtained using the following absorbers: 0.3 cm Pb, 0.05 cm Cd, and 0.075 cm W. The end of the counting period in the crowbar case was chosen so that the time-integrated heating field available from the time of crowbar to the end of the counting period was the same as that which is available, in the normal case, from 60 /usec to the end of the emission period. This is verified to some degree since the maximum energies observed were about 200 kev in both cases. The number and energy distribution of the runaway electrons must have been the same at the time of crowbar in the two cases. Further, an electron at thermal energy at 60 /usec after the heating field was first applied could be accelerated to a maximum energy of only 45 kev by the heating field available after that time, in either case, and these would not be transmitted through the absorbers used. The difference in number of X-rays in the two cases can be due only to a difference in the subsequent behavior of the runaway electrons originally present at 60 /usec.

The ratio of total counts in the normal case to the number counted in the 450 /usec interval in the crowbar case was 0.2, with an estimated error of a factor of 2. This ratio was the same with each absorber.

From these data it seems clear that more X-rays are produced in the crowbarred discharge than in the normal discharge. This implies that, in the normal case, more runaway electrons were lost from the discharge without producing detectable X-rays than in the crowbar case. This would be so if, in the normal case, most of the electrons struck the limiter shortly after the beginning of emission, at which time they would have lower energies than in the crowbarred case and would be detected less efficiently. An alternative explanation, for the normal case, is that runaway electrons are reabsorbed into the plasma in non-radiative interactions; more striking evidence that this phenomenon can occur will be presented in a later section. At present the observations are not sufficient to substantiate either of these two possibilities. However, the mere fact that runaway electrons strike the limiter in abundance after crowbar indicates that even in this situation cooperative phenomena are taking particles to the wall.

CURRENT STEPS

Detailed observations of the residual current after crowbar and associated phenomena have been made. Figure 9 shows the behavior of the current, voltage, X-rays, electron density, and He\(^{4}\) 4921 Å light before and after the (constant-current) heating field is removed. The plasma current is seen to decay to a low constant value and to decay further, abruptly, at about 8 msec. The current is actually constant between steps; the apparent slope seen in the figure is instrumental. Patterns vary from one to several steps per pulse; for a given set of operating conditions the general behavior is reproducible, although the exact time of the steps may not be. X-rays are emitted prior to crowbar; a pronounced peak coincides with the

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**Figure 8.** Oscilloscope traces of plasma current, heating field, and X-rays, for the normal and crowbarred cases.

Sweep speed, 20 /usec/div; confining field, 26.5 kilogauss; pressure, $1 \times 10^{-3}$ mm Hg.

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**Figure 9.** Oscilloscope traces of plasma current, heating field, X-rays, electron density, and He\(^{4}\) 4921 Å light before and after the (constant-current) heating field is removed. The plasma current is seen to decay to a low constant value and to decay further, abruptly, at about 8 msec. The current is actually constant between steps; the apparent slope seen in the figure is instrumental. Patterns vary from one to several steps per pulse; for a given set of operating conditions the general behavior is reproducible, although the exact time of the steps may not be. X-rays are emitted prior to crowbar; a pronounced peak coincides with the...
current step. Some X-rays are emitted continuously, although the rate is low. The electron density shows a steady decrease after crowbar; at the current step, the density increases by almost a factor of two and subsequently decreases again. A peak appears in the He$^1$ 4921 Å light at the time of the current step.

Another pattern of residual current decay after crowbar has also been observed. Here, each current decrease consists of a rapid succession of three or four small steps; the sum of these is about equal to the single step in the previous case. Each of the small steps is accompanied by much more intense X-radiation than accompanied the large step; at the small steps there is no evidence for an electron density increase or excitation of neutral helium. Usually, only one kind of step occurs for a given operating condition depending on the duration of the discharge prior to crowbar. For short discharge durations, less than 0.5 msec, the single step pattern illustrated in Fig. 9 occurs; for longer times the multiple step pattern results.

There was some question as to whether this residual current could be attributed entirely to a circulating beam of runaway electrons. Unfortunately, because of the low-energy cut-off of the X-ray detector, the number of low energy runaway electrons (below, say 50 kev) is unknown; and therefore one cannot obtain an upper limit to the circulating current which would be consistent with the observed X-ray flux. However, a very approximate lower limit can be obtained in the following manner. The X-rays emitted at 90° to the target during a current step were counted on a fast oscilloscope sweep. This was done when the second step pattern (multiple small steps with intense X-ray emission and no density increase) was present; the magnitude of the small step selected was 4 amp. The detector described previously was used. In addition, 3.8 cm Pb was used as absorber to decrease the counting rate until individual pulses were resolved; only X-rays above 400 kev, therefore, were observed. These were all assumed to be produced by 1.4 Mev electrons. This is a reasonable assumption in computing a lower limit to the current consistent with the observed X-ray flux, since any electrons with less energy than 1 Mev would yield fewer X-rays but would contribute almost as much to the circulating current. Furthermore, few electrons above 1.4 Mev can exist because no X-rays above 1 Mev were observed. The bremsstrahlung yield was computed using the data of Miller, Motz, and Cialella, who measured the X-ray spectrum emitted at 90° from a tungsten target by 0.5 ma of 1.4 Mev electrons. These data were corrected for the difference in absorbers used in the two experiments; this correction was rather inaccurate because a large thickness of absorber was used in this experiment and because only narrow-beam X-ray absorption coefficients were used. The corrected spectrum was integrated over energy to obtain the number of photons passing through the Pb absorber per electron striking the limiter. Finally, after correcting for solid angle and detector efficiency (70%) we obtained, as a lower limit to the runaway electron current necessary to produce the observed average number (23) of X-rays during a step, the figure of 0.2 amp, as compared with the observed step current of 4 amp. Since the measurement was limited to X-rays above 400 kev, this very approximate lower limit is entirely consistent with the picture that the current steps after crowbar are caused by the sudden termination of confinement of a group of runaway electrons; at least in the case of the second step pattern, most of these strike the limiter and produce X-rays.

Both kinds of current steps give evidence for cooperative phenomena. In the case of the multiple steps, a group of runaway electrons apparently lose confinement and are abruptly led out of the discharge to the limiter. In the case of the single step, where reionization apparently occurs, it is possible that a cooperative process decelerates the runaway electrons, and they are abruptly reabsorbed into the main body of the plasma. Their energy raises the temperature of the plasma electrons and causes ionization and excitation. Only a few runaway electrons strike the limiter in this case. Although this mechanism is not an expected one, it would be difficult to account for the density and light increase at the time of the step in any other manner; ionization of neutral gas by the
runaway electrons may account in part for the slow decay of electron density after crowbar, but it cannot account for an abrupt increase in density.

CONCLUSIONS

It is obvious that much of the data cannot be explained on the basis of currently understood single particle models or macroscopic plasma physics, and that these data present further evidence for the existence of unexplained collective processes previously referred to as cooperative phenomena. It is possible that many of the inconsistencies observed during ohmic heating, referred to in the accompanying paper¹ may be a result of these cooperative phenomena.

Two major conclusions may be drawn from this work. First, runaway electrons indicate the existence of instabilities at currents well below the Kruskal limiting current. Second, there is evidence that the runaway electrons themselves may affect the development of the discharge. In particular, there have been two indications that energy, at first given to the runaway electrons, can later be re-absorbed into the plasma. It is possible that in addition to the transfer of energy to the plasma electrons, as shown by the increase of ionization and excitation at the current steps, energy may also be transferred to the ions, perhaps causing anomalously fast ion heating. The observed intense microwave generation may have significance in the general field of radio astronomy in that it may help clarify the mechanism producing microwave radiation from the galaxy.

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Hydromagnetic Instability in a Stellarator

By M. D. Kruskal,* J. L. Johnson,† M. B. Gottlieb* and L. M. Goldman;‡

Kruskal and Tuck1 (in a paper hereafter referred to as KT) have examined the influence of a longitudinal magnetic field on the instabilities of the pinch effect. The pinch effect is the confinement of a thin column of plasma by means of the magnetic field due to a high current discharge along the column. Instabilities in the form of lateral "buckling" of the column (in the absence of a longitudinal field) have been predicted theoretically and are well known experimentally.

In KT it was noted that when there is a uniform externally imposed longitudinal field much larger than the field of the discharge current, one should expect instabilities in the form of a lateral displacement of the plasma column into a helix of large pitch. At the wavelength of fastest growth the -folding time approximates the time it takes a sound wave in the plasma to traverse the radius of the plasma column. In successive sections we (a) re-examine this problem under the conditions which might be expected to occur in the stellarator during ohmic heating, including the presence of external conductors; (b) apply this theory to the stellarator; (c) show that the external conductors are in fact unimportant; (d) discuss the important effects due to the finite length of the machine; (e) consider the effects of more general current distributions; and (f) give the relevant experimental results.

It should be emphasized that the considerations of this paper apply only to stellarators in which the rotational transform results from the large scale geometry of the tube (such as a Figure 8 shape) rather than from small local perturbation coils (such as helical windings). It is perhaps worth noting that the theoretical results of the following four sections are given in less condensed form elsewhere; the appearance of instability and the dependence of the critical current, both on the confining field and on the direction of the plasma current, were predicted in this earlier work well in advance of the experimental confirmation.

INFINITE CYLINDER THEORY

We start with the analysis of pinch instability under the conditions considered in KT, but now additionally taking into account the effect of a thin cylindrical sheet conductor coaxial with the plasma. Familiarity with KT is assumed.

The material pressure, density and velocity of the plasma are denoted by \( \rho_p, \rho, \) and \( v, \) the magnetic and electric fields by \( B \) and \( E, \) the current and charge densities by \( j \) and \( \epsilon, \) the permeability and permittivity of space by \( \mu_0 \) and \( \epsilon_0, \) and the ratio of specific heats by \( \gamma. \) (We employ MKS units throughout.) The equations we use for the interior of the plasma (treated as infinitely conductive) are Eqs. (1) through (8) of KT.

At an interface between plasma and vacuum, \( n \) denotes the unit normal to the surface, directed into the plasma; \( \mu, \) the normal velocity of the surface; \( j^* \) and \( \epsilon^*, \) the surface current density and surface charge density; \( \text{brackets}, \) the jump in the enclosed quantity upon crossing the surface from the vacuum into the plasma; a \( \text{bar} \) under a quantity, the arithmetic mean of the values of that quantity on each side of the surface; and subscripts \( P \) and \( V, \) respective values on the plasma and vacuum sides of the interface. The equations we use at the interface are Eqs. (9) through (14) of KT.

Suppose we have a sheet of solid material of small thickness, \( \delta, \) fixed in space with vacuum on both sides. Let \( \sigma \) be the volume conductivity of the material and \( \tau \) a characteristic time for the phenomena to be considered. If \( \delta \) is much less than the penetration distance \( \left( \tau/\mu_0 \sigma \right)^{1/2} \) of the material, the thickness may be disregarded and the sheet treated as a surface of surface conductivity \( \sigma^* = \sigma^* \). With the same notation as at an interface, our equations at the sheet are then

\[
\begin{align*}
\mathbf{n} \times [\mathbf{B}] &= \mu_0 \mathbf{j}^*, \quad (1) \\
\mathbf{n} \cdot [\mathbf{B}] &= 0, \quad (2) \\
\mathbf{n} \times [\mathbf{E}] &= 0, \quad (3) \\
\mathbf{n} \cdot [\mathbf{E}] &= \epsilon^*/\epsilon_0, \quad (4) \\
\mathbf{E} - \mathbf{n} \cdot \mathbf{E} &= \mathbf{j}^*/\sigma^* \quad (5)
\end{align*}
\]

We use cylindrical coordinates \( r, \theta, z. \) Consider the following situation (Fig. 1). Inside the infinite cylinder \( r = r_0 \) we have a uniform plasma with \( \rho = \rho_0, \rho = \rho, \) \( v = 0, B_r = B_0 = 0, B_\theta = B_\theta, E = 0, j = 0, \) \( \epsilon = 0. \) Outside the cylinder \( r = r_0 \) is a vacuum in which \( B_r = 0, B_\theta = B_0/\rho_0 r, B_z = B_\theta, E = 0. \) On the cylindrical interface \( r = r_0 \) we have \( j_r^* = 0, j_\theta^* = j_\theta^*, \)
and hydromagnetic waves, respectively. The general
situation (time-independent solution) if the constants $r_0$, $r_1$, $p_0$, $p_0$, $B_0$, $B_1$, $B_2$, $j_0$, $j_1$, and $s_0$ satisfy

$$
B_0 = \mu_0 j_0, \quad B_1 - B_2 = \mu_0 j_1,
B_0^2 + B_1^2 - B_2^2 = 2\mu_0 p_0.
$$  

We now seek solutions of our equations which are close to the equilibrium solution just described. We suppose that every physical quantity is equal to its equilibrium value plus a small perturbation term. We consider all our equations as equations for these perturbation quantities and linearize them in the usual way. We obtain equations for the seven coefficient constants. The conditions that these equations have a non-trivial solution (i.e., that the determinant of their coefficients vanishes) lead to the characteristic equation, which must be satisfied by the characteristic constants of any solution. Thus we have seven linear homogeneous algebraic equations, linear and homogeneous, with $r$, $s$, and $t$ as independent variables. The coefficients are obviously independent of $r$, $s$, and $t$. Any solution of the equations may therefore be obtained as a superposition of elementary solutions, an elementary solution being one in which each dependent variable is a function of $r$ only (or, in the case of sheet quantities, a constant) multiplied by $\exp(im \theta + ikz + \omega t)$, where $m$, $k$, and $\omega$ are constants, the characteristic constants of the elementary solution. We may therefore restrict ourselves to a search for the elementary solutions. To make physical sense we must require that $m$ be an integer and that $k$ be real. Without loss of generality we may assume that $m$ is non-negative.

We next change our notation, each symbol which originally denoted a physical quantity now denoting the (approximate) characteristic equation is

$$
c^2 = 1/\mu_0 \nu, \quad s^2 = \gamma \beta_0 / \rho_0, \quad h^2 = B_0^2 / \mu_0 \rho_0
$$

$$
\xi_0^2 = \xi_0^2 + \frac{\omega^2}{c^2}, \quad \eta_0^2 = \eta_0^2 + \frac{\omega^2}{c^2} + \frac{\omega^2}{k^2},
\zeta_0^2 = \zeta_0^2 + \frac{\omega^2}{c^2} + \frac{\omega^2}{h^2}, \quad \eta_0^2 = \eta_0^2 + \frac{\omega^2}{c^2},
$$

where $c$, $s$, and $h$ are the velocities of light, sound, and hydromagnetic waves, respectively. The general regular solution of the equations for the plasma is given by Eqs. (19) of KT in terms of an arbitrary constant $p_1$. ($f_m$ is the $m$th order Bessel function of first kind and $f_m^-$ its derivative with respect to its argument; $f_m$ and $f_m^-$ are here always evaluated for the argument $i \nu/\nu_0\xi$.)

The equations for the vacuum are (4) through (7) of KT with $\xi = 0$ and $\nu = 0$. For the region $r > r_1$ (outside the fixed conductor) the general regular solution is given by Eqs. (23) of KT in terms of arbitrary constants $B_1$ and $E_1$. ($H_m$ is the $m$th order Bessel function of first kind and $H_m^r$ its derivative with respect to its argument; $H_m^l$ and $H_m^r$ are here always evaluated for the argument $i \nu/\nu_0\xi$.)

In the general solution for the vacuum region, $r_0 < r < r_1$ (between the plasma and the fixed conductor), we have each magnetic and electric field component given as a sum of two expressions, one the same as in Eqs. (23) of KT except for having $B_1$ and $E_1$ replaced by new constants $B_2$ and $E_2$, and the other again the same except that $B_2$ and $E_2$ are replaced by new constants $B_0$ and $E_0$ and at the same time $H_m$ and $H_m^r$ are replaced by $f_m$ and $f_m^-$ (both evaluated for the argument $i \nu/\nu_0\xi$).

We now have the solution everywhere expressed in terms of the seven so far arbitrary constants, $p_1$, $B_1$, $E_1$, $B_2$, $E_2$, $B_0$, and $E_0$. We obtain relations between these from the interface and boundary conditions. From Conditions (9) through (14) of KT we obtain (only) three independent relations between $p_1$, $B_1$, and $E_1$. From Conditions (1) through (5) we obtain four independent relations between all the constants except $p_1$. Thus we have seven linear homogeneous equations for the seven coefficient constants. The condition that these equations have a non-trivial solution (i.e., that the determinant of their coefficients vanishes) leads to the characteristic equation, which must be satisfied by the characteristic constants of any elementary solution.

We now make the approximation of infinite light velocity by taking $\nu_0 = 0$. We are interested only in unstable solutions, i.e., solutions for which $\omega$ has a positive real part, and we assume that $\omega$ is real. It can be proved (at least for $c^2$ either zero or infinite) that this is no restriction, i.e., that all unstable modes have $\omega$ real. Introducing the dimensionless constants $\alpha_\nu = B_\nu / B_0$, $\alpha_\nu = B_\nu / B_0$ and the functions

$$
W = (ij_0 / 2p_0) \nu, \quad \Sigma = (2p_0 / p_0) \nu_0\nu^2
$$

$$
K_m(y) = \frac{j_m(iy)}{yj_m(iy)}, \quad L_m(y) = \frac{H_m(iy)}{yH_m(iy)},
$$

$$
M_m(y_0, y) = \frac{H_m(iy_0)}{yH_m(iy_0)} \frac{j_m(iy)}{yj_m(iy)}
$$

the (approximate) characteristic equation is

$$
\left[ \frac{K_m(y_0, y)[K_m(y_0) - L_m(y_0)]}{[y^2 + m^2][2W][K_m(y_0) - L_m(y_2)]} \right]
\times \left\{ \frac{L_m(y) - M_m(y_0, y_2)[K_m(y_0) - L_m(y_2)]}{[y^2 + m^2][2W][K_m(y_0) - L_m(y_2)]} \right\},
\text{(9)}
$$

Figure 1. Equilibrium configuration
where the plus or minus sign is to be chosen according to whether \( k \) is positive or negative, and where

\[
y_0 = |k| r_0, \quad y_1 = |k|r_1.
\]

\[
x = \left[ \frac{(y_0^2 + (2/y) W^2)}{\alpha y_0^2 + (1 + \alpha \gamma - \alpha^2) y_0^2 + 1} \right] \quad (10)
\]

Numerically, \( f_m(y) \) and \( f_m^*(y) \) are monotonically increasing functions of \( y \), and \( H_m(y) \) and \( H_m^*(y) \) are monotonically decreasing functions of \( y \). Since \( y_0 < y_1 \) it follows that \( 0 < M_m(y_0, y_1) < 1 \). Also, \( K_m(y) > 0 \) and \( L_m(y) < 0 \). We thus see that the second term in the braces on the right-hand side of Eq. (9), which term we shall denote by \( U \), is negative, as is the first term \( L_m(y_0) \). The left-hand side of Eq. (9) is a monotonically increasing function of \( W \), at least for \( W \) very small, very large, and in the neighborhood of its largest value satisfying Eq. (9), and very likely for all values. In any case, it can be proved easily that the largest value of \( W \) for which the left-hand side of Eq. (9) has a prescribed value is a monotonically non-decreasing function of that prescribed value. Now characteristic Eq. (9) differs from the corresponding equation for the same equilibrium situation without the conducting sheet at \( \tau = r_1 \) (namely Eq. (30) of KT) only in the presence of the term \( U \). It follows, consequently, that the presence of the conducting sheet has, quite generally, the effect of diminishing the rate of instability. As was to be expected, \( U \to 0 \) as \( \Sigma \to 0 \) or as \( r_1 \to \infty \). In the latter case, \( U \to 0 \) very quickly since both functions \( H_m^*(y_1) \) and \( 1/f_m^*(y_1) \) go to zero exponentially.

As pointed out earlier, Eqs. (1) through (5) are valid if the thickness \( \delta \) of the shell is much less than the penetration distance, \((\tau/\mu_0)\delta\), of the shell material, \( \tau \) being a characteristic time for the phenomena under consideration. With the shell in the form of a cylinder of inner radius \( r_1 \), we can, under rough approximations, determine corresponding equations for the opposite limiting case when \( \delta \gg (\tau/\mu_0)\delta \). We take for granted that \((\tau/\mu_0)\delta \ll r_1 \). We do not know \textit{a priori} the distribution of induced eddy currents in the shell, but we assume, for the sake of having something definite to compute, that it is purely in the \( \theta \)-direction. This current distribution turns out to have a characteristic decay time of about \( 1/|r_1(\tau/\mu_0)| \) \( \gg \tau \). Therefore there is no appreciable decay during lengths of time of interest, and the shell may be treated as a perfectly conducting sheet, with a radius perhaps exceeding \( r_1 \) by something of the order of the penetration distance. Thus, with appropriate values of \( r_1 \) and \( \sigma^* \), Eqs. (1) through (5) may still be considered to hold. The validity of this argument is of course questionable because of the arbitrariness in the choice of the current distribution, but in any case the stabilizing action of the shell must be less than it would be for a perfect conductor, which can be treated as a sheet at radius \( r_1 \).

\( Y = |a| y_0, \quad a = r_1/r_0 \). (12)

If \( \Sigma \) is finite, Eq. (11) has a positive real solution \( W \) only for \( 0 < Y < 1 \); for \( Y > 0 \) we find \( W \approx 2aY/\Sigma \), while for \( Y \to 1 \) we find \( W \approx \Sigma/2(1-Y)/1 \). If \( \Sigma = \infty \), it has a positive real solution only for \( \sigma^* \ll Y < 1 \); for \( Y \to \infty \) we find \( W \approx 2Y/\sigma^* \), while for \( Y \to 0 \) we have \( W \approx \Sigma(1-Y)/2 \). In any case, the maximum value of \( W \) and the value of \( Y \) for which it is attained satisfy, Eqs. (11) and (13):

\[ 2Y = -2(Y-1)(1+2\Sigma W/|\Sigma W(a^2-1)+2a|), \] (13)

which is obtained from Eq. (11) by partial differentiation with respect to \( Y \). From Eqs. (11) and (13) we find that

\[ Y = 1 - W^2 \] (14)

and that \( W \) is determined by

\[ W^2(1+\Sigma W/|\Sigma W(a^2-1)+2a|) = \frac{1}{2} \] (15)

UNIMPORTANCE OF EXTERNAL CONDUCTOR

In the absence of the conducting sheet (i.e., for \( a = \infty \) or \( \Sigma = 0 \)) Eq. (15) gives \( W = 1/\sqrt{2} \) or

\[ \omega = (1/r_0)(p_0/p_r)^{1/2} \] (16)
for the maximum rate of instability. In a stellarator we might have a tube of, say, helium plasma of about 2 cm radius. If the ions and electrons were both at temperature \( T \) in degrees Kelvin, Eq. (16) would become

\[
\omega = 3.12 \times 10^4 T^3 \text{sec}^{-1}.
\]  

(17)

Since the time scale for operation of the stellarator is of the order of milliseconds, we see that for \( T = 10^4 \) the instability would grow extremely fast. Even for \( T = 10^4 \) an instability would be serious if its \( W \) were larger than \( 10^{-3} \).

If the ions and electrons were both at \( t \) cm radius. If the ions and electrons were both at temperature \( T \) in degrees Kelvin, Eq. (16) would become

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For the conducting sheet to have the effect of reducing the maximum \( W \) to a very small value, we see from Eq. (15) that it is necessary both for \( a \) to be nearly equal to unity and for \( \Sigma \) to be large. Specifically, it is necessary to have

\[
a - 1 \leq W^3, \quad \Sigma \geq W^{-3}. \tag{18}
\]

For the stellarator, this means that a conducting shell which is to slow up the instability enough to do any good must in the first place be extremely close to the plasma \( (r_1 - r_0 \leq 2 \times 10^{-6} \text{ cm for } T = 10^4) \). This immediately excludes all conductors except those virtually in contact with the plasma, such as stainless steel tubing. The sheet conductivities of such conductors are unlikely greatly to exceed several hundred mhos, which at \( T = 10^4 \) would have to be about \( 10^9 \) to do any good; at higher temperatures the comparison is even less favorable.

Indeed, it would apparently be hopeless to slow up the instability sufficiently by external conductors even if they were designed for that purpose and it were not necessary to worry about inimical effects they might have on the normal operation of the stellarator. For instance, if one had thick walls of silver \( (\sigma = 6 \times 10^7 \text{ mho/meter}) \) arbitrarily close to the plasma, the silver could be treated as a perfectly conducting sheet at a radius greater than the plasma radius \( r_0 \) by approximately the penetration distance of the silver, and computation shows that, even for a plasma temperature as low as \( 0.3^\circ \text{K} \), the instability would then e-fold in a millisecond.

**PERIODICITY CONDITION**

Now that we have seen that external conductors have a negligible effect on the instability, we turn to an examination of the restrictions on perturbations imposed by the geometry of the stellarator. We wish to treat the stellarator tube as if it were straightened out to a right circular cylinder. Put another way, we wish to define coordinates \( r, \theta, z \) in the tube which locally are approximately cylindrical coordinates and in terms of which the \( (\text{inner}) \) surface of the tube is approximately the surface \( r = r_0 \). It is natural to choose the curve \( r = 0 \) to be the magnetic axis of the stellarator (i.e., the magnetic line of force which closes upon itself after one traversal of the length of the tube); \( r, \theta \) to be polar coordinates in each cross section of the tube; \( z \) to be constant on each cross section; and \( z \) to be the arc length along the curve \( z = 0 \) (with the sign of \( dz \) chosen so as to make \( r, \theta, z \) a right-handed coordinate system). It remains only to determine the relative rotation of the polar coordinates in different cross sections, i.e., to determine the direction \( \theta = 0 \) (say) in each cross section. Choosing an arbitrary vector at \( r = 0 \) lying in one cross section to give the direction \( \theta = 0 \) there, we consider a parallel vector at the point \( r = 0 \) of a neighboring cross section. This parallel vector does not in general lie in the neighboring cross section, but we may choose its projection thereon as the direction \( \theta = 0 \). In this way the direction \( \theta = 0 \) may be carried successively around the length of the stellarator. That this is the natural method of relating the values of \( \theta \) in different cross sections may be seen in several ways. One way is to observe that what we have done is equivalent to requiring that the coordinates \( r \) and \( \theta \) be invariant when the \( z \) cross section is projected onto the \( z + dz \) cross section in the direction of the magnetic axis; the resultant values of \( r \) and \( \theta \) in the \( z + dz \) cross section do not exactly constitute polar coordinates, but the deviation is of the order \( dz^2 \) and is therefore negligible. Another way is to observe that the lines of force of the confining magnetic field approximate to curves of constant \( r \) and \( \theta \).

The "cylindrical" coordinates we have defined in the tube are not single-valued functions of position (except for \( r \), which is the distance from the magnetic axis). If we follow the values of \( \theta \) and \( z \) along a closed curve which goes once around the length of the stellarator in the direction of positive \( dz \) we find that when we have returned to the starting point, \( z \) has increased by the length \( L \) of the magnetic axis and \( \theta \) has increased by a definite angle \( \phi \), depending only on the geometry of the stellarator (and not at all on the starting point or the particular curve chosen). This is called the rotational transform angle.

It can be shown by standard methods of the differential geometry of space curves that \( \phi \) is equal to the integral of the torsion of the magnetic axis with respect to its arc length, once around the stellarator. (The torsion of a curve is the negative of the rate of rotation, with respect to arc length, of the osculating plane, i.e., of the plane determined by the tangent and the radius of curvature. The positive direction of rotation is determined by the right-hand rule from the direction along the curve in which the arc length is taken as increasing.) For stellarators of twisted figure-8 shape, \(^4\) let \( \phi \) be the angle through which each end of a plane Figure 8 must be rotated to arrive at that shape, the positive direction of rotation for each end being clockwise as seen from beyond that end. It is then easily seen that

\[
\phi = -4\phi. \tag{19}
\]

Now \( (r, \theta, z) \) and \( (r, \theta + \phi, z + L) \) represent the same point in the tube, so in our perturbation theory we can allow only elementary solutions for which \( m+L \) is an integral multiple of \( 2\pi \). We recall that the unstable
perturbations we are concerned with have \( m = 1 \), \( \delta a < 0 \), \( 0 < \mathbf{Y} < 1 \). Since \( \mathbf{Y} = |\mathbf{a}|r_{0} \), we see that there is one such allowable perturbation for each (positive or negative) integer \( h \) satisfying
\[
0 < (|\mathbf{a}|L)(m + 2h) < 1. \tag{20}
\]
Thus the condition that no instability be allowable is that
\[
a(m + 2h) \geq L/r_{0}, \tag{21}
\]
where \( h \) is an integer which gives the left-hand side of Eq. (21) its smallest positive value.

It is clear that the stability criterion (21) depends not only on the magnitude of \( a \) but also upon its sign, unless \( s \) happens to be an integral multiple of \( \pi \). We note that \( a \) is positive or negative accordingly as the induced longitudinal plasma heating current has the same or the opposite direction as the longitudinal confining magnetic field.

Condition (21) is more conveniently expressed, for application, in terms of the plasma current \( I = 2\pi r_{0}a^{*} = 2\pi r_{0}B_{y}/\mu_{0} \). Since \( a = B_{y}/B_{0} \), Eq. (21) may be written
\[
(B_{y}/I)(m + 2h) \geq \mu_{0}L/2\pi r_{0}^{2}. \tag{22}
\]

**CURRENT DISTRIBUTION EFFECTS**

Some longitudinal current distributions are more general than the purely surface current case first considered above and are treated elsewhere\(^6\) by means of the energy principle.\(^6\) We quote the results without the negligible complications of an external conductor. The fluid pressure \( p \) is taken to be zero, \( B_{y} \) is again taken to be much larger than \( B_{0} \), and the condition for cylindrically symmetric equilibrium,
\[
1/2 \frac{\partial}{\partial r} (B_{y}^{2} + B_{z}^{2}) + B_{z}^{2}/r = 0, \tag{23}
\]
is satisfied for \( B_{y} \) any function of \( r \) by choosing \( B_{z} \) to be the appropriate nearly constant function. The condition then is that there is one allowable mode of instability for each (positive or negative) integer \( h \) satisfying
\[
1/Z < (|\mathbf{a}|L)(m + 2h) < m, \tag{24}
\]
where \( Z > 0 \) depends on the function \( B_{y} \), i.e., on the distribution of current \( j_{z} \), and on \( m \). This is the generalization of Eq. (20).

For \( m = 0 \), there is clearly no instability. For \( m = 1 \), \( Z \) becomes infinite, independently of the current distribution, and thus Eq. (24) reduces to Eq. (20).

For \( m > 1 \), \( Z \) is given as a function of a positive exponent \( v \) in Fig. 2 for \( j_{z} \) proportional to \( 1 - (r/r_{0})^{\alpha} \) and also to \( (r/r_{0})^{\alpha} \). The first type of distribution with \( v = \infty \) and the second with \( v = 0 \) are identical, both representing uniform current, and have \( Z = 1/(m - 1) \).

For both types of distribution \( Z \) is greater than \( 1/m \) and increases monotonically with \( v \). We see that for each \( m > 1 \) there are thus always ranges of values for \( a \) for which there is instability. These ranges increase with \( v \).

As \( v \to \infty \) the second type of distribution approaches the sheet current case considered earlier (and \( Z \) approaches \( (m + 1)/(m - 1) \)). The non-vanishing of the ranges of instability in the limit seems somewhat paradoxical because the limiting sheet current case is stable for \( m > 1 \). The resolution is probably that the unstable perturbations become stabilized by non-linear effects at smaller and smaller amplitudes as one goes to the limit.

These results have applied when the plasma is surrounded by vacuum. If, instead, it is surrounded by pressureless plasma, but the equilibrium fields are the same, then the results for \( m = 0 \) and \( m = 1 \) are the same, but for \( m \geq 2 \) there is now complete stability. It seems uncertain whether vacuum or pressureless plasma is the better approximation to conditions in the region between the main plasma and the tube wall in a stellarator.

**EXPERIMENTAL RESULTS**

It has been shown in the previous sections that a figure-8 shaped stellarator should exhibit an \( m = 1 \) instability for currents greater than a critical value determined by Eq. (22). The magnitude of this critical current depends on the geometry of the system (through \( \mathbf{t} \), the rotational transform angle, and \( L \), the axial length), the plasma cross-sectional area (through \( r_{0} \), the radius), \( B_{y} \) the magnitude of the longitudinal confining magnetic field, and on whether the current direction is along or opposite to the magnetic field. It is interesting to note that the critical current in either of the two directions is just sufficient to cause a resultant rotation of either zero or \( 2\pi \) in the magnetic lines of force just outside the plasma (i.e., to make the lines of force close on themselves once around the machine). This result is independent of the form of current distribution.

The quantities \( t \), \( I \), \( B_{y} \), and \( L \) are all easily measurable. If all the lines of force were exactly parallel to the walls of the discharge tube, then \( r_{0} \) would simply be the discharge tube radius. If this is not the case, \( r_{0} \)
is then the radius of the innermost magnetic surface which anywhere touches the discharge tube walls. By means of a collimated electron beam, it is experimentally possible to determine \( r_0 \) under low-field, steady-state conditions to an accuracy of about 10%. These values are confirmed by measurements of the plasma inductance. In various stellarators the effective radius (called the radius of the aperture) is from 50% to 90% of the discharge tube radius. A summary of some pertinent quantities is given in Table 1.

Here the critical current is calculated from Eq. (22) which becomes, on solving for \( I \),

\[
I = B_v 2 \pi r_0^2 \eta / \mu_0 L, \quad (25)
\]

where \( \eta = 0.911\pi \) and 1.089\pi for the two possible directions of current and \( B_v \), \( r \), \( L \) and \( \mu_0 \) are all in MKS units.

The observational results to be expected as a consequence of this instability are not entirely obvious. The observations on the B-1 Stellarator where the effects seem particularly apparent will first be described in some detail, followed by a summary of similar evidence from other devices.

### Table 1. Some Stellarator Characteristics

<table>
<thead>
<tr>
<th>Stellarator model</th>
<th>Tube inside radius</th>
<th>Aperture radius</th>
<th>Critical current at 10 kilogauss</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-1</td>
<td>196°</td>
<td>450 cm</td>
<td>2.2 cm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.6 cm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>810 or 970 amps</td>
</tr>
<tr>
<td>B-2</td>
<td>196°</td>
<td>600</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>610 or 730</td>
</tr>
<tr>
<td>B-3</td>
<td>196°</td>
<td>600</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>950 or 1140</td>
</tr>
</tbody>
</table>

The ohmic heating electric field is usually applied in the form roughly of a square wave of adjustable amplitude by means of a transformer which links the discharge tube. A more complete description of the characteristics of the stellarator is given by Coor, et al.\(^7\) The duration time is self-limited by saturation of the transformer. The plasma current and applied electric fields are displayed as functions of time on oscilloscopes. Figure 3 shows these data for six different \( E \)-fields, applied to a helium discharge at an initial pressure of \( 6 \times 10^{-4} \) mm Hg. The oscilloscope sweep speed is 1 millisecond per division from left to right and the initially applied electric field in \( v \) per cm is given in each case. The field gradually falls (because of the partial discharge of a capacitor bank) and fluctuations appear because of plasma inductance effects. As successively higher electric fields are applied, the current rises more rapidly and (in the first four cases) to a higher peak value. However, for fields above about 0.06 \( v \) cm there is very little dependency of peak current on applied voltage as shown in Fig. 4, which shows peak current plotted against applied electric field at various gas pressures. The current essentially reaches a plateau, the level of which is roughly independent of the pressure. As shown in the figure, the plateau value of current agrees with that predicted from Eq. (25). Similar sets of data taken at other values of confining field produce similar effects at current levels proportional to the magnetic field in quantitative agreement with prediction. These are the solid circles in Fig. 5. However, the most striking effect is that of reversing the direction of the current with respect to the magnetic field. These data are plotted as open circles in Fig. 5. The ratio of slopes of the two lines is 1.22. On the basis of the twist angle of B-1, one would expect a ratio of 1.19. This difference is well within experimental error.

Another point of interest is that whenever the current rises above this critical limiting value, the current and voltage become quite noisy as may be seen in Fig. 3, and large amounts of impurities appear in the discharge.\(^7\)

Further verification is offered by the fact that the
HYDROMAGNETIC INSTABILITY IN A STELLARATOR

Critical value of current is the same in a hydrogen discharge as in a helium discharge.

In all cases it is possible to drive the current well above the critical current if a high enough electric field is applied. Figure 6 shows, for example, a plot of peak current vs. ohmic heating field for 3 different values of magnetic confining field in the B-2 Stellarator. There is once more a definite leveling off, at a current consistent with prediction, but for high ohmic heating fields the current does continue to rise. However, in this case a slight step or irregularity appears at approximately the critical current (shown by the dotted lines in Fig. 6).

In the case of B-3, much more care was taken in alignment of the field coils and the discharge tube. As a result, the aperture area as measured by the electron-gun is 13 cm² as compared with about 8 cm² in both B-1 and B-2. Correspondingly higher plateau currents are expected and are observed in this case up to a magnetic confining field of 53,000 gauss. The discharge tubes in all the devices previously mentioned are stainless steel with bakable vacuum systems which may be pumped down to pressures of the order of 10⁻¹⁰ mm Hg.

B-1 was formerly operated with relatively “dirty” walls of stainless steel, and later of Pyrex, for which base pressures of only about 10⁻⁶ mm Hg were possible. The metal tube required about 4 times as much electric field to get initial breakdown and showed very little evidence of the current leveling off. In both these respects, on the other hand, the “dirty” glass system was quite similar to the “clean” metal system.

Clearly there is very satisfactory agreement between theory and experiment with regard to the \( m = 1 \) mode of instability. However, there is no experimental indication of the existence of the higher \( m \) modes. One possible explanation of this is that the region between the main body of plasma and the tube wall might contain enough charged particles to act as a good conductor, i.e., as a pressureless plasma, in which case the higher \( m \) modes would be stable. It is of course quite possible that higher \( m \) modes of instability are present but produce effects less easily observable than those due to the \( m = 1 \) mode. For instance, they might merely distort the original equilibrium configuration into a new, not greatly shifted, stable configuration. This would be consistent with the proposed resolution of the paradox in the previous section. It seems highly plausible that for long wave-length modes with \( m \neq 1 \), non-linear terms become significant when the displacement becomes comparable with the plasma radius, whereas for long wave-length \( m = 1 \) modes, the linearized perturbation theory remains valid until the displacement becomes comparable with the wave-length.

REFERENCES


The Divertor, a Device for Reducing the Impurity Level in a Stellarator

By C. R. Burnett,* D. J. Grove,† R. W. Palladino,* T. H. Stix* and K. E. Wakefield*

The divertor is a device, first proposed by L. Spitzer,1 for averting contact between the hot ionized gas and the wall of the main discharge tube. An outer cylindrical shell of magnetic flux is conducted from the main discharge tube into an auxiliary chamber. Ions and electrons diffusing outward from the main plasma enter this shell and follow along the magnetic lines of force to strike a collector plate in the auxiliary chamber rather than the wall of the main discharge tube.

In the Stellarator experimental program, it became apparent rather early that the character of the discharges in helium was dominated by the influx from the walls of impurities of higher atomic number. In the unbaked B–1 Stellarator,2 a correlation was observed during the course of the discharge between the drop in electron temperature and the spectroscopically observed radiation from impurity ions. Measurement of the Doppler width of the $\lambda 4686$ Å line of ionized helium indicated that inelastic collisions with incoming cool impurity ions were responsible for depressed ion temperatures.

Major strides have been made toward overcoming the impurity problem by extended bakeout at 400° C and by use of other ultra-high vacuum techniques.2 A second method for impurity reduction is to use a divertor. The divertor was originally conceived as a solution to a temperature distribution problem; but a device which protects the walls from hot ions can be of use in reducing impurity influx. It is the description of a divertor, the design, and an experimental measure of effectiveness of impurity reduction which will comprise the body of this paper.

THEORY

A divertor introduces an important change in the topology of the magnetic field in a stellarator as is evident from the preceding description and Figs. 1 and 2. The main discharge is, in effect, surrounded by a protecting sheath or scrape-off layer, which leads to an auxiliary chamber. Hot ions diffusing radially outward from the main discharge travel preferentially along the sheath into the collector plate and a reduced number of them will strike the walls of the main discharge tube. Similarly, impurity atoms which are released at the walls of the main discharge tube by photon bombardment (or by those ions and electrons which succeed in traversing the sheath) will become ionized either in the sheath or near the boundary between the sheath and main discharge, and may be led into the auxiliary chamber.

Any beneficial action of a divertor hinges on the possibility of reducing the backflow of those impurities which will be released at the collector plate. Several processes act to reduce this backflow. Most of the impurity atoms released from the collector plate will be neutral and most of the impinging helium ions will be released to the chamber as neutral atoms. For the divertor used in these experiments, the characteristic time for the return of neutral atoms to the discharge is 27 milliseconds, which is long compared to the duration of the discharge. For discharges of much longer duration it is possible, in principle, to pump these atoms away; for the present experiments the required pumping speed is impossibly large and the available speed negligible by comparison. As a result, the pressure of these neutral atoms in the divertor chamber increases as the discharge proceeds. The rate of backflow of the neutral gas into the main discharge region is then determined by the gas kinetic conductance of the divertor-tube opening and the divertor pressure.

A detailed calculation shows that if the thickness of the sheath were sufficiently great and if the return process were solely one of neutral gas flow, this divertor would reduce the impurity level in the discharge by factors of 220 and 30 at times 100 microseconds and 1000 microseconds after the start of the discharge.

While it is unlikely that impurity atoms and helium atoms will leave the collector plate as ions, it is possible that these atoms will be ionized in the divertor by the incoming ions and electrons. These ions could then flow rapidly back into the discharge. However, some of them will be reflected by the strong magnetic mirror at the entrance to the main discharge tube. Those ions which succeed in passing through the mirror are still in the volume of the protective sheath, and may again reach the collector plate before diffusing across
lines of force into the main discharge. A detailed analysis of the ion backflow has not been made.

A smaller reduction in impurity level than that predicted by the simple neutral backflow approach may thus be expected with either inadequate sheath thickness or ion backflow. In such cases there will be an appreciable number of cool helium atoms returning quickly to the discharge and for these cases it is instructive to investigate the expected time dependence of the light from He++. A phenomenological analysis which furnishes an instructive fit to the experimental data on the divertor-no divertor comparison may be obtained as follows. Let \( n_0 \) be the number of neutral helium atoms initially in the tube divided by the volume of the discharge column, and let \( n_1 \) and \( n_2 \) represent the number density of neutral He, He+, and He++ as functions of time. Then we have

\[
\begin{align*}
\frac{dn_1}{dt} &= \gamma n_1(n_2 + n_0) - \alpha_1 n_1, \\
\frac{dn_2}{dt} &= -\alpha_2 n_1 - \xi n_1 + \alpha_1 n_1, \\
\frac{dn_0}{dt} &= \alpha_0 n_1 - \xi n_2,
\end{align*}
\]  

(1)

where \( \alpha_1 \) is the ionization rate for neutral He to He+, \( \alpha_2 \) is the ionization rate for He+ to He++, \( \xi \) is the rate at which charged particles leave the discharge as a result of diffusion, instabilities, etc., and \( \gamma \) is the fraction of ions which re-enter the plasma as neutral atoms after hitting material walls. The rate coefficients \( \alpha_1, \alpha_2, \) and \( \xi \) are assumed independent of time. In the application of the analysis to the experimental data, it will be assumed that the spectral intensity of a line is proportional to the number density of the corresponding species. All spatial variations of these quantities are neglected.

For operation with no divertor, \( \gamma \) is assumed to be unity. The total number of ions and atoms is constant, and in time an equilibrium plateau is reached. If \( \alpha_1 \) is much greater than \( \xi \), it is readily seen from Eq. (1) that the value of \( n_1 \) for the plateau phase is given by

\[
\frac{n_1}{n_0} = \frac{\alpha_2}{\alpha_2 + \gamma \xi} e^{-(\alpha_1 + \gamma) t} + \frac{\gamma}{\alpha_2 + \gamma \xi} e^{-(1 - \gamma) t}
\]  

(2)

where \( n_0 \) is again the initial density of neutral He. For \( 0 < \gamma < 1 \), the decay of singly ionized helium is characterized by two exponential terms. If \( n_1 \) is plotted against time on semi-log paper, the slopes for the two components are simply the exponents in Eq. (2). The measurement of these two slopes plus the peak-to-plateau ratio for the no-divertor operation gives three numbers from which one may determine the ionization rate, \( \alpha_0 \), the confinement time, \( 1/\xi \), and the measure of divertor effectiveness, \( \gamma \).

### DESIGN

For the experiments reported in this paper, the B–65 Stellarator was used. An illustration of this device with its divertor is shown in Fig. 1. Details of the B–65 machine other than the divertor are given elsewhere. The B–65 geometry is a race-track, and the machine is equipped with helical windings. However, the use of helical windings reduces the amount of scrape-off (defined below) furnished by the divertor, and for this reason, the more significant data regarding divertor action were taken with the helical windings not energized. A rotational transform for the magnetic field was then provided entirely by the ohmic heating plasma current flowing along the lines of force. A quarter-section of the divertor is shown in Fig. 2. The coils of the divertor are electrically in series with the confining field coils, but polarized so that current passes through the center coil of the divertor in the reverse direction. The aperture of the machine is defined as the radius of the largest flux tube which passes through the center coil of the divertor, this radius being measured in a section of the machine containing the uniformly wound solenoid. The intersection of this flux tube with a plane through the axis of the coil system is called the diverted line. For each machine arrangement, there is a similar line associated with the smallest flux tube which intersects or is tangent to the vacuum wall at some point in the machine. This line is called the limiting line. The region between these two flux tubes is called the sheath or scrape-off region, and the distance between these two lines is called the scrape-off distance.

The magnetic design of the divertor was performed on a special resistance analogue, capable of accurately representing axially symmetric magnetic fields. The theory and operation of this analogue have been described previously. A group of critical characteristics was met in a trial and error design procedure.
These characteristics were that:
(a) the radius of the aperture should be about 1 in.;
(b) the magnetic field intensity at the center of the divertor should not be less than 20% of the value in the uniform solenoid;
(c) the limiting line should not touch any part of the vacuum wall until well within the divertor chamber;
(d) the distance between the diverted line and the walls of the vacuum system as it passes through the throat should be several radii of gyration for the ions at the local field strength;
(e) the width of the throat should be kept small to minimize the backflow of neutral atoms;
(f) the mirror ratio of the throat should be kept high to reduce the backflow of charged particles, but
there should be no mirror effect for particles entering the divertor from the main discharge tube; and

the coil location and required current density must allow reasonably easy manufacture.

In the B–65 Stellarator, the radius of the limiting surface is 1.59 in. in the sections containing a uniform solenoid, resulting in a scrape-off distance of 0.55 in. When the helical windings are energized, the scrape-off distance is reduced to 0.26 in., because of the deformation of the cross sections of the magnetic surfaces from circles into trefoils. These values of scrape-off distance are derived from the analogue data. Practical difficulties of carrying windings around ports and other obstructions make achievement of the ideal configuration rather difficult and the actual scrape-off distances are probably smaller than these.

The dimensions of the divertor and the general form of the wall are indicated on the scale drawing of Fig. 2, which shows the magnetic configuration. The field strength at the center is 41% of the value in the sections of the machine containing a uniform solenoid. The volume of the divertor is 48 liters, and the gas-field strength at the center is 41% of the value in the magnetic configuration rather difficult and the actual scrape-off distances are probably smaller than these.

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The divertor–limiter comparison was accomplished by taking sets of data alternately for the two cases. A particular study would be taken first with the divertor energized and the limiter retracted. Immediately thereafter, the corresponding data would be taken with the divertor reverse coil not energized and the limiter in place.

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The machine base pressure was about 2 x 10⁻⁸ mm Hg, and the operating pressure was 1 x 10⁻⁴ mm Hg of helium. A confining field of 20,000 gauss and an ohmic heating field of 0.24 v per cm resulted in a plasma current of approximately 3500 amp. This amount of current is 45% of the calculated limiting current for the Kruskal m = 1 instability. ⁶ A 250 kc rf electric field was applied for pre-ionization. Additional information on operation of the B–65 machine is given elsewhere.⁷

Identification of the important impurities and their stages of ionization was made from photographs of the spectra. These were taken on an f/2.8 stigmatic grating spectograph with reflection optics, after a design by Dr. A. B. Meinel. The spectograph has a first-order dispersion of 20 Å/mm. The first-order spectral slit width of 0.3 Å used in these experiments was less than the Zeeman and Doppler widths of the spectrum lines. Exposures of 15 pulses on Kodak T03a–0 spectroscopic film were required.

Time-resolved intensity measurements were made on He⁺ λ6684 Å, He⁺ λ5875 Å, and on selected impurity lines of carbon, nitrogen, and oxygen. For this purpose a Jarrell-Ash monochromator was used with an RCA 1P28 photomultiplier at the exit slit. This is an 0.5 meter instrument with a 1200 mm⁻¹ plane grating 5 cm in width, used in second and third orders. The photomultiplier signal was displayed on a Tektronix Type 531 oscilloscope and photographed with a DuMont Polaroid camera.

Spectral profiles were also obtained with the monochromator. Data were taken in scans of 20 pulses with successive signals at about 0.1 Å intervals. In a compromise between resolution and illumination, a resolving power of 15,000 was used. The light was observed through quartz windows in the axial direction parallel to the confining magnetic field. Ion temperature and intensity measurements were made on the divertor leg of B–65, while photographs were taken both through the divertor leg and through the opposite leg.

EXPERIMENTAL RESULTS AND DISCUSSION

Spectrograms

An initial spectrographic survey produced profuse spectra of carbon, nitrogen, and oxygen in addition to helium. Figure 3 is a reproduction of a divertor–limiter pair of spectra taken at the beginning of the experiment. The spectrogram was made on a 10 in. strip of film which has been reproduced here in three sections. Selected strong impurity lines which were later studied with time resolution have been designated in addition to some of the helium lines. The decreased density of the impurity spectra with the divertor is readily apparent in the original film. A notable exception to this trend for the impurity lines is the enhancement of the O⁺ multiplet 3s 8S–3p 8P in the ultra-violet. In the early operation of the B–65 device, the walls were strongly contaminated with impurities. Under these conditions, the higher electron temperature necessary for the production of strong spectra from highly ionized impurity atoms was achieved only by using the divertor to increase the purity.

The arc lines of chromium appeared at the top and bottom edge of the spectra indicating the sputtering of metal from the limiter and, to a lesser extent, from the divertor. Spectra taken from the opposite leg of the machine were similar except for the absence of the chromium lines.

† The authors are indebted to D. T. Scag for a very clean mechanical design.
**Time-resolved Intensities of Spectral Lines**

Tracings of the light signals for various impurity radiations and helium are shown with the corresponding traces of plasma current in Fig. 4. The time scale for the light signals is identical with that for the plasma current. The ordinates indicate the relative photomultiplier signals for divertor and limiter operation, and in general are different for each transition. In comparing each pair of light signals, allowance must be made for the variation of ionization and excitation cross sections with electron temperature. The electron temperature should increase markedly with relief from impurity influx. Unfortunately, neither the required cross sections as a function of electron energy nor the electron temperature in the plasma are known. Electron temperatures of the order of 20 ev are obtained from plasma resistance measurements; but since the measured resistance includes an unknown contribution from the impedance arising from cooperative phenomena, the technique is not a reliable diagnostic tool. A better clue comes from the strong signals in the Ov spectrum which indicate appreciable numbers of electrons with energies exceeding 77 ev, the energy required to remove the fourth electron.

When the mean electron energy is several times the ionization energy of an ionic species, the excitation and ionization cross sections for the species are not strong functions of the electron temperature. For the divertor–limiter comparison, we thus expect the amount of impurity influx to be better described by the light from the lower stages of ionization. The spectra in Fig. 4 show a factor of 2 to 3 reduction in light from the singly ionized impurities. It seems likely that the divertor reduced the impurity level by at least this factor.

It is believed that the impurities are ionized first near the scrape-off region and that the highest stage of ionization is found primarily in the plasma core. The observed ionization rates and derived confinement time discussed later agree qualitatively with this picture. In Fig. 4, the intensity of the Ov line is reduced by a factor of 10 when the divertor is used, indicating that the purity of the central region may be higher than that indicated by Ov for the scrape-off region. Supporting evidence for this view comes from the measured positive ion temperatures discussed below.

**Positive Ion Temperatures**

Ion temperatures derived from Doppler broadening of the HeII λ4686 Å and Ov λ22781 Å lines are shown in Fig. 5 for the two cases as functions of time during the ohmic heating pulse. The line profiles were compared with the calculated Doppler broadened Zeeman
Line scans on $\text{O}^+4415$ Å indicate the $\text{O}^+$ ions to be at about the same temperature as $\text{He}^+$. From $\text{O}^{IV} \lambda 3063$ Å and the $\text{N}^{IV}$ multiplet $3s^23p \rightarrow 3p^2 \text{P}_0, 1, 2$, it appears that the $\text{O}^{+++}$ and $\text{N}^{+++}$ ions are at temperatures intermediate between the measured temperature for the $\text{He}^+$ and $\text{O}^{+++}$ ions.

The temperature difference between $\text{He}^+$ and $\text{O}^{+++}$ ions may be due to spatial variations in temperature. It is reasonable to assume that the purity of the gas, the electron temperature, and the ion temperature all increase toward the core of the plasma, and that the helium influx is excited and ionized in the outer regions of the plasma. An experimental test of this hypothesis would be desirable, to exclude the possibility that different ions in each small region are not in kinetic equilibrium.

The earliest divertor–limiter comparison data were taken with the helical windings energized. Although the data taken under these conditions have not been used in determining the above measure of divertor effectiveness, the results allow further characteristics of the impurity behavior to be deduced. A clean-up action in B-65 was apparent during these first runs. Light from the low stages of ionization diminished markedly in the first few days of the experiment, and continued to diminish slowly thereafter. A related electron temperature effect was observed on the $\text{O}^+$ intensities. During the same period, the $\text{O}^+$ intensity with limiter operation was observed to grow rapidly from the low value illustrated in the spectrogram of Fig. 3. Also, the limiter/divertor ratio for $\text{O}^+$ increased from much less than unity to greater than unity and by the time the experiments began with the helical windings not energized, the ratio reached 2 to 1. For the low stages of ionization, the limiter/divertor ratio was about 4 to 1 after the first week of operation. This ratio decreased for each successive stage of ionization in contrast to the situation shown in Fig. 4. However, with continued operation with the helical windings energized these ratios gradually approached the situation later observed (Fig. 4) with the helical windings not energized.

From these early data it appears that the divertor action in lowering the impurity influx was somewhat greater when the machine walls had not yet been conditioned by the action of the discharge.
The ion temperatures obtained with the helical windings energized were not as high as those shown in Fig. 5 and the HeII λ4686 Å peak-to-plateau ratio was smaller, as might be expected from the smaller scrape-off distance for this type of operation. Increased heating by use of the divertor was still observed, however.

**Confinement Time, Ionization Rate, and Divertor Effectiveness**

An empirical theory was outlined above, to relate the time behavior of the number density of He⁺ ions to three parameters of direct interest. We make the assumption that the intensity after the initial peak of HeII λ4686 Å is proportional to the number density of He⁺ ions, and plot in Fig. 6 the intensity of this radiation on a logarithmic scale vs. time for the divertor case and the limiter case. In the limiter case, the predicted plateau is readily apparent, and the peak-to-plateau ratio is approximately 3. For the divertor case, the two slopes are equally apparent, with values of $1.8 \times 10^4$ sec⁻¹ and $0.25 \times 10^4$ sec⁻¹. We then obtain:

Conf confinement time, $1/\xi = 200$ microseconnds

Ionization rate of He⁺, $q_\phi = 1.4 \times 10^4$ sec⁻¹

Divertor effectiveness, $\gamma = 0.43$.

We discuss these three parameters in turn.

**Confinement Time**—The value of 200 microseconds may be compared with the 120 microsecond confinement time obtained from the rate of pump-out for H₂ discharges in the baked B–1 Stellarator. Since the B–65 aperture has twice the area of the B–1 device, and since the magnetic fields are comparable, these two results are consistent with one another.

**Ionization Rate of He⁺**—The He⁺ ionization rate may be compared with the ionization rates for the impurities. A crude estimate of these rates was made in the following manner. The time at which the spectral intensity for a given impurity ion reached 0.1 of its maximum was plotted vs. the logarithm of the binding energy of the last electron removed in producing that ion. The points for O⁺, O+++, O++++, C⁺, C++, C+++, N⁺, N+++, and He⁺ were plotted on the same diagram. There was considerable scatter but the straight line of best fit had a slope of $1.7 \times 10^4$ sec⁻¹, which is consistent with the He⁺ ionization rate. Using an estimated cross section of $3 \times 10^{-18}$ cm² for the ionization of He⁺ by 80 ev electrons, we infer an electron density of $1 \times 10^{13}$ cm⁻³. From the known starting pressure, the calculated electron density in the main discharge column is approximately an order of magnitude higher. We surmise that the ionization of the lower states occurs mostly in or near the edge of the scrape-off layer, where the electron density may be much lower.

**Divertor Effectiveness**—The value of $1/\gamma = 2.5$ determined from the time variation of the singly ionized helium line agrees quite well with the impurity reduction ratio (2–3) estimated from the divertor–limiter ratios for the singly ionized impurity spectra. Furthermore, the value 2.5 for $1/\gamma$ appears reasonable when consideration is given to the fact that about 130 microseconds is required for a helium ion of $kT = 50$ ev to travel the average distance of 283 cm to the divertor and only 200 microseconds is required to diffuse out of the aperture.

**CONCLUSIONS**

We conclude that under present conditions the divertor reduces the impurity concentrations in the B–65 Stellarator by a factor of 2–3, with perhaps an even greater reduction in the core of the plasma. The use of a divertor yields an increase in the ion
temperatures from 40 ev to 60 ev for He\(^+\) and to 130 ev for O\(^{++}++\). The latter difference probably represents a higher temperature in the core. The effectiveness of the divertor in these experiments is apparently limited by the diffusion of plasma across the lines of force, which occurs in about the time required for a positive ion to travel around the machine. If better magnetic confinement can be achieved, and if the impurity influx is induced by ion bombardment and not photon bombardment, we may also expect considerably greater effectiveness of a divertor in reducing the impurity concentration in the discharge.

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**REFERENCES**

Stabilization of Plasma by Nonuniform Magnetic Fields

By B. B. Kadomtsev and S. I. Braginsky

STABILIZATION OF PLASMA BY MEANS OF GUARDING CONDUCTORS

The main problem in controlled thermonuclear reactions is the confining of high-temperature plasma, i.e., the obtaining of a stable plasma configuration isolated from the walls. One of the most direct methods for creating a plasma separated from the walls consists in the application of the well-known pinch effect stabilized by a longitudinal magnetic field and a metal casing (this problem is considered in detail by S. I. Braginsky and V. D. Shafranov). It is of interest to study other possibilities of obtaining a stable plasma configuration. The present paper deals with the problem of stabilizing plasma by means of a nonuniform magnetic field of a special kind. The systems considered here can, obviously, be used both separately and in combination with the pinch effect.

Let us consider a case when the magnetic field within the plasma is absent. We shall assume that the conductivity of the plasma is infinite, and all the currents flowing along the plasma are concentrated on its surface.

The criterion of stability of a plasma such as described above can be defined by simple qualitative considerations. Indeed, the instability of the plasma cylinder pinched by the magnetic field of its own current is closely connected with the decrease of the magnetic field, outwards from the boundary of the plasma. This becomes particularly clear if we consider local perturbations of the boundary. If there is no magnetic field within the plasma, the plasma pressure in equilibrium is balanced by the external magnetic field

\[ P = \frac{\gamma P_0}{4\pi} dV + \frac{H_0^2}{4\pi} dV \]

where \( V \) is the volume occupied by the plasma, \( V_e \) is the external volume, \( S_0 \) is the plasma boundary, \( H_0^e \) and \( H_0^i \) are the unperturbed fields outside and inside the plasma respectively; \( \gamma \) and \( H \) are the pressure and field of small perturbations, \( \xi_n \) is the normal component of the displacement on the boundary \( S_0 \) and \( \partial/S_0 \) is a derivative along the external normal to \( S_0 \).

The above equation shows that the observance of the condition

\[ \frac{\partial}{\partial n} \left[ \frac{(H_0^e)^2 - (H_0^i)^2}{\gamma} \right] > 0 \]

at all points of the plasma boundary is sufficient to ensure plasma stability (\( \omega^2 > 0 \)).

If \( H_0^i = 0 \), an increasing field away from the plasma boundary is a sufficient and, as is seen from the previous qualitative consideration, also a necessary condition for its stability. (Similar results were obtained in Ref. 1.) Hence, we see the way to find stable systems. For instance, the infinite flat plasma layer (Fig. 1(a)), outside of which the field is constant, shows no instability. However, a flat conductor of infinite broadness cannot be made and for any finite broadness instability appears on account of the edge effect. As is well known the edge effect in a flat condenser can be eliminated with the help of guard rings. An analogous method can be used in this case too. The system given in Fig. 1(b), in which part of the plasma is substituted by metal conductors, is stable. With increase in the magnetic field the plasma boundary is driven inwards, and in this case the magnetic field increases outward from the plasma boundary (Fig. 1(c)). Such a system is all the more stable.

Original language: Russian.

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One significant fact is that the guard conductors prevent the lines of force from freely moving with the plasma. The lines of force tighten on the conductors like elastic strings and because of this no sausage-type instability and distortions of the cylindrical plasma column appear. This effect can be further intensified if the plasma is surrounded by a lattice of metal rods placed perpendicular to the lines of force (Fig. 1(d)). If the lattice is sufficiently dense, it rigidly determines the plasma boundary, provided the magnetic field pressure outside the lattice is larger than the plasma pressure. Indeed, the magnetic field cannot sag deeply through the dense lattice. Therefore, if the plasma boundary moves away from the lattice appreciably, the magnetic field on it decreases drastically. If the plasma moves outwards, the current from the rods passes on to the plasma which will take up the whole magnetic pressure, greater than the plasma pressure. As a result a certain equilibrium and stable state of the plasma boundary will be established which, roughly speaking, coincides with the lattice surface. In this case part of the magnetic pressure is held by the rods and stability is reached at the cost of some increase in the value of the magnetic field increase balancing the plasma.

If the lattice is flat, it is possible to consider in detail the plasma boundary near the rods. Since outside the plasma and rods, curl $\mathbf{H} = 0$ and div $\mathbf{H} = 0$, and $H^2 = 8\pi p = \text{const}$ on the plasma boundary, the problem of the plasma boundary is analogous to the hydrodynamic problem of potential flow of an incompressible fluid with free boundaries. By known methods of conformal transformations, it is possible to find the magnetic field and the shape of the plasma boundary. If the magnetic field pressure does not exceed the plasma pressure too much, the plasma boundary sags between the rods, as is shown in Fig. 2(a). In this case the magnetic field increases in the outward direction from the plasma boundary, so that it is stable with respect to small perturbations. If there is some magnetic flux between the rods and the plasma, the plasma boundary takes the position illustrated in Fig. 2(b). In this case the plasma never touches the rods, and there is no direct heat exchange between the plasma and the rods. Such a configuration is also stable with respect to small perturbations.

Thus, the guard lattice gives the plasma boundary stability with respect to small perturbations and also leads to a constant stabilizing force for displacements which are large in comparison with the distance between the rods.

The guard conductor systems can differ widely. The simplest system is the plasma cylinder; the lattice can be made either of straight rods arranged along the generatrix of the cylinder or of rings perpendicular to the generatrix. In the first case (Fig. 1(a)) the current in the plasma and in the rods flows in the direction of the cylinder axis. This current produces an azimuthal magnetic field. In the ring system the longitudinal magnetic field is produced by an additional external winding. The azimuthal current flows along the rings and the plasma in a direction reverse to the direction of the current in the winding. Each of these systems can be either transformed into a torus or restricted at the ends. In the rod system, for instance, electrodes can be placed at the ends, and the ring system can be restricted by intensifying the magnetic field near the ends ("magnetic plug") with a simultaneous decrease in the diameter of the extreme rings (Fig. 3). In the last system the presence of a longitudinal magnetic field within the plasma is very undesirable since it facilitates the release of the plasma from the system.
PLASMA STABILIZATION

The guard conductor systems are, generally speaking, not stationary. Their time of action is restricted by the finite value of electrical conductivity of the plasma, as well as of the conductors if the latter are passive, i.e., if they are not fed by energy from the external circuit. Indeed, the entire plasma will move beyond the lattice, where it will be released because of the loss of stability, during the time \( t_L = \frac{4\pi\sigma a^2}{c^2} \), where \( a \) is the transverse plasma dimension and \( \sigma \) is its conductivity. In reality the lifetime of the plasma may be significantly less. For instance, in the system illustrated in Fig. 3, with a sufficiently large frequency of collisions, the plasma in the skin layer will escape along the lines of force with a thermal ion velocity \( v_{\text{e}} \). If the length of the system equals \( L \), the time of escape of particles along the lines of force is \( t_L = L/v_{\text{e}} \).

As a result of the plasma loss from the skin layer a certain thickness of the skin layer independent of the time

\[
\delta = \left[ \left( \frac{c^2}{4\pi\sigma} \right) L^2 \right]^\frac{1}{3}
\]

is established,† and the plasma lifetime \( t \) will become, obviously,

\[
t = t_L \frac{a}{\delta} = \left( \frac{4\pi\sigma a^2}{c^2} \right)^\frac{1}{3} \frac{L}{v_{\text{e}}}
\]

With sufficiently high plasma conductivity \( t \ll t_L \).

A similar situation is found in other systems, viz., as a result of diffusion of the magnetic field, the plasma moves beyond the rod to the line ABC in the solution presented in Fig. 2(a) or to the dotted line of force in the isolated solution presented in Fig. 2(b), so that around each rod there appears a plasma "encasement". But each rod must somehow be suspended from the walls of the chamber; the supports would pass through this plasma "encasement". Consequently, in the system there should occur losses on the supports. If the supports have a diameter \( b \), and the distance between the supports of a given rod is \( l \), the probability for the particle to escape when passing through one "encasement" equals \( b/l \).

Hence, the lifetime of particles in the skin layer is \( t_L = L/v_{\text{e}} \cdot l/b \), where \( L \sim a \) is the distance between the rods. By repeating the above calculations we obtain

\[
t = \left( \frac{4\pi\sigma a^2}{c^2} \right) \left( \frac{L}{v_{\text{e}}} \right) \left( \frac{l}{b} \right)
\]

which, at \( b/l \sim 10^{-2} \), is one order greater than the lifetime of the system presented in Fig. 3. The supports submerged in hot plasma are subject to a powerful plasma ion bombardment which results in their evaporation and in contamination of the system. The evaporation of the supports can be considerably decreased if they are supplied with a larger current, so that the magnetic lines of force near the supports close. Then the ions can reach the supports only by diffusion, and the density of the energy loss will be of the order of

\[
\frac{Tn}{b} \cdot \tau = \frac{nTv_k}{b} \cdot \frac{\rho}{\omega \tau}
\]

where \( n \) is the plasma density, \( T \) is its temperature, \( \rho \) is the mean Larmor ion radius, \( \tau \) is the ion-ion collision time and \( \omega = eH/Mc \) is the cyclotron frequency. If \( b/l \sim 1 \), the energy loss decreases \( \omega \tau \) times in comparison with the energy release \( nTv_k \) on the currentless support. In this case there appears along the supports at some distance from it a region of very weak magnetic field which will play the part of a "canal" for the plasma leakage. But if the width of the canal is of the order of \( b/l \), this leakage, in any case, will not exceed the direct loss to the supports.

It follows from the estimates given that though the plasma lifetime is less than the skin-time, it can reach several hundredths of a second at \( a \sim 10^2 \) and \( T \) of the order of several kev. The estimates show that for a D-T mixture the thermonuclear reaction energy released in this time can reach the value of the thermal energy. It may be expected that the most promising is the use of the guarding conductors in combination with the pinch effect. As is known from Ref. 3, the plasma pinch in the toroidal system possessing an azimuthal symmetry can be in equilibrium only in the presence of a current along the pinch. Therefore, alternation of the current is not permissible in such a system: at the moment of zero current equilibrium is lost. If a torus with a strong magnetic field is supplemented by guard rings, the plasma at the moment of zero current will be retained by the rings, and this will make it possible to extend the discharge over many periods of the axial current. On the other hand the self field of the current compressing the pinch apparently leads to a decrease in loss on the suspenders. These problems require experimental investigation.

MAGNETIC TRAPS

The problem of plasma stabilization may be approached by a different, in some sense opposite, method. The condition for stability obviously becomes more and more severe with an increase in plasma pressure. Therefore, if we consider the case of a plasma

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† The mean skin layer thickness \( \rho_s \) should be taken as a Larmor radius of electrons if \( \delta < \rho_s \).
with a pressure much lower than that of the magnetic field, it will enable us to determine the minimum requirements for the field which is capable of confining the plasma. Under such conditions it is a property of the field itself, since the low-pressure plasma plays only the part of a "test body"—practically, it does not distort the field under investigation. A field capable of confining a quasi-neutral plasma with an infinitesimal pressure will be called a magnetic plasma trap. Consideration of magnetic traps makes it possible to obtain a simple plasma confinement criterion which provides some orientation in more complicated cases.

**Magnetic Particle Traps**

Since plasma confinement implies the confinement of separate plasma particles and since the investigations of the behaviour of charged particles can often predict the behaviour of plasma, it would be of interest to investigate the simpler problem of magnetic particle traps, i.e., magnetic fields that can hold charged particles in a limited space for a long time. We shall start our consideration with these particle traps.

Particle traps naturally fall into two groups: traps with closed lines of force, and adiabatic traps based on the conservation of the transverse invariant \( I = \omega / H \), where \( \omega \) is the velocity component perpendicular to the magnetic field. Only those particles which experience a reflection from the "plugs," i.e., from the strong magnetic field regions, are confined in the adiabatic traps suggested by G. I. Budker. Particles with a small \( \omega / v \) ratio pass out through the "plugs" and one would like by some means to return them. However, an ordinary closing of the lines of force will not bring us to our aim: the particles moving in a closed line will gradually drift to the walls. An analogous drift takes place in all closed-line traps, and the problem of confining the particles leads to the investigation of this drift.

Both in closed-line traps and in adiabatic traps the particles travel quasi-periodically; they move mainly along the lines of force, gradually drifting from one line to another. If the magnetic field is strong enough so that the total drift during one passage of the line of force is small, then during such a movement the longitudinal invariant \( J = \omega / v \), as can be shown, is conserved (\( \omega \) is the longitudinal velocity component, \( dl \) is the element of length of the line of force; the drift velocity itself is proportional to the gradient \( J \)). If the electric field is absent, then \( v = \text{const} \), and the equation for the surface along which the particle drifts can be written in the form

\[
\phi = \frac{1}{v} \int (1 - \frac{JH}{v^2}) dl = \text{const}.
\]

Thus, in order to keep a particle with a given \( v \) and \( J \) within a certain volume, the \( \phi \) surface must be closed and remain entirely inside this volume.

The condition is simplest for particles with \( \omega / v = 1 \), i.e., \( J = 0 \). For these particles \( \phi \) simply denotes the length of the line of force, and consequently, they will be confined if the magnetic field is "corrugated," i.e., if the peripheral lines of force are longer than the internal lines.

As an example, we shall consider two corrugated magnetic field traps. Each of them consists of two corrugated traps with plugs coupled to one another with a "cross-link" A. The presence of plugs with field \( H_m \) will lead to the confinement of a great number of particles in the corrugated parts and only particles with \( \omega / v^2 < H_0/H_m \ll 1 \) will pass over the cross-link, travelling along the closed lines of force.

The first system (Fig. 4(a)), which can be called a closed magnetic plug trap, operates to a certain extent like the ordinary adiabatic trap. If \( L > R \), the length of the line of force is determined by the corrugated part and the share of the cross-link is insignificant. Therefore the particles that have passed over the cross-link will be stabilized; corrugation leads to a compulsory circular drift of particles near the axis of the system which compensates the drift in the cross-links, providing \( \Delta L/\Delta a > 1 \), where \( \Delta L \) is the elongation of the peripheral line in comparison with the central line. Therefore, the main process of particle loss in such a system is due to particle-particle collisions in the cross-links A which will lead to their drifting out of the cone of \( \omega / v^2 < H_0/H_m \) and to a subsequent drifting out to the cross-link wall. But since the density in the cross-link is \( H_0/H_m \) times less than the density in the straight parts and their volume is \( \pi R/L \) times less than the volume in the straight part, the leakage will decrease \( H_m L/HR \gg 1 \) times.

**Magnetic Plasma Traps**

The second system (Fig. 4(b)) is some modification of the torus. Here the bulk of particles in the cross-links is also stabilized. However, the particles with small \( \omega / v \) in the cross-links are able to settle on the wall during the time of their flight over the cross-links.
which leads to an additional energy leakage exceeding the leakage caused by thermal conductivity if \( \lambda > L \), where \( \lambda \) is the mean free ion path.

Besides, in both traps there occurs an increase in the thermal conductivity on account of the radial displacement caused by the drift in the cross-links ("mixing"). Therefore, if the plasma touches the wall, the linear part must be of a very great length to fulfill the condition \( \Delta L/\pi R \gg 1 \).

**Corrugated Traps**

Corrugated magnetic traps can confine individual particles, or at least most of them. But this does not mean that they will confine a quasi-neutral plasma, since there occur in the plasma proper electric fields which can essentially change the character of the individual particle motion. The next problem consists in finding conditions of confinement for a plasma with an infinitesimal pressure. In the hydrodynamic approximation and with the assumption of an infinite conductivity these conditions have a simple form and can be deduced from simple qualitative considerations. For the low-pressure plasma only such motions can exist that shift separate tubes of force almost without distorting the magnetic field. For such motions the tube volume of a tube of plasma changes proportionally to the integral \( \int (1/H) dl \) along the line of force. Plasma trying to expand will induce each tube to tend towards an increase of this integral.

As a result, it appears that the behaviour of low-pressure plasma is analogous to that of a nonuniformly heated gas in gravitational field; the plasma pressure corresponds to the gas density and the function \( U = -\int (1/H) dl \) is the potential energy. If the plasma at the initial moment has an arbitrary distribution, convection sets in: the tubes of force in which the plasma pressure is higher start moving towards \( U \), outsting the tubes with lower pressure. Thus, the plasma tends toward a stable state in which its pressure declines with increasing \( U \). Besides this, other stable equilibrium states exist. Just as a stable equilibrium is possible in the atmosphere of the earth when the air temperature does not drop too fast with height, in a plasma there is no loss of stability if its pressure rises not very quickly with increase in \( U \).

Indeed, let the plasma be in equilibrium and let a certain tube shift towards an increase in \( U \) by some infinitesimal quantity, moving the other tubes apart. If the process occurs adiabatically, the plasma pressure in this case will change by a quantity

\[
\delta p = -\gamma \delta V \frac{\delta U}{|U|} = \gamma \delta U \frac{\delta U}{|U|}
\]

where \( \gamma \) is the adiabatic index. If the pressure in the tube appears to be lower than the surrounding plasma pressure, which is equal to \( p + \delta p \), the tube will move with acceleration towards further increase in \( U \); if \( \delta p > \delta p \), the tube will be forced back and the plasma will be stable. Thus, to obtain stability it is necessary and sufficient that \( dp/\delta U < \gamma p/U \), i.e., the plasma pressure should drop with \( U \) not faster than \( |U|^{-\gamma} \).

Thus, the problem of plasma confinement by a given magnetic field requires an investigation of the potential \( U \) for this field. For a magnetic trap the potential \( U \) should be minimum within a bounded volume and increase towards the walls. It appears that the field of the guard conductor systems satisfies this condition completely. Let us consider, for example, a guard ring system the magnetic field of which has the form presented in Fig. 5. Since the system in Fig. 5 is periodic, \( U \) may be regarded as an integral over one period, \(-a/2 < z < a/2\). The potential \( U \) is a function of the line of force; therefore it is convenient to consider it in the plane \( z = 0 \), crossing all the lines of force. In this plane \( U \) has two "wells" at the points \( A \) and \( C \) located on the dotted line of force that passes through \( A \), where the field vanishes. At \( A \), \( U \) logarithmically approaches minus infinity. Near the rod \( U \) has a maximum and is close to zero. At \( r = 0 \), \( U = -a/H_1 \), \( \Delta H = \sigma - \sigma_0 \), where \( H_1 \) is a certain mean value of the field on the \( z \) axis; and at point \( D \), \( U = -a/H_0 > -a/H_1 \).

Hence, we see that any plasma configuration whose pressure decreases in the direction away from the dotted line of force is certainly stable. Moreover, an arbitrary plasma configuration in such a system changes by itself, so that the pressure decreases in the direction away from the dotted line, i.e., the plasma itself will tend to a stable equilibrium.

Besides states in which there are plasma encasements near the rings, we can find in the ring system a stable plasma state whose pressure differs from zero only within the dotted line of force, and vanishes on this line; for stability it is sufficient that the plasma pressure decreases with \( U \) slower than \( |U|^{-\gamma} \).

However, the actual attainment of such a state is a more subtle matter; it requires a special preparation of the plasma. Moreover, this state actually corresponds
to the plasma touching the wall, since the state assumes that there should be constantly a pressure equal to zero on the dotted line, otherwise plasma enclosures will appear.

Being stable, the system presented in Fig. 5 can be transformed into a torus: for a very small toroidality there will be no loss of stability.

An analogous consideration shows that there are stable low-pressure plasma states in other guard conductor systems. This means that guard conductor systems are also magnetic plasma traps. This circumstance, although not well proven, can serve as an important argument in favour of a possible plasma stabilization in guard conductor systems in a broad pressure region.

The stability conditions for magnetic traps with a corrugated field follow also from the above considerations. Indeed, the magnetic field in the system presented in Fig. 5 is a field of a corrugated type, and the fact that the plasma in such a field tends to take a position along the dotted line shows that, at least for the case of plasma states separated from the wall, magnetic traps with corrugated fields should be practically realized as guard conductor systems.

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REFERENCES

Thermal Insulation and Confinement of Plasma with a High-Frequency Electromagnetic Field

By A. A. Vedenov, T. F. Volkov, L. I. Rudakov and R. Z. Sagdeyev (theory) and V. M. Glagolev, G. A. Yeliseyev and V. V. Khilil (experiment)*

At the Institute of Atomic Energy (Academy of Sciences, USSR) we have studied the problem of creating and thermally insulating a plasma by means of high-frequency electromagnetic fields. Electromagnetic alternating fields which do not penetrate into plasma set up a pressure difference on the plasma boundary. There may be various ways of exciting alternating fields. One of the ways, most convenient from the radio engineering standpoint, is the setting up of a standing electromagnetic wave in a volume resonator partly filled with plasma. Such electromagnetic oscillations can be excited between the conductive walls of the resonator and the surface of plasma in such a way that the electromagnetic pressure, averaged over the high-frequency oscillations, with geometry specially selected, is the same at every point of the plasma surface. (For example, see Ref. 1.) If the amplitude of the high-frequency electromagnetic field is sufficiently large, its pressure may balance the pressure of the plasma. However such a method of plasma containment is unsuitable, as the power required to maintain a large amplitude over the whole surface of the plasma would greatly exceed the thermonuclear reaction energy yield in the plasma, assuming reasonable resonator quality Q.

Those systems appear to be more realizable in which the high-frequency electromagnetic field plays only an auxiliary role; viz., balancing the plasma pressure at individual (most “dangerous”) sections of its surface while the plasma is, in the main, balanced by the steady magnetic field. An example of such an arrangement may be a system with a longitudinal magnetic field, which provides for thermal insulation of plasma across the magnetic lines of force, with volume resonators at the butt ends. Plasma spreading along the lines of force would be limited by the effect of the alternating fields in the resonator.

THEORY

Single Particle Problem

A strict theoretical analysis of the problem should be made within the framework of a kinetic equation. The free path of particles in hot plasma is long, and collisions may be considered to be so rare that their effect on the movement of particles inside the transition region may be neglected, while interaction between the particles is effected by a self-consistent electromagnetic field. It is therefore necessary, first of all, to consider the motion of a particle in a given electromagnetic field. The movement of a charge in a high-frequency field of arbitrary geometry is very complicated, even when the spatial gradients of the fields are small. We shall therefore confine ourselves to a special case where there is a plane standing electromagnetic wave with circular polarization

\[ E_\perp = E_\perp(z)(e_1 \sin \Omega t - e_2 \cos \Omega t) \]
\[ H_\perp = H_\perp(z)(e_1 \sin \Omega t - e_2 \cos \Omega t) \]

and a steady magnetic field \( H_0 \), which may have spatial gradients (here \( e_0, e_1, e_2 \) are orthonormal vectors: \( e_0 = H_0/H_0 \)). The net magnetic field at every point precesses with frequency \( \Omega \) about the direction of the steady field, remaining constant in value.

The full movement of a particle in the prescribed field consists of a fast oscillating motion with frequencies \( \omega_0 \) and \( \Omega (\omega_0 = eH/mc) \) and of a slow motion during time periods longer than \( 1/\omega_0 \) and \( 1/\Omega \). Such a subdivision is possible if the conditions \( R_0 \ll L, a \ll L \) are fulfilled, where \( R_0 \) is the Larmor radius of the particle, \( a \) is the displacement of the particle during \( 1/\Omega \), and \( L \) is the characteristic length in which the magnitudes of the fields change substantially. A particle in a non-uniform field is acted upon by a force whose time average is \( (\mu \cdot \nabla)H \) where \( \mu = (e/2e)(\mathbf{r} \times \mathbf{v}) \). In the simplest case when the frequency and the amplitude are small (\( \omega_0 \ll \omega_0 \), \( H_\perp \ll H_0 \)), the particle, being attached to the precessing magnetic line of force, will move as follows:

\[ \mathbf{v} = \mathbf{v}_\mathbf{0} \left( v_\perp - v_\perp \sin \omega_0 t \right) + v_\perp \left( e_1 \cos \omega_0 t - e_2 \sin \omega_0 t \right) + \frac{H}{H_0}(e_1 \sin \Omega t - e_2 \cos \Omega t), \]

and consequently its magnetic moment is

\[ \mu = \frac{e}{2c}(v_\perp (v_\perp^2 \frac{H_0^2}{\omega_0} - v_\perp \frac{v_\perp}{\omega_0})) e_0. \]
Furthermore, in a standing circularly polarized wave $E$ has a constant component directed along the revolving line of force of the net magnetic field $H$. Its projection on $H$ equals
\[ E_{\parallel} = E \sin(H_{\perp}, H) \approx EH_{\perp}/H_0. \] (3)

The total force acting on the particle in the direction of the steady field $H_0$ equals the sum of the expressions $\mathbf{e}_0 \cdot (\mathbf{E} \times \mathbf{H})$ and $eE_{\parallel}$, and the equation of motion along the magnetic lines of force is as follows:
\[ m\mathbf{v}_i = -\frac{e}{2c} \mathbf{v}_i \times \mathbf{H}_0 \times \mathbf{e}_0 \mathbf{H}_0 + e\mathbf{E}_0. \] (4)

An exact solution of the problem, without the above limitation of the amplitude and frequency of the alternating magnetic field, but retaining the conditions $R_{ce} \ll L < a < L$, results in the following equation for a slow motion:
\[ m\mathbf{R} = (\epsilon/c)\mathbf{R} \times \mathbf{H}_0 + mg. \]

where:
\[
\mathbf{g} = \frac{1}{2} (\mathbf{e}_0 \times (\mathbf{e}_1 \cdot \nabla)\omega_{ce}) - \mathbf{e}_1 \times (\mathbf{e}_2 \cdot \nabla)\omega_{ce})
\times \left\{ \frac{v_i^2}{4} \left( 1 + \cos \theta \right)^2 + \frac{v_i^2}{4} \left( 1 - \cos \theta \right)^2 + \frac{v \sin \theta}{2} \right\} - \frac{E_0}{\omega_{ce}} \mathbf{e}_0.
\]
\[ \omega = \Omega + \epsilon \sin \theta \sqrt{2 + \omega_{ce}^2 \omega_{ce}^2 + \omega_{ce}^2}; \]
\[ \omega_{ce} = \frac{eH_0}{m c}, \omega_{ce0} = \frac{eH_0}{m c}. \]

Let us consider by way of illustration the movement of a charged particle along the lines of force of a non-uniform magnetic field $H_0$ with $\Omega < \omega_{ce}$ and $H_{\perp} < H_0$, neglecting the effect of the electric field (which is permissible in plasma with a large dielectric constant). If $H_0$ changes in space much slower than $H_0$, Eq. (4) is integrated as follows:
\[ \frac{m_0^2}{2} = \frac{(H_0 - H_0)}{H_0} \left[ 1 - \epsilon \frac{\omega_{ce}}{\omega_{ce0}} \left( 1 - \frac{H_0}{H_0} \right) \right] \]

where $H_0$ is the magnetic field at the turning point of the particle, and $\epsilon$ is the kinetic energy of the particle.

The magnitude of the relative drop of the magnetic field $(H_0 - H_0)/H_0$ sufficient to retain particles of any energy with $\Omega \omega_{ce} < 0$ is determined from the condition
\[ (H_0 - H_0)/H_0 = x^{-1} \ln (1 + x) \] (6)

where $x = \omega_{ce}/\omega_{ce0}(\Omega)$.\]

Plasma Problem

When there are no collisions, the distribution function depending on the variables defining slow motions corresponds to a kinetic equation having the equations of motion as its characteristics.\] The electric and magnetic fields are determined from Maxwell’s equations, account being taken of the fact that a current of charged particles is set up both by the “slow” and “fast” motion. Such a system of equations permits one to consider the problem of equilibrium between a standing plane wave and plasma in the presence of a steady uniform magnetic field (directed along the $z$ axis) perpendicular to the boundary of plasma. If the frequency of the alternating field is considerably greater than the Larmor frequency of ions, the effect of the alternating field on the movement of ions may be neglected.

The ions are acted upon only by the longitudinal electric field $E_z$ resulting from the separation of the charges (obtained with due consideration for quasi-neutrality). We are interested in a solution in which there is only plasma at $z \to +\infty$ and there is a standing electromagnetic wave at $z \to -\infty$.

The character of the changes in the electric and magnetic fields inside the transition region is defined by the equations:
\[ \frac{d^2E}{dz^2} - \frac{4\pi e^2}{m^2} \int F_0(\epsilon + e^2E_0^2) d\epsilon = \frac{\Omega^2}{c^2} E = 0; \]
\[ \frac{dE}{dz} = -\frac{\Omega}{c} H. \] (7)

where $F_0(\epsilon)$ is the limiting value of the energy distribution of ions as $z \to +\infty$, and the boundary conditions $E \to 0$ as $z \to \infty$, $E = E_0$ at $z = 0$, are imposed. Here $z = 0$ corresponds to the return point for the ions having maximum energy; plasma occupies the half-space $z > 0$.

Equations (7) define the behaviour of solutions for $E$ and $H$ as $z \to 0$ and $z \to \infty$.

\[ H, \epsilon \sim \exp \left[ -\frac{\omega_p}{c} \left( \Omega \right)^{1/2} \right], \quad z \to +\infty \] (8)

\[ E \approx E_0 \left[ 1 - \frac{\omega_p^2}{c^2} \frac{m \Omega \omega_{ce}}{2} \epsilon_{\max} \right] \left( \frac{dF_0}{d\epsilon} \epsilon_{\max} \right)^{3/2} \]
\[ \approx E_0 \left[ 1 - \frac{\omega_p^2}{c^2} \frac{m \Omega \omega_{ce}}{2} \epsilon_{\max} \right] \left( \frac{dF_0}{d\epsilon} \epsilon_{\max} \right)^{3/2} \]
\[ H \approx \frac{e^2}{c} \frac{m \Omega \omega_{ce}}{2} \epsilon_{\max} \right] \left( \frac{dF_0}{d\epsilon} \epsilon_{\max} \right)^{3/2} \]
\[ \approx 0 \]
\[ z \to 0 \]

Here $\omega_p$ is the Langmuir plasma frequency. It should be noted that from the first integral of Eqs. (7) it is possible to obtain the condition for the balance of pressures of plasma and the standing electromagnetic wave
\[ \frac{H_0^2}{8\pi} + \frac{E_0^2}{8\pi} = 2 \int eF_0(\epsilon) d\epsilon. \] (9)

For this it is necessary to put $E = E_0$ ($H = H_0$) where $E_0$ is the electric field of the wave at the return
Measurement of plasma concentration

Generator of 3 or 0.8 cm waves

Measuring resonator

To magnetron generator

Receier

To the vacuum system

Figure 1. Schematic of apparatus for study of plasma confinement by e-m fields

(1) Measurement of plasma concentration. (2) Generator of 3 or 0.8 cm waves. (3) Measuring resonator. (4) To magnetron generator. (5) Measurement of the amplitude of the electromagnetic field. (6) Receiver. (7) To the vacuum system

point for the ions with the maximum energy $e_{\text{max}}$, so that $e_{\text{max}} = \frac{e^2E_0^2}{2m\omega_{ce}}$. The averaging we have made use of is applicable if the spatial gradients of the field are small. In any self-consistent problem, the characteristic size is the width $\delta$ of the transition region. From (8) $\delta \sim (c/\omega_p)(\omega_{ce}/\Omega)^2$, so that the criterion $R_{ce} \ll L$ appears as follows: $c^2/\omega_{ce}^2 \ll (c^2/\omega_p^2)\omega_{ce}/\Omega$. With $H_0^3/8\pi \sim n_0T$ this coincides with the condition $\Omega \ll \omega_{ce}$ formerly accepted by us.

If the distribution function for energies $\epsilon < e_{\text{max}}$ approximates the Maxwellian one with temperature $T$, then $e_{\text{max}}/T = \alpha_p^3/\omega_{ce}$. Collisions in plasma, however rare they might be, lead to the appearance of particles with $\epsilon > e_{\text{max}}$, which pass through the effective potential barrier $e^2E_0^2/(2m\omega_{ce})$. If, however, $\omega_{ce}/\alpha_p^3 \ll 1$, there will be few such particles, and the leakage determined by the exponential factor of the Maxwellian distribution exp$[-\alpha_p^3/(\Omega\omega_{ce})]$, will be small.

We have considered here a case where the magnetic field has no gradients, i.e., in the force (4) there is no term with $\mu \cdot \nabla H$. Analysis of the movement of a single particle indicates (see Eq. (4)) that in a non-uniform magnetic field forces appear which may considerably increase the potential barrier.

Proceeding from the analysis of a kinetic equation without collisions, we have shown that in the range of frequencies $\omega_{ce} < \Omega < \omega_{ce}$ the pressure of an electromagnetic wave with a wave vector directed along a steady magnetic field may balance the pressure of plasma. In this case the plasma may be localized in a limited volume provided there are no particles with energy exceeding the height of the effective potential barrier.

**Stability**

Let us now consider the stability of the boundary of plasma whose pressure is partly or completely balanced by the time average of the pressure of an electromagnetic field. We shall attempt to solve the problem in a hydrodynamic approximation, considering the plasma merely as a high conductivity fluid. Let us assume that the skin-effect is so strong that the electromagnetic field does not penetrate inside the plasma to a considerable depth (i.e., the wave lengths of the perturbation should be much greater than the thickness of the transition region). Let us consider a two-dimensional problem, all the magnitudes being a function of $x$ and $z$. Let the $x$ axis be directed along the plane surface of the plasma and the $z$ axis be normal to it. The plasma occupies region $z > l$. The plane $z = 0$ is assumed to be rigid and ideally conductive.

The electric field (with frequency $\Omega$) is determined from the equation:

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} + \frac{\Omega^2}{c^2} E_y = 0. \tag{10}$$

The electric field is to vanish both on an ideally conductive wall and on the surface of the plasma. If a travelling electromagnetic wave is excited in the cavity $0 < z < l$ between the wall and the plasma, we obtain from Eq. (10):

$$E_y(0) = E_0 \sin (mz/l) \sin (\Omega t + \kappa x), \quad \kappa^2 = \frac{\Omega^2}{c^2} - \frac{\pi^2}{l^2}.$$
A steady and uniform magnetic field $H_0$ may be applied outside the plasma along its surface. The following conditions must be satisfied on the surface (with $z = l$):

$$8\pi \rho T = \langle H^{(0)}_\varphi \rangle H_0^2.$$  \hspace{1cm} (11)

The symbol $\langle \rangle$ denotes in this case an average in time. Let us consider perturbations of the electromagnetic field caused by the movement of the plasma surface. Suppose the perturbed surface is defined by the equation $z = l + \phi_0 \exp i(\kappa x + \omega t)$, $|\phi_0| \ll l$. Let us assume that the perturbed motion of the surface is slow compared to the frequency of the electromagnetic field. In determining the perturbations of the electromagnetic field $E_y^{(1)}$, we shall therefore assume the surface stationary and regard time as a parameter. For $E_y^{(1)}$ we have an equation similar to (10). The electric field $E_y$ should vanish on the displaced surface. Expanding $\phi_0$ in a series we obtain

$$E_y^{(1)} = -\phi_0 \exp i(\omega t + \kappa x) \frac{\partial E_y^{(0)}}{\partial x}, \text{ at } z = l.$$  \hspace{1cm} (12)

Then, taking the motion of the surface as being prescribed, we determine the movement of the plasma on which no volume forces act, fields being absent inside the plasma in the given case of a strong skin-effect. The boundary condition is obtained from the requirement that the normal component is equal to the velocity of plasma at the velocity of the surface itself. Assuming that the deformation is small, we have

$$v_x = i\omega \phi_0 \exp i(\kappa x + \omega t), \text{ at } z = l.$$  

On the surface $z = l$ we have a boundary condition for the perturbed quantities:

$$8\pi \rho T = \langle H_x^{(0)} H_x^{(1)} \rangle.$$  \hspace{1cm} (13)

The magnetic field is determined from the known electric field with the aid of Maxwell’s equations. On determining the perturbation of the plasma density (regarding the process, for the sake of simplicity, as being isothermal) we obtain from boundary condition (13):

$$\sqrt{k^2 - \frac{\omega^2}{a^2}} = AB(k),$$  \hspace{1cm} (14)

where

$$B(k) = \sqrt{(k - \kappa)^2 - q^2} \coth \sqrt{(k - \kappa)^2 - q^2} l + \sqrt{(k + \kappa)^2 - q^2} \coth \sqrt{(k + \kappa)^2 - q^2} l,$$

$$q = \Omega / c, \text{ } a^2 = T / M$$  \hspace{1cm} (15)

The quantity

$$A = \frac{\pi}{32} \rho^2 E_0^2 M \approx 10^{-6} \frac{E_v c^2}{\rho^2 \Omega^2}$$

($E_v$ is the amplitude of the electric field in volts/cm, $\rho$ is the plasma density in g/cm$^3$).

It can be seen from (14) that the sign of $\omega^2$ coincides with the sign of $B(k)$. Now $B(k)$ is negative provided

$$2\kappa < k < 2\kappa + \Delta,$$  \hspace{1cm} (16)

where $\Delta$ is some positive quantity which is easy to obtain numerically from (14) and (15) for given $\Omega$.

In case $\kappa l \gg 1$ (i.e., the plasma is far away from the wall) we obtain:

$$2\kappa < k < 2\kappa + \frac{\pi^2}{3\sqrt{2 \kappa}} \sqrt{\frac{1 - q^2}{9q^2}}.$$  \hspace{1cm} (17)

The perturbations whose wave numbers are outside the range determined by (16) or (17) are stable. Their propagation velocity is a function of the amplitude of a high-frequency electromagnetic field.

Now let us consider the case of plasma oscillation in the field of a standing electromagnetic field when $E^{(0)}_x = E_0 \sin \left(\frac{\pi x}{l}\right) \sin \Omega t$. The dispersion equation for such a case is obtained from (14) when $\partial E_x / \partial x$ vanishes. It is easy to prove that in this case the oscillations are unstable if

$$k^2 < \frac{3}{2} \left(\frac{\pi^2}{l^2}\right).$$  \hspace{1cm} (18)

Instability appears when the length of the perturbation wave becomes comparable to the distance between the plasma and the ideally conductive wall. Short-wave perturbations remain stable.

We can see that surface oscillations of plasma (or more generally of any conductor, even an incompressible fluid) in the field of a travelling or standing electromagnetic wave possess instability of a resonant
nature with respect to the wave lengths. Instability arises because during deformation of the surface the electromagnetic field increases near the points concave into the plasma, and decreases near the convex ones.

Consequently the presence of a high-frequency electromagnetic field may in some cases reduce the stability of the plasma. To retain stability, the region where the plasma contacts the electromagnetic field should not be too long. This factor should be taken into account when designing installations in which plasma contacts wave fields. Leaving out of consideration the possibility of damping, instability exists at any amplitude of the external electromagnetic field. If, however, the plasma is partly confined by an external magnetic field, the oscillations excited as a result of the above instability may remain small.

It is easy to prove that the criteria of instability so obtained are also valid in the case when there is a steady magnetic field inside the plasma. It should also be noted that analysis of a cylindrical case leads to the similar result.

EXPERIMENTS

To check the conclusions of the theory of the possible confinement of plasma by means of alternating electromagnetic fields, experiments were conducted, whose preliminary results are outlined in this chapter. Electromagnetic fields excited in volume resonators were used in the experiments. Plasma was produced in a longitudinal magnetic field inside a straight quartz tube (15 mm in diameter, 400 mm long), rectangular volume resonators being placed along its butt ends. The apparatus is shown in Fig. 1(2).

The ends of the tube were set into the resonators 15 mm deep through openings. Electromagnetic oscillations of the TE_{101} type were excited from a pulsed magnetron generator in the ten-centimetre range, supplying high-frequency pulses lasting 120 microseconds with a power up to 400 kw. An adjustment of the generator power over a wide range was provided. The maximum amplitude of the high-frequency magnetic field in the resonators attained 60 gauss. The longitudinal magnetic field was varied in a range from 0 to 2000 gauss. The experiments were carried out on various gases (argon, nitrogen and hydrogen) at pressures ranging from \(5 \times 10^{-5}\) to \(5 \times 10^{-3}\) mm Hg.

Measurements of the concentration in time were made both by measuring the shift of the characteristic frequency of the measuring resonator and by measuring the cut-off frequency for electromagnetic wave transmission (the waves being from 3 to 0.8 cm long).

During the magnetron pulse, the amplitude of the electromagnetic field in the resonator was oscillographed. The measurements made it possible to determine approximately the position of the plasma inside the resonator, and consequently provided an answer to the main question: whether the electromagnetic field of the resonator insulates the plasma from the butt end of the tube.

The oscillograms of the field amplitude for various conditions are presented in Fig. 3.

Figure 3. Oscillograms of e-m field amplitude in resonator
(a) \(H = 60\) gauss without discharge
(b) \(H = 30\) gauss; \(p = 3 \times 10^{-4}\) mm Hg
(c) \(H = 60\) gauss; \(p = 3 \times 10^{-4}\) mm Hg
(d) \(H = 60\) gauss; \(p = 1 \times 10^{-3}\) mm Hg

When oscillations were excited in the resonator, a high-frequency discharge was struck at the ends of the tube, and the resulting plasma, spreading along the magnetic field, filled the whole tube. At the initial stage of the process, the formation of plasma in the parts of the tube located inside the resonator, leads to a considerable detuning of the resonator and to a reduced amplitude of oscillations.

Further development of the process depends on the amplitude of the field in the resonator and the concentration of the plasma formed in it. For example, at a concentration of \(10^{13}\) cm\(^{-3}\) (argon, pressure \(3 \times 10^{-4}\) mm Hg) and a field amplitude amounting to 30 gauss, the resonator remains highly detuned during a part of the pulse (Fig. 3b).
With an increase in the amplitude of the field (60 gauss), the shape of the pulse changes drastically. Beginning with a certain moment, the detuning is abruptly reduced and the amplitude of the field approximates the initial one (Fig. 3c).

In this case the concentration of the plasma in the tube, measured near the resonator, increases, reaching a maximum value (~ $10^{13}$ cm$^{-3}$). As the pressure increases, the concentration of the plasma grows, accompanied by a new intensive detuning of the resonator throughout the pulse (Fig. 3d).

The nature of the process does not change in experiments with other gases (nitrogen, hydrogen).

Reduced detuning of the resonator during continued growth of the concentration in the tube indicates that the plasma is forced out of the resonator by the electromagnetic field.

Control measurements of the shift of the natural frequency of the resonator by a plasma which completely fills the end of the tube entering it, made at a small amplitude of the field, have shown that the magnitude of the shift at a concentration of $10^{10}$ cm$^{-3}$ exceeds the transmission band of the resonator. Hence, it may be stated that the concentration of the plasma outside the resonator exceeds $10^{13}$ cm$^{-3}$, it is much less than $10^{10}$ cm$^{-3}$ inside the resonator. This proves that the plasma is practically forced out of the resonator by the electromagnetic field.

An electromagnetic field with an intensity $H$ may confine plasma with a pressure $nT = \langle H^2 \rangle / 8\pi$. With $H = 60$ gauss we have $nT \approx 70$ dyn/cm$^2$, which agrees qualitatively with a pressure corresponding to the measured concentration of ~ $10^{13}$ cm$^{-3}$ and a temperature of electrons estimated in time by the afterglow of plasma (~ 5 eV).

Preliminary experiments point to the possibility of confining plasma by means of electromagnetic fields of large amplitude.

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Summary of UCRL Pyrotron (Mirror Machine) Program*

By R. F. Post*

Under the sponsorship of the Atomic Energy Commission, work has been going forward at the University of California Radiation Laboratory since 1952 to investigate the application of the so-called "magnetic mirror" effect to the creation and confinement of a high temperature plasma. By the middle of 1953 several specific ways by which this principle could be applied to the problem had been conceived and analyzed and were integrated into a proposed approach which has come to be dubbed "The Mirror Machine".1 Study of the various aspects of the physics of the Mirror Machine has been the responsibility of a Laboratory experimental group (called the "Pyrotron" Group) under the scientific direction of the author. During the intervening years of effort, experiments have been performed which demonstrate the confinement properties of the Mirror Machine geometry and confirm several of its basic principles of operation. There remain, however, many basic quantitative and practical questions to be answered before the possibility of producing self-sustained fusion reactions in a Mirror Machine could be properly assessed. Nevertheless, the experimental and theoretical investigations to date have amply demonstrated the usefulness of the mirror principle in the experimental study of magnetically confined plasmas. This report presents some of the theory of operation of the Mirror Machine, and summarizes the experimental work which has been carried out.

PRINCIPLE OF THE MAGNETIC MIRROR

The modus operandi of the Mirror Machine is to create, heat and control a high temperature plasma by means of externally generated magnetic fields. The magnetic mirror principle is an essential element, not only in the confinement, but in the various manipulations which are performed in order to create and to heat the plasma.

Confinement

The magnetic mirror principle is an old one in the realm of charged particle dynamics. It is encountered for example, in the reflection of charged cosmic ray particles by the earth's magnetic field. As here to be understood, the magnetic mirror effect arises whenever a charged particle moves into a region of magnetic field where the strength of the field increases in a direction parallel to the local direction of the field lines, i.e., wherever the lines of force converge toward each other. Such regions of converging field lines tend to reflect charged particles, that is, they are "magnetic mirrors". The basic confinement geometry of the Mirror Machine thus is formed by a cage of magnetic field lines lying between two mirrors, so that configuration of magnetic lines resembles a two-ended wine bottle, with the ends of the bottle defining the mirror regions. This is illustrated in Fig. 1, which also shows schematically the location of the external coils which produce the confining fields. The central, uniform field, region can in principle be of arbitrary length, as dictated by experimental convenience or other considerations. For various reasons, it has been found highly desirable to maintain axial symmetry in the fields, although the general principle of particle confinement by mirrors does not require this.

The confinement of a plasma between magnetic mirrors can be understood in terms of an individual particle picture. The conditions which determine the binding of individual particles between magnetic mirrors can be obtained through the use of certain adiabatic invariants applying generally to the motion of charged particles in a magnetic field. The first of these invariants is the magnetic moment, \( \mu \), associated with the rotational component of motion of a charged particle as it carries out its helical motion in the magnetic field.8 The assumption that \( \mu \) is an absolute constant of the motion is not strictly valid, but, as later noted, it represents a very good approximation in nearly all cases of practical interest in the Mirror Machine.

The magnitude of \( \mu \) is given by the expression

\[
\mu = W_\perp/H = \frac{2}{3}m v_\perp^2 H \text{ ergs/gauss} \tag{1}
\]

which states that the ratio of rotational energy to magnetic field remains a constant at any point along the helical trajectory of the charged particle. In this case the magnetic field is to be evaluated at the line of force on which the guiding center of the particle is moving.

Now, at the point at which a particle moving toward a magnetic mirror is reflected, its entire energy of

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motion, \( W \), is in rotation, so that at this point \( \mu H = W \). The condition for binding charged particles between two magnetic mirrors of equal strength is therefore simply that for bound particles

\[
\mu H_M > W \tag{2}
\]

where \( H_M \) is the strength of the magnetic field at the peak of the mirror. This condition may also be expressed in terms of field strength and energies through use of (1). If \( H_0 \) is the lowest value of the magnetic field between the mirrors, and \( H_M \) is the peak value at the mirror (both being evaluated on the same flux surface) then (2) becomes

\[
W/\bar{W}_0(0) = R_M \tag{3}
\]

where \( R_M = H_M/H_0 \) (the "mirror ratio") and \( \bar{W}_0(0) \) is the value of \( W \) at the point where \( H \) takes on the value \( H_0 \).

It is to be noted that the condition that particles be bound does not depend on their mass, charge, absolute energy, or spatial position, nor on the absolute strength or detailed configuration of the magnetic field. Instead, it depends only on the ratios of energy components and magnetic field strengths. Such an insensitivity to the details of particle types or orbits is what is needed to permit the achievement of a confined plasma; i.e., a gas composed of charged particles of different types, charges and energies. The simplicity of the binding conditions reflects the insensitivity to detailed motion required to accomplish this.

Further insight into the nature of confinement of a plasma by mirrors can be obtained by manipulation of the reflection condition. By differentiation, holding the total particle energy constant, it can be shown that a particle moving along a magnetic line of force (or flux surface) into a magnetic mirror always experiences a retarding force which is parallel to the local direction of the magnetic field and is given by the expression

\[
F_u = -\frac{\partial H}{\partial u} \mu H \tag{4}
\]

where \( u \) is the axial distance from the plane of symmetry.

This force, which is the same as that expected on a classical magnetic dipole moving in a magnetic field gradient, represents the reflecting force of the mirror. Integrated up to the peak of the mirrors, (4) yields again the binding condition (3).

Since \( \mu \) has been assumed constant, (4) may also be written in the form

\[
F_u = -V_u(\mu H) \tag{5}
\]

This shows that the quantity \( \mu H \) acts as a potential so that the region between two magnetic mirrors of equal strength lies in a potential well between two potential maxima of height \( \mu H_{\text{max}} \).

Loss Cones

It is evident, either from (2), (3) or (5), that it is not possible to confine a plasma which is isotropic in its velocity conditions—one in which all instantaneous spatial directions of motion are allowed—between two magnetic mirrors: confinement of an isotropic plasma is only possible to contemplate in special cases where multiple mirrors might be employed. This limitation can be understood in terms of the concept of loss cones. The pitch angle, \( \theta \), of the helical motion of particles moving along a line of force is transformed in accordance with a well-known relationship

\[
\sin \theta(u) = \frac{R(u)}{\sin \theta(0)} \tag{6}
\]

where the angle \( \theta(u) \) is measured with respect to the local direction of the magnetic field. \( R(u) \) is the mirror ratio, evaluated at \( u \), \( R(u) = H(u)/H(0) \).

Expression (6) bears a resemblance to Snell's Law of classical optics, which relates refraction angles within optically dense media. Here \( \sqrt{R} \) is analogous to the index of refraction of Snell's Law. As in the optical case, total internal reflection can occur for those angles larger than a critical angle, \( \theta_c \), found by setting \( \sin \theta \) equal to its maximum value of 1. Thus

\[
\sin \theta_c = R_{\text{max}}^{-1} \tag{7}
\]

This relation defines the loss cones for particles bound between magnetic mirrors, illustrated in Fig. 1. All particles with pitch angles lying outside the loss cones are bound, while all with pitch angles within the loss cone will be lost upon their first encounter with either mirror (for equal mirror strengths). It should be emphasized that the concept of the mirror loss cone
pertains to the velocity space of the trapped particles and has nothing to do with the spatial dimensions of the confining fields.

Although the binding of particles between magnetic mirrors can be accomplished with fields which are not axially symmetric, there are substantial advantages to the adoption of axial symmetry. One of these advantages arises from the fact that all "magnetic bottles" involve magnetic fields with gradients or curvature of the magnetic field lines in the confinement zone, the existence of which gives rise to systematic drifts of the confined particles across the magnetic lines of force. If the particle orbit diameters are small, compared to the dimensional scale of the field gradients or field curvature, these drifts will be slow compared to the particle velocities themselves. If directed across the field, however, they would still be too rapid to be tolerated and would effectively destroy the confinement. Furthermore, drifts of this type are oppositely directed for electrons and ions and may thus give rise to charge separation and electric fields within the plasma. These electric fields then may induce a general drift of the plasma across the field to the walls. However, if the magnetic field is axially symmetric, as for example in the mirror field configuration of Fig. 1, the particle drifts will also be axially symmetric, leading only to a rotational drift of the plasma particles around the axis of symmetry, positive ions and electrons drifting in opposite directions. However, this will not result in a tendency for charge separation to occur, since each flow closes on itself.

Another consequence of the use of axial symmetry is that even though the confined particles may be reflected back and forth between the mirrors a very large number of times, this fact will not lead to a progressive "walking" across the field. In fact, it is possible to show that all particles trapped between the mirrors are also bound to a high order of approximation to the flux surfaces on which they move and may not move outward or inward to another flux tube (apart from the normal slow diffusion effects arising from interparticle collisions).

Once bound, there is no tendency for particles to escape the confining fields, within the assumptions made to this point. However, in predicting the conditions for confinement of a plasma by means of conditions applying to the individual particles of the plasma, one makes the tacit assumption that cooperative effects will not act in such a way as to destroy the confinement. Such cooperative effects can modify the confinement through static or time-varying electric fields arising from charge separation, or through diamagnetic effects which change the local strength of the confining fields themselves. However, if the confining magnetic fields are axially symmetric, and if the presence of the plasma does not destroy this symmetry then, as has already been explained, charge separation effects cannot give rise to systematic drifts across the field. Similarly, in such a circumstance the diamagnetic effect of the plasma can only lead to an axially symmetric depression of the confining fields in the central regions of the confinement zone, but will leave the fields at the mirrors essentially unaltered (since the density falls nearly to zero at the peak of the mirrors, where particles are escaping). Symmetric diamagnetic effects therefore tend to increase the mirror ratios above the vacuum field value. Of course, there will exist a critical value of the ratio of plasma pressure to magnetic pressure, above which stable confinement is not possible. This critical relative pressure will be dependent on various parameters of the system, such as: the mirror ratio; the shape, symmetry and aspect ratio of the fields; and the plasma boundary conditions. Although the precise conditions for stability of plasmas confined by magnetic mirrors are not at this time sufficiently well understood theoretically to predict them with confidence, there is now agreement that it should be possible to confine plasmas with a substantial value of the ratio between magnetic mirrors. This conclusion is borne out by the experiments here reported, which indicate stable confinement. It should be emphasized, however, that, encouraging as this may be, neither the theoretical predictions nor the experimental results are yet sufficiently advanced to guarantee that plasmas of the size, temperature and pressure necessary to produce self-sustaining fusion reactions in a Mirror Machine would be stable. As in all other known "magnetic bottles" the ultimate role of plasma instabilities has not yet been determined in the Mirror Machine.

### LOSS PROCESSES

#### Non-Adiabatic Effects

Although it has been shown that trapping conditions based on adiabatic invariants predict that a plasma might, in principle, be confined within a Mirror Machine for indefinitely long periods of time, it is clear that mechanisms will exist for the escape of particles, in spite of the trapping.

One might first of all question the validity of the assumption of constant magnetic moment, since trapping depends critically on this assumption. It is clear that this assumption cannot be exactly satisfied in actual magnetic fields, where particle orbits are not infinitesimal compared to the dimensions of the confining magnetic field.

It can be seen from first principles, as for example shown by Alfvén that the magnetic moment will be very nearly a constant in situations where the magnetic field varies by only a small amount in the course of a single rotation period of the particle. It might also be suspected, in the light of the analogy between this problem and the general theory of non-adiabatic effects of classical mechanics, that the deviations from adiabaticity should rapidly diminish as the relative orbit size is reduced. Kruskal has shown that the convergence is indeed rapid but his results are not readily applicable to the Mirror Machine. Using numerical methods, however, it has been shown that the fluctuations in magnetic moment associated
with non-adiabatic effects are of importance only for relatively large orbits, (b) that these fluctuations seem to be cyclic in nature, rather than cumulative, and (c) that they diminish approximately exponentially with the reciprocal of the particle orbit size, so that it should always be possible to scale an experiment in such a way as to make non-adiabatic orbit effects negligible. The results may be roughly summarized by noting that, for the typical orbits which were considered, the maximum amplitude of the fractional changes in the magnetic moment, which occurred after successive periods of back-and-forth motion of trapped particles, could be well represented by an expression of the form

$$|\Delta \mu/\mu| = ae^{-b/\nu}$$  \hspace{1cm} (8)

where $\nu = 2\pi R/L$, i.e., the mean circumference of the particle orbit divided by the distance between the mirrors. The mirror fields were represented by the function $H = H_0 \left[1 + \alpha \cos \mu I(p)\right]$ plus the appropriate curl-free function for $H_r$. Here, in dimensionless form, $\nu = 2\pi R/L$ and $\rho = 2\pi r/L$. With $\alpha = 0.25$, typical values of the constants $a$ and $b$ were in the range $4 < a < 6$, $1.5 < b < 2$, for various radial positions of the orbits. From these values it can be seen that provided $\nu < 0.2$, the variations in $\mu$ are totally negligible, so that the adiabatic orbit approximation is well satisfied. For example, if $L = 100$ cm then all orbits with mean radii less than about 3 cm can be considered to behave adiabatically. This imposes restrictions on the minimum values of magnetic fields which can be used, or on the minimum size of the confinement zones, but it is clear that, as the scale of the apparatus is increased, the assumption of orbital adiabaticity becomes increasingly well satisfied.

Collision Losses

Even under conditions where the adiabatic invariants establish effective trapping of particles, there still remains a simple and direct mechanism for the loss of particles from a mirror machine. This is, of course, the mechanism of interparticle collisions, the mechanism which limits the confinement time of any stable, magnetically confined plasma. Collisions can induce changes in either the magnetic moment or energy of a trapped particle, and thus cause its velocity vector to enter the escape cone. The dominant collision cross section in a totally ionized plasma is the Coulomb cross section, which varies inversely with the square of the relative energy of the colliding particles. Thus the rate of losses (if dominated by collisions) can be reduced by increasing the kinetic temperature of the plasma. The basic rate of these loss processes may be estimated by consideration of the "relaxation" time of energetic particles in a plasma, as calculated by Chandrasekhar or by Spitzer. In this case, the relevant quantity is the mean rate of dispersion in pitch angle of a particle as a result of collisions, since, if a given trapped particle is scattered through a large angle in velocity space, the probability is large that it will have been scattered into the escape cone and thus lost. As is usual in a plasma, distant collisions (within a Debye sphere) play a greater role than single scattering events. The time for the growth of the angular dispersion in this (multiple) scattering process for a particle of given energy is governed by the relation

$$\theta^2 = t/t_D$$  \hspace{1cm} (9)

where

$$t_D = \frac{M_z^2}{\pi n e^4} \mathcal{G}(x) \log \Lambda,$$  \hspace{1cm} (10)

$M$ and $v$ are the mass and velocity of the scattered particle, $x^2 = W/kT = \frac{1}{2}Mv^2/kT$ and log $\Lambda$ has the usual value of about 20. $\mathcal{G}(x)$ is a slowly varying function of $x$ and is approximately equal to 0.5 for typical values.

In terms of deuteron energies in kev,

$$t_D = 2.6 \times 10^3 \nu_W^2/\mu s \text{ seconds.}$$  \hspace{1cm} (11)

For $\theta^2 = 1$, the scattering time becomes equal to $t$. Thus $t_D$ represents a rough estimate of the confinement time of ions in a Mirror Machine. It is clear that the confinement time will also depend on the mirror ratio $R$, but detailed calculations show that for large values of this ratio, the confinement varies only slowly with $R$.

Some numerical values are of interest in this connection. Suppose $W = 0.01 \text{ kev}$, and $n = 10^{14}$ a mean particle energy and density which might be achieved in a simple discharge plasma. In this case $t_D = 0.26 \mu s$. This means that the confinement of low temperature plasmas by simple mirrors will be of very short duration, unless means for rapid heating are provided, which would extend the confinement time. On the other hand, for $W = 150$ kev, the same particle density would give $t_D = 0.5 \text{ sec}$, which is long enough to provide adequate confinement for experimental studies and is within a factor of about 20 of the mean reaction time of a tritium-deuteron plasma at a corresponding temperature. Since the energy released in a single nuclear reaction would be about 100 times the mean energy of the plasma particles, it can be seen that the possibility exists for producing an energetically self-sustaining reaction, with a modest margin of energy profit, in this case about 4 to 1.

The problem of end losses may be more precisely formulated by noting that these losses arise from binary collision processes, so that it should be possible at all times to write the loss rate in the form

$$n = -v \langle \sigma v \rangle_a f(R)$$  \hspace{1cm} (12)

where $\langle \sigma v \rangle_a$ represents a scattering rate parameter and $f(R)$ measures the effective fractional escape cone of the mirrors for diffusing particles. If $\langle \sigma v \rangle_a$ remains approximately constant during the decay, then integration of the equation shows that a given initial density will decay with time as

$$n = n_0 \tau/(t + \tau)$$  \hspace{1cm} (13)

where

$$\tau = n_0 \langle \sigma v \rangle_a [f(R)]^{-1}.$$  \hspace{1cm} (14)
The relaxation time approximation consists of setting \( f(R) = 1 \) \( - \) i.e., ignoring the dependence on mirror ratio, thereby overestimating the loss rate—
and, at the same time, approximating the true value of \( n_0(\sigma v) / f_0 \), which tends to underestimate the loss rate. Thus, in this approximation, the transient decay of the plasma in a Mirror Machine is given by

\[
n = n_0 f_0 (t + t_0).
\]

In this approximation, \( f_0 \) represents the time for one-half of the original plasma to escape through the mirrors. Now, the instantaneous rate of nuclear reactions which could occur in the plasma is proportional to \( n^2 \). Integrating \( n^2 \) from (15) over all time, it is found that the total number of reactions which will occur is the same as that calculated by assuming that the density has the constant value \( n_0 \) for time \( f_0 \) and then immediately drops to zero; i.e., number of reactions is proportional to \( n_0 f_0 \).

Actually, although they correctly portray the basic physical processes involved in end losses, estimates of confinement time based on simple relaxation considerations are likely to be in error by factors of two or more. To obtain accurate values of the confinement time more sophisticated methods are required. Judd, McDonald, and Rosenbluth, and others have applied calculations to this problem and have derived results which should be much closer to the true situation, even though it was necessary to introduce simplifying approximations in their calculations. They find that calculations based on the simple relaxation considerations tend to overestimate the confinement times. They are also able to calculate explicitly the otherwise intuitive result that the mean energy of the escaping group of particles is always substantially less than the mean energy of the remaining particles, a circumstance which arises because low energy particles are more rapidly scattered than high energy ones.

The most detailed results in these end loss calculations have been obtained by numerical integration. However, before these results were obtained, D. Judd, W. McDonald, and M. Rosenbluth derived an approximate analytic solution which, in many cases, differs only slightly from the more accurate numerical calculations. Their results can be expressed by two equations which describe the dominant mode of decay of an eigenvalue equation for the diffusion of particles in the velocity space of the Mirror Machine.

The first equation, that for the density, is:

\[
n = -n^2 \pi (e^4/m) \langle v^2 \rangle \langle v^{-3} \rangle \log \Lambda \lambda(R), \]

where the angle brackets denote averages over the particle distribution and \( \log \Lambda \) is the screening factor.\(^8\) The eigenvalue \( \lambda(R) \) is closely approximated by the expression \( \lambda(R) = 1/\log_{10}(R) \), showing that the confinement time varies linearly with the mirror ratio, at small mirror ratios, but that it varies more slowly at large values of \( R \). We see immediately from the form of the equation that one can define a scattering time as in Eq. (14):

\[
\tau_s = \left\{ \left[ \pi (n_0 e^4/m^2 e^2) \log \Lambda \right] \left[ \pi (e^4/v^4) \langle v^3 \rangle \lambda(R) \right] \right\}^{-1}.
\]

The term in the first bracket is almost identical with that for the relaxation time of a particle with the mean velocity \( v_0 \). The velocity averages give rise to small departures from the simple relaxation values but, as noted, the corrections are usually not large.

In a similar way, an equation can be written for the rate of energy transport through the mirror by particle escape. The basic rate for this is, of course, simply given by the value of \( n \langle v^2 \rangle \); but \( \langle v^2 \rangle \), the mean energy of escaping particles, as noted, is not the same as the mean energy of the remaining particles. They find, for the rate of energy loss:

\[
n \langle v^2 \rangle = -n^2 \pi (e^4/m) \langle v^{-3} \rangle \log \Lambda \lambda(R).
\]

Since the value of \( \langle v^{-3} \rangle \) is not particularly sensitive to the velocity distribution, one may calculate this for a Maxwellian distribution without committing excessive error.

The mean energy of the escaping ions, \( \langle v^2 \rangle \), may be evaluated from the equations. Dividing (18) by (16), \( \langle v^2 \rangle \) is seen to be \( \frac{3}{8} m \langle v^4 \rangle^{-1} \). Approximating the actual distribution by a Maxwellian, \( \langle v^2 \rangle \) turns out to be \( \frac{1}{5} kT \), or \( \frac{1}{6} \) of the Maxwellian mean energy. This is roughly the same value as was obtained by the more accurate numerical calculations.

As far as eventual practical applications are concerned, the over-all result of the detailed end loss calculations is to show that although a power balance can in principle be obtained, both with the DT and DD reactions, by operation at sufficiently high temperatures, the margin is less favorable than the rough calculations indicated. Thus, any contemplated application of the Mirror Machine to the generation of power would doubtless require care in minimizing or effectively recovering the energy carried from the confinement zone by escaping particles. Some possible ways by which this might be accomplished will be discussed in sections to follow. Other methods to reduce end losses through magnetic mirrors have been suggested and are under study. One of these, involving a rotation of the plasma to "enhance" the mirror effect, is under study at this Laboratory and at the Los Alamos Scientific Laboratory.

In the present Mirror Machine program, scattering losses impose a condition on the experimental use of simple mirror systems for the heating and confinement of plasmas. This condition is that operations such as injection and heating must be carried out in times shorter than the scattering times. These requirements can be readily met, however, in most circumstances so that end losses do not present an appreciable barrier to the studies.

**Ambipolar Effects**

Since the scattering rate for electrons is more rapid than for ions of the same energy, the intrinsic end loss rate for electrons and ions will be markedly different. In an isolated plasma this situation will not persist,
since a difference in escape rates will inevitably result in establishing a plasma potential which equalizes the rate. In ordinary discharge plasmas, where this effect also appears, this "ambipolar" loss rate is always dominated by the slower species of the plasma. This situation seems to apply also to plasmas confined in the Mirror Machine. Although many different cases are possible, not all of which are understood, the role of ambipolar effects seems mainly to be to introduce a (usually) small correction to the end loss rate as calculated from ion-ion collisions. For example, in the important case where the electron temperature is small compared with the ion temperature, Kaufman has shown that ambipolar phenomena establish a positive plasma potential which, in effect, changes the mirror ratio for the ions to a somewhat smaller value given by the expression

\[ R_{\text{eff}} = R[1 + \gamma(T_e/T_i)]^{-1}, \]  

where \( \gamma \) is a constant of order unity. For cases of practical interest, the resulting correction to the ion loss rates is small. However, the precise role of ambipolar diffusion effects and the electric potentials to which they give rise is not well understood, especially with regard to plasma stability, and must, therefore, be labeled as part of the unfinished business of the Mirror Machine program.

Confinement of Impurities

In the study of magnetically confined plasmas, and in their eventual practical utilization, contamination of the plasma by unwanted impurities presents a serious problem. Since the plasma particle densities which are of present and future interest are about \( 10^{14} \text{ to } 10^{18} \text{ cm}^{-3} \), the presence of impurities to the extent of even a small fraction of a microgram per liter represents a sizeable amount of contamination, percentage-wise. Impurities usually originate from adsorbed layers on the chamber walls, the release of even a small fraction of an atomic monolayer corresponding to a large degree of contamination.

Impurities of high atomic number greatly increase the radiation losses from the plasma. At temperatures of a few million degrees the radiation loss mechanism is principally from electron-excited optical transitions of incompletely stripped impurity ions. In this case, the intrinsic radiation rates per impurity ion can be as high as one million times the radiation rate of a hydrogen ion. At thermonuclear temperatures, the stripping becomes more nearly complete, so that the radiation rate from impurities becomes less intense, finally approaching the bremsstrahlung value, \( Z^2 \) times the loss rate for hydrogen. But even in this case, because of the relatively narrow margin between the thermonuclear power production and the radiation loss rates from even a perfectly pure hydrogenic plasma, the concentration of high-Z impurities must be kept to a minimum. The actual level of impurities in any magnetically confined plasma will be determined by a competition between their rate of influx and the degree to which they are confined by the magnetic fields.

In the light of these facts, it is important to note that in the Mirror Machine, the confinement of impurity ions should be much poorer than for energetic hydrogen ions. Thus there tends to exist a natural "purification" mechanism discriminating against the presence of impurities in the plasma.

This happy circumstance arises from three general causes. First, the scattering rate for stripped high-Z ions is more rapid than for hydrogen ions of the same energy. Second, impurity ions originating from the walls will always have much lower energies than the mean energy of the confined ions. This will increase their loss rate by scattering. Third, in the "normal" high temperature confinement condition in the Mirror Machine, where ion energies are much greater than the electron temperature, the sign of the plasma potential should be positive, so that low energy positive ions cannot be bound at all, but will be actively expelled. These circumstances could be of an importance equal to that of adequate confinement in the eventual problems of establishing a power balance from fusion reactions.

BASIC OPERATIONS

In order to create, heat and confine a hot plasma in any magnetic bottle a series of operations is always required. In the Mirror Machine a somewhat different philosophy of these operations has been adopted from that used in most other approaches, for example, those utilizing the pinch effect. In these approaches, one starts with a chamber filled with neutral gas and then attempts to ionize the gas, heat and confine the resulting plasma. By contrast, in the Mirror Machine a highly evacuated chamber is employed, into the center of which a relatively energetic plasma is injected, trapped and subsequently further heated. Since the Mirror Machine possesses "open" ends through which the plasma can be introduced by means of external sources, injection methods can be employed which are not possible in systems of toroidal topology.

By using the magnetic mirror effect in various ways, it is possible to perform several basic operations on a plasma. These are employed in the Mirror Machine to create, heat, control and study the plasma. In addition to simple confinement the following operations are used.

1. Radial compression—adiabatic compression of the plasma performed by uniformly increasing the strength of the confining fields.

2. Axial compression—adiabatic compression performed by causing the mirrors to move closer together.

3. Transfer or axial acceleration—pushing the plasma axially from one confinement region to another by moving the mirrors.

4. "Valve" action—controlling the direction of diffusion of the plasma by weakening or strengthening one mirror relative to the other.

In the experimental study of these operations most of the emphasis has been placed on achieving conditions where the assumption of adiabaticity is valid and
where collision losses and ambipolar effects are minimized by control of the heating cycle. This operational regime is a relatively simple one to understand, is readily analyzed theoretically, and presents some substantial practical advantages.

The primary method of heating used in the Mirror Machine is adiabatic compression; i.e., the heating occurs when the confining magnetic fields are changed at a rate which is relatively slow compared with the periods of rotation of the plasma ions.

The adiabatic magnetic heating of a plasma can be understood either from an individual particle viewpoint, or from general thermodynamic principles. Consider, first, the behavior of trapped particles in a confining field which is increasing with time. In such a field, the energy of each trapped particle will increase, simply because of the constancy of the magnetic moment, \( \mu = W_\perp/H = \text{constant} \) (Eq. (1)). Thus, rotational energy will increase in direct proportion to the magnetic field:

\[
W_\perp(t) = W_\perp(0)H(t)/H(0) = W_\perp(0)\kappa(t).
\]  
(20)

At the same time it can be shown that adiabatic magnetic heating of this type proceeds so that the flux through each elementary orbit remains constant and also so that the guiding centre of each particle remains on the same flux tube of the increasing magnetic field, i.e., the plasma is uniformly compressed toward the magnetic axis of the confining field.\(^{11}\)

This leads to an increase in the plasma density as well as an increase in its mean energy. This is the radial compression listed above. Since energy is imparted to the rotational component of particle motion, this type of heating leads to more effective binding of the particles.

Axial compression, accomplished by mechanically or electrically moving the mirror fields toward each other, results in the same type of heating as discussed by Fermi in his theory of the origin of cosmic rays.\(^4\) Axial compression leads to an increase in the parallel energy component of each trapped particle by an amount proportional to the square of the compression factor. If the mirrors are separated by a distance \( L(t) \) then

\[
W_\parallel(t) = W_\parallel(0)[L(0)/L(t)]^2 = W_\parallel(0)\kappa^2(t).
\]  
(21)

In this case the heating leads to poorer trapping, so that it can only be used to a limited extent.

The increase in density which results from these two compression processes is given by the product of the radial and axial compression factors. The radial compression is determined by the constant flux condition, which implies that the radius of the plasma will vary as \( H^{-1} \) and its area therefore as \( H^{-2} \). The total compression is then given by

\[
\frac{n(t)}{n(0)} = \frac{H(0, t)}{H(0, 0)} = \kappa = \alpha \kappa.
\]

Both types of adiabatic compression heating can be related to simple thermodynamic gas laws. In radial compression, the gas behaves as a two-dimensional gas, since there are two degrees of freedom associated with rotational energy. For this case, the gas constant, \( \gamma \), has the value 2 and the heating varies linearly with the density \( T_\perp \sim n_\perp \sim n \). In the case of axial compression, there is only one degree of freedom (motion along the lines of force) so that \( \gamma = (2+\nu)/\nu = 3 \) and \( T_\parallel \sim n^3 \). If collisions become important during the compression heating, all three degrees of freedom will be coupled and the heating will proceed as in an ordinary gas; \( T \sim n \).

**Compression**

Magnetic compression processes, as used in the operation of the Mirror Machine, can be calculated by the use of a single integral which incorporates all of the adiabatic assumptions. The adiabatic invariants which apply to the motion of trapped particles of the plasma moving between the mirrors are (1) the magnetic moment, \( \mu = W_\perp/H \) and (2) the action integral of the particle momentum along magnetic lines of force, taken over a period of the bound motion along a line of force, i.e., \( \oint d\mu = A \). The constancy of \( A \) is dependent on the assumption that the magnetic field shall change only by a small relative amount during the time of one period, a condition which is usually satisfied in the experiments. When this is true, the total energy of each particle is also slowly varying so that the angle transformation relationship, Eq. (6), applies to the motion.

All three of the above conditions can be combined in a single equation which predicts the essential character of the compression process.\(^12\) Since we are not interested in the detailed motion of each particle, but only in the salient features of its motion, it is convenient in this equation to represent the orbit of each trapped particle solely by the location of its end point or "high water mark" in the confining mirror fields. The subsequent fate of the plasma can then be predicted by following the evolution of these end points. The equation derived is:

\[
[H(0)]^{1/2} \int_{u_1}^{u_2} [R_m - R(u)]^{1/2} du = S = \text{constant}
\]  
(22)

where \( u_1 \) and \( u_2 \) are the extreme values of \( u \) reached by the particle, and \( R(u) \) is the function, \( H(u)/H(0) \); i.e., the field strength at any point \( u \), relative to its value at \( u = 0 \), where the field has its weakest value. \( R_m \) is the value of \( R(u) \) at \( u_1 \) or \( u_2 \) (i.e., at the high water marks). The value of the constant \( S \) is to be chosen from the initial conditions.

Equation (22) may also be written in other forms which are sometimes more convenient to use. Written as an integral over the variable \( R \), it becomes

\[
[H(0)]^{1/2} \int_{R_m(1)}^{R_m(0)} (R_m - R)^{1/2} \frac{dR}{dR} dR = S,
\]  
(23)

where \( dR/du \) expressed as a function of \( R \). Here, of course, \( R_m(1) \) and \( R_m(2) \) have the same numerical value. When the confining fields have a plane of symmetry at \( u = 0 \) the lower limit of (22)
may be taken as zero, and the upper limit as $u_m$, with corresponding limits $I$ and $R_m$ for Eq. (23).

By use of the compression equation, the motion of the end points, $u_m$ or $R_m$ can be determined. Combining this motion with the constant flux condition, the transformation of the radial distribution can also be obtained. To show that this last statement is true, it is necessary to demonstrate that, in the adiabatic, collision-free limit here assumed, there is no 'mixing' of the distribution from one flux surface to another, even after many reflections have occurred. This can be shown using the compression equation. Consider the possible orbits which a particle of given fixed magnetic moment, $\mu$, energy $W$, and action integral $A$ can execute in the field. These orbits will be describable by the compression equation. Now we may evidently write (23) in the form

$$\int_{H_m(0)}^{H_m(1)} (H_m - H) \left( \frac{du}{dH} \right) dH = S.$$  \hfill (24)

Consider two possible orbits (a) and (b) for the above particle, characterized by integrals $S_a$ and $S_b$ respectively. Only $du/dH$ may differ between the two orbits. Therefore, we may subtract integral $S_a$ from $S_b$ to arrive at a condition which these derivatives must satisfy. This condition is, since $S_a = S_b$,

$$\int_{H_m(0)}^{H_m(0)} (H_m - H) \left( \frac{du}{dH} \right) dH = 0. \hfill (25)$$

If the field system is axially symmetric, then this equation can be satisfied only for those orbits for which the value $\left( \frac{du}{dH} \right)_{a}$ and $\left( \frac{du}{dH} \right)_{b}$ are the same at every point along $u$, i.e., on the surface of flux tubes. Thus, in this case, the particles are constrained to move on flux surfaces so that mixing does not occur, as was to be proved. But even if the magnetic field is only approximately axially symmetric, then, as pointed out by Teller,\textsuperscript{18} Eq. (25) still defines a set of separate closed surfaces on which the particles are constrained to move. These surfaces are generated from any initial given orbit by finding adjacent orbits along flux lines for which (25) is satisfied.

It is evident that, if the magnetic field changes adiabatically, (25) is still valid quasi-statically, so that the motion of the particle orbits is still constrained to lie on fixed but slowly collapsing flux tubes of the system. This allows the prediction of radial compression effects by use of the compression equation. Thus each initial end point of an orbit say $[u_m(0), r(0)]$ will be transformed to a new point, $[u_m(t), r(t)]$ such that

$$\left[ \int_{0}^{(t)} 2\pi r H \phi dr \right]_{u_m(0)} = \left[ \int_{0}^{(t)} 2\pi r H \phi dr \right]_{u_m(0)}.$$  \hfill (26)

To be used with the compression equations is the relationship giving the particle energy as a function of time. From the constancy of $\mu$ it follows immediately that the final kinetic energy of any particle is related to its initial energy by the equation

$$W(t) = W(0) \left[ \frac{R_m(t)}{R_m(0)} \right] \left[ \frac{H(0, t)}{H(0, 0)} \right].$$  \hfill (27)

It is important to note that, under the assumptions made, particle energies do not appear in the integrals (22) or (23), so that if the trapped particle density is represented by its distribution in $u_1$, $u_2$, or $R_m$, this distribution transforms in time in accordance with these equations, independent of the energy distribution. The transformation of the energy distribution which results from the compression can then be found by using (27) together with the results from the compression equation.

When the external applied magnetic field is substantially altered by the pressure of the plasma, it is necessary to use self-consistent field values in computing the compression, found by iteration of the solutions to the compression equation, or by other means. For many purposes, however, this difficult procedure would not seem to be necessary, since its omission does not lead to large errors.

Radial Compression

The heating and the increase in density resulting from a simple radial compression can be found from the compression equations above, if the shape of the confining field is known.

In some cases the strength of field in the central region can be represented by a parabolic distribution, i.e., by the function $R(u) = 1 + (u/\lambda)^2$. Inserting this into the compression equation one finds that, in addition to radial compression, even with the mirrors held stationary, an axial compression occurs in accordance with the relationship

$$[H(0)] (R_m - 1)^2 = \text{constant};$$

i.e., $H(0)u_m^4 = \text{constant}$, so that for the axial compression factor one obtains

$$\kappa = \frac{u_m(0)}{u_m(t)} = \left[ \frac{H(0, t)}{H(0, 0)} \right]^{1/2} = \alpha^4. \hfill (28)$$

This results in a uniform axial compression of the entire trapped-plasma. In those cases where very large compression factors are used, the extra increase in density which results is substantial. Combining the radial compression with the axial compression factor above, the density is seen to vary as

$$n(t) = n(0) \alpha \kappa = n(0) \alpha^5. \hfill (29)$$

Taking an example from some of the experiments, if $\alpha = 10^4$, then the axial compression amounts to about a factor of six, a very noticeable effect.

Combined Radial and Axial Compression

Although the trapping is always improved by a simple radial compression, the trapping may be made less effective, when both radial and axial compression are employed, unless certain restrictions are observed. The effect of a combined radial and axial compression can be predicted from the compression equations (22) or (23) by allowing $R(u)$ to be a function of the time. Consider the case where the behavior of $R(u)$ is
representable by a simple scaling of the axial coordinates of the field. This represents, approximately, a practical situation in which the mirror ratios are left fixed, but the mirrors are moved together as a function of time.

To represent this situation, let \( R(u) \) be given by the function \( g(u/\lambda) \) where \( \lambda \) changes with time (\( \lambda \) decreasing corresponds to axial compression, \( \lambda \) increasing corresponds to expansion); then, if \( g^{-1}(R) \) is the function inverse to \( g(u/\lambda) \), so that \( \lambda g^{-1}(R) = u \), then \( du/\lambda = \lambda g^{-1}(R) \). In this case, \( g^{-1}(R) \) is not, of course, a function of the time, but is constant during the compression. Thus, (23) becomes

\[
\mathcal{A}(H(0)) \int_{R_{m}(0)}^{R_{m}(u)} (R_{m} - R) \left( \frac{dR}{R} \right) dR = S, \tag{30}
\]

but now the integrand is no longer an explicit function of the time.

The condition that all plasma particles remain at least as tightly bound after the compression as before, is simply that \( R_{m} \) does not increase for any particle. From (30) this is seen to be possible only if \( \mathcal{A}(H(0)) \) is constant, or at least does not decrease with time, i.e.,

\[
\lambda H(0, t) \geq \lambda H(0, 0) \tag{31}
\]

or,

\[
\kappa^{2} \leq \alpha \tag{32}
\]

since \( \lambda(0)/\lambda(t) = \kappa \), the axial compression factor. If the mirror ratio is also a function of time, then, in simple cases, this condition takes the form

\[
\kappa^{2} \leq \alpha[R(t) - 1]/[R(0) - 1]. \tag{33}
\]

The case, \( \lambda H = \) constant, is interesting because it represents a uniform compression of the plasma in all three dimensions. For this case, since the values of \( R_{m} \) remain constant for all trapped particles, the energy of each of the particles, as given by Eq. (27), is simply proportional to \( \alpha \). However, the volume of the plasma varies as \( (\alpha \lambda)^{-3} \) and therefore as \( \alpha^{-3} \). Thus, the mean particle energy varies inversely with (volume)\(^{\frac{1}{3}}\), i.e., \( \dot{W} \sim \alpha^{2} \). This is the same as the law of adiabatic heating of an ideal gas, as derived from thermodynamic considerations. Although this correspondence with the classical result might also have been foreseen from Liouville's equation, it is interesting to see it emerge from the equations of a magnetically confined plasma.

If only an axial compression is carried out, with \( H(0) \) remaining constant, the compression equation (30) shows that the particles must climb higher on the mirrors, so that the actual amount of axial compression of the plasma which occurs will be less than the geometrical compression ratio. For example, if the strength of confining fields varies parabolically, so that \( R(u) = 1 + (u/\lambda)^{2} \), then it is found that \( \lambda(R_{m} - 1) = \) constant, so that for every particle, \( \kappa_{m} - \lambda \). Although the compression is uniform, it proceeds only as \( \lambda \) rather than linearly with the geometric compression.

### Competing Effects

**Collisions**

When properly scheduled, magnetic compression results in improved trapping of the plasma, and the ordering effect of the compression acts in opposition to the disordering effect of collisions. Since the effect of heating the plasma is to reduce the mutual Coulomb collision cross section of the heated particles, it is clear that it should be possible to carry out the compression at such a rate that it continues to overcome the collision losses throughout the cycle and thus inhibits the onset of losses through the mirrors.

The condition for this to occur can be estimated from the relaxation time considerations already discussed. It is required that, during the compression, the gain of perpendicular (rotational) energy shall exceed the gain of parallel energy (kinetic energy of motion in the direction of the field lines). Taking as an example the simple case of radial compression, from (20) the gain of rotational energy is obtainable from \( W_{\perp}(t) = W_{\perp}(0)\alpha(t) \), Eq. (20).

On the other hand, the mean instantaneous rate of change of parallel energy of the ions, owing to collisions, will be given approximately by the expression:

\[
\langle dW_{\parallel}/dt \rangle = \frac{1}{2} \langle dW_{\perp}/dt \rangle = \frac{1}{2} W_{\parallel}/\tau.
\]

Now from (11), \( \tau_{D} = AW_{\parallel}/\kappa_{s} \) sec, where \( A = 2.6 \times 10^{10} \) for deuteron energies expressed in kev. If compression heating dominates over dispersion due to collisions, throughout the compression, then we obtain \( W_{\perp} \gg W_{\parallel} \), so that we may set \( W_{\perp} \approx W \). If the duration of the heating operation is equal to \( t \), then we require that \( W_{\perp} \gg W_{\parallel} \), for all times less than \( t \). This requires, therefore, from the relations above, that

\[
\alpha W_{\parallel} \gg \frac{n(0)}{3A[W(0)]^{4}} \int_{0}^{t} [\alpha(t')] dt'. \tag{34}
\]

If \( H \) increases linearly with time, so that \( \alpha(t') = 1 + t'/\tau \), then (34) becomes (dropping the zeroes in \( n(0) \), etc.)

\[
\frac{2n\tau}{9A\dot{W}} \left[ \frac{\alpha^{-1} - 1}{\alpha} \right] \leq 1. \tag{35}
\]

If the compression factor, \( \alpha \), is large, then this can be expressed as a simple condition on the time within which the compression must be accomplished to avoid excessive scattering losses. This condition is

\[
t_{s} < \frac{9A\dot{W}\alpha^{2}}{2n} \text{ seconds.} \tag{36}
\]

It is apparent that if the particle energy is low it is more difficult to satisfy this condition. For this reason, even when it is satisfied for most of the constituent particles of an initial distribution, the lowest energy particles of the distribution are still likely to be lost.

To illustrate a case which might typically be encountered in an experiment, suppose that \( W_{\perp} \approx W = 0.1 \text{ kev} \) and the initial density, \( n_{s} \), is \( 10^{18} \text{ cm}^{-3} \). If it is desired to compress by a factor of 100, so that the
final mean energy becomes 10 kev and the final density \(10^{14}\), then (36) shows that the compression time must be less than about 0.04 sec. This is not an unduly restrictive requirement. Raising the initial energy allows even slower compression rates to be used. Figure 2(a) illustrates the condition imposed by Eq. (36) for various values of compression rates and initial energy. To satisfy the condition, compression times must be shorter than the values given by the boundary lines. The final particle densities reached...
are shown along the right margin, for various values of $\alpha_{\text{max}}$. If the condition of adiabaticity, i.e.,

$$\tau_H = \left[ \frac{1}{H'} \frac{dH}{dt} \right]^{-1} > \tau_g = \frac{2\pi Mc}{eH}$$

(the gyromagnetic period), is to be satisfied throughout the compression, then the initial magnetic field for this to be true can be specified for each total compression time. These values of initial field, $H_0$, are indicated along the upper margin of the plot. It can be seen that this condition could not conceivably be satisfied for the shortest compression times and largest compression ratios indicated. It is worth noting, however, because of the dependence of $\tau_H$ on mass, that the adiabatic condition is likely to be well satisfied at all times for electrons, even when it may not be satisfied for ions.

However, in many cases of practical interest, the compression will depart from adiabaticity only for a short time at its very beginning. In this case the heating and compression may not deviate significantly from the adiabatic values. Because the gyromagnetic period, $\tau_g$, rapidly diminishes, as the field starts to increase, while the doubling period of the field, $\tau_H$, increases; the ratio of these two quantities rapidly increases with time. This ratio is given by the expression

$$\chi = \frac{\tau_H}{\tau_g} = \frac{eH^2}{2\pi McB},$$

where $\chi > 1$ corresponds to the adiabatic limit.

If evaluated for a linearly rising field, and for $M =$ mass of a deuteron, this becomes

$$\chi = \frac{0.8 \times 10^3 H_0 \alpha/(t)^2}{(\alpha_{\text{max}} - 1)}.$$

Figure 2(6) shows the fraction of the total magnetic compression during which the adiabatic assumption fails to be satisfied, i.e., the value of $H/H_{\text{max}} = a/(t)/\alpha_{\text{max}}$ at which $\chi$ rises to unity for various values of $\alpha_{\text{max}}, I_2$ and $H_0$. When this fraction is small, the adiabatic result will be approached.

The general effect of compressing at too slow a rate is to retain the most energetic components of the particle distribution, while losing the lower energy groups before the compression is completed. This tends, therefore, to "upgrade" the energy distribution to a somewhat higher mean energy than would have been expected, albeit at a lower density than predicted by the simple compression equation.

If axial compression as well as radial compression is included, the compression time requirement becomes somewhat more restrictive, because of the increased density and because of the change in parallel energy which accompanies axial compression.

**Ambipolar Diffusion**

Ambipolar diffusion effects must also play a role in the compression of a plasma. However, as has already been indicated, this role seems to be a secondary one in the experiments conducted to date, not interfering appreciably with the observed heating effects. More sophisticated experimental work will, however, have to be done to determine the precise influence of ambipolar diffusion on the plasma history. The exact way in which the ambipolar effects will appear will depend on questions of the relative initial ion and electron mean energies. For example, when the initial ion energies are higher than the electron energies, the energy of these electrons can be expected to lag increasingly behind the ion mean energy, since the collision effects will cause the heating energy to be shared among all degrees of freedom of these electrons, so that their heating rate will tend to be dependent on the $\frac{1}{2}$ power of the compression, rather than the first power. On the other hand, those electrons which start with energies sufficiently above the ion energies will be heated at the same rate as the ions and so will remain at a higher energy than the ions throughout the compression. The ratio of electron and ion energies at which the scattering times become comparable can be estimated from the relaxation time, Eq. (10). From this equation it can be seen that

$$W_e/W_i = (M/m)^{1/2} \approx 15,$$

if evaluated for $M =$ mass of a deuteron.

**Radiation from Impurities**

It has already been noted that, especially at relatively low plasma temperatures, radiation from impurities can represent a potent mechanism for loss of energy from the plasma through the agency of electronic excitation of incompletely stripped impurity ions. The rate of this radiation loss is thus proportional to the product of the electron particle density and the particle density of the impurity ions. But in many cases the impurity density is roughly a constant fraction, $q$, of the plasma ion density itself, so that the radiation rate varies as $q n_{\text{e}}$. The absolute rate of impurity radiation per unit volume is then representable by an expression of the form (for each type of ion):

$$\dot{p}_e = q n_e^2 T_e^{-3} f(hv/kT).$$

On the other hand, the rate of energy input to the plasma depends linearly on the electron density and the rate of rise of the magnetic field. For example, in the collision-free limit for a simple radial compression, Eq. (20) gives for the energy gain:

$$\dot{p}_e = \frac{n_e dH}{dt}.$$

In order to heat the plasma it is obviously necessary to put in energy from the magnetic compression at a rate more rapid than the radiation rate. Since the energy input rate varies linearly with density, whereas the radiation varies as the square of the density for any given rate of increase of the field, there exists an initial density above which the temperature will not increase, the energy from compression being dissipated by radiation. However, since the density can be arbitrarily set by controlling the injection, it is always possible to find a low enough initial density which leads to heating. Fortunately, it has been found
possible to control the impurity levels to the point where radiation imposes no more stringent a limita-
tion on the initial density than does the requirement 
of collision losses discussed previously: in the experi-
ments, electron heating occurs essentially as pre-
dicted by the compression equations. If the electrons 
were not heated, but were maintained at a low 
temperature by radiation losses, the ion energies 
would also tend to be quickly damped by collision 
cooling.

Adiabatic Expansion—Energy Recovery

Before leaving the subject of adiabatic compression, it is important to note that, under the assumptions 
made, all of the operations discussed are reversible: i.e., the same equations apply to a magnetic expansion 
of the plasma, carried out by appropriate manipulation 
of the confining fields. In this case, the roles of 
radiation crystal decompression, in their competition 
with collision effects, are just reversed from the case 
of compression. Axial decompression leads to improved 
confinement of the remaining plasma whereas radial 
compression leads to poorer trapping. An appro-
 priate combination of the two operations is therefore 
indicated. In an adiabatic expansion the energy of the 
plasma will decrease, corresponding to a transfer of 
energy back to the confining fields, and (usually) an 
accompanying flow of electrical energy out of the 
system.1 This fact offers a possible avenue to the direct 
electrical recovery of plasma internal energy, either 
that associated with charged reaction products or 
unreacted plasma. In the Mirror Machine, efficient 
recovery of the kinetic energy of escaping particles, 
which might be accomplished by adiabatic expansion, 
could be a very important means of improving the 
power balance. It enables one to visualize a con-
tinuously operating "plasma engine" which would 
utilize the compression and expansion operations, 
which have been described above, to create, heat, 
confine and recover energy from a reacting plasma. 
The feasibility of such a machine has, of course, not 
yet been proved.

INJECTION

One of the most difficult technical problems of the 
Mirror Machine program has been the problem of 
injection. Because of the nature of plasma confine-
ment by magnetic mirrors, it is not often satisfactory 
to create the plasma by ionizing a cold gas inside the 
confinement volume. Instead, other methods have 
been employed which involve the injection of plasma 
or of streams of energetic particles. The "open ended"
topology of the Mirror Machine allows one to employ 
some unusual methods for injection.

The dilemma which is faced in all injection prob-
lems is the essential impossibility of trapping charged 
particles within a static magnetic field under conserva-
tive conditions; i.e., in order to inject, something must 
change. Various methods for injecting into a Mirror 
Machine have been proposed15 and several of them 
are under study. They employ one or more of the 
following effects: (a) time-varying fields, (b) collision or 
cooperative-particle interactions and (c) change of charge 
state of the injected particles.

Time-Varying Fields

In the first category lie several of the methods which 
have been applied in the program to date. In early 
experiments,16 beams of energetic particles were cap-
tured between the mirrors by injecting them through 
regions where they were subjected to radio-frequency 
fields. These fields produce an irreversible energy gain 
which leads to trapping. This method has not been 
actively pursued because of problems of field pene-
tration but it remains a potentially interesting process.

External Injection

Another method which has been successfully 
exploited might be called "external adiabatic trap-
ping". In this method a plasma is captured by 
injecting it through the mirrors into a rapidly rising 
magnetic field.17 This method is similar to that 
employed in a betatron to accomplish injection. It can 
be understood qualitatively from the binding equa-
tions (2) or (3). Suppose a plasma is injected through 
a mirror from outside the confinement zone. Then 
none of its ions will be initially bound. For these ions, 
an inequality which is the inverse of the binding 
condition will apply, i.e.,

\[ W > \mu H M. \] (41)

If, however, during the transit of the plasma 
through the confinement zone, the strength of the 
magnetic field is increased sufficiently, then because of 
the constancy of \( \mu \), the mirror field strength \( H_M \) may 
increase sufficiently to reverse the inequality so that 
many of the plasma ions are captured. This can be 
accomplished either by strengthening the entire field, 
or simply by changing the field at the mirrors.

The condition that particles should be captured 
under these conditions has worked out.1 It 
shows, as might be expected, that injection can be 
accomplished only as long as the magnetic field 
changes by a sufficient fractional amount during one 
transit of the particles. If the injected ions of the 
plasma flow over the top of the first mirror, making a 
small helix angle, \( \epsilon \), with respect to the plane of the 
Mirror (\( \epsilon = \frac{1}{2} \pi - \theta \), where \( \theta \) is the pitch angle) then injection 
can be accomplished as long as

\[
\frac{dH}{d\epsilon} \geq H \left( \frac{R}{R - 1} \right)^{1/2} \left( \frac{L/L_0} \right)
\] (42)

where \( L \) is the separation between the mirrors 
(assumed large compared to length of the mirror regions), 
\( L_0 \) is the velocity of the injected ions, and \( R \) is the 
mirror ratio. If the field is assumed to rise linearly 
with time, then the maximum acceptance time over 
which injection can be accomplished will be given by

\[
\tau = \frac{L}{v_0} \left( \frac{R}{R - 1} \right)^{1/2} \frac{1}{\epsilon^2}
\] (43)
When only the mirrors are strengthened, rather than the entire field, a similar expression is found. Also, if the far mirror is made much larger than the near mirror, the acceptance time is doubled.

It can be seen that this injection mechanism is most effective for particles entering with small angles, \( \epsilon \). Thus, if a stream of plasma with a wide range of helix angles is injected through the mirrors, then the injection condition will continue to be satisfied longest for the smallest angles \( \epsilon \).

**Internal Injection**

Injection can also be accomplished by sources located partially within the confinement volume. Here the problem is one of preventing the plasma particles from returning and hitting the source. One of the methods studied has been the use of compact ion sources located just inside the confinement volume, emitting energetic ions in a direction perpendicular to the local direction of the field lines. The great advantage of using such sources is that the plasma is initially created with a very high ion energy—many kilovolts—thus over-leaping most of the heating problems. There are serious problems, however, in reducing the method to practice. As injected, the ions miss the source after their first turn, because of the field gradient in which they move. From the force equation (4) it can easily be shown that, in the adiabatic approximation, an ion injected at right angles to a field gradient advances in one turn by a distance

\[
\Delta u = \frac{\pi \rho^3}{\lambda} \tag{44}
\]

where \( \rho \) is the mean radius of curvature of the particle in the field and \( \lambda \) is a characteristic distance associated with the field gradient, defined by

\[
\frac{1}{\lambda} = -\frac{1}{H} \frac{\partial H}{\partial u} \tag{45}
\]

and not to be confused with the eigenvalue, \( \lambda(R) \), used earlier.

Having missed the source on the first turn, the ions may then travel to the far end of the confinement region, where they are reflected (since they were "born" inside the mirrors). While moving along the field lines, the ions will precess slowly around the axis of the machine, because of their finite orbit size. It is possible to arrange conditions so that this precession, though necessarily small at each reflection, since the ions are nearly adiabatic (small orbits), can still be large enough to cause the ions to miss the source on their first return. If this is the case, then they may continue to be reflected back and forth in the volume until they have precessed completely around the axis. This might take, typically, 50 to 100 traversals. At this time they would be likely to hit the source and be lost, thus limiting the injection and confinement time to about the period of precession around the axis, too short a time to be of much interest. If, however, the magnetic field is increased while this is going on, then by the time each ion has precessed around to a position where it might hit the source, the axial compression which occurs in the increasing field may prevent it from returning to the source position, so that it becomes trapped. In this way the time over which injection can be accomplished may be substantially extended over that of a single precession period. Injection will cease either (a) when, because of the increasing field, the orbits become too small to miss the source, on the first turn, because of the \( \rho^3 \) dependence of (44), or (b) when they are not sufficiently compressed axially to miss the source. The most efficient use of the injection time occurs when conditions (a) and (b) fail simultaneously.

The amount of axial compression which occurs when the field is increased can be calculated readily from the compression equation (22). Assuming that the mirrors are alike, this may be written as

\[
H\frac{dJ}{dt} = S
\]

where

\[
J = \int_{u_{\text{in}}}^{u_{\text{max}}} [R_m - R] du.
\]

On differentiating, we obtain

\[
H\frac{dJ}{dt} + \frac{1}{2} \frac{dH}{dt} = dS = 0;
\]

but

\[
dJ = \left( \frac{\partial J}{\partial u_m} \right) du_m + \left( \frac{\partial J}{\partial t} \right) dt.
\]

Hence, if \( \partial J/\partial t = 0 \) (field shape does not change with time), we find that

\[
|\delta u_m| = \frac{1}{\tau} \left[ \frac{1}{2} \left( \frac{\partial J}{\partial u_m} \right)^{-1} \right] \delta t = \lambda \frac{\delta t}{\tau_H} \tag{46}
\]

where \( \lambda \) is the characteristic distance given by the expression in brackets and \( \tau_H = H/(dH/dt) \) represents the instantaneous doubling time of the field as it increases. If the field increases linearly with time, \( \tau_H \) is simply the time in seconds subsequent to the initiation of the field rise. In the equations, \( \delta u_m \) represents the distance which the end point of motion of the particle progresses during time \( \delta t \), if \( \delta t \) corresponds to one of the times of return of the particle to the first mirror.

In simple cases, for example where the mirrors are widely separated, \( \lambda \) \( \lambda \) so that (46) reduces to the simple expression,

\[
|\delta u_m| = \frac{\lambda \delta t}{\tau_H} \tag{47}
\]

where \( \lambda \) is defined as in Eq. (45). If the angle of emission of the ions with respect to the normal to the field lines is not exactly zero, but is \( \pm \epsilon \) then (47) and (44) become

\[
|\delta u_m| = \lambda[(\delta t/\tau_H) - \epsilon^2] \tag{48}
\]

\[
|\Delta u_m| = \pi^3 \rho^3 / \lambda \pm 2\pi \rho \epsilon \tag{49}
\]

In order for injection to be accomplished, both \( \delta u_m \) and \( \Delta u_m \) must simultaneously be larger than \( \Delta x \), the length of the source in the direction of the magnetic field. The maximum injection time available may be estimated by setting \( \epsilon = 0 \) and \( \delta u_m = \Delta u_m = \Delta x \) and solving for \( \tau_{\text{max}} \):

\[
\tau_{\text{max}} = (\lambda/\pi \rho)^3 \delta t. \tag{50}
\]
Traveling Mirror Injection

To conclude the discussion of injection into time-varying fields, mention should be made of the use of traveling mirrors to accomplish injection. If a plasma source is immersed in a moving mirror confinement zone, a capture process similar to the one mentioned in the previous discussions can occur. This process has been observed experimentally.  

In this case, the velocity of translation and the shape of the moving mirror field can be chosen to maximize the injection time. Just as in the case of internal adiabatic injection, if necessary, this time can, in principle, be made much larger than the precession time.

Capture by Collision or Cooperative Particle Interactions

Collision effects can lead to capture of a plasma as well as to losses. This effect has been successfully exploited in some of the experiments. For example, if a relatively dense, low temperature plasma is suddenly discharged into a mirror confinement chamber then there is an appreciable probability that some of the plasma will be captured by mutual collision effects within the volume. The rest of the plasma will then escape through the second mirror, leaving behind a trapped component. Since the capture arises from nonconservative processes, there is no requirement that the magnetic field be changed to accomplish this injection, but the confinement time will necessarily be limited if heating is not employed.

The approximate conditions for collision capture can be established from relaxation time considerations. If the helix angle of particles crossing the mirrors is $\epsilon$, then particles will tend to be captured which approximately satisfy the condition

$$
\epsilon^2 \leq 2.5 \times 10^{-18} \frac{L}{\mathcal{W}} \quad (51)
$$

where $L$ is the distance between mirrors, $\mathcal{W}$ is the mean particle energy in kilovolts and $\epsilon$ is the plasma ion density. It is clear that this condition strongly favors small helix angles and low particle energies.

Another method which involves cooperative phenomena to effect injection has been successfully demonstrated and studied qualitatively. This is the method of injecting plasma streams across the confinement volume and causing them to intersect in the center of the confinement volume. It is well known, from astrophysical theory and other evidence, that a stream of plasma can move across a magnetic field, sustained in an evacuated region, by the process of setting up a state of charge separation on opposing surfaces of the stream. This is the same mechanism that which occurs in a flute instability, for example, where it enables the transport of certain unstable plasma configurations across a magnetic field. Such effects have been reported in the literature.

When such streams enter a conducting medium, for example another plasma disposed along the magnetic field lines, they can no longer freely cross the field and will tend to mix with the other plasma. This is the clue to the injection mechanism. This process has been qualitatively demonstrated in some of the experiments to be reported, but the potentialities of the method have not been fully explored.

In principle, any cooperative mechanism such as collision and polarization effects, just described, or plasma diamagnetic effects, can lead to capture of at least a part of an injected plasma. The utilization of cooperative processes seems to represent a very promising attack on the problem of injection.

Change of Charge State

Another method by which energetic particles can be trapped in a magnetic confinement volume is by change of charge state. For example, beams of energetic neutral atoms can be produced by established techniques. Such beams can freely penetrate a magnetic confinement volume, and would normally pass completely through it. If, however, a mechanism exists which will ionize a part of all of the atoms of the beam as it passes through the chambers, the ions and electrons thus produced could be trapped. One of the main difficulties of the method is the establishment of an effective breakup mechanism. G. Gibson, W. Lamb and E. Lauer are studying the possibility of using a very low residual neutral gas pressure as a means for ionizing very energetic atoms, so that a high energy plasma would slowly accumulate in a dc magnetic confinement zone. Their method is potentially applicable to any steady-state magnetic bottle. Another
method, which might be especially applicable to the
Mirror Machine, would be to use the plasma created by
one of the injection processes already described to
provide efficient breakup for neutral beams of some-
what lower energies than those considered by Lauer
et al.

Molecular ions can also be used in the same way to
accomplish injection. Studies at Oak Ridge National
Laboratory are now being carried forward using an
unusual arc breakup mechanism discovered by Luce.33
In this method, capture of energetic molecular ions
results when they are broken up into their constituent
atomic ions, because of the sudden change of radius of
curvature of the orbits in the field. The potentialities
and difficulties of this method have not been fully
explored, but intensive work is being carried out at
Oak Ridge to generate a high temperature plasma by
these methods. Although Luce's method is, in prin-
ciple, applicable to any dc magnetic bottle, the
present experiments are being done in magnetic
mirror geometry.

SUMMARY OF EXPERIMENTAL RESULTS

The experimental study of the confinement of
plasma by magnetic mirrors began on a small scale in
early 1952. Preliminary experiments conducted by
Post and Steller34 with transient plasmas produced by
rf excitation within dc magnetic mirror fields showed
qualitatively that confinement was possible. This was
followed up by the experiments of Ford18 and others
on rf trapping of a plasma. In 1953, a serious effort
was undertaken to investigate compression heating of
the plasma. In the early experiments of Coensgen in
1953 and 1954,25 plasma produced by means of an
electromagnetic shock tube was magnetically com-
pressed by pulsed mirror fields which rose to peak
values of about 200,000 gauss in a period of a hundred
microseconds or so. Time-resolved pictures of the
compression process were obtained and showed that
the plasma was being stably compressed. It was soon
realized, however, that the magnetic field technology
then developed did not allow effective use of the
magnetic compression process with the high densities
being used. Recently, the Los Alamos group26 and
Kolb37 at NRL have performed similar experiments,
but with greatly improved energy storage and pulsed
field technology and have obtained more striking
results. The early experiments of Coensgen, however,
pointed out the basic feasibility of the magnetic com-
pression process and provided encouragement for
continuation of the research.

Because of a realization that, under the conditions
then achievable, the compression process would
function more effectively with less dense, more highly
ionized plasmas, a search was made for improved
methods of injection. This led to the development, by
Ford and Coensgen, of early forms of the occluded gas
source, a source in which a burst of ionized hydrogen
was released by an electrical discharge on the surface
of a "hydride" of titanium. Using this source, it was
soon possible to show that substantial heating was
occurring and that the plasma was being confined for
periods of the order of a millisecond, in rough agree-
ment with the expected diffusion loss rates. In the
experiments of Coensgen, escaping energetic radiation
and particles were observed that gave direct proof of
the heating. Confinement was established both by the
analysis of the escaping radiation and by direct
measurement with microwave interferometric tech-
niques.35

Theoretical calculations based on certain hydro-
magnetic models were being made at about this time.
These seemed to indicate that plasmas in Mirror
Machines should exhibit hydromagnetic instabilities
and, under the conditions of the experiments then
being conducted, might be expected to be lost in a
very few microseconds. The fact that plasmas were
demonstrably being confined, for periods perhaps a
thousand times as long as the theoretical instability
times, led to the realization that the model used in the
early calculations did not correspond closely enough
to the real situation. Since that time, it has been
realized that stability may be influenced by: (a) the
finite nature of an actual plasma in a Mirror Machine,
as opposed to the infinite periodic plasma of the early
type; (b) the existence of conductors or quasi-con-
ductors at the ends of the machine; (c) the noniso-
tropic nature of the plasma pressure; (d) the non-zero
size of the particle orbits and, perhaps, (e) the existence
of ambipolar effects. Although there is general agree-
ment that these factors may explain the observed
stability, and calculations have been performed6
(with simplifying assumptions) which predict stable
confinement, the problem is so complicated that no
wholly satisfactory theoretical treatment has yet
been made.

DC Mirror Machine "Cucumber"

In 1954, an experiment of another type was also
tried, specifically aimed at the problem of stability
and diffusion across a magnetic field. It was desirable
for many reasons to obtain a situation in which the
particle pressure constituted as large a fraction as
possible of the magnetic pressure. Aside from the
obvious method of pushing the temperature and
density up, one way to accomplish this is to reduce
the magnetic pressure to the lowest practicable value,
so that a low temperature, low density plasma might
suffice. Thus a dc Mirror Machine, called "Cucumber",
was built, with a large-diameter central chamber
(eventually 18 in.) and very low confining fields (as
low as 20 gauss). The machine was "capped" with
strong mirror coils (6000 gauss) so that very large
mirror ratios could be achieved. In Cucumber the
process of collision trapping, described in the previous
section, operated very effectively, so that it was
possible to trap a substantial fraction of an injected
low energy plasma by this process alone. Stable and
long time confinement of the plasma was observed,
under conditions where the plasma pressure was
estimated to be several percent of the (low) magnetic
pressure. These experiments helped qualitatively to confirm the stability picture and to show the influence of high mirror ratios in reducing end losses. They also showed that, under the conditions of the experiment at least, there seemed to be no anomalously large rate of transverse diffusion of the plasma across the magnetic field. An example of the confinement data taken in this machine is shown in Fig. 3. The traces represent signals received on a very thin wire probe. The plasma source was pulsed in all cases, but in only one case were all the confining fields turned on.

Recent Experiments

The recent experimental work in the Mirror Machine program has concentrated on the objectives of (1) studying the heating and confinement process in detail with relatively simple injection techniques, (2) increasing the total usable compression factors to reach higher temperatures and longer confinement times and (3) exploration of new high energy injection methods.

High Compression

The most fruitful experiments to date have been the so-called “high-compression” or “plasma injection” experiments, in which a burst of relatively low energy plasma is injected into an evacuated chamber immersed in an initial weak magnetic mirror field. The field is then rapidly increased to a high final value (in a few hundred microseconds) compressing and heating the plasma in the manner already described. Results of these experiments are being reported so that a brief summary only will be given here.

Figure 4 shows one of the devices with which these experiments have been performed. The machine shown has been used to perform plasma injection studies in which the plasma bursts were trapped and strongly compressed.

These high compression experiments have shown that an initial plasma burst filling the confinement volume with a hydrogen plasma to a density of $10^{11}$ to $10^{13}$ cm$^{-3}$ can be stably compressed to a final density of $\sim 10^{14}$ cm$^{-3}$ by using very large magnetic compression ratios. The compression times used were in the general range of 200 to 500 microseconds. The magnetic compression factors used ranged to 1000 to 1 or even more. The diameter of the compressed plasma column was determined in some of the experiments by means of fine wire probes, and by study of the radial distribution of escaping electrons. It was found that the radial compression of the plasma followed the predicted behavior with acceptable accuracy. Axial compression effects were also observed, in rough agreement with the predictions of the compression equations.

Particle Emission

The effective confinement time of the plasma was deduced by analysis of escaping particle fluxes through the mirror and by other means, and was found typically to be characterized by half-value times of the order of a millisecond, with energetic components of the plasma being observed to remain as long as 20 milliseconds. In some of the experiments, involving very large compression factors, the electron energy distribution of escaping electrons was measured by absorber techniques, over a range of about 3 keV to 120 keV. Figure 5 shows an oscilloscope trace of the escaping electron flux as measured by a collimated scintillator. The sweep time is 5 milliseconds. By
weighting the observed distributions to take account of the influence of particle energy on the escaping particle fluxes, it was possible to fit the distribution to a Maxwellian distribution corresponding to electron temperatures between 10 and 20 keV. These high electron temperatures were confirmed by the use of microwave radiometry, which also served to establish a lower limit on the plasma density and to yield information on the axial compression which had occurred.

As might have been expected, the peak of the flux of escaping electrons occurs almost simultaneously with the peak of the plasma compression. This is in contrast to the time of X-ray emission, as shown in Fig. 6 and discussed below. Although the curves in Figs. 5 and 6 are generally similar, Fig. 5 does show some small periodic fluctuations for which the explanation is not entirely clear.

It was realized that, in the experiments where the highest compressions (lowest initial fields) were used, the ion mean energies could not have been as high as the indicated electron temperatures. In these experiments, it was not at that time possible to measure the ion energies directly. These could be inferred, however, from the confinement data and from the initial conditions, and are believed to have been about 1 keV. In separate experiments, where the conditions were more favorable for measurement, compression heating of ions was observed, with final energies as high as 2 to 3 keV being observed. The small volume of plasma used in all of these experiments, together with the relatively slow time scale of the compressions precluded the possibility of appreciable numbers of neutrons being observed.

In these high compression experiments, as normally carried out, there seem to be no gross plasma instabilities. Measurement of the total energy content of the plasma by scintillator and calorimetric techniques leads one to infer that the plasma pressure at the end of the compression cycle was in some cases about 8 percent of the applied magnetic pressure. This is still not a sufficiently high value, nor was it obtained under sufficiently general circumstances to conclude that large volumes of a plasma at useful temperature and pressures could be stably confined in a Mirror Machine.

**X-ray Emission**

Although the behavior of the major part of the plasma was, on the whole, explicable in terms of the concepts of compression heating and simple collision losses, there remain some interesting effects to be explained. An example of one of these is the fact that it is possible to detect the radial escape of a small fraction of the most energetic electrons of the plasma by the X rays which they produce at the chamber wall in the middle of the machine. This escape rate, though slow, seems too rapid to be explained in terms of ordinary scattering because of the high energies of the escaping electrons. At the midplane of the machine the flux tubes have their largest diameter, so that trapped electrons which are drifting radially will contact the chamber wall first at this point.

In the experiments, a collimated and filtered X-ray signal was detected by a scintillation crystal located at the midplane of the machine. This crystal responded only to X-ray quanta with energies above about 100...
kev. Examples of the signals observed are shown in Fig. 6. The sweep time used was 10 msec. Apart from an initial transient (mostly of electrical origin) it can be seen that the signals are small for a time of the order of a millisecond and reach their maxima at about 2 to 3 msec after the start of the compression cycle. The compressing magnetic field, however, rose to its maximum amplitude of about 40,000 gauss (in the center of the machine) in 0.5 msec then decayed exponentially with a time constant of about 20 msec. Thus, the maximum X-ray signal was received much later than the time of maximum compression. This is evidence of a slow outward radial motion of an energetic component of the heated electrons. Part of this radial motion, and the subsequent decay of the amplitude of the observed signal, can be identified with the decay and consequent slow radial expansion of the confining field. But there remains a slow drift which is not easily explained in this way. Now, in 3 msec, a 100-kev electron will have traveled about $6 \times 10^7$ cm, and will have executed some $3 \times 10^8$ revolutions in the confining field. In the experiments, the radial distance between the chamber wall and the surface of the compressed plasma is about 7 cm. One can therefore estimate that the radial drift velocity of 100 kev electrons could have a maximum value of about $2.5 \times 10^3$ cm/sec. Although this inferred radial drift velocity is very small compared to the electron velocity itself it is still too large to explain by ordinary scattering, both since the Coulomb scattering cross section of 100-kev electrons is very small (about $6 \times 10^{-23}$ cm$^2$ on hydrogen) and because multiple scattering would provide a very strong selective mechanism against any particle reaching the wall prior to being scattered into the escape cone. On the other hand, a quite small electric field, fluctuating at a frequency comparable to or slower than the electron gyromagnetic frequency, could induce a randomly directed crossed electric and magnetic field drift which would cause the observed radial transport. Such a drift would not be expected to interfere appreciably with the normal mirror binding of the energetic electrons. Since it is known that the number of electrons reaching the wall is small compared to the total number, and that the plasma density outside the compressed column is much smaller than in the column it appears that the mechanism causing the transport operates primarily in this region, and not in the main body of the plasma. It is tempting, therefore, to ascribe the transport to fluctuation phenomena associated with the low density of this exterior plasma.

In the high compression experiments it was possible to show that the heating effects were quenched by the introduction of relatively small particle densities of high-Z gases, such as argon. This quenching effect became effective at about the density predicted by the theory of the radiation losses discussed earlier; i.e., at about $3 \times 10^{11}$ cm$^{-3}$ for a field rising in 0.5 msec. Figure 7 illustrates this quenching effect as measured by an 8-mm microwave radiometer. Shown are traces for three argon pressures. Peak signal is proportional to $T_e$.

The quenching is noticeable at $2 \times 10^{11}$ cm$^{-3}$ and is complete at $10^{12}$. It is possible to conclude from these measurements that the normal impurity level had been reduced to the point that it did not substantially interfere with the compression heating process.

**Low-Energy Injection**

Two injection methods were used in these high compression experiments. In the most commonly used method, a low-density, 5- to 10-$\mu$sec burst of
hydrogen plasma was injected in an axial direction through one of the mirrors from a source located just outside the mirror. Capture of the plasma was accomplished, in the manner described in the previous section, by starting with a weak initial field and rapidly increasing it. Continued increase of the field led to the compression effects which have been described.

In another injection method used, synchronized bursts of plasma were injected across the chamber, aimed toward the axis of the machine. By carefully timing the confining fields with respect to the injected bursts, it was possible to capture much of the plasma at the time the beams intercepted each other in the center of the machine. Figure 8 shows two views of a machine used for studies of radial plasma injection.

High-Energy Injection

Other experiments have been carried out, which are aimed at studying the injection of high energy ion streams into a Mirror Machine. Because of the desirability of extending the time scale of the injection phase, close attention had to be paid to the role of charge exchange processes between trapped ions and the background gas, since such processes represent a powerful loss mechanism in this type of experiment. The apparatus developed for this experiment, shown in Fig. 9, was therefore designed with unusual attention to vacuum problems. A thin-walled stainless steel vacuum chamber was used, with metal gaskets throughout. The entire chamber was baked out at elevated temperatures. With these precautions it was possible to reach base pressures below $10^{-9}$ mm Hg.

Observations were conducted both with pulsed and with dc fields. Careful studies of ion orbit trajectories were made with collimated sources at ion energies of about 10 kilovolts. These studies confirmed the general picture of the ion behavior in the machine, and demonstrated the precession effects alluded to in the discussion of injection methods. An attempt was then made to increase the current density and to trap a high current density beam. It was then found that serious space-charge "blowup" effects were occurring in the vicinity of the source leading to the loss (by striking the back of the source) of most of the injected beam. Although it was proved that a small fraction of the beam was actually trapped and continued to be reflected back and forth in the machine for several thousand reflections (several milliseconds), insufficient beam was trapped to call the experiment a success. At this time the blowup process is being studied, and methods are under consideration to improve the degree of space-charge neutralization.

CONCLUSION

One might characterize the theoretical and experimental philosophy of the Mirror Machine program to date by a few words: "An attempt to study phenomena in a high temperature plasma by the extrapolation of ideas and methods already established in the field of charged particle dynamics into the realm of plasma dynamics." Thus the attitude has been to set up situations which were clearly and demonstrably workable at sufficiently low particle densities and then proceed to push the density and temperature as far as possible into a regime where the reactions of the particles on the confining fields is no longer negligible. Although such an approach necessarily implies the use of externally generated confining fields, it has much to recommend it. One deliberately picks those situations where symmetry and the method of confinement tend to minimize the possibility of what might be called "uncooperative" phenomena in the confined plasma, and where collision effects play a secondary role throughout the experiment. The advantages gained are: simplicity of understanding, the possibility of starting with very low densities and working up, and the likelihood that simple conservation laws will be adequate to define the essentials of the behavior of the plasma. The price which one pays for this is to introduce new technical problems of a more difficult nature than those encountered in some of the other approaches, for example, the simple linear pinch, and to be confined to working in a regime where the basic time scale is essentially set by a single collision time of the plasma ions. In the long run, although it clearly represents a compromise, this regime might still turn out to be a necessary one for the attainment of fusion power.

Judged in terms of the eventual goal of fusion power, progress thus far in the Mirror Machine program is limited indeed: where seconds of confinement time will be needed milliseconds have been achieved; where ion mean energies of a hundred kilovolts or more would be needed, perhaps a kilovolt has been attained. But judged in terms of the growth of understanding of the basic processes and problems of injection, heating, and confinement, substantial progress has been made. The goals for the future still lie in the direction of understanding these processes and their

Figure 9. Mirror machine for injection of high energy ions. Entire vacuum chamber may be baked to reach pressure of $10^{-9}$ mm Hg.
limitations more fully. From this understanding an assessment can be made of the future of the Mirror Machine in the quest for fusion power.

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I wish to acknowledge especially the work of several members of the Mirror Machine experimental group: Mr. Post, R. F. Post, Sixteen Lectures on Controlled Thermonuclear Reactions, UCRL-4231 (February 1954). Some of this material was presented at the American Physical Society meeting held at Washington, D.C. (May 1956).


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Figure 10. Equipment schematic and magnetic field sequences in multi-stage experiment

Figure 11. Multi-stage compression and transfer apparatus

which are placed at several points along the machine, it is possible to follow the plasma during its successive compressions, except in the case of the final and most intense compression. It is found that the density increases at each stage in relatively close agreement with theory. This encourages us to believe that these compression and transfer operations do not, as yet, give rise to plasma instability.

Electron heating and confinement effects have also been measured in this machine and have been found to be similar to those observed in the single stage machine. Also, under some circumstances, it has been possible for us to measure the type and the energy distribution of ions escaping from this machine. Figure 12 shows measured energy distributions integrated over the time of the compression pulse. In case (a) the plasma burst was injected parallel to the machine axis, so that only a small fraction of the initial ion kinetic energy appeared initially in rotational motion. Even so, the heating produced by two stages of magnetic compression can clearly be seen. Figure 12(b) shows a similar analysis taken with sources directed nearly across the magnetic field. In this case, even with a single compression and transfer a pronounced ion heating was observed.

These compression experiments have shown us that we can inject and heat a plasma by carrying it through manipulations by means of magnetic mirrors. We feel that these results are not only helpful to us in learning about plasma heating and confinement, but also have a bearing on the possible future practical use of a mirror machine. This is because the operations of transfer and compression, which are carried out adiabatically, should also in principle be reversible, so that energy could possibly be extracted directly from the plasma in electrical form by performing controlled transfers and radial and longitudinal expansions which are just the inverse of the operations we have demonstrated.

In all these compression experiments, although the ion energies reached were substantial, the plasma volumes were too small to yield any appreciable numbers of neutrons. The equipment is now being modified to permit the confinement of larger plasma volumes at higher final ion energies.
Pyrotron Plasma-Heating Experiments

By F. H. Coensgen, F. C. Ford and R. E. Ellis*

R. F. Post has proposed that an ionized gas (plasma) can possibly be contained in a linear magnetic field of circular symmetry. The motion of charged particles along the magnetic field lines is to be limited by strong magnetic fields (magnetic mirrors) at the ends of the system. (See Curve A, Figure 1.) He has discussed the properties of a plasma in such a magnetic field configuration from the premise that an individual charged particle of a tenuous plasma will behave as an isolated ion in a magnetic field. If one also assumes that the gradient of the magnetic field (∇B) is small over a region of the order of an ion orbit and that the time rate of change of the magnetic field (dB/dt) is so slow that the magnetic field is essentially constant for the time of one ion rotation, the motion of an ion conforms with the following relations:

\[ \mu = W/B = \text{constant} \]  

(1)

\[ \int P_x dx = \text{constant} \]  

(2)

The axis of symmetry of the magnetic field is taken as the \( z \) coordinate. For convenience, the symbols used in this paper are defined as follows:

- \( W \) = the rotational energy of an ion associated with the component of velocity perpendicular to the magnetic field.
- \( U \) = the translation energy of an ion associated with the component of velocity parallel to the magnetic field.
- \( v \) = ion velocity.
- \( \theta \) = angle between the magnetic field vector and the ion velocity vector.
- \( v_\perp \) = \( v \sin \theta \) = component of velocity perpendicular to the magnetic field.
- \( v_\parallel \) = \( v \cos \theta \) = component of velocity parallel to the magnetic field.
- \( m \) = ion mass.
- \( P_x = mv_\parallel \) = component of momentum parallel to the magnetic field.
- \( R = B_m/B_0 \) = the instantaneous ratio of the maximum value to the minimum value of the magnetic field. Typically, \( B_0 \) is located midway between two mirrors of value \( B_m \).
- \( \alpha = B_0/B_m \) = the ratio of the magnetic field at time \( t \) to magnetic field at the same spacial point at time \( t = 0 \). In general, \( \alpha \) is a function of position even for simple systems.
- \( n = \) the number of electrons per cm\(^3 \) = the number of ions per cm\(^3 \). Electric charge neutrality is assumed at all points in the plasma.
- \( k = \) Boltzmann's Constant.
- \( T = \) Temperature in degrees Kelvin.
- \( \beta = \frac{n k T}{B^2/8\pi} \), the ratio of particle energy density to magnetic field energy density.
- \( L = \) distance between turning points of an ion, approximately equal to the separation of the magnetic mirrors.
- \( \gamma = L/L_0 \), ratio between turning points of an ion at time \( t \) and time \( t = 0 \).
- \( r = \) radius of plasma column.
- \( \rho = \) radius of curvature of a charged particle in a magnetic field.

As the field changes slowly its value can be considered constant during the time necessary for the ion to move a distance \( L \). Therefore, the total energy of the ions will not change during this interval, i.e.,

\[ W + U = \frac{4}{3} m v_\parallel^2 = \text{constant} \]  

(3)

From (1) and (3) it follows that the angle between the velocity vector of an ion and the magnetic field has the following dependence:

\[ \sin \theta_0 = \frac{\sin \theta_\parallel}{\sqrt{B_0}} = \frac{\sin \theta_\perp}{\sqrt{B_\parallel}} \]  

(4)

where the subscript \( c \) denotes values taken at the field minimum, which is usually the center of a simple system. At the position for which \( \sin \theta_\parallel \) is unity, the velocity of the ion along the field line is zero. Here the ion stops and its motion along the field lines is reversed, that is, the ion is reflected. This behavior is the basis upon which plasma confinement by magnetic fields has been postulated. The condition for ion containment is then

\[ \sin \theta_0 \leq R^{-1} \]  

(5)

If an ion that initially has a rotational energy \( W_1 \) in a magnetic field \( B_1 \) is later found in a magnetic field \( B_0 \), it follows from Eq. (1) that it will have an energy

\[ W_2 = W_1(B_0/B_1) \]  

(6)

From Eq. (2), if the distance between turning points...
of a confined ion be changed from \( L_1 \) to \( L_2 \), the translational energy will be given by

\[
U_a = U_1(L_1/L_2)^2.
\]

Again, from the constancy of the magnetic moment it follows that

\[
B^2 = \text{constant}.
\]

This, in turn, implies that the radius of the plasma in a magnetic field varies as

\[
r_a = r_1(B_1/B_2)^{1/2}.
\]

By use of Eqs. (1) and (2) the density is found to be

\[
\rho_a = \rho_1(B_1/B_2)(L_1/L_2).
\]

For \( L_1 = L_2 \) and \( B_1 > B_2 \), the density is seen to increase. Thus, if the magnetic field throughout a containment region is increased at every point with the same functional dependence, the density is increased, the ion energies are increased and the radius of the plasma column is decreased. Hence the simple operation in which the magnetic field is increased as a function of time is frequently referred to as a radial magnetic compression.

Similarly, an operation which decreases the longitudinal extent of the plasma is referred to as a longitudinal compression. A longitudinal compression can be performed without increasing \( B \) in the central part of the system. Therefore, \( \beta \) can be increased.

\[
\beta_a = \beta_1(L_1/L_2).
\]

\( \beta \) is constant for a radial compression.

**HISTORICAL DEVELOPMENT OF THE HIGH COMPRESSION EXPERIMENTS**

It was first proposed that a plasma be established in a magnetic field of mirror geometry by injecting a beam of energetic ions into an initially evacuated chamber. The ion beam was to be trapped by increasing the magnetic field. Since the density of neutral particles was low, the loss of energetic ions by charge exchange would be low. The loss rate due to scattering could be reduced by use of high energy ions because the Coulomb cross section is inversely proportional to the square of the ion energy. System I (see Table I) was designed to use a beam of 5-kv ions. However, a suitable source had not been developed at the time the vacuum and coil systems were completed in February 1954. Therefore, the experiment was performed using the plasma generated by an electric arc of 10-\( \mu \)sec duration between deuterium-loaded titanium electrodes. The delay between initiating the pulsed magnetic field and striking the arc was varied from zero to a few hundred microseconds. A microwave interferometer was used to determine the electron density and study its decay. Densities of the order of \( 10^{13} \) electrons per cubic centimeter were found. The characteristic decay time was typically 500 \( \mu \)sec. These observations were the first experimental evidence of a plasma magnetically heated and confined. Although accurate determinations of the energy distributions for the ions and electrons were not made, it is known from the time-of-flight measurements that the mean ion energy did not exceed 5 ev, before compression.

**Table 1. Comparison of Systems I, II and III**

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal diameter of vacuum chamber at the center</td>
<td>6&quot;</td>
<td>2&quot;</td>
<td>6&quot;</td>
</tr>
<tr>
<td>Distance between dc mirrors</td>
<td>no dc</td>
<td>24&quot;</td>
<td>76&quot;</td>
</tr>
<tr>
<td>Distance between pulsed magnetic field mirrors</td>
<td>44&quot;</td>
<td>6&quot;</td>
<td>32&quot;</td>
</tr>
<tr>
<td>Pulsed field mirror ratio.</td>
<td>2 : 1</td>
<td>1.5 : 1</td>
<td></td>
</tr>
<tr>
<td>Pulsed field rise time</td>
<td>500 ( \mu )sec</td>
<td>50 ( \mu )sec</td>
<td>500 ( \mu )sec</td>
</tr>
<tr>
<td>Pulsed field decay time</td>
<td>10 msec</td>
<td>3 msec</td>
<td>30 msec</td>
</tr>
<tr>
<td>Capacitor bank energy</td>
<td>( 2 \times 10^4 ) joules</td>
<td>( 3.2 \times 10^4 ) joules</td>
<td>( 4.4 \times 10^4 ) joules</td>
</tr>
<tr>
<td>Approximate peak magnetic field at mirror</td>
<td>( 3 \times 10^4 ) gauss</td>
<td>( 1.14 \times 10^4 ) gauss</td>
<td>( 4.4 \times 10^4 ) gauss</td>
</tr>
</tbody>
</table>

In fabricating System I it was necessary to develop magnet coils of exceptional strength for the high magnetic fields. Continued improvement of the magnet coils made it possible to utilize pulsed magnetic fields up to \( 2 \times 10^5 \) gauss. Thus it appeared possible to produce a plasma with a mean ion energy of a few kilovolts by use of magnetic compression (see Eq. (6)), if the low-energy plasma could be confined in a weak magnetic field.

System II was built specifically to achieve a large value for the ratio of the final magnetic field to initial magnetic field, that is, a high magnetic compression. Radiation from this system was first detected by scintillation counters external to the vacuum and coil systems late in January 1955. The investigation was interrupted at that time to allow the experiments to be moved to larger quarters. However, by July 1955 it was determined that the radiation was primarily x-rays in the energy range from 10 kev to 200 kev. When the 6-inch system was rebuilt in the new laboratory it was provided with dc field coils, System III.

The signal variations from one operation to the next were found to be much smaller for System III than for System II. Therefore, all the detailed investigations have been made in the former system.

**DESCRIPTION OF SYSTEM III**

The main vacuum chamber is a Pyrex glass pipe of 6-inch internal diameter. A thin stainless-steel coating is evaporated on the inner surface, to assure that all points of the vacuum chamber walls will be at the same potential. A base pressure of \( 2 \times 10^{-7} \) mm Hg is achieved with a 6-inch mercury diffusion pump. End ports are provided through which plasma sources and detection equipment can be introduced.

Two sets of magnetic field coils surround the vacuum chamber and are coaxial with it (see Fig. 1). A dc current is passed through the large coil set to provide
a dc magnetic field up to 400 gauss in the mirror regions (Curve A, Fig. 1). Current is supplied to the coils of small diameter by a 1.2 millifarad capacitor bank which can be charged to 40 kv. The pulsed coil and the capacitor bank form essentially a large series circuit of an inductance, a resistance, and a capacitance, as shown in Fig. 2. Each series switch (Si) is a group of four 5555-type mercury ignitrons. The series resistance in the circuit is small so that current rise is nearly sinusoidal (see Curve A, Fig. 2). At the time the current in the coil is a maximum, switch S2 is closed. The current then decays with a time constant equal to the inductance of the short circuit loop divided by the resistance of this loop. S2 is made up of eight 5555-type ignitrons arranged in four parallel paths.

In a typical operation the plasma is injected into the dc magnetic field about 20 µsec before the pulsed magnetic field is applied. The field reaches its maximum value in approximately 500 µsec and decays to 1/e of its maximum value in 30 milliseconds.

**OBSERVATIONS AND DISCUSSION**

**Density Studies**

After the source is fired a charged particle density of $10^{10}$ to $10^{13}$ electrons/cm$^3$ is observed in the central region of the system. The density decays to 1/e of its maximum value in times from 50 µsec to a few hundred µsec. The decay time is found to depend upon the initial density, the magnetic field configuration, details of the plasma source and the orientation of the source. If the source is fired into a uniform magnetic field there is no observable containment of the plasma.

The ions trapped in the dc field must satisfy the condition for reflection (Eq. (5)) and miss the source upon returning to the point of injection. Those ions that collide in the initially dense plasma in the vicinity of the source will return to the point of collision rather than to the source.

As the microwave interferometer will not follow rapidly changing densities or accurately detect the density of small plasma columns, it has been impossible to determine the density as a function of time after applying the pulsed field. Langmuir-type probe studies of a plasma in a weak magnetic field agree with the microwave data.

**Plasma Compression**

As discussed above, the product of the magnetic field and the square of the radius of the plasma column should remain constant for a purely radial compression. If the magnetic field is given by $B_t = B_1 + B_m \sin \omega t$, the radius of the plasma column as a function of time should be

$$r_t = r_s [1 + (B_m/B_1) \sin \omega t]$$

(12)

![Figure 1. Schematic diagram and magnetic field distributions for System III](image1)

![Figure 2. Pulse coil circuit and time dependence of the current in the pulsed coil for System III](image2)
Figure 3. Radial positions of the surface of the plasma column as a function of time for the plasma in System III. The solid curves are plotted from Eq. (12) and the dashed curves from probe measurements.

where \( r_1 \) is the initial radius of the plasma. A small probe, whose radial position could be varied, was inserted at the midplane of the pulsed coil system. The probe signals were found to drop suddenly to zero at a time \( t \) after the pulsed magnetic field was applied. If \( t \) is taken as the time the plasma boundary passed the probe position one obtains a curve as shown in Fig. 3. The discrepancy between the experimental curve and that predicted by Eq. (12) could be due in part to the fact that the plasma which is compressed by a mirror field is also expected to be longitudinally compressed. Thus the individual charged particles gain more energy than in a simple radial compression.

Another set of observations that demonstrate the compression of the plasma column was taken with a small NaI scintillation crystal and a photomultiplier. The crystal was shielded so that X-rays could enter only the front surface. Relative X-ray intensities were then observed outside of the vacuum chamber walls as a function of longitudinal position. Previous investigations have shown that all but a small part of the X-rays are generated at the walls. Thus if it is assumed that the energy distribution of the electrons striking each part of the walls is the same, and that the electrons which strike a portion of the wall traveled along magnetic field lines from the interior of the compressed plasma, the radial distribution of the plasma can be inferred from the measured X-ray signals (see Fig. 4).

Attempts have been made to study the electron energy distribution by introducing electron sensitive emulsions into the vacuum system and locating them at one mirror. It is necessary to coat the emulsions in order to exclude light; consequently, low-energy electrons are excluded from the emulsion. However, the high-energy electrons are concentrated in a cylinder no larger than \( \frac{3}{4} \) inch at the mirror.

**Heating**

To date there is evidence only for electron heating, as techniques for ion measurements are still in the developmental stage.

The earliest evidence for considerable heating of some of the electrons was, of course, the energetic X-rays. Later, as discussed by C. B. Wharton,\(^2\) microwave noise measurements were made using a 25 kilomagcyle narrow band width system. This technique gives a "noise temperature" between 10 and 20 kev. Some information of the electron energy distribution is obtained from the emulsions. Again, large numbers of electrons with energies up to 50 kev are observed. It should be pointed out that none of these techniques will detect the low-energy electrons of the initial electron energy distribution. R. E. Ellis (of this Laboratory) has carried out the most careful study of the electron energy distribution and he discusses the method and results in the following paragraphs.

**ELECTRON ENERGY DISTRIBUTION**

Using a scintillator detector, measurements of the signal produced by electrons escaping through the loss cone of one of the mirrors of System III have been made to obtain information concerning the energy...
distribution of the escaping electrons. Consideration of the processes by means of which particles leak through the mirrors permits these energy distribution measurements to be referred to the plasma inside the mirrors, and the relative numbers of electrons per unit energy interval at a given energy may be determined as a function of time.

**Method**

The data discussed here were obtained by placing a scintillator detector at a position approximately on the magnetic axis of the machine and 3 inches outside the mirror at the end opposite the source. Particles escaping through the loss cone, along the magnetic axis of the machine, passed through thin aluminum and aluminum-coated plastic absorber foils and were collimated by a ¼-inch hole in the stainless steel probe head before entering the detector. The scintillator, of the terphenyl-loaded styrene plastic type, was vapor-coated with 2000 Å of Al for light-tightness. The signal produced in a photomultiplier tube with cathode follower was displayed on an oscilloscope and photographed.

The runs were made by taking 5 to 10 pulses for a given absorber value and capacitor-bank voltage. Figure 5 shows the experimental points obtained for a peak magnetic field of ~32 kilogauss, with 20 µsec delay between source firing and initiation of the field pulse, and an elapsed time of $\tau = 1$ millisecond after the source was fired (peak magnetic field occurs at ~600 µsec).

In addition to the signal from the electron scintillator probe, the signals from two NaI scintillator detectors were used as monitors in a limited sense. One NaI detector viewed the X-rays emerging from the side of the machine at a position midway between the mirrors, and the other was arranged so as to view the X-rays emerging from one end of the machine, more or less axially. Since there appeared to be no consistent correlation between the magnitude of the X-ray pulses and the electron probe pulse, it was not possible to use these detectors as quantitative monitors. Long-term monitoring of machine stability was accomplished by taking zero absorber runs at given machine parameters. Statistical averaging was used to normalize from pulse to pulse.

**Analysis of Data**

The statistical reliability of the raw data is about 50%, based on the number of pulses taken for each absorber value. In order to analyze the data at the present state of the experiment it has been assumed that (1) the loss of electrons to the detector caused by collision diffusion in the absorber foils may be neglected, and (2) the relative scintillation efficiency of the plastic detector for electrons is essentially the same as that of anthracene crystal. An experiment has been designed, but not completed, to test the first assumption.

If we let the quantity $Q(E_0)$ represent the height of the oscilloscope pulse at a time $\tau$ subsequent to firing of the source, then, under our assumptions, $Q(E_0)$ is proportional to the light emitted by the scintillator, in a small time-interval centered about $\tau$, due to the flux of electrons which have passed through the absorber (of thickness equivalent to $E_0$) and stopped in the scintillator. Then for the $j$th absorber, of thickness $E_{0j}$, we have

$$Q_j(E_{0j}) = K \int_{E_{0j}}^{\infty} n(E)g(E_{0j}, E)\epsilon(g) dE.$$  \hspace{1cm} (13)

Here $K$ is a constant relating to the parameters of the detection system and $n(E)$ is the number of electrons per unit energy interval, at energy $E$, in the flux of electrons which has diffused through the mirror along the magnetic axis of the machine. The function $g(E_{0j})$ represents the energy given up to the fluor by an electron of energy $E$. The percentage of the energy transferred to the fluor (by an electron) which is converted into emitted light is represented by the scintillation efficiency $\epsilon$ and is a function of $g$, the energy which an electron has when it enters the fluor.

Equation (13) may also be written as

$$Q_j(E_{0j}) = K \int_{E_{0j}}^{\infty} n(E)F(E, E_{0j})\epsilon(g) dE,$$  \hspace{1cm} (14)

where $F(E, E_{0j}) = g(E, E_{0j})\epsilon(g)$. Now $g(E, E_{0j})$ may
be computed from energy-range curves for electrons in aluminum and $\varepsilon(g)$ taken from the empirical curve for anthracene. The solution of Eq. (14) may be written as

$$Q_j(E_0) \approx K_1 \int_{E_{oj}}^{100 \text{ kev}} n(E) F(E, E_0) dE + \sum_{m=0}^{325 \text{ kev}} \int_{100 \text{ kev}}^{E_m} E^2 e^{-\kappa T E} F(E, E_0) dE \approx K \sum_{m=0}^{j+1} F_{mj}(E_m, E_{oj}) n_m(E_m) + R_j(E_0).$$

Here we have assumed that above 100 kev the distribution will be essentially a three-dimensional Maxwellian and that the contribution, to the signal, of electrons above 325 kev in energy will be negligible. The contribution to the signal from electrons with $E > 100$ kev is represented by $R_j(E_0)$. The contribution from electrons with energy $E \leq 100$ kev is given by the discrete sum over $j+1$ intervals, where $KF_{mj}n_m$ represents the contribution of electrons of energy $E_m = E_{oj} + (j-m+1)\Delta E$; $\Delta E = 10$ kev. The constant $\gamma$ results from the Maxwellian normalization factor and the detector parameters and $\kappa T$ is the kinetic temperature of the electrons in kev. The quantity $n(E_0)$ represents the number of electrons per kev at an average energy $E_0 = 100$ kev in the energy interval 95 kev to 105 kev; $n(E_1)$ represents the number of electrons at $E_1 = 90$ kev in the interval from 85 kev to 95 kev, etc.; $F_{mj}$ represents the light emitted for electrons of energy $E_m$ when absorber $E_{oj}$ is in place. The quantities $K_n$ may be solved for explicitly and the relative distribution $N(E)$ obtained when the $R_j$ have been evaluated. The constant $\gamma$ may be evaluated on the assumption of a high-energy Maxwellian tail for the distribution by taking the ratio of $Q_m(E_{oj})$ to $Q_{n}(E_{oj})$ for two energies and finding by trial substitution the value of $\kappa T$ that satisfies the equation for the ratio of these two experimental quantities.

**Interpretation**

To relate this analysis to the plasma electrons, we substitute for $n(E)$ in Eq. (14) a distribution modified by Coulomb scattering of electrons by plasma ions so that Eq. (14) becomes

$$Q_j(E_0) = K' \int_{E_{oj}}^{\infty} E^{-2n'}(E) F(E, E_0) dE,$$

where the primed quantities are those changed by the scattering process. The value of $\kappa T$ obtained for the plasma electrons on the basis of the smooth curve shown in Fig. 5 is $\kappa T = 21.5$ kev, using $Q(65 \text{ kev})/Q(95 \text{ kev})$. With other equally probable smooth curves.
drawn through the experimental points in Fig. 5, a range of \( \kappa T \) values from 21 to 27 kev is obtained. Using the value of \( \kappa T = 21.5 \) kev and a three-dimensional Maxwellian distribution for \( n'(E) \) in Eq. (16), the values obtained for \( Q(15 \text{ kev}) \) and \( Q(35 \text{ kev}) \) are 94\% and 102\%, respectively, of the experimental quantities derived from the smooth curve in Fig. 5.

The relative distribution \( N'(E) \) based on the smooth curve in Fig. 5 is shown in Fig. 6.

Using the unmodified distribution, \( n(E) \), the relative distribution for the escaping electrons is shown in Fig. 7. A value of \( \kappa T = 17.1 \) kev was obtained for the escaping electrons, using the same data.

The fractional error in the results for the temperature of the plasma electrons is at least of the same order of magnitude as for the experimental quantities, i.e., \( \sim 50\% \). More precise data and more extensive investigation of the type of trial distribution used to compute the value of Eq. (16) should lead to a more dependable estimate of the plasma temperature. The smooth distributions obtained result from the use of a smooth curve through the data points. Even though the \( K_r \) are computed independently, they are relatively small and vary by about a factor of 2.5 between 5 and 85 kev. The \( n_f \) have large absolute errors in this analysis and the use of an iterative technique for their determination may be advantageous.

CONCLUSIONS

The experiments to date have shown that a plasma can be compressed by a pulsed magnetic field and that the plasma column moves as predicted. There is a large amount of evidence for electron heating. However, it has not been shown that the heating follows the adiabatic law (Eq. (6)).

Since the X-ray and electron signals persist for times of the order of 30 milliseconds, containment of a plasma is indicated. Furthermore, the lack of discontinuities or "bursts" in the X-ray and electron signals suggests that the plasma is stable. However, better information of the plasma density, energy content and composition are needed before significant statements can be made concerning containment or stability. If the total energy that is measured by a scintillator or a thermocouple located at one mirror is integrated, it is found that the particle energy density in the confinement volume at the end of the compression was of the order of 8\% of the magnetic field energy density. Further work is in progress to improve the accuracy of the measurement.

Many modifications of the single-stage compression experiment can be made. Radial injection of the initial plasma at the midplane is a simple variation that has recently been introduced. Preliminary operation indicates that the radial injection utilizes the plasma source more effectively, resulting in higher final particle energies in the plasma.

REFERENCES

Pyrotron High-energy Experiments

By C. C. Damm and F. S. Eby*

A possible approach to the problem of creating and confining an energetic plasma is to trap pre-accelerated ions and neutralizing electrons in a magnetic containment volume. Approximate single-particle calculations, based mainly on simple adiabatic theory and neglecting space-charge fields, indicate that ions can be injected into a time-rising magnetic-mirror field in such a way as to follow trajectories that remain within the containment volume. The possibility of trapping large numbers of ions in this way has been investigated experimentally.

In these studies, ions having energies of several keV were extracted from an ion source located inside one of the peaks of the mirror field. Figure 1 shows the ion source, the shape of several magnetic field lines, and the trace of a typical ion trajectory. The extraction slot of the source is located near the field inflection plane, about midway between the mirror peak and the center, and ions are injected perpendicular to magnetic field lines. In their traversal of the distance between the mirrors, the ions are in an inhomogeneous magnetic field and have a component of velocity along curved field lines. This gives rise to a "guiding center" drift in the azimuthal direction and prevents the particle trajectory from intersecting the ion source after the first reflection. The magnetic field is increased during the injection period by an amount sufficient to move the reflection surface inside the ion source region by the time the ions have precessed completely about the symmetry axis.

EXPERIMENTAL EQUIPMENT

The measurements were carried out in a bakable stainless steel vacuum tank 45 cm in diameter, 270 cm long. Following bakeout to 400°C, vacua of about $1 \times 10^{-9}$ mm Hg have been attained. The magnetic field is determined by a set of coils that can be adjusted to provide axial mirror ratios, $B$(Mirror)/$B$(Center), from near unity to about 3:1. The coils can be operated either with direct current to a peak mirror field of 5 kilogauss or from a $10^6$-joule condenser bank to 24 kilogauss. In pulsed operation the field reaches its peak value in 4.7 milliseconds and decays with a time constant of 200 milliseconds.

A pulsed ion source is employed in which the gas is liberated from loaded titanium contained within the source structure, so that only electrical leads need be brought into the vacuum chamber. A monoenergetic beam of H$^+$ or D$^+$ ions, variable in energy from 3 to 15 keV, is extracted radially through a slot 1 1/4 mm wide and several cm long. Depending on the measurement, extracted beam intensities are varied from a few microamperes to more than 100 milliamperes, and pulse durations are varied from a few to several hundred microseconds. The efflux of neutral gas accompanying the ion beam amounts to about 0.1 micron liter per pulse. The source can be adjusted from outside the vacuum system to provide for axial and radial positioning within the volume, rotation about the source axis, and alignment with the magnetic field lines at any position.

MEASUREMENTS

A detailed examination was made of the trajectory of a well-collimated beam of about one milliamphere or ion source positions and beam energies. Since the rise time of the pulsed field is very long compared with the ion transit time, only a negligible error was introduced by making the measurements in a static field. The beam was intercepted at various points along its trajectory by a conducting plate covered with phosphor, and photographs were taken of the luminous trace. Data were obtained on the precession of the "guiding centers," the pitch of the trajectory with respect to the axial direction, and the positions of the reflection planes. These data, together with the orbit diameters, are sufficient to specify the trajectories completely.

It was observed that for source positions within about 10 cm of the mirror peak, even with a well-collimated beam, an appreciable fraction of the injected beam is transmitted through the far mirror on the first transit. Beam trajectories begin to show strong deviations from trajectories calculated on the basis of simple adiabatic theory as the ion source is removed from near the axis to positions at larger radii; this effect being more pronounced the larger the orbit diameters. For suitably small orbit sizes and source positions more distant from the mirror, however, the beam is completely reflected in front of the far mirror peak. When the source is located near the inflection plane, and as far as 4.5 cm off axis, orbits 14 cm in diameter (measured at the center plane) are well behaved. Moving the source still...
farther off axis reduces the acceptable orbit sizes. Under suitable conditions, the trajectory of a well-collimated beam of ions has been followed for as many as 40 transits between the mirrors.

When the injected beam intensity is increased to a few hundredths of an ampere, space-charge forces begin to be of importance. An additional drift motion is imparted to the beam trajectories which displaces them slightly towards the symmetry axis. In addition, the beam spreads in the axial direction, and an appreciable fraction passes through the far mirror on the first transit. Some of the beam, however, is reflected earlier, and a small fraction becomes trapped in the static field and continues to circulate until it is lost by various processes, the dominant one being charge-exchange collisions with the neutral background gas. The trapped component has been investigated by the use of electrical probes, and some of the ions have been observed to undergo several thousand reflections.

It is to be expected from the results of simple adiabatic calculations, neglecting space-charge fields, that all of the injected beam would be trapped in a suitably fast time-rising magnetic field. Measurements were made of the quantity of beam stored in a field rising at the rate of about $5 \times 10^6$ gauss/sec by observing the number of ejected fast atoms that are produced in the charge-exchange process. The flux of neutral particles was determined by using a secondary electron detector placed outside of the containment volume. For maximum obtainable beam intensities, it was found that ions escape from the volume with a mean loss time of about 100 microseconds. Evidently, in the absence of an adequate source of neutralizing electrons, the repulsive space-charge force built up by the trapped positive charge exceeds the axial compressive force exerted by the time-rising field. An investigation of space-charge neutralization mechanisms is currently under way.

**RESULTS AND CONCLUSIONS**

Subject to the conditions of appropriate source position and suitably small orbit size, the trajectories executed by a well-collimated beam of energetic ions in a magnetic-mirror field are in reasonably good agreement with trajectories calculated using the adiabatic approximation. Even for mirror ratios as low as 1.64 : 1, small beams of particles are contained for periods of several milliseconds. For short injection times under high-vacuum conditions, the space charge in the beam remains unneutralized, and the trajectories are seriously perturbed for beams more intense than a few hundredths of an ampere.

**REFERENCES**

Injection into Thermonuclear Machines Using Beams of Neutral Deuterium Atoms in the Range 100 kev to 1 MeV

By G. Gibson, W. A. S. Lamb and E. J. Lauer*

The purpose of the tests proposed in this paper is to create a high-energy plasma for experimental study. This study would clarify the thinking about the possibility of making a machine that releases more fusion energy than it consumes.

INJECTION INTO MIRROR MACHINE

For clarity, consider injection into a static mirror-machine geometry (Fig. 1). Similar analyses can be carried out for this type of injection into other containment geometries, e.g., the torus. Very briefly, a mirror machine is formed by having two identical coils with a separation greater than in the Helmholtz configuration. Charged particles whose velocity vectors make an angle \( \theta \) with the magnetic field \( H \) are contained between the coils if \( \psi > x \), where

\[
\psi = \frac{1}{\sin x} = \frac{1}{(H/H_m)^{1/2}}
\]

\( H_m \) is the magnitude of the field at its maximum. This is true in the adiabatic approximation where the relative change in the field is small across a Larmor radius. In order for an injected particle to be trapped, either the field must be time-varying or the particle must be perturbed.

Trapping of Injected Areas

In this injection scheme, trapping is accomplished by increasing the \( \psi/m \) of the particles. A beam of neutral deuterium atoms is sent into the mirror machine normal to the field lines and in the symmetry plane. A fraction of the beam is trapped in the field as a result of being ionized by collisions with either cold neutral atoms or trapped hot ions. The trapped ions suffer scattering collisions and are lost when their velocity vectors enter a loss cone (of half angle \( x \)), or they are lost by charge-exchanging with cold neutrals so that they are no longer confined by the magnetic field.

The differential equation that describes the change in density, \( \rho \), of the trapped ions is, using angular brackets to denote average values,

\[
\frac{d\rho}{dt} = \rho_0 J_0 a_1 \left( \frac{V_0}{V} \right) + \rho_0 \left[ J_0 \left( \sigma_1 \frac{V}{V_0} \right) \left( \frac{V_0}{V} \right) - \rho_0 \left( \langle \sigma_e v_1 \rangle + \langle \sigma_n v_2 \rangle \right) \right] \rho - \langle \sigma_e v_2 \rangle \rho^2
\]

where

- \( \rho_0 \) = neutral gas density (atoms/cm\(^3\)).
- \( J_0 \) = flux of the incident beam (atoms/cm\(^2\) sec).
- \( V_0 \) = volume of the beam within the machine (cm\(^3\)).
- \( V \) = confinement volume of the machine (cm\(^3\)).
- \( v \) = relative velocity of a particle in the beam and a trapped ion (cm/sec).
- \( v_1 \) = relative velocity of a trapped ion and a cold neutral atom (cm/sec).
- \( v_2 \) = relative velocity of two trapped ions (cm/sec).
- \( \sigma_1 \) = trapping cross section (cm\(^2\)).
- \( \sigma_0 \) = charge exchange cross section (cm\(^2\)).
- \( \sigma_e \) = effective cross section for small-angle multiple-Coulomb scattering to result in the loss of an ion (cm\(^2\)).

Trapping of injected areas is accomplished by increasing the \( \psi/m \) of the particles. A beam of neutral deuterium atoms is sent into the mirror machine normal to the field lines and in the symmetry plane. A fraction of the beam is trapped in the field as a result of being ionized by collisions with either cold neutral atoms or trapped hot ions. The trapped ions suffer scattering collisions and are lost when their velocity vectors enter a loss cone (of half angle \( x \)), or they are lost by charge-exchanging with cold neutrals so that they are no longer confined by the magnetic field.

The differential equation that describes the change

\[
\frac{d\rho}{dt} = \rho_0 J_0 a_1 \left( \frac{V_0}{V} \right) + \rho_0 \left[ J_0 \left( \sigma_1 \frac{V}{V_0} \right) \left( \frac{V_0}{V} \right) - \rho_0 \left( \langle \sigma_e v_1 \rangle + \langle \sigma_n v_2 \rangle \right) \right] \rho - \langle \sigma_e v_2 \rangle \rho^2
\]

where

\( \rho_0 \) = neutral gas density (atoms/cm\(^3\)).
\( J_0 \) = flux of the incident beam (atoms/cm\(^2\) sec).
\( V_0 \) = volume of the beam within the machine (cm\(^3\)).
\( V \) = confinement volume of the machine (cm\(^3\)).
\( v \) = relative velocity of a particle in the beam and a trapped ion (cm/sec).
\( v_1 \) = relative velocity of a trapped ion and a cold neutral atom (cm/sec).
\( v_2 \) = relative velocity of two trapped ions (cm/sec).
\( \sigma_1 \) = trapping cross section (cm\(^2\)).
\( \sigma_0 \) = charge exchange cross section (cm\(^2\)).
trapped ions, respectively. It has been assumed that:

1. losses due to nonadiabatic effects are negligible;
2. attenuation of the neutral beam in crossing the diameter of the machine is negligible; and
3. the plasma density is uniform across a diameter of the machine.

If an energetic neutral beam of constant flux, $J_0$, is switched on suddenly at time zero when $p = 0$, then at this time the first term on the right of Eq. (2) predominates. This represents a constant rate of increase of $p$ due to trapping new beam particles by ionization in collisions with the residual vacuum density of cold neutral gas atoms. The term involving $p$ to the first power is composed of the difference in the rate of increase of $p$ due to trapping hot ions which capture electrons from cold neutral atoms. If the coefficient of $p$ is positive, then, as $p$ increases, this term increases the rate of trapping and, in the limit, this term predominates and $p$ increases exponentially with time.

**Critical Initial Gas Density**

The coefficient of $p$ will be positive if the neutral gas density is less than a critical density, $\rho_{oc}$:

$$\rho_{oc} = J_0 \left( \frac{\sigma_{in} v}{v_0} \right) \left( \frac{V_0}{V} \right) \left( \frac{\sigma_{en} v_0}{v_0} \right).$$

(Over the energy range of interest, $\sigma_{in} \ll \sigma_{en}$.)

If $p_0 \ll \rho_{oc}$, then the $\epsilon$-folding time, $\tau$, of the build-up is given by

$$\frac{1}{\tau} = J_0 \left( \frac{V_0}{V} \right) \left( \frac{\sigma_{in} v}{v_0} \right) \left( \frac{\sigma_{en} v_0}{v_0} \right).$$

The last term (involving $\rho^2$) is the rate of decrease of $p$ due to ions having their velocity vectors scattered into the loss cone, and this loss causes the density eventually to approach a constant. Assuming that, after build-up, the neutral gas density within the hot plasma is negligible, the steady-state density, $\rho_{ss}$, is

$$\rho_{ss} = J_0 \left( \frac{V_0}{V} \right) \left( \frac{\sigma_{in} v}{v_0} \right) \left( \frac{\sigma_{en} v_0}{v_0} \right).$$

Figure 2 shows this build-up of the density in a qualitative way. The ions are initially trapped with their velocity perpendicular to the field direction midway between the mirrors; this starting condition maximizes the time for the velocity vectors to scatter into the escape cone. When a neutral beam atom is ionized, one high-energy ion and one essentially zero-energy electron are added to the plasma; thus the plasma remains electrically neutral, which is a desirable condition to avoid large space-charge forces.

**DESIGN ESTIMATES**

Equations (3), (4), and (5) will now be rewritten in a form suitable for making numerical estimates of conditions near the center of the machine. Equations (3) and (4) pertain to the conditions during build-up when $p \ll \rho_{ss}$ and the plasma ion velocities have not spread much from $v_0$, so the average value $\langle \sigma_{en} v_0 \rangle$ will be replaced by $\sigma_{en} v_0$. Similarly $\sigma_{en} v_0$ is not a very sensitive function of $v$, so $\langle \sigma_{en} v_0 \rangle$ will be replaced by $\sigma_{en} v_0$. Also, $J_0 V_0$ will be replaced by $J_0 L$, where $J_0$ is the neutral beam current in atoms/second and $L$ is the effective trapping path length of the neutral beam in the plasma: $L = 2r$, where $r$ is the radius of the machine. Finally, $V$ will be replaced by $\pi(2.4r)^2\frac{L}{2}$, the volume of a cylinder of radius $2.4r$ and length $Z$. Therefore Eqs. (3), (4) and (5) become:

$$\rho_{oc} = \frac{J_0 L \sigma_{en} v_0}{\sigma_{en} v_0 2\pi 2.4r \frac{L}{2}} = \frac{J_0 L}{4\pi 2.4r Z}$$

$$\tau = \frac{AZ}{J_0 \sigma_{en} v_0},$$

and

$$\rho_{ss} = \frac{J_0 L \sigma_{en} v_0}{\sigma_{en} v_0 \frac{L}{2}}.$$

For $\langle \sigma_{en} v_0 \rangle$ we have used

$$\frac{5.71 \pi e^4}{\left( \frac{1}{2} M_{en} v_0^2 \right)^2} \ln \left( \frac{\text{maximum impact parameter}}{\text{minimum impact parameter}} \right) v_0$$

where $e$ is the electronic charge in esu and $\frac{1}{2} M_{en} v_0^2$ is the kinetic energy of a beam atom in ergs. The above result is obtained by setting Spitzer's formula for the ion relaxation time equal to $\langle \sigma_{en} v_0 \rangle^{-1}$ and setting the mean energy of the plasma ions equal to one-half the ion injection energy. We have used $\ln$ (maximum impact parameter/minimum impact parameter) equal to 20. Spitzer's formula assumes that the velocity distribution of the ions approaches a Maxwellian distribution. This is not correct for the mirror machine since ions whose velocity vectors lie in the escape cone are missing from the distribution; however, on the basis of unpublished estimates it is thought that this effect would not change $\langle \sigma_{en} v_0 \rangle$ by more than a factor of three. If $N_0$ ions of initial kinetic energy $\frac{1}{2} M_{en} v_0^2$ and $N_0$ electrons with zero initial energy interact without change of the total energy, then the average energy

$\S$ Some neutral beam atoms will be ionized in collisions with plasma electrons. Because the plasma electrons are expected to have a high average energy due to the transfer of energy from the ions, this contribution to the trapping is estimated to be smaller than that due to plasma ions and has been neglected.
of the ions and of the electrons tends to approach one-half \( \frac{1}{2} M_{\text{ion}} \) because of equipartition of energy. If the mean containment time of the electrons is equal to that of the ions, then it is estimated that the mean energy transferred from an ion to the electrons during the ion mean containment time is less than one-half \( \frac{1}{2} M_{\text{ion}} \). If, on the other hand, the electrons have a much shorter mean containment time than the ions, so that electrons that have gained some energy from the ions escape through a mirror and are replaced by zero-energy electrons which enter through the mirror, then the energy transfer would be larger.

In the calculations, a cylindrical volume of length one-tenth the distance between mirrors is assumed for the plasma. This seems a reasonable value to use for the time during build-up when \( \rho \ll \rho_{\text{gas}} \); however, it is not obvious that it should be used in determining \( \rho_{\text{gas}} \), since \( \rho_{\text{gas}} \) is determined by the scattering-out time and this is just the time for a particle to be scattered in such a manner that it makes excursions for the total distance between mirrors. However, particles that are just introduced into the plasma spend all of their time in the center of the machine, whereas just prior to scattering into a loss cone the particles spend part of their time in the center: as a result, the density of ions is larger in the center of the machine and decreases at the ends.

**Beam Neutralization**

At 100 kev a neutral beam may be formed by passing a \( D^+ \) beam through a gas target. Charge exchange, \( \sigma_0 \), competes with ionization, \( \sigma_1 \), in the formation of neutral atoms within the target. The fraction of the beam that is neutralized for an A\(^1\) target\(^3\) is \( \sigma_0/(\sigma_0 + \sigma_1) = 3.5/(3.5 + 4.7) = 0.43 \). For a helium or hydrogen target this fraction is about 0.5 and for nitrogen about 0.4. A condensable gas (mercury or pump-oil) jet makes an acceptable target. The fraction does not appear to be sensitive to the magnitude of \( Z \).

A \( D^+ \) beam of 0.3 amp, and a magnetic field strength of 12.6 kilogauss at the center of the machine is assumed. The selection of these values has been in- 

**Beam Neutralization**

1. The critical pressure can be increased by increasing the magnetic field strength.
2. The other assumptions are
   - \( r = 5.14 \) cm
   - \( Z = 17.4 \) cm
   - \( v_0 = 3.10 \times 10^8 \) cm/sec
   - \( \sigma_0 = 8.5 \times 10^{-17} \) cm\(^2\) (Ref. 3)
   - \( \sigma_1 = 2.5 \times 10^{-16} \) cm\(^2\) (Ref. 4)
   - \( \langle \sigma_1 v \rangle = 1.65 \times 10^{-15} \) cm\(^3\)/sec.

Using these values, we obtain the critical pressure, \( p_c = 1.3 \times 10^{-10} \) mm Hg of \( D_2 \) gas; the build-up time, \( \tau = 4 \) sec; and the steady state high energy ion density, \( \rho_{\text{ss}} = 1.5 \times 10^{12} \) D\(^+\)/cm\(^3\). A quantity of interest is the ratio, \( \beta \), of the particle pressure to the magnetic field pressure, since it is a measure of the relative diamagnetic effect and is also indicative of the possibility of co-operative effects in the plasma:

\[ \beta = 8\pi \rho_{\text{gas}} \times \frac{1}{2} M_{\text{ion}} v_0^2 / B^2 = 0.013 \]  

The \( \text{D(d,n)} \text{He}^0 \) reaction rate is \( 1.4 \times 10^{11} \) sec\(^{-1}\). Other reactions would take place of course, such as the \( \text{D(d,p)} \text{H}^0 \) process, which occurs at about the same rate.

In obtaining the reaction rate, an average of the product of the reaction cross section and the velocity of the ions has been obtained for a two-dimensional isotropic velocity distribution and a single speed.

The fraction of the beam that is trapped is \( \rho_{\text{trapped}} L = 0.4 \). For this fraction to approach 100% at this energy, a beam current of the order of 100 times greater than has been assumed would be required.

If the conditions for exponential build-up are not satisfied with regard to critical pressure, the linear build-up on the cold gas would yield a steady-state density of

\[ \rho_{\text{ss}} = \frac{\rho_0 L \sigma_0(v_0)}{A \sigma_e(v_0) v_0} = 3 \times 10^6 \text{ D}^+/\text{cm}^3, \]  

where it is assumed that the cold gas density is much greater than the critical density.

**Vacuum Requirements**

In practice the critical pressure that is required is very difficult to achieve. However, it is essential if large steady-state densities are to be obtained in these tests. One could hope to reduce the rise time by starting to build up on an initial plasma, which might be obtained from some type of pulsing mechanism, rather than on the "vacuum"; however, unless the existing neutral gas partial pressure is less than the critical pressure, there will be no high-energy build-up. Base pressures of the order of magnitude required in large vacuum systems can be achieved, but the large current beam is a source of cold gas at the point where it terminates. The neutral beam does have the advantage of remaining collimated in passing through the magnetic field region. The technology required to maintain the low operating pressures must be developed for a successful experiment.

The critical pressure can be increased by increasing the energy of the beam, since the charge-exchange cross section decreases rapidly with increasing energy. A neutral beam of 1 Mev D may be formed by the dissociation of 2 Mev \( D_2^+ \) in a gas target. It is estimated that one-quarter of the \( D_2^+ \) ions dissociate, yielding a neutral atom for the optimum density of a pump-oil gas jet. A current of 0.3 amp of 100 kev \( D_2^+ \) is assumed, half of which is lost in accelerating the \( D_2^+ \) to 2 Mev by the known technology of high-current accelerators. The other assumptions are

\[ H = 20 \text{ kg} \]
\[ r = 10.2 \text{ cm} \]
\[ Z = 20 \text{ cm} \]
\[ v_0 = 9.80 \times 10^8 \text{ cm/sec} \]
\[ \sigma_1(v_0) = 3.7 \times 10^{-17} \text{ cm}^2 \]  
\[ \sigma_0(v_0) = 6.0 \times 10^{-21} \text{ cm}^2 \]  
\[ \langle \sigma_1 v \rangle = 5.22 \times 10^{-15} \text{ cm}^3/\text{sec}. \]
At this energy, the critical pressure, $p_c$, is $1.2 \times 10^{-8}$ mm Hg of D$_2$ gas; the build-up time, $\tau = 200$ sec; the ion steady-state density, $n_{ss} = 9 \times 10^{11}$ D$^+$/cm$^3$; the D(d,n)He$^+$ reaction rate, $1.8 \times 10^{13}$ sec$^{-1}$; $\beta = 0.03$; and 0.07% of the beam is trapped. If the cold gas pressure is much greater than the critical pressure, the steady-state density of hot ions is $n_{ss} = 4 \times 10^8$ D$^+$/cm$^3$ with a mean life time against charge exchange of $1/\rho \sigma_i n_0 = 2$ sec if the cold gas pressure is $10^{-6}$ mm Hg of D$_2$.

Experimentally, the number of ions contained in the machine would be determined by measuring the number of ions which escape through the mirrors after turning off the beam. The build-up time would be measured by observing the transient build-up of the escape of plasma ions, or by measuring the transient build-up of reaction rate after turning on a constant beam. The containment time would be obtained by measuring these quantities after turning off the beam.

Tests are in progress to: (1) measure how large a beam of 100-kev deuterium atoms can be produced using the MTA injector$^6$ as a source of 100-kev D$^+$ ions and using pump-oil and mercury gas targets; (2) measure the single-particle containment time of electrons of about 1 Mev energy in a mirror machine in order to search for nonadiabatic effects that might limit the containment time; and (3) investigate methods of attaining a pressure less than the critical pressure with the beam turned on.

REFERENCES

Astron Thermonuclear Reactor

By N. C. Christofilos*

1. INTRODUCTION

The basic requirements of an ideal scheme of confining a plasma at an adequate density and temperature to produce thermonuclear reactions on a large scale can be summarized as follows:

(a) A magnetic field pattern must be established whereby the magnetic lines are closed onto themselves within a vacuum vessel, before injection of any plasma within this pattern. In addition, it is desired that this pattern of closed magnetic lines be axially symmetric with no field components in the azimuthal direction. Thus the drifts resulting from field and pressure gradients are all in the azimuthal direction. Hence, no plasma loss can result from such drifts. Since this field pattern must enclose the current distribution, which creates this pattern, the only way to establish such currents is through organized motion of charged particles.

(b) Besides providing a pattern of closed magnetic lines, means must be provided for ionizing neutral gas to establish a plasma and, thereafter, for heating this plasma up to ignition temperature.

We hope to meet the above requirements in the Astron reactor by a long cylindrical layer of relativistic electrons. This layer of rotating relativistic electrons, hereafter called E-layer, is the key feature of the Astron concept. It not only performs the above-mentioned functions but, in addition, its presence is a necessary condition, as will be shown later, in order to obtain an equilibrium solution of the plasma in the steady state, such an equilibrium solution satisfying hydrodynamic, diffusion and electromagnetic equations.

The relativistic electrons ionize neutral atoms and thus a plasma can be established as soon as the pattern of closed magnetic lines is created. The condition for the latter is that the number of relativistic electrons in the E-layer exceed a certain critical number. Then the relativistic electrons as they rotate inside the plasma lose energy by Coulomb scattering to the electrons of the plasma. If this energy transfer, which is an energy gain for the plasma, is higher than the plasma losses by diffusion or other loss processes, the temperature of the plasma increases. Since the electrons of the E-layer are continuously losing energy by scattering, they have a finite lifetime. Hence in order to maintain the E-layer, a continuous injection of electrons from outside is required.

2. ESTABLISHING THE E-LAYER

The E-layer is established within a long cylindrical vessel (Fig. 1). In this vessel a magnetic field is first established by means of external coils. The direction of this field is substantially parallel to the axis of the cylinder but converging at both ends in order to repel the electrons. It is desirable, mostly for mathematical convenience, that the gyration radius of the electrons, as they move along the E-layer, remain constant although their azimuthal momentum varies as a function of position. This condition is satisfied if the vector potential of the external field obeys the equation

\[ A_0 = B_0 \left( \frac{r}{2} + \frac{\alpha}{k} J_1(kr) \cosh(kz) \right), \]

(1)

where \( k = 1.84/r_L \), \( r_L \) is the desired radius of the E-layer, \( 2L \) is the length of the E-layer, \( J_1(kr) \) is the Bessel function of first order and \( \alpha \) is a constant determining the ratio of the total momentum, \( \rho \), to the azimuthal momentum, \( \rho_\theta \), far from the ends

\[ \frac{\rho}{\rho_\theta} = 1 + \alpha J_0(kr_L). \]

(2)

The value of \( B_0 \) is given by the equation

\[ B_0 = \frac{m_0 c^2}{\sqrt{1 + \alpha J_0(kr_L)}}, \]

(3)

where \( m_0 \) is the rest mass and \( \gamma \) is the relativistic mass ratio of the electrons.

The constant \( \alpha \) must be selected so that \( 2\rho_\theta^2/\rho^2 > 1 \) (far from the ends), otherwise the defocusing action of the end fields, in the radial direction, would lead to unstable electron orbits. This method of injection provides the electrons always with a considerable axial momentum; this constitutes an axial pressure and prevents the collapsing of the E-layer, in the axial
direction, from the contractive forces of its own magnetic field. The electrostatic charge of the E-layer is assumed to be neutralized by positive ions.

Since the injection of the electrons must continue, after a steady state is established, in order to maintain the E-layer, a technique is required allowing external injection to a region where the magnetic field is constant in time. This is accomplished by a combination of standing and traveling waves whereby electrons injected from outside, periodically in short pulses, are trapped in the trough of a traveling wave, forming a ring of rotating electrons, and irreversibly injected in the region where the E-layer is to be established. Details of this injection method are beyond the scope of the present general description and will be discussed in a later paper.

With each bunch or ring of electrons injected in the E-layer region the charge per unit length of the E-layer increases. The rotating charges constitute a current which creates within the volume enclosed by the E-layer a magnetic field in a direction opposite to the external magnetic field. Thus the net value of the magnetic field within this volume decreases continuously as the charge of the E-layer increases. When the number of electrons in the E-layer, per cm length of the layer, reaches a critical value

$$N_0 = \gamma/r_e$$  \hspace{1cm} (4)

(where $r_e$ is the classical electron radius), then the magnetic field inside the volume enclosed by the E-layer far from the ends is reduced to zero. Then the field within the enclosed volume can be reversed by increasing the charge per unit length of the layer a little more. At that moment the combination of the external field with the E-layer field provides the pattern of closed magnetic lines.

3. INITIAL PLASMA FORMATION

As soon as the pattern of closed magnetic lines is established, concentration of plasma is possible within this pattern. By injecting neutral particles within the vessel, these particles become ionized as they travel through the E-layer region. The electrons liberated by the ionization process have acquired also a kinetic energy of a few electron volts. Thus as the ionization continues and the plasma is being established its initial temperature is not zero but a few electron volts. However, this temperature further increases as the relativistic electrons lose energy, by Coulomb scattering, to the plasma electrons.

The energy loss of relativistic electrons is given by the well-known Bethe formula

$$\gamma = -4\pi e^2 n_0 \ln A,$$  \hspace{1cm} (5)

where $n_0$ is the density of the plasma electrons and $A$ is the ratio of the maximum to minimum interaction distance. In Eq. (5) it has been assumed that the velocity of the electrons is equal to $c$. Then the energy gain per plasma electron is

$$u = V_0 \frac{N_0 c e}{\alpha n_0 \gamma},$$  \hspace{1cm} (6)

where $V_0 = m_0 c^2/e$, $N_0$ is the number of electrons per cm length of the layer, $\alpha = 4\pi r_0^3 \ln A$, $r_0$ is the plasma radius,

$$\alpha = \frac{2}{r_0^2 n_0} \int_0^{r_0} n d\rho \hspace{1cm} (6a)$$

and $n_0$ is the plasma electron density near the E-layer; $u$ and $V_0$ are expressed in the same units.

The temperature of the plasma continues to rise as long as the rate of losses due to diffusion across the magnetic lines and other processes is lower than $\dot{\gamma}$. The rate of these losses in comparison with the energy gain will be discussed later in Section 6 of this paper. We observe, however, that the energy gain is proportional to the charge per unit length of the layer, which in turn, by Eq. (4), is proportional to the relativistic mass ratio of the E-layer electrons.

4. PLASMA EQUILIBRIUM

As soon as the plasma temperature starts to rise, the plasma pressure also rises. Since the last closed magnetic line constitutes the boundary of the plasma, the pressure is zero at this boundary. Hence a pressure gradient is created across the magnetic lines. Because
of this pressure gradient, the plasma starts diffusing outwards, and furthermore a Hall current is created which crossed with $B$ (the magnetic field) balances the pressure gradient. This Hall current in turn modifies the pattern of closed magnetic lines. This modification, however, tends to enhance the intensity of the magnetic field; thus the pattern of closed magnetic lines becomes denser as the plasma pressure rises. Of course during the buildup of the plasma pressure the external magnetic field must be increased so that, far from the ends, the following equation is satisfied:

$$B_0^3 = 8\pi\rho + B_0^3,$$  \hspace{1cm} (7)

where $\rho$ is the maximum plasma pressure, $B_0$ is the magnetic field at the surface of the layer and $B_0$ is the external magnetic field (also far from the ends). Then the question arises as to whether or not a self-consistent equilibrium of the plasma can exist under these conditions. In order to solve this problem, I started with the assumption that in a cross section of the plasma, on a plane normal to the axis of symmetry and far from the ends, the magnetic field goes through zero within the plasma at an unknown radius $r_1 < r_0$, where $r_0$ is the radius of the plasma cylindrical boundary. Further, I assumed that the magnetic flux through this cross section between $r = r_1$ and $r = r_0$ is equal and opposite to the flux between $r = 0$ and $r = r_1$; the Larmor radius has been assumed negligible in comparison with the physical dimensions of the system. Then, using hydrodynamic, diffusion and electromagnetic equations, a class of solutions has been obtained for the cylindrical part, far from the ends. One of these self-consistent equilibrium solutions has been worked out for the whole volume including the ends. The detailed derivation of this solution is given in Appendix 1. The solution for the vector potential in this case is

$$A_s = B_0[c_1J_1(k_1r) + c_2J_1(k_2r)] \cos(hk_1) + A(r),$$  \hspace{1cm} (8)

where

$$A(r) = \frac{B_0}{2k_2} e^{-kr}, \quad \text{for } r < r_0,$$

$$A(r) = \frac{B_0}{2k_2} e^{-kr}, \quad \text{for } r_0 < r < r_0.$$

$B_0$ is the external magnetic field far from the ends and $\lambda$ is a parameter.

This solution requires at $r = r_1$ a field discontinuity or a field jump from $-B_0 e^{-kr}$ to $+B_0 e^{-kr}$. This discontinuity can be realized only by the presence of a sheath of rotating charged particles where, within the thickness of the sheath, the field goes through zero. The presence, for other reasons, of the E-layer allows the existence of this solution. However, as it turned out, the investigation of this equilibrium solution could have led, eventually, to the discovery of the E-layer, but not this layer been postulated long before the above mathematical solution was obtained.

For a numerical example, the shape of the plasma, the distribution of the plasma pressure and the magnetic field have been computed. The results are shown in the graphs (Figs. 2 and 3). In Fig. 2 a cross section is shown on a plane through the axis of symmetry. The magnetic lines (which are also equipressure lines) are shown as well as the shape of the plasma at the ends. The line where $\rho = 0$ is the last closed magnetic line. Hence it constitutes the plasma boundary. A line just outside this boundary line is open and it goes out along the axis. Two of these lines are shown in Fig. 2. The plasma diffuses out, being guided through these lines to form two collimated beams (one at each end). In this way the diverters which proved necessary in the stellarator do exist inherently in the Astron. The solution given above is continuous inside as well as outside the plasma except for two singular points at the intersection of the line $\rho = 0$ with the axis of symmetry. Since there is no plasma outside, the necessary current, as required by the vector potential solution, will be provided by material coils. At this point it should be noted that since the plasma is diverted out through the open lines, for example between the line $\rho = 0$ and the outermost line shown in Fig. 2, with finite velocity, it follows that the pressure in this region, however small, is also finite and not exactly zero. Consequently even in this region the plasma provides some current; thus the material coils can be located beyond the outermost line shown in Fig. 2.

In Fig. 3, a cross section of the plasma on a plane normal to the axis of symmetry (far from the ends) is shown. In the upper graph (Fig. 3a) the plasma density is plotted as a function of the radius. We observe that the pressure is zero at the axis of symmetry, as well as at the boundary, whereas its maximum occurs in the region of the E-layer. In the lower graph (Fig. 3b) the magnetic field distribution is shown. Since the magnetic field is reversed within the plasma volume, we have $\beta = 8\pi\rho/B_0^3 = 1$ by definition. In addition, because of the fact that the maximum pressure is not at the axis of symmetry, the quantity $\eta$, 

$$\eta = \frac{2}{m_0^2 r_0^2} \int_{r_0}^{r_0} n\rho dr,$$  \hspace{1cm} (9)

which is proportional to the rate of thermonuclear

![Figure 2. Magnetic field geometry](image)
In the above solution \( \nabla \cdot (\rho v) \) is different from zero. Actually, \( \nabla \cdot (\rho v) \) equals \(-S\) in the region \( 0 < r < r_1 \) and \(+S\) in the region \( r_1 < r < r_0 \). If we now assume that all the diffused particles are replaced by ionized neutral atoms in the region of the E-layer, we might question how the sources and sinks required by the solution would be realized. However, on closer examination we observe that these sources and sinks are automatically mutually cancelled or

\[
\int_0^{r_0} Srdr = 0. \tag{10}
\]

Since no actual sources or sinks are present within the plasma, what actually will happen is this: Inside (in the region \( 0 < r < r_1 \)), because of the lack of sinks the density will tend to rise at a rate \( S \). However, along the same magnetic line outside the E-layer the density will tend to decrease at the same rate. This rate of density change is very slow and it is negligible during the time required by the particles to travel a complete circuit along a line. As a result of this density change a small pressure gradient is created along the lines and the particles which are superfluous inside appear as a source on the same line outside. Consequently particles vanish from the inside region, thus creating virtual sinks, and reappear outside as sources.

5. PLASMA STABILITY

Having now ascertained that an equilibrium solution exists, we shall examine the stability of this equilibrium solution with respect to small perturbations. We can divide the possible perturbations into two classes.

1. Perturbations where the magnetic lines are tilted or where Alfvén waves are propagated along the magnetic lines. In this case the perturbation is characterized by a Poynting vector, \( \mathbf{P} \), traveling along the magnetic lines. This condition can be expressed as

\[
\mathbf{B} \cdot \mathbf{P} \neq 0. \tag{11}
\]

2. The second class is characterized by the absence of a Poynting vector along the magnetic lines or

\[
\mathbf{B} \cdot \mathbf{P} = 0. \tag{12}
\]

Since in the first case the lines are tilted and this can be done at the expense of the energy of the perturbation, it seems that in the second case the perturbations are more dangerous. Consequently we confine most of our attention in the present paper to this second case.

Let \( \mathbf{E} \) and \( \mathbf{H} \) be the perturbed electric and magnetic fields, respectively. Then Eq. (12) can be written

\[
\mathbf{P} \cdot \mathbf{B} = \mathbf{E} \times \mathbf{H} \cdot \mathbf{B} = \mathbf{E} \cdot \mathbf{H} \times \mathbf{B} = 0. \tag{13}
\]

Equation (13) can be satisfied if \( \mathbf{E} = 0 \). However, one finds that this condition is possible only in trivial motions of the plasma along the lines or in trivial rotation of the plasma. Hence the case which makes sense is

\[
\mathbf{B} \times \mathbf{H} = 0. \tag{14}
\]

This case has been treated in normal mode analysis as described in Appendix 2. For the purpose of this analysis we assume that the perturbation is of the type

\[
\hat{q} = q(r, z) \sin(m\theta) e^{\int}, \tag{15}
\]

where \( \hat{q} \) is any of the perturbed quantities. Then it follows from the linearized hydromagnetic equations that

\[
E_\theta = \frac{1}{r} \phi(rA_\phi) \cos(m\theta) e^{\int} \tag{16a}
\]

and

\[
v_\theta = rf(rA_\phi) \sin(m\theta) e^{\int}, \tag{16a}
\]

where \( E_\theta \) and \( v_\theta \) are the electric field and perturbed velocity, respectively, in the azimuthal direction, and \( f \) and \( \phi \) arbitrary functions of the flux \( \phi = rA_\phi \). From further investigation (see Appendix 2) it was concluded that \( E_\theta \) and \( f(rA_\phi) \) are zero at the axis of symmetry. Since \( rA_\phi \) is constant along a magnetic line, this means that \( E_\theta = 0 \) at any point in the boundary surface. Thus one can observe that this kind of perturbation is an internal motion where the surface remains at rest. This is due to the fact that all boundary lines return through the axis of symmetry. As we move far from the ends, the flux \( rA_\phi \) degenerates very soon to a function of the radius only. Then the same holds for \( f \) and \( \phi \), and the perturbation degenerates to one of the type

\[
\hat{q} = q(r) \sin(m\theta) e^{\int}. \tag{15}
\]

This perturbation has been investigated in more detail. If we define a new quantity \( w = \hat{p} + (\mathbf{B} \cdot \mathbf{H}/4\pi) \), where \( \hat{p} \) is the perturbed pressure, then from the linearized equations we obtain (see Appendix 2) a second-order differential equation for \( w \):

\[
\frac{\partial^2 w}{\partial r^2} + \frac{2}{r} \frac{\partial w}{\partial r} - \frac{\partial^2 w}{\partial r^2} - \frac{\rho_0^2}{2} \left( \frac{m^2}{r^2} + \frac{1}{r^2} \right) w = 0, \tag{17}
\]

where \( \rho_0 \) is the equilibrium density and

\[
S^2 = \frac{B_0^2}{4\pi \rho} + \frac{\rho_0}{\rho}
\]

the sound velocity of the medium.
Equation (17) is valid for any perturbation where the perturbed velocity is not constant in time. In the latter case \( \omega = 0 \). However, the plasma is subject to rotation since angular momentum is continuously transferred from the E-layer to the plasma electrons. This angular momentum cannot be transferred to the wall because the magnetic lines are closed onto themselves. Hence the pattern of closed magnetic lines rotates together with the plasma. The angular momentum of the plasma is transferred outside as the plasma particles diffuse towards the open magnetic lines. The plasma angular momentum bears the same proportion to the angular momentum of the E-layer as the mean lifetime of a plasma particle bears to the mean lifetime of an electron of the E-layer. Because of this rotation it makes no sense to consider velocities constant in time, for any change of radial position means change of the centrifugal force. However, otherwise this rotation is not taken into consideration in the stability calculations.

Equation (17) has been thoroughly investigated and it has been found that in the case \( E_0 = 0 \) at two different values of \( r \) the admissible eigenvalues of \( \omega^2 \) are always negative, which means stability. From the solutions of Eq. (17) one can compute all the perturbed quantities and then invert these solutions to functions of \( rA_0 \). Thus, the general solutions can be obtained for all the perturbed quantities for the type of perturbation \( \mathbf{B} \times \mathbf{H} = 0 \). Thereafter one can write down the linearized hydromagnetic equations in curvilinear coordinates and insert the solutions obtained into these equations. Then either the equations are not satisfied, which means that perturbations of the type \( \mathbf{B} \times \mathbf{H} = 0 \) are not possible at all in the Astron geometry, or they are satisfied, which means that the perturbation is possible but only stable modes are admissible. Consequently, the conclusion is that perturbations of type \( \mathbf{B} \times \mathbf{H} = 0 \) either are possible in the Astron geometry and are stable or do not exist at all. Hence we are left with perturbations of the type

\[
\mathbf{B} \cdot \mathbf{P} \neq 0.
\]

In order to investigate this perturbation the linearized equations have been written in curvilinear coordinates. One then would observe that terms are added in the equations of motion resulting from the mechanism of the Alfvén waves, and their contribution is stabilizing. However, wherever the magnetic lines are concave towards the plasma they contribute to instability. Direct normal-mode solutions of these equations appear extremely difficult if not impossible. Consequently, it appears that only through an energy principle would it be possible to evaluate this class of perturbations. The Princeton energy principle \(^2\) appears to be the most convenient for this investigation. However, some adaption to the Astron geometry is required. This problem is now under investigation and the results will be reported later.

Finally I would like to note that the first class of perturbations \( \mathbf{B} \times \mathbf{H} = 0 \) includes the so-called flute instability and perturbations in general, where the magnetic lines move parallel to themselves. These are considered as the most dangerous perturbations and it is concluded that at least against such perturbations the Astron geometry is stable.

### 6. Energy Balance During the Plasma Heating Process

The main energy losses of the plasma are diffusion and bremsstrahlung. The first is predominant at low temperature (less than a kilovolt) whereas the second is the more important above ten kilovolts. The rate of diffusion depends on the pressure and field distribution throughout the plasma, which in turn depend on the particular equilibrium solution selected. The solution discussed in Section 4 has been selected as being simple and helpful in obtaining a clear picture of the plasma equilibrium. However, as an actual solution it is somewhat unrealistic. The reason is that it has been assumed that the magnetic field just inside the E-layer is equal to the magnetic field just outside; this is not actually possible as it implies infinite charge in the E-layer. Furthermore, it has been assumed that all the new ions are produced in the region of the E-layer. This is correct only at the very beginning of the plasma formation. Consequently, in order to calculate diffusion losses, a solution for the equilibrium is required without the above restrictions.

The following calculations are not considered accurate; their purpose is to show the possibility of heating the plasma with the relativistic electrons up to ignition temperature and not to compute the parameters of an actual machine. In view of these approximations and considering that the cylindrical part of the plasma is very long, the diffusion loss from the ends is neglected. Thus in what follows we consider, as far as diffusion is concerned, the plasma as being an infinite cylinder.

In an equilibrium solution satisfying the above conditions, as well as the conditions considered in Appendix I, the vector potential \( A_\theta \) and plasma pressure \( p \) are:

\[(a)\] Region \( 0 < r < r_i \)

\[
A_\theta = \frac{B_t}{2\lambda r} \sinh[\lambda(r^2 - r_i^2)],
\]

\[
p = \frac{B_t^2}{8\pi} \left[ \sinh^2[\lambda(r^2 - r_i^2)] - \sinh^2[\lambda(r^2 - r_b^2)] \right],
\]

\[(b)\] Region \( r_i < r < r_0 \)

\[
A_\theta = \frac{B_\omega}{2\lambda r} \sinh[\lambda(r^2 - r_i^2)],
\]

\[
p = \frac{B_\omega^2}{8\pi} \left[ \sinh^2[\lambda(r^2 - r_i^2)] - \sinh^2[\lambda(r^2 - r_b^2)] \right],
\]

where \( B_\omega \) and \( B_t \) are the magnetic field just outside and inside the E-layer, respectively; \( r_0 \) and \( r_i \) the
plasma boundary radius and E-layer radius, respectively, and
\[ \lambda r_e^2 = \sinh^{-1}\left(\frac{(8\pi \rho_0)^{\frac{1}{3}}}{B_t}\right), \]  
(22)
\[ \lambda r_0^2 = \sinh^{-1}\left(\frac{(8\pi \rho_0)^{\frac{1}{3}}}{B_w}\right) + \sinh^{-1}\left(\frac{(8\pi \rho_0)^{\frac{1}{3}}}{B_t}\right), \]  
(23)
and \( \rho_0 \) is the maximum value of the plasma pressure.

The diffusion velocity of the plasma is
\[ v = -\frac{\lambda \rho_0^2}{\sqrt{\pi} \tau n} \tanh[\lambda (\rho_0^2 - r_e^2)]. \]  
(24a)
This equation is valid in both (inside and outside) regions. We are interested in the energy loss due to diffusion at \( r = r_0 \).

The time, \( \tau \), between collisions is not constant throughout the plasma since it is dependent on the density; however, we consider in the following the minimum value of \( \tau \), which occurs at \( n = n_0 \). This value of \( \tau \) is
\[ \tau = c_1 \left(\frac{\mu}{V_0}\right) \frac{1}{\eta \sigma_o \gamma_c}, \]  
(26)
where \( c_1 \approx 1 \). Substituting in Eq. (24a) and from Eq. (25) we calculate the energy loss per cm length of the plasma due to diffusion:
\[ W_d = \frac{v}{r_0} \frac{\sigma_c}{V_0} \left(\frac{16\pi n_0 \rho_0 V_0}{B_w}\right)^\frac{1}{2} \frac{eV_0 \rho_0 F}{B_w} \text{ergs/cm sec}, \]  
(27)
where
\[ F = \frac{\sinh^{-1} y + \sinh^{-1}\left[\frac{(B_w/B_t) y}{1+y^2}\right]}{(1+y^2)^{\frac{1}{2}}}, \]  
(27a)
y = \left(\frac{(8\pi \rho_0)^{\frac{1}{3}}}{B_w}\right)\left(\frac{1+y^2}{1+y^2}\right),
and \( n_0 \) is the density of the plasma electrons in the region of the E-layer.

The energy transferred to the plasma by Coulomb scattering with the relativistic electrons is
\[ W_e = eV_0 N \sigma_c \eta_c n_0 \text{ ergs/cm sec}, \]  
(28)
where \( N \) is the number of electrons per cm length of the E-layer. This can be written
\[ N = s \gamma / r_e, \]  
(28a)
where
\[ s = \frac{B_w+B_t}{B_w-B_t}, \]  
(28b)
and
\[ B_w = V_0 \frac{(s+1)\gamma \cos \delta}{r_t}, \]  
(29)
where \( \delta \) is the ratio of the azimuthal to the total momentum of the E-layer electrons. The plasma temperature can rise to any desired value as long as
\[ W_e > \alpha W_d, \]  
(30)
where \( \alpha \) is the ratio of the rate of the plasma losses from all possible processes to the rate of diffusion. The rate of diffusion is proportional to \( F \), which in turn is a function of the pressure. Assuming \( n_0 \) fixed, we observe that for \( y \ll 1 \) the quantity \( F \) is proportional to the square root of the pressure. This condition \( (y \ll 1) \) is met at the very beginning of the plasma formation. Thereafter, as the plasma pressure increases, \( F \) reaches a maximum, and for further pressure increase the quantity \( F \) decreases almost as \( 1/p_0 \). For \( B_w/B_t = 10 \) this maximum occurs at \( y = 1 \), and \( F_{\text{max}} = 2.75 \). Since the diffusion loss goes through a maximum for fixed plasma density, this density can be calculated from Eq. (27) for \( F = 2.75 \). In view of condition (30) and Eqs. (27), (28) and (29), the plasma density must be less than the value given by the relation
\[ n_0 < \frac{1}{r_0} \frac{s^2(s+1)^2}{16\pi r_t^2} \gamma^4 \cos^2 \delta \]  
(31)
For \( B_w/B_t = 10 \) we have \( s^2(s+1)^2 = 7.5 \), so that Eq. (31) becomes
\[ n_0 < \frac{1}{r_0} \frac{s^4 \cos^2 \delta}{16\pi r_t^2}. \]  
(32)
We observe that the allowed density increases very fast with the value of \( y \). The temperature where the maximum diffusion loss occurs is
\[ n_0 > \frac{V_0(F_{\text{max}}/sy)^2}{r_0} \]  
(33)
For \( B_w/B_t = 10 \) we have \( s = 1.22 \), and Eq. (33) becomes
\[ n_0 > 2.5 \times 10^6 \alpha^2 \gamma^2 \text{ ev}. \]  
(33a)
For a numerical example we assume \( \gamma = 100 \) and \( \cos^2 \delta \) \( \pi r_t^2 = 10^{-6} \).

Then
\[ n_0 > 2.5 \times 10^{14} / \alpha^2, \]
\[ n_0 > 2500 \text{ ev}. \]

The two other main loss processes are the excitation loss and bremsstrahlung. However, the first one is rather negligible in the present case. This loss results from orbital excitation of partially ionized atoms of high atomic number contaminating the plasma. In the present case this loss is small if a moderately good vacuum can be provided. For example, for \( \gamma = 100 \) a contamination influx of about one micron-liter per second is tolerable.

The bremsstrahlung loss per cm length of plasma is
\[ W_b = 2.6 \times 10^{-24} \rho_0 \mu_0 (\eta \gamma \sigma_c^0) \text{ ergs/cm sec}, \]  
(34)
where \( \mu \) is expressed in esu and \( \eta \) is defined in Eq. (9).
We shall now evaluate this bremsstrahlung loss for the values of $n_0$ and $\sigma_0$ as given in relations (32) and (33), at the temperature where the maximum diffusion loss occurs, namely, at $\gamma = 1$, or $\phi = 16\pi n_0 \sigma_0 = B_w^2$. Substituting this value in Eq. (34) we obtain

$$W_b = 2.6 \times 10^{-24} n_0^2 B_w^2 / (16\pi) \gamma \sigma_0^2. \tag{35}$$

Then from Eqs. (27), (29) and (35) we can derive the value of $\alpha$:

$$\alpha = 1 + \frac{2.6 \times 10^{-24}}{16 F_{\max}} V_0 b r_0^2 / (s + 1) \gamma^2 \sigma_0^2 \cos^2 \delta\tag{36}$$

or

$$\alpha = 1 + \frac{\gamma}{2(100)} \cos^2 \delta. \tag{37}$$

The parameters of the numerical example give $a^2 \approx 1.2$. Thus, we conclude that in order to raise the temperature beyond the value where the maximum diffusion loss occurs the allowed density is of the order of $10^{14}$. As the temperature rises above this value the bremsstrahlung loss increases whereas the diffusion loss becomes less important. Thus, at very high temperature we shall compare the energy gain $W_e$ with the bremsstrahlung loss only. Then Eqs. (28) and (34) yield

$$n_0 < 3 \times 10^{17} s V_0 / (\pi \sigma_0^2 \mu^2), \tag{38}$$

where $s$ is expressed in kev. Substituting in this equation the numerical values and $s = 100$ kev the value of the density is

$$n_0 < 3 \times 10^{14}.$$

Thus we can conclude that, provided the energy of the E-layer electrons is high enough (about 50 Mev), we can maintain a plasma density of $10^{14}$ at any desired temperature. This can be done without the help of fusion energy; thus, it is possible to heat a high-density plasma up to a temperature much higher than the ignition temperature.

In addition to satisfying the requirements of plasma heating, the distribution of high energy electrons must provide for equilibrium of the E layer. This requires the E layer to be longer than the active plasma volume; i.e., the pattern of closed magnetic lines. As the E-layer electrons enter this pattern their azimuthal momentum changes in proportion to the change of vector potential, along the E layer, from the tip of the plasma to the region of highest pressure. Consequently the value of $\gamma$ must satisfy the condition

$$\gamma > \frac{B_\varphi r_0}{2 V_0 (\mu r_0^3). \tag{39}}$$

For a power reactor this value is of the same order as in the above numerical example.

### 7. EXPERIMENTAL PROGRAM

Although the concept of the E-layer for confining and heating the plasma had been proposed several years ago (early 1953), several features of the E-layer as described above were not well understood at that time. Consequently, only after almost three years of theoretical work, the Atomic Energy Commission decided that an experimental program was warranted on the Astron proposal. This program started early in 1957. The first purpose of the Astron group was to design a model where it would be possible to demonstrate the feasibility of the basic feature of the Astron concept; namely, that the E-layer can be established and can create the pattern of closed magnetic lines. The electron energy for this model will be of the order of 3 Mev. Because of this low energy no plasma is expected to be established in this first model except under exceedingly good vacuum conditions. The electrons are provided by means of an electron gun capable of producing an electron beam of 1 Mev energy and a peak current of 100 amp under pulsed conditions. This electron gun is now undergoing tests. The other components of the model have been designed and will be constructed as needed for the program.

After completion of the tests with the low-energy electrons, and provided that the E layer can be established, it is planned to increase the E-layer electron energy in successive steps so that, eventually, positive power gain would be demonstrated.

Finally the possibilities of the Astron concept leading to an economical power reactor will be briefly discussed. An engineering group started several months ago to investigate possible designs and parameters of an Astron power reactor under the assumption that the basic principles are sound. The results of this study, thus far, are that it appears to be feasible to build and operate an Astron power reactor competitively with conventional power plants. It should be noted that the particular components associated with the acceleration and injection of the electrons in the E-layer constitute less than 15% of the total cost. The balance of the system involves familiar equipment such as turbogenerators, magnets, power supplies and switchgear, for which the costs are more or less amenable to reliable computation. This engineering study was carried out for an Astron power reactor with the following parameters:

- **E-layer electron energy**: 50 Mev
- **E-layer radius**: 50 cm
- **Plasma radius**: 70 cm
- **External magnetic field**: 40,000 gauss
- **Length of the E layer**: 30 m
- **Diameter of the reaction tank**: 150 cm
- **Plasma temperature**: 25 kev
- **Net electric power output**: 500,000 kw

The above results must be considered at this time as being only indicative, although encouraging, for many difficulties are anticipated in the effort to materialize the Astron concept to a power-producing
thermonuclear reactor. Consequently, one must anticipate that many years will pass before this ultimate goal is achieved.

**APPENDIX 1**

As mentioned above, the vector potential governing the equilibrium of the plasma, in the proposed reactor, should satisfy the requirement within the plasma that

\[ \int_p \mathbf{B}_d \mathbf{d} \mathbf{F} = 0 \]

on any plane \( z = \text{constant} \), where \( z \) is the axis of symmetry and by \( F \) is understood the area which, on any such plane, is occupied by the plasma.

The equations that should be satisfied by a steady-state equilibrium solution are

\[ \mathbf{B} = \nabla \times \mathbf{A}, \tag{A1.1} \]

\[ \frac{4\pi}{c} \mathbf{j} = \nabla \times \nabla \times \mathbf{A}, \tag{A1.2} \]

\[ \nabla \mathbf{p} = \frac{1}{c} \mathbf{j} \times \mathbf{B}, \tag{A1.3} \]

\[ \mathbf{v} = -\nabla \mathbf{p} \cdot \frac{M}{m_0 \omega_e r_p}, \tag{A1.4} \]

\[ S = \frac{\delta \mathbf{p}}{\delta \mathbf{r}} + \nabla \cdot (\mathbf{v} \rho), \tag{A1.5} \]

and for a steady state

\[ S = \nabla \cdot (\rho \mathbf{v}). \tag{A1.5a} \]

The equilibrium solution is assumed symmetric about the \( z \) axis; the coordinates are cylindrical: \( r, \theta, z \); the vector potential \( \mathbf{A} \) and the current \( \mathbf{j} \) have components only in the azimuthal direction.

Further definitions are:

- \( \mathbf{B} \) = the magnetic field having components in the radial and \( z \) directions,
- \( \mathbf{p} \) = the scalar pressure of the plasma,
- \( \mathbf{p} \) = the mass density of the plasma,
- \( \mathbf{v} \) = the diffusion velocity,
- \( M, m \) = the ion and electron mass respectively,
- \( \tau \) = the mean time between Coulomb collisions,
- \( \omega_e = \frac{e}{mc} (B_r^2 + B_z^2)^{1/2} \)
  - the electron gyrofrequency, and
- \( S \) = the strength of sources or sinks within the plasma volume.

It is assumed that diffused particles are replaced by neutral atoms at the region of highest pressure. Consequently, within the plasma volume the condition

\[ \int_p S \delta V = 0 \]

should be satisfied by the solution. The solution for the vector potential is assumed to be of the form

\[ r \mathbf{A} = \phi(r) + f(r) \frac{\cosh(kz)}{\cosh(kL)} \tag{A1.6} \]

and \( k \) to be of the order of \( r_0^{-1} \) (\( r_0 \) is the boundary radius of the plasma), whereas \( 2L \) (the length of the plasma) is assumed to be at least \( 20r_0 \). Thus the second function vanishes very fast as one moves from the end inward, parallel to the \( z \) axis. Consequently in this internal region the magnetic lines are parallel to the axis of symmetry. It is then possible to derive first the solution for the infinite cylinder, namely the function \( \phi(r) \) only, and thereafter to modify the solution by adding the \( z \)-dependent function. In the cylindrical part (far from the ends) we observe that the pressure is

\[ \mathbf{p} = \frac{B_0^2 - B_z^2}{8\pi}. \tag{A1.7} \]

The pressure is constant along a magnetic line as well as the function \( \psi = r \mathbf{A} \). In order to satisfy the basic requirement that

\[ \int_0^{r_0} B_z dr = 0 \]

it is obvious that in a region where the radius is less than a value \( r_\ell \), the value of \( B \) is positive and in the remaining region (where \( r_\ell < r < r_0 \) the value of \( B \) is negative.

From Eq. (A1.7), solving for \( \mathbf{B} \), we obtain

\[ B_z = \pm (B_0^2 - 8\pi \rho)^{1/2}, \tag{A1.7a} \]

which indicates that the same line has a value \( + (B_0^2 - 8\pi \rho)^{1/2} \) in the region where \( r < r_\ell \) and the value \( - (B_0^2 - 8\pi \rho)^{1/2} \) in the regions \( r_\ell < r < r_0 \).

From \( \mathbf{B} = \nabla \times \mathbf{A} \), Eq. (A1.1), we have

\[ B_z = \frac{\partial (r \mathbf{A})}{\partial r}. \tag{A1.1a} \]

Let

\[ \psi_r = r A_r \text{ in the region } r < r_\ell, \]

\[ \psi_0 = r A_0 \text{ in the region } r_\ell < r < r_0. \]

For every value of \( r = r_m \), where \( \psi = \psi_r(r_m) \) in the internal region, there corresponds a value \( r_\ell \) in the external region, where \( r_\ell \) is determined from the equation

\[ \psi_0(r_\ell) = \psi_r(r_m), \tag{A1.8} \]

where \( r_m < r_\ell \) and \( r_\ell < r_0 \).

From Eqs. (A1.7) and (A1.1a) it follows that

\[ \frac{\partial \psi_r(r_m) - \partial \psi_r(r_m)}{r_\ell \partial \ell} = \frac{\partial \psi_r(r_m)}{r_\ell \partial \ell} \tag{A1.8a} \]

The above conditions, (A1.8) and (A1.8a), are satisfied for any value of \( r \) if \( \psi_0 \) and \( \psi_r \) are solutions respectively of the differential equations

\[ \frac{\partial \psi_0}{\partial r} - r \phi_0 = 0, \tag{A1.9} \]

\[ \frac{\partial \psi_r}{\partial r} + r \phi_r = 0. \tag{A1.9a} \]

Solutions satisfying these differential equations are

\[ \psi_0 = c_0 e^{-r^2}, \tag{A1.10} \]

\[ \psi_1 = c_1 e^{r^2}. \tag{A1.10a} \]

Taking into consideration that the field should
decrease as one moves away from the axis of symmetry, we observe that the second solution corresponds to the inside region. At \( r = r_0 \), \( \psi = 0 \) and \( B = -B_0 \), then
\[
\psi_0 = -\frac{B_0}{2\lambda} e^{\phi(r^2 - r_0^2)} \tag{A1.11}
\]
and the second equation becomes
\[
\psi_1 = -\frac{1}{2\lambda} B_0 e^{-r^2}. \tag{A1.11a}
\]
At \( r = r_t \) it is obvious that
\[
\psi_0 = \psi_1,
\]
whence
\[
r_t^2 - r_0^2 = -r_t^2 \quad \text{or} \quad r_t = r_0/\sqrt{2}.
\]
The magnetic field is
\[
B_r = B_0 e^{-r^2} \quad \text{for} \quad r < r_t, \quad \tag{A1.12}
B_r = -B_0 e^{\lambda(r^2 - r_0^2)} \quad \text{for} \quad r_t < r < r_0. \tag{A1.12a}
\]
As we approach \( r_t \) from the inside, we find that
\[
\lim_{r \to r_t} B = B_0 e^{-r_t^2},
\]
and as we approach \( r_t \) from the outside, we find that
\[
\lim_{r \to r_t} B = -B_0 e^{-r_t^2}.
\]
Consequently the existence of a field jump or field discontinuity is required at \( r = r_t \). The value of the intensity of the field jump is
\[
B_0 = 2B_0 e^{-r_t^2}. \tag{A1.12b}
\]
The only way to create such a field jump at \( r = r_t \) is to provide a current sheath at that radius. Since this current sheath is inside a high-temperature plasma, such a current can only be created by an organized motion of charged particles. Hence the relativistic electron layer will fulfill this requirement and create the field jump.

Let
\[
\lambda = \frac{k_0}{2r_t}. \tag{A1.13}
\]
Then
\[
k_0 r_t = 2 \ln \left( \frac{2B_0}{B_0} \right) \tag{A1.14}
\]
and
\[
B_0 = 4\pi c j_0. \tag{A1.15}
\]
where \( j_0 \) is the current per cm of the electron layer. The rate of diffusion is
\[
\nu_r = \frac{Mm_0^2}{8\pi e^2 r_t^3} \ln \left( \frac{2B_0}{B_0} \right) \quad \text{for} \quad r < r_t, \tag{A1.16}
\]
\[
\nu_r = \frac{Mm_0^2}{8\pi e^2 r_t^3} \ln \left( \frac{2B_0}{B_0} \right) \quad \text{for} \quad r_t < r < r_0. \tag{A1.16a}
\]
The strength of the required distributed sources inside the plasma are (assuming \( \tau \) constant)
\[
S = \nabla \cdot (\nu r) = -\frac{Mm_0 k_0}{\tau m_0^2 r_t} = -S_0 \quad \text{for} \quad r < r_t, \tag{A1.17}
\]
\[
S = +\frac{Mm_0 k_0}{\tau m_0^2 r_t} = +S_0 \quad \text{for} \quad r_t < r < r_0, \tag{A1.17a}
\]
and
\[
\int_0^{r_0} S r dr = -S_0 \frac{r_0^3}{3} + S_0 \frac{r_0^3 - r_t^3}{3} = 0.
\]
The complete solution with the \( z \)-dependent function is assumed in the form
\[
A_0 = -\frac{B_0}{k} [c_1 J_1(k_1 r) + c_2 J_1(k_2 r)] \frac{\cosh(kz)}{\cosh(kL)} \tag{A1.18}
\]
\[
\frac{B_0}{2k} e^{\phi(r^2 - r_0^2)} \quad \text{for} \quad r > r_t, \tag{A1.18a}
\]
\[
A_1 = -\frac{B_0}{k} [c_1 J_1(k_1 r) + c_2 J_1(k_2 r)] \frac{\cosh(kz)}{\cosh(kL)} \tag{A1.18}
\]
\[
-\frac{B_0}{2k} e^{-r^2} \quad \text{for} \quad r < r_t. \tag{A1.18a}
\]
The boundary conditions which determine the constants are
(1) at \( r = r_t, z = L, \)
\[
r_t A_0 = r_t A_1 = -\frac{B_0}{2\lambda},
\]
(2) at \( r = r_t, B_z \) goes through zero, or
\[
c_1 k_1 J_0(k_1 r_t) + c_2 k_2 J_0(k_2 r_t) = 0;
\]
(3) at \( r = r_t, \)
\[
\partial \phi/\partial r = 0 \quad \text{(since} \ B_z = 0). \]
Then
\[
\int_0^L \frac{\partial \phi}{\partial z} dz = -\int_0^{r_t} jB_r dz = \frac{B_0^2}{8\pi}
\]
(4) at \( r = r_t, z = L, \)
\[
B_r = eB_0,
\]
where \( \epsilon = k/k_0 \).
Furthermore, the arguments \( k_1 r_t \) and \( k_2 r_t \) should be selected so that \( B_r \) is always finite for every value of \( r \) between zero and \( r_0 \) except \( r = 0 \) where it becomes zero. The complete solution for \( k_0 r_t = 10 \) and \( \epsilon = 0.184 \) is:
\[
-\frac{k_0}{B_0} j_0 \cdot (r A_0)
\]
\[
= 5.435 \times [1.665 J_1(2.12z) + 2.136 J_1(5.385z)]
\]
\[
\times \frac{\cosh(1.84z/r_t)}{\cosh(1.84L/r_t)} + e^{(a^2 - z)} = a \quad \text{for} \quad r > r_t,
\]
\[
-\frac{k_0}{B_0} j_0 \cdot (r A_1)
\]
\[
= 5.435 \times [1.665 J_1(2.12z) + 2.136 J_1(5.385z)]
\]
\[
\times \frac{\cosh(1.84z/r_t)}{\cosh(1.84L/r_t)} + e^{-5z^2} = a \quad \text{for} \quad r < r_t,
\]
\[
\int_0^{r_0} S r dr = -S_0 \frac{r_0^3}{3} + S_0 \frac{r_0^3 - r_t^3}{3} = 0.
\]
where \( x = r/r_t \), \( a = 1 \) at the boundary line where \( p = 0 \), and \( a = 0 \) for \( p = p_0 = B_0^2/8\pi \). The field configuration and the plasma shape have been computed from the above numerical example and plotted in Fig. 2.

The solution is continuous for \( a > 1 \). This implies that the current distribution as given by said vector potential is finite beyond the plasma boundary where no plasma exists to create such a current as required by the vector potential.

Consequently, in order that the above solution can be physically realized, material coils should be placed beyond the plasma boundary. Such coils should be energized in such a way that the current distribution inside these coils is as given by the vector potential. At a certain distance (towards negative \( z \)) from the plasma boundary, the vector potential function should be substituted by another function of the form

\[
A = g(r)e^{iz}\phi + j(r)
\]

so that the vector potential vanishes at infinity.

As mentioned above, the solution given here has been obtained in the frame of certain approximations. This approximation breaks near the boundary within the last few Larmor radii.

In a further approximation there should be taken into consideration that:

1. the mean time between collisions, \( \tau \), is proportional to \( 1/p \),

2. the temperature decreases as the electrons approaching the boundary and encountering higher field lose energy by radiation,

3. the Hall current density at the boundary line goes to zero, since there are no particles there to create such a current.

A further approximation in the cylindrical part can be expressed in the form

\[
\phi = rA_1 = \sum_{n=1}^\infty c_n e^{-\alpha_n r} e^{i\alpha_n z}
\]

for \( 0 < r < r_t \).

\[
\phi_0 = rA_0 = \sum_{n=1}^\infty c_n e^{i\alpha_n (r - r_t)} e^{i\alpha_n z}
\]

for \( r_t < r < r_0 \)

and accordingly, thereafter, to determine the \( z \)-dependent functions.

APPENDIX 2

In the investigation of the stability of the equilibrium state of the plasma, I classify possible perturbations into two categories, the criterion being whether or not a Poynting vector \( (P) \) travels along the magnetic lines. This can be expressed as

(1) \( P \cdot B \neq 0 \) (Alfvén waves) \hspace{1cm} (A2.1)

(2) \( P \cdot B = 0 \) \hspace{1cm} (A2.1a)

In the present appendix our attention will be restricted to the second category.

Equations, Assumptions and Approximations

The equations of the problem are

\[
\frac{dv}{dt} = -\nabla p + j \times \frac{B}{c} + j_0 \times \frac{H}{c}, \hspace{1cm} (A2.2)
\]

\[
E + v \times B = 0, \hspace{1cm} (A2.3)
\]

\[
\nabla \times E = \frac{1}{c} \frac{d}{dt} \frac{B}{c}, \hspace{1cm} (A2.4)
\]

\[
\nabla \times H = \frac{4\pi j}{c}, \hspace{1cm} (A2.5)
\]

\[
\frac{dp}{dt} = \gamma \frac{d\rho}{dt}, \hspace{1cm} (A2.6)
\]

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0, \hspace{1cm} (A2.7)
\]

where \( B, p, \rho \) are the equilibrium (undisturbed) values of the magnetic field, plasma pressure, and plasma density, respectively, \( E, H \) are the perturbed values of the electric and magnetic field, respectively, \( j \) is the Hall current, \( \dot{j} \) is the perturbed current, \( \dot{p}, \dot{\rho} \) are the perturbed values of the plasma pressure and density, respectively, and \( v \) is the perturbed mass velocity of the plasma.

From Eqs. (A2.6) and (A2.7) we eliminate \( \partial \rho/\partial t \) and obtain

\[
\frac{\partial \rho}{\partial t} = -v \cdot \nabla \dot{p} - \gamma \dot{\rho} \nabla \cdot v \hspace{1cm} (A2.8)
\]

and we introduce a new variable

\[
w = \dot{\rho} + (B \cdot H/4\pi). \hspace{1cm} (A2.9)
\]

From the system of six Eqs. (A2.2), (A2.3), (A2.4), (A2.5), (A2.8) and (A2.9) we can determine the six unknown quantities

\( v, \dot{v}, w, H, E, j \).

The plasma has been assumed of infinite conductivity insofar as the ohmic term (not shown here) in Eq. (A2.2) is concerned, so that this term can be considered negligible. However, Coulomb collisions are assumed to bring about a Maxwellian distribution in plasma, and their frequency is assumed to be such as to allow the consideration of the stress tensor to be isotropic.

The solutions are assumed in the form

\[
\dot{q} = q(r, z)e^{im\alpha l+4\pi t}, \hspace{1cm} (A2.10)
\]

where \( \dot{q} \) is any of the perturbed quantities. Further it
is assumed that \(|\omega| < eB/Mc|\), where \(M\) is the ion mass.

The expression (A2.1a) can be written

\[ \mathbf{E} \times \mathbf{H} = \mathbf{E} \cdot \mathbf{H} = 0, \tag{A2.11} \]

which implies

\[ \mathbf{E} = 0 \tag{A2.11a} \]

or

\[ \mathbf{H} \times \mathbf{B} = 0. \tag{A2.11b} \]

The first case yields only trivial motion along the magnetic lines or trivial rotation of the plasma and is not of interest. Consequently the case which makes sense is the second, namely

\[ \mathbf{H} \times \mathbf{B} = 0. \]

Expanding this equation we obtain

\[ \frac{H_r}{B_r} = \frac{B_r}{B_z}, \tag{A2.12} \]

and

\[ \Theta_0 = 0. \tag{A2.12a} \]

Combining Eqs. (A2.12) and (A2.12a) with Eqs. (A2.3) and (A2.4) we obtain two sets of equations:

(a) \[ \frac{\partial E_r}{\partial z} = -\frac{E_r}{E_z}, \tag{A2.13} \]

(b) \[ \frac{H_r}{B_r} = \frac{B_r}{B_z}, \tag{A2.15} \]

\[ -\frac{\omega}{c} H_r = \frac{\partial E_z}{\partial \Theta} - \frac{\partial E_r}{\partial \Theta} = \frac{m}{r} E_z - \frac{E_r}{E_z}, \tag{A2.15a} \]

\[ -\frac{\omega}{c} H_z = \frac{\partial E_r}{\partial \Theta} - \frac{\partial E_z}{\partial \Theta} = \frac{\partial (r E_z)}{\partial \Theta} - \frac{m}{r} E_r, \tag{A2.15b} \]

which in turn yield the partial differential equation

\[ B_r \frac{\partial E_r}{\partial r} + B_z \frac{\partial E_z}{\partial z} = -B_r \frac{E_z}{r}, \tag{A2.16} \]

The general solutions of the partial differential equations (A2.14) and (A2.16) are, respectively,

\[ \nu_0 = r f(r A_\theta) e^{i m \Theta + i \omega t}, \tag{A2.14a} \]

\[ E_\theta = \frac{1}{r} f(r A_\theta) e^{i m \Theta + i \omega t}, \tag{A2.16a} \]

where \(f\) and \(\phi\) are arbitrary functions of \((r A_\theta)\); the value of \(r A_\theta\) within the plasma is given by the equilibrium solution, and according to this solution \(r A_\theta = f_0(r, z)\). However, as soon as we move from the ends inward along the \(z\) direction, the \(z\)-dependent function vanishes exponentially; this results in \((A4)\) being practically a function of \(r\) only in most of the plasma volume, far from the ends.

Consequently the perturbed quantities \(\nu_0\) and \(E_\theta\) degenerate in this region to a function of \(r\) only, and in general the perturbation is degenerated to the form

\[ \frac{\partial \mathbf{e}}{\partial t} = q(r) e^{i m \Theta + i \omega t}. \]

Since \(r A_\theta\) is constant along a magnetic line, it follows that \(f(r A_\theta)\) and \(\phi(r A_\theta)\) are constants along a magnetic line. Then if we find the solutions of Eqs. (A2.2), (A2.3), (A2.4), (A2.5), (A2.8) and (A2.9) in the region where \(r A_\theta\) is practically a function of \(r\) only, then the values of \(E_\theta, \nu_0\) and the other perturbed quantities can be easily obtained for the whole volume. Consequently it is enough to solve the system of the six Eqs. (A2.2), (A2.3), (A2.4), (A2.5), (A2.8) and (A2.9) for a perturbation of the type

\[ \frac{\partial \mathbf{e}}{\partial t} = q(r) e^{i m \Theta + i \omega t}. \]

This form of perturbation, after eliminating from the system of six equations the quantities \(\phi, j, h\) and \(E\), yields the equations

\[ \rho \omega \nu_r = -\frac{\partial \nu_r}{\partial r}, \tag{A2.17} \]

\[ \rho \omega \nu_\theta = -\frac{m}{r} \nu_r, \tag{A2.18} \]

\[ \frac{\partial (r \nu_\phi)}{\partial r} + \frac{B_\phi^2}{4\rho} \frac{\partial \nu}{\partial r} = -\omega \nu_\phi, \tag{A2.19} \]

where \(\frac{\partial (r \nu_\phi)}{\partial r} + \frac{B_\phi^2}{4\rho} \frac{\partial \nu}{\partial r} = S^2\), where \(S\) is the velocity of sound in the medium. By eliminating \(\nu_0\) from Eqs. (A2.18) and (A2.19), we obtain

\[ \omega \rho \frac{\partial (r \nu_r)}{\partial r} = -\left(\frac{\omega^2 + \frac{m^2}{r^2}}{S^2}\right) \nu_r. \tag{A2.20} \]

Finally from Eqs. (A2.17) and (A2.20) by eliminating \(\nu_r\) we obtain

\[ \frac{\partial \nu_r}{\partial r} + \frac{\partial \nu_\theta}{\partial r} - \left(\frac{\partial \nu_r}{\partial r} + \frac{\partial \nu_\phi}{\partial r}\right) \nu_r = 0. \tag{A2.21} \]

From Eqs. (A2.14a), (A2.16a), (A2.17a) and (A2.18) we have

\[ i \frac{m}{r} \nu = \rho \omega f(r A_\theta) e^{i m \Theta + i \omega t}, \tag{A2.21a} \]

\[ \frac{\partial \nu_r}{\partial r} = \rho \omega f(r A_\theta) e^{i m \Theta + i \omega t}. \tag{A2.21b} \]

The last two equations impose restrictions on the behavior of \(\nu_r\) along the radius. Our task now is to determine boundary conditions satisfying these restrictions, as well as the condition that the value of the velocity should be finite at the axis, and thereafter to determine the sign of the eigenvalues of \(\omega^2\) that can satisfy these conditions.

At first we examine the behavior of \(\nu_r\) near the axis.
of symmetry \((r = 0)\). In this region the density of the plasma
\[
\rho = \rho_0(1 - e^{-2r^2}) \to 2\rho_0^2 r^2 \quad \text{as} \quad r \to 0.
\]
Hence \(\rho \partial \rho / \partial r \to 2r\) and \(\omega^2 / S^2\) becomes proportional to \(r^2\) and thus it can be neglected; then as \(r\) tends to zero, Eq. (A2.21) degenerates to
\[
\frac{\partial^2 \omega}{\partial r^2} + \frac{m^2}{r^2}\omega = 0,
\]
yielding solutions
\[
\omega \propto r^n,
\]
where
\[
n = \pm (m^2 + 1)^{1/2} + 1.
\]
From Eq. (A2.17) we have
\[
\nu_r = -\frac{1}{\rho_0} \frac{\partial \omega}{\partial r} \propto r^{n-3}.
\]
Since \(\nu_r\) should be finite at the origin it follows that
\[
n - 3 = (m^2 + 1)^{1/2} - 2 > 0 \quad (A2.24)
\]
or
\[
m^2 > 3.
\]
Thus the modes \(m = 0\) and \(m = 1\) are excluded. The first admissible mode is \(m = 2\), resulting in
\[
n \geq 3.26,
\]
\[
\nu_r = 0, \quad \frac{\partial \omega}{\partial r} = 0 \quad \text{at} \quad r = 0
\]
and by Eq. (A2.21a)
\[
\nu_r = 0, \quad \frac{\partial \omega}{\partial r} = 0 \quad \text{at} \quad r = r_0
\]
Now we have established the boundary conditions and we can proceed to determine the admissible eigenvalues of \(\omega^2\) in Eq. (A2.21).
By changing the variable \(w\) to \(u\) where
\[
u = (r/r_0)^{1/2} w,
\]
Eq. (A2.21) becomes
\[
\frac{\partial^2 u}{\partial r^2} = \left[\frac{\omega^2}{S^2} + \frac{m^2}{r^2} - \frac{3}{4}\left(\frac{\partial \omega}{\partial r}\right)^2 + \frac{1}{2}\frac{\partial \rho}{\partial r} \right] u = 0,
\]
which can be written as
\[
\frac{\partial^2 u}{\partial r^2} \left[\frac{\omega^2}{S^2} + \frac{m^2}{r^2} + \frac{3}{4}\left(\frac{\partial \omega}{\partial r}\right)^2 + \frac{1}{2}\frac{\partial \rho}{\partial r} \right] u = 0,
\]
where
\[
\psi = \frac{\partial \rho}{\partial r} - \frac{3}{4}\left(\frac{\partial \omega}{\partial r}\right)^2 + \frac{1}{2}\frac{\partial \rho}{\partial r} + \frac{\mu^2}{r^2}
\]
and
\[
\frac{\partial \psi}{\partial r} = 0.
\]
For
\[
\rho = \frac{1}{\psi^2}, \quad \psi = 0,
\]
where
\[
\psi(r) = c_1 I_n(r) + c_2 K_n(r)
\]
and
\[
I_n(r) = (-i)^n J_n(r), \quad K_n(r) = (-i)^{n+1} H_n^{(1)}(r).
\]
By this substitution we not only simplify Eq. (A2.26a) but also we can represent piecewise, by appropriate selection of the order and argument of the modified Bessel functions, any density distribution where the density is zero at the axis of symmetry, and at a boundary radius \(r = r_0\).
Then Eq. (A2.26a) becomes
\[
\frac{\partial^2 u}{\partial r^2} \left[\frac{\omega^2}{S^2} + \frac{m^2}{r^2} + \frac{3}{4}\left(\frac{\partial \omega}{\partial r}\right)^2 + \frac{1}{2}\frac{\partial \rho}{\partial r} \right] u = 0.
\]
At \(r = 0\), we have \(u = \partial u / \partial r = 0\) (from Eqs. (A2.22a), (A2.24a) and (A2.25)).
From Eqs. (A2.20) and (A2.25) we have
\[
\omega \frac{\partial (r\nu_r)}{\partial r} = -\left(\frac{\omega^2}{S^2} + \frac{m^2}{r^2} + \frac{3}{4}\left(\frac{\partial \omega}{\partial r}\right)^2 + \frac{1}{2}\frac{\partial \rho}{\partial r} \right) u.
\]
The function \(\partial (r\nu_r) / \partial r\) is alternating in the region \(0 < r < r_0\), as
\[
\int_0^{r_0} \frac{\partial (r\nu_r)}{\partial r} dr = \nu_r(r_0) = 0.
\]
If we now assume that \(\omega^2\) is positive, then by Eq. (A2.31) \(u\) should be an alternating function. However, for positive \(\omega^2\), it results from Eq. (A2.30) that \(u\) is a function starting from zero at \(r = 0\) and increasing monotonically with increasing radius, as \(\partial^2 u/\partial r^2\) remains always positive. This contradicts the requirement imposed by Eq. (A2.31).
Consequently, solutions are possible only for negative eigenvalues of \(\omega^2\), indicating that all the perturbations of the type
\[
H \times B = 0
\]
are positively stable.
An investigation of the lowest possible value of \(|\omega|\) gave
\[
|\omega| > S_0 / r_0,
\]
where \(S_0^2 = \gamma \rho / \rho\), the sound velocity in the plasma in the absence of a magnetic field.

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Controlled Fusion Devices, Part II

FRIDAY MORNING, 5 SEPTEMBER 1958

Chairman: Mr. L. Biermann (Federal Republic of Germany)
Vice-Chairman: Mr. G. Vendryes (France)
Scientific Secretaries: Messrs. T. Coor and I. Rojansky

PROGRAMME

P/1064 A summary of the Berkeley and Livermore pinch programs S. A. Colgate et al. (Presented by S. A. Colgate.)
P/3 Review of controlled thermonuclear research at the A.E.I. Research Laboratory T. E. Allibone et al. (Presented by A. A. Ware.)

DISCUSSION

P/2170 The stellarator concept L. Spitzer Jr. et al. (Presented by L. Spitzer Jr.)

DISCUSSION

P/2212 Stabilization of plasma by nonuniform magnetic fields B. B. Kadomtsev and S. I. Braginsky (Presented by B. B. Kadomtsev.)
P/2501 Thermal insulation and confinement of plasma with a high-frequency electromagnetic field A. A. Vedenov et al. (Presented by B. M. Glagolev.)
P/377 Summary of UCRL pyrotron (mirror machine) program R. F. Post

DISCUSSION

DISCUSSION OF P/1064 AND P/3

Mr. R. S. Pease (UK): Mr. Colgate, did you carry out experiments on the energy loss as a function of different gases?

Mr. S. A. Colgate (USA): The large irreversible energy loss of the electrons only occurs when the characteristic temperature is very high, implying that the collision effects are minor. When you go to a higher gas pressure or gases of higher atomic number, the scattered cross section dominates and the radiative process gets hold of the energy in preference to the electron mode loss.

Mr. J. Kistemaker (Netherlands): Mr. Colgate, concerning the homopolar device, what type of viscosity dissipation are you thinking of?

Mr. Colgate (USA): The theoretical treatment of viscosity in a plasma with magnetic fields has been dealt with in some detail by Kaufmann. When the plasma is not leaning against the wall, so that collision processes of the wall are not dominant, then we expect his theory to apply. Later in the cycle, the plasma may lean against the wall and then we use the ordinary viscosity equations of gas flow. Turbulence should be suppressed by the magnetic field but these are problems more for Chandrasekhar than for us.
Mr. R. Carruthers (UK): Mr. Colgate attributes the failure of the stabilized pinch to the large power loss due to runaway electrons. Has he any evidence of such a limitation with the 'screw dynamic' pinch?

Mr. Colgate (USA): The screw dynamic pinch is a transient hydrodynamic phenomenon which breaks up in very rapid fashion because of corkscrew or \(m = 1\) mode of instability. Certainly, when this mode grows, we expect there to be large and very sudden shears in the magnetic field so that certain lines of force will have extremely large currents flowing on them. My comment yesterday about neutron production related to the suspicion that, on these lines of force where the current density is extremely high, we excite the same form of instability, giving rise to local high electric fields and consequent acceleration of the ions. But I do not expect that the major mode of the screw dynamic pinch is concerned with this phenomenon.

Mr. D. R. Chick (UK): Mr. Colgate, can you, from your oscillograms on Triax, estimate the rate of loss of rotational energy of the plasma?

Mr. Colgate (USA): There is, I think, some misunderstanding. There was no rotation in Triax. This was purely a hydrodynamic plane shock-wave driving a plasma into a sheet. There was no rotation involved. The rotation time in the homopolar is determined by viscosity and observed to be 200 \(\mu\)sec. The current time in Triax is determined by the resistive dissipation of the plasma due to the electric current flowing through it.

Mr. L. S. Dzung (Switzerland): Mr. Colgate, in the case of the linear pinch and Triax pinch, do you have any figures on loss of energy at plasma ends relative to that at the wall?

Mr. Colgate (USA): When we divided the Triax pinch into thirty sections by introducing thirty dummy electrodes, the resistance changed by a very small amount. The neutron yield was suppressed, but not completely, so that one would suspect, under those conditions—the present conditions of operation—that the loss to the electrodes is an insignificant contribution to the total loss from the plasma.

Mr. A. R. Gibson (UK): Dr. Colgate, what methods have been used to detect the soft X-rays to which you referred, and, in particular, how was the energy measured?

Mr. Colgate (USA): The soft X-ray flux is so tremendous, as can be imagined from 200,000 amp reaching the wall, that almost any transparent object will act like a scintillator. What we actually used was a pyrex rod inside a small stainless steel tube, with a 1–2 mil beryllium foil excluding the optical light. The scintillation line in the pyrex glass was then observed with a photomultiplier. To check the energy, we simply did an absorption measurement, introducing one-third mil aluminium foils immediately behind the beryllium. The K-edge absorption coefficient of aluminium and beryllium, as also of carbon, is so sharp that a very accurate differential between X-ray energies is obtained. We would say, roughly, that the energy loss is in the 2–4 kv range and it has not been defined more accurately. It must change during the course of the pinch as the current increases and the voltage decreases; one would expect the spectrum of these X-rays to change. We have not measured this accurately.

DISCUSSION OF P/2170

Mr. V. D. Shafranov (USSR): Mr. Spitzer, from what values of the parameters for the stellarator (e.g., tube radius, current, magnetic field and duration of current) did you start, supposedly, to heat the plasma merely up to \(10^6 \text{ K}\)?

Mr. L. Spitzer Jr. (USA): My remark about one million degrees was rather a general one. If one wishes to push the heating up, assuming a good confinement, one could get to ten million degrees but if you assume that the current is limited by the Kruskal limit, the rate of heating will go down by a factor of one thousand and in our particular device I think you would need a confinement time of a minute or so to get to one hundred million degrees and still stay below the Kruskal limit.

Mr. B. P. Lehnhert (Sweden): Mr. Spitzer, the drift of a charged particle due to magnetic field inhomogeneity has been reduced by "twisting" a torus to a figure-eight configuration. The twisting cannot be performed exactly to 180 degrees in one plane since the figure-eight has a cross-over. Does this imply that no exact cancellation of the particle drift can be obtained?

Mr. Spitzer (USA): In our machine, we deliberately chose a rotational transform angle of a little less than 180 degrees. The theory of the cancellation of the drift we worked out is based on the fact that a certain line of force, when followed many times around, generates an entire magnetic surface, and the volume between two adjacent magnetic surfaces is accessible to all the other parts of that volume by just travelling along the lines of magnetic force. Therefore, any current divergence in one part can be cancelled out by some other part of the volume; it does not have to cancel out by going around once—that is too restrictive—but it will cancel out if the line of force is followed sufficiently many times round the stellarator.

Mr. J. Kistemaker (Netherlands): Mr. Spitzer, do the 200 kev electrons essentially indicate many rotations? Could there not be a relation with runaway electrons?

Mr. Spitzer (USA): These are exactly runaway electrons. When the ohmic heating goes to zero, the main current of runaway electrons disappears rather rapidly because of cooperative phenomena, but a few free particles, in the absence of large current, persist for some 10–15 msec.

Mr. A. A. Ware (UK): Mr. Spitzer, how are you sure that the limiter does not cause the increased impurity with the divertor off?
Mr. Spitzer (USA): I think it is clear that, when the limiter is used, the impurities come from the limiter. If the limiter were taken out, however, and the discharge tube wall were put in its place, the impurities would then come, presumably, from the tube wall. The question is, what are the particles of the gas going to hit as they diffuse away from the magnetic axis? If you have a limiter, they will hit the limiter, and come back into the discharge. If you do not have a limiter or divertor, they will hit the tube wall and again come back into the discharge. If you have a divertor, they then hit the divertor plate, and in that case, we assume, they do not rapidly come back into the discharge. If you do not activate the divertor, then the radius of the entire current channel is changed; the current is increased. The diffusion rate is presumably different when the diameter of the current channel is changed, so we have to keep the same in the two cases.

Mr. R. A. Demirkhanov (USSR): Mr. Spitzer, were the frequency and the length of the region of magnetic pumping chosen with a view to minimizing the perturbation of the plasma?

Mr. Spitzer (USA): The length of the magnetic pumping section is, in principle, arbitrary. We have chosen it for practical convenience to be several tube diameters. The frequency is then determined from theory, in the sense that the period with which the field pulsates should equal one of the two natural periods of the gas, that is to say, a period for maximum heat transfer to the gas. The period of the magnetic pumping should be either the period between collisions or the period required for the positive ion to move the length of the magnetic pumping section.

Mr. B. M. Glagolev (USSR): Mr. Spitzer, do you consider it possible to heat by magnetic pumping without preliminary ohmic heating?

Mr. Spitzer (USA): The theory we developed was deliberately restricted to cases we could treat theoretically, and, for these cases, we had to start with a plasma that was already at half a million degrees. However, the observations of ohmic heating have indicated that, in more extreme conditions, that is to say, where the field pulsates by, say, another ten in magnitude, the gas does in fact break down and is heated by ohmic heating. We do not understand this and we hope to do more work on it.

Mr. W. B. Thompson (UK): Mr. Spitzer, what evidence exists for the production of a stable discharge in stellarator, even below the Kruskal limit?

Mr. Spitzer (USA): We do not as yet have any data on this but hope to obtain some in the near future.

Mr. F. Cap (Austria): Mr. Kadomtsev, do stabilization criteria exist expressing non-uniformity (space dependence) of magnetic field as a function of stability requirements?

Mr. B. B. Kadomtsev (USSR): If there is no magnetic field inside the plasma, then for stability it is necessary that the magnetic field increase away from the plasma boundary. If it is a plasma of very low pressure, $8eP/H^2 \ll 1$, then the condition for stability is $V_p \nu \approx \gamma (\nu u)^2/|\nu|$, where $u = -j\delta /H$. These two conditions supplement each other.

In more general cases the condition for stability is very complicated.

Mr. R. Knechti (USA): Mr. Kadomtsev, in the case of Fig. 2(b) of your paper, does plasma not escape at the places where it forms sharp edges? Can you evaluate the importance of this effect?

Mr. Kadomtsev (USSR): The figure was somewhat idealized. In practice it would be better to give a system such as that shown in Fig. 2(c). Here the plasma sags between the rods. On account of its conductivity, the plasma goes out at the forward edge of the rod and can be lost to the supports attached to the rod.

With straight rods there is no such sagging and additional losses do not arise.

Mr. H. W. B. Skinner (UK): Mr. Post, can you tell us how serious the end losses from the mirrors have proved to be in your experiments?

Mr. R. F. Post (USA): The losses from the ends do occur, and the confinement time seems to be in approximate agreement with those losses, but they are not an impediment to carrying out the operation. In a practical application, however, they are such that one would have to be very careful to control them and minimize them by the methods I described.

Mr. J. M. Adlam (UK): Mr. Post, is there any difference in the particle densities of electrons and ions at any point in the plasma? If there is any difference, does the resultant electrostatic field affect the particle trajectories?

Mr. Post (USA): This has to do with the question of ambipolar diffusion to which I referred. The answer is, yes; in general, there will be an intrinsic difference of diffusion rates between electrons and ions giving rise to electrostatic fields. In the conditions of our experiments and in the conditions we finally hope to achieve, Kaufmann of our laboratory has shown that these effects should not be serious.

The Chairman: There are several questions, addressed to Mr. Christofilos, which cover largely the same ground; therefore, I will read the questions in the form posed by Mr. J. G. Linhart (CERN), although as I say, several people have submitted questions on similar lines. The first question is:

“In the preprint on the Astron machine, the stability of the confined plasma is investigated, but not the stability of the E-layer. From our work and that of Budker, the stability of such streams is doubtful for long-wave perturbations. Have you taken into account the hydrodynamic forces on the E-layer?”

The second question is: “Is it not true that the power loss from the E-layer should be mostly due to
cyclotron radiation, which is many times larger than
the bremsstrahlung?"

Mr. N. CHRISTOFILOS (USA): In the Budker scheme,
there is not a layer of electrons; it is a pinch of
electrons which is subject to some of the pinch per-
turbations. Here we have a very long layer; it is very
thin in the radial direction and very long in the axial
direction. Furthermore, there is a large axial com-
ponent of momentum. It has been found that if we
start to compress the layer, the compression leads to an
increase of the axial pressure, whereas the magnetic
forces are increased more slowly. So that, in at least
simple collapses, it has been found stable. However,
we have not investigated all the possible perturba-
tions of this layer yet.

The bremsstrahlung is completely negligible in this
case. However, the synchrotron radiation is very small
relative to the scattering loss. For example, in the
case of a machine at 50 Mev, the lifetime of the elec-
trons is of the order of 50 msec. In that case, the
radiation constitutes only 2–3% of the total loss, so
it is not an important loss.
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Operational Characteristics of the Stabilized Toroidal Pinch Machine, Perhaspatron S-4


Several investigators have reported initial success in stabilizing a pinched discharge through the utilization of an axial $B_z$ magnetic field and conducting walls, and theoretical work with simplifying assumptions, predicts stabilization under these conditions. At Los Alamos this approach has been examined in linear (Columbus) and toroidal (Perhaspatron) geometries.

Perhaspatron S-3 (PS-3), described elsewhere, was found to be resistance-limited in that the discharge current did not increase significantly for primary voltages over 12 kv (120 volts/cm). The minor inside diameter of this machine was small, 5.3 cm, and the onset of impurity light from wall material in the discharge occurred early in the gas current cycle. It was tentatively concluded that in the small PS-3 torus the flux of energy to the walls had reached an intolerable value. The resulting evaporation or sputtering of wall material then contaminated the discharge causing the resistance of the discharge to rise. For this reason the next in the progression of toroidal machines, Perhaspatron S-4 (PS-4), was constructed (see Fig. 1).

The machine has a quartz torus of 14.0-cm minor inside diameter, 218-cm mean circumference and wall thickness 0.6 cm. An aluminum primary, 1.23 cm thick, surrounds the quartz torus with minimum spacing, ~0.4 cm, and can be energized by 90,000 joules, 20 kv at two feed points, permitting a maximum electric field of 150 volts/cm. A two-ton iron core (laminations 0.002 in. thick) links the primary and secondary and has a rating of 0.2 volt-sec. A solenoid, 1.6 turns per cm, wound around the primary is energized by a 2400 µF capacitor at voltages up to 3 kv and produces an axial $B_z$ magnetic field up to 4000 gauss. A split in the aluminum primary allows rapid $B_z$ field penetration.

**ELECTRICAL CHARACTERISTICS**

The typical behavior of gas current and secondary voltage at 25 kv is shown in Fig. 2. The current-voltage phase relation shows that the gas current is largely inductance-limited and not resistance-limited as observed in PS-3. After gas breakdown about 80% of the condenser voltage appears around the secondary, in agreement with the ratio of source and load inductances. The rate of increase of gas current is at first large, ~1.3 x 10^4 amp/sec, until the gas current contracts to cause an increase in inductance, at which time the gas current is a good approximation to a sine curve. The gas current maximum is found to rise linearly with primary voltage (Fig. 3), deviating as expected at the higher voltages because of saturation of the iron core.

At the discharge current maximum, the secondary voltage is not zero, and if one assumes that there is no large change of inductance, the total resistance of the discharge ($R = V/I$) is computed to be ~28 milliohms.

**EXPERIMENTAL RESULTS**

**Neutron Production**

The total average yield of neutrons is ~4 x 10^6 per discharge at 30 kv, and neutron yields as high as ~10^7 per discharge have been recorded. No circumferential asymmetry in neutron yield has been observed.

The average yield of neutrons is found to vary with pressure as shown in Fig. 4. The maximum neutron yield is found at a lower pressure (~15 µ) than with the smaller machine, PS-3 (~30 µ). The neutron yield also is observed to decrease more rapidly with increasing pressure than for PS-3 and is reduced to one-half of its maximum value at a pressure of 21.5 µ.

The dependence of the average neutron yield from PS-4 on the initial axial $B_z$ magnetic field (Fig. 5a) exhibits a striking similarity to that reported for PS-3 with an optimum value of $B_z$ giving the maximum yield. It is also found that the optimum value of $B_z$ varies with the applied primary voltage as shown in Fig. 5b. Again a comparison with PS-3 can be made in that a smaller value of $B_z$ is required in the larger machine to optimize the neutron yield for a given discharge current. It is of further interest to note that for the $B_z$ field adjusted for optimum neutron yield at various primary voltages, the yield experiences...
essentially a linear rise with increasing primary voltage. These data are summarized in Fig. 6.

The average times of onset, cessation and duration of neutron emission are shown in Fig. 7a and 7b. As the primary voltage is increased the neutrons appear earlier in time and their duration grows shorter. The onset of neutrons at a fixed $B_z$ field may well be associated with a critical value of discharge current ($\approx 180 \text{ka}$) so that the more the primary voltage is increased, with the resulting increase in the rate of rise of gas current, the earlier will this critical current be reached and neutrons emitted. The termination of the neutron burst is correlated with the appearance of radiation from wall materials. At the lower voltages (20 to 24 kv), neutron emission occurs almost symmetrically in time about the current maximum with little or no impurity light (Si II) emitted in this time interval. At the higher voltages (>24 kv), impurity light comes in earlier with an apparent quenching of the neutrons, thus leading to shorter burst duration.

The region of emission of neutrons in the torus has been measured with a large paraffin collimator used
in conjunction with a fast phosphor and photomultiplier detector. The results of the measurement are shown in Fig. 8. Within the experimental errors, the results for the horizontal and vertical scan across the minor diameter are the same, and it is concluded that the source is probably symmetric about the minor axis. From a knowledge of the resolution of the collimator, these data have been obtained with the aid of a 704 computing machine. The results of the calculation (Fig. 9A) show that the neutrons are produced in a hollow cylinder, ~2 cm id and ~3 cm od, with some contribution between 8- and 14-cm diameter. The data, however, are not inconsistent with neutrons being produced uniformly in a rod 3 cm in diameter with a small contribution between this rod and the torus wall, Fig. 9B.

The energy distributions of the emitted neutrons have been measured in two directions tangential to the major circumference: one in the direction in which deuterons are accelerated by the induced electric field, and the other opposed. An expansion-type cloud chamber, 30 cm id, filled with CH₄ at a pressure of 1.5 atmospheres placed one meter from the torus was used as the detector. A paraffin collimator between the cloud chamber and the torus limited the neutrons accepted to those emitted tangentially from the torus. To minimize X-ray background a 0.6-cm thick lead shield surrounded the cloud chamber. The high relative neutron yield from this machine has made it possible to obtain ~870 usable tracks in ~1600 expansions. The results with the machine operating at 27.6 kV, 1800 gauss Bₑ field and 12.5 μ D₂ gas pressure, are shown in Fig. 10 together with a calibration run made

| Figure 5a. Neutron yield as a function of axial Bₑ magnetic field. Primary voltage = 30 kV |
| | Figure 5b. Axial Bₑ magnetic field for optimum neutron yield as a function of primary voltage. Deuterium gas pressure = 12.5 μ |
| Figure 6. Neutron yield vs. primary voltage Axial Bₑ magnetic field adjusted for optimum neutron yield at the individual data points. Deuterium pressure = 12.5 μ |
on a Cockcroft-Walton accelerator. It is seen that the neutron energies measured in the direction in which deuterons may be accelerated in the machine are higher than those in the opposite direction, with pronounced energy peaks at 2.59 MeV in the forward direction and 2.37 MeV in the backward direction. The widths at half-maximum of the neutron energy peaks are 0.3 and 0.35 MeV and only slightly larger than that of the calibration run, \( \sim 0.2 \) MeV. These energy shifts show that the center of mass of the reacting deuterons is moving in the direction of the induced electric field with a velocity of \( \sim 5 \times 10^7 \) cm/sec. If the assumption is made that fast deuterons react with deuterons at rest, the peak of the neutron energy distribution implies a deuteron energy of 10 kev.

Magnetic Field and Pressure Distributions

The \( B_z \) and \( B_r \) magnetic field distributions have been measured across a minor diameter of the torus in the horizontal plane. If the assumption of symmetry of the discharge about the horizontal plane is correct, then only two components of the magnetic field, \( B_z \) and \( B_r \), need be measured as \( B_r \) is zero. Subsequent to these measurements, other results at Los Alamos in cylindrical geometry indicate a gross bodily movement of the discharge with the resulting inference that a more detailed program of measurements including the third component (radial) in toroidal geometry is required.

The magnetic fields are measured by previously described techniques, and the resulting calculated \( j_x \) and \( j_y \) current density distributions at 30 kv primary voltage and 1700 gauss \( B_z \) stabilizing field are shown in Figs. 11a and 11b, respectively. The discharge appears to be confined by the pinch field and is concentrated about the axis with little or no current near the walls after the initial pinching at about one microsecond.

The discharge current along the axis is not zero when the total secondary current passes through zero. This phenomenon has been observed in the linear discharge machine, Columbus S-4, and is of course due to finite velocity of diffusion of currents through the conducting medium.

Pressure and current distributions were calculated from a knowledge of the magnetic fields measured along a minor diameter of the torus. The equation used for calculating the pressures along this diameter is

\[
\frac{\delta + B^2}{8\pi} \frac{B_z}{B_r} = \left\{ \frac{(-2 + 2R - A^2) B_x + 2B_z^2}{R^2 - A^2} \right\} dR + C, \quad (1)
\]

where \( R \) is the major radius to the point under consideration, \( A \) is determined by the condition \( B_\theta = 0 \) at \( R = A \), and the arbitrary constant \( C \) is determined at some point where the pressure is known. The quantity \( (R^2 - A^2)/2R \) is the minor radius of the toroidal coordinate surface through the point. The equations for the current distributions along the diameter are

\[
j_x = \frac{1}{4\pi} \left( \frac{\partial B_\theta}{\partial R} + \frac{2R}{R^2 - A^2} B_\theta \right), \quad (2)
\]

and \( j_\theta = \frac{1}{4\pi} \left( \frac{\partial B_z}{\partial R} + \frac{B_z}{R} \right). \quad (3)

Typical results of the pressure calculations are shown in Fig. 12.
axis. (2) A pressure minimum occurs on the axis and persists during most of the first half-period. Two pressure peaks of varying amplitude are present on each side of the pressure minimum at a radius of \(\sim 2 \text{ cm}\). (3) Appreciable pressures of varying amplitude are present at the inside and outside walls.

These pressure distributions were unexpected as they have not been obtained in toroidal machines heretofore. The possibility exists that the assumptions made in the derivation of Eq. 1 are not valid and a complete mapping of the fields is required. It is interesting to note, however, that one possible configuration for the radial distribution of the emitted neutrons (see Fig. 9A) is a thin shell and that consequently, this shell will be found on the pressure peaks at radius \(\sim 2 \text{ cm}\).

At least three possible explanations exist for the large pressures calculated at the walls: (1) The discharge moves bodily about the tube and large inward and outward accelerations are present as has been observed radially in the linear system\(^\text{12}\) (the present data are not sufficiently precise at this time to make a more detailed analysis); (2) High-energy runaway electrons are contributing to the pressure by their centrifugal force; or (3) Errors in field measurements exist near the walls or near the probe openings in the primary shell.

**Powered Crowbar Operation**

For some conditions it has been found possible to sustain the current maximum for times as long as 30 \(\mu\)sec. The current is maintained by switching an
additional 24 mF of capacitance ($1.1 \times 10^5$ joules at 3 kV) in parallel with the primary energy supply at the time of the current maximum. Because of the unexpected high impedance of the discharge, the discharge current could be held constant by the crowbar bank only at the level corresponding to a total primary voltage of ~14 kV. If switching takes place at the current maximum with initially 7 kV and 3 kV on the primary and sustaining crowbar supplies respectively, the results shown in Fig. 13 are obtained. With an initial applied secondary voltage of ~13 kV, it is observed that the discharge current is held relatively constant, ~180 kA, for ~30 µsec by the crowbar system.

The $B_x$ and $B_z$ magnetic field distributions have been measured as a function of time with a sustained crowbar operation, and the corresponding current densities, $j_x$ and $j_z$, calculated (Fig. 14a, b). Pressure profiles have also been calculated. It is surprising that for times as long as 32 µsec after the first current maximum the magnetic field distributions vary as little as they do. It is apparent that the axial discharge current remains separated from the walls for ~48 µsec after the initiation of discharge current.

A neutron yield of ~$2 \times 10^5$ per burst under normal operating conditions with 13 kV applied secondary voltage increases with crowbar operation at 13 kV to ~$2 \times 10^6$ per burst with duration increased up to ~60 µsec (Fig. 13c). The rate of neutron emission remains relatively constant during this time. This result apparently precludes explanations of neutron production which use shocks originating by rapid rate of rise of discharge currents or sudden applications of high voltages. It appears necessary that the neutron-producing mechanism be continuous for relatively long times and that it involve no permanent break-up of the pinch structure.

During the sustained crowbar operation, a considerable amount of energy ($9.0 \times 10^2$ joules/µsec) is being transferred to the discharge. No appreciable changes, however, are found during these times in the pressure distributions. The conclusion is inescapable that energy is being lost by the discharge as fast as it is being received.

**Spectral Observations**

The intensities of the deuterium $D_2$ and silicon Si II ($4128$ Å) lines vary with time as shown in Fig. 15. The hydrogen line is initially relatively intense. Radiation from the discharge gas becomes much
weaker when ionization has taken place and then becomes very strong, presumably because the ionization of hydrogen released from the walls. The time of onset of the silicon light is a function of primary voltage (Fig. 7a) and is associated with the bombardment of the walls by particles or radiation.

**X-rays**

For primary voltages above ~20 kV a short burst of X-rays, duration ~2 µsec, is detected outside the aluminum primary at the start of the gas current. Little or no X-ray emission of energy >50 keV occurs during the remainder of the discharge cycle. The mean energy of the X-ray(s) outside the aluminum primary is 68 keV, as measured with film badges. With an applied primary voltage of 30 kV, a maximum X-ray energy of 0.6 MeV was determined by lead absorbers. The X-ray intensity varies widely from discharge to discharge with an average reading of ~1 mR per discharge measured with a dosimeter placed alongside the torus primary.

**DISCUSSION**

From the above data some conclusions can be drawn concerning the plasma confinement and heating in Perhapsatron S-4.

**Electrical Resistance**

In the operation of PS-4, the electrical resistance of the discharge was found to be considerably larger than expected. At the discharge current maximum (3 x 10³ amp), for example, the secondary voltage was measured to be 8.4 kV, which gives from Ohm's law a total discharge impedance of 28 milliohms. From the j₂ and j₀ current density distributions (Fig. 11a, b) it would appear that there are no large fluctuations in the current density distributions at the current maximum (t = 12.5 µsec) indicating that at this time there is little change in the secondary inductance and hence the voltage drop is purely resistive. The current channel diameter, ~7.2 cm, is also defined by the j₂ distribution. With the assumption that the electric field is constant throughout the discharge, we calculate a resistivity of 4.0 x 10⁻³ ohm-cm, 45° pitch of the magnetic field lines, and a corresponding electron
temperature of ~6 ev, assuming validity of the resistivity-temperature relation $\rho = 3 \times 10^{-6}/T^{3/2}$.

A second measurement of the discharge resistance is available from the performance of the discharge during the sustained crowbar operation. In this case the discharge current was held essentially constant for 30 $\mu$sec with little observed change in the $j_z$ current distributions (Fig. 14a). The discharge current was $1.8 \times 10^5$ amp with a secondary voltage of ~5.0 kv. From the foregoing crowbar operating conditions, the total discharge resistance is calculated to be ~28 milliohms in agreement with the calculation made at the higher voltages and currents. At the reduced power level required for crowbar operation the discharge remains remarkably free from impurity radiation for ~40 $\mu$sec. This enables us to reject impurities as a serious contributor to the high resistivity.

A more precise and meaningful measurement of the resistivity of the discharge can be made with the knowledge of the $B_z$ and $B_x$ distributions. The electric fields and current densities parallel to the magnetic field can be calculated throughout the discharge and from these data changes in the resistivity obtained as a function of time. These calculations are now being made but no results are available at this time.

In summary, the experimental results indicate that the apparent resistivity of the discharge is large (4.0 $\times$ 10$^{-8}$ ohm-cm) with a corresponding low, ~6 ev, electron temperature. It should be emphasized, however, that these calculations use the simplified Ohm’s law relation for the plasma, ignoring, in particular, velocity terms that would exist in the presence of plasma waves or turbulent conditions.

**Energy Balance**

It is instructive to account for the energy taken from the capacitor banks during the discharge. With PS-4 operating at 15 kv, the energy deposited in the various sections of the machine at a time of 6 $\mu$sec has been calculated. The results are summarized in Table 1.

If the remaining energy, 7180 joules, were used to heat all the gas initially present in the torus, a plasma temperature of ~760 ev would exist at 6 $\mu$sec in the discharge cycle. This result is clearly incompatible with the ~6-ev electron temperatures calculated from the resistivity.

---

**Figure 14a.** Contour plot of axial $j_z$ current density (amp/cm$^2$) as a function of time and radius for sustained crowbar operation. Primary voltage = 14 kv, sustained crowbar voltage = 6 kv, $B_z$ field = 700 gauss, and $P = 12.5 \mu$.

**Figure 14b.** Contour plot of preliminary transverse $j_\theta$ current density (amp/cm$^2$) data as a function of radius and time for sustained crowbar operation. Primary voltage = 14 kv, sustained crowbar voltage = 6 kv, $B_z$ field = 700 gauss, and $P = 12.5 \mu$.
The sustained crowbar operation gives us additional information as to the problem of heating the plasma. With a constant current of $1.8 \times 10^7$ amp the secondary voltage remains essentially constant at 5.0 kv for $\sim 30 \mu\text{sec}$. Energy is then being transferred to the discharge at the rate of approximately $(1.8 \times 10^7) (5 \times 10^3) \times 10^{-4} = 900$ joules/$\mu\text{sec}$. The calculated current density distributions (Fig. 14a, b) indicate that this energy is not appearing in the magnetic field since these distributions do not change appreciably during this time interval. The energy must then be given to the gas at the rate of 95 ev per particle per $\mu\text{sec}$. However, the calculated pressures during this time interval do not rise appreciably nor does the rate of neutron production increase as would be expected from a steadily increasing temperature.

These arguments lead to the obvious conclusion that there are mechanisms at work by which energy is being continuously drained from the discharge. The magnetic probe measurements indicate that the discharge current is at least grossly confined away from the tube walls, but the experimental results demonstrate that the energy transferred to the plasma is not confined. The nature of the loss mechanism is not yet known though it is suspected that loss by escape of energetic particles across the confining magnetic field, particle acceleration by the electric field, and loss by radiation generated by plasma waves and turbulence may be contributing factors.

**Pressure Balance Results**

The pressure distributions calculated from the measured magnetic field distributions in PS-4 show, in general, a pressure maximum at a radius of $\sim 2$ cm. With a pressure minimum on the axis it can be concluded that the pressure difference between the peak pressure and the axial pressure is not a result of forces such as those produced by runaway electrons. The pressure differences between the peaks and the minima at the axis vary up to $\sim 2.5$ atmospheres with corresponding energy densities of 0.25 joules/cm$^3$. It is not clear whether this energy exists as gas turbulence, shock waves, plasma waves, or as thermal motion of the gas particles. If it is assumed that all the gas is confined uniformly in a rod of diameter 3.5 cm, and that an energy density of 0.25 joules/cm$^3$ is expended in thermal motion of the gas particles within the rod, then a temperature of $220 \text{ ev}$ is deduced from the pressure measurements. This temperature is perhaps low since considerable pressures are observed between 3.5 cm radius and the wall, which indicates that all the gas may not be confined within the 3.5 cm radius.

A temperature of 220 ev is not sufficient to explain the PS-4 neutron yield based on thermonuclear reaction considerations although, as stated above, the results of the collimated neutron observations across a minor diameter suggest that the neutrons are produced in the measured high-pressure regions of the discharge.

**Neutron Energy Distributions**

The energy shifts in the neutron energy distributions described earlier in this report show conclusively that most of the neutrons are generated by reacting deuterons having a center-of-mass velocity of $\sim 5 \times 10^7 \text{ cm/sec}$. Two possible mechanisms representing extremes of viewpoints can be considered by which the neutrons may be produced with the observed center-of-mass velocity: (1) a simple acceleration process in which 10-kev deuterons, which are a small fraction of the total deuterons present in the torus, react with the remaining deuterons at rest, and (2) a streaming of deuterons of velocity $5 \times 10^7 \text{ cm/sec}$ reacting with each other by their relative velocities.

(1) In the acceleration process we can calculate the current of 10-kev deuterons required to produce the observed neutron yield. With an initial gas pressure of 12.5 $\mu$ and an assumed compression of ten, a deuteron current of $\sim 370$ amp is required to give the observed rate of neutron production of $\sim 5 \times 10^{11}$ neutrons/sec. (If a smaller compression is assumed, the current...
will increase inversely as the compression.) The percentage of the total number of deuterons present in the torus which would contribute to this current would be small, ~0.02%, and the energy required to accelerate them to 10 kev quite reasonable, ~8 joules.

Possibly the strongest argument that can be made against this process of neutron production is that the required deuteron current is too large. If conservation of momentum is to occur in the discharge, then the ratio of the deuteron and electron currents should be inversely proportional to their masses. With a total discharge current of 300 ka, the deuteron current would be only 80 amp.

The above calculation of the required deuteron current has assumed that all the accelerated deuterons have an energy of 10 kev, corresponding to the peak in the neutron energy distribution. This may well be in error since comparable numbers of lower-energy deuterons may be present, which would not contribute much to the total neutron yield since the cross section is such a rapidly increasing function of energy. Unfortunately the cloud chamber data are not sufficiently precise to permit a deuteron energy distribution to be calculated.

(2) If it is assumed that deuterons are streaming parallel to the axis of the torus it can be shown that the deuterons in the gas cannot all have the indicated velocity of 5 x 10^7 cm/sec (~2.5 kev energy). To accelerate all the deuterons to this energy, a total energy of 1.2 x 10^4 joules is required. In the energy balance consideration discussed above, only 7.2 x 10^3 joules are available for translational kinetic energy. One must then conclude that only a fraction, 60%, at most, of the deuterons could cooperate in this process. A difficulty arises in that this number of deuterons traveling at a velocity of 5 x 10^7 cm/sec would produce a current of 6.5 x 10^6 amp. In order to be consistent with the observed current and momentum conservation, the steady deuteron current must not exceed 80 amp. The pressure distribution also establishes a limit on the maximum temperature and density which can exist. We must then conclude if hypothesis (2) is correct that in order to be consistent with current and energy balance the reaction must proceed at large relative deuteron velocities in a small fraction of the total gas. The necessarily high transient pressures which would then exist would not be observed by the present techniques. This highly localized source of neutrons receives some experimental confirmation in that, during the collimated neutron measurements, at a given collimator position, the numbers of neutrons detected on successive discharges frequently varies by a factor of three.

Further experiments may reveal that the models developed above are gross oversimplifications of the true mechanism for neutron production in PS-4.

SUMMARY

The preceding comments may be summarized as follows:

(1) The discharge has been stabilized in a gross sense (the discharge current confined from the walls) for times ~48 μsec.

(2) From the measured resistivity of the discharge the electron temperature is ~6 ev.

(3) By some as yet unknown mechanism, energy is being lost by the discharge at a rate of ~900 joules/μsec under certain operating conditions.

(4) Maximum gas pressures of ~2.5 atmospheres are measured during the current cycle.

(5) There is a center-of-mass velocity of ~5 x 10^7 cm/sec of the reacting deuterons which produce the neutrons. The mechanism responsible for the production of these neutrons is not clear.

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REFERENCES

6. S. Colgate, University of California Radiation Laboratory, Livermore, California (private communication).
Mr. Phillips presented Paper P/2488, above, at the Conference and added the following remarks:

The first theory is that, although the pinched discharge has been grossly stabilized, there still remain local surface instabilities as observed by Burkhardt and Lovberg.\(^1\) This would be expected to increase the resistivity and give high local energy densities. Recently a possible stable configuration has been derived, using the stability criteria of Dr. B. Suydam\(^\dagger\) for mixed \(B_\theta\) and \(B_z\) fields. The assumptions are (1) uniform axial current density in the pinched discharge and (2) a negative pressure gradient outward everywhere. The results show that a reverse longitudinal magnetic field outside the discharge markedly increases the pressure that can be confined.

The remaining theory is that the energy losses are due to plasma vibrations excited by the electron current, as discussed by Akhiezer, Fainberg, Luchina and Gordayev in the USSR and, recently, by Bunemann and Tuck in the USA. The disturbing implications are that, if such plasma vibrations exist in gas discharge systems, the stabilized pinch discharge is in serious difficulty.

\(^\dagger\) Paper P/1364, Session A-5, Vol. 31, these Proceedings.
Proposed Methods of Obtaining Stable Plasma

By G. Miyamoto, T. Kihara, G. lwata, S. Mori, T. Ohkawa and M. Yoshikawa*

The successful operation of ZETA\(^1\) has shown that the instabilities of the self-pinned plasma column with longitudinal current can be suppressed by confining a longitudinal magnetic flux within the plasma and surrounding the column with a conductor tube. The theory\(^2\) gives the stability conditions for the magnetic flux and the radius of the tube in an idealized model of the plasma. However, it is not clear that the actual plasma column can be held for the length of period required for a thermonuclear reactor.

Two possibilities for obtaining stable plasmas of high temperature are discussed.

**INDUCTION PINCH EFFECT**

A magnetic field increasing rapidly with time is produced in a discharge tube. The electric field induced by the magnetic field will initiate a pole-less discharge in the gas and closed currents will flow in the plasma. An auxiliary r.f. discharge may help to start the discharge. The plasma will be compressed radially by the Lorentz force due to the currents and the magnetic field and form a pinched column as in the ordinary pinch effect. The difference is that the directions of the current and the magnetic field are interchanged and the magnetic field induces the current in this type of pinch effect. Therefore, the theoretical treatments using simplified models, such as those considered by Rosenbluth,\(^3\) can be applied to this type of pinch and give similar results as long as the curvature of the plasma surface is neglected. Magneto-hydrodynamical calculations\(^4\) are made, assuming quasi-static compression and infinite conductivity for the plasma, and show similar results to the ordinary pinch effect. In a two-dimensional model, where the radial and the azimuthal components of the pressure are increased by the compression and hence the ratio of the specific heats is considered to be equal to two, the temperature at any point in the plasma rises, by compression, simply in proportion to the external magnetic field. The importance of ohmic-loss heating is realized, after disclosure of the data of ZETA, and calculations for finite conductivity are being pursued. It is felt that a combined method, in which the plasma is held from the wall by the pinch effect and heated mainly by ohmic loss, may give better results since the high temperature must be maintained for a period of seconds and since a larger radius of the plasma gives stronger coupling between the power supply and the plasma, especially in this type of pinch effect.

The discharge tube may be a cylinder, long enough to neglect end effects, or a torus. A solenoid is wound around it to produce the magnetic field. The power supply may be a condenser bank or a short-circuit testing generator, with spark gap switches. Figures 1 and 2 are diagrams of the tube and the power supply for the two geometries. For the donut-shaped tube, auxiliary coils may be needed to suppress the radial expansion of the plasma.

An alternative method of obtaining a strong magnetic field is also considered. In a static magnetic field, two oppositely rotating discs are placed face to face. They are made of conducting material and have radial cuts to avoid one-turn short-circuit in the azimuthal direction. The axes of the rotating discs are connected electrically to the external chamber, which is filled with low pressure gas. Each disc acts as the rotor of a homopolar generator. A gas discharge may start between edges of the discs. The hollow column of the plasma will be pinched so that the inside magnetic field, being in pressure balance with the outside magnetic field produced by the discharge current, will increase rapidly. A discharge tube may be placed near the central region as shown schematically in Fig. 3.

The plasma column in this type of pinch effect will not show any instabilities. It is hoped that the high temperature and long period required for achieving thermonuclear reactions may be reached by the above method with a power supply of a large capacity. Preliminary experiments are in progress.

**SPATIAL PINCH EFFECT**

When an intense beam of electrons of relativistic energy is introduced into a gas of low density, the beam will soon be neutralized by ions produced by the beam. The magnetic force then pinches the beam. In this stationary beam the temperature of the ions may be sufficiently high to produce nuclear reactions. A rough estimate of ion temperatures may be obtained by using the following naïve model. Assuming that the densities of electrons and ions, \(n_e\) and \(n_i\), are

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uniform within the beam, the radius of which is \( a \), the electrostatic potential, \( V \), within the beam is given by

\[
V = \frac{\epsilon}{4\pi}(n_e - n_0)(r^2 - a^2),
\]
where \( \epsilon \) is permittivity in vacuo. Ions are trapped in this potential valley and, when the ion density increases from \( n_0/\gamma \) to \( n_e \) on neutralizing the beam, the average energy of the ions is given by

\[
\langle kT_i \rangle = 7.5 \times 10^{-3} I(1 - 1/\gamma),
\]
where \( \gamma = (1 - \beta^2)^{-1/2} \) and the current \( I \) and kinetic temperature \( kT_i \) are given in amperes and kev, respectively. Those ions and electrons are confined by the magnetic fields produced by the beam and the total pressure is in balance with the magnetic pressure.

Assuming that the electron pressure, which is determined by the boundary conditions, is small, and using the relation \( P_i = n_i kT_i \), the ion temperature is given by the same formula as (2).

To discuss this kind of plasma, the relativistic theory of plasma is developed. Assuming that each kind of constituent particle is in local equilibrium and has a Maxwellian energy-momentum distribution, the basic equations of motion of plasma are derived relativistically from the transport equation and Maxwell’s equations, as shown in the Appendix. The stationary lines of flow for the above beam are calculated by using these equations. It is shown that thousands of amperes are required in order to obtain a temperature high enough for the D-D reactions.

Experimental arrangements are shown schematically in Fig. 4. Electrons ejected from a large gun may be accelerated through a series of pulse transformers by induction. Each transformer is shielded by grounded copper sheets to prevent leakage of magnetic flux. Sets of quadrupole magnets may be used for focusing the electron beam. The beam is introduced through slits, which maintain a pressure difference, into the reaction chamber which is filled with deuterium gas of low density. After passing through the chamber, the electrons may be decelerated by a series of transformers and their energies fed back to the accelerating transformers, so that the capacity of the power supply can be cut down appreciably.

Preliminary experiments are being planned, on a small scale, to study the behavior of a relativistic electron beam in gases, which plays an essential part, both for the above proposal and for Budker’s radiation-cooled beam.

APPENDIX

Assume that each kind of constituent particle in a gas consisting of charged particles has a Maxwellian energy-momentum distribution,

\[
N = C \exp(\xi r M_r)
\]
where \( N \) is the distribution function of particles in 8-dimensional space-time-momentum-energy space, \( C \) and \( \xi \) are functions of space-time and \( M_r \) is the momentum-energy vector; with \( N \), \( C \), and \( \xi \) not necessarily common for different kinds of particles.
Then the current \( N_r \) and momentum-energy tensor \( T_{rs} \) are given by

\[
N_r = N_0 \lambda_r, \quad r = 1, 2, 3, 4, \quad (A.2)
\]

\[
T_{rs} = m N_0 G(\lambda_2, \lambda_4) \lambda_r + N_0 \delta_{r4}/\xi, \quad (A.3)
\]

where the particle density, \( N_0 \), is given by

\[
N_0 = 4\pi m^2 C K_2 (\xi^2)/\lambda^2; \quad (A.4)
\]

and where \( G \equiv K_3/K_2, \lambda_r = \xi_r/\xi \) and \( \xi^2 = -\xi_r \xi_r \).

Then the equation of continuity and the equations of motion are written as

\[
\frac{\partial N_0 \lambda_r}{\partial x_r} = 0, \quad \frac{\partial T_{rs}}{\partial x_r} = \frac{\epsilon}{c} N_0 A_{sr} \lambda_r, \quad (A.5)
\]

where \( A_{sr} \) is the tensor of the electro-magnetic field and satisfies Maxwell's equations

\[
\frac{\partial A_{sr}}{\partial x_r} = \frac{\partial A_{sr}}{\partial x_s} = 0, \quad (A.6)
\]

\[
\frac{\partial A_{sr}}{\partial x_s} = \mu_0 \Sigma e N_0 \delta_{sr}. \quad (A.7)
\]

From the above equations, the adiabatic law

\[
N_0 L(\xi^2) = \text{const.} \quad \text{(along lines of flow)} \quad (A.8)
\]

is derived. Using these equations, the steady state of the axially symmetric plasma, consisting of ions and relativistic electrons, is calculated when the plasma is uniform in the axial direction. The density distribution is given by

\[
N_0^3 = n_0 f(z), \quad z = \frac{1}{2} A n_0 r^2 \quad (A.9)
\]

where \( f(z) \) is the solution of the equation

\[
\frac{d}{dz} \left( z \frac{df}{dz} \right) = -f^3, \quad f(0) = 1 \quad (A.10)
\]

\( A \) is a constant resulting from boundary conditions and \( n_0 \) is the density at the beam axis. The beam radius, \( a \), is given by using the zero, \( z_1 \), of the function \( f(z) \),

\[
a = 2 z_1^4 A^{-1} n_0^{-1}, \quad z_1 = 1.75. \quad (A.11)
\]

REFERENCES


Mr. Miyamoto presented Paper P/1329, above, at the Conference and added the following remarks:

A plasma betatron, which has been designed for studying the behaviour of a high energy electron beam neutralized by ions, is shown schematically in Fig. 5.

![Figure 5. Proposed plasma betatron](image)

The magnetic field is shaped in such a way that the azimuthal component of the vector potential has two maxima and, between them, a minimum which corresponds to the betatron orbit. Electrons may be emitted from a cathode placed at the center and form a space charge cloud in the magnetic field as in a magnetron. The magnetic field then increases with time and the machine starts as a betatron. Some of the electrons may be captured in the valley of the vector potential and accelerated to relativistic energies. An alternative method of injection is to use a secondary emission discharge. An rf electric field is applied along the magnetic field and slow electrons produced by the discharge fill the chamber. If the accelerating voltage per turn in the betatron is made much higher than the energies of the electrons, most of the electrons may be accelerated stably to the final energy.

After an intense beam of relativistic electrons is built up, the betatron orbit may be made to shrink rapidly towards the axis and to form a cluster in the central region. The temperature of the ions may rise to the thermonuclear temperature by the combined effects of the above two processes; namely, electrostatic and pinch effects.

This machine can also be used to study Budker's radiation-cooled beam.
Plasma Loop in a Transverse Magnetic Field

By S. M. Osovetz, Y. F. Nasedkin, E. I. Pavlov, Y. F. Petrov and N. I. Schedrin

In this paper the theoretical foundations and experimental results of investigations of a plasma loop in an alternating transverse magnetic field are described. As distinguished from other systems which are also designed for gas heating to high temperatures, in this system a plasma is both heated and kept in equilibrium by a single magnetic field of a specially selected configuration.

The system we consider provides an electrodeless ring-shaped gas discharge with the creation of conditions to some degree similar to electron acceleration in a vacuum betatron. While the vacuum betatron accelerates electrons, a gas discharge of the type considered provides a possibility for plasma heating and confinement.

THEORY

Let us consider a plasma loop with a current in an alternating axially symmetric magnetic field which is normal to the plane of the loop (Fig. 1). Let $L_1$ be the self-inductance of the winding which creates the magnetic field $H$ and $L_2$ be the self-inductance of the plasma loop. The alternating magnetic flux created by the primary current crosses the plasma loop and induces current $I_2$ in it. The magnetic flux is equal to

$$\phi = 2\pi \int_0^R H R dR = \frac{1}{c} L_{12} I_1,$$

where $L_{12}$ is the mutual inductance of the winding and gas loop and $R$ is the radius of the loop, which is supposed to be larger than the radius $r$ of cross section of the loop. The self-inductance of the loop is determined by the following well-known expression:

$$L_2 = 4\pi R \left( \ln \frac{8R}{r} - 2 \right) = 2\pi R l \text{ cm},$$

where $l = 2 \left( \ln \frac{8R}{r} - 2 \right)$ is the inductance of the loop per unit length. Assuming that the inductive reactance of the plasma loop considerably exceeds the active resistance, and taking into account relation (1), we obtain

$$L_2 I_2 = -L_{12} I_1 = \pi R c \bar{H},$$

where $\bar{H} = \frac{2}{R^2} \int_0^R H R dR$ is the average value of the field intensity inside the loop. Equation (3) can be written as

$$I_2 = \frac{c R}{2l} \bar{H}.$$  \hspace{1cm} (4)

The plasma loop with a current is influenced by the following forces in a magnetic field: the force of interaction of the current with the external magnetic field which tends to contract the loop toward the centre,

$$F_1 = \frac{2\pi R}{c} I_2 \bar{H},$$  \hspace{1cm} (5)

and the force of interaction of the current with its own magnetic field which tends to increase the radius of the loop,

$$F_2 = -\frac{1}{2l^2} \frac{\partial}{\partial R} L_2 I_2 = -\frac{\pi R c}{2} (2+l).$$  \hspace{1cm} (6)

In a betatron the force $F_1$ is compensated by a centrifugal force

$$F_3 = -2\pi N \frac{m v^2}{R} = -\frac{2\pi m l_2}{2l^2},$$

where $N$ is the number of particles per unit length. The repulsive effect of the current’s self-field in the betatron proves to be small as compared with the centrifugal force and can be neglected. Let us determine the applicability of such an approximation; for this purpose we divide $F_3$ by $F_2$. As a result we find that the centrifugal force is large in comparison with $F_2$ if

$$\frac{2mc^2}{Ne^2(2+l)} \gg 1,$$

i.e., when $N \ll 10^{13}$ particles/cm. When this requirement is met, the equilibrium conditions for the loop will be the same as in the betatron.

We are interested in the opposite extreme—i.e., in the equilibrium conditions at $N \gg 10^{13}$ cm$^{-3}$. In this case the effect of the centrifugal force can be neglected and, using expressions (5) and (6) as well as relation (4), it is easy to obtain

$$H = \frac{\bar{H}}{\bar{f}} \left( 1 + \frac{2}{l} \right).$$  \hspace{1cm} (9)

Thus, we have found the condition that should be satisfied by the magnetic field at the equilibrium radius...
for the existence of a loop in equilibrium. The equilibrium condition for a betatron is known to be
\[ H = \frac{\lambda}{2} R. \]  \hspace{1cm} (10)

It is evident from a comparison between expressions (9) and (10) that equilibrium, in this case, required a steeper fall of magnetic field along the radius than in the case of a betatron, because the value \( 2\lambda \) usually does not exceed 0.2–0.3.

For the existence of a plasma loop in the region of this equilibrium orbit, not only Eq. (9) but also stability conditions should be satisfied.

\[ \frac{\partial F_R}{\partial R} = \frac{\partial F_1}{\partial R} + \frac{\partial F_2}{\partial R} < 0 \text{ when } F_R = 0 \text{ and } \frac{\partial F_2}{\partial z} < 0 \text{ when } F_z = 0. \]

In this case stability with respect to radial perturbations requires that
\[ \frac{R}{H} \frac{\partial H}{\partial R} > \frac{3}{2} \frac{l-2}{l(l+2)} > 0, \] \hspace{1cm} (12)
and stability with respect to perturbations along the \( z \) axis requires that
\[ \frac{\partial H}{\partial R} < 0. \] \hspace{1cm} (13)

The final conditions for stability can be written as follows:
\[ 0 > \frac{R}{H} \frac{\partial H}{\partial R} > \frac{3}{2} \frac{l-2}{l(l+2)} , \] \hspace{1cm} (14)

When conditions (9) and (14) are satisfied, the existence of a stable plasma loop is possible. These conditions seem to provide a sufficiently large energy of the magnetic field of the current running in the loop and then to give this energy to the particles in the plasma loop.

These results were, however, obtained without due account of some additional circumstances which complicate the process and lead to very important consequences. First of all it is necessary to take into account the fact that in reality the conductivity of a loop has a finite value. Hence the equilibrium conditions prove to be dependent on time and fail after a certain period of time.

Indeed, let us consider a plasma loop of finite conductivity in a magnetic field with a configuration which satisfies equilibrium condition (9). The behaviour of the loop is described by the equation of motion
\[ MN \frac{d^2 R}{dt^2} = \frac{2\pi R e^2 \lambda H}{c} + \frac{I_0^2 \partial L_2}{2c^2 \partial R} \] \hspace{1cm} (15)
and by the equation of electrical equilibrium
\[ \frac{d}{dr} \left( L_2 \phi + \phi_0 \right) + \frac{\phi_0}{2} = 0, \] \hspace{1cm} (16)

where \( M \) is the total number of particles in the loop, \( M \) is the mass of one particle, \( \Sigma = \pi r^2 \sigma R \) is the conductivity of the loop and \( \pi r^2 \sigma \) the conductivity of the loop per unit length which we shall consider constant. The inductance \( L \) of plasma per unit length is also assumed constant.

To make the problem definite, it is necessary to express in a direct form the dependence of the magnetic field on the coordinate \( R \) and on the time. The dependence on the coordinate will be given graphically (see Fig. 2). We assume that the conditions for stability are satisfied from \( R_1 \) to \( R_0 \) so that until the loop is within this region, its movement is slow and the inertial term in Eq. (15) can be neglected. It is natural to take the time dependence in the form
\[ H = H_m \sin \Omega t. \]

The solution of these equations allows one to obtain the time \( t_{col} \) of existence of the loop in the region of the stable orbit, the value of current \( I_{col} \) corresponding to the moment of collapse—i.e., to the transition into the unstable region, and the value of the residual flux inside the loop at the moment of collapse \( \Delta \phi \) provided that at the initial moment of time conditions (9) are observed.

The time of collapse is determined from a solution of equations (15) and (16) for \( R = R_0 \) (the term \( d^2 R/dt^2 \) is omitted in (15)). This leads to a condition
\[ \frac{R_0}{R_1} \sin \Omega t_{col} = \frac{\xi \Omega}{1 + (\xi \Omega)^{1/2}} \times \left[ \xi \Omega \sin \Omega t_{col} + \cos \Omega t_{col} - \exp (-t_{col}/\xi \Omega) \right]. \] \hspace{1cm} (17)

To make the formula more concise the following symbols have been introduced:
\[ \xi = 1 - \frac{1 + 2\lambda}{4}, \quad \tau = \frac{\pi \Omega t_{col}}{c^2}. \]

It is evident from Eq. (17) that a loop can exist in the region of a stable orbit during a time interval which is shorter than half the period of the external circuit oscillations if this period exceeds considerably the characteristic inertial time. Since for our purposes a plasma loop must be given considerable energy, the use of high frequencies at which inertial times become
important is hardly promising; we therefore confine ourselves to the study of comparatively small $\Omega$.

The evaluation of the collapse time is simplified when the terms of Eq. (17) are expanded in a power series in $\Omega_{\text{col}}$ up to the second order inclusive. Such an expansion leads to the relation

$$\frac{t_{\text{col}}}{2\xi^2} = 1 - \frac{R_0}{R_1^2}$$

The collapse current is

$$I_{\text{col}} = -\frac{CH(0)R_0 \sin \Omega_{\text{col}}}{2[\xi R_1^2/R_0 + (1 - \xi)]}$$

$$\propto \frac{CH(0)_{\text{max}} \Omega \xi R_0}{[\xi R_1^2/R_0 + (1 - \xi)](1 - R_0/R_1^2)},$$

while the residual flux is

$$\frac{\Delta \phi}{\phi_0} = \left[\frac{\xi (R_1^2/R_0^2 - 1)}{\xi R_1^2/R_0^2 + (1 - \xi)}\right],$$

where

$$\phi_0 = \pi R_0^2 H(0).$$

Evaluation of the time of existence of the plasma loop in the stable orbit zone for real parameters of the apparatus leads to a value of the order of several microseconds. This period of time having passed, the loop is contracted towards its centre and the energy accumulated therein is transformed into kinetic energy of the particles and into contraction energy of the magnetic field which remains inside the loop.

The solution of this problem with the aid of such an apparatus can be sought in two different directions: in the direction of the so-called “fast” process and in the direction of the “slow” process. In the former the electromagnetic energy accumulated in the loop during its presence in the stable orbit zone is transformed during the contraction process into the kinetic energy of progressive motion of the particles towards the centre of the system. Then when the loop becomes a “clump” of plasma situated near the centre of the system, this kinetic energy of particles is transformed into thermal energy as a result of the intensive interaction of the particles in this “clump” where the density is high.

The second way is to preserve the loop in the region of the stable orbit zone during a period which exceeds the collapse time considerably. This can probably be attained in an apparatus with an internal conducting cylinder whose field configuration is similar to that shown in Fig. 3. Here the contraction of the loop is prevented by the influence of the magnetic field “frozen in” between the plasma loop and the internal cylinder. Under these conditions the loop can vibrate near the equilibrium position only with a comparatively small amplitude.

Now we consider the process of contraction of the loop after its collapse from the stable orbit zone.

For the contraction process, which is determined by the inertial time, the residual magnetic flux can be considered “frozen” into the loop and, since the inertial time is much shorter than the time of magnetic field diffusion through the plasma, the term with active resistance in Eq. (16) should be neglected. The contraction time and other values which are of interest are determined by a direct integration of Eqs. (15) and (16). The expression for the speed of contraction has the form

$$v = \frac{\phi_0}{(MN \cdot \xi L_{20})} \left[1 - \frac{R}{R_0}\left(\frac{\xi R_0^2}{R_0^2 + R + 1} + \frac{\Delta \phi}{\phi_0} \left(2\xi - 1\right) + \frac{\Delta \phi}{\phi_0} \left(1 - \xi\right) R_0 \frac{R}{R_0}\right)\right].$$

It is possible to determine from this relation the value of the minimum contraction radius corresponding to $v = 0$. The second root of this expression, $R = R_0$, corresponds to the beginning of contraction. Knowing $R_{\text{min}}$ it is possible to find from the condition $v = 0$ the value of the current at the moment of maximum contraction. It is clear that the sign of the current $I_{\text{col}}$ must be opposite to that of the orbital current, because at that moment the electrodynamic force must have a direction opposite to the initial one—i.e., outward. The value of this current is determined from the relation

$$\left|\frac{I_{\text{col}}}{I_{\text{col}}}\right| = \left[\frac{\Delta \phi}{\phi_0} - \frac{R_{\text{min}}^2}{R_0^2}\right] \left[\left(1 - \frac{\Delta \phi}{\phi_0}\right) \frac{R_{\text{min}}}{R_0}\right].$$

The above considerations show that the existence of the residual flux $\Delta \phi$ due to the finite plasma conductivity prevents the contraction of the loop into a small-sized “clump”. The energy of the loop is partially consumed in the contraction of the residual flux. For a complete consumption of the energy accumulated in the loop and its transformation into the energy of particles in the “clump”, this residual flux should be compensated. The compensation is provided by an additional flux whose direction is opposite to that of the main one.

The compensating flux is created by a circuit whose period of oscillation is very large compared to that of the main process. The compensating field can therefore be considered as quasi-stationary and consequently not leading to the appearance of an additional electro-motive force in the region of the stable orbit.

The accuracy with which the residual flux must be compensated can be evaluated from the condition that during the time when the loop is contracted into a “clump” the residual flux must diffuse through the
plasma to a depth approximately equal to its radius in a contracted state provided that the magnetic field is completely "frozen in". This is reduced to the condition that during the time the loop is contracting, i.e., \( R_0/\sigma \), the skin depth \( \sigma > R_{\text{min}} \).

Using relation (29) and assuming that \( R_{\text{min}} \ll R_0 \), we come to the conclusion that the accuracy with which the residual flux must be compensated is determined by the inequality

\[
\frac{\Delta \phi}{\phi_0} < \left( \frac{\sigma^2}{4\pi \sigma \nu R_0} \right) \frac{1}{2}.
\]

If this inequality is observed, it is possible to consider that the residual flux is compensated and \( R_{\text{min}} \approx 0 \). In this case the maximum value of the kinetic energy of the particles in the "clump" is determined from the relation

\[
\frac{M v_{\text{max}}^2}{2} \approx L_{\text{cl}}^2 c_0 \frac{2N}{2N}
\]

In experiments, where the field was not compensated, approximately one-half of the energy accumulated in the loop by the time \( v\rightarrow v_{\text{max}} \) was consumed in contraction of the residual flux.

When considering the interaction of the particles in the "clump" it is possible, without appreciable error, to assume that the temperature of the "clump" particles is approximately equal to the maximum kinetic energy.

Indeed, the distribution of the velocity vectors when the loop is contracted into a "clump" results in the presence of particles with relative energies from zero to quadruple kinetic energy (because an interaction of particles with oppositely directed velocities takes place). Therefore even without multiple collisions the function of the energy distribution of particles at the moment of complete contraction will not be very different from the Maxwellian one.

The question of maximum particle density in the "clump" and of factors which determine the density of the contracted state, as well as the question of the duration of this state, is not considered here because at present there are no sufficiently reliable experimental results which would allow one to make any definite conclusions.

The experiments, the results of which are given below, in the main confirm the above-mentioned considerations not only qualitatively but to some extent quantitatively. These results show that the process of rapid contraction of a plasma loop from the region of the stable orbit towards the centre allows a considerable part of the energy to be transferred to the particles, and from this point of view the development of such an apparatus is of interest.

**EXPERIMENTAL**

Now we proceed to the results of the experiments. The diagram of a typical experimental device is shown in Fig. 4. The discharge chamber consists of a porcelain cylinder closed at the ends with porcelain discs. The magnetic field is created by current in a system of ring-shaped conductor strips \( A \) and two copper discs \( B \) and \( C \) connected in parallel; the diagram of the magnetic system and its proportions are given in...
Fig. 5. By varying the dimensions $D_1$, $D_2$, and $D_3$, it is possible to obtain a magnetic field of the necessary configuration.

The winding of the discharge chamber was fed by a capacitor bank through an air spark gap. The 140 µF capacitor bank could be charged to a voltage of 50 kv. The inductance of the winding was 0.3 µh, and the inductance of the feeders together with the spark gap was 0.15 µh. The coupling coefficient of the gas loop with the winding measured on a model was equal to 0.33.

The form of the magnetic field as a function of $R$ which satisfies the equilibrium and stability conditions is shown in Fig. 6.

To study the distribution of the gas currents over the vacuum chamber a system of Rogovský measuring belts was introduced into it. These belts measured the total current flowing through the gas, the current in the stable orbit zone, the current near the geometrical centre of the chamber and the currents in other regions of the chamber. The location of the measuring belts is shown in Fig. 7. To measure the magnetic field in the centre of the chamber, a magnetic probe in the form of a small coil was used. The parameters of the belt and of the magnetic coil were selected in such a way that the values of the currents and magnetic field intensities, not their derivatives, could be measured directly. The discharge was carried out in deuterium at various initial pressures. The voltage on the capacitor bank was 33 kv and the voltage on the primary winding reached 22 kv while the voltage per loop in the stable orbit zone was 17 kv.
Figure 8. Oscillograms of the currents at $p = 0.1$ mm Hg, $V_0 = 33$ kv (1, current through belt II (total gas current); 2, current in the winding; 3, current through belt III; 4, current through belt II; 5, current through belt V; 6, current through belt II; 7, current through belt IV; 8, current through belt V) [Fig. 8 (b) is inverted]

In Fig. 8a oscillograms are shown of the primary current and total gas current taken at the pressure of 0.1 mm Hg. The frequency of the process is equal to 22 kc/sec, and the maximum value of the primary current is 550 ka. The gas current appears at almost the same time as the primary current. In about 3 $\mu$s there occurs a drastic decrease of the gas current which corresponds to the collapse of the gas loop from the orbit and its movement towards the centre.

It follows from the curve in Fig. 6 that $R_1/R_0 \simeq 1.3$. Then, according to relation (18), $\tau = 5\mu$s. This means that, with $R = 22$ cm and $r = 1.5$ cm,* the plasma conductivity $\sigma = 3 \times 10^{14}$ gse. This value is close to the results of other independent measurements. The value of collapse current is equal to 60 ka, which also coincides rather well with the calculated value for the collapse time.

It follows from a comparison between the oscillograms for the total gas current and for the current flowing through belt III (Fig. 8b) that these currents are equal up to the moment of collapse. Belt IV, located in the stable orbit zone, registers, at the moment of collapse, a value of current close to the value of the total current. The dimensions of belt IV point to the fact that the radius of the loop with a current inside the belt does not exceed 2 cm. An average current density inside this belt is not less than 1.3 ka/cm$^2$.

After the collapse of the gas loop, the current decreases rapidly to zero and then changes sign. The maximum value of this negative current is 82 ka. Belt V, located near the geometrical centre of the chamber, does not register the current until the loop collapses from the orbit (Fig. 8c and 8d), and then it begins to measure the negative current, whose maximum value was about 80 ka. This means that the minimum radius of the loop lies within belt V—i.e., within the range of values of $R$ from 1.5 to 9 cm.

The calculated value of the minimum radius is $R_{\text{min}} \simeq 0.3R_0 \simeq 6$ cm. The calculated value of $\Delta\phi/\phi_0 = 0.35$. Here $\Delta\phi/\phi$ has been calculated from formula (20) and $R_{\text{min}}/R_0$ from formula (21) on the assumption that $v = 0$.

According to relation (22), $|f_{\text{exp}}/f_{\text{cal}}|$ then equals 1.35, while the measured value is 1.32. The calculated value of the minimum radius does not contradict the measurements either.

The oscillograms of the magnetic field in the centre of the chamber obtained with the aid of a magnetic probe are shown in Figs. 9a and 9b. The former was obtained with no gas current. The maximum field in the centre is 11,300 gauss. In the second oscillogram one can see the effect of the magnetic field contraction.

* Reasons for selecting $r = 1.5$ are given below.

![Figure 9. Oscillograms of the magnetic field in the centre of the chamber; (a) no gas in the chamber, $V_0 = 33$ kv, (b) $p = 0.1$ mm Hg, $V_0 = 33$ kv (1, magnetic field; 2, current in the winding)]
by the plasma loop. In this case at the moment the contraction starts the total field (oscillogram 9b) is 3700 gauss while the field of the primary winding at this moment is equal to 8000 gauss (oscillogram 9a). Thus, the ratio $\Delta H/H(0) = 0.45$, while the calculated value of $\Delta \phi/\phi_0 = 0.35$. The difference is explained by the fact that in the calculations the field intensity inside the unstable zone was assumed constant, while in reality the field is non-uniform. At the moment of maximum contraction the intensity of the field is increased up to 14,500 gauss, i.e. approximately four times.

After the gas loop has contracted towards the centre, it expands and then is again contracted. In the oscillogram of the total current (8a) one can see that after a negative current pulse the positive current is restored and then once again the negative one is observed. It is evident from the oscillogram in Fig. 9b that the contraction of the loop, and consequently of the field, is also repeated in the second half-period of the primary current oscillation, but not in so prominent a form.

Figures 10 and 11 show the oscillograms of the currents at the initial deuterium pressure of 0.06 and 0.03 mm Hg with the same value of the capacitor bank voltage. When the pressure decreased from 0.1 to 0.03 mm Hg, the general character of the oscillogram did not change. But the values of collapse currents decreased considerably, which can be explained by an increase of active resistance of the loop due to its
stronger contraction and more difficult starting of the discharge at low pressures. During the further decrease of the initial pressure the gas current continues to fall in the course of the first half-cycle, and at the pressure of 0.01 mm Hg the discharge starts only at the second half-period of the primary current oscillation (see Fig. 12), because at this pressure the time of development of the discharge is larger than the primary current half-period.

Now we give an estimate of the rate of contraction of the plasma loop towards the centre of the chamber obtained from the oscillograms of total current (Fig. 13) for the pressure of 0.1 mm Hg. The time scale in these oscillograms is increased for greater accuracy in rate measurement. The contraction time was determined as the interval between the moment when the gas current begins to decrease and the moment which corresponds to the maximum value of the negative current.

During this time the radius of the loop is changed by approximately 20 cm. When the initial pressure is 0.1 mm Hg, the average rate of contraction is $1.8 \times 10^7$ cm/sec and, when the pressure is 0.06 mm Hg, it is $2.4 \times 10^7$ cm/sec. These rates correspond to the mean kinetic energies of deuterons of 320 ev and 580 ev, respectively. The maximum energy, of course, will be much larger because the movement of the loop is an accelerated one. It is evident that the kinetic energy reaches a maximum when the current passes through zero.

The maximum rate can be estimated from the equation of electric equilibrium for a loop at the moment corresponding to $\phi = 0$. In this case

$$v_{\text{max}} = -\left(L_{12} \frac{dI_1}{dt} + L_2 \frac{dI_2}{dt}\right) / I_1 \frac{\partial L_{12}}{\partial R}.$$  

(25)

The values of $I_1$, $dI_1/dt$ and $dI_2/dt$ for this moment are known from the oscillograms. The value of the radius of the loop when the current passes through zero is also known from the belt measurements and is, for a pressure of 0.1 mm Hg, $R_{\text{max}} = 11$ cm (see Fig. 7). Consequently at this moment the value of $L_2$ is known. From measurements on a model the values $L_{12}$ and $\partial L_{12}/\partial R$ were determined for a given value of the radius. The maximum rate determined from these data is approximately two times larger than the average value. Thus, the kinetic energy of deuterons in the process of contraction reaches 1500 ev, when $P_0 = 0.1$ mm Hg. It is necessary to mention that, in the further contraction of the loop, the energy of the deuterons is consumed to a great extent by an increase in the energy of the magnetic field and is converted into heat only partially. Knowing the maximum energy of deuterons, it is not difficult to estimate the number of particles in the loop from relation (24):

$$N = \frac{L_2 I_{\text{col}}^2}{M v_{\text{max}}^2}.$$  

In the case we consider $\frac{1}{2} L_2 I_{\text{col}}^2 = 1700$ joules. Here it should be taken into account, however, that approximately one-half of this energy is consumed in compensating for the residual flux. Then $N = 4 \times 10^{18}$ atoms. These measurements allow one to estimate the conductivity of the plasma loop from the following condition:

$$\frac{2\pi R_0}{\pi \varepsilon_0} I_{\text{col}} = -\frac{1}{c} \frac{d\Delta \phi}{dt}.$$  

In the process of contraction $\Delta \phi = L_{12} I_1 + L_2 I_2 = \text{const}$. When the gas current passes through zero, $\Delta \phi = L_{12} I_1$, where $I_1$ is the value of the primary current at the moment when the gas current changes its sign; $L_{12}$ for this moment, as has been mentioned above, was determined from measurements on a model. The value of $\Delta \phi$ determined in this way was $2 \times 10^8$ Max. Hence the resistance of the loop is $2\pi R_0/\pi \varepsilon_0 = 4.5 \times 10^{-3}$ ohms and the conductivity for $R = 22$ cm and $r = 1.5$ cm has the value $\sigma = 5 \times 10^{14}$ cgs, which agrees sufficiently well with the above-mentioned estimate and is in reasonable agreement with the value of plasma conductivity measured in experiments with straight discharge tubes.
The above-mentioned experimental results show that the initial theoretical assumptions are valid. Work was carried out to improve apparatus of this kind. First of all a series of experiments were made to study the compensation of the residual field. These experiments were carried out with a smaller chamber.

The diagram of the apparatus and dimensions of the chamber are given in Fig. 14. The power supply of the...
The principal magnetic system was provided by a capacitor bank with a capacity of 12 \( \mu \text{F} \), which could be charged up to 50 kV. The inductance of the winding was 0.17 \( \mu \text{H} \). The period of oscillation of the circuit was 13 \( \mu \text{sec} \) (the inductance of the whole circuit being 0.37 \( \mu \text{H} \)). The magnetic system which served to compensate the residual magnetic flux consisted of two spiral coils connected in series with a total inductance \( L_2 = 3 \mu \text{H} \). The winding was fed by a capacitor bank with a capacity of 3000 \( \mu \text{F} \) and maximum voltage of 3 kV. The period of oscillation of this coil was \( \sim 9 \times 10^{-4} \text{ sec} \). The timing device was adjusted in such a way that the moment of triggering the main circuit corresponded to the maximum value of the compensating field (see Fig. 15).

The ratio of the frequencies of the two circuits was selected in such a way that during the period of discharge the compensating field could be considered constant. The distribution of the \( H_z \) component of the compensating magnetic field as a function of the radius in the plane of the loop is shown in Fig. 16. Table 1 gives the values of the magnetic fields for both systems.

The measurements of the gas currents and the magnetic field in the centre were carried out by methods similar to those described above. In this case the full current in the gas was almost exactly equal to the current measured by the orbit belt and was about 16 ka, which corresponds to the calculated value for these parameters. In all the measurements the voltage on the primary winding was 12.5 kV.

Table 1

<table>
<thead>
<tr>
<th>Frequency Circuit</th>
<th>Current through the winding, ka</th>
<th>Magnetic field at the orbit, gauss</th>
<th>Magnetic field in the centre, gauss</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;High&quot; frequency circuit</td>
<td>170 1000 6000</td>
<td>&quot;Low&quot; frequency circuit</td>
<td>15 180 3300</td>
</tr>
</tbody>
</table>
Figure 17 gives the results of the measurements (oscillograms) made in the absence of the compensating field at an initial deuterium pressure of 0.08 mm Hg. At the moment of maximum contraction in the centre of the chamber one could observe approximately a fourfold increase in the intensity of the magnetic field as compared with the field at the moment of collapse. An estimate of the average energy and of the number of particles for this apparatus gives $\frac{1}{2}mv^2 \approx 100$ ev and $N = 2 \times 10^{18}$ atoms.

Figure 18 shows the corresponding oscillograms for pressures of 0.015 mm Hg, 0.03 mm Hg and 0.05 mm Hg. When the deuterium pressure is 0.015 mm Hg, the discharge starts at the second half-period of the primary current. Contraction of the plasma loop also leads to a strong contraction of the magnetic field in the centre. During the first half-period an insignificant secondary current is found only when the pressure attains 0.03 mm Hg. When the pressure is 0.03 mm Hg, the field in the centre is hardly reduced at all when the discharge starts. When the density is increased (0.05 mm Hg), the gas current in the first half-period is increased and the field in the centre is reduced by approximately 30% while the residual field contracts more than 1.5 times.

Now we proceed to the results of measurements with a compensating field present. It is evident from the oscillograms of Fig. 17 that in this case the gas current is ring-shaped. The increase of the collapse phase can be explained by some “penetration” of the compensating field into the stable orbit zone which prevents the loop from moving towards the centre. During the first half-period an insignificant secondary current is found only when the pressure attains 0.03 mm Hg. When the pressure is 0.03 mm Hg, the field in the centre is hardly reduced at all when the discharge starts. When the density is increased (0.05 mm Hg), the gas current in the first half-period is increased and the field in the centre is reduced by approximately 30% while the residual field contracts more than 1.5 times.

A decrease of the gas current at low initial pressures is connected with an increase in the time of development of the discharge because of the small density of particles. On the other hand with a given current, the energy per particle is the larger, the smaller the number of particles in the plasma loop. Therefore it seems expedient to develop a device with sufficiently large local gas density, facilitating a rapid development of the discharge, but with a sufficiently small total number of particles in the loop. Such conditions are realized in a system with peripheral gas injection in which the gas spurts through nozzles into the stable orbit zone. The amount of gas in the orbit zone by the moment of discharge can be controlled by varying the time of application of the voltage depending on the front position of the jet.
To maintain a uniform filling of the region where the formation of the loop occurs, 12 nozzles fed by a single gas source are situated over the periphery. Figure 20 shows a diagram of such device.

Figure 21 shows the oscillograms of the currents obtained for the system with peripheral gas injection. These oscillograms are compared with the oscillograms taken at a uniform filling of the chamber at a pressure of 0.1 mm Hg. It is evident from these oscillograms that when the values of collapse current are the same, the time of complete contraction of the plasma loop in the system with peripheral gas injection is approximately one-half that for a system with uniform filling. This points to the fact that the maximum energy of the particles in such a system is approximately four times higher than the energy of particles in a system with uniform filling. Since the electromagnetic energies accumulated on the orbit are equal in both cases, the number of particles in the loop in a system with peripheral injection is one-fourth that in the other system and corresponds to a pressure of 0.025 mm Hg for which in a system with uniform filling the current would be approximately one-third.

The development of the discharge, increase in current, and plasma loop contraction proceed so fast that the gas has no time to spread over the chamber and can be considered immovable. Therefore the loop contraction proceeds in a vacuum and the contracted clump does not interact with the neutral gas (another important advantage of the system with peripheral gas injection).

During the experimental investigations of a plasma loop in a transverse magnetic field with the aid of the above-described apparatus it was found, at a comparatively early stage in 1953, that, under certain conditions, the discharge is accompanied by hard X radiation. This radiation was found in the time intervals when there was no current through the gas, and within the range of initial pressure from $10^{-4}$ to $10^{-3}$ mm Hg. The evaluation of the irradiation energy made by the method of X-ray quanta absorption by

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Figure 21. Oscillograms of currents for the system with peripheral gas injection; (a) stationary conditions, $p = 0.08$ mm Hg, (b) gas injection

Figure 22. Radiation oscillograms phased with the gas current; (a) $p = 5 \times 10^{-4}$ mm Hg, (b) $p = 0.01$ mm Hg

Figure 23. Radiation oscillograms phased with the gas current in the winding, (a) without a lead screen ($p = 8 \times 10^{-3}$ mm Hg, $V_b = 33$ kv), (b) with a lead screen, 3 mm thick ($p = 8 \times 10^{-3}$ mm Hg, $V_b = 33$ kv), (c) with a lead screen 20 mm thick ($p = 8 \times 10^{-3}$ mm Hg, $V_b = 33$ kv)
lead screens of different thickness shows that the maximum energy of these quanta attains a value of the order of 1 Mev. A radiation oscillogram for the pressure of $5 \times 10^{-4}$ mm Hg obtained with a scintillation counter and photomultiplier timed with the secondary current is shown in Fig. 22a. At a pressure of 0.01 mm Hg the pulse of radiation (see Fig. 22b) arises when a current is flowing through the gas.

The presence of hard X radiation at small pressures and currents (below the sensitivity threshold of measuring devices which register currents of 2 ka and up) points to the fact that in such a system there evidently exists a betatron acceleration of the electrons produced by the gas ionization in the chamber. This leads to the conclusion that it is possible, in principle, to create a betatron accelerator without a preliminary injection of electrons but to provide them by the gas ionization in the chamber. To illustrate this we show some scintillation counter oscillograms timed with the primary current for a deuterium pressure of $8 \times 10^{-3}$ mm Hg (Fig. 23) obtained in another more perfect device. In this case there is practically no gas current before the beginning of the third half-period. At the beginning of the third half-period there is a radiation pulse simultaneous with the starting of the gas current.

Figure 23a shows an oscillogram taken when the counter was not protected by a lead screen, while Figs. 23b and 23c show the oscillograms obtained when the counter was protected by a lead screen 0.3 and 2 cm thick, respectively. It is evident that the radiation pulse is preserved, but its beginning is shifted. The beginning of the pulse in Figs. 23b and 23c corresponds to the moment when the plasma loop goes out of the stable orbit zone and contracts towards the centre.

Figure 24 shows similar oscillograms for the pressure of 0.1 mm Hg with a radiation pulse during the first half-period of the current. At these pressures a discharge in the gas starts almost simultaneously with the rise of the primary current, while the moment of appearance of the radiation pulse corresponds to the moment of collapse of the plasma loop.

The nature of the radiation observed is now being studied.
Radio-frequency Thermonuclear Machines

By J. W. Butler, A. J. Hatch and A. J. Ulrich*

"Radio-frequency Thermonuclear Machine", as the term is used here, signifies a thermonuclear reactor in which the main confining force on the reacting plasma is developed by a periodic magnetic or electromagnetic field. Thus, for example, machines in which only heating of the plasma is accomplished by rf fields will be excluded from consideration. According to circumstances, the frequency involved might be from about 10 to 1000 Mc/sec.

There are three reasons why the study of high frequency confining fields may make a contribution toward eventual solution of the thermonuclear power problem.

(a) The existence of vacuum fields with non-zero curl makes it possible to obtain field configurations quite different from those obtainable with conduction currents.

(b) With a periodic confining force there is an opportunity to take advantage of possible dynamic stabilization effects.

(c) Suitable rf fields may simultaneously perform the functions of confinement, heating, and direct electrical power extraction.

Another way of looking at this general idea is that it entails analysis of possible machines in the frequency domain instead of in the time domain. Because of materials limitations and other factors, it is almost certain that practical reactors will have to be operated in a pulsed or repetitive manner anyway, and this approach both suggests new classes of machines and throws fresh light on old ones.

Remark (a) above is, of course, a simple consequence of Maxwell’s equations for the full electromagnetic field. The intention is to suggest that there may be advantages in developing the required magnetic confining fields partially from displacement currents instead of from conduction currents. Closed systems of magnetic lines are thereby permitted, having a tendency to remove the plasma farther from the outer walls of the machine. A more concrete result is that the paths of reactive power flow can be confined to the interior of the machine, thus requiring the external circuitry to deal with only the real power component. The significance of these observations will become more apparent after presentation below of several possible machine configurations.

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DYNAMIC STABILIZATION

Perhaps the most important reason for consideration of alternating confining fields is item (b), which relates to the stability of the plasma–field interface. Dynamic stabilization of otherwise unstable systems does exist in nature, as is evidenced by the frequently cited examples of the inverted pendulum and the alternating gradient focusing system which has been proposed for high energy particle accelerators. It is also possible for a column to support a periodic compression load with an average value greater than the critical buckling force, although in this situation the stability zones are quite narrow. These are all instances of parametrically excited systems in which infinitesimal displacements are governed by differential equations of the Mathieu or Hill type. It does not seem easy to find essentially different examples.

In the present case, a simple argument for the existence of such a stabilizing principle is based upon the fact that the organized motions resulting from instabilities tend to be oriented by the confining magnetic field at the plasma surface. If this field is then caused to vary rapidly, particularly in direction, it is expected that the instability motions will lose coherence with the field structure and will not develop sufficiently to allow escape of the plasma. This problem has indeed been attacked by J. Berkowitz and independently by R. J. Taylor. Berkowitz treats the case of a semi-infinite plasma supported against gravity by a rotating magnetic field tangent to the surface, while Taylor investigates an infinite cylindrical plasma held under compression by the same type of field. Inspection of these two approaches suggests that general conclusions might be drawn by making use of Taylor’s results for long wave-length perturbations and utilizing Berkowitz’ conclusions for shorter wave-lengths. In this manner it may be inferred that there should exist a band of stable perturbation wave-lengths which is, roughly, bounded above by the size of the plasma and below by $C_s/\omega$, where $C_s$ is the sound speed (or Alfvén speed) in the plasma and $\omega$ is the angular frequency of the confining field. This seems quite close to actual stability, if only one is optimistic enough to believe that growth of long and short wave-length deformations might be suppressed, respectively, by the finite size of the apparatus and by viscosity or stiffness effects in the
plasma. The use of conducting walls near the plasma is another well-known expedient for controlling long wave-length instabilities.

It is evident from the above that, if any benefit is to be gained from rf confinement, the angular frequency \( \omega \) must be very large compared to the quantity \( C_s / \Omega \), where \( I \) is the mean linear dimension of the plasma body. Assuming a polytropic equation of state, the ordinary formula for sound speed is,

\[
c_s = \sqrt{\frac{\gamma kT}{\mu e^4}} \frac{1}{\mu},
\]

where \( kT \) is the energy corresponding to temperature \( T \), \( \gamma \) is the polytropic exponent, and \( m_e^2 \) is the energy equivalent of the average particle mass. For a 1–1–2 mixture of deuterons, tritons, and electrons at a temperature of 10 kev, one has \( (\gamma = 2); C_s = 1.24 \times 10^6 \) m/sec, and \( C_p/(2\pi l) = 0.987 \) Mc/sec, for an \( I \) value of 0.2 m. This indicates the use of frequencies in the vicinity of 100 Mc/sec or higher.

**ENERGY BALANCE**

Estimates of minimum power requirements and some other relations of importance can be made without reference to a particular apparatus. In all cases of interest the angular frequency, \( \omega \), of the confining field is much smaller than the plasma frequency,

\[
\omega_p = \left( \frac{e\rho_0/m_e}{} \right) \frac{1}{\mu},
\]

where \( e/m_e \) is the specific electronic charge, \( \rho_0 \) is the neutralized charge density, and \( \mu \) is the dielectric constant of space. Some typical values of this quantity are given in Table 1.

Under these conditions the electromagnetic field quantities will fall off as \( \exp(-x/d) \) with depth \( x \) into the plasma, where \( d \) is very nearly equal to \( C_s / \Omega \).

Neglecting displacement currents in the plasma, the power input per unit area due to ohmic heating may then be approximated by

\[
P_1 = \frac{\eta \mu d}{B_s^2/\Omega},
\]

Here \( \eta \) is the volume resistivity of the plasma and \( B_s^2/\Omega \) is the time average value of the magnetic energy density at the surface. Spitzer\(^\text{6}\) gives a formula for \( \eta \) which is equivalent to

\[
\eta = 52.3 \times 10^{-6} (kT)^{-1} \log \Lambda \ \text{ohm m.}
\]

This value applies to a plasma containing ions of unit charge; \( kT \) should be expressed in electron volts (ev) and \( \log \Lambda \) is the familiar shielding factor which occurs universally in analyses of plasma transport properties. A short table of \( \log \Lambda \) is given on page 73 of Ref. 6.

Taking as representative a spherical plasma of diameter \((l)\) equal to 0.2 m, a graph of the total power input as calculated from Eq. (2) is displayed in Fig. 1. The pressure balance condition

\[
\left( \frac{B_s^2}{\Omega} \right) = nkT
\]

has also been used, in which \( n \) is the total particle density. In Fig. 1(a), the resistive power input is compared to the total energy output from the D-T reaction, the small \((\sim 1\%)\) contribution from D-D reactions being ignored. Values of the reaction cross sections were taken from Tuck\(^7\). This presentation shows that the lower limit of density for a plasma of this particular size is in the neighborhood of \(10^{20}/m^3\), and that at least 0.1 to 1 Mw of power must be involved. These figures appear quite reasonable; it is an unpleasant fact, however, that any contact between the electromagnetic field and ordinary metallic conductors causes the power balance situation to be

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**Table 1. Plasma Frequencies**

<table>
<thead>
<tr>
<th>( \rho_0/e ) electrons/m(^3)</th>
<th>( \omega_p ) radians/( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{21} )</td>
<td>( 5.64 \times 10^{18} )</td>
</tr>
<tr>
<td>( 10^{22} )</td>
<td>( 1.78 \times 10^{18} )</td>
</tr>
<tr>
<td>( 10^{23} )</td>
<td>( 5.64 \times 10^{18} )</td>
</tr>
</tbody>
</table>

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\( \omega_p = (e\rho_0/m_e) \frac{1}{\mu} \), where \( e/m_e \) is the specific electronic charge, \( \rho_0 \) is the neutralized charge density, and \( \mu \) is the dielectric constant of space. Some typical values of this quantity are given in Table 1.
dominated by skin losses in these conductors. The result is to raise the minimum density and power levels into seemingly impractical regions several orders of magnitude higher than in the above example.

Figure 1(b) shows the relation between the power input and bremsstrahlung radiation losses for a hydrogenic plasma of the same size. For a stripped plasma, the specific radiated power is given by the expression

\[ Z^2 n_e n_i g(Z, kT), \]

where \( Z \) is the ionic charge number and \( n_e n_i \) the product of electron and ion densities. At \( kT \) levels in excess of about 10^4 ev, \( g \) is practically independent of \( Z \) and is proportional to \( (kT)^4 \). Actual values were taken from Thompson. The conclusion to be drawn from the figure is that a plasma of the type considered cannot be in equilibrium under pure radiation cooling, unless it is quite large. If necessary, an equilibrium condition can be secured by admixture of a small percentage of high \( Z \) impurities, resulting in increased losses from excitation radiation and enhancement of bremsstrahlung.

The temperature to aim for is in the vicinity of 1 kev. In addition, Fig. 1(b) indicates that it becomes increasingly difficult at lower temperatures to close the gap between power input and radiation loss.

Using these values of density and temperature, with a frequency of 100 Me/sec, the reactive power required to contain the representative plasma volume used above \( (4.19 \times 10^{-3} \text{ m}^3) \) turns out to be \( 1.69 \times 10^4 \text{ Mw} \). This is a large value and evidently can only be attained in a very high-\( Q \) system. A comparatively long time scale is thereby implied, since, in order to take advantage of the high \( Q \), a time of the order of \( Q/\omega \) must be allowed for the field energy to build up to near its final value. It thus becomes important to eliminate any contact between the plasma and material walls or electrodes, because of the attendant large and uncertain cooling effect.

**Spherical Cavity**

To fulfill this latter requirement, the arrangement should be such that no electric field lines connect the plasma with metal walls. If the plasma region is assumed to be simply connected, such a device must appear essentially as in Fig. 2, which depicts a spherical conducting shell surrounding an approximately spherical plasma body. Electric power is supplied to the space between the plasma and the outer shell at a frequency corresponding to resonance in the \( \text{TE}_{110} \) mode. As indicated in the figure, the electromagnetic energy then oscillates between a purely toroidal \( E \) (electric) field and a poloidal \( B \) (magnetic) field. If plasma losses are neglected, the ratio of reactive to real power for this mode is given approximately by

\[ Q = 2.3 \times 10^5 R^4, \]

for a silver cavity, where \( R \) is the wall radius in m.

There are two points, \( L \) and \( L' \), at which the confining force is zero and, indeed, such points will exist for any instantaneous field configuration as long as

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\( \dagger \) Reactive power is here defined as (peak field energy) \times (angular frequency).
the plasma region is topologically equivalent to a sphere. This is true because the containing force is derived from a continuous vector field \((B\) field), and a theorem of Poincaré asserts that such a field on the surface of a sphere always has singularities.\(^{10}\) It may be inferred from this same theorem that a torus is the only surface capable of supporting a non-singular vector field.

Containment can possibly be improved by making use of the three-fold degeneracy of this resonant frequency in a spherical cavity. Thus, if the \(\text{TE}_{110}\) and \(\text{TE}_{111}\) modes are excited in time quadrature, the instantaneous field structure will be as shown in Fig. 3, and will rotate around the axis \(PP'\), causing the time average of the containing force to be positive at all points of the plasma surface. Under these circumstances, it is reasonable to suppose that an equilibrium configuration exists, although this has not yet been proved. If an equilibrium plasma shape does exist, it will certainly not be a sphere, since a spherical plasma would be subjected to a magnetic pressure varying as \((1 + \sin^2\theta)\), (see Fig. 3).

**Figure 4. Fields in a toroidal cavity**

**Toroidal Cavity**

If, for topological reasons, a toroidal plasma should be found desirable, suitable modes for the purpose are essentially as exhibited in Fig. 4. These do not represent actual solutions of the field equations for this case, but are simply drawn as being the same as the lowest non-dc \(\text{TE}\) and \(\text{TM}\) modes in an infinite coaxial cavity. Such modes could be used singly or in combination; they do not, however, have the same resonant frequency.

**Magnetic Pressures**

Figure 5 displays the results of a calculation of the ratio of the magnetic pressure exerted on an inner conducting sphere to that at the outer cavity wall, for the modes pictured in Figs. 2 and 3. This ratio is the same along any radius. The superimposed straight lines reflect the variation of gas pressure with radius for the indicated polytropic gases. Since these lines exhibit a steeper slope than the magnetic pressure curve, it may be inferred that the plasma can very likely be made stable against change of scale, e.g., by maintaining a constant rms value of magnetic field at the outer shell. This is a weak form of stability but is nonetheless necessary.

F. B. Knox\(^{11}\) has proposed an experimental apparatus very similar to that shown in Fig. 3, the difference being that three different resonant frequencies are employed, corresponding to the mode shown in Fig. 2 and two other modes with successively higher radial wave numbers. By establishing these three wave systems at right angles in space, a constant pressure can be exerted over the surface of a sphere. A certain averaging time must be allowed, however, before this pressure can be regarded as constant. If the magnetic pressure is to be uniform within 1%, a rough estimate indicates that about 100 cycles are necessary at the lowest resonant frequency. Higher frequencies are also required, by a factor of about 2\(\frac{1}{2}\), which could be a slight disadvantage from an engineering point of view. Nevertheless, there is the definite advantage that an equilibrium plasma shape obviously exists.

**Scale-up to Power Reactors**

It is interesting to inquire if these devices can be scaled up to actual thermonuclear power reactors. Table 2 contains some calculated power relationships for two different sized machines, wherein the field shown in Fig. 3 is used to confine a plasma at a density of \(2 \times 10^{21}/m^3\) and a temperature of 10 kev. Electrical constants for the walls are taken to be those of silver. It is seen, as stated above, that the power situation is completely dominated by the skin effect in the metal walls. Whenever this is the case, it is easily shown that the power level at which the input to the wall is equal to the thermonuclear output from the plasma is independent of size and depends only on the plasma temperature and the geometry of the system. For the particular temperature and proportions used in calculating the values in Table 2, this "break-even" power is \(0.83 \times 10^{14}\) w. By way of comparison, this is about the power consumption of a 1000-ton spaceship travelling at \(3 \times 10^6\) m/sec under an acceleration of three gravities.
Consideration of the numbers in Table 2, together with the associated discussion, shows that practical use of such machines as thermonuclear reactors must be accompanied by substantial reductions in skin effect power dissipation. Such reduction would be quite effective; for instance, if the skin losses could be reduced by a factor of 100, the break-even power level given above would be lowered by the square of the same factor to the conceivable value of 8900 Mw.

There does exist precedent for such investigations. Litzendraht or “litz” wire has been used for many years in radio service, and accomplishes significant reductions in skin losses for frequencies below about 2 Mc/sec. More recently, A. M. Clogston has developed a method of using laminated conductors to reduce skin effect losses in rf transmission lines. This approach, however, depends on a rather precise matching of phase velocities in the laminated medium and in the exterior space and therefore does not seem applicable to the construction of resonant cavities. In order to be useful, the wall medium would have to possess an intrinsic phase velocity greater than c; it appears that the only known “material” with this property is plasma itself. Superconductors do not either favorably or unfavorably.

Looking at the problem from a systems point of view, it is evident that the ohmic heating in the metal wall is to be regarded as a “loss” only if the power involved must be handled by external devices and fed back into the magnetic field. If it were possible for the field to be maintained directly by the fast charged reaction products, the skin dissipation would then represent useful power output. There does not seem to be any general theorem which states that this cannot happen, as indeed it does, for example, in such electronic devices as klystrons and magnetrons. Energy transfer in the reverse direction, from field to particles, takes place in rf accelerators. On the favorable side, it may be argued that, if containment is indeed effective, then any particles that do escape will have to do work against the confining force and hence give energy to the electromagnetic field.

In order to study the energy exchange question as well as to furnish other needed information about the motions of charged particles in electromagnetic fields, an investigation of these motions is being carried out, both by analytical and numerical techniques. Analytical methods, while necessarily limited in scope, are quite essential for discovering theorems applicable to limiting cases and for suggesting fruitful areas of investigation. Detailed results about specific problems must, in general, be established by numerical integration of the equations of motion. A preliminary program for this purpose has been assembled for the IBM-704 computing machine.

Referring again to Fig. 3, at a typical point on the plasma surface the physical components of the tangential magnetic field may be represented by

$$B = (b_2 e^{j \omega t} - j b_1 e^{j \omega t})$$

(6)

wherein the usual complex notation is used to indicate the time dependence. The numbers $b_1$ and $b_2$ are real.

In the vicinity of the plasma surface, where the displacement current is small, Maxwell’s equations give, for the complex electric vector,

$$E = j \delta \omega n \times B$$

(7)

in which $\delta$ is a small real number and $n$ is the outward normal to the plasma surface. The situation thus brought about is as illustrated in Fig. 6. For rotation in the indicated sense, the angle $\tau$ is always obtuse; otherwise it is acute. At points on the axis of rotation, PP’ (Fig. 3), the ellipses expand into circles and the angle $\tau$ is either 0 or $\pi$, again depending on the direction of rotation.

Exactly this combination of $E$ and $B$ fields can also be obtained in a resonant cavity formed of two infinite parallel plates, by exciting the two orthogonal modes in either 90° leading or lagging time phase. Accordingly, the field configuration chosen for initial study is that shown in Fig. 7, which depicts schematically a resonant cavity formed from a conducting plate on the right and a “plasma” on the left. Assuming the condition $\omega < \omega_p$, the phase velocity in the plasma may be taken to be infinite; the transverse field quantities inside the plasma are then simply those at the surface attenuated by the factor $\exp(-|x|/d)$.

If $\omega$ is small compared to $\omega_p$, implying that $d$ is much
less than \( \lambda \equiv 2\pi c/\omega \), the “extrapolation distance” \( d_e \) is practically equal to \( d \). The transverse fields are of the forms given by Eqs. (6) and (7), where \( b_1 \) and \( b_2 \) are allowed to depend on \( z \) but are required to maintain the same ratio. In anticipation of the possibility that only the electrons might be confined by the rf field, leaving the positive ions to be held in by a resulting negative space charge, a time-independent electric field in the \( Z \) direction is also included. This is assumed to fall off exponentially into the plasma but not necessarily with the relaxation distance \( d \).

The equations of motion to be solved are of the usual form:

\[
\begin{align*}
\dot{\mathbf{v}} &= (q/m)(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \\
\dot{\mathbf{r}} &= \mathbf{v}, \tag{8}
\end{align*}
\]

where \( q/m \) is the specific charge of the particle under investigation, \( \mathbf{v} \) is its velocity, and \( \mathbf{r} \) its position vector.

In the actual machine program, provision is also made for introducing an additional magnetic field which is constant in space and time but may have an arbitrary direction.

A kind of Monte Carlo approach has been used in studying energy exchange phenomena. Particles are emitted from a point inside the plasma more or less uniformly distributed in solid angle. The trajectories are then followed until they terminate in the plasma or on the metal wall and the resulting energy changes are averaged over the group. A considerable number of trajectories have been calculated in which helium nuclei are projected into a field having a magnetic energy density of 3.2 \( \text{MJ/m}^3 \). Results so far are inconclusive, although they do demonstrate that not much action can be expected if the applied frequency differs too much from the Larmor frequency \( qB/m \), evaluated at the plasma surface. This had been indicated by preliminary analytical work, and is definitely confirmed by the detailed calculations. Deuterium ions introduced with zero velocity just to the left of the magnetic nodal point (see Fig. 7) in some cases enter the plasma with energies in excess of their rest mass, the exact value depending on the field frequency. These magnitudes are not quite correct, because of the non-relativistic character of the equations of motion, but do indicate a considerable repulsive force. This conclusion is quite important since it shows that, during the initial excitation of a machine containing neutral gas, events will move toward formation of a plasma core. It appears also that a certain amount of the gas will be pumped into the walls; this is regarded as advantageous since the initial base pressure can thereby be increased.

**PROPOSED EXPERIMENT**

Turning to consideration of an actual experiment with which to test the foregoing ideas, the operating frequency and, hence, the physical scale of the apparatus depend primarily upon the availability of electronic tubes capable of handling the required power. It is difficult to generate large amounts of rf energy at frequencies in excess of 1000 \( \text{Mc/sec} \) (L band). An operating frequency of 800 \( \text{Mc/sec} \) therefore appears
to be a reasonable choice; as is shown in Table 2, this implies a spherical cavity with the convenient radius of 0.3 m. For operation at a density of $2 \times 10^{18}/m^3$ and a temperature of one kev, the power loss shown in the table must be scaled down by a factor of 10, leading to a power requirement of 0.52 Mw.

**Figure 9. Current distribution on inner surface of spherical cavity**

by making the main cavity an integral part of the feedback path for the oscillators. Because of the physical dimensions involved, the arrangement shown in Fig. 8 also requires that the feedback paths contain a non-linear circuit element in the form of an equal phase delay line.

Another problem of some consequence is the synchronization of the two sets of rf drivers in the required 90° time phase. In the face of the inevitable mode coupling action of the plasma, it is not known whether it is possible to maintain the desired relationship of equal stored energy and 90° phasing between the two orthogonal wave systems. This is a stability question which is best answered experimentally. It is worth noting that the method of excitation proposed by Knox would probably not be subject to mode coupling difficulties.

**Vacuum System**

A high quality vacuum system to insure adequate control of gas purity is essential. A critical feature of this system is the design of cavity pumping ports. These ports must have adequate evacuating conductance but should not appreciably affect the cavity nor contribute to the excitation of spurious modes. A possible design is suggested by examining the direction and magnitude of current flow on the inner wall of the cavity as shown by the typical current density vectors in Fig. 9. At the poles, the current density vector $j$ is constant in magnitude and rotates synchronously with the resultant rf field. The vector locus is a circle as shown. At points intermediate between the poles and the equator, the $j$ locus becomes a straight line, the rf current flow being always perpendicular to the equatorial plane. The average magnitude of the current density is a minimum at the equator. The circumstance of minimum current density and one-dimensional current flow at the equator suggests that the pumping ports consist of a large number of small slots at the equator parallel to the current density vector. The main effect of such slots will be to increase the current density by a small amount. They will also tend to suppress any spurious modes in which the current flow in the equatorial zone is not parallel to the slots.

A layout drawing of the cavity and vacuum system is shown in Fig. 10. The cavity and entire vacuum system are metal except for the rf domes, experimental ports and glass pressure gauges. Gold “O” rings are used on all vacuum flanges. An additional guard ring and guard vacuum pump are used on the large cavity flange.

The slotted vacuum pumping ports connect the cavity to a vacuum manifold which extends around the entire equator. The manifold is intended to isolate the cavity from the pumps during test runs. The vacuum system also includes a large liquid nitrogen–copper baffle between the bakable valve and angular adjustment is provided for each loop.

The major problems in the design of the external rf circuit elements are connected with the sizable band-width requirements that must be met. It is intended to operate the A2515D tubes as self-excited grid driven oscillators, so connected that they follow any resonant frequency changes that occur during formation and subsequent confinement of the plasma. The empty cavity resonates at 715 Mc; a fully developed plasma core of one-third the radius of the cavity raises the resonance frequency to 800 Mc, which is a frequency shift of more than 11%. It is expected that the band width problem can be solved by minimizing the stored energy in the external circuit elements and

**Electrical System**

A preliminary design which has been evolved is illustrated in Fig. 8. The tubes shown are RCA A2515D beam power tetrodes, each of which is capable of an output power of about 127,000 w in long pulse service. Four of these tubes are used to drive each of the two orthogonal modes making up the rotating field shown in Fig. 3. The total power available from the eight tubes is then slightly greater than one Mw, furnishing a comfortable margin over the 0.52 Mw mentioned above. A maximum pulse length of about 10 msec is contemplated. The cavity itself is constructed as a pair of flanged hemispheres made of copper-lined steel. The coupling loops transmit rf energy to the evacuated cavity through ceramic domes; it may be necessary to embed the loops in a pressurized atmosphere or in solid insulation to prevent breakdown. Individual micrometer type linear and angular adjustment is provided for each loop.

The coupling difficulties.
RADIO-FREQUENCY THERMONUCLEAR MACHINES

Figure 10. Vacuum system layout

and the diffusion pump, and a smaller liquid nitrogen-
copper trap (not shown) between the diffusion pump
and the mechanical pump. Consideration is also being
given to the use of the recently developed "continuous
gettering" pumps for the lower pressure range.

The volume of the cavity is 113 liters. The pumping
speed of the vacuum system calculated on the basis
of molecular flow is approximately 50 liters/sec.
Outgassing is accomplished by baking the entire
system at 400°C. It is expected that an ultimate
pressure of $10^{-10}$ mm Hg can be attained. Gas is
admitted to a base pressure in the range of $10^{-6}$ to
$10^{-3}$ mm Hg for most test runs.

Diagnostic Facilities

Access to the cavity and plasma for various diag-
nostic purposes is through the pair of experimental
ports shown in Figs. 8 and 10, and through a limited
number of the pumping slots connecting the vacuum
manifold with the cavity. Material probes probably
cannot be inserted into the plasma because of their
excessive cooling effect.

The diagnostic problem can be treated on the basis
of three major episodes which are expected to occur
during a typical 10 msec pulse. These episodes are as
follows: (1) plasma formation time of a few hundred
μsec, (2) thermonuclear reaction time from about one
msec to 10 msec, and (3) plasma decay from the end
of the pulse up to about one sec. The time limits on
these episodes as listed above are order-of-magnitude
values only, and are mentioned merely to facilitate
the discussion which follows.

Knowledge of the cavity filling characteristics is
necessary as a first step in understanding the plasma
formation mechanism. It is planned to determine the
filling characteristics under hard vacuum conditions,
i.e., at pressures of the order of $10^{-10}$ mm Hg. The
superimposed effects of plasma formation can then
be studied by pulsing the cavity at successively higher
pressures. Plasma formation starts in the vicinity of a
spherical shell of approximate radius $R/2$ where the
electric field is maximum. This plasma region then
enlarges both inwardly and outwardly from $R/2$.
Ions outside $R/2$ are driven toward the cavity wall
while those inside $R/2$ are compressed toward the
cavity center to form the plasma core. Observation of
this plasma formation mechanism requires the use of
several diagnostic methods. The most significant
information is expected to come from measurement of
the frequency variation of the cavity field during the
plasma formation episode. In view of the large fre-
quency increase anticipated (10–15%), it appears
desirable to obtain plasma loading characteristics in
small time increments in order to guide adjustment of
the rf driving system. The increase in cavity resonant
frequency gives a direct indication of the effective size
of the plasma core. The outward progress of the ioniza-
tion front during the core formation episode should
be observable by high speed photography. Optical
spectroscopy methods can be used to obtain tempera-
ture and impurity content information during the
early stages of plasma formation. Microwave measure-
ments of electron density and temperature are espe-
cially applicable to this particular experiment; at the
low densities contemplated ($\sim 10^{18}/m^3$), commercial X and K band components will serve very well.

The principal criterion for the identification of a thermonuclear reaction episode is the observation of neutrons of unmistakably thermonuclear origin. This will involve correlation of neutron flux observations with plasma density, temperature, and gas composition. Therefore, in addition to neutron flux measurements, microwave measurement of plasma density and temperature will also be essential. Optical methods appear to have no application during this phase.

The plasma decay episode is expected to furnish information concerning fundamental plasma properties such as recombination rates and relaxation times. Diagnostic methods applicable here include microwave measurement of plasma density and temperature, optical spectroscopy, and photographic observations. The probing rf field methods of measuring decaying plasma properties as developed and used by Brown, et al., and by Biondi should also be applicable during this period. Measurement of gas pressure in the quiescent states immediately preceding and shortly following the rf pulse is expected to indicate the extent of the pumping action resulting from ions being driven toward the cavity wall during the initial plasma formation stages.

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Plasma Studies in a Low Pressure High Frequency Discharge

By A. J. Hatch*

The utilization of high frequency electromagnetic fields for the containment and heating of thermonuclear plasmas appears to present attractive possibilities. The exploitation of these possibilities, however, depends on the solving of many formidable problems. One phase of an attack on these problems is the experimental investigation of basic phenomena related to thermonuclear containment and heating in low pressure high frequency plasmas. This paper presents a description of such experimental work in progress at Argonne National Laboratory.

THE MULTIPACTING PLASMA MECHANISM

Density requirements for a thermonuclear plasma indicate that the pressure range of $10^{-2}$ to $10^{-5}$ mm Hg is of interest for basic studies of relatively cool plasmas. High frequency plasmas in this pressure range are usually produced by the multipacting mechanism. Multipacting is also known as the secondary electron resonance mechanism, as "electrodeless discharge," and as "Pendelvervielfachung der Sekundärelektronen." Multipacting plasmas are sufficiently well understood to be a useful research tool. Furthermore, recently observed phenomena in multipacting plasmas appear to have possible significance for the understanding of high frequency thermonuclear plasmas. Most of the experimental work described in this paper is being done with plasmas produced by the multipacting mechanism.

The principal features of the multipacting mechanism are as follows. The mechanism is operative when the electron mean free path exceeds the electrode separation. The mechanism is controlled mainly by the parameters of frequency, electrode separation and applied voltage, and by the secondary emission characteristics of the electrode surfaces. Electron transit time between electrodes is nominally $\frac{1}{2}$ cycle in the fundamental mode. Higher order modes of $\frac{3}{4}$, $\frac{5}{4}$ and $\frac{7}{4}$ cycle transit times can be excited under appropriate conditions. Electron multiplication is mainly by secondary emission at the electrode surfaces. At the upper pressure limit of multipacting, the contribution of gaseous ionization to electron population can be comparable to that of secondary emission.

This contribution decreases with decreasing pressure. Electrons traversing the inter-electrode gap are well bunched in space, time and energy. Electron energies are typically of the order of tens and hundreds of electron volts. The energy distribution of ions in the plasma, however, is not known.

A useful generalized parameter for multipacting is $fd$, the product of frequency $f$ times electrode separation $d$. The fundamental $\frac{1}{4}$ cycle mode is dominant from cutoff at about 100 Mc cm/sec to about 450 Mc cm/sec. The $\frac{3}{4}$ cycle mode is dominant from 450 to about 650 Mc cm/sec. The higher order modes usually overlap and are resolvable only under special conditions.

EXPERIMENTAL APPARATUS

The demountable discharge tube assembly being used at present is shown in Fig. 1. Rubber gaskets and "O" rings are used throughout. A tubulated glass cylinder 30 cm in diameter and 45 cm long is mounted between a pair of 40 cm diameter metal base plates. Plane parallel metal electrodes 22.5 cm in diameter are mounted inside the cylinder. They are supported by metal tubes which pass through a sliding "O" ring seal in each base plate. The position of each electrode can be axially adjusted over a range of 25 cm. The three tubulations on the top and front of the discharge tube are probe ports. Two of these, A and C, are in the center plane of the discharge tube. Probe port B is axially displaced from A and C by 5 cm. The probes are radially adjustable through sliding "O" ring seals. Fine position adjustment is provided for by the metal bellows. The combined flexibility of the electrode and probe positions permits location of the probes at any desired point in the plasma. Various probes are to be used, including single and double probes. Provision is made for the mounting of electron and ion energy analyzers behind small apertures in the discharge electrode surfaces. Electrode temperature and secondary emission ratio of the electrode surfaces can also be measured. The large tubulation on the bottom of the discharge tube is connected to the vacuum system (not shown). The vacuum system includes a 300 liter/sec oil diffusion pump, a 10 cm diameter throttling valve and a liquid nitrogen baffle. The source of high frequency energy (not shown) is a 1 kw continuously
tunable continuous wave generator covering the range of 3 to 50 Mc. The output terminals of the high frequency generator are connected to the base plates of the discharge tube assembly.

**EXPERIMENTAL PROGRAM**

Three experiments now being performed or planned for the immediate future will be described briefly:

The first experiment is an exploration of the characteristics and fundamental nature of a type of high frequency plasmoid† originally observed by R. W. Wood. The experimental conditions under which these observations were made appear to be those of multipacting. Wood has hypothesized that this type of plasmoid is associated with the longitudinal electron plasma oscillations studied by L. Tonks and I. Langmuir. It is well known that plasma oscillations constitute one of the principal mechanisms whereby a plasma can be shielded from an applied high frequency field. Furthermore, the plasma density limitations inherent in plasma oscillations are generally too low for thermonuclear applications. Hence an understanding of plasmoids and plasma oscillations can be considered as an essential first step in order to overcome these limitations to high frequency methods of plasma heating.

The second experiment is an investigation of a possible containment mechanism in a multipacting plasma. It has been pointed out by C. F. Robinson and by H. B. Williams that the time-average force exerted on a positive ion in a multipacting plasma by the oscillating cloud of electrons is directed toward the center of the plasma volume. Thus, a three dimensional well or trap for ions can be established. This containment mechanism is illustrated schematically in Fig. 2. Although the temperature and density capabilities of this mechanism are estimated to be fairly high, the possible limitations due to effects such as plasma oscillations is not known. A further aspect of the mechanism is that it may be associated with the high frequency plasmoid discussed in the preceding paragraph. Studies of the characteristics of the multipacting containment mechanism are expected to contribute to an understanding of the general problem of high frequency (or dynamic) containment mechanisms.

The third experiment is an investigation of the penetration of a plasma by high frequency fields in the presence of transverse static magnetic fields. Theoretical studies by M. H. Johnson indicate the possibility of greatly enhanced high frequency field penetration of the plasma in such systems. In a simple experimental geometry such as that of the discharge tube shown in Fig. 1 the longitudinal or axial plasma oscillations are converted to transverse plasma oscillations by the transverse magnetic field. This conversion is shown schematically in Fig. 3A. Although this modification provides a substantial enhancement of field penetration, the transverse plasma oscillations still constitute a serious barrier. The transverse plasma oscillations arise from charge separation at the edges of the plasma perpendicular to the electron trajectories. A method of overcoming both the longitudinal and transverse plasma oscillation limitations on high frequency field penetration as proposed by Johnson is illustrated in Fig. 3B. In this coaxial cylindrical geometry, the high frequency electric field is radial and the static magnetic field is axial. There is now no

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† The term “Plasmoid” was originally suggested by R. W. Wood (Ref. 10). He applied the term to the luminous balls, spindles, etc., observed in certain low pressure high frequency plasmas. W. H. Bostick has more recently generalized the term to include any well defined plasma-magnetic entity or compact geometrical configuration, independent of the method of production (see Ref. 11).
azimuthal boundary at which charge separation due to the azimuthal (transverse) motion of the electrons can occur. Charge neutrality can now be maintained throughout the plasma. The resulting high frequency field penetration is now adequate to enable the plasma to accept energy efficiently by the ion cyclotron resonance mechanism. This experiment will be performed in two parts, one using the plane parallel electrode geometry of Fig. 1, the other using the cylindrical geometry of Fig. 3.

**EXPERIMENTAL RESULTS**

A photograph of a typical high frequency plasmoid is shown in Fig. 4. This plasmoid was formed between the plane parallel electrodes of the apparatus shown in Fig. 1. The electrodes were on the left and right hand sides of Fig. 4. The frequency was 15 Mc and the electrode separation was 25 cm, corresponding to $\frac{1}{2}$ cycle multipacting at 375 Mc cm/sec. The plasmoid was formed in air at a pressure of $0.3 \times 10^{-3}$ mm Hg. Both the plasmoid and the surrounding plasma were white. A prominent feature of the plasmoid is the dark boundary which completely surrounds it. The plasmoid thus defined is approximately a spheroid of diameter 12 cm. The plasmoid exists in various forms from about $3 \times 10^{-3}$ mm Hg to about $0.1 \times 10^{-3}$ mm Hg. At pressures above $0.3 \times 10^{-3}$ mm Hg the plasmoid enlarges, becomes more cylindrical in form, and the dark boundary becomes less distinct. At pressures below $0.3 \times 10^{-3}$ mm Hg the plasmoid becomes ellipsoidal. At a pressure of about $0.1 \times 10^{-3}$ mm Hg both the plasmoid and surrounding plasma sometimes suddenly disappear. At other times the plasmoid suddenly becomes quite flat, like a thin double convex lens which tapers off to a thin tenuous edge, and the dark boundary extends all the way to the confining edges of the discharge tube and electrodes. This latter plasmoid configuration is usually quite unstable; it either reverts quickly to the ellipsoidal plasmoid with the narrow dark boundary, or both the plasmoid and plasma suddenly disappear.

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Neutrons from Plasma Compressed by an Axial Magnetic Field (Scylla)

By W. C. Elmore, E. M. Little and W. E. Quinn*

Earlier work\(^1\) at this laboratory established that shock waves of considerable intensity may be produced in deuterium gas by the abrupt application of a strong magnetic field. Such shock waves ionize the gas and can heat the resulting plasma to a temperature of several electron volts or more, provided the shock waves have sufficient intensity. The plasma so produced can then be used as the starting condition for further heating, for instance, by adiabatic compression. The first part of the work reported here deals briefly with an arrangement for magnetically driving high-intensity shock waves in deuterium gas at reduced pressure. The second part deals in more detail with early experimental results in which the shock-heated plasma is adiabatically compressed by an axial magnetic field to a point where neutron emission is observed.

**Single-Coil System**

A simple theoretical analysis of a magnetic driver for shock waves shows that the product of the linear coil dimension and the initial rate of increase of magnetic field intensity should have as high a value as possible. If \( V \) is the potential of a capacitor bank, \( L_0 \) is the external source inductance, and \( L_1 \) is the inductance of a single turn of a closely coupled coil of \( n \) turns of radius \( r \), then for similar geometries one finds that

\[
r \frac{dB}{dt} = \frac{nV}{L_0 + n^2 L_1}
\]

when the bank is suddenly connected to the coil.

Equation (1) is independent of the capacitance of the capacitor bank, except through the contribution of the bank to the source inductance \( L_0 \). Clearly, \( V \) should be made high, and \( L_1 \) and \( L_0 \) kept as low as possible. A lower limit to \( L_1 \) is normally set by other considerations, such as the thickness of the tube walls, the thickness of necessary electrical insulation, and the desired volume of plasma to be shock excited. For given values of \( L_0 \) and \( L_1 \), \( n \) should be chosen so that for \( \infty > L_1/L_0 > \frac{1}{2} \), \( n = 1 \); for \( \frac{1}{2} > L_1/L_0 > \frac{1}{3} \), \( n = 2 \); etc.

Results of experiments with several different coil configurations have been consistent with these design criteria. In a system where \( L_1 > \frac{1}{2} L_0 \), we find that a single band of copper surrounding a cylindrical glass tube makes a satisfactory driver. Shock fronts having measured speeds up to 10 cm/μsec have been produced in a tube of 5 cm inside diameter, containing deuterium gas at pressures in the range 10 to 1000 microns of mercury. In a particular experiment, the driving coil had a radius of 3.0 cm and consisted of a band of copper 0.075 cm thick and 2.5 cm wide. The coil was connected by a short parallel-plate transmission line to a 0.85-μF capacitor of 100 kv rating, surmounted in coaxial geometry by a triggered spark gap. Such a system had a total inductance of 0.23 μH, of which the coil contributed about 0.075 μH. A weak axial magnetic field (500 to 2000 gauss) was applied to the tube over an interval 5 to 20 cm from the driving coil. This field served to keep the shock wave free of the walls of the tube.\(^3\)

**Shock Wave Studies**

Various properties of shock waves produced in the manner described have been studied using magnetic probes, electric probes and a pair of photomultiplier tubes. In addition, a moving-image camera with a writing speed of 0.6 cm/μsec has been used to record axial shock-front motion, and to establish the radial extent of the shock front. Normally, the gas is partially pre-ionized by radio-frequency power applied to external capacitive electrodes situated near each end of the shock tube. Without this provision, the gas often breaks down only after one or more oscillations of the applied field, particularly at low gas pressures.

Moving-image records and magnetic probe measurements show that the shock speed some distance from the driver coil is very nearly proportional to the voltage to which the capacitor is charged. Observations were made in the range 30 to 70 kv. The speed at 70 kv was about 10 cm/μsec at a position 13 cm from the coil. Except near the coil, the shock front is nearly plane, with a small central cusp whose origin is associated with the magnetic field used to keep the shock wave from the tube wall. Near the coil, conditions are not simple and the shock front possesses a converging radial component. A number of unexplained shock

\* Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico. The paper includes an addendum by K. Boyer and the above authors.
phenomena have been observed and deserve further study. However, these exploratory experiments, taken in conjunction with some unpublished calculations of the plasma temperature behind a plane shock front in deuterium gas, indicate that it should be possible to attain plasma temperatures in the 10–20 ev range, immediately after the passage of a magnetically driven shock front.

In the earlier experiments, a study was made of the compression produced by a magnetic field on the plasma in a tube through which successive shock fronts had been driven in opposing directions. It was estimated that temperatures of perhaps 40 ev were attained. In these experiments, the axial compression field reached a peak value of 40 kilogauss in 10 µsec over a 7.5-cm length of a tube having an inside diameter of 8.75 cm. Calculation showed, however, that even if a mirror geometry were used, the escape of particles by collision and deflection into the mirror escape cone would prevent the attainment of a detectable thermonuclear reaction unless the rise time of the magnetic field were held down to a few microseconds.

Scylla Twin-Coil System

This consideration has led to a new but very simple arrangement for shock excitation followed by adiabatic compression. The device has been named Scylla. In it, two identical single-turn coils mounted coaxially are used both to shock excite plasma to a temperature in the 10–20 ev range and, by use of the subsequent rise in magnetic field, to compress adiabatically the plasma formed between the coils. In such an arrangement, the spacing of the coils sets the mirror ratio, that is, the ratio of the maximum axial magnetic field to the axial field at the mid-plane.

A diagram showing the general functional arrangement of the experiment is given in Fig. 1. Two capacitor banks of 4.4 µf each and of 100 kv rating are separately connected through triggered spark gaps to the two single-turn coils. The coils consist of copper bands 0.63 cm thick, 2.5 cm wide and with a separation about equal to the mean coil diameter of 7.0 cm.

The coils give a mirror ratio of about 1.4. The inductance of each coil is 0.08 µh, as compared with a source inductance of 0.13 µh for each bank. Thus only 38% of the capacitor voltage appears initially across the coil terminals. With the capacitors charged to 65 kv (the highest potential that could be used in the present set-up because of sparking difficulties) a maximum central compression field of about 38 kilogauss is obtained 1.5 µsec after the spark gaps fire. Radio-frequency power is used to pre-excite the gas in the tube.

Operation

The arrangement described appears to function in the following manner. When the spark gaps fire simultaneously, an initial circumferential electric field of about 1.2 kv/cm exists near the inner wall of the tube at each coil. The deuterium gas there breaks down and a plasma current arises. Magnetic repulsion between this current and the current in the coils produces a rapid motion of the plasma away from the vicinity of the coils thereby setting up shock fronts. The shock fronts pass through the gas in the region between the coils and also proceed in opposing directions down the tube away from the central region. In the central region, whose volume is about 150 cm³, the gas is ionized and heated to perhaps 20 ev within 0.1 to 0.2 µsec. The resulting plasma is now a good electrical conductor and is further heated by the compression produced by the increasing magnetic field. Just prior to peak compression, neutron emission is observed.

Preliminary measurements made with a lead-shielded liquid scintillation detector show that, under the conditions of operation described, more than 10⁴ neutrons are produced per burst. The neutron yield is very sensitive to capacitor voltage and disappears below about 50 kv. If the firing of the two spark gaps differs in time by 0.2 µsec with the capacitors at 60 kv, the neutron yield is considerably reduced, and it disappears if the time difference is as much as 0.5 µsec. The yield is not very sensitive to initial deuterium pressure in the range 20 to 80 microns of mercury. Below 20 microns it begins to fall off; it disappears at pressures somewhat below 10 microns. The yield also begins to fall off fairly rapidly at pressures above 80 microns, and disappears somewhat above 100 microns.

In addition to neutrons, X-rays have been observed at gas pressures below 50 microns. As in the case of the neutrons, the X-rays first appear approximately at the time of peak compression. A simple absorption measurement in lead shows that many of the X-rays have an energy of at least 200 kev. The X-rays appear to come from regions of the tube about 13 cm each side of the midplane of the coils, as shown by the darkening of photographic film placed along the tube.

In these regions, the walls of the Pyrex tube are darkened and have a bluish tinge suggesting bombardment by electrons that have been accelerated into the central region, and then scattered so that they can escape along magnetic field lines. It is surmised that...
the absence of X-rays at higher initial gas pressures is due to electron-ion collisions that prevent the acceleration of electrons to high energies. There seems to be no correlation between X-ray and neutron yield.

A few observations have been made without radiofrequency pre-excitation. It has been found that, at lower pressures, the neutron yield decreases or vanishes and that the X-ray yield increases considerably both in intensity and in energy. Evidently, gas breakdown is delayed so that a significant fraction of the final compression field exists at the time the ions are formed. Such a field would reduce the ultimate plasma compression. Furthermore, the particle trajectories in such a case have a somewhat higher escape probability.

If the energetic deuterons responsible for neutrons arise by a run-away process, a close correlation between the neutron and X-ray yields should be expected. The lack of such correlation with respect to pressure dependence and with respect to rf pre-excitation makes a thermonuclear origin of the neutrons appear plausible. It is clear, however, that further diagnostic work must be done to establish conditions in the plasma at the time of peak compression. In addition, better information is needed regarding the duration of the neutron burst, the dependence of yield on the peak compression field, the effect of impurities on yield and the geometrical origin of the neutrons. Work along these lines is in progress with a new version of Scylla in which high-voltage insulation has been improved and the source inductance of the capacitor bank lowered by the use of ten spark-gap switches operating simultaneously.

ACKNOWLEDGEMENTS

The authors are greatly indebted to Keith Boyer for encouragement and advice in extending earlier experiments on the adiabatic compression of shock-heated plasmas by employing higher capacitor potentials and shortening the time for compression. They are also indebted to James L. Tuck for emphasizing the importance of the rate requirements in the experiment.

ADDENDUM†

During the past several months a number of advances have been made in the Scylla experiment. The

capacitor bank and its switching have been improved. These changes together with a new coil geometry have greatly increased the neutron yield. With the new Scylla, observations have been made on (1) the dependence of neutron yield on pressure, (2) the time of occurrence of neutron and X-ray bursts, (3) the approximate shape and position of the neutron source, (4) the neutron energy spectrum for both signs of the applied voltage, to determine whether ordered acceleration processes are responsible for neutron production, and (5) the magnetic field configuration at the time of maximum compression.

Equipment

Capacitor Bank and Switching

The original capacitor bank, which had ten 0.88 μF, 100 kv capacitors connected in two equal sections with a spark-gap switch for each section, has been reassembled with one four-element spark-gap switch for each capacitor. From each switch mounted on its capacitor, eight 4-meter lengths of RG-14/U cable (RG-17/U in the Geneva Exhibit model) run to a common oil-immersed junction, connecting with a short parallel-plate transmission line that leads to the compression coil. This arrangement reduces the source inductance of the bank from 0.065 μH to 0.040 μH and permits operation, without sparking, at voltages approaching the limit set by the capacitor ratings. The connecting cables can accommodate a time jitter of 0.04 μsec in the firing of the triggered gaps. After considerable experimentation, the circuit of Fig. 2 was found to trigger the gaps reliably within this time interval. A five-stage cascade capacitor bank (Marx circuit) is used to charge the high-voltage bank to a potential in the range 70-90 kv in 50 μsec. The high-voltage bank is then discharged through the compression coil to produce a peak current of about 10 amperes per volt on the condensers, rising to this value in 1.25 μsec, after which time it continues to oscillate with a damped sinusoidal waveform.

Compression Coil

It has been found that a compression coil having the shape shown in Fig. 3 results in an increase in neutron yield over that produced by the two single-turn coils used originally. In such a coil, the magnetic field lines fail to penetrate the thick metal walls, so that the average mirror ratio of the axial field is approximately set by the ratio of the internal area at the mid-plane to that at the end sections. The predicted ratio is 1.4 for the coil of Fig. 3, whose dimensions have been roughly optimized for greatest neutron yield by trying a series of coils of modified shapes. The measured mirror ratio is 1.33 with a circuit inductance of 0.033 μH.

Results

Dependence of Neutron Yield on Pressure and Contaminants

The neutron yield produced with the modified capacitor bank and shaped compression coil is sufficient to permit the use of a neutron detector consisting of silver foils surrounding four Geiger tubes mounted in a cadmium-shielded block of paraffin. The detector was calibrated using a Cockcroft Walton D-D neutron source, and gives one count for 4500 neutrons from a point source at the distance used with Scylla. The greatest neutron yield appears to occur at a deuterium pressure of about 100 microns. The observed yield, as a function of pressure, for the bank charged to 70 kv is given in Fig. 4. Ten to twenty observations were made at each pressure, and the five highest yields are indicated. At higher bank voltages (75 to 85 kv), occasional yields as high as 2×10^7 are noted. It is found that at 100 microns, an admixture of 1% of dry air reduces the yield by 56% and 5% of air reduces it by 97%. A more detailed study of the effect of impurities is planned.

Time of Occurrence of Neutrons and X-rays

Oscilloscope records made with a multi-channel oscilloscope have shown that the neutrons emerge on the second and to a much smaller extent on the third
compression of the magnetic field. The pulses of neutrons have a symmetrical bell-shaped distribution which is centered on peak magnetic field, and which occupies less than one-quarter period. X-ray pulses, which appear only at low pressures, have an asymmetrical intensity distribution starting near magnetic field reversal and terminating before peak field. Often a smaller but similar pulse of X-rays occurs after peak field and extends to the next field reversal. Evidently the X-rays are strongest when the mean free path is long and during times when there is a strong electric field due to a high rate of change of magnetic flux.

The absence of neutrons on the first current maximum suggests that the first half-cycle is required to ionize the gas fully, and to establish a starting temperature such that magnetic compression is possible during the second half current cycle. This would permit the adiabatic compression to raise the temperature to neutron producing levels. This conclusion is substantiated by the magnetic field measurements.

Shape and Position of Neutron Source

A massive neutron collimator, made from paraffin loaded with lithium to stop scattered neutrons and followed by lead shielding to stop X-rays, has been used to survey the source of neutrons within the compression coil. A plastic scintillator and fourteen-stage photomultiplier coupled to a cathode-ray oscilloscope records neutrons that pass through the 1 cm diameter cylindrical aperture of the collimator. By this means, the neutrons are found to come from a central region in the compression coil roughly 1.5 cm in diameter and 3 cm long, as shown in Fig. 5. These observations clearly establish that the neutrons originate in the gas away from the tube walls and that the compression is three-dimensional.

Neutron Energy Spectrum

A shadow-bar experiment, using nuclear emulsions as detectors, has been performed to look for acceleration processes produced by the induced electric field. A thick paraffin plate was mounted with its upper surface parallel to, and 0.33 cm above a horizontal plane passing through the axis of symmetry of Scylla. Nuclear emulsions were mounted on this plate at distances of 15 and 20 cm from the axis and Scylla was operated 1000 times for each of the two possible magnetic field orientations. Analysis of the nuclear emulsion data shown in Fig. 6 demonstrates an acceleration of deuterons, in the direction of the electric field, due to the rate of change of magnetic flux. If the assumption is made that all the particles are rotating with a uniform velocity, the actual energy shift will be twice the observed shift of 150 ± 60 ev. Analysis of the probe data indicate circulating currents which could produce rotational energies of about 40 ev for the ions, which is approaching the lower limit of the observed energy shift. However, this is too low an energy, by a factor of 10, to have any appreciable effect on the neutron production rate. The neutron energy spectrum does not seem to permit more energetic collisions than those corresponding to 6 kev deuterons incident on deuterons at rest.

Magnetic Probe Measurements

An attempt has been made to determine the magnetic field distribution at the time of peak compression. Preliminary results are given here with some reservations. The difficulty arises from the extreme corrosion experienced by any device introduced into the hot plasma and the disturbing effects of the evaporated material on the plasma. Most of the measurements were taken by a small probe with thin metal walls (stainless steel) extending along a line parallel to the axis and passing to the mid-plane of the plasma region. Probes with quartz or ceramic walls were always destroyed by a single discharge. The surface of the steel probe tube was melted to a depth of about 0.025 cm in 1 or 2 μsec and waves were
The magnetic probe measurements on the medium plane of Scylla at peak compression and the particle pressure calculated from the magnetic field distribution formed on the molten surface by the high velocity gas escaping axially after compression, indicating that the plasma was indeed heated to quite high temperatures. The magnetic field distribution on the mid-plane and the resulting particle pressure deduced from pressure balance are shown in Fig. 7. While these data are preliminary, the major features are reproducible and are believed to be substantially correct.

**SUMMARY**

The neutron yield, the size and shape of the neutron emitting source, the neutron energy spectrum and the magnetic field configuration are all consistent with the assumption that the plasma has a temperature of about 1.3 kev at maximum compression and a density of about $6 \times 10^{16}$ deuterons per cm$^3$. It is also of interest to note that the peak compression lasts long enough for the average ion to make one thermalizing collision. If the neutrons prove to be of non-thermonuclear origin, they must then arise from random acceleration processes acting on the deuterons to give them energies lying between 2.5 and 6 kev in order to be consistent with the data.

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Diffusion of Arc Plasmas across a Magnetic Field

By Albert Simon*

The effect of a magnetic field $B$ is to reduce the coefficients of diffusion, $D^P$, across the magnetic field to the values

$$D^P = \frac{D_0}{1 + (\omega \tau_c)^2},$$

where $D_0$ denotes the coefficient for $B = 0$, $\omega = eB/m_ac$ and $\tau_c$ = mean free time between collisions for ions or electrons. Diffusion in the direction of the magnetic field, $D^P$, is unchanged from its field-free value. Hence

$$D^P = D_0.$$

In many arc experiments, $(\omega \tau_c) > 1$ and, as a result, $D^P > D_0$. In this case the plasma has a highly anisotropic conductivity, since currents may flow in the direction of the field far more easily than in the perpendicular direction. In the analysis which follows, it will be shown that this anisotropy produces a diffusion across the magnetic field which is not ambipolar. The ions and electrons, instead, diffuse at their own intrinsic rates. Space-charge neutralization is maintained by slight adjustments of the currents in the direction of the field lines. These results will then be compared with the experiments of Bohm, Neidigh, and Bickerton.

The analysis below will be valid only for the case of a weakly ionized gas. This means that the dominant mechanisms of diffusion are ion-neutral atom and electron-neutral atom collisions. In typical arc plasmas (ion and electron energies of the order of a few electron volts) this requires the degree of ionization to be smaller than about 1%.

In the final section, the effects of providing magnetic mirrors at the ends of the arc and the effect of increasing the tube length in the direction of the field lines will be considered.

DIFFUSION IN A UNIFORM FIELD OF FINITE LENGTH

Theory

Consider a two-dimensional problem in which the magnetic field is in the $y$-direction and the $x$-direction is perpendicular to the field. The time dependent equations for the ion and electron densities, $n_i$ and $n_e$ are:

$$\frac{dn_i}{dt} = \frac{D_+}{d^2} \frac{d^2n_i}{dy^2} - \mu_i \frac{d}{dx} \left( n_i E_x \right),$$

$$\frac{dn_e}{dt} = \frac{D_+}{d^2} \frac{d^2n_e}{dy^2} - \mu_i \frac{d}{dx} \left( n_e E_x \right) + \frac{D_+}{d^2} \frac{d^2n_e}{dy^2} - \mu_e \frac{d}{dy} \left( n_e E_y \right).$$

Here $\mu$ denotes the mobility and $E_x$ and $E_y$ are electric field components in the plasma. The relation between the mobility across a magnetic field and that parallel to the field is the same as that for the diffusion coefficients in Eq. (1). Hence $\mu^P \leq \mu$. In addition, let us consider a finite system bounded by conducting walls. If $L$ is the dimension of the system in the $y$-direction and $R$ the dimension in the $x$-direction, then one would expect that $E_y/L \sim R/L$. The reason for this is that the electric fields are the derivatives of the scalar potential, which varies monotonically from a central positive value to zero at the walls. Thus, except for the case of a very long and thin container, one may safely neglect the second term on the right side of Eq. (3) compared to the last term. This is the formal way of stating that the essential currents which adjust for space charge neutrality are those which flow in the direction of the magnetic field.

One may now proceed in the usual fashion to eliminate the explicit dependence on $E_y$ by the assumption of approximate charge neutrality $n_1 = n_e = n$. By combining the ion and electron expressions of Eq. (3), there results:

$$\frac{dn}{dt} = \frac{D_+}{d^2} \frac{d^2n}{dy^2} + \frac{\mu_i D_+ - \mu_e D_+}{\mu_i - \mu_e} \frac{d^2n}{dx^2} + \frac{\mu_e D_+ - \mu_i D_+}{\mu_e - \mu_i} \frac{d^2n}{dy^2}.$$

Note that the effective diffusion coefficient in the $y$-direction has the usual ambipolar value,

$$D_{eff} = \frac{\mu_i D_+ - \mu_e D_+}{\mu_i - \mu_e} = D_{AMB}$$

while the coefficient in the $x$-direction is that of the ions since $\mu_i > \mu_e$ and $D_i^P > D_e^P$. Thus

$$D_{eff} = \frac{\mu_i D_+ - \mu_e D_+}{\mu_i - \mu_e} = D_+.$$
In many cases the dimensions of the system in the field direction are small compared to the collision mean free paths. In this case, the streaming of ions and electrons in the $\gamma$-direction may not be represented by the diffusion terms. It may be shown that the new equations corresponding to those of Eq. (3) are now,

$$\frac{d n_i}{dt} = D_{i\gamma} \frac{d^2 n_i}{dx^2} - \mu_i \frac{1}{L} \frac{d(n_i E_x)}{dx} - \frac{n_i}{L} \frac{d^2 n_i}{dx^2} = D_{i\gamma} \frac{d^2 n_i}{dx^2}$$

where $M$ is the ion mass, $m$ the electron mass, and $v_i$ and $v_e$ are the average thermal velocities. Note that there is now free streaming to the end walls as well as electric field accelerations of the ions and electrons through a distance of the order of half the arc length. Once again, it may be shown that the mobility term in the $\gamma$-direction is usually negligible. In the same fashion as before, one finds

$$\frac{d n}{dt} = \left( \frac{M v_i D_{i\gamma} + m v_e D_{e\gamma}}{M v_i + m v_e} \right) \frac{d^2 n}{dx^2} - \frac{n}{L} \left( \frac{M v_i^2 + m v_e^2}{M v_i + m v_e} \right)$$

The effective streaming velocity to the end walls (for comparable electron and ion energy) is $2v_i$ while the effective diffusion coefficient across the field is $D_{i\gamma}$, as before.

It should be noted that the effective diffusion coefficient across the magnetic field, $D_{i\gamma}$, is much larger than the “ambipolar” coefficient, $D_{iAMB}$, which would have been obtained by requiring that the electron and ion currents balance precisely to zero in the $x$- and $y$-directions separately. This coefficient, by analogy to Eq. (5), has the value

$$D_{iAMB} = \frac{\mu_i D_{i\gamma} - \mu_e D_{e\gamma}}{\mu_i - \mu_e} \approx 2D_{i\gamma}. \quad (9)$$

### e-Folding Length

Suppose now that ions and electrons are being produced equally and uniformly along the $y$-axis ($x = 0$). The steady state solution of Eqs. (4), (5) and (6), with inclusion of the source term and boundary conditions, will be of the form $n(x, y) = N(x)G(y)$. With the principal solution having the value $G(y) = \sin \left( \frac{\pi y}{L} \right)$, and with $N(x) = Ae^{-x/\lambda}$ where $A$ is a constant and where

$$q^2 = \frac{L^2 D_{i\gamma}}{\pi^2 D_{AMB}}. \quad (10)$$

The corresponding result for the case of free streaming to the end walls is

$$q^2 = \frac{L^2 D_{e\gamma}}{2v_i}. \quad (11)$$

Note that the ion density decreases exponentially with an $e$-folding length $q$. Note also that the $e$-folding length varies as $P_0/B$ in the diffusion case of Eq. (10) and as $P_0/\lambda B$ in the free streaming case of Eq. (11). Here $P_0(\sim r^{-3})$ is the gas pressure. These results go over exactly in the case of cylindrical symmetry except that

$$N(x) \rightarrow r^{-1}e^{-x/\lambda}. \quad (12)$$

### Experiment

A measurement of the ion diffusion coefficient across a magnetic field was obtained by Bohm et al. at Berkeley.\(^2\) A complete description of the experiments may be found in Ref. 2. In essence, an arc was struck parallel to the magnetic field and along the long axis of a rectangular arc chamber made of graphite. The resultant ion density in the median plane was measured with a cold shielded probe. In most measurements, the chamber was filled with argon gas at a pressure of about $10^{-3}$ mm Hg and the magnetic field strength was 3700 gauss. These observations established that the ion density decreased exponentially from the center outwards with an $e$-folding length of about 0.3 cm.

If one assumes that the ion energy in the plasma is about 2 eV, the resultant mean free path at a pressure of $1.4 \times 10^{-3}$ mm is then $\lambda \approx 5$ cm. The thermal velocity is $v_i \approx 3 \times 10^8$ cm/sec and the arc length $L = 12$ cm. From Eq. (11), using the measured value of the $e$-folding length, one finds that $D_{i\gamma} \approx 4.5 \times 10^8$ cm$^2$/sec. This is to be compared with the theoretical value of $D_{AMB}$ obtained from Eq. (1). Since $\omega_e \tau_e = 14.4$, this value is $2.5 \times 10^8$ cm$^2$/sec, which is in good agreement with the measured value.

Unfortunately, at that time, it was assumed that the diffusion across the magnetic field was ambipolar, and the experimental value was compared with the theoretical value of $D_{AMB}$ which is given in Eq. (9). This value is $D_{AMB} \approx 10$ cm$^2$/sec in complete disagreement with the experiment. As a result of this apparent anomaly, Bohm postulated that the principal mechanism of diffusion was by plasma oscillations rather than by the classical collision mechanism. The
diffusion of arc plasmas

\[ D \approx \frac{10^3 kT_e}{16B^2} \]  

where \( kT_e \) is in electron volts and \( B \) in kilogauss. Note that this coefficient varies as \( B^{-1} \) and is pressure independent, whereas the classical coefficient varies as \( B^{-2} \) and is directly proportional to the pressure.

In order to test these theories, Neidigh\(^3\) has carried out a series of experiments at Oak Ridge. A sketch of his apparatus is shown in Fig. 1. The ion chamber is cylindrical and most experiments were carried out with nitrogen gas at a pressure of about \( 10^{-3} \) mm. The magnetic field could be varied from 2000 to 14,000 gauss. The carbon probe is biased 20 volts negative, which is on the flat portion of its characteristic curve. Measured ion currents varied from 10 \( \mu \)A near the cylinder wall to 10 ma near the arc. It is assumed that ion density is proportional to probe current. Further details of the apparatus may be found in Ref. 3.

Some typical measurements of the \( e \)-folding length as a function of \( B \) are shown in Fig. 2. The resulting values of the reciprocal of the \( e \)-folding length are then plotted as a function of \( B \) in Fig. 3. The plot should be linear if \( D_e P \) varies as \( B^{-2} \) [see Eqs. (1) and (11)] and should vary as the square root if \( D_e P \sim B^{-1} \). The results clearly favor the \( B^{-2} \) behavior. Incidentally, the experimental magnitude of \( D_e P \) (\( \approx 4 \times 10^8 \) cm\(^2/\)sec) is in good agreement with that determined in the experiment of Ref. 2.

In addition to a study of the dependence on \( B \), Neidigh has also investigated the variation with pressure.\(^4\) For a case in which free streaming to the end wall should prevail (\( \lambda \approx 6 \) cm, \( L = 6 \) cm), the results of Fig. 4 indicate a square-root dependence, in agreement with the discussion following Eq. (11). Another case in which diffusion to the end walls has set in (\( \lambda \approx 6 \) cm, \( L = 26 \) cm) is shown in Fig. 5. Here a linear behavior is observed, again in agreement with the theory. These last experiments on the pressure dependence are only preliminary; more careful investigations are planned.

Further confirmation of the \( B^{-2} \) dependence of the diffusion coefficient is provided, for electrons, by the experiments of Bickerton.\(^5\) In this case the spreading parameter \( R \) was found to vary linearly with \( B^2 \) in agreement with the classical theory.

diffusion in a magnetic mirror geometry

The theory presented above indicates that the essential cause of the non-adiabatic diffusion across the magnetic field is the very large conductivity of the system in the direction of the field lines. It is of interest to inquire as to how this conductivity might be reduced and ambipolar diffusion restored. One
possible way is to put magnetic "mirrors" on the ends of the arc. This effect is discussed below.

**Theory**

The theory of the confinement of charged particles by a magnetic mirror indicates that a particle will be reflected from the mirror if its velocity angle $\theta$ is greater than $\theta_a$ where

$$\theta_a = \sin^{-1} \left( \frac{1}{R} \right).$$

(14)

Here $\theta$ is the angle between the velocity vector of the particle and the mirror axis. The ratio of the magnetic field strength in the mirror region to that in the central region is denoted by $R$.

Let us assume that the mean free path $\lambda$ is large compared to the arc length $L$. Those ions with $\theta < \theta_a$ escape immediately. The remaining ions are confined until a collision produces a new $\theta$ which is less than $\theta_a$. If $P$ is the probability that an ion will be scattered into the escape cone ($\theta > \theta_a$) the rate of production of such ions per unit volume per sec is then $n\lambda P/\lambda$ and the total current streaming to the end walls is

$$J = n\lambda P L / \lambda.$$  

(15)

This is to be compared with the corresponding expression for free streaming without mirrors, which is

$$J = n\lambda.$$  

(16)

If one assumed isotropic scattering, the probability $P$ of scattering into the escape cone, by use of Eq. (14) becomes:

$$P = 1 - \left( 1 - \frac{1}{R} \right)^n.$$  

(17)

which has the value $P \approx (2R)^{-1}$ for large $R$. It may be shown that the effect of an electric field potential $\phi$ is to increase the escape cone for the ions and decrease it for the electrons in such a way that

$$P \approx P(1 \pm \frac{\phi W}{L})$$  

(18)

where $W$ is the average kinetic energy of the particles in the region between the mirrors. The combined effect of Eqs. (15) and (18) may be seen to be equivalent to changing the streaming terms in Eq. (7) by the factor $LP/\lambda$, since $\phi L \approx E_p$. This in turn yields a new expression for the e-folding length,

$$q^2 = \frac{\lambda D^2}{2\eta P}.$$  

(19)

It should be noted that the addition of the mirrors reduced the current in the direction of the field by the factor $LP/\lambda$. This will have no effect on the non-ambipolar diffusion, as the discussion in the final section will indicate. However, there is an immediate effect in the behavior of $q^2$ with pressure. Equation (19), together with Eq. (1), indicates that $q^2$ should be pressure independent and that $q \approx r_0/2P^2$ where $r_0$ is the Larmor radius of the ions.

The case of diffusion flow to the end walls is of no interest for a mirror field, since the mirrors will then have only a minor effect on the current flowing in this direction.
DIFFUSION OF ARC PLASMAS

Figure 6. Radial variation of ion density. Magnetic mirrors used.

Experiment

Again there has been a preliminary set of measurements by Neidigh. The mirror field used has a 2:1 ratio. The field was 3500 gauss in the central region and 7000 gauss in the mirror region. The results are plotted in Fig. 6. The $\phi$-folding length seems independent of pressure over the range studied. However, the magnitude of $q$ is about a factor of 4 larger than the Larmor radius. Further experiments are planned.

LONG TUBE LENGTH

It seems clear that if the arc chamber is made long enough the effect of the electric fields parallel to the tube axis will finally become comparable with the radial electric fields; ambipolar diffusion will then be restored. The necessary length may be estimated. From the first part of Eq. (3) the order of magnitude of the perpendicular current is

$$I_\perp \approx \frac{\mu_n P_n E_y}{q} = \frac{\mu_n n_i E_y}{(\omega + \tau) q}$$

where $q$ is the $\phi$-folding length. The magnitude of the parallel current is

$$I_\parallel \approx \mu_n n_i E_y / L.$$  

Finally, since $E_x \approx \phi / R$ and $E_y \approx \phi / L$, one has

$$\frac{I_\perp}{I_\parallel} \approx \frac{L^3}{q R (\omega + \tau)^2}.$$  

Hence, for ambipolar diffusion to be restored, one must have

$$L/R \approx \frac{\omega + \tau}{(\omega + \tau)^2}.$$  

Finally, since it is electron current in the magnetic field direction which neutralizes space charge, it is necessary that this relation be satisfied for the electrons as well. This is the worst case, hence the condition is

$$L/R \geq \frac{\omega - \tau}{(\omega - \tau)^2}.$$  

This condition may be difficult to obtain in practice. For example, in Neidigh's experiments $\omega + \tau \approx 14.4$ and $\omega - \tau \approx 3 \times 10^3$. Since $q/R$ must be appreciably larger than unity, this requires that $L/R > 10^4$. It should be kept in mind that $\omega + \tau$ must be kept appreciably larger than unity if the magnetic field is to play a dominant role in the behavior of the gas.

In the case of free streaming to the end walls, Eq. (7), the corresponding condition is easily shown to be

$$L/R > (\omega R)^2/\lambda.$$  

This condition is incompatible, however, with the necessary requirement for free streaming, $\lambda \gg L$.

It might be thought that the use of insulating end walls would require exact ambipolar diffusion in the field direction and hence produce ambipolar diffusion radially. However, there is no perfect insulator (for example, surface collisions allow electrons to walk radially along the wall) and this probably prevents this scheme from compensating for the huge anisotropy of the plasma conductivity. Some unpublished experiments by Neidigh have shown that an insulating end wall had only a small effect on the radial ion diffusion.
SUMMARY

It is shown that experimental results on diffusion of ions and electrons in a weakly ionized plasma across a magnetic field are in agreement with the classical collision-diffusion mechanism. Hence, no additional mechanisms, such as plasma oscillations, need be postulated. The resultant diffusion rate is not ambipolar, however. The ions and electrons diffuse at their own intrinsic rates.

REFERENCES

Diffusion Processes in the Positive Column in a Longitudinal Magnetic Field

By B. Lehnert*

The motion of a charged particle in a magnetic field may be described as the sum of a gyration around the field lines, a drift motion across the same lines and a motion parallel to the field. External force fields, magnetic field inhomogeneities and inertia forces are responsible for the drift motion in this single-particle picture. In an ionized gas the picture has to be modified by particle interactions, such as collisions and phenomena caused by space charges. Only encounters between nonidentical particles produce a diffusion in first order across the magnetic field. The macroscopic correspondence to a situation where the transverse drift due to particle interactions and inertia forces can be neglected and where the motions are relatively slow is an ionized gas nearly “frozen to” the magnetic field lines. This strong coupling between ionized matter and magnetic fields is of great importance, not only in astrophysics but also for the problem of producing controlled thermonuclear energy in a plasma confined by a magnetic field.

However, it has been emphasized by Bohm, Burhop, Massey and Williams that random fluctuations of charge density and plasma oscillations may produce electric fields which in turn give rise to drift motions across the magnetic field lines. This “drain” diffusion provides an additional mechanism for ionized matter to “slip” across a magnetic field and may, when it dominates over collision diffusion, introduce considerable difficulties into the physics of a plasma in a magnetic field. Bohm and collaborators performed experiments with an arc plasma in a magnetic field and came to the conclusion that the diffusion was not consistent with collision phenomena but could be explained by “drain”. This interpretation was criticized by Simon, who pointed out that the transverse diffusion of the plasma is not necessarily ambipolar. Thus, in the arc experiment, space-charge neutralization can be maintained by the conducting end walls which produce an electron short-circuit across the magnetic field. With this new interpretation Simon was able to show that the arc experiments did not conflict with the binary collision theory.

The purpose of the present investigation is to study diffusion across a magnetic field in a configuration which is free from short-circuiting effects such as those described by Simon. It provides the possibility of deciding whether collision or “drain” diffusion is operative. For the purpose a long cylindrical plasma column with a homogeneous magnetic field along the axis has been chosen as described in Section 5. The theoretical treatment is given in Sections 2 to 4. On the basis of the collision diffusion theory Tonks, Rokhlin, Cummings and Tonks and Fataliev have pointed out that a longitudinal magnetic field will reduce the losses of particles to the walls. Consequently, when the magnetic field is present, a lower electron temperature and a smaller potential drop along the plasma column should be required to sustain a certain ion density. The same conclusions have recently been drawn by Bickerton and von Engel in theoretical and experimental investigations which also include probe measurements in a magnetic field. In the range of the apparatus used by these authors the positive column was found to behave in good agreement with the collision theory. In the absence of a magnetic field the behaviour of the column was consistent with the theory of Tonks and Langmuir, and when the field was present a modified Schottky theory was applicable (next section). The present experiment forms an extension of that of Bickerton and von Engel into a range where the Schottky theory is applicable in the absence of a magnetic field and where the applied magnetic field is still made strong enough to influence the electron temperature.

Theoretical of the Positive Column in a Longitudinal Magnetic Field

A cylindrically symmetric, stationary, ionized gas column is assumed to be situated in a constant, homogeneous, longitudinal magnetic field $B$. The basic macroscopic equations for the ion, electron and neutral gases are assumed to be valid as well as the following conditions:

(a) The mean free paths of ions and electrons are small compared to the tube radius.
(b) The production rate $\xi n_e$ of charged particles is proportional to the electron density $n_e$, where $\xi$ is a

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function of the electron temperature and the neutral gas density. Thus, two-stage ionization is neglected as well as volume recombination. These processes introduce a variation in the longitudinal potential drop with the discharge current.\textsuperscript{11,14} However, since the potential drop varies slowly with the discharge current in the present experiments these approximations do not introduce serious errors.

(c) The ionization degree is low and the frictional coupling between charged and neutral particles does not set the neutral gas into motion.

(d) Electron attachment can be neglected. This is true for a rare gas such as helium which has been used in the present experiment (see Loeb\textsuperscript{15}).

(e) The electron and ion gases have Maxwellian distributions with constant temperatures $T_e$ and $T_i$ across the column. In reality, striated columns often appear with periodically varying conditions in the longitudinal direction. However, Emeléus and Burns\textsuperscript{16} have pointed out that this is not likely to change the theory of the positive column fundamentally. In the present theory the temperatures $T_e$ and $T_i$ can be regarded as symbols for the mean energies of electrons and ions. Not until the next section does the special form of the particle distribution affect the results.

(f) The column is extended very far in the axial direction so as to make end effects and deviations from cylindrical symmetry negligible.

(g) The velocities $v_i$ and $v_e$ of the ion and electron fluids are small and nonlinear terms can be neglected.

**Basic Equations**

With $n_i$ indicating the ion density, $E$ the electric field, $b$ the magnetic field from the current in the plasma, and $v_i$ and $v_e$ the collision frequencies between the charged particles and the neutral gas, the basic equations become

$$\text{div} \left( n_i v_i \right) = \text{div} \left( n_e v_e \right) = \zeta n_e, \tag{1}$$

$$0 = e n_i (E + v_1 \times B) - k T_i \nabla n_i - \nu_1 m_i v_i. \tag{2}$$

$$0 = -e n_e (E + v_e \times B) - k T_e \nabla n_e - \nu_e m_e v_e, \tag{3}$$

$$\text{curl} \ b = \mu (v_i - n_e v_e), \tag{4}$$

$$\text{curl} \ E = 0, \quad E = -\nabla \phi, \tag{5}$$

$$\text{div} \ E = e (n_i - n_e)/\zeta. \tag{6}$$

where Eq. (1) expresses the conservation of mass and charge, and (2) and (3) the conservation of momentum. Cylindrical symmetry is assumed in a coordinate system $(r, \phi, z)$ with the $z$ axis along the column. Equations (5) give

$$E_r = -\partial \phi/\partial r, \quad E_\phi = 0, \quad E_z = \text{const} = E \tag{7}$$

if it is assumed that the component $E_\phi$ should be finite at the axis $r = 0$. Equations (2) and (3) now become

$$0 = e n_i \left( -\frac{\partial E}{\partial r} + v_{ir} B, -v_{ir} B, E \right) - k T_i \left( \frac{d n_i}{d r}, 0, 0 \right)$$

$$- \nu_1 m_i n_i (v_{ir}, v_{ie}, v_{iz}) \tag{8}$$

and

$$0 = -e n_e \left( -\frac{\partial E}{\partial r} + v_{er} B, -v_{er} B, E \right) - k T_e \left( \frac{d n_e}{d r}, 0, 0 \right)$$

$$- \nu_e m_e n_e (v_{er}, v_{eg}, v_{ez}). \tag{9}$$

From the $\phi$ components,

$$v_{ir} = \frac{\omega_i}{v_i} v_{ir}, \quad v_{ez} = \frac{\omega_e}{v_e} v_{er}. \tag{10}$$

where $\omega_i = eB/m_i$ and $\omega_e = eB/m_e$ are the gyro-frequencies of ions and electrons. The macroscopic motions $v_{ir}$ and $v_{ez}$ around the axis of symmetry arise partly from the radial density gradients, and partly from drift motion in the crossed fields $E_r$ and $B = B_z$.

In the axial direction

$$v_{iz} = \frac{e}{\nu m_i} E, \quad v_{ez} = -\frac{e}{\nu m_e} E. \tag{11}$$

**Transverse Diffusion**

The radial components of Eqs. (8) and (9) combine with Eq. (10) to give

$$n_{i} v_{ir} = -\beta_i n_i \frac{\partial v_{ir}}{\partial r} - D_i \frac{d n_i}{d r}, \tag{12}$$

$$n_{e} v_{er} = \beta_e n_e \frac{\partial v_{er}}{\partial r} - D_e \frac{d n_e}{d r}, \tag{13}$$

where we have introduced the notations

$$D_i = \frac{k T_i}{n_i m_i (1 + \omega_i^2/v_i^2)} = k T_i \beta_i/e, \tag{14}$$

$$D_e = \frac{k T_e}{n_e m_e (1 + \omega_e^2/v_e^2)} = k T_e \beta_e/e. \tag{15}$$

$D_i$ and $D_e$ may be regarded as transverse diffusion coefficients, and $\beta_i$ and $\beta_e$ are the corresponding mobilities. Further, the symmetry condition imposed on the radial component of Eq. (4) gives

$$n_{i} v_{ir} = n_{e} v_{er}, \tag{16}$$

which is an exact expression, even if the eigenfield $b$ cannot be neglected. Ecker\textsuperscript{17} obtains the relation (16) from Eq. (1) by use of the condition that the fluxes $n_{i} v_{ir}$ and $n_{e} v_{er}$ have to be finite at $r = 0$.

The notations

$$n \equiv n_i, \quad n' \equiv n_i - n_e \tag{17}$$

are now introduced and Eqs. (12), (13), (1), (16) and (6) become

$$n v_{er} = -\beta_i (n + n') \frac{\partial v_{er}}{\partial r} - D_i \frac{d n}{d r} \tag{18}$$

$$= \beta_i n \frac{\partial v_{er}}{\partial r} - D_i \frac{d n}{d r}$$

and

$$n' = -\frac{e \alpha}{e} \left( \frac{d}{d r} + \frac{1}{r} \right) \frac{\partial v_{er}}{\partial r}. \tag{20}$$

In a subnormal positive column, where the axial current density is very small, considerable deviations from electrode neutrality may arise, i.e., the net space charge $en'$ may be comparable to $en$. This has been discussed by Ecker\textsuperscript{17} in the absence of an external
magnetic field and with the assumption that \( n' \) is nearly proportional to \( n \). For experiments on the normal positive column, however, \( n'/n \ll 1 \) is usually a good approximation in the major part of its cross section. Dropping terms containing \( n' \) in Eqs. (18) gives

\[
\frac{\partial V}{\partial r} = \frac{D_e - D_1}{(\beta_1 + \beta_0)n} \frac{dn}{dr} \tag{21}
\]

and

\[
n v_{er} = -D_a \frac{dn}{dr} \tag{22}
\]

where

\[
D_a = D_0 \beta_e + D_e \beta_1 \tag{23}
\]

is the transverse ambipolar diffusion coefficient. We observe that, when \( P_e/P_i < 1 \), ions will have a stronger tendency to diffuse towards the walls, corresponding to a reversal of the radial potential distribution, compared to the nonmagnetic case. This point is discussed later. Applying \( \frac{d}{dr} + \frac{1}{r} \) to Eq. (22) gives, with Eq. (19),

\[
\frac{d^2 n}{dr^2} + \frac{1}{r} \frac{dn}{dr} + \frac{a}{D_a} n = 0, \tag{24}
\]

with the solution which is finite at \( r = 0 \):

\[
n(r) = n_0 J_0(\alpha r), \quad \alpha = (a/D_a)^{1/2}. \tag{25}
\]

The results (23) and (25) have been obtained in a different way by Bickerton, who starts with the assumption that the diffusion coefficients of ions and electrons are reduced by the factors \( 1/(1 + \alpha^2/\nu^2) \) in a magnetic field. In the present treatments Eqs. (23) and (25) have been deduced directly from the basic equations (1) to (5). From Eq. (21) the radial potential distribution becomes

\[
V = \frac{D_e - D_1}{\beta_1 + \beta_e} \log \left[ J_0(\alpha r) \right] + V_0(z). \tag{26}
\]

**Special Case**

A theory for the positive column in an axial magnetic field has been developed earlier by Tonks under the assumption that the motion of the electrons is affected by the magnetic field, but not that of the ions. This is a special case which is obtained by putting \( \alpha^2/\nu^2 < 1 \) in all equations of this paper. In such a situation Eq. (26) reduces to

\[
V - V_0 = \left( kT_e/\epsilon \right) \log \left[ J_0(\alpha r) \right], \tag{27}
\]

where

\[
\tau = \left( kT_e \right) \frac{D_e - D_1}{\beta_1 + \beta_e} \tag{28}
\]

is a parameter used by Tonks. From expressions (27) and (28), with \( \alpha^2/\nu^2 < 1 \), Tonks concludes that a strong magnetic field may flatten the radial potential distribution or even reverse its sign, i.e., the sign of \( \tau \). However, it has been pointed out by Bickerton that the reverse field may be very small because an introduction of the mean free paths of ions and electrons,

\[
\lambda_i = (3kT_i/m_i)^{1/2} \nu_i, \quad \lambda_e = (3kT_e/m_e)^{1/2} \nu_e, \tag{29}
\]

gives the ratio

\[
\frac{D_e}{D_i} = \frac{\lambda_e}{\lambda_i} \left( \frac{m_i T_e}{m_e T_i} \right)^{1/2} \left( 1 + (eB\lambda_e)^2/(3m_e kT_i) \right) / \left( 1 + (eB\lambda_i)^2/(3m_i kT_e) \right) \tag{30}
\]

It should be observed that \( T_e \) is a slowly varying function of the magnetic field as given in Section 3. A lower limit of \( D_e/D_i \) is obtained by putting into Eq. (30) the smallest value of \( T_e \) and the largest value of \( T_i \) in the range of variation. The function \( (D_e/D_i)_{\text{min}} \) thus obtained varies monotonically with the magnetic field \( B \). The electron temperature in discharge experiments is usually of the order of \( 10^4 \) to \( 10^5 \) K., whereas the ion temperature is considerably lower at moderate ionization degrees and power inputs. Since the mean free paths \( \lambda_i \) and \( \lambda_e \) are of the same order of magnitude, the ratio \( D_e/D_i \) does not necessarily go below unity, and a reversal does not always take place, even in very strong magnetic fields, as shown by Eqs. (30) and (26).

**Space Charge**

Returning to the solutions (25) and (26), the space charge is calculated from Eq. (20) in the quasi-neutral case:

\[
\frac{n'}{n_0} = \alpha^2 \frac{\beta_e \xi_0 - \beta_0 \xi_1}{\beta_1 + \beta_e} \left( 1 + \frac{f_1}{f_0} \right), \tag{31}
\]

where

\[
\xi_1 = \left( kT_e \xi_0 / \epsilon n_0 \right)^{1/2}, \quad \xi_e = \left( kT_e \xi_0 / \epsilon n_0 \right)^{1/2} \tag{32}
\]

are the Debye distances corresponding to the thermal energies of the ions and electrons. Since the electron density is a positive quantity, Eq. (25) shows that \( \alpha^2/\nu_0 < 1 \) is valid throughout the major part of the column as soon as the Debye distances are much smaller than the tube radius \( R \). Only in the immediate neighbourhood of the wall do Eqs. (18) and (19) lose their validity. The density \( n_0 \) at the axis may be calculated from Eqs. (11) and (25) and the total axial current, \( I \), which becomes (cf. von Engel and Steenbeck)

\[
I = \int_0^R 2n_0 \nu_e \left( v_{es} - v_{ex} \right) dr.
\]

Consequently,

\[
n_0 = \frac{\alpha I}{2 \pi e R \tau' \left( \frac{1}{n_0 \nu_0} + \frac{1}{\nu_e} \right) f_1(a R')}, \tag{33}
\]

where \( \tau' = R' \) forms the "boundary" between the quasi-neutral plasma and the wall sheath. Situations where \( (R'-R)/R < 1 \) are of special interest here.
The discussion is now restricted to a nonconducting wall and the boundary condition at \( r = R' \) is given by
\[
\frac{\partial}{\partial r} \left( r n \right) = 2 \pi n \rho R' f_1(aR')/\alpha
\]
(34)
particles inside \( r = R' \). On the other hand, ions leave the plasma edge \( r = R' \) with a mean velocity towards the wall which may be defined by an equivalent "temperature" \( T' \). Thus, the wall absorbs the balance between the particles produced per unit volume, \( \rho \), and the particles absorbed by the wall. Per unit length and time there are created
\[
N = \frac{1}{2} \pi R' n(R')/3kT'(1/m_1)^1 \quad (35)
\]
particles per unit length and time. Eqs. (34) and (35) combine to give
\[
n(R')/n_0 = 4(\pi D_0 m_1/3kT')(aR') \quad (36).
\]
Since \( n(r) \approx 0 \), Eqs. (23), (25), (36) and (29) give the condition
\[
n(R')/n_0 = f_0(aR') \approx \lambda/R' \approx \lambda/R, \quad (37)
\]
where
\[
\lambda = 1.66 \frac{\lambda_0}{a}(m_1(T+T_o)^{3/2}/T)^{1/2} + \lambda_0(1+\omega_e^2/\nu_e^2)(m_1T)^{1/2} \quad (38)
\]
can be regarded as a mean value of the mean free paths and is equivalent to a mean value used earlier by von Engel and Steenbeck.\(^{14}\)

In order to estimate \( \lambda \) the magnitude of \( T' \) has to be determined. For the nonmagnetic case Bickerton\(^\text{10}\) and Bickerton and von Engel\(^\text{11}\) assume that \( T' = T \). On the other hand, Bohm\(^\text{18}\) has pointed out that a stable sheath cannot exist unless the ions leave the plasma edge with a velocity corresponding to \( T' = T_o/2 \). This is caused by an acceleration in a weak electric field which penetrates from the wall region through the plasma edge.

When a magnetic field is present, and in cases of interest in this connection, the Larmor radius of ions is larger than the Debye distances (32) and the thickness of the wall sheath. This implies that the collection of ions by the wall across the sheath is largely unaffected by the magnetic field (Bohm,\(^\text{18}\) Bohm, Burhop and Massey,\(^\text{4}\) Bickerton\(^\text{10}\) and Bickerton and von Engel\(^\text{13}\)). According to Bohm and collaborators \( T' \) should still be of the order of \( T_o/2 \) when a magnetic field is present and \( T_o > T_h \).

In Section 3 the condition \( \lambda/R \ll 1 \) is assumed to be valid. It is seen that a strong reduction of \( \lambda/R \) takes place according to Eq. (38) when the magnetic field is increased (cf. Bickerton\(^\text{10}\)).

DETERMINATION OF ELECTRON TEMPERATURE

The Plasma Balance Equation

When \( \lambda/R \ll 1 \) the balance between charge production and wall losses is given by
\[
\nu R^2 D_a = K_1 \nu, \quad (39)
\]
where \( K_1 = 2.4048 \) is the first zero of \( F_0 \). Combination of Eqs. (36) and (37) gives a corresponding balance equation which leads to a "modified Schottky theory" applicable also to situations when \( \lambda/R \ll 1 \) (cf. Bickerton\(^\text{10}\) and Bickerton and von Engel\(^\text{11}\)). Assuming a Maxwellian distribution, von Engel and Steenbeck\(^\text{14}\) have deduced the number \( \nu \) of ion pairs produced per unit time and per electron:
\[
\nu = \frac{2 \nu}{\sqrt{\pi} e} \left( \frac{2\pi^2}{3kT}\right)^{1/2} \left( \frac{2kT_e}{m_1}\right)^{1/2} \quad (40)
\]
where \( \nu \) is the neutral gas pressure (in newtons/m²; 1 mm Hg = 132.8 n/m²), \( T_o \) the neutral gas temperature, \( V_1 \) the ionization potential and \( a \) is a constant with a numerical value 0.754 times that given by von Engel and Steenbeck when Eq. (40) is expressed in MKSA units. For helium, \( a = 0.0347 \) amp sec²/kg² m² and \( V_1 = 24.54 \) volts. The notations
\[
x = \nu V_1/kT_e, \quad x_1 = \nu V_1/kT_1, \quad (41)
\]
where \( \nu = \nu_0/\nu \), \( \nu_0 = \nu_0(T_o/273), \) \( \nu_1 = \nu_0(T_1/273) \) (42)
are introduced, where \( \nu_0 \) is a function of the electron and ion temperatures given by von Engel and Steenbeck\(^\text{14}\) and Maier-Leibnitz.\(^\text{19}\) Eq. (39) now becomes
\[
F(x) = A(Rp)^3 [L(x, x_1) + y^2 M(x, x_1)], \quad (44)
\]
where
\[
F(x) = x k_{x/2} (1+2/x), \quad (45)
\]
\[
A = 2(6/\pi)^{1/2} k_2 (273/T_o)^{1/2} V_1 (m_1/m_2)^{1/2} K_1^{-2}, \quad (46)
\]
\[
L(x, x_1) = \frac{x}{L_1(1+x/x_1)} \left[ 1 + \frac{L_1(m_1/x_1)}{L_1(x_1)} \right], \quad (47)
\]
\[
y = \omega_e/\nu, \quad (48)
\]
\[
M(x, x_1) = \frac{m_1 x_1^3}{k_2 V_1 (1+x/x_1)} \left[ 1 + \frac{m_1 x_1}{m_1 x_1} \right], \quad (49)
\]
With helium, \( A \) equals 31.7 sec²/kg when \( T_o = 300°K \).

Relation between Ion and Electron Temperatures

Von Engel and Steenbeck\(^\text{14}\) obtain the energies
\[
\frac{dA}{dt}_1 = \frac{A_k}{k_{x/2} (2kT_1)^{3/2}} \left( T_1 - T_o \right) \quad (50)
\]
and
\[
\frac{dA}{dt}_e = \frac{2m_e k_{x/2} (2kT_e)^{3/2}}{k_{x/2} (m_1/m_2)^{1/2}} \quad (51)
\]
which are transferred to the neutral gas by collisions with ions and electrons, respectively. In Eqs. (50) and (51) \( k_1 \) and \( k_e \) are the fractions of the total energy of a particle being lost in a collision. The energy is supplied by the electric field \( E \) which is parallel to the magnetic field \( B \). The energy supply is given by
\[
\frac{dA}{dt}_1 = ev_1 E \quad \text{and} \quad \frac{dA}{dt}_e = -ev_1 E \quad (52)
\]
both in the presence and in the absence of the magnetic field, and from Eqs. (11), (50), (51) and (52)
\[ T_0^* = (\lambda_0^2 k_1^2 \lambda_0^2 \kappa_0) T/t (T_1 - T_n), \]
or
\[ (x_1/x_2)^2 = [\Lambda_0 \lambda_1(x_1)/\Lambda_0 \lambda_1(x_2)]^2 (1 - x_1/x_2), \]
where
\[ x_n = eV_f/kT_n. \]

The temperature of the ions is not very much greater than that of the neutral gas, and ions may be assumed to lose their energy by elastic collisions, i.e., \( k_i \approx 0.5 \). For electrons, however, inelastic collisions will be taken into account and \( \kappa_0 \) becomes a function of \( T_e \) as shown below. The electron temperature is determined by the root \( x \) of Eqs. (44) to (49) and (53).

**LONGITUDINAL ELECTRIC FIELD IN A HELIUM DISCHARGE**

The total fractional loss of energy of an electron in a collision with a neutral gas particle is
\[ \kappa_0 = \kappa_{el} + \kappa_{ion} + \kappa_{exc} + \kappa_{wall}, \]
where \( \kappa_{el} = 2m_e/m_n \) is the energy loss from elastic collisions,
\[ \kappa_{ion} = (eV_f^2)/(h\bar{c} T_e) = x \bar{c}/v_e \]
is the ionization loss, where the rate \( x \) is given by Eq. (40) and
\[ \kappa_{exc} = \sum_s (eV_s^2)/(h\bar{c} T_e) = \sum_s \frac{V_s^2}{v_e} x \bar{c}/v_e \]
is the excitation loss, where \( V_s \) and \( \bar{c} \) are the excitation potential and excitation rate of the \( s \)th level. Since the electrons diffuse in a radial direction there is a wall loss as well. The plasma balance not only requires the ionization work to be done at the rate \( x \), but the electron which is produced must also be "heated" to the temperature \( T_e \). Since electrons of this temperature are lost to the walls per unit time and electron, the fractional wall loss becomes
\[ \kappa_{wall} = (h\bar{c} T_e^2)/(h\bar{c} T_e) = x \bar{c}/v_0 = \kappa_{ion}/x. \]

In order to calculate the excitation loss in helium the experimentally determined excitation cross section (Maier-Leibnitz\(^{19}\)) is used as a starting point. As shown by Fabrikant\(^{20}\) and Karelina\(^{21}\) a good approximation to the experimental results is given by an empirical expression for the total excitation path free cross section:
\[ \lambda_{exc} = \frac{273\phi}{132.8\lambda_n} \sum_s \frac{1}{\lambda_s} \frac{V_s - V_\gamma}{V_{ms} - V_s} \exp \frac{V_{ms} - V_s}{V_{ms} - V_\gamma}, \]
where \( V_\gamma \) is the potential corresponding to the particle energy. In this approximation two "levels", \( s = 1, 2 \), are used with \( \lambda_1 = 7.7 \times 10^{-3} \) m, \( \lambda_2 = 2.7 \times 10^{-3} \) m, \( V_\gamma = 19.25 \) volts, \( V_\gamma = 20.2 \) volts, \( V_{ms} = 20 \) volts and \( V_{ms} = 28 \) volts. If a Maxwellian distribution is assumed, the number of exciting collisions per unit time and electron becomes for "level" \( s \):
\[ \bar{c}_s = \frac{273 \phi}{T_n} \int_0^\infty \frac{4 \pi v^2}{\sqrt{\pi}} \frac{V_s - V_\gamma}{V_{ms} - V_\gamma} \times \exp \left[ \frac{V_{ms} - V_s}{V_{ms} - V_\gamma} \right] dv, \]
where \( v \) is the total velocity of an electron, \( w = (2kT_e/m_0)^{1/2} \) and \( \Lambda_s = 132.8 \lambda_s \). The total loss \( \kappa_0 \) is given by the integrated Eq. (59) and Eqs. (54) to (57), (60) and (40) to (43):
\[ \kappa_0 = \frac{2m_e}{w_n} + \frac{8}{3} \frac{\lambda_{exc}}{3 \pi \bar{c}} \Lambda_0 \bar{V}_e \left[ \frac{1 + (xV_s/2V_f)(1 + V_f/(x(V_{ms} - V_\gamma)))}{(1 + V_f/(x(V_{ms} V_f)))^3} \right] \exp \left[ -xV_s/\bar{V}_f \right]. \]
The result is shown in Fig. 1 and agrees fairly well with experimental results of Bickerton\(^{10}\) and Bickerton and von Engel.\(^{11}\)

Finally, the longitudinal electric field is obtained from Eqs. (51), (52) and (11) as a function of \( x \):
\[ E = (9k/\pi)^{1/2} \left( \frac{273\phi}{T_n} \right) V_f (\kappa_0)^{1/2} x \Lambda_0. \]
The factor in front of the right-hand member of Eq. (61) is somewhat uncertain because it is based upon
elementary kinetic methods in the calculation of the frictional coefficients in Eqs. (2) and (3). A more rigorous kinetic theory gives a factor $(64/\pi)^1$ in front of Eq. (61) instead of $(96/\pi)^1$ (von Engel and Steenbeck). However, this source of error should play a minor role in the relative magnitude of the electric field at varying magnetic field strengths.

Theoretical results for $R_p = \frac{\sqrt{n}}{\mu} = 1.46$ cm mm Hg and $4.52n/m = 3.40$ cm mm Hg with $T_n = 300^\circ$K are shown by the full curves in Figs. 3a and b, as calculated from Eqs. (44) to (49), (60) and (62). For the mean free path of ions defined by Eqs. (42) and (43) a value $\frac{d}{2}$ times that of neutral particles given by Kennard has been taken. A source of error may be introduced by this value but it does not change the position of the theoretical curve fundamentally; appreciable changes in the rate of diffusion are required to cause a noticeable change in the electron temperature since the production rate $\gamma$ is a sensitive function of $T_n$ according to Eq. (40). In Figs. 3a and b a change of the diffusion coefficient has also been simulated by substituting the function $M$ of Eq. (49) by $\gamma M$. The value $\gamma = 1$ corresponds to the actual situation, whereas $\gamma = 0.5, 0.1$ and 0.01 (broken lines) give diffusion coefficients which are about twice, ten and one hundred times larger than the present in a magnetic field, which is strong enough to satisfy the condition $\gamma^2 y M \gg L$. In the present measurements this is the case only for $\gamma \geq 0.1$ when the strong fields of the right-hand sides of Figs. 3a and $b$ are considered.

EXPERIMENT

Apparatus

The experimental arrangement is shown in Fig. 2. A discharge tube of inner radius $R = 1$ cm has been placed inside a magnetic coil of four meters length. In one of the runs being made (Fig. 3b) the magnetic field was varied up to $B = 0.53$ V sec/m² ($=5300$ gauss), corresponding to a power input of about $3 \times 10^5$ watts. Since every measurement required only a few seconds' time it was sufficient to cool the coil with fans. The tube ends with anode and cathode were both extended far outside of the coil. The diverging field at the coil ends, the large ratio between the tube length and tube radius, and the relatively high pressure used in the experiments ($\geq 1.46$ mm Hg) make impossible short-circuiting effects of the type discussed by Simon.

Measurements of the longitudinal electric field were made with an electrostatic voltmeter connected between two floating platinum wire probes, one meter apart and both far from the tube ends. The probes had the shape of circular sectors which followed the tube wall closely. Their connections were screened electrostatically all the way out to the voltmeter. The discharge current was closed through an inductance-free resistor over which the voltage could be examined with an oscilloscope and a wave analyser.

Before the measurements the electrodes were outgassed and the tube run with 180 ma current. Impurities in the helium discharge were easily detected with a small spectroscope. Narrow slits in the coil made observations possible over the entire length of the tube. It was found that the most rapid way to clean the tube of impurities was to place the cathode at the opposite side of the vapor trap filled with carbon.
(Fig. 2). With a strong current the discharge acted as an ion pump and the gradual disappearance of the impurities could be followed from the anode end to the cathode by means of the spectroscope. Measurements were not performed until the tube was observed to be clean over its entire length.

The pressure was measured with a McLeod gauge.

**Results**

Two series of electric field measurements were made at the pressures 1.47 and 3.26 mm Hg as shown by Figs. 3a and b. The results at zero magnetic field are given in Table 1 together with theoretical results calculated from Eqs. (44) to (47), (53) and (61), which are also compared to results by Kareh\(^2\) and by Klarfeld\(^2\) for \(R_p = 3.40\) cm mm Hg. The agreement between the experimental results of these authors and the present experiments is as good as can be expected. The present theoretical values of \(E/p\) for \(R_p = 1.47\) cm mm Hg is closer to the experimental than the value calculated by Kareh\(^2\), but there still remains a discrepancy which may be due partly to an erroneous factor in front of the expression for the mobility.

Figures 4a and b show the corresponding measurements at \(R_p = 1.46\) and 3.40 cm mm Hg of the relative magnitude \(\theta\) of the noise voltage over the resistor in the discharge circuit. The measurements have been made with a discharge current of 160 mA and at 5 × 10\(^3\) and 10\(^4\) cycles per second with the wave analyser adjusted to a band width of 145 cycles per second. Measurements at lower frequencies (10\(^3\) c/sec) were difficult to interpret; even in the absence of a magnetic field there was a strong and rapidly changing noise voltage. It is seen from Figs. 3 and 4 that the experimental points form a "knee", which indicates that the state of the discharge is changing at a magnetic field \(B_0 \approx 0.23\) and 0.27 v sec/m\(^2\) for \(R_p \approx 1.46\) and 3.30 cm mm Hg, respectively.

**DISCUSSION**

**Experimental Conditions**

In the absence of the magnetic field and at the pressures of the present experiments the electron temperature is about 4 × 10\(^4\) K and the ion temperature about 1000° K. From Eq. (38) the values \(\lambda/R \approx 0.06\) and 0.03 are obtained at \(R_p = 1.46\) and 3.30 cm mm Hg when \(T_i'\) is put equal to \(T_i/2\) according to Bohm.\(^1\) With \(T_i' \approx T_i\) according to Bickerton and von Engel\(^1\) the values \(\lambda/R \approx 0.27\) and 0.14 are obtained, which can be regarded as upper limits since the stability of the wall sheath requires \(T_i' \approx T_{e}/2\). Even with \(\lambda/R = 0.3\) the change in the theoretical value of the electron temperature is only a few per cent.

At the lowest pressure and current density used in the experiments the charge density is calculated from Eq. (33) to 4 × 10\(^16\) m\(^{-3}\), corresponding to the Debye distances \(h_0 = 1.4 \times 10^{-4} \text{ m}\) and \(h_1 = 3 \times 10^{-5} \text{ m}\) as given by Eq. (32). Equation (31) gives \(n'/n < 0.2\) at a distance greater than 1 mm (= 0.1\(R\)) from the wall. Consequently, the quasi-neutral approximation is valid throughout the major part of the tube. Conditions are improved with increasing current density and pressure; for \(I = 0.16\) amp and \(R_p = 4.33\) n/m the results are \(h_0 = 1.0 \times 10^{-5} \text{ m}\) and \(n'/n < 1.0 \times 10^{-3}\) at \(r = 0.1R\).

Finally, the smallest radius of gyration of the ions in the present experiments has been 2.8 × 10\(^{-4}\) m (at \(B = 0.53\) v sec/m\(^2\)), which is greater than the largest occurring Debye distance \(h_0 = 1.4 \times 10^{-4}\) m.

From these considerations it is seen that the basic

![Figure 4a](attachment:image1.png)

**Figure 4a.** The ratio \(\theta\) between the noise voltage in the presence of the magnetic field and the voltage in the absence of the same field. (a) \(R_p = 1.94\) cm mm Hg = 1.46 cm mm Hg. (b) \(R_p = 4.52\) cm mm Hg = 3.40 cm mm Hg.

![Figure 4b](attachment:image2.png)

**Figure 4b.** The noise voltage in the presence of the magnetic field and the voltage in the absence of the same field for \(R_p = 1.47\) and 3.26 cm mm Hg.

### Table 1. Values of \(E/p\) in volts/cm mm Hg

<table>
<thead>
<tr>
<th>(R_p) (cm mm Hg)</th>
<th>(I) (mA)</th>
<th>(E/p)</th>
<th>(\text{Present results})</th>
<th>(\text{Earlier results})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.47</td>
<td>20</td>
<td>3.66</td>
<td>—</td>
<td>3.66</td>
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<td></td>
<td>40</td>
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<td>80</td>
<td>3.94</td>
<td>5.61</td>
<td>—</td>
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<td>3.4</td>
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</tr>
<tr>
<td></td>
<td>160</td>
<td>—</td>
<td>3.23</td>
<td>—</td>
</tr>
<tr>
<td>3.40</td>
<td>60</td>
<td>1.75</td>
<td>—</td>
<td>1.75</td>
</tr>
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<td></td>
<td>160</td>
<td>1.71</td>
<td>3.20</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
conditions underlying the theory given earlier have their correspondence in the present experiments.

Conclusions

The left-hand parts of the theoretical curves in Figs. 3a and b show a satisfactory agreement between theory and experiment; the small discrepancies may be due to an error in the mean free paths used in the calculations. An increase in the diffusion coefficient by a factor of ten ($\gamma = 0.1$) or more is, in any case, too large not to be distinguished from the present results. Consequently, the left-hand parts of Figs. 3a and b show that the diffusion is taking place according to the binary collision theory and that a strong reduction in the diffusion coefficient is caused by the magnetic field which traps the particles effectively in its transverse direction. There seems to be no difference between the measurements at varying current densities and from this it is concluded that deviations from electrical neutrality as well as two-stage processes and recombination do not influence the results noticeably. Correspondingly, the left-hand parts of Figs. 4a and b show no increase in the noise level caused by the magnetic field.

However, when the magnetic field reaches a certain critical value which is about $B_c = 0.23$ and 0.27 v-sec/m$^2$ at $R_p = 1.94$ and 4.52 m/\(\text{msec}\) the diffusion coefficient starts to increase rapidly and the noise level is suddenly increased at the same time. The deviations between the binary collision theory and the experiments are considerable in this region as shown by the right-hand parts of Figs. 3a and b. In Fig. 3a the diffusion coefficient exceeds its value due to the collision theory by more than a factor of ten when the magnetic field exceeds about 0.3 v-sec/m$^2$ and in Fig. 3b this occurs at a field strength of about 0.4 v-sec/m$^2$. These results support strongly the existence of the “drain” diffusion mechanism suggested by Bohm, Burhop, Massey and Williams.4 It should also be observed that the right-hand part of Fig. 3a clearly indicates an increase in the diffusion rate at increasing current densities, i.e., at increasing charge densities, $n$. This also speaks in favour of such a mechanism which is based upon the effect of local electric fields caused by deviations from electric neutrality. The values of $\theta$ somewhat above unity at strong fields in Fig. 3b are not unimaginable if the same mechanism is operative, i.e., the diffusion may even be accelerated by the magnetic field. Whether the oscillations consist only of a broad spectrum of “electromagnetic turbulence” or include plasma oscillations concentrated around some special frequencies cannot be judged at this stage and requires further investigation. It should be pointed out that noise and oscillations in the presence of a magnetic field have also been observed by Åström24 and Webster25 in an electron gas and by Block26 in model experiments on the auroral discharge.

Finally, the conclusions drawn here are also consistent with the results obtained by Bostick and Levine27 for a toroidal discharge in a toroidal magnetic field. These authors find a considerable decrease in the diffusion time as well as the presence of oscillations in a limited range of magnetic field strengths. However, as pointed out by Bostick and Levine27 and later by Biermann and Schlüter28 and Lehnert,29 particle losses are also caused by the gradient in a toroidal magnetic field and no definite conclusions can be drawn about the magnitude of the diffusion coefficient in this experiment.

The diffusion caused by oscillations and “electromagnetic turbulence” provides a mechanism for ionized matter to slip across a magnetic field. When this mechanism is acting, modifications of the present theories of an ionized gas have to be undertaken, both in astrophysics when the motions of magnetic fields and matter are considered, and in the thermonuclear problem when the confinement of a hot gas in a magnetic field is discussed. It is desirable to extend the present investigations to a fully ionized gas in stronger magnetic fields than those being used here.

ACKNOWLEDGEMENTS

The author is indebted to Mr. Bertil Andersson for building the apparatus for this experiment and for valuable assistance during the measurements. A grant from Statens Naturvetenskapliga Forskningsråd for covering the costs of the experimental equipment is gratefully acknowledged.

REFERENCES

Spectroscopic Study of High-Temperature Plasma

By S. Yu. Lukyanov and V. I. Sinitsin*

The subject of investigation of physicists in their attempts to realize a controlled thermonuclear reaction is the most abundant substance in nature and yet the substance that has been studied least—completely ionized plasma. This is the state of matter in stellar interiors where reactions of synthesis proceed under natural conditions. It is therefore not surprising that, since the very first stages in the development of controlled thermonuclear reaction problems, repeated attempts were made to utilize in plasma studies the variety of spectroscopic methods of investigation so well tested in astrophysics. One of the chief advantages of these methods is their negligible, if any, effect on the subject investigated during the optical measurements. Their versatility, the possibility of electronic registration, and their complete comprehensiveness open up truly unlimited possibilities for application both in the investigation of pulse and quasi-stationary processes and in the study of the plasma state in magnetic traps of all forms and types. Indeed, practically all the information that we possess to date concerning the physical conditions in the stars has been obtained using spectroscopic techniques. Furthermore, whereas studies of the short-wave portion of solar radiation require complicated apparatus to be sent outside the earth’s atmosphere, the investigation of ultraviolet and soft X radiation of high-temperature plasma obtained in the laboratory can be accomplished by far more simple means.

Spectroscopic techniques make it possible to determine the basic plasma characteristics \( (T_e, T_i, n) \) for any element of space at any moment, throughout the time of plasma existence. Naturally the reliability and accuracy of the results obtained may turn out to be quite different, depending on the specific conditions of the experiment, and at present the problem of the spectroscopic analysis of plasma in both its experimental and its theoretical aspects cannot yet be considered completely solved.

In this paper we give a description of the application of some spectroscopic methods to studies of heated plasma obtained in pulse discharges.

The rapid alternation of absolutely different states of the plasma in pulse discharges makes it necessary to use a continuous time registration of any of the plasma optical characteristics (intensity or contour of spectral lines, the form of the radiating volume, etc.). However, the main attention in such experiments has always been directed to two characteristic moments in the life of the discharge, namely, the moments of first and second contraction of the plasma column. These are the moments when the discharge plasma reaches the extreme conditions—it is just then that hard X radiation emerges in light-gas discharges, while in deuterium plasma neutrons appear in addition. These unexpected effects discovered in pulse discharges are usually observed at the instant of the second contraction. Naturally, investigators have been interested in them; the plasma state at the first contraction is no less interesting, and a considerable part of the paper will be devoted to a detailed description of this phase of the discharge.

METHOD

We will consider briefly the methods used in this work for determining plasma parameters. To estimate the electron temperature, the energy distribution in the continuous spectrum of the plasma has been studied and the density of charged particles has been determined from its absolute intensity; and, finally, to evaluate the ion temperature, the Doppler broadening in spectral lines of artificially introduced impurities has been measured.

As we know, with increasing ionization the lines of the hydrogen spectrum should gradually broaden more and more, and at last, at full ionization (which naturally takes place at high temperature), the line spectrum vanishes by merging into the continuous radiation. There exist several different possible mechanisms that result in the appearance of a continuum: bremsstrahlung (free–free transitions) recombination in the ground or excited levels of the neutral atom (free–bound transitions), and recombination with the formation of negative ions. The last mechanism may be important only at relatively low temperatures and high densities of charged particles and is highly improbable under the conditions in which we are interested. The values of spectral density of bremsstrahlung and recombination radiation of hydrogen plasma in the energy scale may be represented as follows:

\[
I_1(v, T_e) = a_1 n^2 T_e^{3/2} L \exp\left(-\frac{h v}{T_e}\right)
\]

\[
I_2(v, T_e) = a_2 (n^2/m^2) T_e^{3/2} \exp\left[-\frac{h (v - v_1)}{T_e}\right].
\]

Original language: Russian.

* Academy of Sciences of the USSR, Moscow.
Here \( n = n_0 \approx n_e \) is the density of charged particles, \( T_e \) is the electron temperature, \( m \) is the principal quantum number of the level onto which the recombination occurs, \( h\nu \) is the energy of this level, \( \alpha_1 \) and \( \alpha_2 \) are combinations of universal constants, and \( L \) is a correction factor which increases logarithmically when \( T_e \gg h\nu \).

Let us consider a radiating volume of hot plasma \( V \). The total radiation of this volume per energy interval \( (\epsilon, \epsilon + \Delta \epsilon) \) in the visible region of the spectrum (recombination on level \( m = 3 \)) intercepted by the recording instrument can be calculated from the relation

\[
I(\nu, T_e) = KV n^2 \exp(-h\nu/T_e) \times \left[ \alpha_1 T_e^{-1}L + (\alpha_2/27) T_e^{-4} \exp(h\nu/T_e) \right] \Delta \epsilon, \tag{2}
\]

where \( K \) is a geometric factor. The energy distribution in the visible part of the spectrum at low temperatures is completely determined by the exponential factor \(-\exp(-h\nu/T_e)\), and if this distribution is obtained in relative units at any fixed instant of time when values of \( h \) and \( T_e \) are known, the slope of the straight line

\[ \ln \left( \frac{I}{I_0} \right) = f(h\nu) \]

gives the electron temperature of the plasma at this moment.

Unfortunately, if the electron temperature is rather high, the method is effective only when measurements are carried out over a very broad spectral interval. To be more exact, the interval of values \( \nu \) must be of the same order as \( T_e \). This is the reason why the measurements over the visible and near-ultraviolet region allow for the rough estimation of the electron temperature. However, it should be pointed out that even extremely approximate data of the electron temperature do not prevent the rather pointed out that even extremely approximate data of the electron temperature. Indeed, as follows from the calculations, the electron temperature varies very slowly in the visible region.

This is the reason why the electron temperature does not prevent the rather accurate determination of the density of charged particles. Indeed, as follows from the calculations, the electron temperature, varies very slowly in the visible region of the spectrum for values \( T_e > 10 \) ev. To illustrate this, Table 1 shows several values of \( \phi(T_e) \) when

\[
\phi(T_e) = \exp(-h\nu/T_e) \times \left[ \alpha_1 T_e^{-1}L + (\alpha_2/27) T_e^{-4} \exp(h\nu/T_e) \right], \tag{3}
\]

appearing in (2), which is a function of the electron temperature, varies very slowly in the visible region of the spectrum for values \( T_e > 10 \) ev. To illustrate this, Table 1 shows several values of \( \phi(T_e) \) when

\[
h\nu = 2.48 \, \text{ev} \quad (\lambda = 5000 \, \text{Å}).\]

As may be seen from these figures, a thirtyfold variation of \( T_e \) produces only a 20% change in the value of \( \phi(T_e) \), and hence \( n \). Thus, if the intensity of the continuum in a specific spectral interval is measured in absolute units, and if the electron temperature is estimated and the value of the radiating volume is known, the density of charged particles may be calculated from Eq. (2).

The dimensions of the radiating volume have been determined in independent experiments by means of streak photographs of the discharge channel in the spectrally dispersed light of the continuum. As may be shown by appropriate calculations, processes of self-absorption in the region of density and temperature under consideration may be ignored (with the geometry used). A direct experimental proof of the validity of this assumption will be given below.

As has already been mentioned, the line spectrum in a highly heated hydrogen plasma vanishes and, therefore, in order to determine the ion temperature from the Doppler broadening of the spectral lines it is necessary to introduce a proper impurity into the discharge space. The linear Stark effect in lines of high temperatures attained in plasma. Indeed, the next light gas is nitrogen; it should be taken into account that the Doppler broadening is inversely proportional to the square root of the molecular weight. That is why we selected the lines of the bright triplet \( 3^3S-3^3P \), which is very convenient for analysis. The most intense line in this triplet is that of \( \text{N}^\text{IV} \) with \( \lambda = 3479 \, \text{Å} \). We would like to note, by the way, that this same line was used as a nitrogen thermometer in the recent British studies on high-temperature plasma.

Among many causes resulting in a broadening of spectral lines (Doppler effect, Holtzmark broadening, Zeeman effect), in this particular case the chief role is played by Doppler broadening, corresponding to high temperatures attained in plasma. Indeed, Zeeman splitting under the action of the magnetic field associated with the discharge current (flowing through the plasma in the central zone) cannot exceed 0.05 Å.

Broadening of the \( \text{N}^\text{IV} \) lines caused by molecular fields is due to the quadratic Stark effect. However, in this case besides broadening a shift of the line maximum should be observed. The shift was not observed within the limits of experimental accuracy (0.07 Å), and this may be regarded as confirmation of the fact that the quadratic effect is small under the conditions of the experiment. The data on the values of the Stark constants for the \( \text{N}^\text{IV} \) line are not available and no calculations are given. On the other hand, the Stark broadening should increase with the density of the charged particles. This is not observed experimentally. Finally, a convincing proof of the fundamental role of Doppler broadening is the coincidence of the experimental profile of the line with the Gaussian curve within the limits of 3%.

**APPARATUS**

The object of investigation was a small quantity (about 3/4 litre) of high-temperature plasma obtained by an intense discharge in hydrogen at the instant of its maximum contraction. The discharge was produced in a circuit (described many times in the literature; see, for example, Ref. 1) with the following parameters:

<table>
<thead>
<tr>
<th>( T_e )</th>
<th>( \phi(T_e) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4.22 × 10^{-26}</td>
</tr>
<tr>
<td>30</td>
<td>4.10 × 10^{-26}</td>
</tr>
<tr>
<td>100</td>
<td>3.80 × 10^{-26}</td>
</tr>
<tr>
<td>300</td>
<td>3.08 × 10^{-26}</td>
</tr>
</tbody>
</table>

\( h\nu = 2.48 \, \text{ev} \quad (\lambda = 5000 \, \text{Å}).\)
The discharge chamber tube was 40 cm diameter ($D$) and 90 cm long ($L$). The first contraction of the discharge channel occurred about 4 $\mu$s after the breakdown.

The values of intensity of the continuous background necessary for estimating $T_e$ and the subsequent determination of $n$ were measured photoelectrically. The monochromator with the recording element—a FEU-12 photomultiplier—was installed at a considerable distance from the discharge chamber (up to 20 metres) to ensure reliable collimation of the light beam from the selected radiating volume. The apparatus made it possible to reproduce on the screen of a double-beam oscillograph the time variation of the continuous background intensity in any selected wavelength interval. From a preliminary calibration using the radiation of incandescent tungsten the absolute value of the intensity of the continuum emitted by a definite volume of the discharge plasma was determined.

As may be seen from Eq. (2), to calculate the concentration from the measured value of the continuous background intensity $I$, it is necessary to have the exact value of the radiating volume $V$. The apparatus described below made it possible to obtain streak photographs of the luminous discharge channel in a narrow spectral range.

The ion temperature at the instant of the maximum contraction (from the Doppler broadening of the lines of the artificially introduced impurities) was determined by means of the streak photographs of the spectrum.

The optical system used was essentially the same as that described in the papers published previously (see, for example, Refs. 1 and 2). In this work an ISP-28 quartz spectrograph was used as the spectral instrument. The optical system described in Ref. 1 was slightly modified; quartz objectives made it possible to carry out the measurements over the ultraviolet region down to 2200 Å. We studied the light emitted along the axis of the discharge column, and thus any effect of collective radial motion of the plasma particles on Doppler broadening was excluded. Figure 2 shows a general view of the apparatus designed for observation of the discharge spectrum over a period of time.

RESULTS AND DISCUSSION

Figure 3 shows streak photographs of the discharge spectra for two characteristic cases. Spectrogram 3(a) corresponds to a discharge in pure hydrogen with initial pressure $p_0 = 0.1$ mm Hg. Spectrogram 3(b)
gives the spectrum of a discharge in a mixture of hydrogen and nitrogen (95% D₂ + 5% N₂). As has already been pointed out, at the instant of the maximum contraction a burst of the continuum is observed on the spectrogram. Spectrogram (b) shows the continuum and the lines of highly ionized nitrogen, which appear and exist only at the moment of the highest contraction, that is, when the electron temperature reaches its maximum value. In the visible part of the spectrum there are, for example, several N\textsuperscript{v} lines greatly broadened probably because of the linear Stark effect. The nitrogen triplet 3 \textsuperscript{3}S-3 \textsuperscript{3}P is the most prominent in the ultraviolet region. Table 2 contains a list of the brightest nitrogen lines observed. The wavelengths of some of them are given on the spectrogram. It should be stressed that the excitation energies given in Table 2 are taken from the ground level of the corresponding ion.

The appearance of these lines may be regarded as an indication of the sharp rise in electron temperature at the instant of maximum contraction of the plasma column. As has already been mentioned, a low limit of T\textsubscript{e} may be obtained from an analysis of the energy distribution in a continuous spectrum. Figure 4 shows on a logarithmic scale the values of the relative intensity of the continuum as a function of the photon energy. Experimental errors due to the heterochromatic character of the measurements and the narrow spectral region investigated do not permit one to obtain reliable data for the estimation of the electron temperature. The cross-hatched area shows the region over which the experimental points are spread. It may be seen from Fig. 4 that the electron temperature in all cases is at least 10 ev.

We proceed now to the results of the determination of the charged particle density. The dimensions of the radiating volume were determined from the pinch streak photographs obtained by the method of high-speed photography in spectrally dispersed light. As an example, a few of these photographs are shown in Fig. 5. The region of the spectral transparency of the apparatus is indicated nearby. As may be seen from Fig. 5 the lifetime of heated plasma which emits continuous radiation is equal to approximately 0.5 \mu sec at \rho\textsubscript{0} = 0.05 mm Hg, and 0.7 \mu sec at \rho\textsubscript{0} = 0.1 mm Hg.

A measurement of the diameter d of pinched plasma leads to values of d \approx 35-40 mm. The photographs obtained are in complete agreement with the results of spectral scanning, thus showing that the hot plasma emitting continuous radiation exists only at the instant of the maximum contraction. The photograph of a discharge in light of the H\textalpha line demonstrates the behaviour of the plasma prior to contraction.

It should be pointed out that the diameter of the heated plasma column was also determined independently by a photoelectric method. Narrow beams of light, the intensity of which was registered as a function of time, were collimated by a number of special diaphragms (\phi 4 mm) placed at different distances from the axis of the discharge. From the oscillograms obtained, a curve was synthesized that gave the intensity distribution of luminosity along the radius of the plasma column at any fixed instant of time. Figure 6 shows such a curve corresponding to the moment of the first contraction. The slanting lines mark the borders of the light outlet made in the electrode for optical measurements.
Figure 5. Streak photographs, expanded in time, of the discharge column in spectrally dispersed light:
(a) $p_0 = 0.1$ mm Hg, (b) $p_0 = 0.05$ mm Hg
Table 3

<table>
<thead>
<tr>
<th>$T_e$ (eV)</th>
<th>$n_{abs}$ (cm$^{-3}$)</th>
<th>$n/n_0$</th>
</tr>
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<tbody>
<tr>
<td>10</td>
<td>$1.13 \times 10^{17}$</td>
<td>32</td>
</tr>
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<td>30</td>
<td>$1.15 \times 10^{17}$</td>
<td>33</td>
</tr>
<tr>
<td>100</td>
<td>$1.20 \times 10^{17}$</td>
<td>34</td>
</tr>
<tr>
<td>300</td>
<td>$1.33 \times 10^{17}$</td>
<td>38</td>
</tr>
</tbody>
</table>

Naturally, the value $T_e = 300$ eV cannot be realized under the conditions investigated, and is included in the table only to illustrate once again the weak dependence of $n$ on $T_e$. The physical significance of the ratio $n/n_0$ is evident. Assuming 100% ionization this represents the degree of contraction. It is worth while noting that the experimental data are in good agreement with the theoretical calculations (see, for example, Ref. 3).

Figure 8 gives the dependence of the square root of the continuous radiation intensity in arbitrary units on the initial pressure. As may be seen from this figure the experimental points (at low pressures) make a good fit with the linear relation. The self-absorption should have led to a relative reduction in the intensity of the continuum in the region of high initial pressures. But this is not the case in our experiment: on the contrary, as the pressure increases, the experimental points are displaced towards higher intensities, i.e., to higher values of $n$. Perhaps this curious fact may be interpreted as an indication that the enhanced role of free-bound transitions or even recombination with the negative ion formation should be taken into account.

The correctness of measurements of the quantity $n$ was confirmed in independent experiments in which the continuum was observed across the pinch. In this case, the low value of the optical depth completely eliminated the possibility of self-absorption. Agreement between these two series of measurements is within the limits of experimental accuracy (15%).

Now we will turn to the estimation of the ion temperature in the plasma. Figure 9 shows a photograph and a photomicrograph of the nitrogen triplet $3\,^3S_3\rightarrow 3\,^3P$ at the instant of maximum contraction at $p_0 = 0.05$ mm Hg + 5% addition of nitrogen. Figure 10
show a Gaussian contour with the experimental points for the N IV line at 3479 Å. In determining the experimental points the "instrument width" of the spectrograph, dependent on the finite dimensions of the slit, was taken into account. After introducing the correction for the width of the diffraction maximum according to Bruck and Minkowsky the half-width obtained for the line contour leads to values of ion temperature close to $1.2 \times 10^6$°K.

During the intense interaction of electrons and ions at the instant of the maximum contraction of the plasma a rather good equalization of electron and ion temperatures might occur. In this case, the following physical conditions are realized in the plasma: $T_i \approx 10^6$°K; $n \approx 1.2 \times 10^{17}$ cm$^{-3}$; degree of contraction $\approx 35$.

In conclusion, we would like to note that the actual value of $T_i$ may be less than indicated above; this would be the case if at the moment of the first contraction the bulk of the particles take part in the collective motion along the axis. This assumption seems to be rather artificial. On the other hand, the actual value of the temperature may be slightly higher for two reasons. First, the addition of 5% of nitrogen results in a 35% increase in the mass of the gas, and reduces the kinetic temperature of the contracting plasma. The second effect is of purely instrumental character. The resolving time of spectral scanning slightly exceeds the period during which the maximum temperature in the column is sustained. For this reason, the experimental contour is formed as a result of the superposition of instantaneous contours (which correspond to a slightly lower temperature), and its half-width is reduced. Ways of overcoming the above difficulties are clear to the authors and they hope to return again to this problem.

REFERENCES
5. See, for example, S. Tolansky, High-resolving-power Spectroscopy, p. 349. Izdavatelstvo Inostrannoi Literatury 1955.
Diagnostic Techniques used in Controlled Thermonuclear Research at Harwell


Experiments at the Atomic Energy Research Establishment, Harwell, on the production of high temperatures for thermonuclear research have been mainly along the lines described by Thonemann¹ and Pease et al.,² in which gas is heated by the discharge of a unidirectional electric current pulse and confined by the pinch effect. The discharge tube is toroidal in shape and the gas forms the short-circuited secondary winding of a pulse transformer.

Diagnostic measurements in these experiments aim at obtaining as complete a picture as possible of the physical conditions existing in the gas, but because of the extreme complexity of such systems, and because they are inhomogeneous and varying rapidly in time, the measurements which are possible are limited mainly to such properties as temperature, density, electric and magnetic fields, resistivity, impurity content and degree of ionization. Ideally these should be measured as a function of both position and time, but it is frequently possible only to observe the variation with one of these parameters, or to obtain average values.

In addition, information is required on the energy loss from the gas by radiation and conduction, and on nuclear reactions, collisions between electrons and ions, acceleration of ions and electrons to high energies and co-operative phenomena.³

This paper is a general survey of the lines along which diagnostic methods are developing at Harwell, and indicates the results which have been obtained so far. Many of the techniques are in a rudimentary state, and most of the Zeta experiments are incomplete.

Diagnostic techniques used at Harwell may be divided broadly into three groups: spectroscopy, electrical measurements, and the study of high energy radiations. The first includes the measurement of electromagnetic radiations over a very wide range of frequencies, falling into the subdivisions, microwave, visible, quartz ultra-violet, vacuum ultra-violet, and X-ray regions. The intermediate regions mentioned above give rise to special technical difficulties. However, it is expected that they will give valuable information when they are investigated.

SPECTROSCOPY

Visible, Ultra-violet and Vacuum Ultra-violet Spectra

Up to the present, only emission spectra of the discharge have been studied. Both photographic and photoelectric recording is used.

Apparatus

For photographic work, the spectrographs used are Hilger medium glass (f/12), Hilger medium quartz (f/12, 10–20 Å/mm), and a 1-metre normal incidence concave grating vacuum instrument (f/18, 16 Å/mm) covering the range 300–2900 Å. Visible and quartz ultra-violet radiation is observed through a quartz window covering a transverse slit in the torus. The light undergoes three reflections from surface-aluminized mirrors in order to reach the spectrographs. An electromechanically operated shutter synchronized with the discharge allows the exposure to be made during any required part of the current pulse. For the majority of exposures with Zeta it was set so that a period of 1.0 ± 0.1 ms, centered at current maximum within 0.1 ms, was accepted.

The vacuum spectrograph is mounted directly on the torus by means of a vacuum-tight coupling, no solid material being interposed in the light path. The optical slit of the spectrograph forms the only path between torus and spectrograph, and a pressure of 10⁻⁵ mm Hg is maintained in the spectrograph by differential pumping. In addition, two Fastie-Ebert grating monochromators of focal lengths 24 in. (aperture f/4, dispersion 22 Å/mm) and 3 metres (aperture f/30, dispersion 2.7 Å/mm) with photomultiplier detection are used for examining the intensity of single lines as a function of time in the visible and quartz ultra-violet regions.

Measurements

The spectroscopic work on Zeta has been directed primarily towards: (i) determination of impurities in the discharge; (ii) measurement of ion temperature from the Doppler broadening of spectral lines; (iii) estimation of electron temperature.

Impurities

The identification of impurities follows standard procedure. In a discharge in nominally pure deuterium,
strong lines of carbon, nitrogen, oxygen, aluminium and fluorine are observed. These are believed to originate from residual gases, gases occluded in the walls and driven out by the discharge, pump oil, and materials used in the construction of the tube. An interesting feature of the spectra observed with ZETA, particularly at high gas currents, is the appearance of lines of highly ionized atoms, for example N V, C V, O VI. An estimate of the amount of impurity present has been made by comparing the intensities of equivalent lines of isoelectronic sequences of C, N, O and F, and lines for which the transition probabilities are known, on a plate taken when the discharge was in deuterium under “clean” conditions with a known nitrogen impurity of 5% added. Assuming that the nitrogen content remained constant at 5%, the estimated percentages of impurities present are:

<table>
<thead>
<tr>
<th>Element</th>
<th>Estimated Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon</td>
<td>5%</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>5% (assumed)</td>
</tr>
<tr>
<td>Oxygen</td>
<td>1%</td>
</tr>
<tr>
<td>Fluorine</td>
<td>10%</td>
</tr>
<tr>
<td>Aluminium</td>
<td>8%</td>
</tr>
<tr>
<td>Silicon</td>
<td>2%</td>
</tr>
</tbody>
</table>

These figures are estimates only, and may be in error by an order of magnitude. Typical spectrograms obtained in the quartz ultra-violet and vacuum ultra-violet regions are shown in Figs. 1 and 2.

**Ion temperature**

By “temperature” is meant the temperature corresponding to the mean kinetic energy of the ions, at a given position and time. Most of the actual measurements give a value averaged over both space and time, but in some cases limited resolution was
obtained. Ion and electron temperatures are not necessarily equal.

A measurement of the mean kinetic energy of the deuterium atoms by observing Doppler broadening of the spectral lines emitted by them is not possible at the highest temperature in Zeta because all deuterium atoms are ionized and emit no light. However, such measurements are possible on impurity ions such as oxygen, nitrogen and carbon which can exist at this temperature in highly ionized states. The possibility of deducing the temperature of the deuterium ions from the temperature of these impurity ions is discussed in the accompanying paper by Pease.2

A large number of lines from highly ionized atoms were observed to be considerably broadened on photographic plates taken at high currents, in both the quartz and vacuum ultra-violet regions. A list of some of these lines is given in Table 1.4

Table 1. List of Broadened Impurity Lines

<table>
<thead>
<tr>
<th>Ion</th>
<th>Wavelength, Å</th>
<th>Number of lines</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>NIV</td>
<td>3479-3485</td>
<td>3</td>
<td>3s3P→3p3P</td>
</tr>
<tr>
<td>OIV</td>
<td>3381-3389</td>
<td>2</td>
<td>3s2P→3p2D</td>
</tr>
<tr>
<td>FIV</td>
<td>3114-3123</td>
<td>4</td>
<td>3s3P→3p3P</td>
</tr>
<tr>
<td>OIV</td>
<td>3063-3072</td>
<td>2</td>
<td>3s3P→3p3P</td>
</tr>
<tr>
<td>Ov</td>
<td>2781-2790</td>
<td>3</td>
<td>3s3P→3p3P</td>
</tr>
<tr>
<td>Cv</td>
<td>2271-2278</td>
<td>3</td>
<td>2s3S→2p3P</td>
</tr>
<tr>
<td>IV</td>
<td>1548-1551</td>
<td>2</td>
<td>2s2F→2p3P</td>
</tr>
<tr>
<td>Nv</td>
<td>1238-1242</td>
<td>2</td>
<td>2s3P→2p3P</td>
</tr>
<tr>
<td>Ovi</td>
<td>1032-1038</td>
<td>2</td>
<td>2s2P→2p1D</td>
</tr>
<tr>
<td>NIV</td>
<td>1371</td>
<td>1</td>
<td>2s3S→2p3P</td>
</tr>
<tr>
<td>Ov</td>
<td>765</td>
<td>1</td>
<td>2s2S→2p1P</td>
</tr>
<tr>
<td>Ov</td>
<td>630</td>
<td>1</td>
<td>2s2S→2p1P</td>
</tr>
</tbody>
</table>

The line breadths were measured with a microdensitometer and (in the quartz ultra-violet) converted into intensity distributions using the plate characteristic given by a subsidiary exposure on the same plate. The procedure has yet been found. The calibration exposure was made using a 7-step rhodium-on-silica neutral filter. For the vacuum ultra-violet region no satisfactory calibration procedure has yet been found. The calibration exposure was made with a steady light source: a separate instrument lamp) and that the true profile of the spectrograph is a dispersion function on the assumption that the instrumental half-width of the spectrograph (0.25 Å) on the assumption that the instrumental profile of the spectrograph is a dispersion function with the experimentally determined half-width (obtained with a Hg198 isotope lamp) and that the true line profile is Gaussian.

Effects other than Doppler broadening may contribute to the width of the spectral line. The most important are pressure broadening and Zeeman effect. The main contribution to pressure broadening comes from Stark broadening by electron impact. No experimental data are available on the Stark effect of the highly ionized atoms under consideration but a calculation using hydrogen-like wave functions, on the basis of the Lindholm theory4 has shown that the effect is very small for the transitions considered. For the line Ov 2781 the estimated broadening by electron impact is only $10^{-3}$ Å, compared with the observed maximum width of about 1 Å. Further evidence for the absence of Stark broadening comes from the failure to observe any shift in wavelength greater than 0.02 Å as the gas current is increased, compared with an observed broadening of about 1 Å.

Figure 3. Part of ZETA spectrum in quartz ultraviolet showing Doppler broadening of lines: OV 2781 (3s3p→3p3P); OV 2787 (3s3p→3p3P)

Figure 4. Variation, through ZETA current pulse, of (upper) Doppler broadening interpreted as ion temperature and (lower) gas current

Gas, $\mu D_2 + 5\% N_2$; current, 180 ka; voltage, 23 kv; $B_0 = 160$ gauss; NIV $\lambda 3478.7$ Å; gate width ~1 millisecond

$\lambda = \lambda 2781$ press $\frac{1}{6} \mu D_2$

$100$ ka

$160$ ka
The Zeeman effect produced by magnetic fields of the order of 1000 gauss (as indicated by magnetic probe measurements in ZETA; see below) would produce a broadening of about $10^{-2}$ Å. The contribution to Doppler broadening from mass motion of the plasma is not known. Experiments designed to reveal mass motion spectroscopically have given ambiguous results and no reliance can be placed on them. While magnetic and Langmuir probes have so far failed to indicate mass motion sufficient to contribute appreciably to the Doppler broadening, it must be noted that signals at frequencies higher than about 250 kc/s would not have been observed. It has not yet proved possible to obtain data on mass motion from streak photographs. Experiments are in hand to determine the mass-motion contribution. The methods to be used are (i) measurement of Doppler broadening of lines from atoms of different masses present simultaneously in the discharge; (ii) Doppler broadening measurements on light leaving the discharge in different directions.

Electron temperature

Because the plasma in Zeta is not in radiative equilibrium, it is not possible to use the Boltzmann and Saha equations for determining electron temperature from the relative intensities of spectral lines. However, electron temperatures can be estimated from spectroscopic data in two ways: (i) from consideration of intensities and ionization potentials of lines observed; (ii) from relative line intensities, considering the processes of ionization, excitation and recombination in detail. Because of a lack of data on cross sections and excitation functions it has not been possible to apply any of the above methods to the measurements on ZETA with any precision.

In an experiment in helium, for which ionization cross sections have been measured, a measurement, using the method of Cunningham, of the relative intensities of the singlet and triplet lines at 3889 Å and 5016 Å yielded electron temperature values rising linearly to $4 \times 10^6$ °K at a time 0.5 msec after the start of the discharge. The condenser was charged to 15 kv and the helium pressure was 1.25 × 10⁻⁴ mm Hg. In the above experiments, the lines were selected by the 3-metre monochromator and by a Hilger medium glass spectrograph with 6 exit slits in the focal plane and 6 Ediswan Type 27 M1 photomultipliers, the outputs of any two of which were fed to a 2-beam oscilloscope and photographed. The intensities were measured from the smoothed average of a number of exposures. Tracings of typical oscilloscope records obtained in this way are reproduced in Fig. 5.

Electron Temperature—Bremsstrahlung

The energy radiated as Bremsstrahlung by a hot plasma is related to the electron temperature $T_e$ by an expression given by Cilliè.

$$E_{\nu}d\nu = CN_1N_2Z^2T_e^{-4} \exp\left(-\frac{\hbar\nu}{kT_e}\right)d\nu$$

where $C$ is a constant, $\nu$ is the frequency, $N_1$, $N_2$ are the number densities of ions and electrons and $Z$ is the ionic charge. This expression assumes an optically thin plasma and is based on the Born approximation. The electron temperature can thus, in principle, be found in two ways from Bremsstrahlung measurements: first, by measuring the absolute amount of energy radiated in a known energy interval, the number densities of electrons and ions and the ionic charge; or, secondly, by measuring the variation of energy radiated as a function of frequency. In practice the second method involves only relative intensity measurements and is likely to be the more reliable. Using the first method, this measurement can be made in any spectral region which is free of line spectra or continuous spectra from other sources. For the second method, to measure temperature of the order of 10⁶ °K, it is necessary to make the measurement in a spectral region where the exponential term is varying significantly, i.e. the X-ray region of a few kev energy, and the far vacuum ultra-violet (wavelength 100–1000 Å). Preliminary measurements in the X-ray region using a xenon-filled proportional counter with a 0.008 cm beryllium window have shown the necessity of preventing paralysis of the equipment by high-energy non-thermal X-rays.

Total Radiation Intensity

A measurement of the total energy loss from the ZETA discharge by radiation was made using a Cambridge Instruments thermopile, with a sensitivity of 50 mv (w/cm²), connected to a galvanometer. The thermopile was inserted in a side tube forming part of the vacuum envelope of the torus, and viewed a limited region of the discharge defined by slits. It was thermally insulated from the walls. The time constant was 1.5 sec and the overall sensitivity 10⁻⁴ joules/cm² for 1 mm deflection. The sensitivity was measured using a calibrated tungsten ribbon lamp and a rotating slotted disk to simulate ZETA.

\* Correct only if the electrons have a Maxwellian velocity distribution.
To estimate the heating effect of particles bombarding the thermopile, and molecular recombination on its surface, films of celluloid and silicon monoxide could be placed between the discharge and the thermopile by remote control. The thickness of the films was about 10^{-8} cm, so that a part of the radiation in the vacuum ultra-violet was absorbed. No estimate of the fraction has been made, but measurements made with and without the films give upper and lower limits for the energy radiated. Apart from this uncertainty, an error was introduced in the calibration, giving a possibility that the values are 50% too high. There is no fundamental reason why the accuracy of the experiment should not be improved. The results of the experiment, made at two settings of condenser voltage $V_c$, are given in Table 2 below.

<table>
<thead>
<tr>
<th>Table 2. Total Energy Radiated by ZETA Discharge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_c$ = 15 kV</td>
</tr>
<tr>
<td>Total radiated energy, film in</td>
</tr>
<tr>
<td>Total radiated energy, film out</td>
</tr>
<tr>
<td>Stored energy in condenser</td>
</tr>
<tr>
<td>Integrated product of $V_{gas} \times J_{gas}$</td>
</tr>
</tbody>
</table>

* All quantities are in units of 10^4 joules/pulse.

**Streak Photography**

Resolution in time and space but with limited spectral resolution (by means of filters and colour photography) is carried out with drum cameras and image converters, focused on transverse slits in the torus. Since such systems are at present limited to visible and quartz ultra-violet, they show mainly light from neutral or weakly ionized atoms. Very little visible light is emitted from the hottest part of the discharge. The need is evident for devices of this sort operating at shorter wavelengths.

A system employing a mechanical scanner and photomultiplier in conjunction with a monochromator promises to combine reasonably good time and space resolution with good spectral resolution.

**Microwaves**

The range of frequencies $10^{10} - 10^{12}$ sec$^{-1}$ is of special interest in the study of hot plasmas because of the strong interaction between the electron gas and the radiation. By studying the optical properties of the plasma the electron density and collision frequency can be determined; by measuring the radiation from it, the electron temperature can be deduced. In these measurements resolution can be obtained in both position and time.

In the discharges with which we are concerned, the critical or plasma frequency $v_p = (ne^2/m_e)^{1/2}$ (where $n$, $e$, $m$ represent electron number density, charge and mass respectively), is in the range $10^{10} - 10^{11}$ sec$^{-1}$. The refractive index of the plasma to radiation of frequency $v$ is given by $1 - \left(v/v_p\right)^2$, so that below the plasma frequency the plasma is reflecting, whilst well above it, propagation is as in free space, provided the electron cyclotron frequency is outside the range of interest.

The electron–ion collision frequency in the present case is a factor of about $10^4$ lower than the plasma frequency, so that attenuation is small except near to the latter.

Direct transmission of microwaves through a plasma can be observed and interruption of this transmission will indicate either strong absorption or refraction in the intervening plasma. For practical reasons the measurements have at present to be made at fixed frequencies. The method of investigation is to vary the conditions of the discharge and observe the manner in which reflection, transmission and refraction of the radiations occur. In discharges which vary in time such as those in ZETA, this can be done for each pulse by displaying the output of the receiver on an oscillograph. Thus, by using several fixed frequencies to cover the range of interest, a picture of the density structure of the discharge can be built up.

Electron temperatures can be determined by measuring, in a sensitive receiver, the microwave noise emitted by the plasma. It has been shown by Delli$^5$ that in a narrow frequency band near the critical frequency, a plasma emits radiation corresponding to a black body. The energy radiated per unit frequency interval is then given by

$$E_{\nu, dv} = h \nu \tau_{\nu} dv.$$  

The effective aperture of the aerial does not enter into this relation provided that it is small compared to the size of the discharge. In practice the measurement is made by comparing the signal received from the plasma with that from a standard noise source, using an attenuator. It is then not necessary to know the band-width and gain of the receiver. Subsidiary tests are necessary to ensure that the plasma is absorbing (and therefore emitting), radiation of the frequency used.

Measurements have been made with the Mark II Torus (35 cm bore, 105 cm mean diam.) and with ZETA. For the former, a frequency of $3.5 \times 10^{10}$ sec$^{-1}$ was used for electron density and temperature measurements in hydrogen and argon. A diagram of the apparatus is shown in Fig. 6 and the results of temperature measurements in hydrogen in Fig. 7. The arrows at the top and bottom of the curve in the figure indicate the points at which the plasma became reflecting and non-absorbing respectively.

In ZETA, the higher density has necessitated the use of a higher frequency. Experiments made with the highest frequency at present obtainable, $7.4 \times 10^{10}$ c/s, have shown that this frequency is still not sufficient to penetrate the densest regions at maximum current. A series of transmission measurements indicated the electron density increasing with current and reaching a maximum value of $3 \times 10^{13}$ cm$^{-3}$. Figure 8 is a reproduction of oscillograph traces obtained in a series of transmission measurements at $7.4 \times 10^{10}$ sec$^{-1}$. The transmitting klystron was modulated at 1 Mc, and the vertical thickness of the band is a measure of the power transmitted through the plasma. The fact that
the interval of non-transmission increases with the current suggests that the density rises to a higher value than the minimum necessary for absorption.

Electron temperature measurements have been made only at the lower frequency $3.5 \times 10^{10}$ sec$^{-1}$, and give values corresponding to the less dense regions of the discharge. The results found are in the range $1-5 \times 10^5$ °K.

These measurements are at present limited by the equipment available. It is hoped that in the near future the technique of microwaves will be developed sufficiently for such measurements to be made at frequencies up to $3 \times 10^{11}$ sec$^{-1}$. Above this frequency, the technique of infra-red research may be more suitable, and would have the advantage of variable frequency operation.

ELECTRICAL MEASUREMENTS

External Measurements

The current through the gas is measured by a Rogowski coil and the electric field applied round the torus is measured by a loop of wire circling the core. These waveforms are fed to a recording oscilloscope in which 8 signals can be photographed automatically for every discharge.

Information about the condition of the discharge is deduced from such external measurements. Instability is inferred qualitatively from fluctuations of the voltage and current waveforms, which are due to changes in channel diameter and position causing changes in inductance. The radius of the current channel is found from the measured inductance of the plasma, and electron temperature from its resistivity.5, 10

Internal Measurements

Magnetic Probes

By inserting small pick-up coils into the discharge it is possible to measure the magnetic field vectors and plot the field configurations associated with the current channel over a limited region of the interior of the discharge tube. In this way it is possible to find the position, size and pressure of the current channel as a function of time, averaged from pulse to pulse. For toroidal geometry, a pressure balance equation has been given by Honsaker et al.11 These measurements also indicate instabilities in the plasma.

The use of these probes is subject to the limitation that the presence of the probe itself may perturb the discharge and lead to spurious results. These effects were examined (i) by observing the electrical characteristics, spectra and neutron emission and (ii) by inserting a second probe and finding its effect on the first. It was found that the presence of a 1 in. quartz probe increased the resistance of the discharge, greatly reduced the neutron yield, and increased the emission of impurity lines, in particular of silicon and oxygen. The magnetic configuration was not changed in general shape by a second probe at a distance of 70 cm.

The experimental arrangement used with ZETA consists of 16 coils having about 700 turns of 0.004 cm dia. wire wound on formers of 0.4 cm dia. The coils are arranged in a line with their axes parallel and spaced 3.2 cm apart; outside the coils is a silvered glass tube for electrostatic screening, and the whole assembly is placed inside a 1 in. dia. quartz tube which is inserted radially or tangentially into the torus through a vacuum seal. The coils are cooled by a jet of air. One set of coils measures the azimuthal and axial components of magnetic field, by rotating the assembly through 90°, while a second set of coils measures the radial component. The outputs of 8 of the 16 coils can be recorded simultaneously. The coils
Figure 8. Microwave transmission in ZETA. Top trace: power in receiver. Centre trace: gas current. Lower trace: background with local oscillator off.

Gas, deuterium + 5% nitrogen; pressure, $1.25 \times 10^{-4}$ mm Hg; axial magnetic field, 160 gauss; condenser voltage: (a) 8 kv, (b) 10 kv, (c) 17 kv, (d) 21 kv. The "hash" on the traces is due to electrical interference.
are damped by 1000 ohm resistors and connected via twin twisted feeders to amplifiers and Miller integrators and thence to an 8 channel recording oscilloscope. The resonant frequency of the coils is 300 kc/sec and the upper frequency cut-off of the complete system 250 kc/sec. The integrators have an integrating time constant of 10 msec and a differentiating time constant of 100 msec. The complete system is calibrated by discharging a condenser through a specially-shaped inductance into which the probe is inserted, the time constants being arranged to simulate the ZETA pulse. The sensitivity of the system is such that a change of magnetic field of 100 gauss causes a deflecting potential of 1 volt to be applied to the oscilloscope.

Typical records taken during a magnetic probe measurement on ZETA are shown in Figs. 9 and 10. Analyses of these results are given in Ref. 2.

**Electrostatic Probes**

For some years, a standard method in plasma research for determining electron temperature and particle density has been to use electrostatic probes following the method of Langmuir. The electron temperature is obtained from the slope of the voltage-current characteristic, and the density from the saturated ion current.

In the present experiments the double-probe of Johnson and Malter has been found more satisfactory than single probes, for two reasons: (1) the maximum current which can be drawn by the probe is the saturation ion current which is an order of magnitude less than the saturation electron current; this means that there is less danger of destruction of the probe by the formation of arcs; (2) a reference electrode is not needed, eliminating a source of error in electrodeless discharges where the plasma is not necessarily at constant potential.

Experiments on the Mark II Torus were made with double probes consisting of 1 or 2 mm dia. tungsten wires embedded in glass except for their ends. The potential difference between the probes was varied from -30 to +30 volts and saturation currents of the order 1-10 amp/cm² were obtained. Because of the instability of the discharge, the probe current at each potential setting was integrated for about 60 pulses. A condenser of 3000 μF was connected across the probe to keep the potential constant.

A characteristic obtained in this way is shown in Fig. 11. The electron temperature obtained from this characteristic was 81,000 °K. A series of such measurements in hydrogen at a pressure of 1.7 × 10⁻³ mm Hg gave the following results:

---

**Electrostatic Probes**

- Gas current, kA 3 4 5 6 7 8 9
- Electron temperature, °K × 10⁻⁴ 6.4 8.1 10.0 12.6 16.0 20.0 27.0

The accuracy of these measurements, from internal consistency, was about 10%.

When this technique was applied to ZETA it was found that the large potentials needed (up to 1000 volts) resulted in the frequent formation of arcs which destroyed the probe. A number of attempts were made to overcome this problem, none of which has so far proved completely successful. As a result of these experiments, however, a system has been devised which holds some promise. This consists of a double probe made of tungsten wires enclosed, except for their ends, in a quartz tube which is surrounded by another open-ended quartz tube projecting beyond the probe for a distance of the order of its diameter, to act as a plasma attenuator. The effect of this is to reduce the particle density, and therefore the current drawn by the probe, by a factor greater than the reduction of electron temperature. The potential difference between the probes is swept through the necessary range (0–1000 volts) in 100 μsec, so that the probability of arc-formation is further reduced.

**HIGH ENERGY RADIATIONS**

Under this heading is included the detection and measurement of neutrons, protons, other particles arising from nuclear reactions, γ-rays and X-rays. The most direct evidence of the occurrence of the nuclear reactions is obtained by detection of the reaction products. The reactions of greatest interest are those between the heavy isotopes of hydrogen:

- D + D → He³⁺ + n + 3.26 Mev
- D + D → H⁺ + p + 4.04 Mev
- D + T → He⁴⁺ + n + 17.6 Mev

the first two occurring with approximately equal probability at all energies.

**Neutrons**

The emission of neutrons in the first and third of these reactions provides a very convenient means of determining whether nuclear reactions are taking place: the neutron detectors may be placed outside the discharge tube, and the time and spatial distribution found. For the ZETA experiments, five kinds of neutron detector have been used, each having special advantages in a particular application.

**Detectors**

**Activation counters**

The neutrons are slowed down in paraffin and captured in a substance which then becomes radioactive, emitting particles which are counted by standard methods. The method used was to place the substance to be activated as a lining to Geiger counters mounted on the torus, and to count the activity in the intervals between discharges (10 secs). To eliminate interference and the effect of the magnetic field on the counters and associated electronic equipment, counting was started 1½ secs after, and stopped ¼ sec before, each pulse.

High detection efficiency is obtained by using a substance with a large activation cross section and short half-life, but the half-life should not be so short that most of the activity has decayed before counting.
Simultaneous 8-channel magnetic probe oscillograms in each case but with the two sets obtained on different discharges in one experiment. Gas current, 140 ka; gas, deuterium + 5% nitrogen; pressure, $1.25 \times 10^{-4}$ mm Hg; applied axial field, 160 gauss; distance of coil from axis, $r$, as shown in the common scale is started. The choice of $\text{In}^{115}$ (cross section 50 barns, half-life 13 secs) was made because of its high detection efficiency, suitable half-life, and availability. The 13-sec half-life was very convenient because saturation activity was reached after only a few pulses. None of the experiments lasted long enough for the 54 min activity to give a significant contribution to the count.

The counters were calibrated by exposing them to a standard Pu–Be source for a given time and counting the activity with the source removed. The overall efficiency was $1.2 \times 10^{-4}$; the practical lower limit for detection of neutrons in an experiment lasting 5 min was $10^5$ neutrons/pulse. The advantages of activation counting are that the counting is done after the discharge has occurred, eliminating the possibility of spurious counts due to interference; and that no confusion with other radiations can arise. Nuclear reactions in ZETA were first identified in this way.
Boron Trifluoride Proportional Counters

These counters possess the advantages of high efficiency, simplicity and good discrimination against γ-rays. The neutrons give a characteristic pulse height by which they can be identified.

The arrangement used consisted of four cylindrical copper-walled proportional counters with a sensitive volume of 800 cm$^3$ of BF$_3$ gas at atmospheric pressure, mounted as a single unit and surrounded by 5 cm of paraffin wax, 3 mm of lead to absorb X-rays and 6 mm of aluminium for electrical and magnetic screening. The mean life of the neutrons against capture was estimated to be 200 μsec; the counter was therefore not suitable for measuring the time of emission, except very approximately. The pulses were amplified and fed to a scaler via a variable electronic gate which, for most experiments, was set to open for 1.4 msec, centered at the current maximum. In this way the neutrons associated with the large voltage transient at the end of the discharge were not counted.

The counter was calibrated in its operating position, near to the torus, by counting with a standard Pu–Be source at a number of positions on the Z axis§ of the torus. The total neutron yield from the discharge was then calculated assuming that the neutrons were emitted by a uniform line source on the Z axis.§ The counter had an efficiency of $10^4$ neutrons (total yield) per count. The accuracy of calibration was estimated to be ±20%.

Recoil Proton Scintillation Counters

For measuring the time at which the neutrons were emitted, scintillation counters using terphenyl-loaded polystyrene (with TPBD shifter) were used. The plastic was in the form of a cylinder 12.5 cm dia. and 2.5 cm thick, and was in contact with an EMI Type 6099 B photomultiplier. The resolving time of the counters was $10^{-8}$ sec, but this was limited to $10^{-6}$ sec by the amplifiers and sealers. In some experiments, where a high counting rate was encountered, a fast sealer was used to reduce the resolving time to $2.5 \times 10^{-7}$ sec.

For measuring total neutron yield per discharge, a gate similar to that described above was used.

The photomultiplier was surrounded by three layers of mumetal for magnetic screening, the whole counter was enclosed in 6 mm of lead to absorb X-rays, and two insulated layers of aluminium 6 mm thick to exclude electric and magnetic fields.

The counter was calibrated with 2.5 Mev neutrons from the Li$^7$(p,n) reaction, using the Harwell Van-de-Graaff machine and comparing it with a standardized “long” counter. The geometrical factor for neutrons from ZETA was calculated. The overall efficiency was $(1.2 \pm 0.1) \times 10^5$ total neutron yield per count.

Nuclear Emulsions

Neutron energies can be measured using nuclear emulsions if the direction of the neutrons is defined and the range and angle of the recoil proton measured. The method has low detection efficiency (see Table 3) and the neutron flux from ZETA was too low to allow effective collimation to be used in an experiment lasting a reasonable time. The measurements were therefore limited to determining the maximum

<table>
<thead>
<tr>
<th>Table 3. Characteristics of Neutron Detectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scintillation counter with Li$^4$ (Eu)</td>
</tr>
<tr>
<td>Cross section (barns) at 2.5 Mev: 0.18</td>
</tr>
<tr>
<td>Effective volume (cm$^3$): 29</td>
</tr>
<tr>
<td>Density (atoms/cm$^3$): $1.8 \times 10^{22}$</td>
</tr>
<tr>
<td>Utilization factor: 1</td>
</tr>
<tr>
<td>Energy resolution (keV): 0.09</td>
</tr>
<tr>
<td>Min. deuterium energy (keV): 100</td>
</tr>
<tr>
<td>High pressure diffusion cloud chamber with H$^8$</td>
</tr>
<tr>
<td>Cross section (barns) at 2.5 Mev: 2.5</td>
</tr>
<tr>
<td>Effective volume (cm$^3$): 5700</td>
</tr>
<tr>
<td>Density (atoms/cm$^3$): $5.1 \times 10^{20}$</td>
</tr>
<tr>
<td>Utilization factor: $1$</td>
</tr>
<tr>
<td>Energy resolution (keV): 2.4</td>
</tr>
<tr>
<td>Min. deuterium energy (keV): 20</td>
</tr>
<tr>
<td>Ionization chamber filled with He$^8$</td>
</tr>
<tr>
<td>Cross section (barns) at 2.5 Mev: 0.5</td>
</tr>
<tr>
<td>Effective volume (cm$^3$): 2600</td>
</tr>
<tr>
<td>Density (atoms/cm$^3$): $8.1 \times 10^{19}$</td>
</tr>
<tr>
<td>Utilization factor: 1</td>
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<tr>
<td>Energy resolution (keV): 0.10</td>
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<tr>
<td>Min. deuterium energy (keV): 5</td>
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<tr>
<td>Proportional counter with BF$_3$</td>
</tr>
<tr>
<td>Cross section (barns) at 2.5 Mev: 0.02</td>
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<tr>
<td>Effective volume (cm$^3$): (2600)</td>
</tr>
<tr>
<td>Density (atoms/cm$^3$): $2.7 \times 10^{19}$</td>
</tr>
<tr>
<td>Utilization factor: 1</td>
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<tr>
<td>Energy resolution (keV): 0.001</td>
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<tr>
<td>Effective volume (cm$^3$): 0.1</td>
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<td>Density (atoms/cm$^3$): $3 \times 10^{22}$</td>
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<td>Utilization factor: $1$</td>
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<tr>
<td>Energy resolution (keV): 0.0025</td>
</tr>
<tr>
<td>Min. deuterium energy (keV): 50</td>
</tr>
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</table>

§ The circle which corresponds, in a torus, to the axis of a straight tube.
neutron energy. Ilford C2 emulsions were exposed for 5000 discharges and the processing and measurement followed standard procedure. The maximum neutron energy was found to be $2.6 \pm 0.1$ Mev.

**High Pressure Hydrogen-filled Diffusion Cloud Chamber**

This was used for neutron energy measurements (see below) on account of its high overall detection efficiency and good energy resolution (see Table 3).

**Measurements in Zeta Experiments**

**Yield**

The mean yield per pulse was measured under different conditions of the discharge, e.g. gas current, pressure and axial magnetic field. Results of some of these measurements are shown in Figs. 12-14.

**Time Variation**

The neutron emission varied with time through the current pulse as shown in Fig. 15.

**Determination of Place of Origin of the Neutrons**

For this experiment a large shielded neutron counter was used, having an angular acceptance of $\pm 4^\circ$, and a scan made across a minor diameter of the torus. The experimental results are shown in Fig. 16 and on the same figure are plotted some calculated distributions with different assumptions. The calculations include the effect of scattering in the 1 in. aluminium walls of the torus. Because of the low counting rate the statistical error in this measurement is large, but it is clear that the result is not consistent with either the 20 cm or 50 cm channel, nor with the emission of all the neutrons from the walls.

**Time Correlation between Emission of Neutrons and Other Phenomena**

It was observed that under certain operating conditions the gas current ceased abruptly at the end of the pulse, this occurrence being accompanied by a large instantaneous emf around the torus and by a simultaneous burst of neutron emission. Since a large number of smaller pulses of emf were observed throughout the current pulse, an experiment was made to find out whether there was any correlation in time between these pulses and the neutron emission.

The two sets of pulses were photographed on adjacent traces of a 2-beam oscillograph and an analysis made of the time elapsing between a pulse of emf and the next neutron pulse. The result is shown in Fig. 17, which also includes a theoretical distribution for a perfectly random emission. Not more than 10% of the neutrons are correlated with the preceding emf pulse. A similar experiment was made to investigate correlation between neutrons and sudden increases of visible light emission observed in swept image photographs of the discharge, with negative results.
The reaction is due to a deuteron with an energy $E_1$ colliding with a stationary deuteron, the neutrons have energies, at $0^\circ$ and $180^\circ$ to the direction of motion of the deuteron, given by

$$E_n = 2.45 \text{ Mev} + \frac{1}{2} E_1 \pm 1.1(E_1)$$

The neutron energy is very sensitive to deuteron energy, hence quite a small value of the latter can be detected.

In Table 3 are listed several possible methods of measuring neutron energy in the 2.5 Mev region, with typical values of effective volume which may be obtained in practice and a figure of merit which is the product of the reaction cross section $\sigma$, the effective volume $V$, the number density of reacting atoms $N$ and a utilization factor $U$ expressing the fact that, in recoil proton range measurements, only about one-third of the tracks are suitable for measurement. In the last two columns are given typical values for the energy resolution possible with the detector, and the corresponding minimum deuteron energy which could be observed.

It can be seen from Table 3 that where the neutron flux is low, as in ZETA, the diffusion cloud chamber is an obvious choice. The He$^3$ ionization chamber promises to give excellent energy resolution for measurements in a higher flux, and has the advantage over the cloud chamber that time resolution is also possible.

The energy measurement of the neutrons from ZETA has been fully described elsewhere and only a brief description will be given here. A diffusion cloud chamber filled with hydrogen to a pressure of 7 atmospheres was shielded so that only neutrons emitted tangentially from one section of the torus were admitted to the sensitive volume. The neutrons were collimated within an angle of $\pm 6^\circ$, and the range and angle of recoil proton tracks were measured, from which the neutron energy was calculated with an error of 4%. After a sufficient number of tracks had been observed, the direction of the current in the
gas was reversed and the experiment repeated. In a series of experiments, 47 tracks suitable for measurement were obtained with the normal current, and 41 with the current reversed. The mean neutron energies were found to be $2.66 \pm 0.02$ Mev and $2.33 \pm 0.02$ Mev, respectively, indicating that the neutrons were not of thermonuclear origin. A calibration experiment, in which the neutrons emitted from a D-D source were observed at 90° to the deuteron beam, gave a neutron energy of $2.45 \pm 0.022$ Mev.

### Charged Particles

In addition to neutrons, the charged particles from the nuclear reactions listed above may be detected and used as indicators. The most important case is that of the protons emitted in the second reaction. An experiment was made to observe the protons in a nuclear emulsion covered with a thin aluminized nylon foil to absorb the light. Up to the present, no results have been obtained, because of fogging of the plate by X-rays. If this difficulty can be overcome, the method promises to be very useful because of its high detection efficiency and the ease with which the particles can be collimated.

### X-rays

The importance of X-rays in gas discharges is that they indicate the presence of non-thermal processes. Experiments were carried out on ZETA to determine the number and energy of X-rays, as a function of gas current and axial magnetic field, and to investigate correlation in time with the emf pulses referred to above. X-rays of energy greater than about 50 kev are able to penetrate the walls of the discharge tube, but for lower energies an aluminium window 0.4 mm thick was inserted in the torus. The methods used for observing the X-rays were photographic films and NaI (TI) scintillation counters. Energies were measured by absorption in metal foils and by observing single quanta in the counters.

---

**Figure 16.** Variation of neutron yield across minor diameter of ZETA Torus. Experimental points obtained with shielded neutron counter. Curves are calculated distributions for the cases shown.

**Figure 17.** Correlation between neutron pulses and pulses of emf round ZETA torus. Full line: measured number of neutrons in time interval shown after an emf pulse. Broken line: calculated distribution of neutrons assuming random emission, normalized to same total number of neutrons.
X-rays, of energies up to 100 keV, were observed in the main part of the pulse: the total number of quanta per pulse having energies in excess of 60 keV was of the order of \(10^8\). There were larger numbers of softer quanta, e.g. \(5 \times 10^{10}\) harder than 40 keV and \(2 \times 10^{12}\) harder than 10 keV. A strong correlation between high energy X-rays and emf pulses was found. A second group of X-rays, having energies up to approximately 500 keV, was associated with the terminal voltage transient.

To find the spatial distribution of the high energy electrons, a tungsten target was placed diametrically across the torus and the X-rays from it recorded photographically with a pinhole camera. The intensity of X-ray emission was highest on the axis of the torus and fell to 1/10 of its maximum value at a radius of 34 cm when the axial magnetic field was 160 gauss, and 32 cm when the field was 360 gauss.

**CONCLUSION**

In attempting to elucidate the mechanisms in heating and confining plasmas and to determine their physical condition, a wide variety of diagnostic techniques is being developed and used. The most important physical properties and the methods of measuring them are summarized below.

**Ion temperature**, defined as the mean kinetic energy of the ions, is measured from the Doppler broadening of spectral lines of highly ionized impurity atoms. The contribution of mass motion has not yet been measured, and the influence of ionic charge on the measured temperature has to be ascertained. It has not yet been possible to measure the mean deuteron energy by measuring the nuclear reaction rate, because of the occurrence of non-thermonuclear reactions.

Magnetic probes are used to measure the sum of electron and ion temperatures, subject to the limitation that the probe itself perturbs the discharge, and the result is dependent on knowing the density.

**Electron temperature** is found from the resistivity of the plasma, by Langmuir probes, and by measurement of microwave noise. In the conditions in ZETA, experimental difficulties have so far prevented reliable results from being obtained. Measurements of bremsstrahlung and the relative intensities of spectral lines are under investigation, but these are restricted by lack of basic data on processes taking place under conditions very far removed from those usually studied in terrestrial systems.

**Electron density** has been measured by microwave transmission measurements; there is at present an upper limit of about \(3 \times 10^{13}\) cm\(^{-3}\) due to experimental difficulties.

**Ion density** can be measured by Langmuir probes but for the conditions in ZETA the technique is not sufficiently developed. Confinement and stability have been studied by magnetic probes and high-speed photography.

**Collision processes**, ionization excitation has been studied spectroscopically.

**Nuclear reactions** and non-thermal processes were detected and studied by means of the particles and radiations emitted.

**ACKNOWLEDGEMENTS**

We wish to express our gratitude to Dr. P. C. Thonemann and Mr. R. S. Pease for their interest and encouragement. Important contributions to the work described were made by W. M. Burton, P. G. Dawson, H. W. Jones, M. C. Rusbridge, J. Schofield, B. A. Ward, and D. H. J. Wort.

**REFERENCES**

Theory of Microwave Diagnostics of Hot Plasmas

By James E. Drummond*

REVIEW OF THEORIES

The theory of microwave diagnostics of plasmas really began in 1926 with Dittmer's proposal that high frequency oscillations spontaneously arise in an electric arc and are responsible for the existence of a group of extraordinarily high energy electrons which Langmuir had discovered. Before Dittmer's paper was published, Penning published observations of high frequency radiations from a controlled arc and established a definite correlation between the existence of the group of high energy electrons and the radiations.

Three years later, Tonks and Langmuir published their now famous theory of plasma oscillations and their much less well-known theory of microwave propagation in plasmas. This opened two possibilities for quantitative measurements of plasma properties. The index of refraction for microwave signals propagating through the plasma was found to depend markedly on the plasma electron density; a region of extinction occurring for all frequencies below a certain critical frequency proportional to the square root of the electron density. This critical frequency was the frequency of the signals which were observed to be correlated in intensity with the number of high energy electrons present in the discharge plasma. Thus both electron density and energy distribution were related to external observables of the plasma.

The subsequently improved and extended theories were applied to such varied problems as ionospheric propagation, radio noise from the sun, noise fluctuations in long electron beams, anomalous diffusion of plasma across magnetic fields, energy exchange between electrons in metals, and many others.

Recently Dellis has given a thermodynamic argument showing that one should expect on the basis of Kirchoff's law, enhanced emission from thin hot plasmas in the neighborhood of critical frequencies. This is because of the very greatly reduced optical penetration depth exhibited by a plasma for radiation in these bands. One need not look far for physical processes which could be responsible for this enhanced radiation. Cerenkov radiation and its inverse process can explain it nicely, giving peaks of radiation and absorption just in those bands where the phase velocity and hence also the optical depth are small. This effect seems also to be connected with transfer of energy between plasma and waves noted by Bohm and Gross in reference to the "trapping" of electrons traveling near the wave velocity. For these cases, however, the coupling of the plasma to the electromagnetic radiation field is not always clear; seeming to depend upon inhomogeneities and boundary effects.

The coupling between a plasma and the radiation field appears more directly for cases in which a static magnetic field is present. For these cases, resonances occur in the index of refraction near multiples of the electron cyclotron frequency; and, corresponding to these, enhanced power emission is observed. The width of the emission resonances is a measure of electron temperature and the location of the onset of transmission absorption is a measure of the plasma heat density, relative to the magnetic pressure. Again Cerenkov radiation is involved, but let me start from the basic kinematics and equations of motion and outline how the microwave properties of a hot magnetoplasma can be derived and applied to the diagnostics problem.

THEORETICAL DEVELOPMENT

In order to bring out the essential features of the density and temperature effects on the microwave properties of a plasma and to quote some important results for limiting cases, the analysis will here be restricted to homogeneous plasmas in uniform time-independent magnetic fields. The mean electron kinetic energy may be taken as small compared with its rest energy and large compared with the plasma quantum. Also, the motion of ions will be neglected. The latter has been shown to be valid for calculation of plasma properties above the ion resonance frequencies. Certain entropy producing effects such as those represented by equations of the Fokker-Planck type will be approximated by a small collision frequency term, producing relaxation to a steady state distribution. Only first order microwave perturbations around this steady state need be considered for diagnostics of the steady state. Later comments will be made on how these restrictions can be modified.

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The integrand contains the Fourier components of the current density which is an operator which time in the past; \( \omega_0 S \) the past orbital positions of the electrons; \( v \) is the present velocity of an electron and \( \xi \) its velocity at a time \( \xi S \) in the past; \( \mathbf{R}(\omega_0 S) \) is an operator which rotates its vector operand through an angle \( \omega_0 S \), where \( \omega_0 \) is the angular electron-cyclotron frequency, and \( v \) is the small effective close-encounter collision frequency.

The important point to note about this integral is the displacement of the argument of the electric field strength. The term labeled \( A \) represents the effect of the continual exploration of an electron through the electric field of frequency \( \omega \), and term \( B \) represents the drift of electrons along the magnetic field. The effect of term \( A \) is illustrated by the circular orbit, Fig. 1, of an electron exploring a sine wave electric field which has intensity variations indicated by the vertical arrows. Here the magnetic field points toward the viewer. It will be appreciated that this effect will become especially important when \( 2\pi/\omega_0 \), the orbital period, is equal to \( 2\pi/\omega \), the microwave field oscillation period, or to an integral multiple of this.

The term \( B \) represents the effects of the drift of electrons along the magnetic axis. This gives rise to a diffusion damping\(^{25} \) of the Landau type as well as phase changes in the current response of the plasma to axially varying electric fields. In addition, transverse electric fields can give rise to longitudinal currents through this term. This is illustrated in Fig. 2. Here the magnetic field is vertical. At the top of the drawing converging arrows are shown, indicating electron bunching due to electric fields, there, which are assumed to exist normal to the magnetic field. At the bottom of the drawing diverging arrows are shown, indicating the opposite effect of transverse electric fields 180 degrees out of phase with the fields at the top. Midway between these planes is shown a plane receiving bunched thermal electrons from the top and fewer than average thermal electrons from the rarefied region below. Thus when these groups of electrons reach the central plane as a result of their thermal motion, they will produce non-canceling vertical current pulses at the frequency of the transverse electric fields. In addition, it is easy to see from such diagrams as this how predictions of Landau diffusion damping and phase changes in transverse as well as longitudinal currents can arise from term \( B \) which represents electron drift along the magnetic field. It will be appreciated that these effects will become especially important when the wave length becomes very small.

Term \( A \) becomes very important for propagation of microwaves across the magnetic field in hot plasmas and term \( B \) becomes very important for propagation of microwaves along the magnetic field. The striking effects of these terms near certain resonances form the basis of the modern theory of high temperature microwave diagnostics.

Returning to Eq. (2), we observe that because of the indicated position displacements, Ohm's law does not generally hold for an arbitrary form of electric field in a plasma. We can, however, ask for those forms of electric field for which Ohm's law will be valid; i.e. for which \( \mathbf{J}_w \) at position \( \mathbf{r} \) will be proportional to \( \mathbf{E}_w \) at the same position, \( \mathbf{r} \). Such field configurations then must satisfy Eq. (2) with \( \mathbf{J}_w(\mathbf{r}) \) equal to \( \mathbf{E}_w(\mathbf{r}) \). This is then a homogeneous integral equation of the following form

\[
\sigma_0 \mathbf{E}_w(\mathbf{r}) = \int_{-\infty}^{\infty} \mathbf{K}(\mathbf{r} - \mathbf{r}') \cdot \mathbf{E}_w(\mathbf{r}') \, d\mathbf{r}'
\]
where the kernel $K$ is an even difference kernel. This equation is to be solved simultaneously with Maxwell's equations. The eigenvalues of this system are the desired conductivity tensors for the corresponding eigenfunctions, which form a complete set. The tensors are too lengthy to state explicitly here but are published. From the conductivity tensors can be calculated the indices of refraction for various cases of microwave propagation. This has been done with the aid of digital computers.

**RESULTS**

Figure 3 shows the result of the use of the standard zero-temperature conductivity tensor in standard transmission formulas. This shows the index of refraction, $n$, for circularly polarized waves propagating along the magnetic field vs. the ratio of electron cyclotron to microwave oscillation frequencies. The parameter for the various curves is proportional to the plasma electron density. It is the square of the ratio of the plasma frequency to the microwave frequency. The collision frequency for these curves was taken to be $10^{-4}$ of the microwave frequency.

Notice the sharp change of the index of refraction from real values plotted above the axis to imaginary values plotted below the axis at $\omega_p/\omega = 1$ for right-hand polarization. When thermal effects are allowed for, the imaginary part will begin farther to the right and extend past unity with a finite slope due to the Landau damping. This effect begins at very low electron temperatures because of the extremely large indices of refraction and hence extremely short plasma wave lengths just to the right of unity. The effects of successively higher temperatures are shown in Fig. 3(b), (c) and (d).

The intersections of the imaginary parts of the refractive index with the abscissae depend almost entirely upon a single plasma parameter, $\beta_e$, the ratio of electron kinetic energy density to magnetic energy density. This is shown in Fig. 4 for a range of $\beta_e$ of more than three orders of magnitude.

Even a rather gentle slope of the imaginary part of the index of refraction represents extremely rapid changes in the attenuation of a microwave signal sent
Figure 4. Frequency at which attenuation begins for right hand circularly polarized waves propagating along the magnetic field in a plasma.

Figure 5 shows the index of refraction for microwaves propagating across the magnetic field in 5.8 and 58 million degree plasmas. The microwave is polarized normal to the magnetic field. Notice the region of abnormal transmission which is seen to occur for $|\omega/e| < 1$. Within this region, the curves for a given temperature and various densities hug a single curve. The individual curves jump back to the extinguishing region at various frequencies determined by both density and temperature of the electron distribution.

Notice that in the region just to the right of $|\omega/e| = 1$, the indices have both real and imaginary parts. This again is a region of power exchange between the plasma and the radiation field.

Figure 6 shows the indices of refraction for microwaves propagating across the magnetic field with polarization parallel to the magnetic field and electron temperatures of 5.8 and 58 million degrees. A region of abnormal transmission is seen to occur for $|\omega/e|$ slightly greater than unity. The index of refraction within this band as well as the width of the band depend upon both electron density and temperature.

APPLICATIONS
Electron Densities and Temperatures

Having now outlined the kinematic basis and the results of the theory, let us review the possibilities for density-temperature measurements.

Consider one example. A high intensity rf discharge has been established between two large plates separated by 10 cm. A magnetic field of 10,000 gauss is normal to the plates. The nature of the discharge makes the electron density nearly uniform between the plates, but not radially from the axis of the system. Two small circular wave guides terminated in horns pierce the end plates. The index of refraction of the plasma is measured to be about 5 on the axis at 50 kMc ($10^9$ sec$^{-1}$). Reference to the curves shows that the electron density is about $10^{14}$ cm$^{-3}$, nearly independent of the electron temperature. At 30 kMc the attenuation through a few inches of plasma with changes in $|\omega/e|$. This makes possible a sensitive microwave transmission measurement of the mean thermal kinetic energy density of electrons in a plasma.

Furthermore, the existence of both real and imaginary parts to the index of refraction in certain frequency ranges indicates that power can be exchanged between the radiation field and the kinetic energy of plasma electrons for these frequencies. Thus, enhanced radiation might be expected from a high temperature plasma in the region $|\omega/e| \approx 1$. This has been predicted and observed in both the United States and the United Kingdom.$$^{14,28-30}$$

As already mentioned, one simple and effective way to view the radiation is to regard it as Cerenkov radiation by the high speed electrons into the band of frequencies that have very small phase velocities.
becomes very large. Reference to the now known proper density curve shows the mean electron kinetic energy to be about 5 ev.

Measurements along other axes parallel to the magnetic field will define the radial variation of both electron density and temperature.

It appears that either transmission or passive monitoring of high temperature plasmas can be used to measure electron temperature or kinetic energy density. The band width of the Cerenkov radiation and the onset of attenuation of transmitted radiation are both measures of temperature. These temperature effects are of importance from tens of thousands of degrees for radiations propagating along the magnetic axis to tens of millions of degrees for radiation propagating across the magnetic field. Thus it appears practical to build a microwave thermometer-densitometer to cover the critical temperature range leading up to fusion temperatures.

Extension to Ion Properties

The physical principle, upon which these measurements are based, is that the absorption, transmission and radiation properties of a plasma depend markedly upon the ratio of electron orbit diameter to the distance over which the oscillating electric field changes appreciably. If we contemplate trying to extend the methods to the measurement of ion temperature, we will recognize that the wave lengths of frequencies at which significant interaction between electric fields and ions occurs are far too long to provide the spatial variations upon which these methods depend. Thus, these variations must be introduced by special probing techniques. To illustrate, I shall give an example of a technique that appears applicable to the measurement of the temperature of ions just outside the boundary of a strong pinch discharge. The basic method is independent of whether the pinch is linear toroidal or "pretzular."

The few ions that have been thrown outside the pinch find themselves in a magnetic field \( \mathbf{H} = \left(2I/r_e\right) \mathbf{L}_e \) where \( I \) is the pinch current (esu), \( r \) the radius of the ion from the local pinch axis and \( \mathbf{L}_e \) the unit azimuthal vector about this axis. Some rather simple manipulations of the equations of motion yield the approximate equation for the harmonic content of the radial position of the ion:

\[
\begin{align*}
r &= r_0 \left[ \frac{M^2 V_{\perp}^2}{2eI} + 2r_0 \sum_{n=1}^{\infty} I_n \left( \frac{M^2 V_{\perp}^2}{2eI} \right) \cos \left( \omega n - \frac{\pi}{2} \right) \right] \end{align*}
\]

where \( r_0 \) is the mean radius of ion considered and the \( I_n \) are the Bessel functions of imaginary argument

\[
\pi \quad \text{and} \quad \pi 
\]

\[
V_{\perp} \quad \text{is the root mean square ion speed in the radial-axial plane. From this can be computed, for instance, the ratio of second harmonic to fundamental amplitudes of the ion cyclotron frequency motion:
\]

\[
\frac{r_{2\omega}}{r_{\omega}} \approx \left( \frac{M^2 V_{\perp}^2}{4eI} \right) \frac{c^2 (MKT)^{1/2}}{4eI}. \]

For deuterium at a temperature of \( 10^6 \) degrees, with a discharge current of \( 10^8 \) amp, this gives

\[
\frac{r_{2\omega}}{r_{\omega}} = \frac{1}{30}. \]

Since the magnetic fields encountered just at the outer pinch radius will be the largest in the system (so large in fact that the existence of any externally produced magnetic field will cause a negligible correction), there will arise no confusion between this \( r_{\omega} \) or \( r_{2\omega} \) and the \( r_{\omega} \) or \( r_{2\omega} \) at some other radius. Thus the ratio of radio frequency power radiated or absorbed by this ion oscillation and its harmonic will yield a direct measure of ion temperature at the outer edge of the pinch. The gas and ion damping collision frequency will be negligible in the relatively low density that should exist just outside the pinch, so a great many ion oscillation periods should occur during
any significant time interval for the pinch. Thus the ion temperature of various isotopes can be followed as a function of time during the pinch.

This illustrates the general methodology as applied in a particularly simple set of cases. More basic work is now underway to extend the work to treat realistically some problems of inhomogeneous plasmas allowing for entropy production effects.

CONCLUSIONS

I have presented here a brief review of microwave diagnostic theory. Professor Brown of the Massachusetts Institute of Technology will present a review of some of the workable schemes of experimental microwave diagnostics. Dr. Wharton will present a review indicating the relative merit of various other experimental diagnostic techniques.

For the future, there remain three important theoretical problems: (1) a systematic and thorough-going analysis of microwave properties of inhomogeneous plasmas; (2) a generalization of rf probing techniques for measuring ion isotope temperatures; (3) a basic study of entropy production effect in high temperature plasmas. This last topic may be of importance in plasma shock-wave studies. Guidance for this theory might be derived from equations of state for magnetically confined plasmas as observed by use of the currently available theory of microwave diagnostics.

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REFERENCES

Cyclotron Radiation from a Magnetized Plasma

By S. Hayakawa,* N. Hokkyō,† Y. Terashima* and T. Tsuneto*

For the investigation of physical properties of a plasma and for the interpretation of solar radio outbursts, an important clue may be provided by the cyclotron radiation emitted by electrons gyrating around a static magnetic field. With regard to this we treat two particular problems concerning the effect of fluctuating electric fields: one is the purely random fluctuation which brings about the resonance width and the other the organized plasma oscillation which may give rise to induced emission. The resonance width is shown to be related to the slowing-down time of an electron and the spectral distribution for the dipole radiation is derived under the approximation in which both the higher moment of the resonance shape and the coherence of radiations from a number of electrons are neglected. The angular distribution of the induced emission is expressed in a closed form that can be readily reduced to the formula neglecting the effect of the plasma oscillation. Since the general formula is too lengthy, we give explicit expressions only at particular directions.

I. INTRODUCTION

It is well known that a high-temperature plasma loses energy by bremsstrahlung, according to the formula calculated with the Born approximation, as

$$\frac{dW_b}{dt} = \frac{16}{3} \left( \frac{Z}{\pi} \right) \frac{Z^2 e^2}{m} \frac{kT}{e} \left( \frac{e^2}{mc^2} \right)^2 n_e n_l,$$  

where $Ze$, $m$, $2\pi$ and $k$ are the ionic charge, electron mass, light velocity, Planck constant and Boltzmann constant, respectively. Measuring the electron temperature, $T$, in degrees Kelvin and the electron and ion densities, $n_e$ and $n_l$ respectively, in cm$^{-3}$, the rate of energy loss is expressed by

$$\frac{dW_b}{dt} \approx 1.6 \times 10^{-27} Z^2 n_e n_l T^1 \text{ erg sec}^{-1} \text{ cm}^{-3} \quad (1.1')$$

If there exists a magnetic field in a plasma, the so-called cyclotron radiation is expected to compete with the bremsstrahlung. The energy loss due to this process would be

$$\frac{dW_c}{dt} = \frac{4e^2 kT}{3e^2 m} \omega_e n_e$$

$$\approx 5.4 \times 10^{-25} n_e H_0^2 T^1 \text{ erg sec}^{-1} \text{ cm}^{-3} \quad (1.2)$$

The radiation has a line spectrum, the emission frequencies being the integral multiples of the cyclotron frequency,

$$\nu_e = \frac{\omega_e}{2\pi} = \frac{eH_0}{2\pi mc} \approx 2.8 \times 10^6 H_0 \text{ sec}^{-1} \quad (1.3)$$

where $H_0$ is the magnetic field strength in gauss. In most practical cases, however, $\omega_c$ is smaller than the plasma frequency $\omega_p = (4\pi e^2 n_e/m)^{1/2}$, so that the cyclotron radiation cannot occur, except for its higher harmonics of weak intensity.

The relative importance of the above two modes may be compared by observing that

$$\frac{dW_b}{dW_c} \approx 77 \frac{\omega_p}{\omega_c} \quad (1.4)$$

and

$$\frac{(\omega_p/\omega_c)^2}{\omega_p^2} \approx 4\pi mc^2 n_e H_0^2 \approx 1.0 \times 10^{-5} n_e H_0^2 \quad (1.5)$$

The comparison indicates that the cyclotron radiation becomes important if the magnetic energy density is comparable to the electron mass energy density. Such a condition seems to exist in active celestial regions, such as in a part of the Crab nebula and in the vicinity of an active sun spot. It is not impossible to provide such a condition in laboratory experiments.

If the cyclotron radiation from a plasma is observable, it is also feasible to observe the cyclotron absorption of electro-magnetic waves by the plasma. The absorption coefficient for the dipole mode is given by

$$\kappa(\nu) = \frac{\omega_p^2/6c}{\nu_0} \delta(\nu - \nu_e). \quad (1.6)$$

This indicates absorption at a sharply defined frequency, but actually there is a finite absorption width due to the modulation of the gyrating motion of electrons caused by fluctuating electric fields. The width is closely connected with the slowing-down time of an electron. In Sec. 2 we shall show that the reciprocal width is essentially equal to the slowing-down time for dipole radiation; the relation between them becomes complicated for multipole radiations.

The modulation of the electron motion by fluctuating electric fields gives rise not only to the dissipation which results in the emission width or absorption width, but also to a forced motion of electrons that converts the energy of plasma oscillations to radiation, provided that the fluctuation forms an organized pattern.
motion over a long period. This mechanism has been discussed by Field, aiming at the interpretation of solar radio outbursts. In Sec. 3 we shall extend his work to a more general case and give an expression, the latter having also been regarded as a possible mechanism for the solar radio outbursts.

2. COLLISION BROADENING OF CYCLOTRON RADIATION

Since in transport phenomena in a plasma the distant collisions play a more important role than the close collisions, we may expect that the distant collisions would be dominant in bringing about the broadening of the cyclotron radiation of electrons. The width due to the close collisions is estimated, according to the Lorentz theory of collision broadening, to be of the order of \(1/\tau_c\), the collision frequency of the Rutherford scattering between electrons. In what follows, therefore, we shall consider only the cumulative effect of the distant collisions on the spectrum of the cyclotron radiation of electrons in a plasma.

As is usually done in calculating transport coefficients for fully ionized plasma, it would be most convenient for our purpose to treat the problem as a sort of Brownian motion. Thus we start from the equation of motion for individual electrons gyrating under an external magnetic field.

\[
\frac{d\mathbf{v}}{dt} = -\frac{e}{mc}[\mathbf{v} \times \mathbf{H}_0] - \frac{e}{m} \mathbf{E}(t), \tag{2.1}
\]

where \(\mathbf{H}_0\) is the external field and \(-e\mathbf{E}(t)\) is the randomly fluctuating force exerted by field particles. The vector potential of radiation emitted by an electron and received at time \(t\) is given by

\[
\mathbf{A} = -\frac{e}{cR_0 \tau} \int_{-T/2}^{T/2} \mathbf{v}(\tau) \delta(\tau-t+R/\mathbf{v}) d\tau, \tag{2.2}
\]

where \(R_0\) and \(R\) are the distances from the point of observation to the center of gyration and to the electron respectively, and \(T = 2\pi/\omega_0\). We shall restrict our discussions to the dipole radiation, in which case \(\mathbf{A}\) will become

\[
\mathbf{A} = -\frac{e}{cR_0 \tau} \int_{-T/2}^{T/2} \mathbf{v}(\tau) \sum_{n=-\infty}^{\infty} \exp[i\omega_0(\tau-t+R_0 \mathbf{v})] d\tau. \tag{2.3}
\]

The expression for the total intensity of the dipole radiation turns out to be, on averaging over the period \(T\), which is taken to be sufficiently long

\[
I = \frac{2}{3} e^2 \frac{c}{\tau} R_0^2 |\mathbf{A}|^2
= \frac{2}{3} \frac{e^2}{c^3} \frac{1}{2\pi} \left( \frac{1}{T} \right)^2 \sum_{n=-\infty}^{\infty} n^2 \omega_0^2 \times
\int_{-T/2}^{T/2} \langle \mathbf{v}(\tau_1) \cdot \mathbf{v}(\tau_2) \rangle_{\mathbf{A}'} \exp[i\omega_0(\tau_2-\tau_1)] d\tau_1 d\tau_2. \tag{2.4}
\]

Here we have taken the average of the quantity \(\mathbf{v}(\tau_1) \cdot \mathbf{v}(\tau_2)\) over the great number of electrons. Thus the problem is reduced to finding this autocorrelation of the velocities of electrons from the equation (2.1), the proper treatment of which would be quite difficult and beyond the scope of the present work. In order to obtain the approximate value for it, we rewrite Eq. (2.1) in the form of the Langevin equation, separating the systematic part from \(\mathbf{F}\):

\[
\frac{d\mathbf{v}}{dt} = -\frac{e}{mc}[\mathbf{v} \times \mathbf{H}_0] - \eta \mathbf{v} + \mathbf{F}. \tag{2.5}
\]

where \(\mathbf{F}\) is supposed to be purely random and the friction coefficient, \(\eta\), is assumed to be constant. Further we shall take into account only the average behavior represented by \(\eta\), discarding the effect of diffusion due to \(\mathbf{F}\) in the velocity space. Then it is easily shown that

\[
\langle \mathbf{v}(\tau_1) \cdot \mathbf{v}(\tau_2) \rangle_{\mathbf{A}'} = \langle \mathbf{v}^2 \rangle_{\mathbf{A}'} \cos \omega_0(\tau_2-\tau_1) e^{-\eta |\tau_2-\tau_1|}. \tag{2.6}
\]

Therefore, the width due to the collisions is approximately given by \(\eta\). For electrons moving with the mean velocity, \(\eta\) is equal to \(\langle \Delta \mathbf{v}^2 / \langle \mathbf{v} \rangle \rangle\), where \(\langle \Delta \mathbf{v}^2 \rangle / \langle \mathbf{v} \rangle\) being the average rate at which moving electrons are slowed down by the distant collisions. Here we use the value of \(\langle \Delta \mathbf{v}^2 \rangle\) calculated for a plasma in the absence of a magnetic field, although it must be admitted that for electrons gyrating under a strong magnetic field the exact value is likely to be different from it. Considering only electron-electron collisions we get with the aid of the result obtained by Chandrasekhar

\[
\eta \approx 3.5n_e T^{-3/2} \ln(D/\tau_R), \tag{2.7}
\]

where \(D\) and \(\tau_R\) are the Debye shielding radius and the Rutherford scattering radius, respectively, for an electron with mean kinetic energy. The width obtained in this way is apparently \(\tau\) times greater than the one due to the close collisions, where \(\tau = 1/\eta\) denotes the slowing down time.

As was emphasized before, the above discussions are only of a qualitative nature, so that it might be relevant here to add some remarks about the assumptions we have made, thereby pointing out possible improvements in the theory. In the first place, the radiation emitted from individual electrons has been assumed to be entirely incoherent, the correlation between the motions of electrons resulting from mutual collisions ignored. It seems that, if taken into account, this will make the width narrower. Secondly, we have taken the expression (2.5) for \(\langle \mathbf{v}(\tau_1) \cdot \mathbf{v}(\tau_2) \rangle_{\mathbf{A}'}\), that is, we have from the start assumed the Lorentzian type of resonance shape, neglecting the higher moments. Also, diffusion in velocity space has not been considered. Finally, an approximate value has been used for \(\eta\). In order to improve the theory it is
necessary to carry out the detailed analysis of binary collisions under a magnetic field and to find the phase shift of the cyclotron radiation during a collision. We may note that this will enable us at the same time to see how the Larmor gyration would affect the spectrum of the bremsstrahlung. This problem is of some interest because both kinds of radiation compete with each other in usual cases.

As a last remark, we add that it would be worthwhile to evaluate in a more satisfactory manner the autocorrelation of electron velocities, which is known to be of importance in the statistical mechanics of irreversible processes.

3. CYCLOTRON RADIATION INDUCED BY PLASMA OSCILLATIONS

In Eq. (2.1), \( E(\theta) \) in the right hand side may be a space charge wave propagating with frequency \( \omega \) and wave vector \( \mathbf{k}_L \):

\[
E = E_L \exp [i(\mathbf{k} \cdot \mathbf{r} - \omega t)].
\] (3.1)

If the angle between \( \mathbf{k}_L \) or \( E_L \) and \( \mathbf{H}_0 \) is \( \theta_L \), \( \omega \) obeys the dispersion relation

\[
\omega^2 = \left( \frac{3kT}{m} \right) \frac{\sin^2 \theta_L}{\omega^2 - \omega_c^2} + \frac{\cos^2 \theta_L}{\omega^2}.
\] (3.2)

which is an extension of the result obtained by Newcomb for zero temperature.

The electric field (3.1) causes a distorted helical motion of an electron which in turn radiates electromagnetic waves. The \( n \)th Fourier component of the vector potential of the radiation field is given by

\[
A_n = (\omega/2\pi \epsilon R_0) e^{ikr \cos \theta} (\omega/\epsilon \mathbf{r}) d\mathbf{r},
\] (3.3)

where the integral goes over a gyration period; \( \mathbf{k} \) is the wave vector of the emitted radiation and makes an angle \( \theta \) with respect to \( \mathbf{H}_0 \).

The expression for the radiation intensity in a general direction is so complicated that here we give only the intensities in three special directions. We choose the \( z \) axis along \( \mathbf{H}_0 \) and the \( x \) axis parallel to \( \mathbf{H}_0 \times E_L \). The azimuthal angle, \( \phi \), of \( \mathbf{k} \) is measured from the \( x \) axis.

(1) For \( \phi = 0 \), the angular distribution for the \( n \)th harmonic is expressed as

\[
\frac{dI_n}{d\Omega} = \frac{n^2 e^2}{2\pi^2 m^2} \left[ \omega^2 \sin^2 \theta_L |J_n(x)|^2 + \left( \frac{\alpha}{\omega^2 - \omega_c^2} \sin \theta_L \cos \theta - \cos \theta_L \sin \theta \right)^2 \right] \left( \frac{\omega}{\omega^2 - \omega_c^2} \right)\frac{n^2}{x^2} |J_n(x)|^2,
\] (3.4)

with

\[
x = \frac{neE_L}{m\omega} \left( \frac{\omega^2 - \omega_c^2}{\omega^2} \sin \theta_L \sin \theta + \cos \theta_L \cos \theta \right).
\] (3.4')

(2) For \( \phi = \pi/2 \), we have

\[
\frac{dI_n}{d\Omega} = \frac{n^2 e^2}{2\pi^2 m^2} \left[ \left( \frac{\omega^2}{\omega^2 - \omega_c^2} \sin^2 \theta_L \cos^2 \theta + \left( \frac{\omega^2}{\omega^2 - \omega_c^2} \sin \theta_L \cos \theta \right)^2 \right) \frac{n^2}{x^2} |J_n(x)|^2 \right]
\] (3.5)

with

\[
y = \frac{neE_L}{m\omega} \left( \frac{\omega^2 - \omega_c^2}{\omega^2} \cos \theta_L \cos \theta + \frac{\omega^2}{\omega^2 - \omega_c^2} \sin^2 \theta_L \sin^2 \theta \right), \quad \alpha = \tan^{-1} \left( \frac{\omega^2 - \omega_c^2}{\omega^2} \cot \theta_L \cot \theta \right).
\] (3.5')

(3) For \( \theta = \pi/2 \), we have

\[
\frac{dI_n}{d\Omega} = \frac{n^2 e^2}{2\pi^2 m^2} \left[ \left[ \cos^2 \theta_L \cos^2 \beta + \left( \frac{\omega^2}{\omega^2 - \omega_c^2} \sin \theta_L \omega \sin \phi \cos \beta \right)^2 \right] \frac{n^2}{x^2} |J_n(x)|^2 \right]
\] (3.6)

with

\[
z = \frac{neE_L}{m\omega} \left( \frac{\omega^2}{\omega^2 - \omega_c^2} \cos \theta + \frac{\omega^2}{\omega^2 - \omega_c^2} \sin^2 \phi \right), \quad \beta = \tan^{-1} \left( \frac{\omega/\omega_c \cot \phi} {\alpha} \right).
\] (3.6')

In this case, however, there occur complicated stop bands, which should be taken into account when our results are compared with observations.

ACKNOWLEDGEMENTS

Our thanks are due to Professor S. Nakajima and Dr. M. Yokota for their critical but helpful discussions.

REFERENCES


Plasma Diagnostic Developments in the UCRL Pyrotron Program

By C. B. Wharton, J. C. Howard and O. Heinz

At the outset of the Berkeley and Livermore programmes in the study of dense, high temperature, transient plasmas of large physical extent, it was apparent that the existing diagnostic techniques could not yield a complete picture of physical processes in the plasma. High temperature extensions of diagnostic theories were not understood. Few theories had been substantiated experimentally. Worse yet, some of the instruments which had become standards for plasma research (such as Langmuir probes and probing electron beams) introduced such serious perturbations by their presence that the validity of the results was questionable.

This paper describes techniques developed in connection with the Pyrotron programme to fill in some of these voids.

SURVEY OF DIAGNOSTIC METHODS

During the course of the various experiments numerous diagnostic approaches were tried. It would require considerably more space than the scope of this paper permits to present explicit discussions of more than a representative group of these techniques. However, for completeness, a brief tabulation of essentially all of the important techniques tried is presented in Tables 1, 2 and 3. A more complete discussion of the most commonly used methods is presented in the remaining sections of this paper and in a group of papers, P/377–380, of the preceding session, A-9.

Microwave Interactions

Diagnostic techniques involving microwave interactions with plasmas have been in use at this laboratory since 1952. Our approach has been to measure the phase shift, the absorption and the scattering of a wave propagating through the plasma region, determine the plasma spatial distribution by a second method (light emission, probes, etc.) and calculate the electron density and collision frequency. Simultaneously the “white” noise radiation from the plasma is monitored by a calibrated microwave superheterodyne receiver. If the opacity or geometrical conditions of the plasma are determinable and if the electron energy distribution is known, a meaningful value for electron kinetic temperature may be calculated.

Since a plasma is highly dispersive in the microwave part of the spectrum the chosen frequency is determined by the plasma density and size. The diversity of experiments undertaken in the Pyrotron programme has necessitated utilizing microwave equipment in the following frequency bands: 8–12 kMc, 19–26 kMc, 32–48 kMc and 68–77 kMc (kMc = kilomegacycles—10⁹ cycles per second). Very recently some experiments at 130–150 kMc have been contemplated. In order to determine density and collision frequency, the plasma must be transparent. In order to obtain sufficiently intense radiation from the plasma to be measurable the region should be nearly opaque. Therefore, in order to make both measurements simultaneously, they must be performed at different frequencies.

The pyrotron configuration permits a variety of modes of coupling into the discharge. Transmission across the discharge region with the rf electric field either parallel or perpendicular to the machine magnetic field allows coupling with or decoupling from the electron cyclotron interactions. Propagation along the magnetic lines with circularly-polarized waves allows coupling to either the σ = σ₁ or σ + σ₁ modes. The inhomogeneity of the magnetic field is usually a source of ambiguity but in some cases it permits the position of the plasma boundary to be determined by observing the location at which gyroresonance occurs.

Figure 1 shows a sketch of a typical microwave diagnostics system. To measure absorption, the 60-foot reference path is removed and the klystron is modulated by random noise in order to average out any internal reflections within the discharge chamber. The noise spectrum is some 200 Mc wide.

The radiation receiver is a wide-band (10 Mc) receiver with a threshold sensitivity of about 0.1 micro-microwatt. Detected noise is viewed directly, without time averaging, to preserve the wide bandwidth.

In order to seek correlation between the microwave and other measurements, a control experiment has...
been devised. Plasma is generated within a stable pig (Philips Ion Gauge) configuration by passing pulses of 4–8 msec duration, 0.1–2 amp magnitude, 60 times per second through a gas (He, H₂, A, etc.) at pressures ranging between 0.1 and 20 μ Hg. Electron densities as high as 10¹⁰ per cm² are easily obtained, and electron kinetic temperatures as high as 25 ev have been measured. Figure 2 shows a sketch of the equipment and Fig. 3 shows a composite oscilloscope display of data. It is apparent that the microwave radiation appears only when the plasma becomes opaque,² even though the electron temperature remains high during a large fraction of the pulse.

The opacity dependence of the radiation has been demonstrated also in the high compression pyrotrons. The right-hand circularly polarized wave suffers large absorption by the plasma, and the right-hand circularly polarized wave suffers large absorption by the plasma at a frequency slightly below the gyrofrequency, even when the density and collision frequency are relatively low. The absorption (and radiation, by Kirchhoff’s law) cross sections due to

<table>
<thead>
<tr>
<th>Plasma diagnostic technique</th>
<th>Employed ion</th>
<th>Experimental application and comments</th>
<th>Performance valuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ion energy analyzer</td>
<td>t-t</td>
<td>Magnetic momentum analysis, followed by electrostatic energy analysis. Min. ion current 1 ma; ion energy range 16 ev to 20 kev. Yields distributions only if conditions are reproducible from discharge to discharge.</td>
<td>B-1</td>
</tr>
<tr>
<td>Time-of-flight measurement</td>
<td>Q-C</td>
<td>Velocity measurement of plasma burst from pulsed source using probe or microwave sampling.</td>
<td>A-1</td>
</tr>
<tr>
<td>Calorimetric measurement</td>
<td>P-4</td>
<td>Measures temperature increase of target when immersed in plasma. Employs thermocouples, optical instruments, or rate of deterioration of target.</td>
<td>D</td>
</tr>
<tr>
<td>Fluorescent screen</td>
<td>ALB, Fel.</td>
<td>Determination of plasma position and ion orbit sizes. Qualitative density measurement. Time-resolved or integrated presentation.</td>
<td>A-2</td>
</tr>
<tr>
<td>Fluorescent screen</td>
<td>t-t</td>
<td>Determination of plasma position and ion orbit sizes. Qualitative density measurement. Time-resolved or integrated presentation.</td>
<td>B-2</td>
</tr>
<tr>
<td>Charged particle collectors</td>
<td>All</td>
<td>Faraday cups, biased and unbiased to determine particle escape rates. Single and double probes to determine ion and electron densities.</td>
<td>C-2</td>
</tr>
<tr>
<td>Neutral particle detector</td>
<td>Fel.</td>
<td>Determines high energy neutral particle flux by measurement of secondary electron current. Used for determination of charge exchange loss rate.</td>
<td>B-2</td>
</tr>
<tr>
<td>Optical spectrum analysis</td>
<td>All, Fel.</td>
<td>Doppler shift and broadening measured by spectrograph or interferometer. Determines random and collective ion velocities. Stark broadening to determine ion density. Spectral line identification.</td>
<td>B-2</td>
</tr>
</tbody>
</table>

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**Optical Observations**

Spectroscopic measurements have been made, wherever possible, in connection with the pyrotron experiments. The general procedure has been to identify first the light emitters in a particular plasma by means of photographic spectra covering as wide a frequency range as is feasible. Next, the time dependence of atomic or molecular features to which some particular interest is attached, e.g. those of D, He⁺, C⁴⁺⁺, Ti⁺, C₂, CH, etc., is obtained. An attempt is then made to learn something about the conditions in the plasma from spectral line intensity ratios or the intensity distribution within a given line. Often the thermal Doppler broadening in the case of the
ionized helium emission at $\lambda 4686$ Å can provide a direct measure of the mean kinetic energy of the ions.

For atoms which exhibit first-order Stark effect, some idea of the ion density often can be derived from measurements on the extent of the profile wings. Furthermore, it should be possible to get an indication of the magnitude of magnetic fields existing in a plasma from the Zeeman splitting. Use has been made of such auxiliary apparatus as a Fabry-Perot étalon, for obtaining line profiles, and electro-optic shutters and rotating mirror or drum cameras for achieving time resolution. However, nearly all of the optical observations have been plagued by one common problem: low light output when the temperature is high.

The method of electron temperature determination by observation of the degree of ionization attained is straightforward if the pertinent cross sections and transition probabilities are known. Observation of the relative intensities of ion and neutral emission lines presents enough information to at least bracket the electron temperature. Theories for $\text{He}^+/\text{He}$ concentrations, although not complete in detail, have permitted interpretation of measured intensity ratios, to yield results which are in semiquantitative agreement with microwave temperature determinations on the diagnostics correlation experiment. The lines viewed were $\text{He}^+\, \lambda 4686$ Å and $\text{He}\, \lambda 5015$ Å.

Because of the high ion densities ($> 10^{14}$/cm$^3$) and the high electron kinetic temperatures (1–6 kev) achieved in the Saturn experiment, the high light output during initial ionization makes possible some interesting optical measurements. A determination of ion density by observation of Stark broadening of the H$_\alpha$ line was accomplished by utilizing a Wratten No. 25A filter, as a monochromator, and a Fabry-Perot interferometer. The fringes were observed with a camera directly and also through an image converter to permit time resolution of the fringe broadening. Kodak No. IF-3 photographic plates provided sufficient sensitivity and yet small enough grain size to facilitate microscopic scanning. Since the optical system is all transparent, the camera is able also to observe the interior of the discharge region to record the location from which radiation emanates. Figure 6 illustrates the experiment geometry and Fig. 7 shows the broadened fringes with the discharge chamber visible through the interferometer. Apparently the light comes from a ball at the center of the chamber.

A composite data analysis is shown in Fig. 8. The microwave observations were of the same nature as described previously. No electron energy analysis was made for Saturn, but a Maxwellian distribution is to be expected after the first 100 microseconds or so.

Streak camera pictures were taken, looking through a slot into the end of the experiment to observe the radial extent of the light-emitting region as a function of time. A contraction was clearly visible until the light intensity dropped below the threshold.
Table 2. Plasma Electron Diagnostics

<table>
<thead>
<tr>
<th>Diagnostic technique</th>
<th>Employed on</th>
<th>Experimental application and comments</th>
<th>Performance evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical measurements</td>
<td>P-4, L-P</td>
<td>Electron temperature by measurement of intensity ratio of spectral lines of ionized and neutral helium in plasma. Bracketing of electron temperature by calculation of degree of ionization. Hot plasmas have low light intensity.</td>
<td>D</td>
</tr>
<tr>
<td>Current collecting probes</td>
<td>Q-C, P-4, L-P</td>
<td>Electron temperature and density by measurement of characteristics of single and double probes. Distribution profiles. Perturbs plasma badly.</td>
<td>C-3</td>
</tr>
<tr>
<td>X-ray energy analysis</td>
<td>t-t</td>
<td>Collimated detector behind graded absorbers. High energy response limited by crystal thickness.</td>
<td>B-1</td>
</tr>
<tr>
<td>Nuclear plates</td>
<td>t-t, T-T, Sat.</td>
<td>Samples energy and density of escaping electrons by means of nuclear emulsions behind graded absorbers. Scanned for track length and density. Also useful for X-rays and heavy particles.</td>
<td>A-1</td>
</tr>
<tr>
<td>Emitted microwave radiation</td>
<td>Gup, T-T, Sat.</td>
<td>Time-resolved kinetic electron temperature by measurement of intensity of radiation. Requires knowledge of microwave refractive index and opacity. Min. detectable temp. ~0.5 ev.</td>
<td>A-2</td>
</tr>
<tr>
<td>Microwave transmission and reflection</td>
<td>All but Fel.</td>
<td>Time-resolved average electron density and collision frequency. Approx. spatial extent and density distribution may be inferred. Index and opacity determined directly. 10^9 to 10^14 electrons/cm^3 measurable.</td>
<td>A-2</td>
</tr>
</tbody>
</table>

Miscellaneous Techniques

The theories of probes in transient plasmas confined by magnetic fields are not well understood. However, for some conditions, in which the orbits are larger than the sheath dimensions and no collisions occur in the sheath, the ordinary probe theories yield results which are reasonable. A method by which the probe V-vs.-I characteristic is displayed on an oscilloscope has been developed, utilizing a sawtooth sweep voltage of variable period and amplitude. The probe current is transferred from the high voltage environment by means of a transistorized FM telemeter unit of subminiature construction. This probe system has been applied to the diagnostics correlation experiment with moderate success. Attempts to utilize probes in high compression experiments have failed, in general, for two reasons: first, the probe introduces serious perturbations and, second, under extreme conditions the probe melts.

A very simple diagnostic tool, which utilizes one of the shortcomings of a probe, is the floating ball. A small sphere (~1 mm diameter) of a refractory metal

Table 3. Miscellaneous Diagnostic Techniques

<table>
<thead>
<tr>
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<th>Employed on</th>
<th>Experimental application and comments</th>
<th>Performance evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ion resonance absorption</td>
<td>Gup.</td>
<td>Ion identification (e/m ratio). Gyroresonance absorption from electromagnetic field by plasma at frequencies, ( \omega_i = Q_i B / 2m_i ).</td>
<td>D</td>
</tr>
<tr>
<td>Current loops and probes</td>
<td>T-T</td>
<td>Indicate changes of magnetic field due to presence of plasma (diamagnetism).</td>
<td>B-3</td>
</tr>
<tr>
<td>Fast shutter photography</td>
<td>Sat., T-T</td>
<td>Location and relative intensity of plasma light at various times. Min. exposure time ~0.1 ( \mu )sec. Requires intense light.</td>
<td>B-3</td>
</tr>
<tr>
<td>Streak photography</td>
<td>Sat., T-T</td>
<td>Time-resolved portrayal of relative intensity and one-dimensional extent of plasma light, either monochromatic or general. Requires intense light.</td>
<td>B-2</td>
</tr>
</tbody>
</table>

a, b Codes at bottom of Table 1.
Figure 4. High compression pyrotron magnetic field variation with time and corresponding typical microwave radiometer response. $B_c$ is the field strength necessary to produce gyro-resonance.

Figure 5. Radiometer response and corresponding absorption of a noise-modulated transmitted signal for a pyrotron, to demonstrate the relationship between radiation intensity and opacity. Scan 0.5 msec/div; peak temperature, $\sim 25$ kev.

Figure 6. "Saturn" experimental configuration, showing diagnostic attachments.

Figure 7. Interference fringes for H$_\alpha$ line in the Saturn experiment. Stark broadening indicates an ion density of $\sim 2 \times 10^{14}$/cc. Source of light is visible through the interferometer.

Figure 8. Saturn experiment composite data presentation. Electron temperature obtained from microwave radiometers at various frequencies is fused to a slender quartz rod and inserted into the plasma. The temperature attained by the ball is an indication of the total energy transferred by the collisions it suffers, which bears a relationship to the plasma thermal energy $nKT$, when the ball is electrically neutral.
Mr. Wharton presented Paper P/381, above, at the Conference and added the following information on recent measurements of microwave transmission through plasmas and microwave radiation from plasmas.

It has been demonstrated that a plasma will absorb microwaves of a frequency equal to the electron cyclotron resonance frequency. This “gyroresonance” absorption is observed in addition to the plasma microwave absorption characteristics already reported (Fig. 3).

The plasma also emits radiation at the absorption frequencies. Figure 9 shows the variation of the radiated noise intensity as the magnetic field in the discharge chamber is varied. The peak amplitude occurs when the cyclotron resonance frequency and the characteristic frequency of the receiver coincide.

With proper geometry corrections and receiver calibration, the peak amplitude can be related to the average electron temperature. Since the receiver responds linearly with the radiated electric field, the observed signal should vary as the square root of the temperature. The signal amplitude in Fig. 9 corresponds to an electron temperature of roughly 100 ev.

In theory, if the magnetic field were uniform, the width of the resonance absorption curve should bear a relationship to the ratio of the plasma energy density to the magnetic energy density, i.e. the $\beta$ of the plasma, and also be proportional to the product of electron density and temperature. This suggests a method of determining the $\beta$ factor for electrons.

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The plasma also emits radiation at the absorption frequencies. Figure 9 shows the variation of the radiated noise intensity as the magnetic field in the discharge chamber is varied. The peak amplitude occurs when the cyclotron resonance frequency and the characteristic frequency of the receiver coincide.

With proper geometry corrections and receiver calibration, the peak amplitude can be related to the average electron temperature. Since the receiver responds linearly with the radiated electric field, the observed signal should vary as the square root of the temperature. The signal amplitude in Fig. 9 corresponds to an electron temperature of roughly 100 ev.

In theory, if the magnetic field were uniform, the width of the resonance absorption curve should bear a relationship to the ratio of the plasma energy density to the magnetic energy density, i.e. the $\beta$ of the plasma, and also be proportional to the product of electron density and temperature. This suggests a method of determining the $\beta$ factor for electrons.

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Microwave Studies of Gas Discharge Plasmas

By S. C. Brown*

DIAGNOSTICS IN ABSENCE OF MAGNETIC FIELD

The theory of microwave diagnostics of plasmas not in a static magnetic field makes use of the fact that in such a plasma the effective dielectric coefficient of the plasma is given by

\[ K = 1 - \eta \left( \frac{1 + j \beta}{1 + \beta^2} \right) = K_{re} - j K_{im} \]  

(1)

where \( \eta = \omega_p^2/\omega^2 \); \( \omega_p \) is the plasma frequency, \( \omega \) is the applied radian frequency while \( \beta = v_e/\omega \), where \( v_e \) is the electron collision frequency. This analysis is generally limited to the case \( \eta < 1 \). When free space propagation is used as the diagnostic tool, Eq. (1) relates the relative phase shift (radians/wavelength), and the attenuation of the wave (nepers/wave)

\[ \Delta \theta = 2 \pi \left( 1 - \frac{1}{\sqrt{2}} \left( \frac{|K| + K_{re})}{1 + \beta^2} \right) \right)^1 \]

(2)

and the attenuation of the wave (nepers/wave)

\[ \alpha = 2 \pi \sqrt{2} \left( \frac{|K| - K_{re})}{1 + \beta^2} \right)^1 \]

(3)

to \( v_e \) and the plasma density (Fig. 1). Experimentally, only that portion of the phase shift curve which is linear with \( \eta \), and for which the attenuation is negligible, is easy to interpret. This corresponds, as can be seen in Fig. 1, to low values of \( \eta \) which, for high-density plasma, means the use of very high-frequency microwaves. This has been used as a diagnostic tool using millimeter wavelengths and measuring the phase shifts by interferometric methods.

When a microwave cavity is the diagnostic tool, the shift in the resonance frequency of the cavity,

\[ \left( \frac{\Delta f}{f} \right) = \frac{1}{2} \int_{c}^{p} \frac{1}{E^2 du} = \frac{1}{2} \int_{c}^{p} \frac{\eta E^2 du}{E^2 du} \]

(4)

and the change in its loaded \( Q \) value,

\[ \Delta \left( \frac{1}{Q} \right) = \frac{1}{2} \int_{c}^{p} \frac{K_{im} E^2 du}{E^2 du} = \frac{\beta}{1 + \beta^2} \int_{c}^{p} \frac{\eta E^2 du}{E^2 du} \]

(5)

where \( p \) and \( c \) denote integration over the plasma and cavity, are related to \( \eta \) and \( \beta \). The cavity method is also usually restricted to plasmas for which \( \eta \) is less than unity, although by the use of special configuration of the microwave field, (E field everywhere perpendicular to the density gradient), the method can be extended to plasmas for which \( \eta \) is greater than unity. In the low-density plasmas, measuring the real and the imaginary dielectric coefficients leads to the collision frequency of electrons with gas atoms. Measuring the frequency shift in the afterglow has led to many determinations of recombination and diffusion processes.

![Image of Eq. 1](https://via.placeholder.com/150)

DIAGNOSTICS IN PRESENCE OF MAGNETIC FIELD

The application of a static magnetic field to a plasma complicates the theory in that the dielectric coefficient of the plasma becomes a tensor function of the magnetic field. This implies that the refractive index of the plasma depends not only on the electron density and on the magnitude of the magnetic field, but also on the geometrical configuration of the microwave mode and its direction of propagation with respect to the direction of the static magnetic field. As an example, we quote the effective dielectric coefficients of the simplest problem encountered in the interaction of microwaves with a magnetized plasma, namely, the propagation of a plane wave in an infinite uniform plasma.

Propagation along the B field:

1. Right-handed circularly polarized plane wave

\[ K_r = 1 - \eta \frac{1}{(1 - \gamma) - j \beta} \]

(6)

2. Left-handed circularly polarized plane wave:

\[ K_l = 1 - \eta \frac{1}{(1 + \gamma) - j \beta} \]

(7)

Propagation perpendicular to the B field.

3. E-field parallel to B field:

\[ K_z = 1 - \eta \frac{1}{1 - j \beta} \]

(8)

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4. *E*-field perpendicular to *B* field:

\[
K_{\perp} = \frac{2K_xK_y}{K_x + K_y} = 1 - \frac{\omega_p^2 - \omega_0^2}{\omega_0^2 + \omega_0 - \omega}; \quad \nu_c = 0. \tag{9}
\]

Here \(\omega_p\) is the plasma frequency, \(\omega\) the applied radian frequency, \(\omega_0\) the cyclotron frequency, and \(\nu_c\) the collision frequency. In Eq. (9) there is also a component of \(E\) field along the \(B\) field so that \(\text{div} \ E\) is not zero.

In a guiding structure such as a microwave cavity, the dielectric coefficient must be represented by one or more of the coefficients above. As a result, it is not possible, even for low electron densities, to obtain a general theory in a form suitable for experimental verification and use in the microwave diagnostics of a magnetized plasma, and valid for all possible configurations of the microwave field. Consequently, we shall analyze only a few special configurations of the microwave field. We restrict the discussion to a narrow cylindrical plasma column placed coaxially in a cylindrical microwave cavity. We shall consider the behavior of the following modes: the \(\text{TM}_{111}\), \(\text{TE}_{011}\), and \(\text{TM}_{020}\) or \(\text{TM}_{010}\). The static magnetic field is in all cases applied along the axis of the cavity.

**Examples**

\(\text{TM}_{111}\) Mode (degenerate modes)

*Low electron densities*—When the measuring mode is degenerate in frequency, the nonisotropic plasma removes the degeneracy. The physical reason for this can be seen when one considers the \(\text{TM}_{111}\) mode. When the radius of the plasma column is small compared with the cavity radius, the \(E\) field of the \(\text{TM}_{111}\) mode can be considered to be linearly polarized in the plasma region. A linearly polarized field can in turn be considered composed of two circularly polarized fields rotating in opposite directions. As mentioned earlier, the refractive index of the plasma is different for the two fields. As a result, the resonant frequency of the cavity splits into two frequencies, the frequency shifts being given by

\[
\left(\frac{\Delta f}{f}\right)_+ = \frac{a}{2^3} \frac{(1-\gamma)}{(1-\beta^2)\gamma^2 + \beta^2};
\]

\[
\left(\frac{\Delta f}{f}\right)_- = \frac{a}{2^3} \frac{(1+\gamma)}{(1+\beta^2)\gamma^2 + \beta^2};
\]

\[\gamma \ll 1 \tag{10}\]

where \(a\) is a geometrical constant, and \(\gamma\) is the ratio of the cyclotron frequency to the applied radian frequency, \((\omega_p/\omega)\). Note that \((\Delta f/f)_-\) is always larger than zero while \((\Delta f/f)_+\) is greater or smaller than zero, depending upon whether \(\gamma\) is smaller or larger than unity. The ratio of the frequency shifts is independent of the electron density and when \(\beta\) is very much smaller than 1 and \(\gamma\) is not equal to 1, depends only on the static magnetic field. Since the magnetic field in the plasma is usually known, this ratio serves as an experimental check on the validity of Eq. (10). The frequency shifts given by Eq. (10) are shown in Fig. 2(a) for the particular case of \(\gamma^2 = \frac{1}{2}\) and \(\beta \ll 1\).

The larger inset circle in Fig. 2(a) represents the physical boundaries of the resonant cavity; the
smaller dotted circle, a cross section of the plasma; and the set of solid lines, the electric field: the magnetic field is perpendicular to the plane of the figure. In Fig. 2(a) the frequency shift is shown as a function of the electron density, assuming constant magnetic field strength. Equations (10) can also give the curves of Fig. 2(b), in which the frequency shift is plotted as a function of magnetic field strength, assuming constant electron density.

High electron densities—When the electron densities are high, \( n_p > n_e \), the relations for the frequency shift are invalid mainly because the field in the plasma cannot be approximated, at high electron density, by the field in the absence of a plasma. The approximation made in the former case is to consider only one diagonal component of the tensor needs to be considered. For very low pressures, the frequency shift becomes

\[
\frac{\Delta f}{f} = a \frac{1}{2(1 - \gamma^2)}
\]

When pressures are not negligible, the frequency shift is

\[
\frac{\Delta f}{f} = a \frac{\omega_p^2 + \omega_e^2 - \omega_0^2}{[\omega_0^2 + (\omega + \omega_e)^2][\omega_0^2 + (\omega - \omega_e)^2]}
\]

Equation (12) exhibits an interesting property of the TE\(_{011}\) mode; namely, that the frequency shift is zero when \( \omega_0^2 = \omega_e^2 + \omega_0^2 \), independent of the electron density. This can be used as a direct measure of \( \omega_e \).

High electron densities—The TE\(_{011}\) mode is the mode which is ideally suited for measurements of high electron densities in the absence of static magnetic fields. This is so because the azimuthal field of this mode does not excite an ac space charge in an axially symmetric plasma. In the presence of a static magnetic field, this is no longer true. The nonisotropic nature of the plasma causes radial currents and radial fields. These contribute to the frequency shift. A first-order correction to the perturbation formula can be obtained by using a pseudostatic approximation in order to compute the radial microwave field in the plasma. In the limit when the radius of the plasma is negligible compared with the free space wavelength, the field outside the plasma is assumed unchanged. Matching normal components of the displacement \( D \) across the plasma boundary yields an equation for \( E_r \) in terms of \( E_\theta \).

\[
K_T E_r + K_p E_\theta = 0.
\]

Using this, the following expressions are obtained for

\[
\frac{\Delta f}{f} = a \frac{\omega_p^2 + \omega_e^2 - \omega_0^2}{[\omega_0^2 + (\omega + \omega_e)^2][\omega_0^2 + (\omega - \omega_e)^2]}
\]
the frequency shift and for the change in Q value:

\[
\left( \frac{\Delta f}{f} \right) = \frac{a}{2} \eta \left\{ \frac{(1 - \eta)(1 - \beta^2 - \gamma^2 - \eta) + \beta^2(2 - \gamma)}{(1 - \beta^2 - \gamma^2 - \eta)^2 + (2 - \eta)^2 \beta^2} \right\}, \quad (14a)
\]

\[
\Delta \left( \frac{1}{Q} \right) = a \eta \left\{ \frac{(1 - \eta)(2 - \gamma) - (1 - \beta^2 - \gamma^2 - \eta)}{(1 - \beta^2 - \gamma^2 - \eta)^2 + (2 - \eta)^2 \beta^2} \right\}. \quad (14b)
\]

Plots of the frequency shift obtained from Eq. (14a) are shown in Fig. 3(a), solid curve, and are compared with the plot obtained from the simple perturbation formula which neglects the radial fields, dashed curve. A striking feature is the oscillation in \( \Delta f/f \) and a resonance minimum in \( \Delta(1/Q) \). The magnitude of the resonance is larger the smaller \( \nu_c \) is, as shown in Fig. 3(b). We are also interested in the behavior of the \( Q \) value since this controls the sensitivity of the cavity method. A calculation of the \( Q \) as a function of the electron density leads to the results shown in Fig. 4. Here we see a resonant maximum in \( \Delta(1/Q) \). In the vicinity of the resonance, the \( Q \) value of the cavity is so low that accurate measurements of \( \Delta f/f \) are difficult. A quantitative experimental verification of Eqs. (14) has not yet been made although a resonance in the \( Q \) value of the cavity has been observed. Although Eqs. (14) may predict qualitatively the behavior of \( \Delta f/f \) and \( \Delta(1/Q) \) with the electron density, it is known that quantitatively they are not correct. In a nonisotropic medium, a pure TE mode is impossible. This is also true for the region of the cavity outside the plasma. In the present case, the field in the cavity and in the plasma is some superposition of the fields of the \( \text{TE}_{011} \) and the \( \text{TM}_{020} \) modes. An exact solution of this problem is possible. It results in a transcendental equation for the complex resonant frequencies of the cavity which is in the form of a 6x6 determinant, and it must be computed by numerical means.

\( \text{TM}_{010} \) (E parallel to B)

Low electron densities—When the microwave mode is such that the E field is parallel to the B field, the effective dielectric coefficient is given by Eq. (9). The frequency shift is independent of the magnetic field in the limit of zero electron temperature:

\[
\left( \frac{\Delta f}{f} \right) = a \eta \frac{1}{1 + \beta^2}. \quad (15)
\]

The solution is shown as “Perturbation Theory” in Fig. 5.

High electron densities—Since the E field of these modes can be made to coincide with the direction of the static magnetic field, the \( \text{TM}_{020} \) modes do not suffer from the disadvantages which the nonisotropic nature of the plasma imposes on all other modes which have a component of the E field at right angles to the B field. The \( \text{TM}_{020} \) modes, although not as ideal as the \( \text{TE}_{011} \) mode, are also well-suited for measuring high electron densities in those plasmas which do not possess density gradients in the axial direction. The disadvantage of the \( \text{TM}_{020} \) modes lies in the fact that the shift in the resonant frequency of the cavity is large when the plasma density is high. Consequently, the perturbation formula is again inadequate and exact analysis must be resorted to. In this case, however, this is fairly straightforward, and Fig. 5 shows the frequency shift of a \( \text{TM}_{020} \) mode cavity as a function of plasma density, assumed uniform, whose radius is 1/10 that of the cavity.

**Measurement of Electron Density Distribution**

Since the E fields of the three modes discussed have different radial and axial functional dependence, the simultaneous use of two of the three modes yields information about the electron density distribution along the appropriate directions. Figure 6 is a summary of the modes which have been discussed. The use of the \( \text{TM}_{111} \) mode, \( E \) approximately constant, and the \( \text{TE}_{011} \) mode, \( E \) varying as \( R \), yields information about the density distribution along the radius, while the use of the \( \text{TM}_{111} \) mode, \( E \) varying as \( \sin (\pi z/L) \), and the \( \text{TM}_{010} \) mode, \( E \) approximately constant, gives the distribution along the axis of the plasma. By using resonant cavities which support all these modes at once, measurements of the electron density distributions are fairly straightforward and are convenient experimental methods of determining the electron density distribution.
The Dissociation of Diatomic Hydrogen Ions

By C. F. Barnett*

The thermonuclear effort at the Oak Ridge National Laboratory has as a primary interest the development of methods for the trapping of a deuteron beam in a container employing magnetic mirrors. Luce\textsuperscript{1, 2} has proposed that deuterons could be placed in a stable orbit by dissociating diatomic ions with a concentrated gaseous discharge in the form of a high-current arc. With a good efficiency of dissociation of the diatomic ions, an energetic plasma may be formed provided the energy of the trapped particles is large enough to insure that the electron capture cross section is as low as about 10\textsuperscript{-19} cm\textsuperscript{2}, and provided that the neutral particle density in the background gas can be reduced in accordance with the possibilities of modern vacuum technique. The use of dissociation for orbital injection has also been independently proposed for high-energy accelerators,\textsuperscript{3} although the use of an arc as an agent for the dissociation does not appear to have been contemplated in that connection.

The promise of such a proposal and the choice of the best energy for the injection of the ions into the trapping geometry depend upon certain interaction cross sections of D\textsubscript{2}\textsuperscript{+} ions which heretofore have not been measured.\textsuperscript{4} General information on the behavior of the D\textsubscript{2}\textsuperscript{+} dissociation cross section by collisions in various gases is of physical interest, but more direct importance attaches to measurement of the probability that a D\textsubscript{2}\textsuperscript{+} ion be dissociated in passage through an arc column, and it is also important to know the probability that a D\textsuperscript{+} ion capture an electron in passing through the arc, because this represents a loss mechanism for the trapped beam. In this paper we describe the measurement of these quantities in the energy range 20 keV to 2.25 MeV. H\textsubscript{2}\textsuperscript{+} ions were used rather than D\textsubscript{2}\textsuperscript{+}; under the assumption that the cross sections will be equal at equal velocities. The first part of the paper describes the measurement of H\textsubscript{2}\textsuperscript{+} dissociation cross sections, and the second part describes the experiments in which the ion beams were shot through the column of a 300-ampere vacuum carbon arc.

DISSOCIATION BY IMPACT WITH GAS ATOMS

The dissociation cross section of H\textsubscript{2}\textsuperscript{+} in collisions with gases has been studied theoretically by Salpeter\textsuperscript{5} using the Born approximation. Results are given for energies in excess of 2 MeV, which is the energy region wherein the Born approximation might be expected to be valid. The calculation indicates that dissociation is largely produced by excitation of the covalent electron into a repulsive state, and the cross section is predicted to vary inversely as the square of the particle velocity. Gerjuoy\textsuperscript{6} has treated the problem of dissociation by impact of H\textsubscript{2}\textsuperscript{+} with protons in the energy interval 10 to 500 keV. The cross section shows a maximum at 100 keV, but thereafter decreases also as 1/\textit{v}\textsuperscript{2}.

The dissociation cross section was first measured by Efstat\textsuperscript{7} for energies of 9 and 18 MeV. More recently, Fedorenko\textsuperscript{8} has measured the cross section at low energies (5-30 keV) and Damodaran\textsuperscript{9} has studied the energy interval 100-200 keV. Previous papers from this Laboratory\textsuperscript{10-12} have described the apparatus used here for the determination of atomic collision cross sections, and inasmuch as the same equipment served for the measurements of molecular ion dissociation, a detailed description will not be necessary. Suffice it to say that, as is customary in such experiments, a H\textsubscript{2}\textsuperscript{+} beam of adjustable energy was passed through apertures at the ends of a differentially-pumped gas cell, and the emergent beam was analyzed to give the currents of H\textsuperscript{+}, H\textsubscript{2}\textsuperscript{+} and neutral species.

In the low energy range (20-200 keV) of incident H\textsubscript{2}\textsuperscript{+} ions, the measurements present some difficulties, one of which arises from the fact that the neutral components H\textsubscript{2}\textsuperscript{0} and H\textsubscript{2}\textsuperscript{+} are inseparable with ordinary detectors. Under such circumstances, it is usually best to take the sum of the emergent charged beam currents, H\textsuperscript{+} and H\textsubscript{2}\textsuperscript{+} as an approximation to the incident beam current, thereby neglecting the fraction of the beam that undergoes simple electron capture and emerges as H\textsubscript{2}\textsuperscript{0}. Under this limitation, remembering that every H\textsuperscript{+} signalizes a dissociation, a "working" cross section can be recognized through the relation

\[
\frac{I}{I_0} = 1 - e^{-\sigma_a x n} \tag{1}
\]

which in this application takes the form

\[
\frac{I(H^+)}{I(H^+ + I(H_2^+))} = 1 - e^{-\sigma_a x n} \tag{a}
\]

where the \(I(H^+)\) and \(I(H_2^+)\) represent emerging ion currents, \(x\) is the length of the gas cell, and \(n\) is the gas particle density therein. Such a treatment also assumes that the dissociations proceed in the simple...
fashion \( \text{H}_2^+ + \text{M} \rightarrow \text{H}^+ + \text{H}^0 + \text{M} \). Actually, some may proceed according to the reaction \( \text{H}_2^+ + \text{M} \rightarrow \text{H}^+ + \text{H}^+ + e + \text{M} \). This process is assumed to occur infrequently; if, however, it were to equal the first process in probability, then the assigned dissociation cross sections would have to be reduced by 20 to 40%.

Experimentally, it was easy to measure \( \sigma \) by the attenuation of a proton beam sent through the gas cell, knowing the electron capture cross section from previous work. Without changing the pressure in the gas cell, the incident ions were then switched to \( \text{H}_2^+ \), and the transmitted beams were measured. The exit aperture of the gas cell and the detector aperture were made sufficiently large to accept most of the particles scattered by gas collisions or by the dissociation reaction. In addition, the particle density in the gas cell was sufficiently low to prevent multiple collisions. The results for hydrogen as a target gas are shown in Fig. 1, where the cross section for dissociation in units of square centimeters per gas atom is plotted as a function of the particle energy. It is seen that the cross section attains a maximum of \( 5.7 \times 10^{-17} \text{ cm}^2 \) per gas atom at 150 kev. Shown also are the experimental results obtained by Federenko and Damodaran which pertain also to "working" cross section in the sense used above, and for comparison the theoretical results of Gerjuoy are indicated also, despite the fact that they are absolute in character and pertain to a different type of dissociation collision. As is seen, all of these results are considerably greater than the currently determined values. In Fig. 2 are shown the results for a target gas of argon. The cross section does not attain a maximum, but is still increasing at energies of 220 kev. Again the experimental results of Federenko and Damodaran are much higher. At the present time, the reason for the large discrepancies is unknown. Considerable work has been done to distinguish between fast \( \text{H}_2^0 \) and \( \text{H}^0 \) particles by using a scintillator crystal and determining the pulse height distribution of the pulses from a phototube. This will provide means of accurately measuring the cross section at the lower energies.

![Figure 1. Dissociation cross section of \( \text{H}_2^+ \) in hydrogen gas](image1)

![Figure 2. Dissociation cross section of \( \text{H}_2^+ \) in argon gas](image2)

![Figure 3. Apparatus used to measure dissociation cross section at high energies](image3)
At energies above 500 kev the measurements become significant in an absolute sense, because of the reduction in the electron capture cross section for H$_2^+$. Figure 3 is a schematic diagram of the apparatus used in this energy region. The ion beam from a 3 MeV Van de Graaff accelerator was incident upon a differentially-pumped gas cell in which the pressure was sufficiently low to prevent multiple collisions. This pressure was determined by measuring the ratio of H° to H$^+$ in the dissociated beam. At pressures conducive to multiple collisions, the H°/H$^+$ ratio decreased as a result of the ease with which the hydrogen neutral particles lose an electron. Emerging from the gas cell was a mixture of H°, H$^+$, and H$_2^+$. Using the set of electrostatic deflection plates, the beam was analyzed into its various components. The zero detector was used as a secondary electron detector to measure both the neutral component and the charged component of the beam. The Number 1 detector was used as both a secondary electron detector and a Faraday cup. The secondary electron emission produced by H° from a brass target was corrected for the difference in the mean number of secondary electrons emitted by neutral particle impact as compared with proton impact. This correction was determined by measuring the rise in temperature of the target with only ions or atoms impinging, while at the same time measuring the secondary electron emission. The ratio of the relative secondary electron emission for hydrogen atoms and ions increased from 1.36 at the lowest energy to 1.59 at the highest energy. The total beam available for dissociation was found by summing the emergent H$_2^+$ with the number of reactions occurring, found by particle balance between H° and H$^+$. Using the same particle or charge balance, one can also determine the fraction of the reactions going by simple dissociation (i.e., H$_2^+ + M \rightarrow H^+ + H° + M$), or an ionizing dissociation (i.e., H$_2^+ + M \rightarrow 2H^+ + e + M$). From these measurements one determined the cross section from Eq. (1), in which $I$ is equal to the total number of reactions and $I_0$ equals the total H$_2^+$ available for dissociation.

To determine the extent to which scattering might be influencing the measured cross section, the exit pin hole was enlarged. The results indicated that the cross section is independent of the geometry used. Gas pressure was measured with an accurately calibrated McLeod gauge. The path length for dissociation was taken as the geometric length of the gas cell. The usual precautions were taken in regard to the purity of the target gases. Background gas and aperture collisions gave a background of less than 1% which was subtracted from the collisions occurring in the gas cell. It is estimated that the accuracy of the results should be within ± 15%.

The results obtained are shown in Fig. 4. The cross section is plotted as a function of the particle energy for target gases of hydrogen, helium, nitrogen, and argon. For hydrogen and helium, there is an approximate $v^{-1}$ dependence. For nitrogen and argon, the velocity dependence is less marked, the cross section decreasing only 15 to 20% over the entire energy range. Shown also are the theoretical results derived by Salpeter for nitrogen. The predicted theoretical values and the experimental values seem to be in agreement within a factor of two; however, the velocity dependence is seen to be quite different. Shown in Fig. 5 is the fraction of the reactions proceeding by simple dissociation (as contrasted with ionizing dissociation) for hydrogen and argon. The mode of dissociation apparently is dependent only on the target gas, and is independent of the particle energy.

**Dissociation in a Vacuum Carbon Arc**

The measurements described above indicated that the cross section for dissociation of the diatomic hydrogen ion by impact with a gas atom is a slowly varying function of the particle velocity. In a search for an efficient medium for dissociation, Luce has used a carbon arc in a magnetic field. The arc is a discharge of hundreds of amperes operating in a vacuum region at a pressure of about $10^{-5}$ mm mercury. Pre-
previous reports gave a dissociation efficiency of 40% at an energy of 20 kev. Since no adequate explanation had been advanced for the high dissociation efficiency, it was difficult to predict the dependence upon energy. Accordingly, an experiment was performed to determine the dissociation efficiency at energies ~ 600 kev.

If it is assumed that dissociation of the diatomic ions in the carbon arc is the result of particle interaction, then one may write the following as probable reactions leading to dissociation:

\[
\begin{align*}
H_2^+ + C^\text{n} &\rightarrow H^0 + H^+ + C^\text{n} \\
H_2^+ + C^\text{n} &\rightarrow 2H^+ + e + C^\text{n} \\
H_2^+ + C^\text{n} &\rightarrow 2H^+ + C^\text{n-1} \\
H_2^+ + e &\rightarrow H^0 + H^+ + e \\
H_2^+ + e &\rightarrow 2H^+ + 2e
\end{align*}
\]

where \( n \) may vary from 0 to the highest state of ionization in the arc. The first two reactions and the last two involve electron excitation, while reaction (4) is a charge exchange reaction with the carbon ion capturing the electron from the diatomic ion. The present experiment cannot distinguish between reactions (2) and (5) or between (3), (4) and (6). However, by using a charge balance between \( H^0 \) and \( H^+ \), one can determine the fraction of the reaction proceeding by (2) plus (5) and the fraction proceeding by (3) plus (4) plus (6).

For simplicity, the dissociation efficiency will be defined as the ratio of the \( H^+ \) current to the sum of the \( H^+ \) and the \( H_2^+ \) current. A schematic diagram of the apparatus is shown in Fig. 6. The ions were formed in a conventional radiofrequency ion source and accelerated to 600 kev by the ORNL cascade accelerator. Mass analysis was accomplished by means of a 90° magnetic analyzer. The analyzed ion beam was defined by a movable \( \frac{1}{2} \)-inch aperture placed directly in front of the solenoid containing the carbon arc chamber. On the exit side of the arc chamber was a fan-shaped container in which an ion detector could be moved vertically to measure the various emerging beams. The backing plate of the chamber consisted of quartz windows so that the various beams could be observed visually. The initial beam was analyzed into five separate beams by passing through the solenoid. In the region between the solenoid and the analyzing magnet a fraction of the diatomic ions was dissociated into protons and neutral atoms. The protons from the dissociation were deflected through a large angle by the solenoid field, whereas the neutral particles traveled in an undeflected trajectory. In addition to these two beams, there was also a proton and neutral particle beam resulting from dissociation in the arc. This neutral beam was separated from the other neutral beam, because the \( H_2^+ \) was deflected by the magnetic field before suffering a dissociation collision in the arc. There was also present a fraction of the initial beam of \( H_2^+ \). The dissociation process was found to take place in an extended region of the arc, so that protons and atoms were formed at different positions in the magnetic field. This resulted in a vertical spread of both the proton and neutral beams of approximately one inch, which was larger than the detector aperture.

In measuring ion beam intensities, the detector was moved at a constant linear rate across each beam by a Brown recorder. The area under the recorder trace was integrated with a planimeter to determine beam intensity. The procedure consisted in measuring in turn \( H_2^+ \), \( H^+ \), \( H^0 \), and \( H_2^+ \). When the two \( H_2^+ \) recordings differed more than 10%, the run was discarded.

A more detailed diagram of the solenoid and arc chamber is shown in Fig. 7. The chamber was an evacuated stainless steel tube, 6 ft. long and 4 in. in diameter. Surrounding this tube was the magnetic coil consisting of 7 layers of water-cooled copper tubing, each layer having 3.2 turns per in. The coil was separated in the center to provide a gap of 2 in. for the beam entrance and exit. Inside the tube was a \( \frac{3}{4} \) in. diameter water-cooled copper liner to dissipate the power radiated from the arc. The arc electrodes were Union Carbide C-18 grade graphite, pressed into water-cooled copper sleeves. Quartz tubing was placed around the cathode assembly to prevent arc-over in the fringing magnetic field region. Various anode configurations were used, and these will be discussed below. The anode and cathode assemblies were mounted so that the arc length could be varied between 6 in. and 4 ft.
The detection of the energetic particles was complicated by high radiofrequency fields produced by the arc and also by a copious supply of photoelectrons ejected from the detector and from walls surrounding the detector region by the intense ultra-violet radiation from the arc. The detector found most useful consisted of a Faraday cup and a bias suppressor ring mounted inside a completely shielded box. The beams entered through a 50 μin. nickel foil soldered over a 1/8-in. aperture. The foil was sufficiently thick to insure an equilibrium charge distribution between energetic positive ions and neutral particles emerging from the back surface. The characteristic charge distribution was independent of the initial charge state and depended only on the particle energy and foil thickness. The incident H$_2^+$ beam was immediately dissociated at the surface of the foil resulting in two particles of half energy, each of which registered with the same efficiency as the H$^0$ and H$^+$. The procedure used in striking a carbon arc was the one customarily used with shorter dc arcs. To the arc electrodes were applied 350 v dc (open circuit voltage of the four series connected welding generators which supplied the arc power) and a comparable rf voltage. When gas was admitted through a passage in the cathode, a radiofrequency arc appeared, followed immediately by the main discharge. The rf voltage and the gas input were removed as soon as the dc arc had fired. Thereafter, the arc was supported solely by carbon vaporized from the electrodes. Three hundred amperes was the customary arc current and voltages ranged from 70 to 100 v depending upon the arc length. The voltage relationship is shown in Fig. 8, in which the slope of the curve is found to be 1.0 v/in.

For a 600-kev H$_2^+$ particle, the average dissociation efficiency obtained was 10.6(±2.1, −0.9)% which may be compared to an average efficiency of 16.4 (+1.6, −1.2)% for a D$_2^+$ particle of the same kinetic energy. These efficiencies were measured at a distance of 6 in. from the arc anode. The arc conditions were: (1) 300-amp arc current, (2) 3/8-in. diameter cathode, and (3) 1/2-in. diameter anode.

Attempts to measure the velocity dependence of the efficiency with an H$_2^+$ beam were unsuccessful because of difficulties encountered with the detector at lower particle energies. If it is assumed that the deuterium ion and hydrogen ion have the same inelastic collision cross section at identical velocities, then the above values are consistent with a 1/ε dependence for the efficiency. It is not readily possible to compare these figures with those measured at lower energies in another geometry because the distance from the anode is a critical factor, as will be seen.

By making a charge or particle balance between the dissociated beams of H$^0$ and H$^+$ and the undissociated H$_2^+$, it is possible to determine a fraction of the reaction proceeding by simple dissociation or by an ionizing dissociation plus charge exchange. For 600 kev H$_2^+$ particles, the fraction going as simple dissociation was found to be 80%. For a deuterium particle, this fraction decreased to 65%. In the present experiment there could not be excluded the possibility of the ionization in the arc of some of the H$^+$ produced by dissociation in the arc.

In a magnetic containment device, it is desirable that the dissociated H$^+$ particle, with its multiple traversals of the arc, does not capture an electron from either the free electrons or the charged or neutral components of the arc. To determine the probability for electron capture, a 300-kev proton beam was passed through the arc and the appropriate region was scanned for the presence of neutral atoms. None were found, and the upper limit for the probability of capture was estimated at 10$^{-9}$. The limitations of sensitivity were determined by the detector and the noise in the detector circuit.

Numerous changes were made in the geometry of the electrodes to determine the optimum conditions for dissociation. Initially a cathode 3 in. in diameter and 10 in. long was used with 1/2 in. longitudinal bore to admit gas for striking the arc. The anode was 1 1/2 in. in diameter and 1 1/2 in. long. Visual
observation showed the arc to be diverging to a diameter of approximately 1 in. in the center of the solenoid. This swelling was a result of the divergence of the magnetic field at the 2-in. gap in the solenoid windings, which had to be provided to permit entrance and exit of the particle beam. The visual observation of the swelling in the arc was confirmed by placing a carbon disc in the center of the solenoid and measuring the diameter of the hole burned through it. The swelling was undesirable because it would be expected to reduce the degree of dissociation if the latter were due to the product of particle density in the arc and diametrical path length. Therefore an effort was made to concentrate the arc by using more slender electrodes; a decrease in cathode diameter to \( \frac{1}{16} \) in. did in fact reduce the arc diameter to \( \frac{1}{8} \) in. at the point of beam intersection. Attempts further to decrease the diameter of the cathode to \( \frac{1}{32} \) in. were unsuccessful, because the cathodes shattered when current was passed through them. Previous experiments\(^6\) had indicated that the efficiency for dissociation at low particle energy is dependent on the temperature of the anode. By decreasing the anode diameter to \( \frac{1}{16} \) in., the radiant heat was decreased with a gain of a factor of 1.3 in the dissociation efficiency.

The efficiency of dissociation was found to be dependent upon the distance between the anode and the point of intersection of the \( \text{H}_2^+ \) beam. Figure 9 shows the efficiency as a function of the distance to the anode (for a 1\( \frac{1}{2} \)-in. anode diameter). A movement of the anode from a position 3 in. from the \( \text{H}_2^+ \) beam to 21 in. caused a decrease in the efficiency of dissociation by a factor of two. Also, shown in Fig. 9 is the radiant energy measured at 90° from the arc column as a function of the anode distance. This was measured by a thermal detector placed in the same position as the particle detector. The thermal detector consisted of a copper ring with a 25 \( \mu \)in. nickel foil soldered to the front surface. Soldered to the center of the nickel foil was a fine copper wire, which formed a thermocouple indicator with low heat loss, short time constant, and sensitivity of the order of microwatts. Glass, quartz, and fluorite of thickness 1 to 3 mm were found to absorb more than 90% of the radiant energy emitted by the arc. It will be noticed in Fig. 9 that the slope of the radiant energy curve is similar to the slope of the dissociation efficiency. Both the efficiency and the radiant energy were independent of the cathode position. It may be inferred from the radiant energy curve either that the density of radiating particles is increasing or that the level of excitation of the particles is increasing as regions closer to the anode are examined.

Figure 10 gives a plot of the dissociated \( \text{H}^+ \) as a function of the measured arc current. The arc current was varied from 160 to 325 amp with a corresponding 1.3-fold increase in the \( \text{H}^+ \) current.

Attempts to measure the attenuation of various beams by measuring current with and without the arc were unsuccessful because of the pumping action of the carbon arc. The pressure in the solenoid was reduced by a factor of 3 with the carbon arc operating, following an initial outgassing period. The lowest pressure obtained in the vacuum system was \( 4 \times 10^{-6} \) mm Hg, as measured by a VG1A Ionization Gauge placed 8 in. from the arc along the beam entrance tube. Experiments are continuing on the dissociation produced by the carbon arc at lower particle energies. Also, energetic particles of various kinds are being studied to learn more about the nature of the processes taking place.

CONCLUSION

The measurements obtained in these experiments serve to show that the probability of dissociation of \( \text{H}_2^+ \) ions on passing through a vacuum carbon arc remains sufficiently large to be of usefulness in a trapping device when the energy of the incident ions is raised as high as 600 kev. The trapped \( \text{H}^+ \) (or \( \text{D}^+ \)) beam would apparently neutralize itself to only a small extent as it circulates through the carbon arc in an experimental device. Significant additions have been made to cross section information concerning the dissociation of \( \text{H}_2^+ \) ions by collisions in several gases.

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Free Hydrogen Atom Collision Cross Sections of Interest in Controlled Thermonuclear Research

By W. L. Fite*

For the heating of deuterium gas to thermonuclear temperatures, it is convenient for certain purposes to think in terms of three temperature regimes. The first is characterized by the presence of substantial numbers of molecules and molecular ions in the gas; the second, by the virtual absence of molecules and molecular ions, but the presence of neutral deuterium atoms; and the third, by the presence of only deuterons and electrons, i.e., a fully ionized plasma.

While this breakdown is somewhat artificial, for the boundary lines between the three regimes are not at all clearly drawn, it is useful for envisioning the types of extranuclear processes likely to be most important during the heating history of the gas. It is especially useful for those devices in which, during the early phases of heating, electrical energy is fed into the gas by means of extranuclear processes, as in the case of a gas discharge device.

In the first regime, where the most important processes involve molecules and molecular ions, a great deal of experimental information on pertinent cross sections is available. In the third regime, where ionization is complete, atomic processes are apparently of little importance except insofar as they bear on the problems of final heating and confinement of the plasma.

The second regime has received comparatively little attention to date. To understand it, one must know the cross sections for extranuclear processes involving the free deuterium atom (or hydrogen atom, since the extranuclear collision properties of these two atoms are the same). While a number of theoretical predictions for the pertinent cross sections are available, the validity of the scattering approximations used in the calculations has not been accurately established. Very little experimental information on extra-nuclear-collision phenomena involving the free atom is available.

For the past year, an experimental program to measure certain hydrogen-atom cross sections of interest in controlled thermonuclear research has been under way at General Atomic. Its purpose is twofold: (a) to provide experimental values for cross sections in certain pertinent atomic processes; and (b) to provide experimental information by which to judge the merits of the various scattering-theory approximations, and thus make possible improved calculations of cross sections. The present paper summarizes the results of measurements of the cross sections of hydrogen atoms for: (1) ionization, (2) excitation of Lyman-alpha radiation on electron impact, and (3) elastic scattering of electrons of energy less than 10 ev; and also describes the approach of measurements now in progress on (4) charge-exchange between deuterons and deuterium atoms and (5) ionization of the hydrogen atom on proton impact.

EXPERIMENTAL APPROACH

It is well known that hydrogen and deuterium gas can be dissociated to an arbitrary degree in a low-pressure furnace. The use of such furnaces as sources of arbitrarily pure beams of ground-state atoms is common practice. The difficulty in using such beams for the study of collision processes is that by the time the beam is far enough removed from the source to allow convenient experimentation, the density of atoms in the beams (~10^9 atoms/cm^3) is much less than the density of molecules in the best vacuums producible by straightforward diffusion pumping (~10^10 molecules/cm^3). Since electrons, ions, etc., are scattered more readily by the molecules of this residual gas than by the atoms in the beam, a basic signal-to-noise problem exists. Indeed, signal-to-noise ratios of the order of 1:40 would be expected in a dc experiment. The basic approach in all the experiments described here is to modulate the neutral atomic beam, while allowing the current of particles colliding with the atoms to be run dc. In this case, interactions with the background gas will give rise to a dc signal, plus noise, while the signal arising from interactions of the colliding particles and the hydrogen atoms in the beam can be identified by its appearing at the frequency of modulation of the atomic beam and at a specific phase. This combination of atomic-beam and modulation techniques allows measurements of several cross sections of the hydrogen atom, with signal-to-noise ratios varying from ten to several hundred.

In the present experiments, the neutral beam was
modulated at 100 cps by a mechanical chopper wheel, located in the second of three differentially pumped vacuum chambers. The source of atoms was in the first chamber. All experiments were carried out in the vacuum chambers. The source of atoms was in the third chamber and used a crossed-beam configuration in which the beam of colliding particles intersected the time-modulated neutral atomic beam.

Figure 1 shows a typical experimental arrangement and illustrates the essential features of the experimental approach. Although this diagram shows the specific arrangement for only the experiment on excitation of Lyman-alpha radiation on electron impact, the following features are common to all experiments: (1) the mechanical chopping; (2) the use of an electron gun and a mass spectrometer to monitor the contents of the beam; and (3) the use of phase-sensitive detection, with the phasing signal locked directly to the frequency, from ultraviolet photon counter; for elastic scattering of electrons below 10 ev and total scattering above 10 ev, the scattered electrons themselves were detected. The mass spectrometer is used in measurements of charge exchange between ions and hydrogen atoms; and liberated electrons are detected in ionization of the atom on ion impact.

Two types of measurements are made when any given process is under consideration. First, relative cross sections are determined by comparing the signal per unit colliding-particle current at different energies of the colliding particle, for the same neutral atomic beam. Second, it is possible to make direct measurements of the ratio of the atomic to the molecular cross sections by observing signals as the temperature of the furnace is varied with constant flow of mass in the neutral beam. Given this ratio, absolute cross sections for the atom are determined from existing knowledge of the absolute cross sections for the molecule.

To make this last point more explicit, it can be shown from the experimental definition of a cross section and from elementary kinetic theory considerations that the ratio of the atomic to molecular cross sections, \( \frac{Q_1}{Q_2} \), can be related to the signal, \( S_1 \), arising from atoms in a mixed beam of atoms and molecules and to the signal, \( S_2 \), arising from the molecules in the beam by

\[
\frac{Q_1}{Q_2} = \frac{1}{\sqrt{2}} \left( \frac{S_1}{S_0 - S_2} \right),
\]

where \( S_0 \) is the signal which would have been observed with the same mass flow and furnace temperature had the molecule not dissociated. The hypothetical signal \( S_0 \) may be determined by the extrapolation of molecular signals observed with furnace temperatures below that at which dissociation begins, using

\[
S_0 \propto T^{-\alpha},
\]

where \( T \) is the furnace temperature in degrees Kelvin. For discriminating detectors (e.g., the mass spectrometer), these formulas may be used directly, since both \( S_1 \) and \( S_2 \) are determinable. For nondiscriminating detectors (e.g., the scattered-electron detector, which does not specify whether the electron was scattered by an atom or a molecule), only the sum of \( S_1 \) and \( S_2 \) is measured. In this case, the appropriate equation is

\[
\frac{Q_1}{Q_2} = \frac{1}{\sqrt{2D}} \left[ \frac{S_1 + S_2}{S_0} - 1 + D \right],
\]

in which \( D \), the dissociation fraction, is defined by

\[
D = 1 - \frac{S_2}{S_0},
\]

where the two signals indicated in Eq. (4) are those obtained by observing the molecular peak on the mass spectrometer, which is always used to monitor the neutral beam. Thus, with a fixed mass flow in the beam, the beam's constitution can be varied, by varying the temperature, from pure molecular to over 99% pure atomic, and \( \frac{Q_1}{Q_2} \) for any constitution can be determined. The ratio was constant in all cases, irrespective of constitution of the beam, as would be expected.

**ELECTRON IMPACT RESULTS**

Ionization of the Hydrogen Atom

Measurements in this experiment were made using the mass-spectrometer peak heights and Eqs. 1 and 2. Although a complication arose because the collection efficiency of the mass spectrometer was weakly dependent on the furnace temperature through the incident energy of the neutral-beam particles which were ionized, the use of simulants gases to determine mass-spectrometer collection efficiencies as a function of furnace temperature made possible any necessary corrections to Eq. 2. The results of this measurement are shown in Figs. 2 and 3. These graphs also show the molecular cross section obtained by Tate and Smith and Born approximation calculations for the ionization cross section of the atom.
Excitation of Lyman-alpha Radiation

In this experiment, the detector was an iodine-vapor-filled ultra-violet photon counter, filtered by molecular oxygen in a lithium-fluoride-bounded gas cell immediately in front of the counter. The absorption properties of oxygen make this combination responsive only to radiation at 1216 Å (Lyman alpha, which results from a transition of the atom from the n = 2 state to the ground state) and six other narrow “windows” between the cutoff of lithium fluoride and the ionization potential of gaseous iodine. No atomic radiation except Lyman alpha is detected by these counters.

As indicated in Fig. 1, the unorthodox procedure of treating the counter output as an ac current had to be used to overcome signal-to-noise problems caused by electrons exciting molecular hydrogen in the background gas. While this introduced remarkable shot noise, since the quantum of charge was the counter pulse (about 10^10 electrons) rather than a single electronic charge, it resulted in quite usable signals.

Since the angular distribution of the radiation emitted is anisotropic and depends on electron energy, it was necessary to measure this angular distribution in order to obtain total cross sections for excitation of the radiation. Curves of relative cross sections were taken directly, and these were normalized to fit the first Born approximation calculations for the 1s–2p excitation of the hydrogen atom at energies in excess of 200 ev, which fit was found to be quite satisfactory. This procedure was necessary because calculations of absolute cross sections for excitation of the molecule by vacuum ultraviolet radiation apparently have not been made.

Figure 4 shows the experimental results of this measurement, together with calculations for the 1s–2p excitation using the first Born approximation, the second Born approximation, and the distorted-wave approximation. The 90° data were taken with the photon counter looking in a direction perpendicular to the electron beam, and were corrected for angular distribution of the photons. The “magic-angle” data were taken with the photon counter looking at 54.5° from the electron beam direction. It can be shown that,
from this angle, relative cross sections are always proportional to the total cross section, regardless of the angular distribution of radiation.

**Elastic Scattering of Electrons below 10 ev Energy**

This measurement\(^9\) was made primarily to resolve the discrepancy between theory and experiment resulting from recent measurements of this cross section by Bederson, Malamud and Hammer.\(^{10}\) In this measurement, the detector was a shielded electrode which collected all scattered electrons into a cone of 45° half apex angle, whose axis was perpendicular to the electron beam direction. In order to obtain absolute cross sections for the atom, the molecular signals were normalized to fit the data of Ramsauer and Kollath\(^{11}\) for scattering into this angular range. Experimental points from this measurement are shown in Fig. 5, which also shows three theoretical curves\(^{12}−^{14}\) and the earlier experimental results.\(^{10}\) The experimental points were obtained using the assumption of isotropic scattering.

**Discussion of Results on Electron Collisions**

The first conclusion that may be drawn from these measurements is that first Born approximations work quite well for electron energies in excess of about 200 ev. Below this energy, the error introduced by this approximation is somewhat less serious than is often supposed. Indeed, calculations of electron–hydrogen atom interactions in thermonuclear devices, using the first Born approximation, are probably entirely adequate.

The second conclusion, based on the Lyman-alpha excitation data, is that going to higher approximations than the first Born approximation does not appear to be worth the computational effort. The error remaining in the cross-section calculations appears not appreciably less than that associated with the first Born approximation.

**PROTON–HYDROGEN ATOM INTERACTIONS**

In these measurements, currently in progress, the general technique is the same as was used in the electron–atom interactions. The only essential change is the replacement of the electron gun by an ion source. The ion beam is mass-analyzed before it crosses the atomic beam. Energies up to 25 kev are being used in the measurement of (1) charge exchange, (2) ionization, and (3) the appearance of Lyman-alpha radiation in proton–atom interactions, as well as in interactions of atoms with molecular ions.

**REFERENCES**

Thermodynamics of Deuterium–Tritium Mixtures

By G. Bouleque, P. Chanson, R. Combe, M. Feix and P. Strasman

We propose to study deuterium–tritium mixtures which, when heated to an elevated temperature, produce a considerable amount of thermonuclear energy. Such mixtures will constitute the active part of a fusion reactor. This reactor could operate in a stationary manner, a feed of fusionable materials replacing the material used up by the thermonuclear reaction, and the inert waste products being removed in such a manner that the various concentrations as well as the temperature remain constant. It is, however, doubtful that one could construct an apparatus of this type and interest at present seems to be in the direction of a cycle in which the mixture is heated to a given temperature with liberation of thermonuclear energy during the period in which the temperature is sufficiently high. After cooling of the system the cycle begins all over again. We will present rather briefly a problem of a stationary plasma and will then study in greater detail the time behaviour of a D–T mixture of known initial composition taken to a temperature $T_0$. In particular, we will calculate the energy released during such a cycle as well as the time necessary for its completion.

We wish to treat as thoroughly as possible the following two points:

(i)—the importance of “secondary” reactions, i.e., reactions between initial nuclei and products of the primary reaction. (In particular, in those plasmas having no tritium initially, the D–T reaction between deuterons and the tritons formed during the reaction D–D plays a decisive role as will be shown later. The importance of these secondary reactions has been pointed out by Lacombe et al.);

(ii)—the importance of radiation, which results in a cooling of the system, thereby limiting the time during which the release of thermonuclear energy is important.

Basic Assumptions

The following hypotheses are therefore made:

(a) There is a Maxwell distribution of particle velocities for nuclei and electrons, corresponding to a unique temperature which characterizes the environment.

(b) The environment is transparent to neutrons: this hypothesis is quite justified in view of the small density of thermonuclear plasmas. The neutrons escape from the system, therefore, carrying with them a part of the reaction energy in the form of kinetic energy.

(c) The surroundings are not in radiative equilibrium. At the high temperatures considered ($>10^6$ °K) the plasma will be totally ionized. The processes of radiation emission or absorption are discussed below.

(d) The pinch effect is perfect, i.e. no charged particle can escape from the plasma, whose actual dynamic behaviour is thus ignored.

Radiation

Most of the radiative energy loss, see (c) above, is due to bremsstrahlung (mainly of the electrons in the field of the ions), an emission process corresponding to free-free absorption (in French: absorption par une particule libre). The absorption cross section is given by Spitzer. For a photon of frequency $\nu$ the inverse of the mean free path $K_\nu$ is given by

$$K_\nu = \frac{4\pi}{3} \frac{2\pi}{3kT} N_e N_i Z e^6 \left[1 - \exp \left( \frac{h\nu}{kT} \right) \right]$$

where $N_e, N_i$ are the densities of electrons and nuclei of charge $Ze$, and the other letters have the usual definitions. With the usual values of $N_e \simeq N_i \simeq 10^{10}$ to $10^{11}$ electrons (or nuclei) per cm$^3$, the mean free path $1/K_\nu$ is in the range $10^6$ to $10^8$ cm for photons whose energy is of the order of $kT$, i.e. “average” plasma photons ($kT = 10$ kev). Therefore, in view of the inadequate size of the apparatus used in the laboratory, no equilibrium can be obtained.

The Compton effect might also contribute to energy losses. It is usually considered to be an absorption effect, since the photon energy is ordinarily far greater than the thermal energy of the electrons which undergo the Compton collision; but the situation is different in a thermonuclear plasma, since both the electron and photon energy are of the order of $kT$. If one takes into account the electron motion, one realises that the impact may cause the photon either to lose or gain energy. The Compton effect should, therefore, be capable of inducing an equilibrium, according to Planck’s law for photons. Nevertheless, because of the low plasma densities, the mean free paths are far greater than the dimensions of the system.
and, unlike conditions in the stars, there is not sufficient space for an equilibrium to be established.

We will take into account, therefore, only the bremsstrahlung: the power radiated is proportional to the volume, and varies with temperature as the average speed of the electrons, that is, in proportion to $T^3$.

Reactions

We have said that we would take into consideration the "secondary" reactions, but, of course, if we had to consider all possible reactions we should steadily be led on to study all nuclear reactions, which would make our calculation impossible. We also confine ourselves to reactions with reasonably large cross sections, taking into account only the following five:

\[ \begin{align*}
1D^2 + 1D^2 & \rightarrow 1^1T^3 + 1^1p + 4.03 \text{ Mev} \\
1D^2 + 1D^2 & \rightarrow 2^9He^{3} + qn^1 + 3.25 \text{ Mev} (2.44 \text{ Mev carried away by the neutron)} \\
1D^2 + 1^3T^3 & \rightarrow 2^9He^{4} + qn^1 + 17.58 \text{ Mev} (14.06 \text{ Mev carried away by the neutron)} \\
1D^2 + 2^9He^3 & \rightarrow 2^9He^{4} + qp^1 + 18.34 \text{ Mev} \\
1^3T^3 + 1^3T^3 & \rightarrow 2^9He^{4} + 2qn^1 + 11.32 \text{ Mev} (7.55 \text{ Mev carried away by the neutrons)}
\end{align*} \]

The rates of the reactions $\langle \alpha \sigma \rangle$ will be called respectively $\alpha, \beta, \gamma, \delta, \epsilon$ (in the same order as in the above list).

Calculation of the Reaction Rates

If $\sigma$ stands for the cross section, and $v$ for the relative speed of the two nuclei it is known that the number of reactions per unit volume and time will be $n_1n_2\langle \alpha \sigma \rangle$, where $n_1$ and $n_2$ are the particle densities; or, if one is dealing with reactions between identical nuclei of density $n$, the reaction rate will be $\frac{1}{2}n^2\langle \alpha \sigma \rangle$.

The evaluation of $\langle \alpha \sigma \rangle$ has been given in an article by Arnold.\(^8\) It is obtained by calling the cross section $\sigma(v)$, a function of the relative speeds of the two nuclei, and taking into consideration the Maxwell distribution of velocities, whose direction is assumed to be isotropic:

\[ \langle \alpha \sigma \rangle = 4\pi \left( \frac{m}{2\pi kT} \right)^{\frac{3}{2}} \int_0^\infty v^2\sigma(v) \exp \left( -\frac{mv^2}{2kT} \right) dv \] (2)

where $m$ is the reduced mass, $m_1m_2/(m_1 + m_2)$. The quantity $\langle \alpha \sigma \rangle$ is thus given by an integral of the curve $\sigma(v)$. Some experimental results are given by Arnold et al.\(^4\)

In Eq. (2) the integral can be completely evaluated, if one assumes that $\sigma$ depends on the relative energy $\frac{1}{2}mv^2 = E$, as in Gamow's formula:

\[ \sigma = (A/E) \exp (B/E). \] (3)

Therefore, it is only necessary to determine $A$ and $B$, which is what Thompson has done, by choosing $A$ and $B$ in such a way as to conform to the experimental results of Arnold et al.

Unfortunately, it is difficult to present these results exactly in an energy range large enough for the proper calculation of the integral in Eq. (2). We have, for our part rejected Gamow's formula, and have integrated (2) numerically with the help of the experimental values of $\sigma$. These values extend down to about 13 kev. If we study more closely the variations of the product $v^2\sigma(v) \exp (-\frac{mv^2}{2kT})$, which becomes $E\sigma(E) \exp (-E/kT)$ by change of variable, we find that it is impossible to calculate lower than $kT = 10$ kev.\(^\dagger\)

Table 1 shows our results for the two $D-D$ reactions and the $D-T$ reaction, temperatures varying from 10 to 100 kev. These values are slightly lower than those given by Thompson, but we use his results for reactions for which we have no accurate cross section values and for energies lower than 10 kev (0, 0.01, 0.1, 1 and 10 kev); for intermediate values we interpolate in a log-log representation.

Calculation of the Radiated Power

There still remains to be calculated the power lost by radiation, taking into account the bremsstrahlung spectrum and the Maxwell distribution of the particles. Spitzer has obtained the equation:

\[ P_{rad} = \frac{3}{8} Z^2N e^4 \left( \frac{mE}{mc^2h} \right)^{\frac{3}{2}} \text{sec}^{-1} \] (4)

Post\(^k\) has given a numerical formula derived from the results of Heitler. One finds, on applying the formula to a non-relativistic electron:

\[ P_{rad} = \frac{3}{8} Z^2N e^4 \left( \frac{8kT}{mc^2} \right)^{\frac{3}{2}} \text{sec}^{-1} \] (5)

corresponding to Spitzer's expression multiplied by a factor $2\sqrt{3}/\pi \approx 1.09$. The discrepancy is due to the fact that the first equation is derived from a semicalculation making use of a uniform energy spectrum, for the bremsstrahlung of the electron, while Heitler treats the phenomenon in a quantum electrodynamical manner.

\[ \begin{array}{|c|c|c|c|}
\hline
\text{Temperature} & \text{D}(p[Dp])T & \text{D}(Dn)He^* & \text{T}(DN)He^* \\
\text{(energy $kT$, kev)} & \text{(cm$^3$/sec)} & \text{(cm$^3$/sec)} & \text{(cm$^3$/sec)} \\
\hline
10 & 0.574 & 0.554 & 1.075 \\
15 & 1.389 & 1.359 & 2.653 \\
20 & 2.388 & 2.353 & 4.265 \\
30 & 4.648 & 4.595 & 6.995 \\
40 & 7.081 & 6.954 & 8.054 \\
60 & 12.20 & 11.84 & 8.962 \\
80 & 17.32 & 16.92 & 8.947 \\
100 & 22.36 & 22.21 & 8.445 \\
\hline
\end{array} \]
We shall use the equation of Post which, for a mixture of nuclei, becomes:

\[ P_{\text{rad}} = 3.37 \times 10^{-18} (kT) N_x \Sigma_n N_i Z_i^2. \]  

(6)

The radiated power is expressed in Mev/sec cm\(^3\) if \(kT\) is expressed in kev.

**Equations of the System**

We designate the densities of the nuclei of D, T, He\(^3\), H, and He\(^4\) by \(x, y, z, u\) and \(w\), respectively. There may be sources (positive or negative) of relative intensities \(S_x, S_y \ldots S_w\) (number of nuclei/sec cm\(^3\)). Using the reaction rates, \(\alpha, \beta \ldots \epsilon\), previously defined, the equations may be written:

\[
\begin{align*}
\dot{x} &= S_x - (\alpha + \beta)x^2 - \gamma xy - 8xz \\
\dot{y} &= S_y + \frac{1}{2} \alpha x^2 - \gamma xy - cy^2 \\
\dot{z} &= S_z + \frac{1}{2} \beta x^2 - 8xz \\
\dot{u} &= S_u + \frac{1}{2} \alpha x^2 + \delta xz \\
\dot{w} &= S_w + \frac{1}{2} cy^2 + \delta xy + \delta xz
\end{align*}
\]

(7)

**STUDY OF DEUTERIUM–TRITIUM MIXTURES**

**Static Case**

The first problem investigated is that of a stationary mixture. It is assumed that the sources are regulated in such a way that the concentrations and temperature are constant; this eliminates the left sides of Eqs. (7). Under these conditions one can calculate the concentrations as a function of the sources. It is, of course, necessary for the concentration to be positive, in order to make sense physically. It is therefore necessary to have available a positive source of deuterium and a negative source of protons and of helium–4. This was foreseeable \textit{a priori}.

We will now determine the intensity of the sources, subject to other conditions. First, the energy flows must be balanced; that is, the radiated power must be equal to the released thermonuclear power. In addition, we will assume that the feed and extraction, i.e. the concentrations, can be controlled at will. This will lead, then, to the complete elimination of protons and helium–4, inert products which contribute only to the radiation without taking part in the thermonuclear reaction. The helium–3 case is different since it reacts with the deuterium, liberating a large quantity of energy which, moreover, is completely transmitted to the plasma (although in the case of the D-T reaction a considerable part is carried off by the neutron). Unfortunately, helium–3 makes an important contribution to the radiation while having only a small cross section for Reaction 8, He\(^8\)(D, p)He\(^4\), so that it is only for very high temperatures that it becomes advantageous not to extract it completely. The temperature limit is calculated to be \(kT = 36\) kev. For lower temperatures we will assume, therefore, that the concentrations of H, He\(^3\) and He\(^4\) are zero. We calculate, then, the feed of deuterons and tritons as a function of operating temperature, for a given release of energy. The values of the feed will be expressed in deuterons or tritons per Mev.

The minimum temperature below which there is no possible solution is in the range 4.5–4.6 kev. Table 2 gives the values of the feed for temperatures above this limit.

In particular, operation without a supply of tritium takes place for \(kT = 24.6\) kev. For higher temperatures the reactor will be able to produce tritium. A part of this tritium, produced by the D(D, p) reaction could be extracted from the reactor; whereas, up to this temperature, it was necessary not only to leave all the tritium formed, but even to supply some of it.

This problem corresponds, unfortunately, to not very realistic experimental conditions. Its only interest is to point out rather simply the economic conditions of operation for a stationary reactor.

**Dynamic Case**

The following problem is much closer to the planned laboratory experiments. A deuterium–tritium mixture is taken to an initial temperature \(T_0\). We make the same assumptions as previously. The temperature varies as the reaction proceeds, the system becoming progressively poisoned by the accumulation of the waste products He\(^4\) and H. When the power dissipated by bremsstrahlung is greater than the thermonuclear power the system cools itself very quickly. During this process a certain amount of energy has been liberated and it is interesting to compare this with the energy necessary to heat the plasma to its initial temperature. Introducing into the mixture a certain proportion of tritium is very advantageous and we will see that a rather small quantity permits a considerable reduction of the initial temperature necessary.

The equations of the problem are the five equations (7), with the terms representing the sources eliminated, together with the following expressions for the energy balance:
\[
\frac{d}{dt} (NkT) = \frac{1}{2} (P_{th} - P_{rad}) P_{th} = \frac{1}{2} \left( \alpha W_a + \beta W_b \right) \sigma^2 + \gamma xy W_x + \delta x W_t + \frac{1}{4} \gamma^2 W_e \tag{8}
\]

where \( W_a, W_b, W_x, \) and \( W_t \) are the energies given to the charged particles; i.e., the energies of reaction minus the energy carried off by the neutrons.

If \( N \) is the total number of particles and \( N_e \) is the electron density, which stays constant, then

\[
N = N_e + x + y + z + u + w;
\tag{9}
\]

the neutrons escaping from the plasma.

In the following it will be helpful to introduce two reduced variables, the product \( N \sigma \) and the quotient \( W/N_e, W \) being the energy released at time \( t \). Since we are looking at deuterium–tritium mixtures, this quantity \( W/N_e \) is the average energy released per nucleus.

One usually considers, for the initial temperature \( T_0 \), the critical temperature, \( T' \), above which

\[
\frac{d}{dt} (NkT) = 0, \quad \frac{d}{dt} (N_{eq} kT) < 0.
\]

Table 3. Dependence of Critical Temperature on Initial Tritium Concentration

<table>
<thead>
<tr>
<th>Conc.</th>
<th>Temp.</th>
<th>( kT' )</th>
<th>Conc.</th>
<th>Temp.</th>
<th>( kT' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_T )</td>
<td>( % )</td>
<td>kev</td>
<td>( C_T )</td>
<td>( % )</td>
<td>kev</td>
</tr>
<tr>
<td>0</td>
<td>42.4</td>
<td>3.43</td>
<td>9</td>
<td>0.032</td>
<td>36</td>
</tr>
<tr>
<td>0.109</td>
<td>32</td>
<td>5.82</td>
<td>7.5</td>
<td>0.183</td>
<td>28</td>
</tr>
<tr>
<td>0.289</td>
<td>24</td>
<td>8.70</td>
<td>6.5</td>
<td>0.461</td>
<td>20</td>
</tr>
<tr>
<td>0.594</td>
<td>18</td>
<td>13.59</td>
<td>5.8</td>
<td>0.789</td>
<td>16</td>
</tr>
<tr>
<td>1.09</td>
<td>14</td>
<td>25.05</td>
<td>5.0</td>
<td>1.0</td>
<td>12</td>
</tr>
<tr>
<td>1.5</td>
<td>12</td>
<td>44.95</td>
<td>4.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The notion of a critical temperature is interesting, but allows the evolution of the system to be followed only during the initial phase. Later, this evolution can continue in various ways.

Let us suppose, for example, that a plasma is composed initially only of deuterium. If the initial temperature \( T_0 \) is less than 42.4 kev, the radiated power is greater than the thermonuclear power, and the temperature decreases. Nevertheless some tritium is formed after a certain time, the power \( P_{th} \) increases because of the large cross section of the DT reaction, and the temperature rises after having passed through a minimum.

On the other hand, let us consider an environment relatively rich in tritium \( (C_T \approx 1 \%) \) and heated to the temperature \( T_0 = 15 \) kev. The derivative \( T' \) is positive initially, but the complete investigation shows that the tritium "burns" before the temperature attains a value sufficient to "ignite" the deuterium. After having passed through a maximum this time, the temperature decreases and, in all, the released thermonuclear energy is small, in this case 171 keV/nucleus. The yield of the process is mediocre, since it was necessary to supply 45 keV in order to heat the plasma; if the energy is extracted in heat form, these 216 (171 + 45) keV will just allow the production of the 45 keV of electrical energy necessary for the heating of the system.

We have calculated the thermal evolution of a deuterium–tritium mixture for different values of \( T_0 \) and \( C_T \). Table 4 shows the results, namely the thermonuclear energy released, \( W/N_e \), and the reduced time, \( N_{eq} \), necessary to complete the process. The calculations were discontinued when the temperature fell below 1 keV.

We can now define a new critical temperature, \( T_{cw} \) (or rather, a family of new critical temperatures). \( T_{cw} \) is, for a given initial concentration of tritium, the initial temperature necessary in order that the released energy have a given value \( W \). It is, in general, a function of \( C_T \) and \( W \).

An examination of Table 4 shows that, at lower temperatures, the critical temperature is lower, but the critical density is higher. The critical density is higher because, in this case, the energy released by the neutrons is greater; the critical temperature is lower because, in this case, the energy released by the neutrons is greater.

\[
W/N_e, W_{\text{net}} = a + b T_c + c T_c^2 + d T_c^3 + e T_c^4
\]
THERMODYNAMICS OF D-T MIXTURES

peratures \(kT_0 < 9 \text{ kev}\), \(W/N_e\) varies very rapidly in the neighbourhood of a certain critical concentration. \(T_{CW}\) is therefore a function of \(C_T\) but practically independent of \(W/N_e\). Further, in comparing Tables 3 and 4, one notes that \(T_{CW} \simeq T'_c\). The classical notion of a critical temperature \(T'_c\) is therefore still applicable under these conditions.

Table 5 shows \(kT_{CW}\) as a function of \(C_T\) for \(W/N_e = 1\) Mev. Figure 1 shows the variation of both \(kT'_c\) and \(kT_{CW}\) as functions of \(C_T\). For \(C_T = 0\), \(kT'_c = 42.4\) kev and \(kT_{CW} = 27.7\) kev. The curves

Table 6. Temperature and Energy Release in Pure Deuterium

<table>
<thead>
<tr>
<th>Net</th>
<th>(kT_s = 32) kev</th>
<th>(kT_s = 29) kev</th>
<th>(kT_s = 27) kev</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x10^{-18}) sec/cm(^a)</td>
<td>(kT)</td>
<td>(W/N_e)</td>
<td>(kT)</td>
</tr>
<tr>
<td>0.2</td>
<td>31.82</td>
<td>10.1</td>
<td>28.71</td>
</tr>
<tr>
<td>0.4</td>
<td>31.75</td>
<td>21.9</td>
<td>28.50</td>
</tr>
<tr>
<td>0.6</td>
<td>31.77</td>
<td>35.2</td>
<td>28.35</td>
</tr>
<tr>
<td>0.8</td>
<td>31.87</td>
<td>49.6</td>
<td>28.27</td>
</tr>
<tr>
<td>1.0</td>
<td>32.04</td>
<td>65.0</td>
<td>28.22</td>
</tr>
<tr>
<td>1.2</td>
<td>32.27</td>
<td>81.3</td>
<td>28.21</td>
</tr>
<tr>
<td>1.4</td>
<td>32.54</td>
<td>98.3</td>
<td>28.24</td>
</tr>
<tr>
<td>1.6</td>
<td>32.86</td>
<td>116.9</td>
<td>28.28</td>
</tr>
<tr>
<td>1.8</td>
<td>33.22</td>
<td>134.4</td>
<td>28.35</td>
</tr>
<tr>
<td>2.0</td>
<td>33.63</td>
<td>153.3</td>
<td>28.44</td>
</tr>
</tbody>
</table>

\(^a\) In kev. \(^b\) In Mev/nucleus.

In the cases corresponding to the following portion of the curves \((0.24\% < C_T < 6\%)\), if \(T'_c < T_0 < T_{CW}\), the temperature rises, at first, but the tritium disappears before attainment of a temperature sufficiently high to assure even a partial combustion of the deuterium.

Finally, for the high concentrations \((C_T > 6\%)\), the two curves overlapping, one can really speak of a critical temperature \(T_0 = T'_c = T_{CW}\): the mixture heats itself as soon as the thermonuclear power is greater than the power radiated and there is enough tritium for the combustion to be more or less complete.

Finally, we present some results (Table 6 and Fig. 2) concerning the evolution of a pure deuterium plasma, i.e. the values of the temperature and released energy, as functions of the reduced time, for three values of \(kT_0\).

REFERENCES

The Role of Materials in Controlled Thermonuclear Research

By J. L. Craston,* R. Hancox,* A. E. Robson,* S. Kaufman,† H. T. Miles,† A. A. Ware† and J. A. Wesson†

Attempts to achieve very high temperatures, with the ultimate goal of producing thermonuclear power, have led to the extensive study of high current electrical discharges in gases at low pressure. Interest is focused mainly on the processes occurring in the plasma, and the walls of the discharge tube are usually regarded as merely providing boundary conditions for these processes. Already, however, it is clear from experiments that the walls can exert considerable influence on the plasma by the impurities they introduce, and there is good reason to believe that the physical properties of the wall material may set definite limits to the design of a working reactor. The choice of material for the discharge tube is therefore an important problem. It is the purpose of this paper to examine the processes occurring at the wall and to discuss their importance in the choice of materials both for present equipment and for future designs.

The principal problem is contamination of the plasma as a result of erosion of the wall. Since the nuclear power anticipated from a deuterium plasma at the optimum reacting temperature of $10^9 \, ^\circ K$ is only about nine times greater than the power which will be unavoidably radiated as bremsstrahlung, it is important to keep the plasma as clean as possible, since only a small concentration of heavier impurity atoms results in a serious increase in the radiation loss. This will reduce the efficiency of the reactor and, if the impurities are present in the early stages of heating, may even prevent the achievement of reacting temperatures. Another important consideration is that erosion of the wall may limit the useful life of the reactor.

The term erosion is used here to include the several ways in which material may be removed from the wall. Although a reacting plasma must be contained for the most part by magnetic fields, the walls will receive a large flux of radiation which, from a plasma at $10^8$-$10^9 \, ^\circ K$, will be mostly in the soft X-ray region. The absorption of this radiation may raise the temperature of the walls sufficiently for thermal evaporation to take place. Also, in any practical device, it is unlikely that the magnetic containment will be perfect and the walls may be bombarded by energetic deuterons and reaction products which have diffused across the confining fields. As well as contributing to the heat input, these will cause erosion by sputtering. A third form of erosion, which is peculiar to metal walls, is the formation of arc spots; this is likely to occur on any metal surface exposed to a hot plasma and has been the most serious source of contamination in present equipment. Experimental work directed towards understanding and controlling this phenomenon will be described.

In what follows, the emphasis is laid primarily on plasma contamination but other effects are considered, such as thermal stress fatigue and radiation damage of the wall. These have not yet been encountered but may be of importance in power reactors.

PLASMA CONTAMINATION

The power radiated by a pure deuterium plasma of density $n$ ions/cm$^3$ at temperature $T \, ^\circ K$ is:

$$E_0 = 1.42 \times 10^{-3} n^2 T \, \text{watts/cm}^3.$$ (1)

At $T = 10^9 \, ^\circ K$, the ratio of the nuclear power $E_n$ to the radiated power has a maximum value of about 9. If a proportion $f$ of fully ionized atoms of charge $Z$ is introduced into the plasma, the radiation loss becomes:

$$E_r = E_0(1 + 2Zf)/(1 + 2Zf).$$ (2)

The increased radiation loss lowers the efficiency of the reactor; a net power gain is impossible if $E_n < 2E_r$ that is, if $E_r > 4.5E_0$. Table 1; (a) gives values of $f$ for different materials corresponding to $E_r = 4.5E_0$. In practice the impurity level must be kept well below this limit, and in the following calculations it is assumed that only a 10% increase in bremsstrahlung can be tolerated. The corresponding values of $f$ are given in Table 1 (b). It can be seen that not only must $f$ be small, but such impurities as are unavoidable should be of low atomic number.

These figures assume that the impurity atoms are completely stripped and that the only radiation is bremsstrahlung. If an atom retains any of its orbital electrons it will also be a source of line radiation; the

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energy radiated from an unstripped atom will then be many times greater than from the stripped atom. It is therefore important that impurities should be easily stripped. As an impurity atom reaches successively higher stages of ionization the probability of further ionization decreases, while the probability of radiative recombination increases. By considering the balance of these two processes, an approximate calculation shows that in a plasma at $10^9$ K about 10% of atoms of atomic number 25 will not be completely stripped. For heavier atoms this proportion will be greater, and for atoms of atomic number 28 or less, corresponding to a wall temperature of about 1000°C, it may be possible to avoid complete stripping.

To estimate the maximum permissible erosion rate, a hypothetical reactor is considered, consisting of a stable cylindrical pinched discharge of density $n$ ions/cm$^2$ and radius $r$ cm inside a tube of radius $R$ cm. If atoms leave the wall at a constant rate $S$ atoms/cm$^2$ sec and are trapped in the plasma, the fractional impurity concentration at time $t$ is

$$ f = \frac{2RS}{\nu n}. \quad (3) $$

Taking $n = 4 \times 10^{15}$ ions/cm$^3$, corresponding to a reaction power density of about 600 w/cm$^3$ in deuterium at $10^9$ °K, and a pulse length of $t = 10$ sec (thereby satisfying Lawson's criterion$^3$ that $nt > 10^{16}$) and with $R = 4r = 30$ cm, we obtain

$$ S = 3.75 \times 10^{14} f \text{ atoms/cm}^2 \text{ sec}. \quad (4) $$

The erosion rates which will give 10% increase in bremsstrahlung at the end of the pulse are given in Table 1(c). Since impurity ions have to diffuse across magnetic fields, it may be possible in a pulsed device of large dimensions to complete an efficient reaction in the centre of the plasma before it has become heavily contaminated. These calculations must not therefore be regarded as design data for a thermonuclear reactor, but rather as illustrations of the principles involved in estimating the effect of wall erosion.

### Table 1. Impurity Concentration and Erosion Rate

<table>
<thead>
<tr>
<th>Materials</th>
<th>Atomic number</th>
<th>$f$ for $E_p = 4.5E_x \times 10^4$</th>
<th>$f$ for $E_p = 1.1E_x \times 10^4$</th>
<th>$S$ atoms/cm$^2$ sec $\times 10^{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beryllium</td>
<td>4</td>
<td>1250</td>
<td>62.5</td>
<td>234</td>
</tr>
<tr>
<td>Magnesium</td>
<td>12</td>
<td>185</td>
<td>6.9</td>
<td>28</td>
</tr>
<tr>
<td>Aluminium</td>
<td>13</td>
<td>170</td>
<td>5.9</td>
<td>22</td>
</tr>
<tr>
<td>Titanium</td>
<td>22</td>
<td>61</td>
<td>2.1</td>
<td>7.9</td>
</tr>
<tr>
<td>Vanadium</td>
<td>23</td>
<td>56</td>
<td>1.9</td>
<td>7.1</td>
</tr>
<tr>
<td>Chromium</td>
<td>24</td>
<td>52</td>
<td>1.7</td>
<td>6.9</td>
</tr>
<tr>
<td>Iron</td>
<td>26</td>
<td>45</td>
<td>1.5</td>
<td>5.6</td>
</tr>
<tr>
<td>Cobalt</td>
<td>27</td>
<td>42</td>
<td>1.4</td>
<td>5.3</td>
</tr>
<tr>
<td>Nickel</td>
<td>28</td>
<td>39</td>
<td>1.3</td>
<td>4.9</td>
</tr>
<tr>
<td>Copper</td>
<td>29</td>
<td>36</td>
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<td>4.5</td>
</tr>
<tr>
<td>Zirconium</td>
<td>40</td>
<td>20</td>
<td>0.82</td>
<td>2.3</td>
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<td>Molybdenum</td>
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<td>18</td>
<td>0.57</td>
<td>2.1</td>
</tr>
<tr>
<td>Beryllia</td>
<td>—</td>
<td>292</td>
<td>10.9</td>
<td>41</td>
</tr>
<tr>
<td>Alumina</td>
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<td>48</td>
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<td>6.4</td>
</tr>
<tr>
<td>Silica</td>
<td>—</td>
<td>65</td>
<td>2.8</td>
<td>10.5</td>
</tr>
</tbody>
</table>

### Table 2. Maximum Internal Wall Temperature, Heat Flux Through Wall and Wall Thickness

<table>
<thead>
<tr>
<th>Material</th>
<th>$\theta_{max}$°C</th>
<th>Heat flux through 1 cm wall w/cm$^2$</th>
<th>Heat flux for flux of 100 w/cm$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beryllium</td>
<td>670</td>
<td>300</td>
<td>3.0</td>
</tr>
<tr>
<td>Magnesium</td>
<td>140</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Aluminium</td>
<td>510</td>
<td>240</td>
<td>2.4</td>
</tr>
<tr>
<td>Titanium</td>
<td>840</td>
<td>80</td>
<td>0.6</td>
</tr>
<tr>
<td>Vanadium</td>
<td>1080</td>
<td>160</td>
<td>1.6</td>
</tr>
<tr>
<td>Chromium</td>
<td>690</td>
<td>120</td>
<td>1.2</td>
</tr>
<tr>
<td>Cobalt</td>
<td>910</td>
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<tr>
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<td>260</td>
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</tr>
<tr>
<td>Alumina</td>
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<tr>
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</tr>
<tr>
<td>Silica</td>
<td>610</td>
<td>4</td>
<td>0.04</td>
</tr>
</tbody>
</table>

#### Thermal Evaporation

If evaporation occurs at the wall, the erosion rate $S$, corresponding to a wall temperature $\theta$, is given by the relation (temperatures in °K):

$$ \theta_B \frac{10^4 - \theta}{10^4 - \theta_B} = 1.70 - 0.243 \log_{10}[(2.84 \times 10^{-23} S \theta_M t)], \quad (5) $$

where $M$ is the molecular weight and $\theta_B$ the boiling point of the wall material. This has been derived using the simplified vapour-pressure data of Loftness.$^4$ Assuming that the wall temperature is constant during the 10-second reaction period, and with the values of $S$ in Table 1, the maximum permissible wall temperature $\theta_{max}$ for 10% increase in radiation at the end of the period can be derived. The results are given in the first column of Table 2. On account of the logarithmic term in Eq. (5), the values of $\theta_{max}$ are insensitive to the assumed reactor parameters and thus allow a valid comparison between materials.

For the heat removed at the wall to be used efficiently for power production, the outside temperature of the wall should be at least 400°C. The heat flux through a wall 1 cm thick whose external temperature is 400°C and whose internal temperature is $\theta_{max}$ is given in the second column of Table 2. The third column gives the thickness of material which will allow a flux of 100 w/cm$^2$ to be transmitted under the same conditions. Some advantage might be obtained from a thin insulating layer on a metal base; for example, because of its low vapour pressure and comparatively good thermal conductivity a layer of beryllia up to 1.5 cm thick would increase the heat loading capacity of a copper wall cooled at 400°C.

The above figures have been calculated on the assumption that all the energy is absorbed on the inside surface of the wall. If a material of low X-ray absorption coefficient is used, however, this is not necessarily true. For example, in a reactor operating at $5 \times 10^8$ °K with beryllium walls 3 mm thick, about
70% of the radiation would be transmitted through the wall and could be absorbed directly in the coolant. The thermal loading of the wall could be increased about five times in this case. Under the same conditions, a 1 mm thick beryllia layer would absorb less than 30% of the incident bremsstrahlung thus allowing the wall loading to be increased about four times.

In addition to the radiation from the plasma during a long reaction period, heat pulses of short duration may also strike the walls of a reactor. In the pinched discharge machine, for example, there may be a high thermal flux to the walls at the beginning of the cycle before the discharge contracts. Any wall material evaporated and trapped during this initial stage will cause a constant radiation loss throughout the reaction period.

If a heat pulse of constant flux \( W \) watts/cm\(^2\) and duration \( t \) sec is incident on a surface initially at temperature \( \theta_0 \), the surface temperature at any time \( t \) during the pulse is

\[
\theta = \theta_0 + \frac{W}{2.1} \left( \frac{t}{\pi K \rho c} \right) ^{1/4}
\]

where \( K \) is the thermal conductivity, \( \rho \) the density and \( c \) the specific heat. At the end of the pulse, the temperature will fall according to the relation

\[
\theta = \theta_0 + \frac{W}{2.1(\pi K \rho c)^{1/4}} \left[ 1 - \left( \frac{t}{t_i} \right)^4 \right]^{1/2}
\]

By combining Eqs. (5), (6) and (7) and integrating with respect to time, the total amount of impurity introduced by a pulse has been calculated for several materials, and that heat flux derived which will lead to 10% increase in bremsstrahlung. In some cases, large pulses of short duration will melt the surface before significant evaporation occurs, and the limit to the permissible flux is then set by melting. This is illustrated in Fig. 1 where the permissible flux on a copper wall is plotted as a function of pulse duration for various initial temperatures.

A comparison between materials is given in Table 3.

![Figure 1. Maximum pulsed power loading of a copper wall as a function of pulse duration](image)

Table 3. (a) Maximum Thermal Flux, \( W \), During a 100 µsec Pulse and (b) Heat Pulse Parameter

<table>
<thead>
<tr>
<th>Material</th>
<th>( W ) watts/cm(^2)</th>
<th>2.1 (( \pi K \rho c )) ( \times 10^5 ) watts sec/cm(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beryllium</td>
<td>180 (m)</td>
<td>2.05</td>
</tr>
<tr>
<td>Aluminium</td>
<td>60 (m)</td>
<td>2.28</td>
</tr>
<tr>
<td>Titanium</td>
<td>56 (e)</td>
<td>0.55</td>
</tr>
<tr>
<td>Copper</td>
<td>220 (m)</td>
<td>3.25</td>
</tr>
<tr>
<td>Nickel</td>
<td>120 (e)</td>
<td>1.35</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>200 (e)</td>
<td>1.74</td>
</tr>
<tr>
<td>Alumina</td>
<td>64 (e)</td>
<td>0.61</td>
</tr>
<tr>
<td>Beryllia</td>
<td>140 (e)</td>
<td>0.88</td>
</tr>
<tr>
<td>Glass</td>
<td>9 (m)</td>
<td>0.18</td>
</tr>
</tbody>
</table>

(m) Flux limited by surface melting.  
(e) Flux limited by excessive evaporation.

where the maximum allowable heat flux for a 100 microsecond pulse is tabulated. In the last column of this table the parameter \( 2.1(\pi K \rho c) \) is given. This is the heat pulse \( W^{1/4} \) which will raise the temperature of the surface one degree; the size of this parameter is therefore an indication of the ability of a material to withstand heat pulses. The superiority of metals over insulators is again clearly seen. It would seem therefore that evaporation will not be an insuperable problem in a stabilized thermonuclear reactor if metal walls are used. Walls of an insulating material appear less feasible unless in the form of a thin layer on a metal base.

**Sputtering**

Sputtering is a form of erosion which occurs whenever ions of sufficient energy strike a surface. It appears to be independent of the surface temperature. Since a proportion of the energy reaching the walls of a thermonuclear reactor may be carried by energetic deuterons and reaction products, erosion may take place by sputtering even though the wall temperature is insufficient for significant thermal evaporation to occur.

The phenomenon of sputtering has been known for over a century, but few of the results obtained in the earlier experiments are quantitatively reliable. Accurate information is available for only a small number of gas–metal combinations, and over only a limited range of bombarding energy, but some general features of the phenomenon are well established. The sputtering ratio, defined as the number of atoms ejected per incident ion, is zero for bombarding energies less than a threshold energy (which lies in the region 5–50 eV) and then rises less than linearly with energy. The ratio is generally about 1–10 atoms/ion for energies of several keV and, for a given surface, increases with the mass of the incident ion.

Sputtering is an entirely different process from thermal evaporation. The energy of the incident ion is transferred locally to a small group of target atoms, some of which may be ejected from the surface before they reach equilibrium with the lattice. Earlier theories of sputtering regarded it as thermal evaporation on a sub-microscopic scale, but a more satisfactory
Table 4. Calculated Sputtering Ratios for Deuterium Ions

<table>
<thead>
<tr>
<th>Material</th>
<th>Sputtering ratio, atoms/ion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10 kev ions</td>
</tr>
<tr>
<td>Beryllium</td>
<td>0.065</td>
</tr>
<tr>
<td>Titanium</td>
<td>0.025</td>
</tr>
<tr>
<td>Iron</td>
<td>0.039</td>
</tr>
<tr>
<td>Copper</td>
<td>0.043</td>
</tr>
</tbody>
</table>

approach has been developed by Keywell,\(^7\) in which interaction of the incident ion with the lattice is treated on a similar basis to the theory of radiation damage. The incident ion produces a number of primary knock-on atoms; each of these produces a number of secondary displaced atoms, some of which may reach the surface with sufficient energy to be ejected. Keywell’s theory, which considers all collisions as if between hard spheres, gives reasonable agreement with experimental results for the bombardment of several metals by rare-gas ions of low energy (\(< 6\) kev). Pease has pointed out, however,\(^6\) that, as the incident energy is increased, a limit is reached, above which it is no longer permissible to use the hard-sphere analogy for the primary collisions; they should rather be regarded as Coulomb interactions of the ionic charge with the weakly-screened target nuclei. At still higher energies the collisions should be regarded as completely unscreened.

Since displaced atoms produced more than a few atomic layers below the surface will not be ejected, the sputtering ratio should be roughly proportional to the number of displaced atoms in the first few atomic layers. As the Coulomb cross-section, and hence the number of primary collisions, decreases with increasing energy, the sputtering ratio should show a corresponding decrease.

By considering that half the atoms displaced in the first three atomic layers are ejected, Pease has derived the following expression for the sputtering ratio when the primary collisions may be regarded as unscreened:

\[
X = \frac{2n_0\sigma_p}{1 + \log \left(\frac{1}{E} / |E_d|\right)} \text{ atoms/ion} \tag{8}
\]

where \(E\) is the energy of the ion, \(E_d\) the displacement energy of an atom in the target, \(\sigma_p\) the cross-section for a displacing collision, \(\sigma_p / E\) the maximum energy transferable to a target atom, and \(n_0\) the number of atoms/cm\(^2\) in an atomic layer. The displacement cross-section is given by\(^9\)

\[
\sigma_p = \frac{4E_d^2n_0^2M_1Z_1^2Z_2^2}{E^3} \left(1 - \frac{E_d}{\lambda E}\right) \tag{9}
\]

where \(M\) and \(Z\) are the atomic mass and charge respectively, the subscripts 1 and 2 refer to the incident and target ions respectively, \(E_d\) is the Rydberg energy and \(\sigma_0\) is the Bohr radius.

This treatment is valid for deuterons of more than 10 keV energy on materials with \(Z_2 < 30\). Some calculated sputtering ratios for deuteron bombardment are given in Table 4. The only available experimental result is for deuterons of 9.25 keV on silver.\(^10\) The measured sputtering ratio of 0.07 atoms/ion agrees quite well with the value 0.045 calculated on the foregoing theory, using in this case the weak screening model.

A feature brought out by Eq. (9) is that the sputtering ratio is roughly proportional to the atomic number of the target material. To compare the plasma contamination produced by sputtering with that produced by thermal evaporation, the current density of 100 keV deuterons which will cause the same erosion as evaporation at the maximum permissible wall temperature (see Table 2) is \(5 \times 10^{-4}\) amp/cm\(^2\) for beryllium, \(10^{-5}\) amp/cm\(^2\) for aluminium, and \(10^{-6}\) amp/cm\(^2\) for copper. It seems, therefore, that unless the magnetic containment of the plasma is extremely good, sputtering will be a more serious problem than evaporation. Good confinement is, however, a necessary condition for an economical thermonuclear reactor. For example, in the hypothetical reactor considered here, a current to the walls of about \(3 \times 10^{-5}\) amp/cm\(^2\) would represent a heat conduction loss equal to 10% of the radiation loss; the impurities sputtered by this current would increase the radiation loss by more than 10% only if \(Z_2\) was greater than about 10. This emphasizes the advantage of using a material of low atomic number for the wall.

Erosion by sputtering is not uniform but takes place preferentially from close-packed crystal planes. This leads to an etching of the surface similar to that produced by chemical action. Sputtering occurs on insulators as well as on metals; for example the etching of glass\(^11\) by ion bombardment is similar to etching by hydrofluoric acid. On account of the nature of the effect, it seems unlikely that any material could be developed which would resist sputtering by energetic ions.

Sputtering has not been specifically detected from the walls of present discharge tubes, where the ion energies are comparatively low, and if it has occurred it has been masked by impurities introduced by arc formation.

Arcing in Metal Toruses

When high currents were passed in small toruses of quartz and porcelain, appreciable wall evaporation occurred after 5 and 20 microseconds respectively. The heat input to the walls was estimated as about \(10^8\) w/cm\(^2\) and these times are consistent with the surface being raised to boiling point by the heat pulse. Most subsequent experiments have been performed in larger metal tubes, which have superior thermal properties. Another consideration, by no means negligible, is the ease of fabrication of large systems in metal. Furthermore, for reasons of discharge stability, it is desirable that the walls should be electrically conducting.\(^12\)

Most of the metal toruses have been made from
aluminium, the choice being dictated by convenience, although a few small copper tubes have been used. Although it appears that general thermal evaporation of the walls has been eliminated, the use of metal walls is accompanied by its own particular problem of arcing, and this has been responsible for most of the plasma contamination experienced in present systems.

**Power Arcs**—If a toroidal discharge tube is to be made of metal, a gap must be left in its circumference to prevent the induced emf from being short-circuited. When a discharge passes in the torus, this emf appears across the gap, which is at the same time exposed to the plasma. If the voltage across the gap is greater than the running voltage of an arc (about 10–20 v), a cathode spot may form on the negative side, and an arc will short-circuit the gap. Most of the energy being supplied to the system will then go into this arc rather than into the main discharge. The first metal toruses incorporated two gaps and it was found impossible to pass an unstabilized discharge of more than 10,000 amp before arc breakdown occurred at the gaps.

A way out of this difficulty seemed to be to divide the torus by a larger number of gaps, thereby distributing the induced emf so that the voltage across each gap is insufficient to support an arc. This assumes that the electric field is uniform around the torus whereas, with an unstable discharge, considerable fluctuations may occur along its length. Toruses with 48 and 64 gaps, built at AEI, showed evidence of arcing across the gaps even though the mean voltage per gap was only about 10 v; this was most probably due to larger transient voltages arising from discharge instability. Although currents of up to 75,000 amp were achieved in the 64-gap torus, the plasma was heavily contaminated by aluminium vapour from the arcs. Figure 2 shows the damage to the torus caused by such arcs; melting of both sides of the gaps has taken place. This type of arcing across gaps is referred to as power arcing, since the energy for the arc is derived directly from the power source supplying the system.

In the Harwell Mk III (35 cm bore) torus and in ZETA, a system of aluminium liners, which is described elsewhere, is employed inside the main torus body. This is equivalent to providing the torus with 24 gaps in the case of Mk III and 48 gaps in the case of ZETA. In addition, an axial magnetic field is applied to give stability to the discharge. With a stable discharge, no power arcs occur, but reducing the axial field to the point at which the discharge is highly unstable results in considerable damage by power arcing. In SCEPTRE III at AEI, which is an eight-gap torus with an axial magnetic field, some power arcing still occurs, although this has been reduced by covering the aluminium gaps with copper shields.

**Unipolar Arcs**—When Mk III torus and ZETA are operated under conditions in which no power arcs occur, the liners are still found to be covered with the distinctive tracks of cathode spots (see Fig. 3). These have eroded the layer of aluminium compounds which builds up on the liners during operation of the machine. (The layer is principally aluminium nitride which forms from the nitrogen introduced as a controlled impurity.)

The direction of the branching pattern of the tracks indicates that the spots are being driven by the prevailing magnetic field at the torus wall in the so-called retrograde direction. The current density in the cathode spot is known to be of the order of $10^5$ amp/cm$^2$, or greater, so from the width of the tracks it appears that currents of the order of $10^8$ amp have been flowing. From the length of the tracks it can be inferred that the arcs have existed for almost the entire duration of the main current pulse. Since the voltage at each gap is insufficient to maintain an arc between liners, some explanation other than gap breakdown must account for these arcs. It has been suggested by Thonemann that each liner has been simultaneously both cathode and anode of a "unipolar arc", which is a form of discharge peculiar to a
metal wall in contact with a high temperature plasma. The mechanism of this arc is as follows.

An electrically isolated metal plate in a plasma will acquire a negative potential, so that it attracts ions and repels all but the fastest electrons, and in equilibrium receives an equal current of each. This negative potential is known as the wall potential and is given by probe theory \(^{15}\) as

\[
V_w = \frac{kT^-}{2e} \ln \frac{M^+T^-}{M^-T^+},
\]

(10)

where \(T^-, T^+\) are the temperatures of the electrons and ions respectively and \(M^-, M^+\) are their masses. For a deuterium plasma with \(T^+ = T^- = T\), Eq. (10) becomes:

\[
V_w = -3.5 \times 10^{-4} T \text{ volts}. \quad (11)
\]

(This treatment neglects secondary electron emission and photoemission, which will tend to reduce the wall potential.) If \(T\) is greater than about 30,000°K, \(V_w\) will be greater than 10 v, that is to say, greater than the cathode fall of an arc (\(V_e\)). If an arc spot is now initiated at some point on the plate, the potential difference between plate and plasma will decrease by the emission of electrons from the spot; the plate will now be held at \(-V_e\) with respect to the plasma. Since this is less negative than wall potential, a net electron current will flow to the plate from the plasma, balancing the emission current from the spot; the plate will now be at \(-V_e\). The energy for maintaining it is drawn from the thermal energy of the plasma; this distinguishes it from the power arc which draws its energy directly from the electric field applied to the discharge.

The arc spot not only emits electrons; it is a copious source of metal vapour: it is estimated that \(10^{17}-10^{18}\) atoms are evaporated per coulomb passed by the arc, and it seems that the greater part of the plasma contamination in ZETA and SCEPTRE III originates from arc spots. In a thermonuclear reactor it will be essential to eliminate arcing completely, as a single unipolar arc carrying only a few amperes will provide more impurities than can be tolerated.

It is to be expected that if a plasma were contained by magnetic fields the walls would acquire a positive potential with respect to the plasma. This is because the magnetic field acts primarily on the electrons; the ions are contained by electric fields which arise from charge separation and act towards the centre of the discharge. If the walls were positive it would be impossible for cathode spots to form on them. The existence of arc tracks on the walls of present discharge tubes indicates that the walls are in fact negative with respect to the plasma, probably on account of the imperfect containment so far achieved. Similar conditions may exist at the walls of a thermonuclear reactor in the early stages of establishing the plasma.

One method of eliminating unipolar arcs relies on the fact that a minimum current, usually of the order of one ampere, is required to maintain an arc spot. If the area of each metal surface exposed to the plasma is insufficient to collect this current, the arc will not form. This has been demonstrated by hanging aluminized glass plates of different sizes in the plasma of Mk III torus. The appearance of a plate 15 cm x 5 cm after 20 pulses of the discharge is shown in Fig. 4, the arc spots having evaporated the aluminium along their path. An identical plate in which the aluminium layer had been divided by rulings into segments 1 cm square showed no arc tracks after a much longer period in the torus.

It seems, therefore, that it may be possible to eliminate unipolar arcs by applying the “minimum area principle” to the construction of either torus liners or a torus itself. This, however, presents considerable technical difficulty and has not yet been attempted on a large scale. Instead, work has been directed towards understanding the mechanism of formation of arc spots to see if they can be prevented by other means.

THE ARCING PROPERTIES OF MATERIALS

It appears that, at the walls of present devices, conditions exist such that a unipolar arc, once initiated, may be maintained. In order that the arc may start, however, additional conditions are necessary at the surface of the metal. The closest analogy to this situation is to be found at the anode of a mercury-arc rectifier during the non-conducting half-cycle of current, when it is at a negative potential with respect to an established plasma. It is known that a cathode spot may sometimes form on the anode and the rectifier will then conduct in the reverse direction; such
an occurrence is obviously undesirable and can largely be avoided by making the anode of carefully prepared carbon, subsequently outgassed and "conditioned" by methods well known in rectifier technique. To make a large torus in this way presents formidable difficulties and the question arises whether unipolar arcs might be prevented by suitable choice and treatment of some other material.

Experimental work has been carried out at AEI Research Laboratory and at AERE on the initiation of arcs on various materials in a plasma. The principle of the experiments is as follows. Small specimens of the material to be investigated are immersed in a comparatively low temperature plasma generated by a pulsed toroidal discharge. An external power source is used to bias the specimens negative with respect to the plasma, thus simulating the conditions which may exist at the walls of a device containing a plasma at much higher temperatures. The occurrence of arcs on the specimens is detected by the large currents drawn from the biasing source.

Experimental work at AEI Research Laboratory

Experiments were carried out in a pulsed hydrogen ring discharge in a Pyrex glass torus (bore 10 cm and diameter 30 cm). The discharge current was a damped oscillation of 1000 amp peak and period $10^{-4}$ sec. A pair of test electrodes was immersed in the discharge plasma and a capacitor connected between them. The capacitor was charged to a specified voltage and if an arc occurred during the discharge pulse it was rapidly discharged.

In preliminary experiments, the capacitor voltage was raised in fixed steps and 50 discharge pulses produced at each step. The number of pulses which produced arcs within each set of 50, expressed as a percentage, was termed the percentage arcing and was taken as an indication of the probability of arc formation at that particular voltage. Table 5 shows the percentage arcing, over the range 15–400 volts, for cathodes of different metals. These cathodes were polished and vapour-degreased before test. It was found that nickel and copper showed less arcing than aluminium and stainless steel.

A feature brought out by this type of experiment was the independence of the percentage arcing on the number of previous arcs. At higher voltages (500–2000), all the metals tested were found to exhibit a "conditioning" effect, and the percentage arcing fell from 100% to a very much lower value. The number of arcs observed before the percentage arcing fell to less than 5% is shown in Table 6 for various cathodes and experimental conditions. The electrodes had a fine machined finish and were vapour-degreased.

The conditioning was more rapid the higher the condenser energy, but a limit was reached at about 5 joules, which coincided with the anode's showing signs of melting. At this energy, the current in the arc was of the order expected for the saturation electron current to the anode; above this limit, therefore, any extra energy would be dissipated at the anode rather than at the cathode.

Conditioning effects have been widely observed in discharge phenomena where breakdown occurs, and conditioning is a standard treatment in the manufacture of many high voltage electrical devices. To apply such treatment to a large torus might require a very large number of arcs, so there was need to search for some more practical surface treatment which could be applied to large areas of metal. The effect of a wide range of cleaning treatments on the arcing behaviour of copper was therefore investigated; copper was chosen because it was one of the metals showing least arcing in the initial tests, and because its good electrical and thermal properties made it seem a likely torus material.

It was found that electrodes which had been treated by careful chemical cleaning, by polishing, or by heating in vacuo (either by electron or ion bombardment,
or by an internal heater) showed greatly improved arcing behaviour over electrodes which had been simply liquid- or vapour-degreased. This is shown in Table 7, which gives the number of arcs to condition an electrode after each treatment. The only effective chemical treatment was an ammonium persulphate etch\textsuperscript{16} which reduced by an order of magnitude the number of arcs required for conditioning. Similar improvement was obtained by polishing and by heating, but no further improvement resulted from combining these techniques. Differences in the number of arcs to condition were shown statistically to have a low level of significance for all the tests in Table 7 except those of groups 1 and 2. The ammonium persulphate treatment and heating in vacuo also raised the onset voltage for arcing from 25 to about 200 volts.

In further tests, copper electrodes of three different grades of purity were compared, namely phosphorus de-oxidized, oxygen-free high conductivity and vacuum cast, but no significant differences were observed. In other experiments the whole vacuum system was baked, as well as the electrodes, and the base pressure reduced to $10^{-7}$ mm Hg but no further improvement was obtained.

Once an electrode had been conditioned, it was sometimes found to retain this state for an appreciable length of time. After conditioned copper and aluminium electrodes had been exposed for several hours to clean air they needed only a few arcs to return them to the conditioned state. A copper electrode remained conditioned after exposure for sixteen hours to the vapours present in an untrapped oil-pumped vacuum system, and even when it was taken out and fingered.

**Experimental work at AERE**

A series of experiments has been carried out to investigate the arcing properties of 15 different metals under identical conditions. Specimens of the metals in the form of cylinders, 12.5 mm diameter and 45 mm long with hemispherical ends, were polished by hand to mirror finish using successively finer grades of emery and finishing with fine diamond powder. After degreasing in trichlorethylene the specimens were mounted on the end of a probe and lowered into the Mk III (35 cm bore) torus. The probe was biased negatively with respect to the nearest liner (which served as the anode) and a capacitor of 64 µF connected between the liner and the probe.

A pulsed discharge of 10,000 amp peak current in hydrogen at $10^{-3}$ mm Hg was passed in the torus, the pulse length being 800 µsec and the repetition rate, once per second. An axial magnetic field of 100 gauss was applied to give the discharge a small degree of stability. The experimental procedure was to subject

---

### Table 6. Number of Arcs to Condition

<table>
<thead>
<tr>
<th>Metal</th>
<th>Capacitor µF</th>
<th>Voltage (mm Hg)</th>
<th>Number of Arcs to Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>With Hg Diffusion Pump</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aluminium</td>
<td></td>
<td>1000</td>
<td>534</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>500</td>
<td>686</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>105</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2000</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>Copper</td>
<td></td>
<td>10</td>
<td>477</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>168</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>186</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>1000</td>
<td>78</td>
</tr>
<tr>
<td>Nickel</td>
<td></td>
<td>10</td>
<td>114</td>
</tr>
<tr>
<td>Molybdenum</td>
<td></td>
<td>10</td>
<td>186</td>
</tr>
<tr>
<td>Stainless steel</td>
<td></td>
<td>10</td>
<td>324</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2000</td>
<td>78</td>
</tr>
<tr>
<td>Aluminium + 0.005 inch nickel plate</td>
<td>10</td>
<td>1000</td>
<td>1980</td>
</tr>
<tr>
<td>Aluminium + molybdenum sprayed coating</td>
<td>10</td>
<td>1000</td>
<td>1440</td>
</tr>
<tr>
<td>Moly-rhenium</td>
<td></td>
<td>10</td>
<td>882</td>
</tr>
<tr>
<td><strong>With Oil Diffusion Pump</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Copper</td>
<td></td>
<td>10</td>
<td>288</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>132</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>Aluminium</td>
<td></td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

---

- Electrode area 6 cm\(^2\). Hydrogen pressure $6 \times 10^{-3}$ mm Hg.
Table 7. Conditioning Tests in Non-baked Vacuum System for ICI Vacuum-cast Copper Electrodes

<table>
<thead>
<tr>
<th>Group No.</th>
<th>Surface preparation before cleaning</th>
<th>Surface cleaning</th>
<th>Pre-heating</th>
<th>Maximum Cathode temp. during test °C</th>
<th>Number of arcs observed before arcing percentage reached 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fine machined</td>
<td>Liquid degreased</td>
<td>None</td>
<td>80</td>
<td>360</td>
</tr>
<tr>
<td>2</td>
<td>Fine machined</td>
<td>Vapour degreased</td>
<td>None</td>
<td>55</td>
<td>396</td>
</tr>
<tr>
<td>3</td>
<td>Fine machined</td>
<td>Vapour degreased, ammonium persulphate etch</td>
<td>Held at 200°C for 1 hour</td>
<td>65</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>Fine machined</td>
<td>Vapour degreased, ammonium persulphate etch</td>
<td>Held at 200°C throughout experiment</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Fine machined</td>
<td>Vapour degreased, ammonium persulphate etch</td>
<td>Held at 200°C for 1 hour</td>
<td>45</td>
<td>34</td>
</tr>
<tr>
<td>6</td>
<td>Polished with silicon carbide and &quot;Silvo&quot;</td>
<td>Vapour degreased, ammonium persulphate etch</td>
<td>Held at 200°C throughout experiment</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Buffed and polished with jeweller's rouge</td>
<td>Vapour degreased</td>
<td>None</td>
<td>50</td>
<td>43</td>
</tr>
</tbody>
</table>

The preliminary results of these experiments have revealed both similarities and differences in the behaviour of different materials. The similarities are illustrated in Fig. 5 by typical results for aluminium, copper and molybdenum, in which the number of arcs occurring in each 1000 pulses is given in histogram form. The familiar process of conditioning can be seen in each case; this was observed in all the materials studied. In most cases the initially high arcing rate had fallen to a much lower and approximately constant rate within the first 1000 pulses. When the voltage on the specimen was increased to 1000 v a second conditioning was observed, although in this case fewer arcs occurred before the final arcing rate was established.

The most significant differences between materials are in the number of arcs in the first thousand pulses ($N_1$) and in the final arcing rate, averaged over the last 3000 pulses ($N_f$). In Table 8 the values of $N_1$ and $N_f$ are given from a series of 28 tests. The differences between values of $N_f$ from different specimens of the same material are on the whole smaller than differences between materials, but values of $N_1$ are less consistent.

Discussion of the results

In order to understand the initiation of the arc, the mechanism of the established arc must be considered.
The arcs observed here are of the so-called cold-cathode type in which electron emission at high current density ($10^5 - 10^6$ amp/cm$^2$) takes place from a localized region of the cathode, which is clearly visible as a bright spot on the surface. Strong evaporation of the cathode material accompanies the electron emission. There are two principal theories of how electrons are emitted from the cathode. The field theory of Langmuir suggests that they are drawn out by the strong electric field of the positive ion space-charge above the cathode; this theory, although widely held, is open to criticism on quantitative grounds. A more recent theory by von Engel and Robson suggests that the emission is due to bombardment of the cathode by excited atoms formed in the vapour just above it. Without going into the respective merits of the two theories, it can be said that both require a high density of vapour to exist in the cathode spot, and so one of the necessary stages in the initiation of an arc is the establishment of a localized vapour cloud at the cathode surface. The vapour density must be high ($\sim 10^{19} - 10^{20}$ atoms/cm$^3$) but need exist only over a region of dimensions $10^{-3}$ cm.

The density of plasma in the neighbourhood of the specimen under test is only of the order $10^{14}$ ions/cm$^3$; so, for an arc to occur, this must be augmented by vaporization of the surface. A simple calculation shows that the heat input to the surface during the pulse (approx. 1 kw/cm$^2$ for 1 msec) is inadequate to cause significant thermal evaporation of the metal. However, if there is some more volatile material on the surface, this may provide the necessary vapour to start the arc. This may be, for example, absorbed gas layers, dissolved gas, residual greases and chemical layers such as oxides. It is well known that surface contamination and oxide layers increase the probability of glow-to-arc transitions, although the role of the oxide layer is not precisely known. It may either enhance the initial electron emission from the surface, or may vaporize more easily than the base metal under ion bombardment. It is to be expected therefore that the initial rate of arcing will be a function of the surface of the material; it is significant that the specimens which show the largest values of $N_i$ are those which are known to possess tenacious oxides (e.g. aluminium, tantalum, uranium, thorium). Vacuum baking and chemical cleaning of the surface

<table>
<thead>
<tr>
<th>Material</th>
<th>$N_i$</th>
<th>$N_f$</th>
<th>Mean $N_i$</th>
<th>Mean $N_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>255</td>
<td>14.6</td>
<td>550</td>
<td>18.3</td>
</tr>
<tr>
<td>Copper</td>
<td>87</td>
<td>4.3</td>
<td>82</td>
<td>4.8</td>
</tr>
<tr>
<td>Titanium</td>
<td>98</td>
<td>4.3</td>
<td>97.5</td>
<td>6.0</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>44</td>
<td>3.6</td>
<td>70</td>
<td>3.0</td>
</tr>
<tr>
<td>Nickel</td>
<td>212</td>
<td>8.0</td>
<td>268</td>
<td>6.6</td>
</tr>
<tr>
<td>Iron</td>
<td>137</td>
<td>12.7</td>
<td>170</td>
<td>13.0</td>
</tr>
<tr>
<td>Silver</td>
<td>422</td>
<td>256</td>
<td>429</td>
<td>595</td>
</tr>
<tr>
<td>Tantalum</td>
<td>744</td>
<td>3.3</td>
<td>632</td>
<td>3.5</td>
</tr>
<tr>
<td>Niobium</td>
<td>99</td>
<td>5.7</td>
<td>84</td>
<td>6.0</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>51</td>
<td>3.1</td>
<td>66</td>
<td>2.5</td>
</tr>
<tr>
<td>Zirconium</td>
<td>109</td>
<td>3.3</td>
<td>124</td>
<td>5.0</td>
</tr>
<tr>
<td>Uranium</td>
<td>737</td>
<td>3.0</td>
<td>489</td>
<td>3.5</td>
</tr>
<tr>
<td>Thorium</td>
<td>685</td>
<td>1.7</td>
<td>509</td>
<td>2.9</td>
</tr>
<tr>
<td>Tungsten</td>
<td>115</td>
<td>4.3</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Carbon</td>
<td>384</td>
<td>24.7</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Table 8. Initial and Final Arcing Rates for Different Metals (Number of arcs per 1000 pulses)

Figure 6. Arc damage on metal specimens. Left to right: iron, nickel, zirconium, molybdenum, tungsten, copper
before test reduces \( N_1 \) but no treatment has yet been applied which reduces it to the order of the final arcing rate. It seems that the final conditioning of the surface must always be left to the discharge itself.

The fact that conditioning is not perfect but only tends to a final arcing rate is more difficult to understand. Once the surface has been cleaned, the calculated heat input at each pulse is insufficient to cause vaporization. The presence of copper and silver in the plasma around specimens of these materials biased at 1000 v has been detected spectroscopically, even though the specimen was not arcing. It seems likely that this is due to sputtering, and that a steady erosion of the surface is therefore taking place. At each pulse, about \( 5 \times 10^{15} \) ions/cm\(^2\) strike the specimen. If the sputtering ratio is of the order of unity, this means that at least one atomic layer is eroded every pulse. The nature of the surface is therefore continually changing, and it is possible, for example, that the erosion may reveal inclusions of impurities in the metal which, by producing bursts of vapour, may lead to arcs.

While the initial arcing rate is a function of the initial surface condition, it seems that the final arcing rate is probably a property of the bulk material. The results reported here must be regarded as preliminary and more experimental work is needed before the precise origin of the arcs can be decided.

**INSULATING LAYERS**

In an attempt to prevent arcing while still retaining the advantages of metal walls, the application of thin insulating layers to metals has been considered. If a perfect layer is obtained, a unipolar arc cannot occur unless breakdown of the dielectric occurs at two points simultaneously, thereby allowing current to enter and leave the metal base. If the layer contains cracks or pinholes, arcing may still be prevented on the "minimum area principle" if the total area of the holes is sufficiently small. For good thermal properties the layer should be as thin as is consistent with its electrical strength, and tests have been carried out to determine the breakdown potentials of thin insulating layers when they are held at a negative potential in a plasma.

Several types of anodically formed oxide layers on aluminium have been tested, using pairs of electrodes in a glass torus. The breakdown potential depends on both thickness and porosity of the layer and the best results were obtained with composite layers in which a thick (.025 mm) but porous layer was formed in oxalic acid and its impervious barrier layer thickened by further treatment in a boric acid/borax electrolyte. A mean breakdown potential of 515 v was obtained for pairs of anodized electrodes, and 235 v when only the cathode was anodized. This must be compared with the minimum breakdown potential of 500 v per layer in air, which suggests that testing in a plasma is a more severe test than testing in air.

The method of formation of anodic layers limits the breakdown potential to about 500 v, whereas wall potentials of kilovolts may occur in high temperature discharges. In an attempt to obtain stronger layers, alumina has been flame-sprayed onto small metal specimens. The breakdown potential of a layer 0.1 mm thick was, however, only 200 v; the layer was slightly porous and drew an electron current of several ma per cm\(^2\) when made positive in a plasma. A flame-sprayed test liner in ZETA was extensively damaged by unipolar arcs.

Glass layers have proved more successful. These have melting points which are sufficiently low to allow the glass to be fused directly to the metal, and their coefficients of expansion can be matched to the base metal to reduce mechanical strain. Layers have been applied to aluminium which, when tested in an electrolyte, withstood 6 kv. An aluminium rod coated with 0.15 mm of glass, placed diametrically across the discharge in a 35 cm bore torus, successfully withstood 2 kv, although there was spectroscopic evidence of some decomposition or evaporation of the layer. A liner of a 35 cm bore torus, which was coated with glass and known to have several microscopic holes, was biased at 500 v negative with respect to an adjacent uncoated liner; arcing ceased after a few initial breakdowns.

Until techniques for applying impervious layers of insulators such as beryllia and alumina are developed, glass layers may be a useful way of preventing arc formation in present equipment, although they have not yet been applied on a large scale. In a reactor, however, the bombardment of the walls by energetic deuterons, either during or after the reaction pulse, and the intense X-radiation, would probably lead to the decomposition of the majority of insulators. It seems that pure metal walls will be necessary in final designs.

**OTHER CONSIDERATIONS**

**Thermal Stress Fatigue**

Since a thermonuclear reactor of the pinched discharge type must be operated with a pulsed or alternating current, the heat flux to the walls will exhibit cyclic variations. The resulting fluctuations in the temperature difference across the walls may lead to cracking of the surface by thermal stress fatigue if the stresses produced by differential expansion during each cycle exceed the elastic limit of the material.

The maximum variation of the temperature difference across a wall, if it is to withstand indefinite thermal cycling, is given by

\[
\Delta T = \frac{2E_1}{Y} \alpha \theta
\]

where \( Y \) is Young's modulus, \( E_1 \) the elastic limit and \( \alpha \) the coefficient of linear expansion. Some values of \( \Delta T \) for different materials are shown in Table 9. In practice these values could be exceeded since only limited life is required, and cold working of the material by cyclic stressing may increase its strength. However, thermal stress fatigue will probably be an
Table 9. Maximum Allowable Temperature Difference across the Wall under Cyclic Conditions

<table>
<thead>
<tr>
<th>Material</th>
<th>Mean operating temperature °C</th>
<th>Maximum temperature difference °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beryllium</td>
<td>20</td>
<td>70</td>
</tr>
<tr>
<td>Magnesium</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Aluminium</td>
<td>20</td>
<td>9</td>
</tr>
<tr>
<td>Titanium</td>
<td>20</td>
<td>780</td>
</tr>
<tr>
<td>Vanadium</td>
<td>400</td>
<td>890</td>
</tr>
<tr>
<td>Cobalt</td>
<td>20</td>
<td>105</td>
</tr>
<tr>
<td>Nickel</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>Copper</td>
<td>600</td>
<td>60</td>
</tr>
<tr>
<td>Zirconium</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>20</td>
<td>260</td>
</tr>
<tr>
<td>Beryllia</td>
<td>600</td>
<td>185</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>20</td>
<td>150</td>
</tr>
</tbody>
</table>

important factor in the design of apparatus to run for long periods under cyclic conditions.

An aluminium specimen was subjected to 10,000 discharges in the Mk III (35 cm bore) torus whilst at a mean temperature of about 800°C. Each discharge resulted in a flux of 1.5 kw/cm² for 10⁻³ sec, which would give a periodic variation of the surface temperature of about 30°C. The resulting damage due to arcing and thermal stress fatigue is shown in Fig. 7.

Radiation damage

The subject of radiation damage of materials, with the consequent change in their physical properties, has been extensively studied in connection with fission reactors. Whereas the radiation encountered in a fission reactor is made up of hard γ-rays and both fast and thermal neutrons, the wall of a thermonuclear reactor will be subjected only to fast neutrons and soft X-rays (bremsstrahlung). The bremsstrahlung will not damage metal walls other than by raising their temperature, but may lead to photo-decomposition of insulating materials.

The neutrons from the D-D reaction have an energy of about 2.5 Mev and from the D-T reaction about 14.2 Mev. In a D-D reaction in which all the tritium formed is burnt, one neutron is produced for every 12.4 Mev released. Since, at the optimum reacting temperature, the energy in bremsstrahlung is about one-ninth of the reaction energy, a radiation flux of 100 w/cm² to the wall will be accompanied by about 7 x 10¹⁴ neutrons/cm² sec. This neutron flux is appreciably higher than in a conventional fission power reactor.

The knock-on nuclei produced by fast neutrons passing through the walls lead to cascades of point defects which may eventually cause a loss of strength of the material. This may however be compensated by annealing if, as is anticipated, the walls are run hot. A more serious damage may arise from nuclear reactions taking place in the walls, which lead to the formation of stable gas atoms; these may cause swelling and embrittlement of the material. In particular, beryllium, which in other respects seems to be a very favourable wall material, may suffer from reactions in which helium is produced. For example, after a year’s continuous operation at the above neutron flux, the helium concentration in the wall would be about 0.1%; the resulting embrittlement might reduce the material’s ability to withstand thermal stress fatigue.

CONCLUSIONS

The principal problems associated with the choice of wall material for a high current discharge tube have
been discussed, both under the conditions which exist in present systems and under the conditions which are anticipated in a thermonuclear reactor.

It is clear that metals possess marked superiority over insulators in their thermal properties and ease of fabrication, although some advantages might be obtained from a thin refractory layer on a metal base. The principal disadvantage of metals is their tendency to form arc spots when exposed to a hot plasma, and this has been the main cause of plasma contamination in existing toruses in which the imperfect containment has led to appreciable contact of the plasma with the walls.

Experiments on a wide range of metals have revealed only small differences in the ability of most metals to withstand arcing, metals of higher boiling point showing slight superiority. After these metals have been thoroughly cleaned and conditioned, the probability of arc formation even under intense ion bombardment is very small. It is to be expected that in a final reactor, where the containment of plasma must necessarily be very good, the bombardment of the wall will have been reduced to such a low level to minimize conduction losses that the probability of arcing will be virtually eliminated. For the present, however, arc formation represents a serious obstacle to the production of a clean deuterium plasma in a metal tube, and it seems that its elimination will depend not only on the choice of material, but on the design of the torus and the effective containment of the plasma. The conditions experienced in present tubes may also be encountered in a final reactor in the early stages of the discharge, when a relatively cold plasma will be in contact with the wall.

It appears that thermal evaporation need not be a serious problem in metal apparatus of large dimensions but, as higher temperatures are reached, sputtering of the walls may be important unless materials of very low atomic number are used. There is clearly need for more research on this subject.

ACKNOWLEDGEMENTS

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8. R. S. Pease, private communication.
Propulsion of Plasma by Magnetic Means

By W. H. Bostick*

BUTTON-TYPE PLASMA GUN

It has been demonstrated\(^1\) that a small button gun (Fig. 1) can project plasma consisting of metallic ions, deuterium ions, and electrons at speeds up to \(2 \times 10^7\) cm/sec. These speeds are measured in a vacuum chamber by time-of-flight methods, using a probe and an oscilloscope. The first arriving plasma signal corresponds to this high speed of \(2 \times 10^7\) cm/sec. There are later signals corresponding to other portions of the plasma which are traveling more slowly. However, from the predominantly positive sign of these slower signals, it can be inferred that some of this slower plasma encountered the walls of the cylindrical vacuum chamber and thereby was slowed down.

Since it is thus difficult to measure, by means of a probe and oscilloscope, the velocity profile of the plasma from such a plasma gun, a ballistic pendulum method has been devised to measure the momentum of the plasma coming from a pulsed gun. The pendulum bob is a cup made from thin plastic or aluminium foil, with its opening oriented to receive the plasma. In principle, if the plasma is composed entirely of metallic ions, the mass of the plasma can also be measured by determining the loss of weight of the gun or the gain of weight of the collecting cup after the gun has been fired a specified number of times. Such measurements are in progress.

From the point of view of projecting high speed plasma in a given direction these button guns suffer from the following weaknesses.

(a) The back emf (the gun is essentially a linear motor, whose armature delivers a back emf) delivered by the guns is, in general, small compared with the voltages which are suitable for capacitors. Consequently, the discharges of the capacitor are not anywhere near critically damped, and the capacitors ring for many cycles, thus dissipating much of their energy in circulating currents rather than storing that energy as kinetic energy of the plasma.

(b) The region where the magnetic forces are concentrated and hence effective is within only a few millimeters of the gun. The time taken for a fast moving plasma (~\(10^7\) cm/sec) to travel this short distance is a small fraction of a microsecond. It is impractical to try to have capacitors which store reasonable amounts of energy (~\(10^3\) joules) discharge in this short time. Slower capacitors cannot discharge their energy efficiently into the kinetic energy of the plasma with these guns.

(c) Crude measurements indicate and theory predicts that the button source, at least, is not at all unidirectional in its plasma pattern, but fires over a fairly wide angle.

RAIL-TYPE PLASMA GUN

A more efficient arrangement is to accelerate a sample of plasma by passing a current through the plasma as it rides on rails, as indicated in Fig. 2. This scheme for accelerating plasma by the current in the rails is essentially the electromagnetic gun,\(^†\) except that the bullet in our case is a mass of plasma or ionized gas. The analysis of acceleration of a bullet by a rail system has been dealt with elsewhere.\(^4\) Russian investigators\(^5\) have made a theoretical analysis of the acceleration of plasma on a rail system where an externally-excited magnetic field is applied to the plasma. However, they have not considered the effect of the magnetic field due to the current in the rails. Russian investigators\(^6\) have also conducted rail propulsion experiments, involving the acceleration of a plasma produced by evaporation of a metallic wire, and have achieved speeds of \(10^7\) cm/sec. A similar type of experimental arrangement has been used at Temple University.\(^7\)

A simple analysis that assumes an effective current \(I\) gives a relatively easy way of assessing the effect of various parameters in a rail-type plasma motor (or gun) without the laborious task of numerical integration of the equations of motion.

The Series Plasma Motor

If, as is indicated in Fig. 2, \(m\) is the mass of the plasma sample which is placed between two rails of

\(^1\) The development of experimental equipment for accelerating pellets has been carried on very successfully by Mr. Morton Levine at the Air Force Cambridge Research Center.
The integration of the equations of motion is somewhat tedious and the results are unwieldy when the inductances of the condenser, switch and leads are included. Let us assume, for purposes of simplicity, that the capacitor with its internal inductance can be replaced by a battery of voltage $V_0$ and internal impedance $Z_0 = (L_0/C_0)^{1/2}$ as in Fig. 3. Under these circumstances an approximate load impedance, $Z_{\text{load}}$, representing the plasma and the rails can be assigned:

$$Z_{\text{load}} \approx 4.24 \times 10^{-19} (I/m) (\log b/a)^2.$$ (4)

Fairly efficient transfer (about 50%) of energy to the load occurs when $Z_{\text{load}} = Z_0$. This energy is shared between the kinetic energy of plasma and the inductive energy stored in the two-wire transmission line. Almost complete transfer of energy to the plasma occurs when $Z_{\text{load}} \gg Z_0$. Under these circumstances the back emf due to the plasma traveling in $H_{AV}$ is sufficient to reduce the current almost to zero, and there is then very little magnetic energy left in the transmission line. In Table 1 we have inserted some practical numbers. It can be seen that with a pair of rails 50 cm long, $b/a = 10$ and a current $I$ of $10^4$ amp, a $10^{-7}$ g sample can be given a speed of $4 \times 10^7$ cm/sec. The effective plasma impedance, $Z_{\text{load}} = 0.42$ ohm, is fairly high—it is easy to obtain a capacitor with $C_0 = 2.4 \mu F$, and $L_0$ low enough so that $Z_0 < 0.42$ ohm.

### A Combination Series-shunt Plasma Motor

An obvious step is to add an externally-excited magnetic field $H$, as indicated in Fig. 2, in order to obtain the analogue of a series-shunt wound motor. The effect of such an additional shunt field $H$ is to increase the field in which the current $I$ in the sample is flowing. The velocity is given by

$$v^2 \approx 10^{-19} [2I \log (b/a) + 10Hd] I/m \text{ cm/sec}. \quad (5)$$

The approximate load impedance is given by

$$Z_{\text{load}} \approx 10^{-19} [0.92 I \log (b/a) + H^2]$$

$$x [4.67 \log (b/a) + 10 HD] / m \text{ ohms} \quad (6)$$

from which we obtain the back emf, $V_{x=0} = I Z_{\text{load}}$, and the energy input, $E_1 = \frac{1}{2} I^2 Z_{\text{load}}$.

In Table 1 we have tabulated the appropriate values for $v$, $I$, $V_{x=0}$, $E_1$, $Z_{\text{load}}$, $C_0$ and $L_0$ for the same values as for the series plasma motor with the value of $H$ equal to $10^4$ gauss.

### Table 1. Characteristics of Plasma Motors

<table>
<thead>
<tr>
<th>$H = 0$ (series motor)</th>
<th>$H = 10^4$ (series shunt motor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v$</td>
<td>$1.46 \times 10^8$ cm/sec</td>
</tr>
<tr>
<td>$I$</td>
<td>73 cm</td>
</tr>
<tr>
<td>$V_{x=0}$</td>
<td>$2.8 \times 10^4$ v</td>
</tr>
<tr>
<td>$E_1$</td>
<td>140 joules</td>
</tr>
<tr>
<td>$C_0$, from $E_1 = \frac{1}{2} C_0 V_0^2$</td>
<td>$0.36 \mu F$</td>
</tr>
<tr>
<td>$L_0$, from $L_0 C_0 = \frac{1}{2}$</td>
<td>$2.8 \Omega$</td>
</tr>
<tr>
<td>$Z_{\text{load}}$</td>
<td>$2.8 \Omega$</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>$0.42 \Omega$</td>
</tr>
<tr>
<td>$d = 1 \text{ cm}$</td>
<td>$0.42 \Omega$</td>
</tr>
<tr>
<td>$t = 10^{-6} \text{ sec}$</td>
<td>$73 \text{ cm}$</td>
</tr>
</tbody>
</table>
If the magnetic field due to the current in the rails is negligible compared with the externally applied magnetic field, $H$, it is possible to use the equivalent circuit of Fig. 4 where $C_0$ is the storage capacitor and where the plasma is effectively the capacitance $C_L$, into which a certain fraction of the energy of $C_0$ will be discharged, depending upon the ratio $C_L/C_0$. If $C_L' = C_0$, or is made so by the insertion of a pulse transformer, all of the energy of $C_0$ can be transferred through the inductance to $C_L'$. The physical analogue is the complete transformation of electrostatic energy in the capacitance $C_0$ to kinetic energy of motion of the plasma in one-half cycle. If $C_L' = C_0$, then $C_L' = 2E_0/V^2 = 10^9 Wb/H^2$ farad.

Presumably the most efficient way to operate the motor is to adjust the parameters so that the back emf reduces the current to zero (and hence leaves no energy stored in the transmission line) just as the plasma leaves the end of the rails. Under these circumstances all of the energy stored in the capacitor is transformed to kinetic energy of motion of the plasma during the first half-cycle of current. Moreover, no arc will be drawn at the end of the rails as the plasma leaves because no current will be flowing.

The series and series-shunt motors diagrammed in Fig. 2 put their energy predominantly in the forward direction. They also are capable of developing adequate back emf's. It can thus be seen that they do not suffer from the same difficulties as the button sources.

**Initial Experiments with Rail-Type Motors**

Initial experiments on the operation of a rail-type, series-shunt plasma motor have been performed with the experimental arrangement shown in Fig. 5. The cup-shaped rails have proved more suitable than either wire rails or thin strip (3 mm wide) rails in confining the plasma in the $Z$ direction and preventing the plasma from "jumping" the rails in the $y$ direction (see Fig. 5). The plasma is produced by an arc across the insulator between the two copper wires (just as the plasma is produced in a button gun). The wires are electrically attached to the rails. With a storage capacitance of 0.12 $\mu F$ charged to 14 kv, a resistance of about 3 ohms for critical damping of the current pulse, and a current pulse duration of about 0.6 $\mu$sec, the average plasma speed for the distance from $x = 0$ (at the breech of the gun) to $x = 10$ cm (5 cm beyond the muzzle of the gun) is $10^7$ cm/sec. Although this speed is not so spectacular, the encouraging feature is that all of the plasma seems to have this speed, since probe measurements indicate the plasma to be fairly tightly bunched in the $x$ direction. The externally excited magnetic field, $H = 3000$ gauss, pervades the entire trajectory of the plasma (both in and beyond the gun). When the plasma leaves the muzzle it is observed to remain much more tightly bunched in the $Z$ direction than the plasma from a button gun when fired across a magnetic field. It is hoped that with longer rails and higher values of $H$ the plasma speed can be substantially increased.

Equipment is nearing completion for the operation of a plasma motor which employs gaseous ions instead of metallic ions. Figure 6 shows an arrangement where water vapor or carbon dioxide can be frozen on a chilled insulating column. The capacitor discharge can be expected to vaporize and ionize these substances and then propel them down the rails. It is hoped eventually to try the scheme with frozen deuterium.

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**Figure 4. An equivalent circuit for a pure shunt plasma motor**

**Figure 5. Experimental arrangement for accelerating plasma generated by a high current arc between metal electrodes**

**Figure 6. Experimental arrangement to be used for accelerating plasma from $H_2O$ or $CO_2$**

**Figure 7. Arrangement for a rotary shunt plasma motor where the rotational velocity of the plasma ring becomes transformed to linear velocity $v$ as the ring is ejected from the motor**
ROTARY SHUNT PLASMA MOTOR

A variation on the shunt plasma motor is to arrange the magnetic field as shown in Fig. 7. Here the plasma will pick up rotational kinetic energy and the duration of the application of the current can be chosen to be as long as one pleases. Hence, the plasma can be accelerated in principle to speeds which are limited only by the mechanical strength of the materials used in the apparatus. The rotational kinetic energy will be transformed to translational kinetic energy as the plasma ring is propelled to the right by the gradient in the magnetic field.

The copper parts serve as flux concentrators, as well as electrodes, so that the magnetic field can be made as high as $10^8$ gauss without any great difficulty.

BARRAGE OF BUTTON GUNS

The matching of the impedance of the power source can be accomplished by making an array of button sources and connecting them all in series. Although such an arrangement would seem a priori to be inferior to the rail-type plasma motors described in Fig. 2, it nevertheless presents some interesting phenomena involving the manner in which the individual pieces of plasma ejected from the individual guns interact with one another. Furthermore, such an array when suitable shaped and operated in a gas at a pressure of about 10 mm, is capable of generating shock waves of various shapes.

REFERENCES


† The development of experimental equipment for accelerating pellets has been carried on very successfully by Morton Levine at the Air Force Cambridge Research Center.


Energy Balance in a Thermonuclear Reacting Plasma containing Deuterium, Tritium and Reaction Products under Isothermal Pulsed or Steady-State Conditions

By T. Hesselberg Jensen, O. Kofoed-Hansen and C. F. Wandel*

The energy balance in a thermonuclear reacting plasma containing deuterium and tritium has been discussed in several places. By comparing the power escaping as electromagnetic radiation with the power transferred to charged reaction products in the nuclear reactions, a lower limit on the plasma temperature necessary to maintain a self-sustaining reaction is obtained. The limiting temperatures are 350 million °K in the case of pure deuterium and 50 million °K in the case of deuterium and tritium in equal amounts.

Especially when only small fuel burnups are considered, it is important to take into account, in the energy balance, the amount of energy spent in heating the fuel to the reaction temperature. This point has been investigated by Lawson where a pulsed system has been considered in which the fuel is instantaneously heated to the reaction temperature and then allowed to react in a definite time interval after which the plasma is again cooled to essentially zero temperature. If one can make the apparently realistic assumption that the heat content of the plasma, like the radiative power, can be transformed into plasma heat (in a new pulse) only with an efficiency considerably less than unity, then it can be concluded that not only must the temperature exceed a certain limit, but also the reaction must be sustained long enough for a definite fraction of the fuel to be burnt.

It is clear that as soon as a considerable fraction of the fuel is burnt the reaction rates will decrease and, because of the higher nuclear charge of the reaction products, the radiative power will increase. Both of these effects will tend to reduce the power economy of the system. The first of these effects has been treated by Lacombe et al., who have considered a pulsed system and investigated the time dependence of the power densities due to primary and secondary nuclear processes.

In Ref. 4, we have tried to incorporate all of the above-mentioned effects by treating the problem of a steady-state thermonuclear reaction involving deuterium, tritium and their reaction products. To keep the reacting plasma in a steady state it is in general necessary to exchange matter and energy continuously with the surroundings. The rates of these exchanges are completely determined by only three independent parameters. These parameters are the temperature of the plasma, the tritium enrichment in the fuel and the product of the deuterium density and the plasma renewal time. This renewal time corresponds roughly to the pulse time in a pulsed thermonuclear reaction. Instead of the density-time product, a burnup parameter indicating the fraction of the deuterium supplied that is actually burned in the reaction can be used.

It is conceivable that a thermonuclear reactor could be constructed so that the charged reaction products would be retained long enough to reach the average particle energy, while it is less likely that the neutrons and the electromagnetic radiation can be prevented from escaping from the reaction region. Thus, an adequate measure of the ability of the thermonuclear reaction to become self-sustaining (in the sense that the amount of energy transferred to the charged reaction products is sufficient to supply the energy required for the electromagnetic radiation and to heat the fresh fuel to the reaction temperature) is given by the reinjection fraction, *e*, defined as the ratio between the power that must be injected into the plasma to keep the system in a steady state and the total power emitted from the plasma in the form of kinetic energy of neutrons, heat in extracted plasma and electromagnetic radiation. The reaction will obviously be self-sustaining when *e* is equal to zero, while positive values of *e* correspond to situations where energy must artificially be supplied to the reaction region in order to keep the plasma in a steady state.

When the conditions of a self-sustaining thermonuclear reaction have been reached it may be necessary to extract more energy than is emitted as kinetic energy of neutrons, heat in extracted plasma and electromagnetic radiation in order to maintain the desired reaction temperature. It is possible to extend the definition of *e* to this case also. In this way, *e* describes the criticality condition for the thermonuclear reaction: *e* > 0 meaning subcritical conditions and *e* < 0 supercritical conditions.

A survey of the dependence of *e* on the three parameters has been made in the case of the steady state.
and it is shown that optimum values of all three parameters can be found. An investigation of the isothermal non-steady state shows great similarity with the behaviour in the steady state, as has been demonstrated by a few examples. The survey of steady-state systems can thus be said to cover the case of pulsed isothermal systems, when they are compared in terms of the deuteron burnup or in terms of the density–time product.

Since the actual rate of transfer of kinetic energy between the different kinds of ions and the electrons in the plasma is comparable with the rate of energy production through nuclear reactions and with the rate of energy exchange with the surroundings, it must be expected that corrections must be made to the results derived by assuming the energy exchange between particles to take place instantaneously. Calculations in the steady-state case, with these effects taken into account, show that the most important effect is a lowering of the electron temperature with respect to the ion temperature, in the cases where the reaction is nearly self-sustaining. This effect increases somewhat the range of parameter values for which a self-sustaining reaction is possible.

**BASIC EQUATIONS**

The reactions to be considered are listed in Table 1. In this table we have also indicated the notation to be used in the following. For the reaction rate parameters \( \langle \sigma v \rangle \) we have used Roman numerals according to the numbering of the reactions, and for the average energies of the particles participating in the reactions we have, for example, used \( E_r^4 \) for the triton in Reaction 1. It is then obvious that for a reaction between particles of kinds \( i \) and \( k \) giving particles of kinds \( r \) and \( s \),

\[
E_i + E_k + Q = E_r + E_s.
\]  

Further, we use \( n_i \) for the number of particles, per \( \text{cm}^3 \), of type \( i = \) D, T, p, 3, 4, e, n for deuterons, tritons, protons, \( \text{He}^3 \), \( \text{He}^4 \), electrons and neutrons respectively. The \( Q \) values\(^5,6\) for the reactions are also given in Table 1.

**Table 1. Reactions of Interest, Q-values, Reaction Rates and Energy Partitions**

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Q-value, Mev</th>
<th>Reaction rates</th>
<th>Average energy of particles involved in reactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. D + D ( \rightarrow ) T + p</td>
<td>4.04</td>
<td>( \frac{1}{2} \nu_{H}^{II} )</td>
<td>( E_r^2 ) ( E_s^2 ) ( E_r^3 ) ( E_s^3 )</td>
</tr>
<tr>
<td>2. D + D ( \rightarrow ) ( \text{He}^3 ) + n</td>
<td>3.27</td>
<td>( \frac{1}{2} \nu_{H}^{III} )</td>
<td>( E_r^{III} ) ( E_s^{III} ) ( E_r^{IV} ) ( E_s^{IV} )</td>
</tr>
<tr>
<td>3. ( \text{T} + \text{D} \rightarrow \text{He}^3 ) + n</td>
<td>17.38</td>
<td>( \nu_{T}^{IV} )</td>
<td>( E_r^{IV} ) ( E_s^{IV} )</td>
</tr>
<tr>
<td>4. ( \text{He}^3 ) + ( \text{D} \rightarrow \text{He}^4 ) + p</td>
<td>18.34</td>
<td>( \nu_{T}^{V} )</td>
<td>( E_r^{V} ) ( E_s^{V} )</td>
</tr>
<tr>
<td>5. ( \text{T} + \text{T} \rightarrow \text{He}^4 ) + 2n</td>
<td>11.32</td>
<td>( \nu_{T}^{II} )</td>
<td>( E_r^{II} ) ( E_s^{II} )</td>
</tr>
<tr>
<td>6. ( \text{T} + \text{He}^3 \rightarrow \text{He}^4 ) + D</td>
<td>14.31</td>
<td>( \nu_{T}^{VI} )</td>
<td>( E_r^{VI} ) ( E_s^{VI} )</td>
</tr>
<tr>
<td>7. ( \text{T} + \text{He}^3 \rightarrow \text{He}^4 ) + p + n</td>
<td>12.08</td>
<td>( \nu_{T}^{VII} )</td>
<td>( E_r^{VII} ) ( E_s^{VII} )</td>
</tr>
</tbody>
</table>

**Table 2. Reaction Rate Parameters as Functions of Temperature**

<table>
<thead>
<tr>
<th>Temperature ( T^* ) (kev)</th>
<th>( 5 )</th>
<th>( 10 )</th>
<th>( 25 )</th>
<th>( 50 )</th>
<th>( 100 )</th>
<th>( 150 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{I} \cdot 10^{17} ) ( \text{cm}^3/\text{sec} )</td>
<td>0.00882</td>
<td>0.0575</td>
<td>0.351</td>
<td>0.981</td>
<td>2.26</td>
<td>3.42</td>
</tr>
<tr>
<td>( \text{II} \cdot 10^{17} ) ( \text{cm}^3/\text{sec} )</td>
<td>0.00898</td>
<td>0.0613</td>
<td>0.400</td>
<td>1.17</td>
<td>2.78</td>
<td>4.20</td>
</tr>
<tr>
<td>( \text{III} \cdot 10^{17} ) ( \text{cm}^3/\text{sec} )</td>
<td>1.30</td>
<td>11.1</td>
<td>55.8</td>
<td>85.4</td>
<td>84.5</td>
<td>70.4</td>
</tr>
<tr>
<td>( \text{IV} \cdot 10^{17} ) ( \text{cm}^3/\text{sec} )</td>
<td>0.00727</td>
<td>0.0223</td>
<td>0.776</td>
<td>5.68</td>
<td>17.85</td>
<td>24.6</td>
</tr>
</tbody>
</table>

Reaction 5 of Table 1 needs only to be considered for fuel that is extremely rich in tritium. Reactions 6 and 7 need only to be considered for very high temperature and for fuel that is very rich in both tritium and \( \text{He}^4 \). These reactions can, therefore, be neglected for our present purpose. At low temperatures (i.e. \( T \leq 10 \text{ kev} \)) even Reaction 4 may be neglected.

For the calculation of the quantities mentioned in Table 1 we have assumed a Maxwellian velocity distribution of each kind of particles with the temperature \( T_i \) of the \( i \)th kind of particle. The most important of these quantities, the reaction rate parameters, are functions only of a proper average temperature, \( T^* \), for the temperatures of the reacting particles given by

\[
k T^* = \frac{m_k T_j + m_j T_k}{m_k + m_j} = \frac{m_i}{\mu_{ij}} \quad (2)
\]

where \( m_i \) and \( m_j \) are the masses of the reacting particles and \( T_i \) and \( T_j \) are their temperatures. Furthermore, \( m_{ij} \) is the reduced mass, and \( \mu_{ij} \) defined by (2) will be used later on and termed the temperature reduced mass. In Table 2 the reaction rate parameters are given as functions of \( T^* \). In the calculation\(^4,7\) of the reaction rate parameters, and the average energy of particles participating in reactions, we have used cross sections compiled by Jarmie and Seagrave\(^8\) and by Bame and Perry.\(^9\)

**Particle Densities**

First, we shall give a general set of equations covering the kinetics of the ion densities in the plasma. At time \( t = 0 \) one \( \text{cm}^3 \) of plasma contains \( n_i \) ions of type \( i \) and, if we impose the condition of charge
neutrality, \( n_e = \Sigma n_i Z_i \). All summations over \( i \) mean summations over ions only. In order to enable us to discuss the steady state situation we may assume fresh fuel to be added at a rate of \( n_D \Delta \) deuterium atoms, and \( n_T \Delta \) tritium atoms per cm\(^3\) sec. We may also assume that it is possible to extract a representative mixture of the gas at a rate of \( n_\delta \) for the ions and \( \Sigma n_i Z_i \) for the electrons, again per cm\(^3\) sec. Under these conditions, the reaction kinetic equations for the ion densities are as shown in Table 3.

Furthermore, for the number of neutrons emitted per cm\(^3\) sec we have

\[
\frac{dn_n}{dt} = \frac{1}{2} n_D^2 II + n_D n_T III
\]

and for the electrons we have

\[
\frac{dn_e}{dt} = -\delta \Sigma n_i Z_i + n_D (1 + \eta) \Delta
\]

With \( n_e = \Sigma n_i Z_i \) this last equation is already contained in Eqs. (3)-(7), but it gives a convenient and often used relation between \( \delta \) and \( \Delta \).

### Energy Content

Next, we shall set up the kinetic equations describing the energy content in each type of charged particles. In order to do this, we assume that the fresh fuel is added at zero temperature and as neutral atoms, and that the ionization energy is negligible. The ions and electrons extracted from the plasma are assumed to have the same average energy as the corresponding type of particles in the plasma. Furthermore, we shall assume that the charged reaction products are retained in the plasma while the neutrons and the bremsstrahlung escape from the reaction region.

The charged particles are assumed only to exchange energy by Coulomb encounters. A formula covering this energy exchange is given by the expression\(^{10}\)

\[
\frac{\partial}{\partial t} \left( \frac{3}{2} n_k T_k \right) = 4(2\pi)^1 Z^2 Z^2 e^4 n_m \mu_n \mu_i \frac{1}{m_n m_i} (k T_j - k T_i) \ln \Lambda
\]

\[
= n_m f_0 (k T_j - k T_i)
\]

(10)

giving the average energy transfer per cm\(^3\) sec to the particles of type \( i \) with density \( n_i \) from particles of type \( j \) of density \( n_j \).

The term \( \ln \Lambda \) is given by

\[
2 \ln \Lambda = 2[\sin A] \left( \frac{\pi}{2} - \sin(A) \right) - 2(\cos A) \cosi(A) + e^{2i} - B
\]

(11)

where

\[
A = \frac{1}{2} \mu_n Z_i Z_j^2 / m_i D
\]

(12)

\[
B = \frac{1}{2} \mu_i (2Z_i Z_j^2 / n_i^2)^2
\]

(13)

and finally the Debye shielding distance, \( D \), is given by

\[
D = (k T_e / 4\pi n e^2)^{1/3}
\]

(14)

In general, it will be necessary to inject or extract energy from the plasma in order to initiate and control the thermonuclear reaction. The plasma will gain energy from nuclear heating and energy will be lost as kinetic energy of neutrons, heat in extracted gas and radiated energy. Let us specify the injected or extracted power as \( p_{in} \), the sign being positive for injection and negative for extraction. Furthermore, let us assume that this power density is distributed with a fraction, \( \alpha_i \), to each type of charged particles.

### Table 4. Energy Density Equations

\[
\frac{d}{dt}(n_D^2 k T_D) = \alpha_{Din} - n_D^2 (k T_D \delta + \Sigma n_i k[Z_i(T_D - T_e)/f_D + (T_D - T_i)/f_D]) - n_D k T_D \delta - n_D^2 k T_D \delta
\]

(15)

\[
\frac{d}{dt}(n_T^2 k T_T) = \alpha_{Tin} - n_T^2 (k T_T \delta + \Sigma n_i k[Z_i(T_T - T_e)/f_D + (T_T - T_i)/f_D]) + \frac{1}{2} n_D^2 k T_D \delta - n_D n_T k T_D \delta
\]

(16)

\[
\frac{d}{dt}(n_p^2 k T_p) = \alpha_{pin} - n_p^2 (k T_p \delta + \Sigma n_i k[Z_i(T_p - T_e)/f_D + (T_p - T_i)/f_D]) + \frac{1}{2} n_D^2 k T_D \delta + n_D n_T k T_D \delta
\]

(17)

\[
\frac{d}{dt}(n_3^2 k T_3) = \alpha_{3in} - n_3^2 (k T_3 \delta + \Sigma n_i k[Z_i(T_3 - T_e)/f_D + (T_3 - T_i)/f_D]) + \frac{1}{2} n_D^2 k T_D \delta - n_D n_3 k T_D \delta
\]

(18)

\[
\frac{d}{dt}(n_4^2 k T_4) = \alpha_{4in} - n_4^2 (k T_4 \delta + \Sigma n_i k[Z_i(T_4 - T_e)/f_D + (T_4 - T_i)/f_D]) + \frac{1}{2} n_D^2 k T_D \delta + n_D n_3 k T_D \delta
\]

(19)

\[
\frac{d}{dt}(\Sigma n_i Z_i k T_i) = \alpha_{pin} - n_T - \Sigma n_i Z_i k[T e \delta + (T_e - T_i)/f_D]
\]

(20)
With these assumptions, we may write down the equations for the energy densities for the various types of particles as shown in Table 4.

The α’s fulfill the condition

$$\sum c_\alpha + c_\text{e} = 1$$  \hspace{1cm} (21)

and the radiation power density $p_r$ is given by Thompson:

$$p_r = 0.33 \times 10^{-14} (k T_e) [\sum m_i Z_i] [\Sigma m_i Z_i^2] + 0.0151 k T_e [\sum m_i Z_i^2]^2$$  \hspace{1cm} (22)

with $k T_e$ in kev.

Energy Balance

In order to discuss the energy balance of the system we shall add up the Eqs. (15)-(20), taking into account Eqs. (1) and (21), and obtaining

$$\int p_m dt = p_m + p_r - p_{\text{nucl}} + d U_G/dt$$  \hspace{1cm} (23)

where $p_r$ is defined by Eq. (22); $p_m$ is the neutron kinetic power density,

$$p_m = \frac{1}{2} n_p^2 I_n T_I^{11} + n_D n_T I_n T_I^{11}$$  \hspace{1cm} (24)

$p_m$ is the power emitted as heat in extracted gas,

$$p_m = (\Sigma \delta \frac{g}{k} T_m + \frac{1}{2} k T_e \sum m_i Z_i \delta)$$  \hspace{1cm} (25)

and $p_{\text{nucl}}$ is the nuclear power density,

$$p_{\text{nucl}} = \frac{1}{2} n_D I_Q I + \frac{1}{2} n_D I_I Q I + n_D n_T I Q I$$

Finally, $U_G$ stands for the total heat energy present in the gas at any given time $t$.

$$U_G = \frac{1}{2} (\sum \delta k T_m + \frac{1}{2} k T_e \sum m_i Z_i)$$  \hspace{1cm} (27)

Power Injection

Equation (23) determines the value of the injected power, $p_m$, at any time $t$ if the temperature conditions are defined. The magnitude $p_m$ can assume negative or positive values as a function of time, according to whether the reaction is momentarily self-sustaining or not.

In order to analyze the over-all power balance in a period of time $t = 0$ to $t = t'$ we define a power reinjection fraction, $e$, through the equation

$$\int_{\Delta t} [p_m + p_mdt]dt = e \int_{\Delta t} [p_m + p_m + p_r - p_{\text{nucl}} + d U_G/dt]dt$$  \hspace{1cm} (28)

The left-hand side of this equation is the total energy that must be injected into the plasma in order to initiate and control the thermonuclear reaction, while the brackets on the right show the total energy emitted by the plasma to the surroundings during the same time interval.

When a pulsed system is considered, $p_m$ includes the power shot necessary to achieve the high plasma temperature and also the power recovered from the plasma at the end of the pulse. These contributions will appear as positive and negative delta-functions in time at $t = 0$ and $t = t'$ respectively.

Solving Eq. (28) for $e$ and using Eq. (23), while assuming $U_G$ to be equal at $t = 0$ and $t'$, one gets

$$1 - e = \frac{\int_{\Delta t} [p_m + p_r - p_{\text{nucl}} + d U_G/dt]dt}{\int_{\Delta t} [p_m + p_r + p_{\text{nucl}} + d U_G/dt]dt}$$  \hspace{1cm} (29)

The importance of the power reinjection fraction, $e$, can most readily be seen for the steady state. In this case, we may simply substitute the integrands for the integrals in Eq. (29) since they are constant in time. It is immediately seen that $e$ can only assume values in the interval $0 \leq e \leq 1$. $e = 0$ implies a self-sustaining reaction since in this case $p_m \leq 0$. Since the $e$ defined by (29) does not reflect to what extent $p_m$ differs from zero when the reaction is self-sustaining, it is convenient to modify (29) by introducing an $e'$ defined by

$$1 - e' = \frac{\int_{\Delta t} [p_m + p_r - p_{\text{nucl}} + d U_G/dt]dt}{\int_{\Delta t} [p_m + p_r + p_{\text{nucl}} + d U_G/dt]dt}$$  \hspace{1cm} (29')

For positive values of $e'$ the two parameters will be identical in the steady state while for negative values of $e'$ one will have $e = 0$.

For optimizing the parameters, $e'$ is more convenient than $e$ since it has a smooth variation in changing from positive to negative values, thus permitting a minimum point to be determined instead of a minimum region.

In the pulsed isothermal case, $e'$ is the most convenient for comparison with the steady state. Although $e$ will always be larger than zero in this case, a momentarily self-sustaining reaction will be achieved in a period of time in which $p_m \leq 0$.

The calculation of $e'$ is our ultimate aim and Eqs. (3-7) and (15-20) are our basic coupled equations. To solve this set of equations in all detail can only be attempted by means of electronic computers. This falls outside our present program and we have only treated problems to which analytical solutions may be found; namely, the steady-state problem and the isothermal kinetics, neglecting all temperature difference $T_1 - T_B$ which means assumption of infinite energy transmission parameters. A survey of this work is presented in the following sections.

RESULTS

Steady-state Systems

In this section we shall assume a steady-state operation in time. This means that Eqs. (3-7) and (15-20) are equated to zero. We shall also make the simplifying assumption that all temperatures are equal to a common temperature $T$. Mathematically, this means assuming $\int_{x=-\infty}^{x=\infty} T_k dx = 0$. Thus, Eqs. (15-20) are of no consequences except for their sum leading to the definition of $e'$ by Eq. (29').

We then solve Eqs. (3-7) and use the results for the computation of $e'$. In Ref. 4 we give the explicit expressions for the solutions of Eqs. (3-7) in this case.

The solutions depend on three parameters only: the temperature $T$, the tritium enrichment $n$ and the
magnitudes \( n_D/\delta \) which is a product of density and time since \( 1/\delta \) is the mean renewal time for the plasma. Instead of the third parameter we may also use the deuterium burnup, \( \beta \), defined by

\[
\beta = (\Delta - \delta)/\Delta.
\]

The parameter \( \beta \) is related, in a one to one correspondence, to \( n_D/\delta \) through Eqs. (3-7).

Some of the results of the calculations are illustrated in Figs. 1 and 2. In Fig. 1 we shall first discuss the case for \( \eta = 0 \). For this case, we have shown the \( \varepsilon'(\beta) \) curves for the cases \( T = 25, 50, 100 \) and 1500 kev. In each of these cases, an optimum in \( \beta \) is found and if a curve is drawn through these optima an optimum in \( T \) results. The total optimum in \( \beta \) is further illustrated by the fat curve which is the envelope of all the \( \varepsilon'(\beta) \) curves. It is easy to understand the appearance of optimum values for \( T \) and \( \beta \) from physical arguments. If the fuel is pumped through the reactor at a fast rate, relative to the reaction rates, the burnup ratio will be low and the heating and cooling of the fuel will dominate the power balance, with the result that \( \varepsilon' \rightarrow 1 \). On the other hand, if the fuel is permitted to stay for a long time in the reactor the burnup will be high, but there will also be a high build-up of passive reaction products leading to severe radiation losses; the power balance will then be dominated by the radiation losses and the heating necessary to compensate for these losses, again with the result that \( \varepsilon' \rightarrow 1 \). Thus, a minimum in \( \varepsilon' \) must be expected for intermediate burnup ratios. Similarly, for the optimum temperature. For low temperatures, radiation losses dominate over the nuclear heating as demonstrated by Post.\(^1\) On the other hand, for very high temperatures, the reaction rate parameters \( \langle n_D \rangle \) level off as a function of temperature while at the same time both the radiation loss from electron-electron collisions, increasing as \( T^2\gamma \), and the fuel heating, increasing as \( T \), become more important and finally dominate for the highest temperatures. Again, in the two extremes, \( \varepsilon' \rightarrow 1 \) and at intermediate temperatures an optimum must exist.

We have already mentioned that an envelope of the \( \varepsilon'(\beta) \) curves for different temperatures can be found. In the remaining part of Fig. 1 we show these envelopes for different values of \( \eta \). Again an optimum for this parameter is found, as illustrated by the trend in the entire valley created by these envelopes as functions of \( \eta \) and \( \beta \). The slopes of the valley are rather steep for small values of \( \eta \) and because of this and in order not to have too many intersecting lines in the figure we have chosen to plot the envelopes as a function of \( \eta^2 \) rather than \( \eta \). Also, in the case of \( \eta \), a simple physical interpretation of the optimum can be given. That tritium enrichment improves \( \varepsilon' \) is a result of the fact that \( III \gg I + II \). However, if pure tritium is used, Reaction 3 disappears again and only Reaction 5 contributes to the power balance and again \( III \gg V \). Thus, an optimum is found for such values of \( n_T \) and \( n_D \) that the condition \( n_T^2V < n_Tn_DIII \) and at the same time \( n_Tn_DIII > n_D^3(I + II) \). However, the optimum is not very pronounced, since the valley is very flat as a function of \( \eta \). In the optimum the D-T reaction dominates, and it is thus obvious that the optimum condition is reached when \( n_T = n_T \), which means \( \eta = 1 \).

In Fig. 2 we give a further illustration of the optimization. Here, we have given contour curves for \( \varepsilon' \) in the \( (n_D/\delta, T) \)-plane for two values of \( \eta \), namely \( \eta = 0 \) and \( \eta = 1 \). Since we know from Fig. 1 that the optimum \( \eta \) is close to 1 we see from Fig. 2 that the optimum temperature lies near 25 kev and the optimum in \( n_D/\delta \) near \( 10^{15} \) sec/cm\(^3\) corresponding, for that case, to a burnup of \( \sim 20\% \). From Fig. 2 we also see that in spite of the fact that the reaction rate parameter \( III \) for any given temperature is much larger than \( I + II \), only a factor of 10 in the optimum \( n_D/\delta \) is obtained with equal amounts of deuterium and tritium.

The region in \( \eta, T \) and \( \beta \), in which \( \varepsilon' < 0 \), is not given with any reasonable precision by the rather unrealistic steady-state calculations with the same temperature for all particles. If we use the extended set of equations, with individual ion and electron
temperatures, \( \varepsilon' \) will be diminished; also, if we turn to pulsed operation, the region of \( \varepsilon \approx 0 \) is shifted towards higher burnups.

In Refs. 4 and 12 we have given a more detailed description of the steady-state calculations.

**Effects of Finite Energy Transmission Rates for Charged Particles**

The aim of this section is to investigate the influence on the reinjection fraction, \( \varepsilon' \), of leaving out the assumption of a common temperature for all particles in the plasma. However, a Maxwellian velocity distribution of each kind of particles is still assumed.

We now consider the full set of equations, (3)–(9) and (15)–(21), where expressions (3)–(7) and (15)–(20) are equated to zero. This set of equations is rather complicated and will only be solved approximately.

In order to describe a steady state in this case we may again use the parameters \( \eta, n_D/\delta \), and a characteristic temperature. As it is seen, however, we must also ascribe definite values to the \( \alpha's \), indicating how the injected power \( P_{in} \) is distributed between the different kinds of charged particles. Thus, it is seen that this problem contains more parameters than the previous one.

It is possible to avoid a specific choice of the \( \alpha's \) by considering only the special case \( P_{in} = 0 \), or \( \varepsilon' = 0 \).

If we compare two systems with the same average ion temperatures, one in which all particles are assumed to have the same temperature and another where this assumption is not made, then it is found that \( \varepsilon' \) differs in the two cases mainly because of the following two effects. First, the different ion temperatures modify the reaction rate parameters, thereby shifting the relative density of the different kinds of ions and thus changing the value of \( \varepsilon' \). Secondly, the electron temperature is lowered, as discussed by Post,\(^1\) which modifies \( P_{rad} \) and \( P_{f} \) and, consequently, \( \varepsilon' \).

It is found that the second effect is the stronger. We therefore assume, as a first approximation, that all the ions have the same temperature, which we now choose as our temperature parameter. Eqs. (3)–(8) can then be solved separately as in the previous section, and Eqs. (15)–(19) can be neglected, since \( T_e \) is determined by Eq. (20) alone. In this equation it is convenient to put \( \alpha_D = 1 \); \( \varepsilon' \) is then calculated and only the cases where \( \varepsilon' = 0 \) are considered since, for these particular cases, the results are independent of the choice \( \alpha_D = 1 \). These results are, however, slightly dependent on the density of particles and the temperature through the energy transmission rates between ions and electrons. In the calculations we have set \( \ln \Lambda = 14 \), which, for the actual temperatures, roughly corresponds to particle densities between \( 10^{14} \) and \( 10^{18} \) cm\(^{-3} \). For further details see Ref. 10.

To justify the assumption of equating all ion temperatures, we have, in some characteristic cases, solved Eqs. (15)–(19) for the ion temperatures in a first order approximation. To solve this set of equations we used the previously found electron temperature. To calculate the energy transmission rate parameters, the mean energy of reacting particles and the reaction rate parameters, it is also necessary to assume a temperature for the ions. This temperature was chosen as the average ion temperature. Using the values of the ion temperatures found in this way one gets a first order correction to \( \varepsilon' \), a correction found to be insignificant. Also, one can, from Eq. 20, find a correction to the previously found electron temperature, but this correction was also found insignificant. For further details, see Ref. 7.

The results are given in Fig. 3. Here, contour curves corresponding to \( \varepsilon' = 0 \) are given in the \((n_D/\delta, T)\)-plane for \( \eta = 0 \) and 1. Curves A correspond to the assumption of different temperatures for the ions and electrons; and B for comparison, to the case where the same temperature is assumed for all particles. As \( \varepsilon' \) did not reach zero for \( \eta = 0 \) in the latter case, the contour curve for \( \varepsilon' = 0.1 \) is shown instead.

First, it is seen that modifications are not very significant. For the parameter \( n_D/\delta \) the interval with \( \varepsilon' \leq 0 \) is somewhat broadened, because both \( P_{f} \) and \( P_{rad} \), which dominate the energy balance at large and small values of \( n_D/\delta \), are lowered by the lower electron temperature. This effect is somewhat more pronounced at the higher temperatures, mainly because the relative difference between the ion and electron temperatures is larger at higher temperatures since the energy transmission rate parameters between ions and electrons are proportional to \( T_e^{3/2} \).

**The Pulsed Isothermal System**

Since some of the features of the steady-state system seem rather unrealistic from the point of view of present-day ideas and lines of development, we thought it appropriate to investigate a system more closely connected with the physical situation encountered in a pulsed gas discharge.

In the model chosen, a mixture of tritium and deuterium in the ratio \( \eta \) to 1 is instantaneously heated, at time \( t = 0 \), to a uniform temperature, \( T \), and then kept at this temperature until the plasma is instantaneously cooled down to essentially zero temperature at \( t = t' \). The function \( P_{in} \) in Eqs. (15)–(20)

![Figure 3. Contour curves for the power reinjection fraction, \( \varepsilon' \), in the \((n_D/\delta, T)\)-plane. Curves A include the effects of finite energy transmission rates. Curves B are taken from Fig. 2 for comparison](image)
has thus the character of a delta-function at\( t = 0\) and
\( t = t'\) while it is adjusted in between so as to keep the
temperature of the mixture of fuel and reaction pro-
ducts constant. Since we do not, in this case, extract
or inject matter we have \( \Delta = 0 \) in Eqs. (3)-(7)
and (15)-(20). Accordingly, we will not have a steady
state and \( \frac{dn_i}{dt} \) will, in general, be different from zero.
It is further assumed, for the sake of simplicity, that
there is a common temperature for all kinds of par-
ticles in the plasma. These assumptions considerably
simplify Eqs. (3)-(9) and (15)-(20). In Ref. 13 it is
shown how Eqs. (3)-(9) can be solved in this case.
Only the main results will be presented here.

Three linearly independent integrals of the type
\[ \sum_i \lambda_i n_i + \lambda_0 n_0 = N_f \]
can be obtained where the \( \lambda_i \)'s are constants satisfying the conditions
\[ \lambda_d = -\lambda_p + \lambda_T + \lambda_3 \]
\[ \lambda_0 = 2\lambda_D - \lambda_T \]
\[ \lambda_n = 2\lambda_D - \lambda_3, \]  
where \( \lambda_d, \lambda_T \) and \( \lambda_3 \) can be chosen arbitrarily, while
the \( N_f \)'s are arbitrary constants to be determined from
the initial conditions
\[ n_D = n_0, \quad n_T = n_0\eta, \quad n_p = n_3 = n_4 = n_n = 0. \] (32)

The integrals can, for example, be chosen to be

(a) particle conservation:
\[ \sum_i n_i + n_n = n_0(1 + \eta) \]

(b) charge conservation:
\[ \sum_i m_i n_i = n_0(1 + \eta) \] (33)

(c) "tritium conservation":
\[ n_T - n_p + n_4 = n_0\eta \]

It is not possible to express the \( n_i \)'s in terms of
real time in any simple form, but if the parameter
time, \( \tau \), defined by
\[ \tau = \int_0^{t'} \frac{(n_D/n_0)}{d\tau} \] (34)
is introduced, three further integrals are obtained, of
the type
\[ n_D = \frac{1}{\gamma_1 + \gamma_4} n_T - \frac{1}{1 + \gamma_1} u_3 = C_i e^{\gamma_1 n_0}, \]
\[ u_3 = C_i e^{\gamma_1 n_0}, \] (35)
where \( \gamma_4 \) is any one of the three roots in the equation
\[ \gamma^3 + (I + II + III + IV)\gamma^2 + [III - IV + (I + II)(III + IV) + \frac{1}{2}(II - IV + I + III)]\gamma + \frac{1}{2}(I + II - III) - IV = 0 \] (36)
while \( C_i \) is an integration constant determined by
the initial conditions (32),
\[ C_i = n_0[1 - III(II + \gamma_1)] \] (37)
As may be seen from Eq. (34), the parameter time, \( \tau \),
is approximately equal to \( t' \) for small deuterium
burnups.

Solving for \( n_i \) in Integrals (33) and (35), one gets
solutions of the form
\[ n_i = \sum_{k=1}^{3} K_i \rho e^{\mu_0 n_0} + L_i \] (38)
where the \( K_i \)'s and \( L_i \)'s are constants.

In order to compare the pulsed system with the
steady state it is necessary to consider the over-all
energy balance in the time interval \( t = 0 \) to \( t = t' \).
This is reflected in the definition of \( \epsilon \), in Eqs. (28) and
(29) by the fact that time-integrals of the power
densities are considered. Thus, it is perfectly possible
that \( \rho_{in} \) can be negative during a part of the pulse
time, corresponding to nuclear "ignition," whereas
the pulsed system is never self-sustaining in the more
exact over-all sense defined by \( \epsilon = 0 \) because of
the necessity of injecting power initially.

All the integrations necessary to determine \( \epsilon \) can
be performed analytically except for the integration of
the electromagnetic radiative power which must be
evaluated numerically. As in the steady state, \( \epsilon \) is a
function of the temperature, \( T \), and of the tritium
enrichment, \( \eta \). The third parameter can either be
either chosen as the burnup, \( \beta \), defined by
\[ \beta = \frac{n_0 - n_D(t')}{n_0} \] (39)
which can directly be compared with the burnup in
the steady-state calculations, or it can be chosen as
\( n_0\tau \), which must then be compared with the parameter
\( n_0/\delta \) in the steady state.

Computations of \( \epsilon \) as a function of the deuterium
burnup, \( \beta \), has been made in the two cases:
1. \( \eta = 0 \) and \( kT = 100 \text{ kev} \); and
2. \( \eta = 0.983 \) and \( kT = 5 \text{ kev} \).

In the last case it was permissible to neglect Reaction 4
because of the relatively low temperature. The results
are plotted in Fig. 4, together with the corresponding
curves for the steady-state case. The great similarity
between the non-steady and the steady case is
immediately noticed. The largest differences are to be

\[ \begin{align*}
\text{Figure 4. The power reinjection fraction, } \epsilon', \text{ as a function of the deuterium burnup, } \beta, \text{ for the isothermal pulsed cases where } \\
\eta = 0, kT = 100 \text{ kev and } \eta = 0.983, kT = 5 \text{ kev} \\
\text{The corresponding steady-state curves for } \eta = 0, kT = 100 \text{ kev and } \eta = 1, kT = 5 \text{ kev are given for comparison.}
\end{align*} \]
found for the large burnups corresponding to long pulse times and slow renewal rates respectively. This is due to the fact that the pulsed system, because of the retarding effect of the finite reaction times, is relatively richer in the active primary reaction products, \( T \) and \( \text{He}^3 \), while it is poorer in the passive secondary product \( \text{He}^4 \) compared with the equilibrium values of these isotopes in the steady-state system. Nevertheless, it is obvious that both the trends and the actual values of \( \epsilon \) as a function of the parameters \( T \), \( \eta \) and \( \beta \) are quite well reproduced by the steady-state calculations. In particular, it will be true that optimum values for the three parameters will exist also for the pulsed system and that they will not be very much different from the steady-state values. For all practical purposes, at the present time, the analysis of the steady-state problem given above can be taken as valid also for the pulsed system when the comparison is made in terms of the deuterium burnup or the equivalent parameters \( n_D/\delta \) and \( n_{\text{He}^3} \) respectively.

**DISCUSSION**

To be able to treat the problem above it has been necessary to make some approximations and assumptions that limit the results. In this section we shall discuss some of these limitations.

In order to solve the reaction kinetic equations it has been necessary to neglect Reactions 5 to 7. Fortunately, these reactions contribute very little to the power economy in the cases treated here (see Refs. 3, 4 and 12): over most of the parameter ranges considered they contribute less than \( 10^{-8} \) in \( \epsilon \).

Since not all of the power losses conceivable have been taken into account in the power balance calculations, the results must be considered as optimistic. The kind of losses which have been neglected are, for instance, energy dissipated in magnetic fields necessary for containment of the plasma, excessive radiation due to contamination of the plasma with heavy atoms, and losses due to particle collisions with the walls in a finite reactor. Also, the optimum conditions can only be achieved in a restricted region in space and time since temperature gradients and finite heating times are likely to be features of a more realistic fusion reactor.

Although we have tried to keep the number of specific assumptions as low as possible, in order to conserve the generality of the results, it has still been necessary to make some more or less arbitrary decisions. This concerns especially the fuel cycling. The power balance would obviously be improved if a non-representative mixture of the plasma could be extracted, containing mainly passive reaction products like protons and \( \text{He}^4 \) at a temperature lower than the average. Since this possibility did not seem very likely to us we chose to assume a representative mixture at the average temperature to be extracted. The steady-state concept also implies that the fuel is homogeneously and continuously injected in the plasma. The unrealistic character of this assumption is largely removed, we think, by the demonstrated similarity with the pulsed system. It has also been necessary to make assumptions about the energy distribution of the particles involved. For obvious physical reasons, the Maxwellian distribution was chosen; apart from the fact that considerable mathematical simplicity was thereby also achieved.

**CONCLUSIONS**

Finally, we state a few conclusions which can be drawn from the material presented.

(a) The similarity between results for the steady state and the isothermal pulsed case indicate the generality of the results derived in the former case. Comparisons can be made in terms of the parameters \( T \), \( \eta \) and \( \beta \) indicating the temperature, the tritium enrichment and the deuterium burnup respectively. The burnup parameter can be replaced by the parameter \( n_{\text{He}^3} = \int_0^{\infty} n_{\text{He}^3} dt \) which, in the steady state, reduces to \( n_D/\delta \).

(b) The effects of finite energy transfer rates between the charged particles slightly increase the parameter range for which self-sustaining thermonuclear reactions can be achieved, the extension being most marked in the high temperature region, above 50 kev.

(c) An optimum value of the power-reinjection fraction, \( \epsilon' \), can be obtained for the following values of the temperature, \( T \), the tritium enrichment, \( \eta \), and the deuterium burnup, \( \beta \); viz., \( kT = 25 \text{ kev} \), \( \eta = 1 \) and \( \beta = 0.3 \). A self-sustaining reaction can be achieved at a temperature \( kT \approx 5 \text{ kev} \) in the case of a one-to-one deuterium-tritium mixture, with a corresponding value of \( \int_0^{\infty} n_{\text{He}^3} dt \approx 10^{16} \text{ sec cm}^{-3} \) and \( \beta = 0.1 \).

(d) Even a slight tritium enrichment considerably lowers the temperature necessary to achieve a self-sustaining reaction. (From about \( kT = 50 \text{ kev} \) for \( \eta = 0 \) to about \( kT = 9 \text{ kev} \) for \( \eta = 0.1 \) in the steady-state case.) In itself, this has not great consequences since, at these temperatures, the reactor will mainly be a tritium burner, but it opens up the possibility of achieving the higher temperatures by nuclear heating and thereby initiating the D-D reaction. The copious neutron production could then be used to produce the tritium necessary for the nuclear ignition.

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Tritium Production and Cycling in a Fusion Reactor with Lithium Blanket

By Karl O. Hintermann* and Rolf Wideröe†

The T(d,n)He\textsuperscript{4} reaction has the advantage of a larger reaction cross section and greater energy release than the D(d,p)T and D(d,n)He\textsuperscript{3} reactions. Tritium cycling means the use of the neutrons for tritium production in a lithium blanket in order to support the T(d,n)He\textsuperscript{4} reaction in the plasma.

TRITIUM PRODUCTION

The first part of this paper deals with the calculations for neutrons slowing down in lithium. The results show that the neutrons have no chance to become thermal, but are involved in a Li\textsuperscript{6}(n,t)He\textsuperscript{4} reaction before reaching energies below 10\textsuperscript{4} ev in pure Li\textsuperscript{6} and 10 ev in natural lithium.

The spatial distribution of the reaction rates throughout the lithium blanket is then treated for slab geometry, using two methods: (1) Fermi age treatment and (2) application of the fast neutron diffusion equation employing an average value of the Fermi age. The second method gives a good approximation and by a special choice of the average Fermi age, good results may be obtained. Because of the relatively small scattering cross section of lithium, a blanket of natural lithium becomes at least as thick as a blanket of a good moderating material with lithium lining or lithium channels.

Finally, the relationships between the blanket efficiency, the T(d,n)He\textsuperscript{4} reaction rate and the relative numbers of T and D atoms in the plasma are considered.

Passage of Neutrons through Lithium

Cross Sections

The neutrons of the T(d,n)He\textsuperscript{4} reaction have an energy of 14.1 Mev, those of the D(d,n)He\textsuperscript{3} reaction 2.45 Mev.\textsuperscript{1} The prevailing reactions of neutrons at 14.1 Mev in lithium are elastic and inelastic scatterings. No measurements were available regarding the relative magnitudes of elastic and inelastic scattering cross sections in lithium at this energy. The comparison with other elements (Be, Al) leads to the assumption that these two cross sections have the same order of magnitude, so that an inelastic scattering will occur among the first few collisions. Therefore, other reactions, for instance (n,t), (n,d) and (n,p), which occur at high energy but with much lower cross sections than the scatterings, can be neglected. The energy region of these reactions will be largely bypassed by the energy loss in an inelastic scattering collision.

Initial Energy for Elastic Slowing Down

Let us estimate the energy lost in an inelastic collision. The spacing between energy levels of Li\textsuperscript{7} in the region of 14 Mev is approximately 1 to 2 Mev.\textsuperscript{2} The cross sections for excitation of the different levels are not known, but it seems reasonable for us to assume that the neutron energy after an inelastic scattering collision is somewhat more than these level spacings. Two Mev is assumed as initial energy for the elastic slowing down. The neutrons which arrive at the blanket with 2.45 Mev from the (d,n) reaction in the plasma have a good chance of making an inelastic collision during their first 5 collisions,\textsuperscript{3} leaving 0.48 Mev for excitation. For these neutrons, 2 Mev is also a reasonable value for their initial energy.

In the case of Li\textsuperscript{6}, no measurements of the excitation levels at 14 Mev are available, but comparison of the energy level schemes\textsuperscript{9} of Li\textsuperscript{6} and Li\textsuperscript{7} leads to the conclusion that the assumption of 2 Mev as initial energy for elastic slowing is also reasonable for pure Li\textsuperscript{6}.

The influence of the choice of the initial energy on the reaction rate distribution is discussed later.

First Collision Correction

As most neutrons enter from the direction of the plasma pinch, the prevailing flight direction is orthogonal to the blanket. The influence of the direction component parallel to the pinch is neglected. The first collision density resulting from these assumptions is taken as neutron source and the total source strength is standardized to one neutron per second:

\[ \text{First collision source} = \Sigma_{0} = \Sigma_{0}\text{e}^{\Sigma_{0}} \]

\( \Sigma_{0} \) is the macroscopic cross section for elastic and inelastic scattering at 14.1 Mev.\textsuperscript{4} The dimension of the source comes out as \( \text{cm}^{-1} \), corresponding to unit slab thickness.

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Because most neutrons are absorbed before $\Sigma_a$ reaches the same order of magnitude as $\Sigma_a$, diffusion theory for weak capture is applied.

**Boundary Conditions**

It is assumed that no neutrons are absorbed in the space enclosed by the lithium blanket. The reflected neutrons strike the opposite blanket, from which an equal number of neutrons are reflected. Thus the neutron flux and, therefore, the slowing down density have a horizontal tangent on the inner side of the blanket ($x = 0$). The reaction rate is calculated for a blanket of infinite thickness, so that the outer boundary condition is zero flux at infinity. At zero lethargy or Fermi age (neutrons of 2 Mev), the slowing down density is identical with the first collision source distribution.

All calculations are made in slab geometry, and for the total source strength (integrated throughout the slab thickness from 1 to $\infty$) of 1 neutron per second.

**Reaction Rate per Unit Lethargy Interval**

The reaction rate per unit lethargy interval is:

$$ R(u) = \Sigma_a(u) \phi(u), \quad (1) $$

where the lethargy, $u = \ln(E_0/E)$, $E_0$ being the initial energy.

The relationship between the flux per unit lethargy interval and the slowing down density with absorption is:

$$ \phi(u) = \frac{q'(u)}{\xi \Sigma_a + \gamma \Sigma_a}, \quad (2) $$

where: $\gamma = 1 - \alpha - \alpha e - \frac{1}{2} \alpha e^2 / (1 - \alpha - \alpha e)$; $\xi = \langle \ln(E_1/E_2) \rangle$, average logarithmic energy decrement per elastic collision; $\alpha = \left( \frac{D - 1}{D + 1} \right)$, minimum value of $E_2/E_1$ (head-on collision); and $E_1$ and $E_2$ are the energies before and after the collision, respectively.

The slowing down density with absorption,

$$ q'(u) = q(u) \exp \left( - \int_u^\infty \frac{\Sigma_a}{\xi \Sigma_a + \gamma \Sigma_a} du \right), \quad (3) $$

where $q(u)$ is the slowing down density without absorption.

Without considering the spatial distribution of the neutrons ($q(u) = q(0) = \text{constant}$) and standardizing on the source strength of one neutron per second ($q(0) = 1$), the reaction rate per unit lethargy interval becomes:

$$ R(u) = \frac{\Sigma_a}{\xi \Sigma_a + \gamma \Sigma_a} \exp \left( - \int_u^\infty \frac{\Sigma_a}{\xi \Sigma_a + \gamma \Sigma_a} du \right). \quad (4) $$

On the substitution

$$ w(u) = \int_0^u \frac{\Sigma_a}{\xi \Sigma_a + \gamma \Sigma_a} du. \quad (5) $$

† See Glossary of Symbols at end of paper.
Equation (4) becomes
\[ R(u) = \frac{d\omega}{du} e^{-u}. \] (6)

Because \( q(0) = 1 \), the overall absorption must also be unity:
\[ \int_0^\infty R(u) \, du = \int_0^\infty e^{-u} \, du = 1. \]

In Figs. 1 and 2, \( R(u) \) is plotted for natural lithium (7.5% Li\(_6\), 92.5% Li\(_7\)) and pure Li\(_6\). It will be recognized that a correct value of the initial energy is of relatively minor importance especially in the case of natural lithium. A factor 2 in the initial energy value corresponds to a lethargy interval of \( \ln 2 = 0.693 \), while the reaction region extends over a lethargy interval of about 10 in the case of natural lithium and about 4 in the case of pure Li\(_6\).

### Spatial Distribution of Reaction Rate in the Lithium Blanket

#### Fermi Age Treatment

The Fermi age equation with absorption is
\[ \nabla^2 q(x, \tau') = \frac{\partial q(x, \tau')}{\partial \tau'} \] (7)
with the following definitions for age, diffusion coefficient and average cosine of the scattering angle
\[ \tau' = \int_0^{u} \frac{D'}{\xi_{\Sigma} + \gamma_{\Sigma}} \, du \] (8)
\[ D' = \left[ 3\Sigma_\tau (1 - \bar{\mu}_0) \left( 1 - \frac{4}{5} \Sigma_\tau + \Sigma_\Sigma \frac{\bar{\mu}_0}{1 - \bar{\mu}_0} + \ldots \right) \right]^{-1} \]
\[ \bar{\mu}_0 = \frac{2}{3\lambda}. \]

The general solution of Eq. (7) is\(^8\)
\[ q(x, \tau') = \frac{1}{(\pi \tau')^{-\frac{1}{2}}} \int_{-\infty}^{\infty} q(t, 0) \exp\left[-(x-t)^2/4\tau'\right] dt. \]

**Boundary conditions**—The slowing down density at age zero is identical with the source distribution of the first collision correction,
\[ q(x, 0) = \Sigma_0 e^{-x/\tau_0}. \] (9)

As it is assumed that no neutrons are absorbed in the space enclosed by the lithium blanket, the reflected neutrons all re-enter the blanket somewhere. The inner boundary condition is then:
\[ \left( \frac{dq(x, \tau')}{dx} \right)_{x=0} = 0. \]

This condition can be satisfied by assuming a symmetric source distribution on both sides of the inner boundary, \( x = 0 \). With this assumption and in view of Eq. (9), the solution of (7) fulfilling the boundary conditions becomes
\[ q(x, \tau') = \frac{\Sigma_0}{2(\pi \tau')^{\frac{1}{2}}} \left\{ \int_0^{\infty} \left[ \exp\left(-\Sigma_\tau - \frac{(x-t)^2}{4\tau'}\right) \right] dt \right\} + \int_0^{\infty} \left[ \exp\left(-\Sigma_\tau - \frac{(x+t)^2}{4\tau'}\right) \right] dt. \]

After some calculation this becomes
\[ q(x, \tau') = \frac{1}{\tau'} \Sigma_0 e^{-x/\tau'} \left[ (1 - H(a)) \right] \left[ \left( 1 - H(b) \right) e^{b^2} \right], \] (10)

where
\[ a = \frac{z}{2\sqrt{\pi}} \left[ \frac{1}{2} \right] \left[ \frac{1}{2} \right] \]
\[ b = \frac{z}{2\sqrt{\pi}} \left[ \frac{1}{2} \right] \left[ \frac{1}{2} \right], \]
\[ H(a) = \frac{2}{\sqrt{\pi}} \int_0^a e^{-t^2} dt, \] the probability integral.

Now the spatial distribution of the reaction rate \( R(x) \) can be calculated:
\[ R(x) = \int_0^{\infty} R(x, u) \, du = \int_0^{\infty} \Sigma_\tau (u) \phi(x, u) \, du. \]

Using Eqs. (2) and (3),
\[ R(x) = \int_0^{\infty} \frac{\Sigma_0}{\xi_{\Sigma} + \gamma_{\Sigma}} q(x, \tau'(u)) \exp\left(-\int_0^u \frac{\Sigma_\tau}{\xi_{\Sigma} + \gamma_{\Sigma}} \, du\right) \, du. \] (11)

Now again, with substitution (5), Eq. (11) becomes
\[ R(x) = \int_0^{\infty} \frac{\Sigma_0}{\xi_{\Sigma} + \gamma_{\Sigma}} q(x, \tau'(u)) e^{-u} \, du. \] (12)

where Eq. (5) allows for \( \tau'(u) = \tau'(u(x)) \). The results of Eq. (12) are plotted as curves 1a and 1b in Fig. 3 for natural lithium and pure Li\(_6\), respectively.

#### Fast Neutron Diffusion Treatment

Because the evaluation of Eq. (12) includes a numerical integration for each value of \( \tau' \) (\( \tau' \) is an empirical function), the possibility of an approximation by the fast neutron diffusion equation, using a uniform mean Fermi age, is investigated. This approximation shall be based on the following model:

1. The neutron source is given by Eq. (9), with the boundary conditions as before.
2. Because of inelastic scattering at high energies, the Fermi age of the neutrons is counted from 2 Mev as initial energy, as before.
3. No neutrons are absorbed until they have slowed down to a certain uniform Fermi age, where all of them are absorbed.

Comparing with the two-group diffusion theory, we treat the neutrons considered in the same way as we would for the fast group equation. The thermal neutron diffusion equation of the two-group theory would in our model include the neutrons at the chosen mean Fermi age, where all neutrons are absorbed. As these neutrons are no longer subject to diffusion but are absorbed immediately when they enter the "slow group", the slow group diffusion equation has no more meaning.

The fast group diffusion equation with the first collision neutrons as source term is:
\[ D \nabla^2 \phi(x) - \Sigma_1 \phi(x) + \Sigma_\tau e^{-x/\tau_0} = 0. \] (13)

In view of the fact that one absorption collision...
requires all slowing down collisions of one neutron, we obtain the definition of the "slowing down cross section", \( \Sigma_1 \): \( \Sigma_1 = D/\tau \).

The Fermi age, \( \tau \), and the diffusion coefficient, \( D \), are now defined without absorption.

With the same boundary conditions as before, the solution of Eq. (13) is

\[
\phi(x) = \left( \frac{1}{2} \right)^{1/2} \left\{ \frac{1}{\int_0^\infty e^{-xw} - \tau e^{-x/\sqrt{\tau}}} \right\} \] (14)

and the reaction rate becomes:

\[
R(x) = \Sigma_1 \phi(x) = \frac{D}{\tau} \phi(x)
= \left( \frac{1}{2} \right)^{1/2} \left\{ \frac{1}{\int_0^\infty e^{-xw} - \tau e^{-x/\sqrt{\tau}}} \right\}. \] (15)

Now the question arises of what average Fermi age should be chosen. Two different ways of selecting the average Fermi age are investigated:

(a) In order to obtain a good match to the curve from Fermi age treatment, an average age, designated by \( \tau \), is calculated by postulating coincidence of both reaction rate curves at the inner boundary:

\[
R(0)_{\text{Age Eq.}} = R(0)_{\text{Diffusion Eq.}} \] (16)

The slowing down density at \( x = 0 \) is:

\[
g(0, \tau) = 2\rho_0 [1 - H(\Sigma_0 \sqrt{\tau})] e^{\rho_0 \rho_0^2 \tau} \] and from Eqs. (12) and (15) at \( x = 0 \), Eq. (16) yields:

\[
\bar{\tau}_1 = \left( \int_0^\infty [1 - H(\Sigma_0 \sqrt{\tau})] e^{\rho_0 \rho_0^2 \tau} \right)^{-1} \] (17)

The results of (15), with \( \bar{\tau}_1 \) as average Fermi age, are plotted in Fig. 3 as curves 2a and 2b. It will be recognized, that with this method of calculating the age, the fast neutron diffusion equation yields a very good approximation to the Fermi age treatment. This is a great advantage because (15) is a handy equation, whereas Eq. (12) is tedious to solve, including a numerical integration over \( w \) for every value of \( x \).

(b) Another method is to average the Fermi age in the usual way:

\[
\bar{\tau}_2 = \int_0^\infty \frac{\tau(u) R(u) du}{\int_0^\infty R(u) du} \] (18)

The results of (15) with \( \bar{\tau}_2 \) for Fermi age are plotted in Fig. 3 as curves 3a and 3b. This approximation, as seen in the case of natural lithium, is much poorer than the first one.

The numerical values of the average Fermi ages calculated by the two methods and the corresponding lethargies are shown in Table 1.

The lack of consistency in the relationship between \( \bar{\tau}_1 \) and \( \bar{\tau}_2 \) in the case of natural lithium and \( \text{Li}^6 \) can be explained by the fact that most neutrons in natural lithium pass the resonance at 255 keV, while in pure \( \text{Li}^6 \) most neutrons are absorbed before passing the resonance. The resonance at 255 keV appears in both isotopes, \( \text{Li}^6 \) and \( \text{Li}^7 \).

Conclusions about Production

Lithium Blanket

The Fermi ages of lithium are much larger than those for the usual moderators for the same lethargy interval. Compared to graphite, lithium has only half the atomic mass, but its average scattering cross section is much smaller. The average slowing down power of lithium is therefore considerably poorer than that of graphite and other usual moderators. A blanket of natural lithium would have to be at least 100 cm thick in order to have a good efficiency for converting the neutrons to tritium. Pure \( \text{Li}^6 \) would be more advantageous to use, but more expensive too.

Table 1.—Average absorption Fermi ages and their corresponding lethargies

<table>
<thead>
<tr>
<th></th>
<th>( \bar{\tau}_1 )</th>
<th>( \bar{\tau}_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Li</td>
<td>2595 cm(^2)</td>
<td>5.13</td>
</tr>
<tr>
<td>Li(^6)</td>
<td>428 cm(^2)</td>
<td>.918</td>
</tr>
<tr>
<td>Li(^7)</td>
<td>1985 cm(^2)</td>
<td>4.17</td>
</tr>
</tbody>
</table>

* For neutrons with initial energy of 2 Mev in lithium
Moderator Blanket

It seems useful to have a moderator blanket of graphite or beryllium oxide for the purpose of slowing down the fast neutrons before they undergo the \( \text{Li}^6(n,t)\text{He}^4 \) reaction. The reaction cross section follows a \( 1/v \) law below the resonance at 255 kev, and the thermal neutron cross section is more than \( 10^9 \) times as large as for the energy region where the neutrons would react in a pure lithium blanket.

If a moderator blanket is lined with lithium both inside and outside, no thermal neutrons can escape. The mean free path for absorption of thermal neutrons is 0.3 cm in natural lithium and 0.023 cm in pure Li\(^6\). Depending on the albedo of the moderator blanket, the inner lining will receive more thermal neutrons than the outer one.

Since lithium may be used as the heat-carrying medium, it would be an advantage as regards the technical feasibility if the lining were replaced by channels in the moderator. The distance of the channels should be of the same order of magnitude as the slowing down length of the moderator, and there should be more than one row of channels in the direction orthogonal to the blanket. Uranium rods for neutron multiplication may also be provided.

The advantage of a moderator blanket is not only the higher efficiency for the same blanket thickness, but also the lower lithium quantity which allows a higher concentration of tritium after the same irradiation dose. This facilitates the recovery of lithium.

As most moderators have a density several times higher than that of lithium, more energy of the inelastic scattering gamma rays will be recovered within the blanket. The energy of the inelastic scattering gamma rays will be more (about 12 Mev for the \( T(d,n)\text{He}^4 \) reaction) than the energy from slowing down (about 2 Mev) and the \( \text{Li}^6(n,t)\text{He}^4 \) exothermic reaction energy (4.8 Mev) together if the efficiency of the tritium cycle exceeds about 50%.

**TRITIUM CYCLING**

The following reactions take place in the plasma:\(^1\)

\( (a) \) \( D(d,n)\text{He}^3 \) denoted as DD reactions

\( (b) \) \( D(d,p)T \)

\( (c) \) \( T(d,n)\text{He}^4 \) denoted as DT reaction

The first and the second reaction occur with about the same probability, while the cross section for the DT reaction is about 100 times larger.

We define the symbol \( r \) as the number of DT reactions per DD reaction and \( \eta \) as the number of tritium atoms recovered per neutron. With these definitions the following relationship can be established:

\[ r = \frac{1+\eta}{2(1-\eta)} \]

The first term on the right-hand side stands for the tritium atom from reaction \( (b) \), and the second term takes into account the tritium produced in the lithium by neutrons of the reactions \( (c) \) and \( (a) \). The explicit expression for \( r \) becomes:

\[ r = \frac{1+\eta}{2(1-\eta)} \] (18)

From the definition of \( r \) the relationship between the reaction rates becomes:

\[ r R_{\text{DD}} = R_{\text{DT}}. \]

On the other hand, the reaction rates expressed by the average product of cross sections and velocity are:\(^4\)

\[ R_{\text{DD}} = \frac{1}{4} N_D \langle \sigma V_{\text{DD}} \rangle \]

\[ R_{\text{DT}} = N_D N_T \langle \sigma V_{\text{DT}} \rangle ; \]

from this, the ratio of tritons to deuterons in the plasma becomes:

\[ \frac{N_T}{N_D} = \frac{r \langle \sigma V_{\text{DD}} \rangle}{2 \langle \sigma V_{\text{DT}} \rangle} \]

(19)

Numerical example:

kinetic temperature of the plasma = 20 kev

\( \eta = 0.72 \) (with uranium rods, \( \eta \) may have a value of 1 or more)

\[ \langle \sigma V_{\text{DD}} \rangle/\langle \sigma V_{\text{DT}} \rangle = 8.05 \times 10^{-3} \text{ Ref. 1.} \]

\[ N_T/N_D = 0.0121. \]

This result demonstrates that it is only necessary to invest a relatively small quantity of tritium in the reaction gas.

**ACKNOWLEDGEMENT**

The authors express their gratitude to Mr. W. Zünti for useful suggestions, and to Messrs E. Jantsch, W. Lindt, R. Meier, J. M. Patry and W. Winkler for helpful discussions.

**GLOSSARY OF SYMBOLS**

\( A \) Atomic mass number.

\( D \) Diffusion coefficient without absorption.

\( D' \) Diffusion coefficient with absorption.

\( E_1; E_2 \) Energy before and after an elastic collision.

\( N \) Number of atoms per cm\(^3\).

\( H \) Probability integral.

\( R \) Reaction rate.

\( q \) Slowing down density without absorption.

\( q' \) Slowing down density with absorption.

\( r \) Number of DT reactions per DD reaction.

\( \omega \) Lethargy.

\( \nu \) Velocity.

\( w \) Function of cross sections defined in (5).

\( x \) Coordinate orthogonal to the blanket.

\( \phi \) Scalar neutron flux.

\( \Sigma \) Macroscopic cross section.

\( \Sigma_1 \) Slowing down cross section of fast neutron group.

\( \Sigma_0 \) Macroscopic scattering cross section at 14.1 Mev.

\( \Sigma_s \) Macroscopic scattering cross section.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$\Sigma_a$</td>
<td>Macroscopic cross section for the $\text{Li}^6(n, t)\text{He}^4$ reaction.</td>
</tr>
<tr>
<td>$\Sigma_r$</td>
<td>Total macroscopic cross section.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Minimum value of $E_2/E_1$ (head-on collision).</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Number of tritium atoms recovered per neutron.</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Average cosine of scattering angle.</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Microscopic cross section.</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Fermi age.</td>
</tr>
<tr>
<td>$\gamma'$</td>
<td>Modified Fermi age for absorption.</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Average logarithmic energy decrement per collision.</td>
</tr>
</tbody>
</table>

**REFERENCES**

Megatron Scheme for Producing Relativistic Plasmas

By D. Finkelstein*

Budker\(^1\) has proposed that the Bennett\(^2\) effect be used in a new type of strong-focusing accelerator that would use uniform strong focusing in distinction to the alternating strong focusing used up to now. His work aroused interest at many accelerator groups,\(^3\)-\(^7\) but it did not appear in 1957, when this work began, that anyone was pursuing an experimental study of the self-focusing of relativistic beams, as Bennett\(^8\) has termed the new principle. These studies concern a state of matter which does not exist in nature: the relativistic plasma. In undertaking to study this state of matter, the group at Stevens Institute set as its first goal the construction of an apparatus that would produce a ring plasma of 1000 amp of 100 Mev electrons and an equal number of neutralizing ions. Such a plasma is theoretically much more stable than a nonrelativistic plasma of similar geometry. Megatron, the name of the apparatus, is short for \emph{Mega-gauss betatron}.\(^9\)

The design has of necessity some novel features, such as: the use of a single one-turn coil to provide the entire accelerating and guiding fields of the apparatus; guiding magnetic fields and accelerating electric fields larger than those used in accelerators at present; and plasma injection to get the necessary number of particles into the place where they will be accelerated.

Others have considered accelerators with some of the features adopted here. The use of extremely high fields for a small-orbit accelerator was pointed out by Furth, Levine and Waniek\(^10\); air-core accelerators have been initiated by Blewett\(^11\) (with ultimate completion) and Oliphant\(^12\) (not yet completed); and Budker and Naumov\(^13\) list “an ironless betatron-type set, involving a very high potential difference per turn”\(^9\) among methods they have considered for electron acceleration in a gas discharge.

Some of the elementary energetics of converting electrostatic energy into kinetic and magnetic energy of a relativistic beam is discussed in a feasibility consideration circulated at the start of our design study.\(^9\) At the present time (March 1958) our effort is changing from design and testing to construction. The physical principles of the megatron are presented below and the main parts of the apparatus are described in succeeding sections.


PHYSICAL CONSIDERATIONS

Under some circumstances, a charged particle in a magnetic field acquires kinetic energy when the field increases. Is it possible to bring a substantial number of the electrons in a plasma to highly relativistic energies in this way? Figure 1 is a sketch of the sequence of processes to be carried out by the megatron. In this part we consider briefly the physical basis of these processes in the order of their occurrence.

Energetics of Induction Acceleration

If electrons travelling at speed \(v\) are to be bent in a circle of radius \(R\) the magnetic field \(B_0\) at the orbit satisfies the relation

\[
mv^2/R = eB_0, \tag{1}
\]

where \(m\) is the total mass \(\gamma m_0\), and \(\gamma = (1-\nu^2/c^2)^{-1/2}\). This is a special case of the more general conservation relation

\[
\mathcal{F}/R = B_0 I \tag{2}
\]

where \(\mathcal{F}\) is the tension, or integrated stress-tensor, along the beam and \(I\) is the beam current. For a plasma beam there are contributions to the tension from the self-magnetic field, the ion flow, and the transverse motion of the particles in the plasma. It will be seen that, in the initial stages of the process we are trying to carry out, these contributions are small and we may use \(\mathcal{F} = \nu m v^2/R\), where \(\nu\) is the number of electrons per unit length in the beam, thus recovering (1). For a highly relativistic beam, this gives an electron energy

\[
E = e\gamma B_0 R. \tag{3}
\]

On the other hand, the energy stored in the magnetic field for an air-core coil is

\[
E_M = \frac{1}{2\mu_0} \int B^2 dV = \frac{\alpha}{2\mu_0} B_0^2 R^3 \tag{4}
\]

At first, the shape parameter \(\alpha\) may be estimated by assuming that a field \(2B_0\) prevails over a region of radius \(R\) and height \(R\), and that an equal amount of energy is stored in the return flux: \(\alpha \sim 24\). The exact calculation of H. Goertzel gives \(\alpha = 21\) for a typical coil shape.\(^14\) A relation due to Blewett\(^11\) between particle energy \(E\) and field energy \(E_M\) is found by eliminating \(R\) between (3) and (4):

\[
E_M = \left(\frac{\alpha}{2\mu_0 c^2 v^2}\right) B_0^3 = \frac{1.5 \times 10^{-30} \alpha}{B_0} \left(\frac{E}{\gamma}\right)^3 \tag{5}
\]
in mks units. Evidently, economy suggests the use of the highest \( B_0 \) attainable. This is probably achieved in a one-turn coil of careful design, where a conserva- tive limit may be 50 weber/m\(^2\) at some point on the coil wall or \( B_0 = 10 \) weber/m\(^2\) at the orbit. These estimates are based on the work of Furth, Levine and Waniek.\(^10\)

Using \( B_0 = 10 \) weber/m\(^2\), and \( E_M = 10^9 \) joule, which is consistent with the energy storage available at Stevens Inst., (5) gives \( E = 150 \) Mev and (8) gives \( R = 5 \) cm. The field coil determined by these choices possesses a rather high inductance of \( 1.3 \times 10^{-6} \) h. The capacitor bank need only have a much lower self-inductance than this for its energy to be transferred efficiently.

Evidently the goal of 100 Mev is well within the theoretical limits of what is possible with the available energy storage. The design value \( R = 5 \) cm has been adopted for the orbit radius of the Megatron.

**Plasma Injection**

The number of electrons in a relativistic beam of 1000 amp and 5 cm orbit radius is \( 6 \times 10^{12} \). We must inject these many particles at low energy into the acceptance region of the accelerator in a time \( \ll 10^{-6} \) sec. We will use a group of plasma guns of the general kind discovered by Bostick.\(^14\) Each plasma gun produces about \( 10^{17} \) particles, according to

\[
\text{Figure 1. Sequence of operations of the Megatron: 1, Capacitors C charged; 2, plasma injected (S1): } t \sim -1 \mu \text{sec; 3, acceleration begins (S2): } t = 0; 4, \text{ acceleration ceases (S3): } t \sim 6 \mu \text{sec}
\]

measurements of electrode depletion and also according to ballistic measurements of the momentum carried by the plasma and measurements of the plasma velocity. A battery of 25 guns makes a convenient unit, and injects about \( 10^{17} \) particles. The composition of the plasma is the same as that of the electrodes of the gun, and the mean speed can be regulated between \( 10^6 \) and \( 10^7 \) cm/sec. According to the Saha relation, most of the plasma is ionized at the time of creation. Thus there is no difficulty in injecting the necessary number of electrons and ions into the vacuum chamber of the accelerator. It is equally important not to inject too many particles; for then the plasma would shield the central region from the magnetic flux and violate the betatron 2-to-1 condition.

The acceleration of the electrons by the betatron field gives them an energy of \( \sim 100 \) kv for each revolution, at first. The initial thermal energy of the electrons in the plasma is \( \sim 1 \) ev, corresponding to a momentum range of \( \sim 1000 \) ev/c, centered about a much smaller mean momentum. The corresponding values of the initial guide fields, \( B_0 \), for a 5 cm orbit radius fall in a range of \( 10^{-4} \) weber/m\(^2\). Evidently the magnetic field can be zero at the time of injection without appreciable loss of particles from the immediate vicinity of the orbit. The acceptance area of the field is being computed by H. Goertzel.

**Collision and Radiation Losses**

In this section we use rationalized electron-optical units (reu) which are obtained from mks units by setting

\[
e = m = c = \epsilon_0 = \mu_0 = \kappa = 1. \tag{6}
\]

For convenience, we list the reu units of length: \( 3.52 \times 10^{-4} \) m, time: \( 1.17 \times 10^{-2} \) sec, current: 1370 amp, temperature: \( 5 \times 10^3 \) deg K.

Let the relativistic electron current be \( I \), the minor beam radius \( b \), the major beam radius \( R \), and the electron density in the laboratory frame \( n_+ \); then \( I = \pi b^2 n_+ \). Since we start with a neutral plasma there is also an ion density \( n_- = n_+ \).

The probability, per unit laboratory time, of large angle Coulomb scattering of an electron whose energy is \( y \), where \( 0 < y < m_+ \), is \( 1/\tau_0 = \pi s^2 n_+ \), where \( s \), the impact parameter for large angle scattering, is defined by equating kinetic and Coulomb energies:

\[
y = 1/4\pi s. \tag{8}
\]

For \( I = 1000 \) amp, \( b = 10^{-3}R = 0.05 \) mm and \( y = 200 \), we find \( \tau_0 = 10^4 \) sec. For the 10 \( \mu \)sec acceleration time we will deal with, all Coulomb collisions can be neglected, as far as particle loss is concerned.

Centripetal acceleration causes the electrons to radiate an energy-per-revolution, \( E_R = y^4/3R \) (reu). When \( R = 5 \) cm this gives \( E_R = 200 \) ev. This is small compared to the energy gained by the particle in each revolution, which will have fallen to 50 kev by the time the electron reaches 100 Mev. The energy spectrum of the emitted radiation has a peak near the wavelength \( \lambda \sim R/\gamma^2 = 20 \) A, for \( \gamma = 200 \). The total energy radiated during the acceleration to 100 Mev is quite small for a beam of 1000 amperes (\(< 0.1 \) joule), while the peak power radiated is \( W_m \sim 10^8 \) w.

**Field Shaping**

The correct space shape for betatron fields is well known. The circular orbit defined by \( R = \text{const} \), \( \varphi = eBR, \) is an exact solution of the equations of motion in an axially symmetric time-dependent magnetic field, if the 2-to-1 condition

\[
B(R) = 1/2 B(R) \tag{9}
\]
Figure 2. (a) Sketch and (b) cross section of torus producing a field which satisfies the betatron conditions \( n = 0.667 \)

is satisfied, where

\[
\pi R^3 B(R) = \int_0^R 2\pi r B(r) dr.
\]  

We propose to have the injection take place with \( J \approx 0, B \approx 0 \). In view of the short rise-time of the magnetic field, the skin effect in the coil is important: it can be assumed that, during the acceleration, the field vanishes inside the copper of the coil and is tangent at the surface. This can be used to shape the field so that (9) is satisfied. The necessary computation has been done by H. Goertzel of N.Y.U. and will be described elsewhere, but a typical field coil is shown in Fig. 2. At the same time, an experimental determination has been carried out by R. Phillips of Brookhaven National Laboratory. Because of the large guiding field, the self-field of the beam cannot de-stabilize the beam until the minor radius \( b \) of the beam is extremely small. The betatron stability criterion

\[
0 < n = -\frac{d \ln B}{d \ln R} < 1
\]

is valid during the acceleration time. The coil shown in Fig. 2 corresponds to the value \( n = 0.666 \).

The time shape of the betatron field is not critical. The field should build up as rapidly as possible (high accelerating electric field) to minimize collision and radiation losses and should then be held approximately constant to preserve the beam. To prevent the field from collapsing as rapidly as it builds up, it is customary to short-circuit the field coil at maximum current, or approximately zero voltage ("crowbar"). This action surrounds the flux with a good conductor, thus trapping it for the \( L/R \) decay time of the new circuit.

**Diagnostics**

The most interesting properties of the beam are its energy, intensity, and major and minor radii. The most distinctive product of the beam is the radiation discussed above. It is evident that time and spectrum studies of the emitted light can provide the information desired. Also, during the crowbar time, the beam will shrink in major radius. If it strikes a target fixed inside the beam radius \( R \), resulting 100 Mev bremsstrahlung can be readily observed through the coil wall. The presence of \( \pi \)-mesons will show that electrons of energy greater than 140 Mev have been produced.

The most interesting question is whether the pinch effect can occur in this device. Equilibrium between the electron-gas pressure and the self-magnetic pressure is expressed by

\[ T = \frac{2}{\pi} \gamma I \]  

(in reu). This is the relativistic counterpart of the pinch-effect relation

\[ 2\gamma T = \frac{1}{2L} \left( \frac{I}{2\pi} \right)^2 \]

A final transverse temperature \( T \) of the electron beam can be estimated by starting from a temperature of 1 ev at the start of the acceleration and applying the theory of adiabatic invariants: then \( T \sim 10^4 \) ev \( \sim 10^{-2} \) reu when \( \gamma \) reaches 200. This means a high degree of collimation. On the other hand, the righthand member of (12) is \( 10^2 \) reu for \( I = 1000 \) amp, and thus dominates. We cannot discuss the vital question of the time-scale of the pinch effect in this paper; moreover, the estimated final value of \( T \) is only a lower bound, since there are many effects which increase the transverse energy of the beam.

**EXPERIMENTAL EQUIPMENT**

**Energy Storage**

The inductance of the coil of Fig. 2 is \( 10^{-6} \) h. There are few novel problems in constructing a capacitor bank operating at 100 kv and delivering \( 10^5 \) joules to a \( 1 \mu \)h load. (The capacitance of the bank must be \( 20 \mu \)F to store this energy, and the quarter-period of the pulse is then about 6\( \mu \)sec. For comparison, the capacitor bank being assembled at Stevens Institute has a predicted inductance \( <5 \times 10^{-9} \) h, and should be able to deliver its energy in a quarter-period of \( <2 \mu \)sec.) Therefore we will not describe the capacitor bank, C, and its switches, S (see Fig. 1) in this report.

**Injection**

**Plasma Gun**

After the capacitor bank has reached its rated voltage, plasma must be injected into the vacuum chamber of the accelerator. This is to be done with
a group of coaxial plasma guns (Fig. 3) with central electrodes initially of deuterium-loaded titanium wire. The plasma guns will be about 15 cm from the equatorial plane and operate with a pulse derived from a 0.1 /µsec capacitor discharge. The firing of the guns is initiated by a solenoid-operated spark gap. Plasma transit time is between 1 and 10 /µsec.

Synchronization
Phantastrons triggered by the gap mentioned above determine the times between the plasma injection, the discharge of the capacitor bank into the field coil, and the crowbar. The first delay time is adjustable, to trap the maximum number of electrons in the beam. The second delay time is adjustable to maximize the energy of the circulating beam.

Field Coil
Operating Conditions
The coil has the following design parameters:
- Applied voltage: $10^6$ v
- Peak current: $5 \times 10^5$ amp
- Radial force:
  - Rise-time, $6 \times 10^{-6}$ sec
  - Decay-time, $3 \times 10^{-4}$ sec
- Peak value, 200 tons
- Inner radius: 5.5 cm

The magnetic field must satisfy the requirements discussed above. We have selected the value $n = 0.666$ and the orbital magnetic field $B_0 = 10$ weber/m², maximum, for design purposes.

Coil Construction
The mechanical design of the coil was worked out with I. Polk of Brookhaven National Laboratory, and is shown in Fig. 4. In the one-turn coils of Furth, Levine and Waniek the bars that carry current to the coil also support it. With the more restrictive requirements of the present application it was decided to separate the electrical and mechanical circuits. The points where current enters the coil make severe perturbations on the field. These perturbations are reduced if the coil is driven with 50 kv at two points instead of 100 kv at one point. The coil thus consists of two C’s facing each other. The perturbations are further reduced if the gaps between the C’s are small. To permit this, a vacuum ($10^{-6}$ mm Hg) insulates the two C’s from each other. Four force-bearing rings surround the two C’s and take up the 200 tons of impulsive force tending to separate the C’s. The force is transmitted between the force-bearing wall and the C’s through a vacuum gap by a collection of ceramic bearings. The four force-bearing rings form extensions of the transmissions lines from the capacitor bank, and also make up the wall of the vacuum chamber containing the C’s. The floor and ceiling of this vacuum chamber are transparent thermosetting plastic discs. The material of the coil will be a high conductivity beryllium-copper alloy selected by Waniek. The acceleration occurs in a second vacuum chamber inside the first vacuum chamber. The second chamber is of quartz and fits inside the C’s, following their contours closely. A Faraday shield on the outer walls of the second chamber helps to isolate the beam from the electrostatic perturbation of the gap between the C’s.

Crowbar
The coil will be short-circuited at maximum field by a group of vacuum plasma switches, S3 (see Fig. 1), similar to the switches, S2, used to discharge the condenser bank.

Dimensions
The accelerator has an outer diameter of 61 cm and an overall height of 20 cm, weighing somewhat less than ½ ton. However, the associated vacuum pumps, capacitor bank, and power supply occupy the remainder of a 3 m x 4 m floor space and weigh approximately 5 tons. It is hoped that performance results will be available in December, 1958.
ADDENDUM


REFERENCES

Steadily Running Self-Focusing Streams

By W. H. Bennett*

A proposal was submitted several years ago for an experiment in which a steadily flowing self-focusing stream would be produced and maintained in a closed loop by accelerating particles into that stream, using a new kind of acceleration. This was to be a high current accelerator, as distinguished from the more usual high energy accelerator, and was to be used in exploiting the peculiar properties of self-focusing streams.

Later, in 1956, Budker proposed that pulsed self-focusing streams be produced by accelerating very large electron currents in a closed loop in a betatron, pointing out that the motion of the electrons in a self-focusing stream in directions transverse to the axis of the stream would produce radiation which would damp such motion and that such damping would oppose the thermal dispersion of the stream. Any method which depends upon accelerating the electrons after their injection into the stream has one fundamental difficulty: the combined action of the accelerating electric field and the self-magnetic field of the current in the stream excessively drives the electrons toward the axis of the stream. The continued application of the accelerating electric force produces a continually increasing density of electrons with their associated space-charge-neutralizing ions near the axis, and the rate at which energy must be supplied to compensate for the loss of energy of the electrons due to their collisions with the ions increases correspondingly. Budker indicates that this may be "thousands of kilowatts," and that the difficulties are formidable.

Further searches for methods of producing and utilizing self-focusing streams have led to the concept of radiative entrapment which should be more readily attainable than the 1953 proposal and which is presented herewith as a revision of that proposal.

The advances made in the last few years in the development of electron linear accelerators make it practicable to inject electrons into a guide-field at full energy and thus avoid the excessive pinching down of the stream and the attendant difficulties which result from accelerating the electrons after they have joined the stream. The principal difficulty experienced in the past, in injecting particles into a closed loop orbit for a long enough time to build up the current in the loop to the minimum critical current for self-focusing, is that particles injected into any kind of a static or periodically varying magnetic guide-field come back out of that guide-field too soon after injection. In radiative entrapment, this difficulty is overcome by allowing the electrons injected in each injection pulse to radiate energy and so to shrink in orbital radius between pulses enough to avoid being thrown back out of the guide-field during the following injection pulses.

There are several alternative methods and combinations of methods which can be used for accumulating the minimum critical current for magnetic self-focusing in a closed loop stream. Only one of these will be described here. The kind of machine to be used is named the Epitron, from επιτροπή meaning "to intensify," because the purpose of the machine is to intensify the current enough to produce self-focusing streams. In this machine, a magnetic field similar to that of a cyclotron is used. The magnetic field is made to decrease in magnitude with increasing radius. An electron linear accelerator is used for injecting electrons peripherally from the outside at an energy large compared with the rest-energy of an electron.

The figure illustrates schematically the form of epitron used in this description. The two large coils, 1 and 2, are used for keeping the pole pieces steadily magnetized. The discharge chamber, 3, is held between the pole pieces, and high energy electrons are injected through the magnetically shielded tube, 4. Located at 5 is a small coil through which a current can be passed during the electron injection pulses.

The operation of this form of epitron will be described in its three phases: (1) injecting electrons into the guide field; (2) producing a self-focusing stream; (3) maintaining the self-focusing stream.

FIRST PHASE—INJECTION

The high energy electrons from the electron linear accelerator enter the magnetic guide field through a tube, 4, which magnetically shields the beam from the guide field until the beam emerges from the tube. Adjacent to this shielding tube and toward the axis of the guide field, at a point such as 6, the guide field is weakened by the presence of the shielding tube. Located at approximately 270 degrees around the axis from the shielding tube is a small coil, 5, through
which a current is passed during the injection pulse in order to intensify the guide field locally at a position such as 7. The local reduction in guide field at 6 and the local increase in guide field at 7 will be recognized as the Tuck and Teng peeler and regenerator field bumps in reverse order, although of course the action is not strictly the reverse of the Tuck and Teng system. These means could be referred to as the unpeeler but instead will be called the piler.

At the end of each electron injector pulse, the current is removed from the little coil, 5, and the injected electrons find themselves trapped in orbits which are approximately coaxial with the steady guide field, and disturbed by a small reduction bump due to the shielding tube, 4. The steady guide field decreases with radial distance from the axis much more rapidly than enough to keep the injected electrons entrapped and to overcome the tendency of the reduction bump to eject the electrons. The centripetal acceleration of the electrons in the approximately circular loop causes the electrons to radiate some of their energy, and the reduction in electron energy produces a corresponding reduction in loop radius. The injected pulse is allowed to radiate until it has decreased in radius enough to permit putting another pulse of current through the little coil, 5, without the piler throwing the previously injected electrons out of the field. Let

\[ x_0 = \text{energy of a freshly injected electron}, \]
\[ x = \text{energy of the electron after a time, } t, \]
\[ r = \text{radius of curvature of an electron whose energy is } x, \]
\[ a_0 = \text{radius at which electron is injected}, \]
\[ \omega_0 = \text{injected current of electrons during injection pulse}, \]
\[ T = \text{duration of an injection pulse}, \]
\[ f = \text{injection frequency}, \]
\[ H = \text{average magnetic field intensity}, \]
\[ p = \text{momentum of the electron whose energy is } x, \]
\[ e = \text{charge on the electron}, \]
\[ m = \text{rest mass of the electron}. \]

Electron injection energies of interest here are much greater than \( mc^2 \). The radius of curvature of the electron's orbit for these relativistic energies is approximately

\[ r = \frac{pc}{eH} \approx \frac{x}{eH} \quad \text{and} \quad a_0 \approx \frac{x_0}{cH}. \]

While each freshly injected electron is traveling in the loop in which it has been trapped by removal of current from coil 5, it will be radiating energy at a rate which can be obtained from a relation derived by Landau and Lifschitz

\[ \frac{dx}{dt} = -\frac{2e}{3m^2} H^2 x^2. \]

The loss in energy between pulses is

\[ \delta x = \frac{1}{f} \left( \frac{dx}{dt} \right) = \frac{0.67c}{e} \left( \frac{e}{m^2} \right)^4 H x^2. \]

This will produce a reduction in radial distance

\[ \delta r = \frac{1}{eH} \delta x = \frac{0.67c}{e} \left( \frac{e}{m^2} \right)^4 H x^2. \]

In order to prevent the electrons in the first pulse from being thrown out of the stream by the piler field bumps during the successive pulses of current in coil 5, the reduction \( \delta r \) must be made greater than some distance, \( a \), which is proportional to the size of the piler bumps

\[ a \leq \frac{0.67c}{e} \left( \frac{e}{m^2} \right)^4 H x_0^2. \]

Substituting for \( H \) from \( H = \frac{x_0}{ea_0} \) gives a relation between the maximum useful injection frequency, \( f_0 \), and the injection energy, \( x_0 \), which of course is independent of pulse length,

\[ f_0 = \frac{0.67c}{e} \left( \frac{e}{m^2} \right)^4 x_0^3. \] (1)

For example, if \( a_0 = 25 \text{ cm} \), \( a = 1.0 \text{ cm} \), and \( f_0 = 60 \), the injection energy must be at least 32 Mev. For an injection energy of 80 Mev, the injection frequency may be as much as 930 pulses per second. This design relationship can be written as

\[ x_0 \text{ (in Mev)} = 2.8(a_0/f_0)^4 \]

for \( a_0 \) and \( a \) in cm and \( f_0 \) in sec\(^{-1}\).

The electrons continue to lose energy by radiation. The reciprocal of change in radial distance between pulses gives the number of pulses in unit range of radial distance,

\[ 1 = \frac{\delta r}{r} = \frac{0.67c}{e} \left( \frac{e}{m^2} \right)^4 H x^2. \]

The leading electron in a freshly injected pulse will make \( n \) trips around a loop with loop radius \( a_0 \) in a pulse injection time \( T \), where \( n = cT/2m \omega_0 \) and the current introduced into the loop by an injection current \( \omega_0 \) is \( \omega_0 = c \omega T/2m \omega_0 \). Following injection, and while the electrons still have energies large compared with \( mc^2 \), the velocity of the electrons remains approximately equal to \( c \). As the radial distance for a pulse decreases from \( a_0 \) to \( r \), the current in the loop due to
that pulse increases from \( n_0 \) to \( n_0 a_0/r \) and the current in unit range of radial distance is this times the number of pulses in unit range of radius, or

\[
\frac{ds}{dr} = \frac{a_0}{r} n_0 \frac{ef}{0.67c} (\frac{mc^2}{e})^{1/4} \ \frac{1}{Hx^2}.
\]

Substituting for \( H \) from \( H = x_0/ea_0; x \) from \( x = x_0/r_0; f_0 \) from Eq. (1); and \( n \) from \( n = cT/2na_0 \) gives

\[
\frac{di}{dr} = \frac{fi_0 T c a_0^2}{2\pi f_0 a_0^3}.
\] (2)

The rate at which the gas in the discharge chamber is ionized by the high energy electrons can be estimated, using a curve by Hereford,\(^5\) to be between 7 and 6 ionizations per centimeter of path length in hydrogen at normal temperatures and pressure, for electrons with energy between 10 and 50 Mev. If the electron is moving in hydrogen at a pressure of \( 10^{-5} \) mm, it will produce ions at a rate of approximately 2600 ions per second. If the electron loses 36 ev of energy for each ionization, the energy is lost, for this cause, at a rate of 0.09 Mev/sec, which is entirely negligible compared with the radiation loss rate. It is difficult to maintain a much better vacuum than \( 5 \times 10^{-6} \) mm in this kind of machine, and at a residual gas pressure of the order of \( 5 \times 10^{-6} \) mm, ions are produced rapidly enough to permit space charge neutralization with time lags of the order of one millisecond. Because of the above-mentioned rapid production of ions, the density of ions will everywhere be approximately equal to the density of the high energy electrons.

The rates at which the energies of the high energy electrons change because of Coulomb collisions with the ions can be found, using some equations given by Thomas.\(^6\) The electrons lose kinetic energy \( x \) of forward motion by Coulomb collisions with the ions at a rate

\[
\frac{dx}{dt} = -\frac{4\pi e^4}{mc} L n_2,
\]

where \( n_2 \) = numerical density of ions, and

\[
L \approx \log(6x^3/mc^2n_2).
\]

This energy is almost entirely converted into kinetic energy \( y' \) of the electrons because of momenta transverse to the direction of motion in the stream

\[
\frac{dy'}{dt} = \frac{4\pi e^4}{mc} L n_2.
\]

For systems of interest here, \( L \) will be between 60 and 75 and the loss of energy by a high energy electron will be at a rate

\[
\frac{dx}{dt} = 2 \times 10^{-18} n_2 \text{ ergs/sec}.
\]

The cross section for bremsstrahlung loss, \( \Phi \), may be taken as approximately \( 10^{-26} \) cm\(^2\). The loss of energy by the high energy electron as bremsstrahlung is at a rate

\[
\Phi_{ecn_2} = 1.4 \times 10^{-28} n_2,
\]

which is many orders of magnitude smaller than the Coulomb collision loss rate and may be neglected.

During the first phase build-up of current toward the critical self-focusing current, where the current in unit range of radial distance is

\[
\frac{di}{dr} = \frac{fi_0 T c a_0^2}{2\pi f_0 a_0^3},
\]

the current can be supposed to be confined by the guide-field within a slab whose thickness is \( b \), and the density of ions is approximately

\[
n_2 = \frac{1}{ecb} \frac{di}{dr} = \frac{fi_0 T c a_0^2}{2\pi f_0 a_0^3} \frac{(a_0)^3}{r}.\]

The rate at which an electron loses energy by Coulomb collisions is

\[
\frac{dx}{dt} \bigg|_c = \frac{fi_0 T c^3 L}{mc f_0 a_0^3 b} \frac{(a_0)^3}{r},
\]

which is to be compared with the rate of electron energy loss by radiation,

\[
\frac{dx}{dt} \bigg|_r = \frac{f_0 a_0}{a_0} \frac{r}{(a_0)^2}.
\]

From this, it is seen that, as the electron loses energy and its maximum radial distance, \( r \), decreases, the rate of energy loss by radiation decreases with the square of the distance while the rate of energy loss by Coulomb collisions increases, varying inversely with the cube of \( r \).

**SECOND PHASE—SELF-FOCUSING**

The first phase build-up of current in the orbits spiralling inward toward the center of the guide field continues until the total current exceeds the minimum critical current for self-focusing given in the 1955 paper.\(^7\) In the present case, the expression for the critical current can be written in the form

\[
i_0 = (2mce/3)^{1/2} y',
\]

where \( y' \) is the average energy of an electron due to momenta transverse to the direction of the stream, as seen in the laboratory system of coordinates and is approximately equal to \( x_0 \omega_0^2 \) at injection, where \( \omega_0 \) is the mean angular divergence.

As the electrons decrease in energy by radiation, the value of \( \omega \) increases in the ratio \( x_0/x \) and the ‘mean transverse energy,’ \( y' \), increases to \( x_0 \omega_0^2 x^2 \). The critical current for self-focusing becomes

\[
i_e = \frac{2\pi c x_0^3 \omega_0^2}{x^2}.
\] (3)

For example, electrons injected at 50 Mev with mean angular divergence of 0.0002 radians, as soon as their space charge is neutralized, become self-focusing at a current greater than about 0.5 amp.

If the current injected in one injection pulse exceeds the above critical value, the injected stream shrinks in minor radius adiabatically in a manner similar to that discussed on page 1588 of the 1955 paper.\(^7\) As
these electrons radiate and lose energy from \( x_0 \) to \( x \) while shrinking in major radius from \( a_0 \) to \( r \), the value of the current in the injected pulse increases in the ratio \( x_0/x \), but the value of \( t_e \) increases in the ratio of \( x_0^2/x^2 \) thereby adiabatically expanding the minor radius at the same time the major radius is shrinking.

Continued injection of pulses of electrons increases the current in unit radial interval according to Eq. (2) and increases the total current much more rapidly than enough to sustain the continued adiabatic shrinkage of the mean minor radius of the stream.

The value for \( \gamma' \) at injection is further increased by the recoils of the electrons in the several kinds of processes by which the electrons lose energy of directed motion in the stream. The radiation of electron energy by virtue of the acceleration in the magnetic guide field is in quanta and at each quantum there is an electron recoil impulse equal and opposite to the momentum of the emitted photon. The radiation is within a small solid angle around the direction of momentum of the emitted photon. The radiation is magnetic field of the stream make the electrons radiate molecules can also contribute to \( f \). Electrons and ions contribute to the value of \( \gamma' \). Collisions between the electrons and neutral residual gas molecules can also contribute to \( \gamma' \).

THIRD PHASE—STEADY STATE

As soon as the critical current is exceeded, the moving electrons may begin to draw together into a self-focusing stream under the effects of the self-magnetic field, and the accelerations of the electrons by the self-magnetic field of the stream make the electrons radiate energy of motion transverse to the direction of the stream. This will be referred to as transverse radiation to distinguish it from the radiation by the electrons due to their acceleration in the guide field. At this same time, the Coulomb collisions of the electrons with each other in the coalescing stream tend to thermalize the electrons. Recalling that \( v \approx c \), the "mean transverse energy" \( \bar{v} \) of the electrons is found to be \( (L/35) \text{m}^2 \text{c}^4 \). As mentioned previously, the value of \( L \) for systems of interest here is between 60 and 75. The value of \( \bar{v} \) is approximately 630,000 ev. This same steady-state "mean transverse energy," as seen by an observer in the laboratory system of coordinates, is reduced approximately in the ratio \( 1 - v^2/c^2 \) = \( mc^2/x \). The critical current corresponding to this value of \( \gamma' \) is

\[
i_c = 2\pi \left( \frac{L}{35} \right)^{1/2} \frac{m c^5}{e x^2}.
\]

Integrating Eq. (2), between \( a_0 \) and the radial distance \( r \) at which the critical current of Eq. (4) is attained, and equating it to Eq. (4) gives

\[
\frac{r}{a_0} \left( 1 - \frac{r^2}{a_0^2} \right)^{-1} = \frac{f_0 T c x_0}{8\pi (L/35) f g [\alpha (mc^2)]^2}.
\]

For example, injecting 0.125 amp pulses of 50 Mev electrons in one-microsecond pulses at 180 pulses per second at an injection radius of 25 cm, the steady-state stream forms at a radius of 5.1 cm.

CONCLUSION

The essential distinction between the machine proposed in this revised proposal and previous machines is that electrons are to be injected at full energy and accumulated in the stream without being further energized before the critical current for self-focusing is reached. Rather different conditions are required for maintaining the steady-state stream than are required for obtaining it in the first place, and the steadily running stream should be rather different from the stream which has just become critical for self-focusing.

A number of applications of steady-state self-focusing streams can be investigated. One is the use of the stream as a strong magnetic guide-field for ions while the ions are being accelerated to very high...
energies by means with which the electrons in the stream are not resonant.

Ions can be injected into the stream to run in the opposite direction around the loop. The ions cannot be stored in the stream by the kind of process using radiation which was used for storing electrons because the radiation rate from ions is very much smaller. If the ions are injected so that a part of the first few loops lie inside the concentrated self-focusing stream, Coulomb collisions between ions and electrons can deflect a few of the ions through the small angle needed to put those ions in the stream. Those ions will stay in the stream until their transverse energy has become much greater than the ions which have little motion in the direction of the stream. This kind of ion injection can be made rapid enough to keep the stream filled with high energy ions of the species being injected and thus prevent the ions formed by ionization of residual gas from remaining in the stream and forming any important part of the stream.

APPENDIX

Steady-state Thermal Energy of Electrons

The equations for a thermalized self-focusing stream may be written for an observer moving with the center of mass of the electrons in any short section of the stream. For numerical density of electrons

\[ n_1 = \left(1 - \frac{T_1}{T_1 + T_2} \frac{v^2}{c^2}\right) n_0 (1 + r^2/|r_0|^2)^{-2}, \]

and for the numerical density of ions

\[ n_2 = n_0 (1 + r^2/|r_0|^2)^{-2}, \]

where \( n_0 \) is the ion numerical density at the axis, given by

\[ n_0 = \frac{2e_b^2 (T_1 + T_2)}{r_0^2 e^2 v^2}. \]

The number of electrons per unit length of stream in the electron coordinate system is

\[ N_1 = \int_0^\infty n_1 2\pi r dr = \frac{2e_b^2 (T_1 + T_2)}{c^2} \frac{v^2}{v^2} \left(1 - \frac{T_1}{T_1 + T_2} \frac{v^2}{c^2}\right). \]

The electric field in this moving coordinate system is

\[ E = \frac{2e_b}{r} u_\phi \int_0^\infty (n_2 - n_1) 2\pi r dr = \frac{4\pi e_b^2 T_1}{r_0^2 e^2} \frac{r}{1 + r^2/|r_0|^2} u_\phi. \]

The self-magnetic field is

\[ H = \frac{2e_b v}{cr} u_\phi \int_0^\infty (n_2 - n_1) 2\pi r dr = \frac{4\pi e_b^2 (T_1 + T_2)}{r_0^2 e^2} \frac{c}{v} \frac{u_\phi}{1 + r^2/|r_0|^2}. \]

The transverse motions of the electrons will be incoherent and the transverse radiation by an electron can be calculated using the Landau-Lifschitz equation:

\[ \frac{dy}{dt} = -\frac{2e^4}{3mc^3} \left(\left(E + \frac{1}{c} [w \times H]\right)^2 - \frac{1}{c^2} (E \cdot w)^2\right) \left(1 - \frac{w^2}{c^2}\right)^{-1}, \]

where \( y \) is the energy of the electron (this is thermal energy, because the radiation rate is being examined by an observer moving with the center of mass of the electrons); \( m \), the rest-mass of the electron; and \( w \), the velocity of the electron, isotropically directed.

The average rate at which an electron radiates energy because of thermal motion in the stream is calculated for \( v \approx c, kT_1 > mc^2, \) and \( T_2 \ll T_1, \) as follows.

For components of velocity in the \( z, r, \) and \( \phi \) directions, chosen so that \( w_\phi \) is parallel with the axis of the stream; \( w_r \) radially away from the axis; and \( w_\phi \) normal to the other two, the radiation rate becomes

\[ \frac{dy}{dt} = -\frac{2e^4}{3mc^3} \left(E^2 - 2EHw_\phi^2 + H^2w_\phi^2/\xi^2 + H^2w_\phi^2/\xi^2 - E^2w_\phi^2/\xi^2\right) \left(1 - \frac{w^2}{c^2}\right)^{-1}. \]

The total electron energy, including rest-energy, is

\[ y = mc^2 \left(1 - \frac{w^2}{c^2}\right)^{-1} = c(p_z^2 + p_r^2 + p_\phi^2 + m^2c^2)^{1/2}, \]

where

\[ p_z = \frac{mv_z}{1 - \frac{w^2}{c^2}}, \quad p_r = \frac{mv_r}{1 - \frac{w^2}{c^2}}, \quad p_\phi = \frac{mv_\phi}{1 - \frac{w^2}{c^2}}. \]

Substituting gives

\[ \frac{dy}{dt} = -\frac{2e^4}{3mc^3} \left(E^2 - 2EHw_\phi^2/\xi^2 + H^2w_\phi^2/\xi^2 + H^2w_\phi^2/\xi^2 - E^2w_\phi^2/\xi^2\right) \left(1 - \frac{w^2}{c^2}\right)^{-1}. \]

The energy loss rate per unit volume is obtained by integrating this times the distribution function over momentum space. The distribution function giving the number of electrons with magnitude of momentum in \( dp \) at \( p \) is

\[ A e^{-y^2 kT_1 4\pi p^2 dp}, \]

where the constant \( A \) is obtained from

\[ n_1 = \int_0^\infty A e^{-y^2 kT_1 4\pi p^2 dp}, \]

in which \( y^2 = c^2 p^2 + m^2c^4, \) \( dp \) \( = (y/c^2) dy \) and

\[ p = \frac{y}{c} \left(1 - \frac{m^2c^4}{y^2}\right)^{1/2}. \]

Integrating

\[ n_1 = A \frac{4\pi}{c^3} \int_0^\infty e^{-y^2 kT_1 4\pi 2y^2 + 4y^4 + \ldots} dy \]

gives

\[ n_1 = 8\pi (kT_1)^3 A e^{-mc^2 kT_1} \left[1 + \frac{mc^2}{kT_1} + \frac{mc^2}{kT_1} \frac{mc^2}{kT_1} + \ldots\right]. \]
When integrating for \( \frac{dy}{dt} \) over momentum space, the distribution in momentum is isotropic in direction

\[
\{E^2c^2\delta^{4\pi}\delta^3p = \frac{3E^2c^2\delta^{4\pi}\delta^3p}{9r^2m^4c^7},
\]

\[
\{H^2c^2\delta^{4\pi}\delta^3p = \frac{3H^2c^2\delta^{4\pi}\delta^3p}{9r^2m^4c^7}.
\]

Integrating the last term in the expression for \( \frac{dy}{dt} \) requires use of the distribution function in terms of the orthogonal momentum components, so that the number of electrons with momentum components in \( dp_z \) at \( p_z \), in \( dp_r \) at \( p_r \), and in \( dp_\phi \) at \( p_\phi \) is

\[
B \tilde{v} e^{-\nu/kT_4} \delta^3p \delta dp_z,
\]

where \( B \) is a constant. In carrying out the integration of the term \( 2EH\tilde{v}c^2 \) times this distribution function, the step in the integration

\[
2EH\tilde{v}c^2 \int_0^{2\pi} \int_0^\infty \int_{-\infty}^{\infty} \rho \delta^3p \delta dp_z
\]

is zero and the integral of the last term vanishes.

Integrating for \( \frac{dy}{dt} \) in unit volume,

\[
\frac{dy}{dt} = \frac{-2e^4A}{3m^2c^5} \left( \int_0^{2\pi} \int_0^\infty \int_{-\infty}^{\infty} \rho \delta^3p \delta dp_z \right)
\]

gives

\[
\frac{dy}{dt} = \frac{16\pi^4(kT_4)^4(E^2 + H^2)^2 n_1}{3m^2c^7} \alpha,
\]

where

\[
\alpha = 1 + \frac{1}{8} \left( \frac{1 + \frac{1}{1 + H^2/E^2}}{1 + \frac{m^2}{kT_1^2} \left( \frac{m^2}{kT_1^2} + \cdots \right)} \right) + \cdots
\]

and, for \( v \approx c, T_1 > T_2 \) and \( kT_1 > mc^2, 1.11 > \alpha > 1. \)

Integrating for the loss of energy by transverse radiation, per unit length of stream,

\[
\frac{dy}{dt} \mid_{L} \approx \frac{16\pi^4(kT_1)^4(E^2 + H^2)^2 n_2 \theta r d\phi}{3m^2c^7}
\]

gives

\[
\left( \frac{dy}{dt} \right) \mid_{L} \approx \frac{512\pi^4k^2(kT_1)^4(E^2 + H^2)^2 n_2 \theta r d\phi}{9r^2m^4c^7} \times \left( \frac{c^3}{c^2} \right)^2 \left[ 1 - \frac{T_1}{T_1 + T_2}(v^2) \right].
\]

Dividing by the number of electrons per unit length of stream gives the average rate of loss of energy of an electron by transverse radiation

\[
\left( \frac{dy}{dt} \right) \mid_{AV} = \frac{256\pi^4\rho^2(kT_1)^4}{9r^2m^4c^7}.
\]

For the rate of gain of an electron's energy due to an increase in transverse momentum resulting from Coulomb collisions with the ions, use is made of Thomas' Eq. 4.33, as mentioned previously,

\[
\frac{dy}{dt} = \frac{4\pi^4L}{mc^2} n_2.
\]

The electrons, per unit length of stream, gain energy at a rate

\[
\left( \frac{dy}{dt} \right) \mid_{AV} = \int_0^{2\pi} \int_0^\infty \frac{4\pi^4L}{mc^2} \theta r d\phi.
\]

Dividing by the number of electrons in unit length of stream gives the average rate of gain in energy by an electron due to Coulomb collisions

\[
\left( \frac{dy}{dt} \right) \mid_{AV} = \frac{8\pi^4L^2(kT_1 + T_2)^2}{3r^2m^4c^7}.
\]

Setting the loss rate equal to the gain rate gives

\[
kT_1 \approx \left( \frac{3L}{32\pi r^2} \right) \frac{1}{mc^2}
\]

from which it is seen that the steady-state electron "transverse temperature" in the electron system of coordinates is approximately independent of the radial spread of the stream and of the directed energy (from the laboratory standpoint) of the electrons.

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Electrical Characteristics of High Density, High Purity Titanate Ceramics

By D. A. Lupfer*

This report is concerned with the electrical behavior of cubic (Ba,Sr)TiO₃ ceramics at very high values of the electric field. The work was undertaken to develop a dielectric system to be used in capacitors for the storage and discharge of electrical energy. Objectives for the finished system were to store large amounts of energy per unit volume, to release at least 75% of the energy in 0.2 × 10⁻⁶ seconds, and to operate over a limited temperature range above 20°C. The work is incomplete, but the results to date show that (Ba,Sr)TiO₃ ceramics can store more electrical energy per unit volume than any other known dielectric system.

Since energy storage per unit volume of a dielectric may be represented as:

\[ U \text{ (joules/cm}^3) = 4.427 \times 10^{-\phi} \epsilon' E^2 \]  

where \( \epsilon' \) = dielectric constant,  
and \( E \) = electric field intensity, in kv/cm,

then, obtaining a material with a high dielectric constant and a high dielectric strength would give maximum energy storage. The problem is to obtain these two parameters simultaneously in the same dielectric.

**COMPOSITION**

The cubic (Ba,Sr)TiO₃ system was chosen for study, selecting compositions well above the Curie temperature. Figure 1 shows the dielectric constant (for low voltage alternating fields) as a function of the Ba : Sr ratio in (Ba,Sr)TiO₃ solid solutions. It will be noted that a wide range of dielectric constants may be obtained between 30% and 45% (by weight) of SrTiO₃ in the solid solution. All compositions to the right of the dielectric constant peak are cubic; those to the left are tetragonal. The two compositions whose properties are reported here are BaTiO₃ (60 weight %) -SrTiO₃ (40 weight %), and BaTiO₃ (65 weight %) -SrTiO₃ (35 weight %). According to the data shown in Fig. 1, these would have dielectric constant values of 1800 and 3200 respectively.

It was felt that impurities and second phases should be eliminated as much as possible so that the properties studied would be those of the lattice and not of a distribution of flaws. The BaTiO₃ and SrTiO₃ starting materials were obtained from the Titanium Alloy Manufacturing Division of the National Lead Company, and were that company’s chemically pure grade. Of the batches obtained to date, the impurity content has always been less than 0.5%. The major impurities were Group II oxides, SiO₂, and Al₂O₃.

The two titanates were mixed by wet ball milling and then dried, screened and dry pressed. It was found, in air firing to 1300°C, that some local reactions resulted which produced TiO₂, Ba₂TiO₄, and BaTiO₃. Accordingly, the ceramics were crushed, pulverized by ball milling, and fired again. In order to obtain flaw-free high density ceramics, it was necessary to control closely the atmosphere-time-temperature relation during the final firing.

With the optimum control of the atmosphere-time-temperature relation, ceramics with striking physical properties were obtained. The pieces were cream-colored, translucent and of very high density. Light could be transmitted through sections up to 8 mm in thickness. The densities ranged from 98.0% to 98.7% of theoretical density as determined from X-ray diffraction patterns. The grain boundaries were sharp and quite narrow. Most ceramic pieces were in the form of disks ranging from 15 to 45 mm in diameter and 1.2 to 8 mm in thickness.

**SAMPLE GEOMETRY**

Upon obtaining ceramics whose physical properties indicated that major flaws had been eliminated, it became necessary to develop a dielectric geometry from which meaningful electrical data could be obtained. The field had to be reduced at the electrode edge so that the maximum electrical stress could be applied to a thin uniform field section. An edge contour which satisfies this requirement may be developed from the field plot for an edge-to-plane configuration in a uniform dielectric medium. This field plot is shown in Fig. 2. The limiting equipotential surface for which the stress is everywhere equal to or less than the stress in the uniform field region is represented by \( \psi = 0.5\pi \). This derivation was first made by W. Rogowski,¹ and the surface is usually called the 0.5 Rogowski contour. Electrodes of this shape (or of

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* Electronics Laboratory, General Electric Company, Syracuse, New York.
lower values to reduce further the edge stress) have been used in the dielectric strength testing of liquids.

The contour selected for use on ceramic samples was the 0.375 Rogowski section shown in Fig. 3. An arbitrary selection of the boundary of the uniform field section was made where the thickness increased by 10%. There was no reason for selecting the edge of the finite plane as a boundary; at this limit the ceramic thickness increased by 25.4%, which appeared excessive.

The resulting ceramic design is shown in cross-section in Fig. 4. As given above, the effective diameter was measured between the regions where the thicknesses increased by 10%. With an edge thickness at least four times that of the effective central thickness, the problem of flashover (when immersed in oil) was minimized.

A very practical problem was obtaining this contour in ceramics to a nominal precision. It was solved by surface grinding the ceramic disks to obtain uniform thickness and parallel faces, and then grinding the contours ultrasonically. A 500-watt magnetostrictive transducer drove the steel cutting tool. The tolerances on the tool contour were maintained between ±0.01 mm; the ground faces were parallel within 0.02 mm; the thickness was maintained between +0.2 mm. Grinding was done with boron carbide powder (of less than 0.04 mm particle size) suspended in water.

After grinding, a heavy deposit of silver was sputtered over the contoured surface by the technique described by Belser. The ceramics, ready for test, are shown in Fig. 5. A guard electrode, formed on some samples by masking with a circular ring, is displayed on the ceramic on the right.

**LOW VOLTAGE TESTS**

Low voltage electrical tests were first performed. The dielectric constant and dissipation factor values were nearly constant over the frequency range of 100 cycles per sec to 1 megacycle. Direct current resistivity values were obtained with 100v applied to the samples. Representative data for the two compositions studied are listed in Table 1.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Static dielectric constant</th>
<th>Small-signal dielectric constant</th>
<th>Dissipation factor</th>
<th>Resistivity (ohm-cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60/40 Ba : Sr</td>
<td>2500</td>
<td>2120</td>
<td>.0021</td>
<td>10 (10^3)</td>
</tr>
<tr>
<td>65/35 Ba : Sr</td>
<td>4700</td>
<td>4040</td>
<td>.0016</td>
<td>10 (10^3)</td>
</tr>
</tbody>
</table>

![Figure 1. Dielectric constant vs. composition](image1)

![Figure 2. Field plot for edge-to-plane configuration](image2)

![Figure 3. Detail of contour section](image3)

![Figure 4. Cross-section of sample with final design](image4)
The values shown are conservative; on some samples dissipation factor values of 0.0003 were measured. It should be noted that the small signal dielectric constant values are higher than indicated in Fig. 1. As progressively higher densities were obtained it was found that the dielectric constant increased proportionally. The latest results, at 98.7% of theoretical density, show an increase of approximately 20% over previous data as plotted in Fig. 1.

An unusual trend was noted with the dissipation factor values as the sample density increased. Initially, the dissipation factors were quite low (about 0.0008) but they increased with increasing density. This trend was reversed by regulating the atmosphere carefully during the cooling period following sintering. Optically, it appeared that either excess or deficient oxidation states could be obtained in the high density materials unless great care was exercised in atmosphere control. Undoubtedly the stoichiometric condition is desired, but this is a difficult condition to measure and to obtain throughout a thick specimen. Work is proceeding on this problem.

**HIGH VOLTAGE TESTS**

High voltage testing included dielectric strength measurements and corona testing for voids or unusual conduction mechanisms with both alternating (60 cycles per sec) and static fields. High sensitivity resonant circuit corona test methods were used. It was found that no corona (or any other erratic conduction) occurred with theapplication of dc fields to breakdown. To date, the dc dielectric strength values, using a rate-of-rise of 100v per second, have ranged from 40 to 180 kv/cm. With the highest density samples the range of values has been 80–180 kv/cm. Higher values than these have been obtained with increased rates of voltage increase. The testing is incomplete; variations within each batch and from batch-to-batch have not yet been isolated so that meaningful distribution curves could be drawn.

To date, the ac corona test appears sensitive to both the microstructure and the macrostructure of the ceramics. At the least, it is a very sensitive test for voids. Corona starting stresses have ranged from 25 to 80 kv/cm.

Some measurements of the dielectric constant and loss have also been made as a function of the applied alternating field. Results obtained to date show a large decrease in dielectric constant and a slight increase in dissipation factor as the field increases. Measurements have not been taken of the low-voltage dielectric constant and loss as determined from a signal superposed on a dc field.

The most important measurements have been those of the stored charge. In this test, the sample was charged by a dc field and then discharged through a ballistic galvanometer. Because of the magnitudes of the potentials and capacitances, the ballistic galvanometer had to be placed in a resistance divider circuit. As this was not a standard technique for ballistic measurements, the resistors were selected with care, and the final assembly was calibrated to ±0.3% using precision air capacitors. In testing samples, the voltage was raised at the rate of 100v per sec to the prescribed level. The sample capacitor was then rapidly disconnected from the supply and discharged through the galvanometer. Two tests were made at each voltage. The voltage was then raised to succeeding test levels until the sample failed.

![Figure 5. Contoured ceramics with sputtered silver electrodes](image-url)

The results, obtained in units of charge, were readily converted into values of dielectric constant. This is a static dielectric constant and must not be confused with the dielectric constant determined with an alternating field. Values of this static dielectric constant are plotted as a function of the field strength in Fig. 6. It will be noted that the dielectric constant, even while decreasing with increasing field, still has high values at the highest fields shown. This is a most important and gratifying result. It demonstrates that the (Ba,Sr)TiO₃ system is capable of storing large quantities of electrical energy at high stress.

Finally, the polarization was calculated and plotted as a function of the field in Fig. 7. From this can be drawn the important conclusion that the system under study does not reach saturation polarization at the fields applied. It is obvious that, if saturation does occur, it is at much higher fields. Although the polarization may seem small to one accustomed to the values obtained for the maximum spontaneous polarization for piezoelectric BaTiO₃, it is an extremely large number for a non-piezoelectric material. No measurable remanence has been found; the dipoles appear completely dependent on the external field.
DISCHARGE CHARACTERISTICS

It then remained to determine how much of the stored charge was available in very short discharge time intervals. This was determined indirectly using the data in Table 1. Values of the static dielectric constant extrapolated to zero field are listed, and these quantities may be compared with the small signal high frequency dielectric constant values. The static dielectric constants for both compositions are approximately 20% higher than those measured with alternating fields. This means that interfacial polarizations account for only 20% of the stored static charge. Electronic and atomic polarizations intrinsic to the (Ba,Sr)TiO₃ structure account for the bulk of stored energy. All measurements to date give no evidence of relaxation mechanisms (up to 10 megacycles frequency) which would inhibit energy release in very short time intervals. If one pictures the energetic system as a charged particle in a potential well, it appears that the well has smooth sides with no traps. Further, the sides of the well do not rise so steeply as once thought.

Using the values shown in Fig. 6, the energy stored per unit volume may be calculated from Equation (1). For example, at 80 kv/cm the 60/40 Ba : Sr ceramic has an energy storage of 0.48 joules/cm³, while the 65/35 Ba : Sr ceramic stores 0.65 joules/cm³. At 120 kv/cm stress the 60/40 Ba : Sr ceramic stores 0.89 joules/cm³ and the 65/35 Ba : Sr ceramic stores 1.15 joules per cm³. These values are large by comparison with conventional dielectric systems.

A final word must be said concerning sample conditioning. All data reported are for samples in equilibrium with a 23°C ambient at 20-45% relative humidity. Both the dissipation factor values and the interfacial polarization term should be decreased by careful drying. Data are being taken to assess the importance of such conditioning.

It may be concluded that the (Ba,Sr)TiO₃ system has unique properties for energy storage and for low-loss capacitor applications. The importance of purity and control of the sintering operation is established, and may be extended to other ceramic studies. Finally, some interesting applications and new directions of research may be stimulated by this work.

REFERENCES

Record of Proceedings of Session A-10

Special Topics and Instrumentation in Fusion

FRIDAY AFTERNOON, 5 SEPTEMBER 1958

Chairman: Mr. H. Alfvén (Sweden)
Vice-Chairman: Mr. M. A. Leontovich (USSR)
Scientific Secretaries: Messrs. C. Sanchez del Rio and W. B. Woollen

PROGRAMME

P/2488 Operational characteristics of the stabilized toroidal pinch machine—Perhapsatron S-4.................................................. J. A. Phillips et al.
(Presented by J. A. Phillips.)
P/1329 Proposed methods of obtaining stable plasma................................. G. Miyamoto et al.
(Presented by G. Miyamoto.)
P/2225 Plasma loop in a transverse magnetic field................................ S. M. Osovets et al.
(Presented by A. M. Andrianov.)
P/356 Neutrons from plasma compressed by an axial magnetic field (Scylla)........ W. C. Elmore et al.
(Presented by K. Boyer.)

DISCUSSION

P/366 Diffusion of arc plasmas across a magnetic field........................... Albert Simon
P/146 Diffusion processes in the positive column in a longitudinal magnetic field........ B. Lehnert
P/2228 Spectroscopic study of high-temperature plasma.......................... S. Y. Lukyanov and V. I. Sinitsin
(Presented by V. I. Sinitsin.)
P/1520 Diagnostic techniques used in controlled thermonuclear research at Harwell......................................................... G. N. Harding et al.
(Presented by G. N. Harding.)
P/381 Plasma diagnostic developments in the UCRL pyrotron program........ C. B. Wharton et al.
(Presented by C. B. Wharton.)
P/387 Microwave studies of gas discharge plasmas................................ S. C. Brown
P/2506 Energy balance in a thermonuclear reacting plasma, containing deuterium, tritium and reaction products, under isothermal pulsed or steady-state conditions........................................ T. Hesselberg Jensen et al.
(Presented by O. Kofoed-Hansen.)

DISCUSSION

DISCUSSION OF P/2488, P/1329, P/2225 AND P/356

Mr. M. A. LEONTOVICH (USSR): I wish first to make one remark from which flows a question for all three speakers. It concerns the fact that in the work of Osovets (presented by Andrianov), in the work of Boyer, and in one of the cases mentioned in the report of Dr. Miyamoto, external-field configurations and phenomena are considered which are identical in principle. The difference between the cases of Osovets and Miyamoto, on the one hand, and Boyer on the other, is that the first two cases show the perfectly definite configurations of field which are necessary to form a plasma ring in some equilibrium. If we look into this question (it may not be immediately obvious)
the configuration of the external net field will be very nearly identical in all these cases.

Now for my question to the speakers. Since, in the experiments of Osovets, the presence of a frozen-in field considerably hinders the effect, measures were taken to compensate it. In the experiments of Boyer, a fast compression was obtained with no method of compensation whatever. On what does this depend—on the form of the field or on the apparatus?

Mr. K. Boyer (USA): The velocity at which the front progresses, in our system, is roughly the Alfvén speed and we have no idea at this time whether instabilities develop further or not. The time-scale is extremely short in our case. The idea of the compensating field was tried, where one biases the main field so as to pass through the zero field at the varying compressions. We do trap some field in the discharge at the time when the external field is going from zero. However, this field is not completely frozen in because it goes to zero at a later time. I think it depends rather critically on the rate of compression.

Mr. S. A. Colgate (USA): Mr. Andrianov, how can the rate of collapse be faster than the rate at which the trapped flux in the plasma turn escapes, i.e., diffuses out? This should be slower for high temperatures.

Mr. A. M. Andrianov (USSR): As I understand the question, it is: how does the discharge plasma go into a clump? In comparison with pinches, it goes more slowly. The plasma is pulled along the lines of magnetic field into the centre of the system and takes on a longitudinal configuration of a column along the axis of the tube. This process is now being studied.

Mr. Colgate (USA): I was essentially asking for an evaluation of the energy in the distortion of the trapped flux. If, during collapse, the trapped flux is distorted by a large factor, this should slow down collapse until the trapped flux can leak out of the plasma turn, and perhaps the compensating field is accurate enough so that this energy is trivial.

Mr. R. S. Pease (UK): Mr. Boyer, is the neutron yield affected by the probes?

Mr. K. Boyer (USA): The answer is that it is affected quite strongly. If the probe is terminated so far into the discharge, the neutron yield goes down by a factor of about 10. If it goes completely through the discharge, the neutron yield practically vanishes and the field distribution will also change rather drastically if one runs the probe too far into the discharge. We have some reservation about the probe measurement.

DISCUSSION OF P/366, P/146, P/2228, P/1520, P/381, P/387 AND P/2506

Mr. A. C. Kolb (USA): Mr. Sinitsin, I should like to point out that a spectral line broadened by the second order Stark effect does not necessarily have a shift due to inelastic electron scattering. Therefore the lack of shift does not prove there is no Stark broadening. Furthermore, the width depends on temperature as well as density in Stark broadening. These effects could compete with one another and account for the apparent insensitivity of width as a function of electron density for various ambient gas pressures. This is because the temperature will also vary with ambient pressure. Would you care to comment on this?

Mr. V. I. Sinitsin (USSR): I agree with these remarks. I wish only to say that it is necessary to begin measurements of ion temperature. This will be more valid for investigations of atoms of different mass according to different lines. Then the question may become self-evident.

Mr. S. Kaufman (UK): Mr. Sinitsin, were the Doppler widths measured in mutually perpendicular directions?

Mr. Sinitsin (USSR): Line widths were measured in directions perpendicular to the axis. They correspond to higher temperatures than those found by measurements along the axis. This is understandable, since there is a significant contribution from the radially directed motion of the plasma.

Mr. R. S. Pease (UK): Mr. Sinitsin, how did you estimate Stark effect for N IV lines? What is the half-width Stark effect at an electron density, n_e, of 10^17 cm^-3?

Mr. Sinitsin (USSR): As pointed out in the paper, the authors do not know the Stark constants. For those lines, such as N IV, for which measurements have been made, it appears that there is not a linear but a quadratic Stark effect. However, it does not seem to be of decisive importance.

Mr. D. W. Kerst (USA): Mr. Sinitsin, do you know with certainty that collision broadening is not troubling your Doppler broadening temperature measurements when n is of the order of 10^17?

Mr. Sinitsin (USSR): I would be happy to discuss this question with the questioner. I think that this question is controversial and requires a very careful approach.

Mr. V. M. Glagolev (USSR): Mr. Brown, have you compared the resonance method of measuring the concentration with other methods such as the probe method?

Mr. S. C. Brown (USA): No, we have been using microwave methods only.

Mr. V. D. Shafranov (USSR): I wish to make some remarks about microwaves; i.e., about the report of Mr. Brown, specifically about the calculations in today's report by Mr. Brown, in which he said that the imaginary part of the dielectric permittivity is defined by collisions.

It is very important to investigate the question of whether, in the plasmas with which we experiment at present, one could say that not collisions, but Doppler effect could be determining.

It is well known that the Doppler effect plays an important role. For electromagnetic waves propagat-
ing along the field, I would like to call your attention to the fact that (even for EM waves propagating perpendicular to the magnetic field, in the case where the electric sector of the wave is parallel to the wave) the importance of the relativistic Doppler effect can be shown.

Mr. Brown (USA): Perhaps I did not make it quite clear what I meant about the imaginary and real components of my calculation. The imaginary component is not the collision: it is the frequency shift.

Mr. Shafranov (USSR): I would like to remark that the effect I have mentioned may be important as well as collisions.

Mr. H. Dreicer (USA): Mr. Brown, how much penetration is possible into a plasma containing $10^{12} - 10^{13}$ electrons per cm$^3$ with the mode $TE_{011}$?

Mr. Brown (USA): This depends on the shape of the field. If the electric field is perpendicular to the gradient of the electron density it penetrates without attenuation.

The Chairman: We are now at the end of our discussions about controlled thermonuclear research.

At the First Geneva Conference there were rumours about the existence of this field of research, but very little was known with certainty—darkness prevailed and the atmosphere was cold.

This Second Conference has produced an enormous change due to the declassifications. The ice is broken, spring has come, full light has flooded over the whole field of research and the temperature gets higher and higher. A multitude of wonderful projects has been presented to us like beautiful flowers in the spring. It is possible that some of these projects will be fruitful and lead to final success, releasing fusion energy with enormous consequences for mankind. However, these fruits, like all fruits, belong to the autumn; before autumn, we have to pass a long summer which will be very hot—millions and millions of degrees.

But now we are still in the springtime of our field of research. Let us enjoy the spring and thank all the speakers for the wonderful flowers they have shown us during these memorable days.
The following is a listing of the thirty-three volumes in the English-language edition of the Proceedings. The titles of the sessions included in each volume are given to show the main subjects dealt with therein.

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E-14 & E-15. Properties of Reactor Materials

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