Heavy quark pair production in gluon fusion at next-to-next-to-leading $\mathcal{O}(\alpha_s^4)$ order: One-loop squared contributions

B. Kniehl* and Z. Merebashvili†
II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany

J. G. Körner‡
Institut für Physik, Johannes Gutenberg-Universität, D-55099 Mainz, Germany

M. Rogal§
Deutsches Elektronen-Synchrotron DESY, Platanenallee 6, D-15738 Zeuthen, Germany

(Dated: September 23, 2008)

We calculate the next-to-next-to-leading order $\mathcal{O}(\alpha_s^4)$ one-loop squared corrections to the production of heavy quark pairs in the gluon-gluon fusion process. Together with the previously derived results on the $q\bar{q}$ production channel the results of this paper complete the calculation of the one-loop squared contributions of the next-to-next-to-leading order $\mathcal{O}(\alpha_s^4)$ radiative QCD corrections to the hadroproduction of heavy flavours. Our results, with the full mass dependence retained, are presented in a closed and very compact form, in dimensional regularization.

PACS numbers: 12.38.Bx, 13.85.-t, 13.85.Fb, 13.88.+e

I. INTRODUCTION

It has been already twenty years since the next-to-leading-order (NLO) corrections to the hadroproduction of heavy flavors were first presented in the seminal work [1]. These results were confirmed yet in another seminal work [2].

In the last few years there was much progress in describing the experimental results on heavy-flavour production. For instance, in a recent work [3] it was shown that a NLO analysis of the transverse-momentum distributions does in fact properly describe the latest bottom quark production data [4] in a surprisingly large kinematical range. The improvement in the theoretical prediction is mainly due to advances in the analysis of parton distribution functions and the QCD coupling constant. We also point out the progress in dealing with numerically large mass logarithms that spoil the convergence of the perturbative expansion in the high energy (or small mass) asymptotic domain. In this respect we mention the work [5] where also the charm pair production is reconciled with the experimental data. Data on top quark pair production also agrees with the NLO prediction within theoretical and experimental errors (see e.g. Ref. [6]). However, in all of these NLO calculations there remains, among others, the problem that the renormalization and factorization scale dependences render the theoretical predictions to have much larger uncertainties than today's standards require. This calls for a next-to-next-to-leading-order (NNLO) calculation of heavy-quark production in hadronic collisions, which is expected to considerably reduce the scale dependence of the theoretical prediction when NNLO partonic amplitudes are folded with the available NNLO parton distributions. For example, by approximating the NNLO corrections with the fixed-order expansion of the next-to-leading-log (NLL) prediction, one finds a projected NNLO scale uncertainty of about 3% [7], which is below the parton distribution uncertainty, and in line with the anticipated experimental error.

Recently there was much activity in the phenomenology of hadronic heavy-quark pair production in connection with the Tevatron and the CERN Large Hadron Collider (LHC), which had its start-up this year. There will be much experimental effort dedicated to the discovery of the Higgs boson. There will also be studies of the copious production of top quarks and other heavy particles, which serve as a background to Higgs boson searches as well as to a possible new physics beyond the Standard Model. Therefore, it is mandatory to reduce the theoretical uncertainty in phenomenological calculations of heavy-quark production processes as much as possible.

Several years ago the NNLO contributions to hadron production were calculated by several groups in massless QCD (see e.g. Ref. [8] and references therein). The completion of a similar program for processes that involve massive quarks requires much more dedication, since the inclusion of an additional mass scale dramatically complicates the whole calculation.

At the lower energies of Tevatron II, top-quark pair production is dominated by $q\bar{q}$ annihilation (85%). The remaining 15% come from gluon fusion. At the higher energies of the LHC, gluon fusion dominates the production process (90%) leaving 10% for $q\bar{q}$ annihilation (percentage figures from Ref. [6]). This shows that both $q\bar{q}$

*Electronic address:kniehl@desy.de
†Electronic address:zakaria.merebashvili@desy.de
‡Electronic address:koerner@thep.physik.uni-mainz.de
§Electronic address:Mikhail.Rogal@desy.de

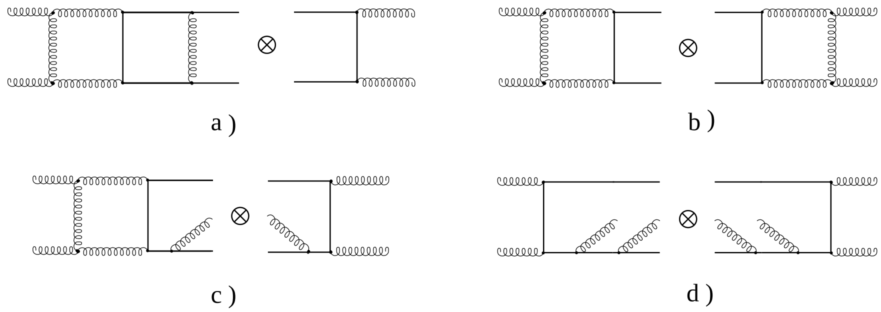

FIG. 1: Exemplary gluon fusion diagrams for the NNLO calculation of heavy-hadron production.

annihilation and gluon fusion have to be accounted for in the calculation of top-quark pair production. Since gluon fusion makes up the largest part of the heavy-quark pair production cross section at the LHC it is important to reduce renormalization and factorization scale uncertainties in the gluon fusion process as much as possible in view of the fact that the large uncertainties in the gluonic parton distribution functions translate to large cross section uncertainties at the LHC.

There are four classes of contributions that need to be calculated for the NNLO corrections to the hadronic production of heavy-quark pairs. In Fig. 1 we show one generic diagram each for the four classes of contributions that need to be calculated for the NNLO corrections to the gluon-initiated hadroproduction of heavy flavors. The first class involves the pure two-loop contribution (1a), which has to be folded with the leading-order (LO) Born term. The second class of diagrams (1b) consists of the so-called one-loop squared contributions (also called loop-by-loop contributions) arising from the product of one-loop virtual matrix elements. This is the topic of the present paper. Further, there are the one-loop gluon emission contributions (1c) that are folded with the one–gluon emission graphs. Finally, there are the squared two-gluon emission contributions (1d) that are purely of tree-type. The corresponding graphs for the quark-initiated processes are not displayed.

Bits and pieces of the NNLO calculation for hadroproduction of heavy flavors are now being assembled. In this context we would like to mention the recent two-loop calculation of the heavy-quark vertex form factor [9] that can be used as one of the many building blocks in the first class of processes. There is also a very promising numerical approach applied to the calculation of the pure two-loop diagrams [10]. Recently, an analytic calculation of a subclass of the two-loop contributions to $q\bar{q} \to Q\bar{Q}$ was published [11]. The authors of Ref. [12] have calculated the NLO corrections to $t\bar{t}$+jet production with contributions from the third class of diagrams. However, this result needs further subtraction terms in order to allow for an integration over the full phase space. We would also like to mention the recent work on the two-loop virtual amplitudes that are valid in the domain of high energy asymptotics, where the heavy-quark mass is small compared to the other large scales. In this calculation [13], mass power corrections are left out, and only large-mass logarithms and finite terms associated with them are retained. Much work was also done in relation to the resummation of soft contributions. In this respect we refer the reader to recent publications where some different approaches to the resummation are advocated [7, 14].

The authors of the present paper have been involved in a systematic effort to calculate all the contributions from the second class of processes, i.e. the one-loop squared contributions. The NNLO one-loop squared amplitudes for the quark-initiated process were recently presented in Ref. [15]. In this paper, we report on a calculation of the NNLO one–loop squared matrix elements for the process $gg \to Q\overline{Q}$. The calculation is carried out in dimensional regularization [16] with space-time dimension $n = 4-2\varepsilon$. We mention that we have presented closed-form, one-loop squared results for heavy quark production in the fusion of real photons in Ref. [17]. With the present paper the program of calculating the one-loop squared contributions to heavy-quark pair hadroproduction has now been completed.

Let us briefly describe some of the main features of the calculation of the one-loop squared contributions. The highest singularity in the one-loop amplitudes arise from infrared (IR) and mass singularities (M) and is thus, in general, proportional to $(1/\varepsilon^2)$. This in turn implies that the Laurent series expansion of the one-loop amplitudes has to be taken up to $\mathcal{O}(\varepsilon^2)$ when calculating the one-loop squared contributions. In fact, it is the $\mathcal{O}(\varepsilon^2)$ terms in the Laurent series expansion that really complicate things [18], since the $\mathcal{O}(\varepsilon^2)$ contributions in the one-loop amplitudes involve a multitude of multiple polylogarithms of maximal weight and depth 4 [19]. All scalar master integrals needed in this calculation have been assembled in Refs. [18, 19]. Reference [18] gives the results in terms of so-called L functions, which can be written as one-dimensional integral representations involving products of log and dilog functions, while Ref. [19] gives the results in terms of multiple polylogarithms. The divergent and finite terms of the one-loop *amplitude* for $gg \to Q\overline{Q}$ were given in Ref. [20]. The remaining $\mathcal{O}(\varepsilon)$ and $\mathcal{O}(\varepsilon^2)$ amplitudes have been written down in Ref. [21]. We shall

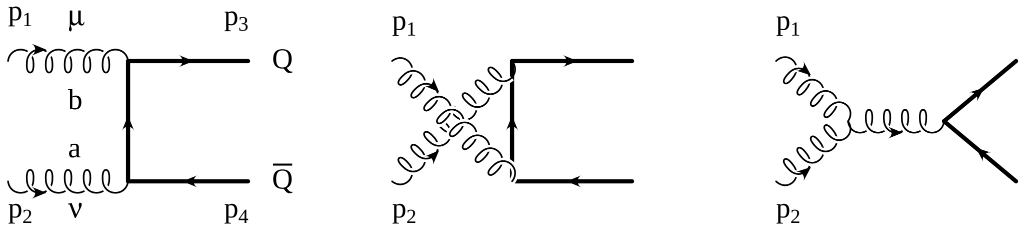

FIG. 2: The t-, u- and s-channel LO graphs contributing to the gluon (curly lines) fusion amplitude. The thick solid lines correspond to the heavy quarks.

rewrite these matrix elements in a representation more suitable for the purposes of the present application.

In our presentation, we shall make use of our notation for the coefficient functions of the relevant scalar one-loop master integrals calculated up to $\mathcal{O}(\varepsilon^2)$ in Refs. [18, 19]. For the case of gluon-gluon and quark-antiquark collisions, one needs all the scalar integrals derived in Refs. [18, 19], e.g. the one scalar one-point function A, the five scalar two-point functions B_1, B_2, B_3, B_4 and B_5, the six scalar three-point functions C_1, C_2, C_3, C_4, C_5 and C_6, and three scalar four-point functions D_1, D_2 and D_3. Taking the *complex* scalar four-point function D_2 as an example, we define successive coefficient functions $D_2^{(j)}$ for the Laurent series expansion of D_2. One has

$$D_2 = iC_\varepsilon(m^2)\Big\{\frac{1}{\varepsilon^2}D_2^{(-2)} + \frac{1}{\varepsilon}D_2^{(-1)} + D_2^{(0)} + \varepsilon D_2^{(1)} + \varepsilon^2 D_2^{(2)} + \mathcal{O}(\varepsilon^3)\Big\}, \tag{1.1}$$

where $C_\varepsilon(m^2)$ is defined by

$$C_\varepsilon(m^2) \equiv \frac{\Gamma(1+\varepsilon)}{(4\pi)^2}\left(\frac{4\pi\mu^2}{m^2}\right)^\varepsilon . \tag{1.2}$$

We use this notation for both the real and the imaginary parts of D_2, i.e. for $\mathrm{Re}D_2$ and $\mathrm{Im}D_2$. Similar expansions hold for the scalar one-point function A, the scalar two-point functions B_i, the scalar three-point functions C_i and the remaining four-point functions D_i. The coefficient functions of the various Laurent series expansions were given in Ref. [18] in the form of so-called L functions, and in Ref. [19] in terms of multiple polylogarithms of maximal weight and depth 4. It is then a matter of choice which of the two representations are used for the numerical evaluation. The numerical evaluation of the L functions in terms of their one-dimensional integral representations is quite straightforward using conventional integration routines, while there exists a very efficient algorithm to numerically evaluate multiple polylogarithms [22].

Let us briefly summarize the main features of the scalar master integrals. The master integrals $A, B_1, B_3, B_4, C_2, C_3$ and D_3 are real, whereas $B_2, B_5, C_1, C_4, C_5, C_6, D_1$ and D_2 are complex. From the form $(AB^* + BA^*) = 2(\mathrm{Re}A\,\mathrm{Re}B + \mathrm{Im}A\,\mathrm{Im}B)$ it is clear that the imaginary parts of the master integrals must be taken into account in the one-loop squared contribution. The master integrals B_2, B_5, C_1, C_4, C_5, and C_6 are $(t \leftrightarrow u)$ symmetric, where the kinematic variables t and u are defined in Sec. II.

This paper is organized as follows. Sec. II contains an outline of our general approach and discusses renormalization procedures. Sec. III presents LO and NLO results for the gluon fusion subprocess. In Sec. IV one finds a discussion of the singularity structure of the NNLO squared matrix element for the gluon fusion subprocess. In Sec. V we discuss the structure of the finite part of our result. Our results are summarized in Sec. VI. In the Appendices, we present expressions for various coefficients that are used in Sec. III to write down the NLO result.

II. NOTATION AND RENORMALIZATION

Heavy flavor hadroproduction proceeds through two partonic subprocesses: gluon fusion and light-quark-antiquark annihilation. The first subprocess is the most challenging one in QCD from a technical point of view. It has three production topologies already at the Born level (see Fig. 2). The second subprocess, where there is only one topology at the Born level, was considered in Ref. [15]. Irrespective of the partons involved, the general kinematics is, of course, the same in both processes. In particular, for gluon fusion, Fig. 2, we have

$$g(p_1) + g(p_2) \to Q(p_3) + \overline{Q}(p_4), \tag{2.1}$$

The momentum flow directions correspond to the physical configuration, e.g. p_1 and p_2 are ingoing whereas p_3 and p_4 are outgoing. With m being the heavy-quark mass, we define

$$s \equiv (p_1+p_2)^2, \qquad t \equiv T - m^2 \equiv (p_1-p_3)^2 - m^2,$$
$$u \equiv U - m^2 \equiv (p_2-p_3)^2 - m^2, \tag{2.2}$$

so that one has the energy-momentum conservation relation $s + t + u = 0$.

We also introduce the overall factor

$$\mathcal{C} = \left(g_s^4 C_\varepsilon(m^2)\right)^2 , \tag{2.3}$$

where g_s is the renormalized strong-coupling constant and $C_\varepsilon(m^2)$ is defined in Eq. (1.2).

As was shown e.g. in Refs. [20, 21] the self-energy and vertex diagrams contain ultraviolet (UV), infrared and collinear (IR/M) poles after heavy-mass renormalization. The UV poles need to be regularized.

Our renormalization procedure is carried out in a mixed renormalization scheme. When dealing with massless quarks, we work in the modified minimal-subtraction ($\overline{\rm MS}$) scheme, while heavy quarks are renormalized in the on-shell scheme defined by the following conditions for the renormalized external heavy quark self–energy graphs:

$$\Sigma_r(\not{p})|_{\not{p}=m} = 0, \qquad \frac{\partial}{\partial\not{p}}\Sigma_r(\not{p})|_{\not{p}=m} = 0. \tag{2.4}$$

In the on-shell scheme, the first condition in Eq. (2.4) ensures that the heavy-quark mass is the pole mass.

For completeness, we list the set of one-loop renormalization constants used in this paper. One has

$$\begin{aligned}
Z_1 &= 1 + \frac{g_s^2}{\varepsilon}\frac{2}{3}\left\{(N_C - n_l)C_\varepsilon(\mu^2) - C_\varepsilon(m^2)\right\},\\
Z_m &= 1 - g_s^2 C_F C_\varepsilon(m^2)\frac{3-2\varepsilon}{\varepsilon(1-2\varepsilon)},\\
Z_2 &= Z_m,\\
Z_{1F} &= Z_2 - \frac{g_s^2}{\varepsilon}N_C C_\varepsilon(\mu^2),\\
Z_{1f} &= 1 - \frac{g_s^2}{\varepsilon}N_C C_\varepsilon(\mu^2),\\
Z_3 &= 1 + \frac{g_s^2}{\varepsilon}\left\{(\frac{5}{3}N_C - \frac{2}{3}n_l)C_\varepsilon(\mu^2) - \frac{2}{3}C_\varepsilon(m^2)\right\}\\
&= 1 + \frac{g_s^2}{\varepsilon}\left\{(\beta_0 - 2N_C)C_\varepsilon(\mu^2) - \frac{2}{3}C_\varepsilon(m^2)\right\},\\
Z_g &= 1 - \frac{g_s^2}{\varepsilon}\left\{\frac{\beta_0}{2}C_\varepsilon(\mu^2) - \frac{1}{3}C_\varepsilon(m^2)\right\},
\end{aligned} \tag{2.5}$$

with $\beta_0 = (11N_C - 2n_l)/3$ being the first coefficient of the QCD beta function, n_l the number of light quarks, $C_F = 4/3$, and $N_C = 3$ the number of colors. The arbitrary mass scale μ is the scale at which the renormalization is carried out. The above renormalization constants renormalize the following quantities: Z_1 for the three-gluon vertex, Z_m for the heavy-quark mass, Z_2 for the heavy-quark wave function, Z_{1F} for the $(Q\overline{Q}g)$ vertex, Z_{1f} for the $(q\overline{q}g)$ vertex, Z_3 for the gluon wave function and Z_g for the strong-coupling constant α_s. For the massless quarks, there is no mass and wave function renormalization.

Let us sketch the two alternative ways of getting the final one-loop-renormalized amplitude from the mass renormalized amplitude:

i) Take the given mass-renormalized matrix element or the square of that matrix element and multiply all the self-energy graphs by a factor 1/2. Then renormalize the coupling constant in the LO Born amplitude.

ii) Take the given mass-renormalized matrix element and apply the corresponding counterterms obtained from the LO matrix element by inserting the relevant $Z-1$ factors into the *internal* propagators and vertices. All the renormalization constants we need are presented in Eq. (2.5). We will get the renormalized vertex function $\Gamma_R^{(N)}$ where (N) denotes the set of N external particles. The renormalized matrix element is obtained from

$$M_R = \Gamma_R^{(N)}\prod_{i=1}^{N}\left(Z_R^{(i)}\right)^{\frac{1}{2}}, \tag{2.6}$$

where $Z_R^{(i)}$ are the residues of the renormalized propagators at the poles for all the particles under consideration. They are related to the residues of the unrenormalized propagators via

$$Z_R^{(i)} = Z_U^{(i)} Z_i^{-1} \tag{2.7}$$

where the Z_i are the respective external wave function renormalization constants.

Working at the one-loop order, we note that in the on-shell scheme $Z_R^{(i)} = 1$. This is a direct consequence of the second condition in Eq. (2.4), which effectively cuts off the external massive lines. For the case of external massless partons $Z_U^{(i)} = 1$. It is important to note that the gluon wave function renormalization constant Z_3 is a mixture of two parts: the part which multiplies $C_\varepsilon(\mu^2)$ is derived in the $\overline{\rm MS}$ scheme, while the last term due to the heavy-quark loop is derived in the on-shell scheme. For this reason, this last term has to be omitted in Z_3 when using it as an external field renormalization constant in Eq. (2.7). Since in our case we have two gluon and two heavy-quark fields, we therefore obtain

$$M_R = \Gamma_R^{(N)} Z_3^{-1}. \tag{2.8}$$

The final result should not depend on which of the two ways has been chosen to do the renormalization. We have checked that, in both ways, one arrives at the same renormalized matrix element.

In order to fix our normalization, we write down the differential cross section for $gg \to Q\overline{Q}$ in terms of the squared amplitudes $|M|^2$. One has

$$d\sigma_{gg\to Q\overline{Q}} = \frac{1}{2s}\frac{d(\mathrm{PS})_2}{4(1-\varepsilon)^2}\frac{1}{d_A^2}|M|^2_{gg\to Q\overline{Q}}, \tag{2.9}$$

where the n–dimensional two–body phase space is given by

$$d(\mathrm{PS})_2 = \frac{m^{-2\varepsilon}}{8\pi s}\frac{(4\pi)^\varepsilon}{\Gamma(1-\varepsilon)}\left(\frac{tu - sm^2}{sm^2}\right)^{-\varepsilon}\delta(s+t+u)dtdu\,. \tag{2.10}$$

We explicitly exhibit the flux factor $(4p_1p_2)^{-1} = (2s)^{-1}$, and the spin $(n-2)^{-2} = (2-2\varepsilon)^{-2}$ and color d_A^{-2} averaging factors for the initial gluons. Here $d_A = N_C^2 - 1 = 8$ is the dimension of the adjoint representation of the color group SU(N_C).

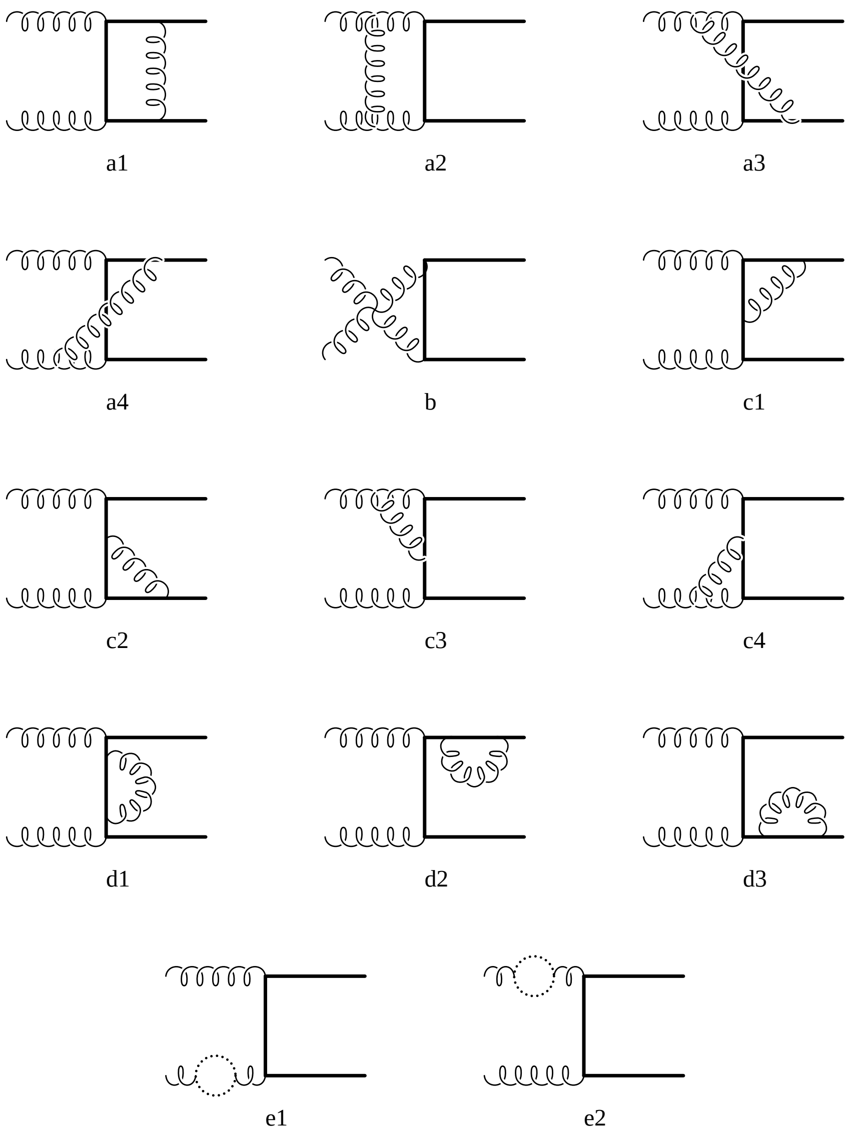

FIG. 3: The t-channel one-loop graphs contributing to the gluon fusion amplitude. Loops with dotted lines represent the gluon, ghost, and light and heavy quarks.

III. LEADING AND NEXT-TO-LEADING ORDER RESULTS

At LO for $gg \to Q\overline{Q}$, we shall use a representation which differs from the one given in Refs. [20, 21]. First note that there are only two independent color structures for this subprocess. The s-channel matrix element is a sum of two parts, each of which are proportional to one of the two independent color structures. We combine terms with the same color structures of the three (e.g. s, t and u) production channels. Finally, we remove the heavy-antiquark momentum p_4 using energy-momentum conservation and use on-shell conditions for the gluons ($p_1 \cdot \epsilon_1 = 0$ and $p_2 \cdot \epsilon_2 = 0$) and the heavy quark ($\bar{u}_3 \not{p}_3 = \bar{u}_3 m$). We then obtain the two color-linked LO matrix elements

$$M_{\rm LO,t} = iT_b T_a \hat{M}/t, \qquad M_{\rm LO,u} = iT_a T_b \hat{M}/u, \tag{3.1}$$

with

$$s\hat{M} = \gamma^\mu \not{p}_1 \gamma^\nu s + 2\gamma^\mu p_1^\nu t - 2\gamma^\nu p_2^\mu t - 2\gamma^\nu p_3^\mu s - 2\not{p}_1 g^{\mu\nu} t. \tag{3.2}$$

It can be verified that the function $\hat{M}$ is $t \leftrightarrow u$ symmetric, and consequently the color-linked Born amplitudes $M_{\rm LO,t}$ and $M_{\rm LO,u}$ turn into one another under $t \leftrightarrow u$.

We then square the full Born matrix element $M_{\rm LO,t} + M_{\rm LO,u}$ and do the spin and color sums to obtain the LO amplitude

$$|M|^2_{\rm LO} = \frac{d_A}{2}\left(C_F \frac{s^2}{tu} - N_C\right) |\hat{M}|^2 \equiv B, \tag{3.3}$$

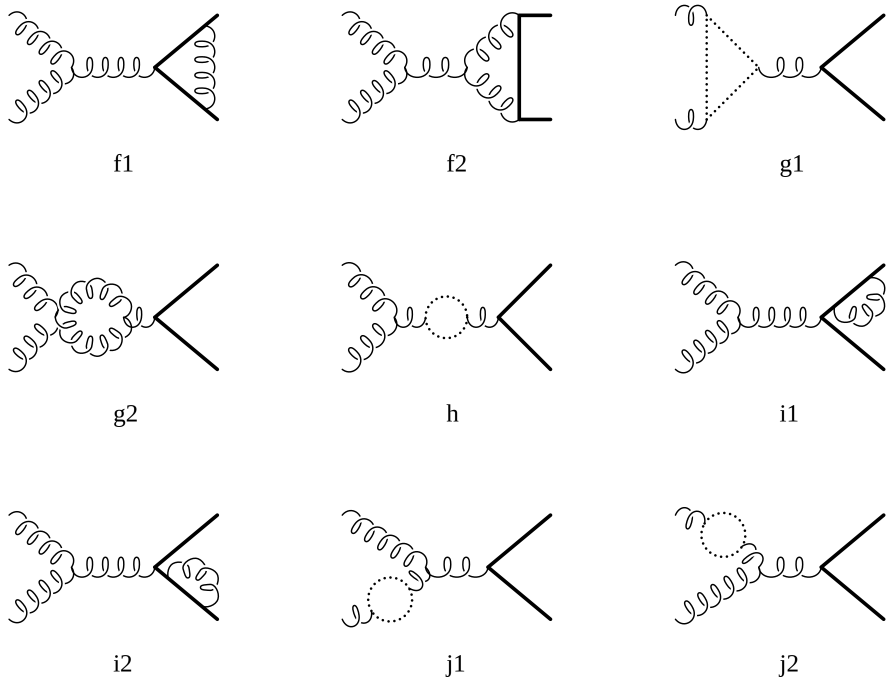

FIG. 4: The s-channel one-loop graphs contributing to the gluon fusion amplitude. Loops with the dotted lines as in g1, h, j1 and j2 represent the gluon, ghost, and light and heavy quarks. The four-gluon coupling contribution appears in g2.

where we have factored out a color-reduced Born term $|\hat{M}|^2$, which reads

$$|\hat{M}|^2 = 8\Big\{\frac{t^2+u^2}{s^2} + 4\frac{m^2}{s} - 4\frac{m^4}{tu} -\varepsilon\, 2(1-\frac{tu}{s^2}) + \varepsilon^2\Big\} \equiv \hat{B}. \qquad (3.4)$$

The expression in Eq. (3.3) for the LO amplitude agrees with the well-known result in n dimensions (see e.g. Ref. [2]). Note that, by using the precription of Ref. [23], we were able to avoid the introduction of ghost contributions which would otherwise arise from the square of the right-most three-gluon coupling amplitude in Fig. 2. In our case the precription of Ref. [23] consists in the use of on-shell conditions for external gluons, i.e. $p_1 \cdot \epsilon_1 = 0$ and $p_2 \cdot \epsilon_2 = 0$, and the exclusion of the heavy-antiquark momentum via $p_4 = p_1 + p_2 - p_3$. When squaring amplitudes, we sum over the two helicities of the gluons using the Feynman gauge, i.e. we use

$$\sum_{\lambda=\pm 1} \epsilon^{\mu}(\lambda)\epsilon^{\nu}(\lambda) = -g^{\mu\nu}\,. \qquad (3.5)$$

The use of the framework set up in Ref. [23] has the advantage in the non-Abelian case that one can omit ghost contributions when squaring the amplitudes. Using the above on-shell conditions already at the amplitude level means that one takes full advantage of the gauge invariance of the problem when squaring the amplitudes. Thus, in general, the results for the different channels will not be identical to the ones which would be obtained using 't Hooft-Feynman gauge throughout.

Folding the one-loop matrix elements (see Figs. 3 and 4) with the LO Born term (see Fig. 2) one obtains the virtual part of the NLO result.

As concerns the one-loop matrix elements, we shall use the one-loop matrix elements of Refs. [20, 21] to compute the virtual NLO contribution up to $\mathcal{O}(\varepsilon^2)$ in terms of the coefficient functions (1.1) of the scalar master integrals. However, in Ref. [20], where expressions for the NLO matrix elements up to $\mathcal{O}(\varepsilon^0)$ are given, the values for the scalar coefficient functions in terms of logarithms and dilogarithms are substituted directly. Therefore, we had to recalculate the corresponding expressions from Ref. [20] for the matrix elements in order to have a uniform result in terms of scalar coefficient functions. This has allowed us to retrieve and use relations between coefficients of the scalar coefficient functions in the result for different orders of the Laurent series expansion in ε. We will comment on these relations later on.

We also mention that we had to regroup and rearrange various terms in the one-loop amplitudes from Refs. [20, 21] according to the three independent color structures in order to bring the pole terms into agreement with the form suggested in Ref. [24]. In the gluon fusion case treated here, there are three independent color structures in the one-loop amplitudes, e.g. T^bT^a, T^aT^b and δ^{ab}. As in the LO case, one also has to exclude the heavy-antiquark momentum p_4 from the one-loop amplitude expressions. As a result of the above two steps, the pole terms of our new matrix elements became proportional to the LO color-linked amplitudes (3.1). In all our subsequent calculations, we shall use only these matrix elements.

Although the NLO virtual corrections to heavy-flavor hadroproduction were calculated before for the $gg \to Q\bar{Q}$ case, one cannot find explicit results for this subprocess in the literature. We have therefore recalculated the virtual NLO contribution to gg fusion. In fact, we have calculated the virtual NLO results up to $\mathcal{O}(\varepsilon^2)$. As it turns out, use of the expressions for the NLO virtual $\mathcal{O}(\varepsilon^1)$- and $\mathcal{O}(\varepsilon^2)$-contributions considerably simplify the presentation of the corresponding NNLO results in as much as they appear as important building blocks in the NNLO results.

Next we fold the pole, finite, $\mathcal{O}(\varepsilon^1)$ and $\mathcal{O}(\varepsilon^2)$ terms of our NLO matrix element with the LO matrix element. In dimensional regularization, the trace evaluation in $n = 4 - 2\varepsilon$ dimensions will lead to terms of order $\mathcal{O}(\varepsilon^1)$ and $\mathcal{O}(\varepsilon^2)$ when multiplied with the pole and finite terms, as well as to the terms of $\mathcal{O}(\varepsilon^3)$ and $\mathcal{O}(\varepsilon^4)$ when multiplied with the $\mathcal{O}(\varepsilon^1)$ and $\mathcal{O}(\varepsilon^2)$ terms of the squared amplitude, respectively. In the following we will disregard terms of $\mathcal{O}(\varepsilon^3)$ and $\mathcal{O}(\varepsilon^4)$ as they do not contribute to the finite part of the NNLO result.

Before presenting our result for the NLO matrix element, we would like to comment on its color structure. We have decomposed our matrix elements according to the following three independent color structures:

$$\begin{aligned} \delta^{ab}\ \mathrm{Tr}(T^aT^b) &= \frac{d_A}{2}, \\ \mathrm{Tr}(T^bT^a)\ \mathrm{Tr}(T^bT^a) &= \frac{d_A}{2}C_F, \\ \mathrm{Tr}(T^bT^a)\ \mathrm{Tr}(T^aT^b) &= \frac{d_A}{2}(C_F - \frac{N_C}{2}). \end{aligned} \tag{3.6}$$

At NLO, the final spin and color summed matrix element can be written as a sum of five terms:

$$\begin{aligned} |M|^2_{\mathrm{Loop}\times\mathrm{Born}} = g_s^2\sqrt{\mathcal{C}}\ \mathrm{Re}\Big[&\frac{1}{\varepsilon^2}W^{(-2)}(\varepsilon) + \frac{1}{\varepsilon}W^{(-1)}(\varepsilon) \\ &+W^{(0)}(\varepsilon) + \varepsilon W^{(1)}(\varepsilon) + \varepsilon^2 W^{(2)}(\varepsilon)\Big], \end{aligned} \tag{3.7}$$

where $\mathcal{C}$ has been defined in Eq. (2.3). The notation $|M|^2_{\mathrm{Loop}\times\mathrm{Born}}$ means that one is retaining only the $\mathcal{O}(\alpha_s^3)$ part of $|M|^2$.

The first two coefficient functions in Eq. (3.7) have a rather simple structure:

$$W^{(-2)}(\varepsilon) = -4N_C B\,, \tag{3.8}$$
$$\begin{aligned} W^{(-1)}(\varepsilon) = d_A\hat{B}\Big(&\frac{s^2}{tu}f_\delta + (C_F - \frac{N_C}{2})(f_t + f_u) \\ &+C_F\frac{u}{t}f_t + C_F\frac{t}{u}f_u\Big)\,, \end{aligned}$$

where B and $\hat{B}$ are the LO terms defined in Eqs. (3.3) and (3.4). We have also introduced new functions

$$f_\delta = \frac{1}{2}\ln\frac{s}{m^2} + \frac{t}{s}\ln\frac{-t}{m^2} + \frac{u}{s}\ln\frac{-u}{m^2} + \frac{2m^2 - s}{2s\beta}\ln x,$$
$$\begin{aligned} f_t &= N_C\ln\frac{s}{m^2} + 2N_C\ln\frac{-t}{m^2} - 2C_F - \beta_0 \\ &\quad +(2C_F - N_C)\frac{2m^2 - s}{s\beta}\ln x, \\ f_u &= f_t|_{t\leftrightarrow u}, \end{aligned} \tag{3.9}$$

where $\beta = \sqrt{1 - 4m^2/s}$ is the heavy-quark velocity and β_0 is defined after Eq. (2.5).

One should keep in mind that the overall Born term factors B and $\hat{B}$ contain terms multiplied by ε and ε^2. Therefore, if the expressions for B and $\hat{B}$, given in Eqs. (3.3) and (3.4), are substituted in $W^{(-2)}$ and $W^{(-1)}$, we will obtain additional $\mathcal{O}(\varepsilon^{-1})$ and finite terms from the first two terms of Eq. (3.7).

The third term in Eq. (3.7) reads

$$W^{(0)}(\varepsilon) \equiv F^{(0)}_{\mathrm{NLO}}\,, \tag{3.10}$$

where we have constructed the following generic functions

$$F^{(j)}_{\mathrm{NLO}} = \mathcal{W}^{(j)}_1 + \mathcal{W}^{(j)}_2\,, \tag{3.11}$$

with

$$\begin{aligned} \mathcal{W}^{(j)}_1 &= -\frac{d_A}{2}\Big[\frac{s}{tu}F^{(j)}_1 + \Big\{\frac{1}{u}\Big(\frac{s}{t}C_F + \frac{N_C}{2}\Big)(F^{(j)}_2 + F^{(j)}_3) \\ &\qquad + (t\leftrightarrow u)\Big\}\Big], \\ \mathcal{W}^{(j)}_2 &= -\frac{2B\beta_0}{(1+j)!}\ln^{1+j}\frac{m^2}{\mu^2}\,. \end{aligned} \tag{3.12}$$

The three functions F_1, F_2 and F_3 are defined as follows:

$$\begin{aligned} F^{(j)}_1 &= \sum_I (a_I + \varepsilon a^{(\varepsilon)}_I + \varepsilon^2 a^{(\varepsilon^2)}_I) I^{(j)}, \\ \text{with}\quad & I^{(j)} = \{B_2, B_5, C_1, C_2, C_{2u}, C_3, C_{3u}, C_4, C_5, C_6, \\ &\qquad D_1, D_{1u}, D_2, D_{2u}, D_3\}^{(j)}; \\ F^{(j)}_2 &= \sum_I (b_I + \varepsilon b^{(\varepsilon)}_I) I^{(j)}, \\ \text{with}\quad & I^{(j)} = \{1, B_2, B_5, C_1, C_4, C_5, C_6\}^{(j)}; \\ F^{(j)}_3 &= \sum_I (c_I + \varepsilon c^{(\varepsilon)}_I + \varepsilon^2 c^{(\varepsilon^2)}_I) I^{(j)}, \\ \text{with}\quad & I^{(j)} = \{1, B_1, B_2, B_5, C_1, C_2, C_3, C_4, C_5, C_6, \\ &\qquad D_1, D_2\}^{(j)}\,. \end{aligned} \tag{3.13}$$

For $I = 1$ one has $I^{(j)} \equiv 1$, otherwise $I^{(j)} \equiv B^{(j)}_1, C^{(j)}_2$ etc. In other words, the summation index I runs over the scalar integral coefficient functions, while the coefficient functions $a_I, a^{(\varepsilon)}_I, a^{(\varepsilon^2)}_I$ etc. denote the explicit dependence on s, t and m^2. These coefficient functions are presented in Appendix A. Note that index j takes the same value for all these coefficient functions throughout as in Eqs. (3.13) as well as in similar equations that will follow.

The additional subscript "u" in some of the scalar coefficient functions in the expression for $F_1^{(j)}$ (such as $C_{2u}^{(j)}$) is to be understood as an operational definition prescribing a $(t \leftrightarrow u)$ interchange in the argument of that function, i.e. $C_{2u}^{(0)} = C_2^{(0)}\big|_{t\leftrightarrow u}$ etc.

Note that $\mathcal{W}_2^{(j)}$ is only contributed to by the renormalization procedure. Of course, all the remaining $\mathcal{O}(\varepsilon)$ terms (e.g. $W^{(1)}(\varepsilon)$ and $W^{(2)}(\varepsilon)$, as well as those coming from $W^{(-1)}(\varepsilon)/\varepsilon$ and $W^{(0)}(\varepsilon)$) should be disregarded in the NLO final result in Eq. (3.7). It is important to note that $F_{\rm NLO}^{(0)}|_{\varepsilon=0}$ is not formally the full finite part of the NLO result in dimensional regularization, but it results from folding the finite part of our original NLO matrix element with the LO one. Another part of the finite result comes from the first two terms in Eq. (3.7), as mentioned before Eq. (3.10). However, one should realize that the first two terms in Eq. (3.7) would be cancelled with the corresponding parts from the real bremsstrahlung diagrams. Given the overall factor Eq. (1.2) the term $F_{\rm NLO}^{(0)}$ evaluated for $\varepsilon = 0$ represents the finite part of the virtual one-loop NLO result.

Our $\mathcal{O}(\varepsilon^{-2})$, $\mathcal{O}(\varepsilon^{-1})$ and $\mathcal{O}(\varepsilon^{0})$ NLO results in Eq. (3.7) were analytically compared with the corresponding results obtained in Ref. [1] which were kindly provided to us in a Schoonschip format by the authors [25]. We obtained complete agreement.

The fourth term in Eq. (3.7) is a result of folding the $\mathcal{O}(\varepsilon)$ term of the matrix element with the Born term. Due to the n-dimensional traces, one also obtains terms of $\mathcal{O}(\varepsilon^2)$ and $\mathcal{O}(\varepsilon^3)$. As mentioned before, we will only retain terms of $\mathcal{O}(\varepsilon)$ and $\mathcal{O}(\varepsilon^2)$. We have

$$W^{(1)}(\varepsilon) = F_{\rm NLO}^{(1)} + F_{{\rm NLO},\varepsilon}^{(0)}, \qquad (3.14)$$

where

$$F_{{\rm NLO},\varepsilon}^{(j)} = d_A\Big[F_4^{(j)} - \Big\{\Big(C_F + \frac{N_C}{2}\frac{t}{s}\Big)(F_5^{(j)} + C_F F_6^{(j)} + N_C F_7^{(j)}) + (t \leftrightarrow u)\Big\}\Big]. \qquad (3.15)$$

Here

$$\begin{aligned}
F_4^{(j)} &= \sum_I (d_I^{(\varepsilon)} + \varepsilon d_I^{(\varepsilon^2)}) I^{(j)},\\
&\text{with} \quad I^{(j)} = \{B_2, B_5, C_1, C_2, C_{2u}, C_3, C_{3u}, C_4, C_5, C_6, D_1, D_{1u}, D_2, D_{2u}, D_3\}^{(j)};\\
F_5^{(j)} &= \sum_I (e_I^{(\varepsilon)} + \varepsilon e_I^{(\varepsilon^2)}) I^{(j)},\\
&\text{with} \quad I^{(j)} = \{1, B_2, B_5, C_5\}^{(j)};\\
F_6^{(j)} &= \sum_I (g_I^{(\varepsilon)} + \varepsilon g_I^{(\varepsilon^2)}) I^{(j)}, \qquad (3.16)\\
&\text{with} \quad I^{(j)} = \{1, B_1, B_2, C_2, C_5, C_6, D_1\}^{(j)};\\
F_7^{(j)} &= \sum_I (h_I^{(\varepsilon)} + \varepsilon h_I^{(\varepsilon^2)}) I^{(j)},\\
&\text{with} \quad I^{(j)} = \{1, B_1, B_2, B_5, C_1, C_2, C_3, C_4, C_5, C_6, D_1, D_2\}^{(j)}.
\end{aligned}$$

The coefficients d_I, e_I, g_I, h_I are presented in Appendix B. Note that the first term in Eq. (3.14) in nothing but the NLO term of Eq. (3.10) with indices of the coefficient functions of the scalar master integrals and the power of the logarithm that multiplies β_0 shifted upword by one.

The last term in Eq. (3.7) is a result of folding the $\mathcal{O}(\varepsilon^2)$ term of the matrix element with the Born term. Due to the n-dimensional traces, one also obtains terms of $\mathcal{O}(\varepsilon^3)$ and $\mathcal{O}(\varepsilon^4)$, which are omitted as before. For the $\mathcal{O}(\varepsilon^2)$ terms we obtain:

$$W^{(2)}(\varepsilon) = F_{\rm NLO}^{(2)} + F_{{\rm NLO},\varepsilon}^{(1)} + F_{{\rm NLO},\varepsilon^2}^{(0)}, \qquad (3.17)$$

where

$$F_{{\rm NLO},\varepsilon^2}^{(j)} = d_A\Big[F_8^{(j)} - \Big\{\Big(C_F + \frac{N_C}{2}\frac{t}{s}\Big)(F_9^{(j)} + C_F F_{10}^{(j)} + N_C F_{11}^{(j)}) + (t \leftrightarrow u)\Big\}\Big]. \qquad (3.18)$$

Here

$$\begin{aligned}
F_8^{(j)} &= \sum_I k_I^{(\varepsilon^2)} I^{(j)},\\
&\text{with} \quad I^{(j)} = \{C_1, C_2, C_{2u}, C_3, C_{3u}, C_4, C_5, C_6, D_1, D_{1u}, D_2, D_{2u}, D_3\}^{(j)};\\
F_9^{(j)} &= \sum_I l_I^{(\varepsilon^2)} I^{(j)},\\
&\text{with} \quad I^{(j)} = \{1, B_2, B_5, C_5\}^{(j)};\\
F_{10}^{(j)} &= \sum_I m_I^{(\varepsilon^2)} I^{(j)}, \qquad (3.19)\\
&\text{with} \quad I^{(j)} = \{1, C_2, C_5, C_6, D_1\}^{(j)};\\
F_{11}^{(j)} &= \sum_I n_I^{(\varepsilon^2)} I^{(j)},\\
&\text{with} \quad I^{(j)} = \{1, B_5, C_1, C_2, C_3, C_4, C_5, C_6, D_1, D_2\}^{(j)}.
\end{aligned}$$

The coefficients k_I, l_I, m_I, n_I are presented in Appendix C. We mention that the functions F_1, F_4 and F_8 are $(t \leftrightarrow u)$ symmetric.

IV. SINGULARITY STRUCTURE OF THE NNLO SQUARED AMPLITUDE

The NNLO final spin and color summed squared matrix element can be written down as a sum of five terms:

$$\frac{1}{\mathcal{C}}|M|^2_{\rm Loop\times Loop} = {\rm Re}\Big[\frac{1}{\varepsilon^4}V^{(-4)}(\varepsilon) + \frac{1}{\varepsilon^3}V^{(-3)}(\varepsilon) + \frac{1}{\varepsilon^2}V^{(-2)}(\varepsilon) + \frac{1}{\varepsilon}V^{(-1)}(\varepsilon) + V^{(0)}(\varepsilon)\Big], \qquad (4.1)$$

where $\mathcal{C}$ has been defined in Eq. (2.3). Note that Eq. (4.1) is *not* a Laurent series expansion in ε since the coefficient functions $V^{(m)}(\varepsilon)$ are functions of ε as explicitely annoted in Eq. (4.1). It is nevertheless useful to write the NNLO one-loop squared result in the form of Eq. (4.1) in order to exhibit the explicit ε structures. All five coefficient functions $V^{(m)}(\varepsilon)$ are bilinear forms in the coefficient functions that define the Laurent series expansion of the scalar master integrals (1.1). Some of these coefficient functions are zero and some of them are just numbers or simple logarithms. In the latter case, we have substituted these numbers or logarithms for the coefficient functions $V^{(m)}$ in the five terms above. This has been done for all the scalar coefficient functions that multiply poles, i.e. for scalar functions with negative subscripts $I^{(-2)}$ and $I^{(-1)}$, as well as for the whole scalar functions $A^{(i)}$, $B_3^{(i)}$ and $B_4^{(i)}$.

We found that a significant part of the NNLO results can be expressed in terms of the ε expansion of the NLO contribution. In particular, we will need the NLO expansion up to ε^2. Therefore, in this section, we will make full use of the results derived in Sec. III.

Before proceeding further, we note that there are no additional color structures appearing in the NNLO calculation for gg fusion in addition to the ones already presented in Eq. (3.6): they are just linear combinations of the ones in the NLO case. This is in contrast to the $q\bar{q}$ subprocess, where the NNLO color structures exhibit much higher complexity and richness [15] relative to the NLO ones.

The two most singular terms in Eq. (4.1) are proportional to the Born B and color-reduced Born $\hat{B}$ terms defined in Eqs. (3.3) and (3.4), respectively. One has

$$\begin{aligned} V^{(-4)}(\varepsilon) &= 4N_C^2 B, \\ V^{(-3)}(\varepsilon) &= -2N_C W^{(-1)}(\varepsilon)\,, \end{aligned} \tag{4.2}$$

where $W^{(-1)}(\varepsilon)$ is given in Eq. (3.8) and is nothing but the full coefficient of the single pole NLO result.

For the $1/\varepsilon^2$ term we obtain

$$\begin{aligned} V^{(-2)}(\varepsilon) \,=\, & d_A\hat{B}\Big[\frac{s^2}{tu}|f_\delta|^2 + \frac{1}{2}C_F\Big(\frac{u}{t}|f_t|^2 + \frac{t}{u}|f_u|^2\Big) \\ & -sf_\delta^*\Big(\frac{1}{t}f_t + \frac{1}{u}f_u\Big) + (C_F - \frac{N_C}{2})f_t^* f_u\Big] \\ & -2N_C F_{\rm NLO}^{(0)}\,, \end{aligned} \tag{4.3}$$

where the functions f_δ, f_t and f_u above are the same as those in Eq. (3.9), but now with the imaginary parts retained, i.e. one has the following replacements:

$$\ln\frac{s}{m^2} \to \ln\frac{s}{m^2} - i\pi, \qquad \ln x \to \ln x + i\pi. \tag{4.4}$$

This reflects the fact that, contrary to the NLO calculation, one has to keep the imaginary parts in the NNLO calculation as emphasized in the Introduction. It should be clear that the completion (4.4) has to be done everywhere in the NNLO calculation whenever the logarithms (4.4) appear in bilinear forms multiplying complex functions.

The last term $-2N_C F_{\rm NLO}^{(0)}$ in Eq. (4.3) is obtained from folding the $\mathcal{O}(\varepsilon^{-2})$ singular term of the matrix element with its finite part, while the remaining parts result from folding the single poles. Note that when one substitutes the Laurent expansions for $\hat{B}$ and $F_{\rm NLO}^{(0)}$ one gets additional $1/\varepsilon$ poles and finite terms in Eq. (4.3).

The structure of the fourth term in Eq. (4.1) is somewhat more complicated. One has

$$V^{(-1)}(\varepsilon) = \frac{\beta_0}{2N_C}\ln(\frac{m^2}{\mu^2})V^{(-3)}(\varepsilon) + \mathcal{S}_1^{(0)} - 2N_C W^{(1)}(\varepsilon), \tag{4.5}$$

where we have introduced new functions

$$\mathcal{S}_1^{(j)} = -\frac{d_A}{4}\frac{s}{tu}\Big(L_1^*F_1^{(j)} + L_2^*F_2^{(j)} + L_2^*F_3^{(j)} + (t \leftrightarrow u)\Big), \tag{4.6}$$

with

$$\begin{aligned} L_1 &= 2f_\delta - \frac{u}{s}f_t - \frac{t}{s}f_u\,, \\ L_2 &= 2f_\delta - 2C_F\frac{u}{s}f_t - (2C_F - N_C)\frac{t}{s}f_u\,. \end{aligned} \tag{4.7}$$

The first two terms in Eq. (4.5) arise from folding the single-pole terms in the original matrix element with its finite $\mathcal{O}(\varepsilon^0)$ part. The last term is due to the interference of $\mathcal{O}(\varepsilon^{-2}) \times \mathcal{O}(\varepsilon)$ terms in the original matrix element. This pole term is due to the Laurent series expansion of the original matrix element and cannot be deduced from the knowledge of the NLO terms alone. The function $W^{(1)}(\varepsilon)$ is defined in Eq. (3.14), while the functions $F_1^{(j)}, F_2^{(j)}$ and $F_3^{(j)}$ are given by Eqs. (3.13).

When one substitutes the Laurent expansions for $F_1^{(0)}$, $F_2^{(0)}$, $F_3^{(0)}$ and $W^{(1)}(\varepsilon)$, one gets finite and $\mathcal{O}(\varepsilon)$ terms in Eq. (4.5). However, since we are only interested in the Laurent series expansion up to the finite term, these $\mathcal{O}(\varepsilon)$ contributions can be omitted as before.

V. STRUCTURE OF THE FINITE PART

In this section, we present the finite part of our result. In the course of our calculation, we have made full use of the results presented in Sec. III, e.g. of our detailed study of the NLO structure of the Laurent series expansion up to $O(\varepsilon^2)$. As a consequence, we can present a large part of our results for the finite part in a surprisingly concise and closed form. We decompose the finite part into several pieces, as

$$V^{(0)}(\varepsilon) = {\rm Re}\left[V_{11}^{(0)} + V_{22}^{(0)} + V_{00}^{(0)}\right]. \tag{5.1}$$

The first two terms originate from the interference of the $\mathcal{O}(\varepsilon^{-1}) \times \mathcal{O}(\varepsilon)$ and $\mathcal{O}(\varepsilon^{-2}) \times \mathcal{O}(\varepsilon^2)$ pieces of the initial matrix element. Each of them can be conveniently presented in a very compact form:

$$V_{11}^{(0)} = \frac{d_A}{2}\hat{B}\beta_0 \ln(\frac{m^2}{\mu^2})\Big[-\frac{s^2}{tu}f_\delta + \Big(\frac{s}{t}C_F + \frac{N_C}{2}\Big)f_t + \Big(\frac{s}{u}C_F + \frac{N_C}{2}\Big)f_u\Big] + \mathcal{S}_1^{(1)} + \mathcal{S}_2^{(0)}, \quad (5.2)$$

where we have introduced one more function

$$\mathcal{S}_2^{(j)} = d_A\Big[L_1^* F_4^{(j)} - \Big\{\frac{L_2^*}{2}(F_5^{(j)} + C_F F_6^{(j)} + N_C F_7^{(j)}) + (t \leftrightarrow u)\Big\}\Big], \quad (5.3)$$

Similarly, for the second term in Eq. (5.1), we write

$$V_{22}^{(0)} = -2N_C W^{(2)}(\varepsilon), \quad (5.4)$$

with $W^{(2)}(\varepsilon)$ defined in Eq. (3.17). Note again that the $\mathcal{O}(\varepsilon)$ and $\mathcal{O}(\varepsilon^2)$ terms in the above expressions for $V_{11}^{(0)}$ and $V_{22}^{(0)}$ can be disregarded. We mention that the scalar coefficient functions with the superscript "2" above involve multiple polylogarithms of weight and depth 4.

We emphasize that the quasi-factorized forms of all the expressions given in this paper hold only when one retains the full ε dependence in the Born and NLO terms.

The last term in Eq. (5.1) comes from the square of the $O(\varepsilon^0)$ term of the matrix element which can be written as

$$V_{00}^{(0)} = -\beta_0 \ln(\frac{m^2}{\mu^2})\Big[F_{\rm NLO}^{(0)} - \frac{1}{2}W_2^{(0)}\Big] + Y\,, \quad (5.5)$$

where $F_{\rm NLO}^{(0)}$ and $W_2^{(0)}$ are given in Eqs. (3.11) and (3.12). We found that the last term Y in Eq. (5.5) also possesses the quasi-factorization properties discovered in a recent paper [15]. For instance, the result can also be written down as a sum of bilinear products, where each of the factors are linear combinations of scalar integral coefficient functions multiplied by some combinations of kinematic variables. However, because of the great number of Laurent structures appearing in the original matrix element for the gg fusion subprocess, the length of the final expressions does not allow us to present the results in this paper. Also, we were not able to find the optimal way to organize the different contributions in Y as in Ref. [15], as not all the powers of common numerators and denominators cancel out. Therefore, we have opted to supply the results on the finite term Y in a separate electronic file.

In the finite contribution of Eq. (5.1), one notices the interplay of the product of powers of ε resulting from the Laurent series expansion of the scalar integrals (cf. Eq. (1.1)) on the one hand and powers of ε resulting from doing the spin algebra in dimensional regularization on the other hand. For example, for the finite part one has a contribution from $C_6^{(-1)}B_1^{(0)*}$ as well as a contribution from $C_6^{(-1)}B_1^{(1)*}$. Terms of the type $C_6^{(-1)}B_1^{(0)*}$, where the superscripts corresponding to ε powers do not compensate, would be absent in regularization schemes where traces are effectively taken in four dimensions, i.e. in the so-called four-dimensional schemes or in dimensional reduction.

We emphasize that all our factorized results given in this paper (except for the expression for Y in Eq. (5.5)) take up about 22 Kb of hard disk space. This has to be compared with the length of the original, untreated FORM output. The original computer output for the corresponding one-loop squared cross section of the $gg \to Q\overline{Q}$ subprocess turned out to be very long and took up about 85 MB of hard disk space. Therefore, the reduction is of the order of 10^3–10^4 in the present case.

As a final remark we want to emphasize that we have done two independent calculations using REDUCE [26] and FORM [27] when squaring the one–loop amplitudes. Casting the results into the compact forms presented in this paper was done with the help of the REDUCE Computer Algebra System.

VI. CONCLUSIONS

We have presented analytical $\mathcal{O}(\alpha_s^4)$ NNLO results for the one-loop squared contributions to heavy-quark pair production in the gluon-gluon fusion reaction. The corresponding result for photon-photon fusion has already been presented in Ref. [17], while results for the photon-gluon fusion process can be obtained from Ref. [21] after some color factor adjustments. As concerns hadroproduction of heavy quarks, the results of the present paper, together with a recent publication on $q\bar{q}$ production [15], complete the derivation of the one-loop squared contributions to the hadroproduction of heavy quarks at NNLO with the heavy-quark mass dependence fully retained. Our results form part of the NNLO description of heavy-quark pair production relevant for the NNLO analysis of ongoing experiments at the TEVATRON and the LHC.

A large part of our analytical results are presented in a very compact form. The singular contributions proportional to $\varepsilon^{-4}, \varepsilon^{-3}$ and ε^{-2} are entirely given in terms of LO and NLO contributions, whereas the ε^{-1} contributions contain some true NNLO structure in addition to LO and NLO structures. Since the LO and NLO terms are themselves expanded in Laurent series, this implies that our singular contributions are not true (in a mathematical sense) Laurent series in ε. We believe that our representation of the singular contributions has structural advantages in as much as it will be simpler to match our singular structures onto the singular structures of the other classes of contributions. Also, our representation is convenient if one wants to convert our expressions to different regularization schemes such as DRED (see e.g. Ref. [28]). If needed, our singular contributions can easily be converted into true Laurent series expansions since our expressions are very compact.

Due to our representation of the singular parts, we obtained quasi-factorized expressions for a large part of the finite contributions. Writing our analytical results in factorized forms led to a reduction of the length of the original output by a factor of 10^3–10^4, which will lead to a dramatic reduction of the CPU time needed in numerical evaluations.

The present paper deals with unpolarized gluons in the initial state and unpolarized heavy quarks in the final state. Since our results for the original matrix elements contain the full spin information of the process, an extension to the polarized case with polarization in the initial state and/or in the final state including spin correlations would be possible.

Analytical results in electronic format for the coefficients given in the Appendicies as well as for the term Y in Eq. (5.5) are readily available [29].

Note added: While finalizing our manuscript for publication, we became aware of the preprint arXiv:0809.1355 by C. Anastasiou and S. Mert Aybat, who also discuss the NNLO one-loop squared gluon fusion production of heavy quark pairs.

Acknowledgments

We would like to thank J. Gegelia, A. Kotikov, G. Kramer and O. Veretin for useful discussions. We are very grateful to R.K. Ellis and P. Nason for swift response and for providing the electronic files of their analytical one-loop virtual NLO results. We also acknowledge helpful communications with W. Beenakker, I. Bojak, I. Schienbein, J. Smith and H. Spiesberger. Z.M. would like to thank the Particle Theory group of the Institut für Physik, Universität Mainz for hospitality, where this work has started. The work of Z.M. was supported in part by the German Research Foundation DFG through Grants No. KN 365/7-1 and No. KO 1069/11-1, and by the Georgia National Science Foundation through Grant No. GNSF/ST07/4-196. M.R. was supported by the Helmholtz Gemeinschaft HGF under Contract No. VH-NG-105.

APPENDIX A

First, we write down a few abbreviations that we use throughout the paper:

$$\begin{aligned} \beta &= \sqrt{1-4m^2/s}, \qquad D = m^2 s - tu, \\ z_2 &= s+2t, \qquad z_{2u} = s+2u, \\ z_t &= 2m^2+t, \qquad z_u = 2m^2+u. \end{aligned} \tag{A1}$$

Note that D in Eq. (A1) is *not* the space-time dimension.

Here we present the expressions for all the coefficients a_I, b_I, c_I appearing in Eqs. (3.13):

$$\begin{aligned}
a_{B_2} &= 16D/(s\beta^2)\ , \\
a_{B_5} &= -a_{B_2}\ , \\
a_{C_1} &= 4(8m^4 - z_2^2/s(2m^2 - s + 2m^2/\beta^2))\ , \\
a_{C_2} &= 8t/s(4m^2 z_t + 2st + t^2)\ , \\
a_{C_{2u}} &= a_{C_2}(t \leftrightarrow u)\ , \\
a_{C_3} &= 8t/s(4m^2 z_t + t z_2)\ , \\
a_{C_{3u}} &= a_{C_3}(t \leftrightarrow u)\ , \\
a_{C_4} &= 4(4m^2 s + 3s^2 - 8tu)\ , \\
a_{C_5} &= 4(8m^4 - 3s^2 + 2tu)\ , \qquad (A2) \\
a_{C_6} &= -4\beta^2(2m^2 s + s^2 + 2tu)\ , \\
a_{D_1} &= 4(2m^2(2D + s z_t\beta^2 - t^2\beta^2) + s^2 t\beta^2 + t^3)\ , \\
a_{D_{1u}} &= a_{D_1}(t \leftrightarrow u)\ , \\
a_{D_2} &= 4(8m^2 D - stu\beta^2 + 2t^2/s(t^2+u^2))\ , \\
a_{D_{2u}} &= a_{D_2}(t \leftrightarrow u)\ , \\
a_{D_3} &= 8(8m^2 D - 8m^4 tu/s - stu\beta^2 + 2t^2u^2/s)\ ;
\end{aligned}$$

$$\begin{aligned}
a^{(\varepsilon)}_{B_2} &= 4(2s - z_2^2/(s\beta^2))\ , \\
a^{(\varepsilon)}_{B_5} &= -a^{(\varepsilon)}_{B_2}\ , \\
a^{(\varepsilon)}_{C_1} &= 2(\beta^2(s^3(8m^2+s) - 8t^2u^2)/D \\
&\quad + 16m^2/s(s^2 + tu + D/\beta^2)\ , \\
a^{(\varepsilon)}_{C_2} &= -4t^2(10 - t/s(2tu\beta^2 + 2s^2 - 3t^2)/D)\ , \\
a^{(\varepsilon)}_{C_{2u}} &= a^{(\varepsilon)}_{C_2}(t \leftrightarrow u)\ , \\
a^{(\varepsilon)}_{C_3} &= -4t^2(6 - t/s(2tu\beta^2 + 2s^2 - 4st - 5t^2)/D)\ , \\
a^{(\varepsilon)}_{C_{3u}} &= a^{(\varepsilon)}_{C_3}(t \leftrightarrow u)\ , \\
a^{(\varepsilon)}_{C_4} &= -2(s^3(2m^2 - s) - 8tu(m^2 s + t^2 + u^2))/D\ , \\
a^{(\varepsilon)}_{C_5} &= 2(\beta^2(s^3(6m^2 - s) + 4t^2u^2)/D \qquad (A3) \\
&\quad + 8s^2 - 12m^4 z_2^2/D)\ , \\
a^{(\varepsilon)}_{C_6} &= 2(\beta^2(s^3(8m^2 - s) - 4t^2u^2)/D \\
&\quad + 8m^2 s - 4m^4 z_2^2/D)\ , \\
a^{(\varepsilon)}_{D_1} &= -2t(2s^2\beta^2 - s^2 t\beta^2(2m^2 z_2^2/s^2 + 4m^2 + t)/D \\
&\quad + 2t z_t + 2s^2)\ , \\
a^{(\varepsilon)}_{D_{1u}} &= a^{(\varepsilon)}_{D_1}(t \leftrightarrow u)\ , \\
a^{(\varepsilon)}_{D_2} &= -2t(2s(s-u) + t^2(s^2 + 8tu - 8u^3/s)/D)\ , \\
a^{(\varepsilon)}_{D_{2u}} &= a^{(\varepsilon)}_{D_2}(t \leftrightarrow u)\ ,
\end{aligned}$$

$$a_{D_3}^{(\varepsilon)} = 4tu(4s - t^2u^2/s^2(8m^2-7s)/D) ;$$

$$\begin{aligned}
a_{B_2}^{(\varepsilon^2)} &= 0 , \\
a_{B_5}^{(\varepsilon^2)} &= 0 , \\
a_{C_1}^{(\varepsilon^2)} &= 8s(s - 2m^2z_2^2/D) , \\
a_{C_2}^{(\varepsilon^2)} &= -8t^2(3u/s + t(m^2-u)/D) , \\
a_{C_{2u}}^{(\varepsilon^2)} &= a_{C_2}^{(\varepsilon^2)}(t \leftrightarrow u) , \\
a_{C_3}^{(\varepsilon^2)} &= 8t^2(2 + tu(1-3t/s)/D) , \\
a_{C_{3u}}^{(\varepsilon^2)} &= a_{C_3}^{(\varepsilon^2)}(t \leftrightarrow u) , \\
a_{C_4}^{(\varepsilon^2)} &= -16s^2tu/D , \qquad \text{(A4)} \\
a_{C_5}^{(\varepsilon^2)} &= -4s(s + 2m^2z_2^2/D) , \\
a_{C_6}^{(\varepsilon^2)} &= 4s(s - 2m^2z_2^2/D) , \\
a_{D_1}^{(\varepsilon^2)} &= -4st(2m^2 - s + t(\beta^2tu + m^2z_2^2/s)/D) , \\
a_{D_{1u}}^{(\varepsilon^2)} &= a_{D_1}^{(\varepsilon^2)}(t \leftrightarrow u) , \\
a_{D_2}^{(\varepsilon^2)} &= 4st(s - 4t^2u/D) , \\
a_{D_{2u}}^{(\varepsilon^2)} &= a_{D_2}^{(\varepsilon^2)}(t \leftrightarrow u) , \\
a_{D_3}^{(\varepsilon^2)} &= -8tu(s + 3t^2u^2/(sD)) ;
\end{aligned}$$

$$\begin{aligned}
b_1 &= -16/3z_2/s(m^2(n_l+1) + (2C_F - N_C)3D/(s\beta^2) \\
&\quad - N_C(m^2 + D6(10m^2-s)/(s^2\beta^4))) , \\
b_{B_2} &= -8z_2/s^2(8m^4 - (2C_F - N_C)D(2 + 1/\beta^2)) , \\
b_{B_5} &= -N_C8z_2(D(16m^2-s)/(s\beta^4) + tu)/s^2 , \\
b_{C_1} &= -N_C16m^2Dz_2(8m^2+s)/(s^3\beta^4) , \\
b_{C_4} &= N_C4z_2(D - 2tu)/s , \qquad \text{(A5)} \\
b_{C_5} &= -32m^4z_2/s , \\
b_{C_6} &= -(2C_F - N_C)16Dz_2(2m^2-s)/s^2 ;
\end{aligned}$$

$$\begin{aligned}
b_1^{(\varepsilon)} &= 16/3\, z_2(tu(n_l+1) + (2C_F - N_C)3D/\beta^2 \\
&\quad - N_C(36m^2D/(s\beta^4) - tu(4m^2-7s)/(s\beta^2))))/s^2 , \\
b_{B_2}^{(\varepsilon)} &= 8z_2(8m^2tu/s + (2C_F - N_C)(2tu - D/\beta^2))/s^2 , \\
b_{B_5}^{(\varepsilon)} &= N_C8z_2(3m^2z_2^2/(s\beta^4) - 2(D + 2m^2tu/s)/\beta^2)/s^2 , \\
b_{C_1}^{(\varepsilon)} &= N_C16m^2z_2(3D/\beta^4 + 2tu/\beta^2)/s^2 , \\
b_{C_4}^{(\varepsilon)} &= N_C12tuz_2/s , \qquad \text{(A6)} \\
b_{C_5}^{(\varepsilon)} &= 32m^2tuz_2/s^2 , \\
b_{C_6}^{(\varepsilon)} &= -(2C_F - N_C)16tuz_2(2m^2-s)/s^2 ;
\end{aligned}$$

$$\begin{aligned}
c_1 &= 16(C_F(D\beta^2(8m^2T/t^2 + 2) - D(6z_t/t - 2 - t/s) \\
&\quad + 2m^2(4z_t(m^2/s - 1) - m^2) - D(1 + 4t/s)/\beta^2) \\
&\quad - N_C(D2m^2(2D + tu)/(st^2) - 2m^2tu/s \\
&\quad - D4m^2(s + 4t)/(s^2\beta^2)))/T , \\
c_{B_1} &= 16(C_F(2m^2\beta^2(T - 2s - D(2T + t)/t^2) \\
&\quad + D(3z_t/t + t/s) - 2m^2u(2 + 5t/s))/T \\
&\quad + N_C2D(D/s - t)/t^2) , \\
c_{B_2} &= (2C_F - N_C)16D/(s\beta^2) , \qquad \text{(A7)} \\
c_{B_5} &= N_C8(-8m^2D/(s^2\beta^2) - t\beta^2 + t^2z_2/s^2) , \\
c_{C_1} &= N_C8(t^3 + u^3 - 4t^2T - sD/\beta^2 - s^2\beta^2(m^2 - t))/s , \\
c_{C_2} &= -(2C_F - N_C)16(2m^2z_2(m^2s/t - z_t) \\
&\quad + t(s^2 + t^2))/s , \\
c_{C_3} &= N_C16t(4D/s - t\beta^2 + s) , \\
c_{C_4} &= N_C4(-s^2\beta^2 + 3z_2(m^2s - t^2)/s - 3su + 2t^2) , \\
c_{C_5} &= (2C_F - N_C)8(2T(2m^2 + s) - u^2) , \\
c_{C_6} &= -(2C_F - N_C)8(4m^2D/s - 4m^2t\beta^2 + 3tz_t - z_2^2) , \\
c_{D_1} &= -(2C_F - N_C)8(m^2s^2\beta^4 - 2m^2t\beta^2(s - t) + st^2\beta^2 \\
&\quad - t^3 - sD) , \\
c_{D_2} &= N_C8(8m^2D - stu\beta^2 + 2t^2(t^2 + u^2)/s) ;
\end{aligned}$$

$$\begin{aligned}
c_1^{(\varepsilon)} &= 16(C_F(D(16m^2D/(st^2) - 24m^4/t^2 + 4 - t/s \\
&\quad + 2t/(s\beta^2)) + 2m^2(4m^2 - 6t - 9t^2/s) \\
&\quad + 4m^2t^2z_2/(s^2\beta^2))/T \\
&\quad + N_C2(2m^4s/t^2 + t + Dz_2/(st) \\
&\quad - D(4m^2 + 3s)/(s^2\beta^2) + tz_2/(s\beta^2))) , \\
c_{B_1}^{(\varepsilon)} &= 16(C_F(4m^4D/t^2 - 6TD/t - 2m^2D/s - tD/s \\
&\quad - 5m^2z_t + t^2) \\
&\quad - N_C(2m^2D^2/(st^2) + 2tD/s - m^4 + t^2))/T , \\
c_{B_2}^{(\varepsilon)} &= -(2C_F - N_C)8t(2 + z_2/(s\beta^2)) , \qquad \text{(A8)} \\
c_{B_5}^{(\varepsilon)} &= N_C8(2D/\beta^2 + 6m^2z_2/\beta^2 - 3t^2 - 2t^3/s)/s , \\
c_{C_1}^{(\varepsilon)} &= -N_C4(2m^2z_2^2/s - 4s^2 - 4t^2 - 4m^2(4z_t \\
&\quad + tz_2/s)/\beta^2 + 2tuz_t(4s + 3t^2/s + u^2/s)/D
\end{aligned}$$

$$+ st^2(2t\beta^2 - z_2)/D) \,,$$

$$c_{C_2}^{(\varepsilon)} = (2C_F - N_C)8(\beta^2 t(6sD - 4m^2tu - st^2) + 2D(2m^4 s/t - sz_t + Dt/s - t^2) - 4m^2t^3 z_2/s)/D \,,$$

$$c_{C_3}^{(\varepsilon)} = -N_C 8t^2(4s/t + 14 - st(4\beta^2 - 8tT/s^2 + 5)/D),$$

$$c_{C_4}^{(\varepsilon)} = -N_C 4(2s^2 + 2t^3/s - 2su + m^2 st(9s + 7t)/D - t^4(9 + 8t/s)/D) \,,$$

$$c_{C_5}^{(\varepsilon)} = -(2C_F - N_C)4(2m^2 z_2^2/s - 2t^2 - 4s\beta^2(s - m^2tu/D) + t^2(8m^2t + s^2)/D) \,,$$

$$c_{C_6}^{(\varepsilon)} = -(2C_F - N_C)4st(7\beta^2 + 2u^2\beta^2/(st) + 2s/t + 5 + \beta^2(2m^2 z_2^2/s - 3st - 4t^2)/D) \,,$$

$$c_{D_1}^{(\varepsilon)} = -(2C_F - N_C)4st^2(2s\beta^2/t + 2s/t + 2t/s + 4m^4 z_2^2/(s^2 D) - \beta^2(6m^2 s - 4m^2tu/s + st)/D),$$

$$c_{D_2}^{(\varepsilon)} = -N_C 4t(4s^2 + 2st + t^2(z_2^2 + 12tu - 8u^3/s)/D) \,;$$

$$c_1^{(\varepsilon^2)} = -16(C_F(2m^2(6D/t + 4m^2 + t) - 4m^2(3D + tz_2)/(s\beta^2) + D/\beta^2)/T - N_C 2(u - 4m^2 z_u/(s\beta^2))) \,,$$

$$c_{B_1}^{(\varepsilon^2)} = 16(C_F z_t(3D/t + 2m^2)/T - N_C 2m^2) \,,$$

$$c_{B_2}^{(\varepsilon^2)} = -(2C_F - N_C)32m^2 z_2/(s\beta^2) \,,$$

$$c_{B_5}^{(\varepsilon^2)} = -N_C 64m^2 z_2/(s\beta^2) \,,$$

$$c_{C_1}^{(\varepsilon^2)} = -N_C 8s(4t + 2m^2t(s + 4t + z_2^2/(s\beta^2))/D + s/\beta^2) \,,$$

$$c_{C_2}^{(\varepsilon^2)} = (2C_F - N_C)16(2m^2 s + tu + 2m^2t(m^2 s - t^2)/D) \,,$$

$$c_{C_3}^{(\varepsilon^2)} = N_C 16t(s + 2t(m^2 s + tu)/D) \,,$$

$$c_{C_4}^{(\varepsilon^2)} = N_C 8s(s + 2t(m^2 s + tu)/D) \,, \qquad \text{(A9)}$$

$$c_{C_5}^{(\varepsilon^2)} = (2C_F - N_C)8s(u - 2m^2tz_2/D) \,,$$

$$c_{C_6}^{(\varepsilon^2)} = -(2C_F - N_C)8(2m^2 z_2(m^2 s - t^2)/D + su) \,,$$

$$c_{D_1}^{(\varepsilon^2)} = -(2C_F - N_C)8t(sz_u + 2m^2tz_2^2/D) \,,$$

$$c_{D_2}^{(\varepsilon^2)} = N_C 8st(s - 4t^2u/D) \,.$$

APPENDIX B

In this Appendix, we present the expressions for all the coefficients d_I, e_I, g_I, h_I appearing in Eq. (3.16):

$$d_{B_2}^{(\varepsilon)} = 2s(4m^2 + z_2^2/(s\beta^2))/(tu) \,,$$

$$d_{B_5}^{(\varepsilon)} = -d_{B_2}^{(\varepsilon)} \,,$$

$$d_{C_1}^{(\varepsilon)} = s(s^2\beta^2(8m^2 s + s^2 + 2D) + 4s^2 D - 16m^2 D^2/(s\beta^2) - 8m^4 z_2^2)/(tuD) \,,$$

$$d_{C_2}^{(\varepsilon)} = 2t(2u(D + m^2 s) + st^2 + \kappa_d t/s)/(uD) \,,$$

$$d_{C_{2u}}^{(\varepsilon)} = d_{C_2}^{(\varepsilon)}(t \leftrightarrow u) \,,$$

$$d_{C_3}^{(\varepsilon)} = -2t(2m^2 s^2 + st^2 - \kappa_d t/s)/(uD) \,,$$

$$d_{C_{3u}}^{(\varepsilon)} = d_{C_3}^{(\varepsilon)}(t \leftrightarrow u) \,, \qquad \text{(B1)}$$

$$d_{C_4}^{(\varepsilon)} = s^2(\kappa_d + 3sD - s^2(m^2 - s))/(tuD) \,,$$

$$d_{C_5}^{(\varepsilon)} = -s(\kappa_c + 2m^2 s z_2^2)/(tuD) \,,$$

$$d_{C_6}^{(\varepsilon)} = -s\kappa_c/(tuD) \,,$$

$$d_{D_1}^{(\varepsilon)} = st(z_2 + \beta^2(\kappa_d - s^2(m^2 - t))/D)/u \,,$$

$$d_{D_{1u}}^{(\varepsilon)} = d_{D_1}^{(\varepsilon)}(t \leftrightarrow u) \,,$$

$$d_{D_2}^{(\varepsilon)} = st(\kappa_d + sD + s^2(m^2 - t))/(uD) \,,$$

$$d_{D_{2u}}^{(\varepsilon)} = d_{D_2}^{(\varepsilon)}(t \leftrightarrow u) \,,$$

$$d_{D_3}^{(\varepsilon)} = 2tu\kappa_d/(sD) \,,$$

with

$$\kappa_c = 4D^2 - s\beta^2(8m^2 s^2 - 8m^2tu - s^3) \,,$$

$$\kappa_d = 10m^2 s^2 - 8m^2tu - 3stu \,;$$

$$d_{B_2}^{(\varepsilon^2)} = -8m^2 z_2^2/(stu\beta^2) \,,$$

$$d_{B_5}^{(\varepsilon^2)} = -d_{B_2}^{(\varepsilon^2)} \,,$$

$$d_{C_1}^{(\varepsilon^2)} = -s((22m^2 s^2 - 16m^2tu + s^3)s\beta^2 + 4m^2 sD - 16m^2 D^2/(s\beta^2))/(tuD) \,,$$

$$d_{C_2}^{(\varepsilon^2)} = 2t(6sD + 4m^2 s z_2 + t^2 z_2 - \kappa_d t/s)/(uD) \,,$$

$$d_{C_{2u}}^{(\varepsilon^2)} = d_{C_2}^{(\varepsilon^2)}(t \leftrightarrow u) \,,$$

$$d_{C_3}^{(\varepsilon^2)} = -2t(\kappa_d t/s - s(2m^2 s - 2st - t^2))/(uD) \,,$$

$$d_{C_{3u}}^{(\varepsilon^2)} = d_{C_3}^{(\varepsilon^2)}(t \leftrightarrow u)\,, \tag{B2}$$
$$d_{C_4}^{(\varepsilon^2)} = -s^2(\kappa_d + sD + s^2(m^2+s))/(tuD)\,,$$
$$d_{C_5}^{(\varepsilon^2)} = -s(\kappa_c - 2m^2 s(4D + z_2^2))/(tuD)\,,$$
$$d_{C_6}^{(\varepsilon^2)} = -s\kappa_c/(tuD)\,,$$
$$d_{D_1}^{(\varepsilon^2)} = st(2uD - \beta^2(\kappa_d - 4stu - st^2))/(uD)\,,$$
$$d_{D_{1u}}^{(\varepsilon^2)} = d_{D_1}^{(\varepsilon^2)}(t \leftrightarrow u)\,,$$
$$d_{D_2}^{(\varepsilon^2)} = -st(\kappa_d + s(2m^2 s - 2st - t^2))/(uD)\,,$$
$$d_{D_{2u}}^{(\varepsilon^2)} = d_{D_2}^{(\varepsilon^2)}(t \leftrightarrow u)\,,$$
$$d_{D_3}^{(\varepsilon^2)} = -2tu\kappa_d/(sD)\,,$$

with

$$\kappa_c = 4tuD + s\beta^2(18m^2s^2 - 16m^2tu - s^3)\,,$$
$$\kappa_d = 18m^2s^2 - 16m^2tu + stu\,;$$

$$e_1^{(\varepsilon)} = 2s(n_l+1)\kappa_1\kappa_2\,, \qquad e_{B_2}^{(\varepsilon)} = 3(8m^2+s)\kappa_1\kappa_2\,,$$
$$e_{B_5}^{(\varepsilon)} = 3sn_l\kappa_1\kappa_2\,, \qquad e_{C_5}^{(\varepsilon)} = 18m^2 s\kappa_1\kappa_2\,,$$
$$e_1^{(\varepsilon^2)} = e_1^{(\varepsilon)}/\kappa_2\,, \qquad e_{B_2}^{(\varepsilon^2)} = e_{B_2}^{(\varepsilon)}/\kappa_2\,, \tag{B3}$$
$$e_{B_5}^{(\varepsilon^2)} = e_{B_5}^{(\varepsilon)}/\kappa_2\,, \qquad e_{C_5}^{(\varepsilon^2)} = e_{C_5}^{(\varepsilon)}/\kappa_2\,,$$

with

$$\kappa_1 = 8z_2/(9s^2)\,,$$
$$\kappa_2 = -m^2 s/(tu)\,.$$

Next, we introduce common factors that appear in the various coefficients g_I and h_I. They are multiplied by one power of ε and read:

$$\begin{aligned} s_{b2} &= 2m^2 s - tz_2^2/(s\beta^2)\,, \\ s_{c2} &= ts_{c5} - 4m^2 suD\,, \\ s_{c5} &= 2D(D + s(8m^2+t)) + 2st\beta^2(2m^2u - t^2) + st^2 z_2\,, \\ s_{c6} &= D(3s\beta^2 + z_2) + \beta^2(6m^2s^2 - 8m^2tu + s^2t)\,. \end{aligned} \tag{B4}$$

For the coefficients $g_I^{(\varepsilon)}$, we have

$$\begin{aligned} g_1^{(\varepsilon)} = {} & 8(2m^2 s\beta^2(4sT^2/t + tz_t - 2tu) + D(10m^2u - 5sz_t \\ & - 2t^2) - 2m^2t(s^2+u^2) - D^2 6t/(s\beta^2) \\ & - 3Dt^2z_2/(s\beta^2))/(t^2uT)\,, \end{aligned}$$
$$g_{B_1}^{(\varepsilon)} = -8(D(4m^2u - st) - t^3(2s\beta^2 + 3z_t))/(t^2uT)\,,$$
$$g_{B_2}^{(\varepsilon)} = -8s_{b2}/(tu)\,, \qquad g_{C_2}^{(\varepsilon)} = 8s_{c2}/(Dtu)\,, \tag{B5}$$
$$g_{C_5}^{(\varepsilon)} = 4ss_{c5}/(Dtu), \quad g_{C_6}^{(\varepsilon)} = 4ss_{c6}/(Du), \quad g_{D_1}^{(\varepsilon)} = -tg_{C_6}^{(\varepsilon)}.$$

Finally, we introduce factors that are common to various coefficients g_I and h_I that are multiplied by two powers of ε:

$$\begin{aligned} c_{b2} &= 2m^2u + D\,, \\ c_{c2} &= 2D(4m^2su - tD - st(17m^2+3t)) \\ & \quad - 4st^2\beta^2(2m^2u - t^2) + st^2(3sz_t + 4m^2z_2)\,, \\ c_{c5} &= 2D(20m^2s - st + t^2) + 2st\beta^2(4m^2u + st - 2t^2) \\ & \quad + 5st^2z_2\,, \\ c_{c6} &= 2D(s\beta^2 - u) + \beta^2(16m^2(s^2 - tu) + s^2t)\,. \end{aligned} \tag{B6}$$

For the coefficients $g_I^{(\varepsilon^2)}$, we have

$$\begin{aligned} g_1^{(\varepsilon^2)} = {} & 8(D^2 12t/(s\beta^2) + D^2 16m^2/t + D4(2sz_t - tu) \\ & - Dt(14m^2+3t)/\beta^2 - 2m^2(12m^2s^2T/t + 5t^3) \\ & - 4m^2t^2z_2/\beta^2)/(t^2uT)\,, \end{aligned}$$
$$g_{B_1}^{(\varepsilon^2)} = 8(D(4z_t/t^2 + 1/u) + 2m^2(t/u - 2))/T\,,$$
$$g_{B_2}^{(\varepsilon^2)} = -16z_2c_{b2}/(stu\beta^2)\,, \qquad g_{C_2}^{(\varepsilon^2)} = 8c_{c2}/(Dtu)\,,$$
$$g_{C_5}^{(\varepsilon^2)} = -4sc_{c5}/(Dtu)\,, \qquad g_{C_6}^{(\varepsilon^2)} = -4sc_{c6}/(Du)\,,$$
$$g_{D_1}^{(\varepsilon^2)} = -tg_{C_6}^{(\varepsilon^2)}\,. \tag{B7}$$

For the remaining coefficients h_I, we get

$$\begin{aligned} h_1^{(\varepsilon)} = {} & 8(tz_2(2m^2 + D/(s\beta^2))/\beta^2 - m^2(4sD/t - 2s^2 \\ & + tz_2/9))/(t^2u)\,, \end{aligned}$$
$$h_{B_1}^{(\varepsilon)} = -8(3m^2s + tz_2)/(tu)\,,$$
$$h_{B_2}^{(\varepsilon)} = 4s_{b2}/(tu)\,, \tag{B8}$$
$$\begin{aligned} h_{B_5}^{(\varepsilon)} = {} & 4(2D(8m^2t + s^2)/(s^2\beta^4) + 4/3m^2(s-u) \\ & - sz_t/\beta^2)/(tu)\,, \end{aligned}$$
$$\begin{aligned} h_{C_1}^{(\varepsilon)} = {} & -2s(s^2\beta^4t/D + 2s\beta^2(2m^2z_2 - tu)/D \\ & - 8m^2s^2z_t/(tD) - D6/t + 8t + 2tz_2/(s\beta^2) \\ & - D4z_t/(st\beta^4))/u\,, \end{aligned}$$
$$h_{C_2}^{(\varepsilon)} = -4s_{c2}/(Dtu)\,, \qquad h_{C_3}^{(\varepsilon)} = 2t/sh_{C_4}^{(\varepsilon)}\,,$$
$$h_{C_4}^{(\varepsilon)} = 2s(8m^2u^2/D + 4s - st(8m^2+s)/D)/u\,,$$
$$h_{C_5}^{(\varepsilon)} = -2ss_{c5}/(Dtu)\,, \qquad h_{C_6}^{(\varepsilon)} = -2ss_{c6}/(Du)\,,$$
$$h_{D_1}^{(\varepsilon)} = -th_{C_6}^{(\varepsilon)}\,, \qquad h_{D_2}^{(\varepsilon)} = -th_{C_4}^{(\varepsilon)}\,;$$

$$\begin{aligned} h_1^{(\varepsilon^2)} = {} & 8(4m^4s^2/t - s^2z_t - 10/9st^2 - t^3/3 - 2/9t^4/s \\ & - Dtz_2/(s\beta^4) + t(uz_2 + 8m^2/sD)/\beta^2)/(t^2u)\,, \end{aligned}$$
$$h_{B_1}^{(\varepsilon^2)} = 16(Dz_2 + t^2u)/(t^2u)\,,$$

$$h_{B_2}^{(\varepsilon^2)} = 8z_2c_{b2}/(stu\beta^2) , \quad \text{(B9)}$$

$$h_{B_5}^{(\varepsilon^2)} = -8z_2(4m^2D/(s^2\beta^4) + tu/(3s) - 2m^2u/(s\beta^2))/(tu) ,$$

$$h_{C_1}^{(\varepsilon^2)} = -2s(40m^2s/t + (8m^2su\beta^2 + 20m^2su + 16m^2t^2 - s^2t)/D + 2(2m^2s^2/t + 4m^2t - s^2)/(s\beta^2) + 8m^2D/(st\beta^4) + 4m^2(1/s^2 + \beta^2/D)z_2^2/\beta^4)/u,$$

$$h_{C_2}^{(\varepsilon^2)} = -4c_{c2}/(Dtu) , \qquad h_{C_3}^{(\varepsilon^2)} = 2t/sh_{C_4}^{(\varepsilon^2)} ,$$

$$h_{C_4}^{(\varepsilon^2)} = -2s(20m^2s^2 + 4stu\beta^2 + stz_2)/(Du) ,$$

$$h_{C_5}^{(\varepsilon^2)} = 2sc_{c5}/(Dtu) , \qquad h_{C_6}^{(\varepsilon^2)} = 2sc_{c6}/(Du) ,$$

$$h_{D_1}^{(\varepsilon^2)} = -th_{C_6}^{(\varepsilon^2)} , \qquad h_{D_2}^{(\varepsilon^2)} = -th_{C_4}^{(\varepsilon^2)} .$$

APPENDIX C

In this Appendix, we present the expressions for all the coefficients k_I, l_I, m_I, n_I appearing in Eq. (3.19) using the following abbreviations:

$$\kappa_{c1} = -s\beta^2(18m^2s^2 - s^3 + 2(2m^2 + s)z_2^2) - 8m^2sD ,$$
$$\kappa_{c6} = -s\beta^2z_2^2 + 16sD + 6stu . \quad \text{(C1)}$$

We have:

$$k_{C_1}^{(\varepsilon^2)} = -s\kappa_{c1}/(tuD) ,$$

$$k_{C_2}^{(\varepsilon^2)} = 2t(\kappa_{c6}t/s + 2m^2su - s^2z_t + t^3)/(uD) ,$$

$$k_{C_{2u}}^{(\varepsilon^2)} = k_{C_2}^{(\varepsilon^2)}(t \leftrightarrow u) ,$$

$$k_{C_3}^{(\varepsilon^2)} = 2t^2(\kappa_{c6}/s + 2s^2 + u^2)/(uD) ,$$

$$k_{C_{3u}}^{(\varepsilon^2)} = k_{C_3}^{(\varepsilon^2)}(t \leftrightarrow u) , \quad \text{(C2)}$$

$$k_{C_4}^{(\varepsilon^2)} = s^2(\kappa_{c6} + 2s(u^2 - st))/(tuD) ,$$

$$k_{C_5}^{(\varepsilon^2)} = -s(\kappa_{c1} - 2s^2(2D - tu\beta^2 - z_2^2))/(tuD) ,$$

$$k_{C_6}^{(\varepsilon^2)} = s^2\beta^2\kappa_{c6}/(tuD) ,$$

$$k_{D_1}^{(\varepsilon^2)} = st\beta^2(\kappa_{c6} - suz_2)/(uD) ,$$

$$k_{D_{1u}}^{(\varepsilon^2)} = k_{D_1}^{(\varepsilon^2)}(t \leftrightarrow u) ,$$

$$k_{D_2}^{(\varepsilon^2)} = st(\kappa_{c6} + s^3 - s^2t)/(uD) ,$$

$$k_{D_{2u}}^{(\varepsilon^2)} = k_{D_2}^{(\varepsilon^2)}(t \leftrightarrow u) ,$$

$$k_{D_3}^{(\varepsilon^2)} = 2tu(\kappa_{c6}/s - st + u^2)/D ;$$

$$l_1^{(\varepsilon^2)} = 4/3s(n_l + 1)\kappa_1\kappa_2 , \qquad l_{B_2}^{(\varepsilon^2)} = (16m^2 + 5s)\kappa_1\kappa_2 ,$$

$$l_{B_5}^{(\varepsilon^2)} = 5sn_l\kappa_1\kappa_2 , \qquad l_{C_5}^{(\varepsilon^2)} = 18m^2s\kappa_1\kappa_2 ; \quad \text{(C3)}$$

$$m_1^{(\varepsilon^2)} = 32(2m^2(2m^2u^2/t^2 - s^2/t + 2t - 8DT/t^2) - s\beta^2(2D/t + m^2) + D4U/(s\beta^2))/(tu) ,$$

$$m_{C_2}^{(\varepsilon^2)} = 2z_t/(s\beta^2)m_{C_6}^{(\varepsilon^2)} ,$$

$$m_{C_5}^{(\varepsilon^2)} = z_t/(t\beta^2)m_{C_6}^{(\varepsilon^2)} , \quad \text{(C4)}$$

$$m_{C_6}^{(\varepsilon^2)} = 4s\beta^2(\kappa_{c6}/u - sz_2)/D ,$$

$$m_{D_1}^{(\varepsilon^2)} = -tm_{C_6}^{(\varepsilon^2)} ;$$

$$n_1^{(\varepsilon^2)} = -16m^2(18s^2z_t/t^2 + 82/3s + 2/3t - 9sz_2/(s\beta^2) + 144z_uD/(s^2\beta^4))/(9tu) ,$$

$$n_{B_5}^{(\varepsilon^2)} = 16m^2z_2/(9tu) ,$$

$$n_{C_1}^{(\varepsilon^2)} = z_t/t\, n_{C_4}^{(\varepsilon^2)} ,$$

$$n_{C_2}^{(\varepsilon^2)} = 2z_t/(s\beta^2)n_{C_6}^{(\varepsilon^2)} ,$$

$$n_{C_3}^{(\varepsilon^2)} = 2t/s\, n_{C_4}^{(\varepsilon^2)} , \quad \text{(C5)}$$

$$n_{C_4}^{(\varepsilon^2)} = 2s(\kappa_{c6} + s^3 - s^2t)/(Du) ,$$

$$n_{C_5}^{(\varepsilon^2)} = z_t/(t\beta^2)n_{C_6}^{(\varepsilon^2)} ,$$

$$n_{C_6}^{(\varepsilon^2)} = -2s\beta^2(\kappa_{c6} - suz_2)/(Du) ,$$

$$n_{D_1}^{(\varepsilon^2)} = -tn_{C_6}^{(\varepsilon^2)} , \qquad n_{D_2}^{(\varepsilon^2)} = -tn_{C_4}^{(\varepsilon^2)} .$$

[1] P. Nason, S. Dawson and R. K. Ellis, Nucl. Phys. **B303**, 607 (1988); Nucl. Phys. **B327**, 49 (1989); **B335**, 260(E) (1990).

[2] W. Beenakker, H. Kuijf, W. L. van Neerven, and J. Smith, Phys. Rev. D **40**, 54 (1989); W. Beenakker, W. L. van Neerven, R. Meng, G. A. Schuler and J. Smith, Nucl. Phys. **B351**, 507 (1991).

[3] B.A. Kniehl, G. Kramer, I. Schienbein, and H. Spiesberger, Phys. Rev. D **77**, 014011 (2008);

[4] D. Acosta *et al.* (CDF Collaboration), Phys. Rev. D **71**,

032001 (2005); A. Abulencia *et al.* (CDF Collaboration), Phys. Rev. D **75**, 012010 (2007).

[5] B.A. Kniehl, G. Kramer, I. Schienbein, and H. Spiesberger, Phys. Rev. Lett. **96**, 012001 (2006).

[6] D. Chakraborty, J. Konigsberg and D. L. Rainwater, Ann. Rev. Nucl. Part. Sci. **53**, 301 (2003).

[7] S. Moch and P. Uwer, Phys. Rev. D **78**, 034003 (2008).

[8] E. W. N. Glover, J. High Energy Phys. 04 (2004) 021.

[9] W. Bernreuther, R. Bonciani, T. Gehrmann, R. Heinesch, T. Leineweber, P. Mastrolia and E. Remiddi, Nucl. Phys. **B706**, 245 (2005).

[10] M. Czakon, Phys. Lett. B**664**, 307 (2008).

[11] R. Bonciani, A. Ferroglia, T. Gehrmann, D. Maître and C. Studerus, J. High Energy Phys. 07 (2008) 129.

[12] S. Dittmaier, P. Uwer, and S. Weinzierl, Phys. Rev. Lett. **98**, 262002 (2007).

[13] M. Czakon, A. Mitov and S. Moch, Phys. Lett. B**651**, 147 (2007); Nucl. Phys. **B798**, 210 (2008).

[14] M. Cacciari, S. Frixione, M.L. Mangano, P. Nason and G. Ridolfi, arXiv:0804.2800; N. Kidonakis and R. Vogt, arXiv:0805.3844.

[15] J.G. Körner, Z. Merebashvili and M. Rogal, Phys. Rev. D **77**, 094011 (2008).

[16] C.G. Bollini and J.J. Giambiagi, Phys. Lett. **40B**, 566 (1972); G. 't Hooft and M. Veltman, Nucl. Phys. **B44**, 189 (1972); J.F. Ashmore, Lett. Nuovo Cimento **4**, 289 (1972).

[17] J.G. Körner, Z. Merebashvili, and M. Rogal, Phys. Rev. D **74**, 094006 (2006).

[18] J.G. Körner, Z. Merebashvili, and M. Rogal, Phys. Rev. D **71**, 054028 (2005).

[19] J.G. Körner, Z. Merebashvili and M. Rogal, J. Math. Phys. **47**, 072302 (2006).

[20] J. G. Körner and Z. Merebashvili, Phys. Rev. D **66**, 054023 (2002).

[21] J.G. Körner, Z. Merebashvili, and M. Rogal, Phys. Rev. D **73**, 034030 (2006).

[22] J. Vollinga and S. Weinzierl, Comput. Phys. Commun. **167**, 177 (2005).

[23] W.C. Kuo, D. Slaven, and B.L. Young, Phys. Rev. D **37**, 233 (1988).

[24] S. Catani, S. Dittmaier, and Z. Trocsanyi, Phys. Lett. **B500**, 149 (2001).

[25] R.K. Ellis and P. Nason, private communication.

[26] A. Hearn, *REDUCE User's Manual Version 3.7* (Rand Corporation, Santa Monica, CA, 1995).

[27] J.A.M. Vermaseren, *New features of FORM*, math-ph/0010025.

[28] A. Signer and D. Stöckinger, arXiv:0807.4424 [hep-ph].

[29] All the relevant results are available in Reduce format. The results can be retrieved from the preprint server `http://arXiv.org` by downloading the source of this article or can be obtained directly from the authors.