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Quark-mixing renormalization effects on the W-boson partial decay widths

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Abstract

We briefly review existing proposals for the renormalization of the Cabibbo-Kobayashi-Maskawa matrix and study their numerical effects on the W-boson partial decay widths. The differences between the decay widths predicted by the various renormalization schemes are generally negligible, while their deviations from the $\overline{\mathrm{MS}}$ results are very small, except for $W^+ \to u\overline{b}$ and $W^+ \to c\overline{b}$, where they reach approximately 4%.

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The renormalizability of the Standard Model (SM) without quark-flavor mixing was proved in the early seventies [1]. Since the elements of the mixing matrices appear as basic parameters in the bare Lagrangian, they are subject to renormalization, too. This is a problem of old vintage [2], the solution of which was first realized for the Cabibbo angle in the SM with two fermion generations in a pioneering paper by Marciano and Sirlin [3] in 1975. The extension to the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix of the three-generation SM was addressed fifteen years later [4].

In the subsequent years, interest on the subject increased significantly, and new renormalization prescriptions were proposed. The on-shell (OS) prescription of Ref. [4] is compact and plausible, but the proposed expression for the CKM counterterm matrix, δV, is gauge dependent, as was noticed later [5–7]. In Ref. [5], an alternative OS-like prescription was proposed that avoids this problem at one loop. The characteristic feature of this prescription is that the quark self-energies that enter the definition of δV are not evaluated on their respective mass shells, but at the common subtraction point $q^2 = 0$.

Genuine OS renormalization conditions for the CKM matrix, which satisfy the criteria of ultraviolet (UV) finiteness, gauge-parameter independence, and unitarity have been found more recently, first by explicit construction with reference to the corresponding theory without mixing [8] and later based on a novel procedure to separate the external-leg mixing corrections into gauge independent self-mass and gauge-dependent wave-function renormalization contributions [9]. Very recently, a variant of the prescription of Ref. [9] was proposed that is flavor democratic and is formulated in terms of the invariant self-energy functions appearing in the quark mixing amplitudes [10].

The aim of this paper is to apply several of these renormalization schemes to the numerical evaluation of the one-loop corrected partial decay widths of the W boson into quark-antiquark pairs, in order to compare them and ascertain their quantitative effects.

We consider the two-particle decay of the W^+ boson to generic quarks,

$$W^+(k) \to u_i(p_1)\overline{d}_j(p_2). \tag{1}$$

The partial decay width in the Born approximation is given by

$$\Gamma_0^{Wu_i\overline{d}_j} = \frac{N_c\alpha\left|V_{ij}\right|^2}{24s_w^2m_W^3}\kappa(m_W^2, m_{u,i}^2, m_{d,j}^2)\left[2m_W^2 - m_{u,i}^2 - m_{d,j}^2 - \frac{(m_{u,i}^2 - m_{d,j}^2)^2}{m_W^2}\right], \tag{2}$$

where $N_c = 3$, $\alpha = e^2/(4\pi)$ is the fine-structure constant, and

$$\kappa(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 2(xy + yz + zx)} \tag{3}$$

is Källén's function.

The one-loop corrected partial decay width is calculated by including the renormalization constants for the parameters e, s_w, and V_{ij}, those for the W^+, u_i, and $\overline{d}_j$ fields, and the proper vertex corrections. The results can be expressed in the form:

$$\Gamma_1^{Wu_i\overline{d}_j} = \Gamma_0^{Wu_i\overline{d}_j}(1 + \delta^{\text{ew}} + \delta^{\text{QCD}}), \tag{4}$$

where δ^{ew} and δ^{QCD} are the electroweak and QCD corrections, respectively. Analytical expressions for δ^{ew} and δ^{QCD} in the R_ξ gauges may be found, for example, in Ref. [6].

We now proceed with our numerical analysis of Eq. (4). We perform all the calculations with the aid of the `LOOPTOOLS` [11] package embedded into the `MATHEMATICA` environment. As a check, we reproduce the numerical results of Ref. [6] when adopting the definition of δV_{ij} and the values of the input parameters employed in that paper.

In our analysis, we use the following input parameters [12]:

$$
\begin{array}{lll}
\alpha = 1/137.035999679, & G_F = 1.16637 \times 10^{-5}\ \text{GeV}^{-2}, & \alpha_s^{(5)}(m_Z) = 0.1176, \\
m_W = 80.398\ \text{GeV}, & m_Z = 91.1876\ \text{GeV}, & \\
m_e = 0.510998910\ \text{MeV}, & m_\mu = 105.658367\ \text{MeV}, & m_\tau = 1776.84\ \text{MeV}, \\
m_u = 2.4\ \text{MeV}, & m_d = 4.8\ \text{MeV}, & m_s = 100\ \text{MeV}, \\
m_c = 1.25\ \text{GeV}, & m_b = 4.25\ \text{GeV}, & m_t = 172.4\ \text{GeV}.
\end{array}
$$

The standard parameterization of the CKM matrix, in terms of the three mixing angles θ_{ij} and the CP-violating phase δ, reads [12]:

$$
V = \begin{pmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{5}
$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. The choice

$$
s_{12} = \lambda, \qquad s_{23} = A\lambda^2, \qquad s_{13}e^{i\delta} = \frac{A\lambda^3(\overline{\rho} + i\overline{\eta})\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}[1 - A^2\lambda^4(\overline{\rho} + i\overline{\eta})]} \tag{6}
$$

ensures that the CKM matrix written in terms of λ, A, $\overline{\rho}$, and $\overline{\eta}$ is unitary to all orders in λ. In our analysis, we evaluate the CKM matrix elements from Eqs. (5) and (6) using the values $\lambda = 0.2257$, $A = 0.814$, $\overline{\rho} = 0.135$, and $\overline{\eta} = 0.349$ [12].

As stated above, we investigate here the quantitative significance of various definitions of δV_{ij}. Toward this end, we compare in Table 1 the one-loop corrected partial widths of the various hadronic W-boson decay channels for different definitions of the CKM counterterm matrix δV, assuming $m_H = 100$ GeV.

The first two columns in Table 1 describe the partial widths of the W boson when adopting the CKM matrix renormalization conditions proposed in Refs. [4,5], respectively. This has been already done in the literature, for example in Ref. [6]. We emphasize that we find full agreement, provided we adopt the same values for the input parameters. Note that the prescription of Ref. [4] leads to a gauge-dependent result, so that the gauge choice must be specified. We perform the calculation in 't Hooft-Feynman gauge.

New results are those from the third, fourth, and fifth columns, which refer to the three genuine OS renormalization proposals of Refs. [8–10], respectively. The prescription of Ref. [8] entails the minor complication that one needs to consider a reference theory

Partial width	Ref. [4]	Ref. [5]	Ref. [8]	Ref. [9]	Ref. [10]	$\overline{\text{MS}}$-scheme
$\Gamma(W^+ \to u\overline{d})$	0.6696902	0.6696902	0.6696902	0.6696902	0.6696902	0.6696885
$\Gamma(W^+ \to u\overline{s}) \times 10$	0.3594543	0.3594543	0.3594543	0.3594543	0.3594543	0.3594743
$\Gamma(W^+ \to u\overline{b}) \times 10^4$	0.0934565	0.0930905	0.0934530	0.0934564	0.0934566	0.0904054
$\Gamma(W^+ \to c\overline{d}) \times 10$	0.3589685	0.3589685	0.3589686	0.3589685	0.3589685	0.3589678
$\Gamma(W^+ \to c\overline{s})$	0.6684705	0.6684705	0.6684706	0.6684705	0.6684705	0.6684154
$\Gamma(W^+ \to c\overline{b}) \times 10^2$	0.1211291	0.1211291	0.1211244	0.1211299	0.1211297	0.1266297
$\Gamma(W \to \text{hadrons})$	1.4112237	1.4112237	1.4112237	1.4112237	1.4112237	1.4112235

Table 1: Partial widths (in GeV) of the hadronic W-boson decay channels at the one-loop level.

with zero mixing. It is important to note that the proposals of Refs. [8–10] have the important property that they lead to renormalized amplitudes that are non-singular in the limit in which any two fermions become mass degenerate and are thus suitable for the generalization to theories where maximal mixing could appear. A generalization of Ref. [9] to lepton mixing in Majorana-neutrino theories was recently carried out in Ref. [13]. For reference, we have included in the last column of Table 1 the results based on the $\overline{\text{MS}}$ scheme with 't Hooft mass scale $\mu = m_W$.

A comparison of the various columns in Table 1 shows that the differences between the decay widths predicted by the various renormalization schemes are negligible (of $\mathcal{O}(10^{-3}\%)$ or less), with the single exception of $W^+ \to u\overline{b}$ in the scheme of Ref. [5]. Also, the deviations of these predictions from the $\overline{\text{MS}}$ results are very small (of $\mathcal{O}(10^{-3}\%)$ or less), except for $W^+ \to u\overline{b}$ and $W^+ \to c\overline{b}$, where they reach approximately 4%. Although the CKM matrix renormalization plays a crucial role in the cancellation of UV divergences, it is clear that the phenomenological significance of these corrections is very small, except for final states involving b quarks. This may be understood by observing that, in the approximation of neglecting the masses of the down-type quarks relative to the W-boson mass, the CKM matrix can be taken to be unity, so that it does not need to be renormalized at all. The situation is possibly very different for lepton mixing in a non-minimal SM with massive Dirac neutrinos or in extensions of the SM involving Majorana neutrinos.

We note that the differences in the results presented in Table 1 should not be viewed as an argument to decide which of the renormalization prescriptions is to be preferred, but rather as a quantitative comparison between the W-boson partial decay widths predicted in the various renormalization schemes. On the other hand, some of the renormalization schemes discussed above are endowed with very desirable theoretical properties that make them especially attractive. In summary, we have reviewed several possibilities for the definition of the CKM counterterm matrix and studied their effects on the partial widths of the hadronic W-boson decays. The differences in the decay widths induced by the various definitions are generally negligible, while their deviations from the $\overline{\text{MS}}$ results are very small, except for final states involving b quarks.

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