

Supersymmetric solutions for non-relativistic holography

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Abstract

We construct families of supersymmetric solutions of type IIB and $D = 11$ supergravity that are invariant under the non-relativistic conformal algebra for various values of dynamical exponent $z \geq 4$ and $z \geq 3$, respectively. The solutions are based on five- and seven-dimensional Sasaki-Einstein manifolds and generalise the known solutions with dynamical exponent $z = 4$ for the type IIB case and $z = 3$ for the $D = 11$ case, respectively.

1 Introduction

There has recently been much interest in finding holographic realisations of systems invariant under the non-relativistic conformal algebra starting with the work [1], [2] and discussed further in related work [3]-[32]. Such systems are invariant under Galilean transformations, generated by time and spatial translations, spatial rotations, Galilean boosts and a mass operator, which is a central element of the algebra, combined with scale transformations. If x^+ is the time coordinate, and \mathbf{x} denotes d spatial coordinates, the scaling symmetry acts as

$$\mathbf{x} \rightarrow \mu \mathbf{x}, \quad x^+ \rightarrow \mu^z x^+, \quad (1.1)$$

where z is called the dynamical exponent. When $z = 2$ this non-relativistic conformal symmetry can be enlarged to an invariance under the Schrödinger algebra which includes an additional special conformal generator.

The solutions found in [1], [2] with $d = 2$ and $z = 2$ were subsequently embedded into type IIB string theory in [8],[9],[10] and were based on an arbitrary five-dimensional Sasaki-Einstein manifold, SE_5 . The work of [9] also constructed type IIB solutions with $d = 2$ and $z = 4$ and again these were constructed using an arbitrary SE_5 . It was also shown in [9] that the solutions with $z = 2$ and $z = 4$ can be obtained from a five dimensional theory with a massive vector field after a Kaluza-Klein reduction on the SE_5 space [9]. This procedure was generalised to solutions of $D = 11$ supergravity in [31]: using a similar KK reduction on an arbitrary seven-dimensional Sasaki-Einstein space, SE_7 , solutions with non relativistic conformal symmetry with $d = 1$ and $z = 3$ were found.

The type IIB solution of [8],[9],[10] with $z = 2$ do not preserve any supersymmetry [9]. One aim of this note is to show that, by contrast, the type IIB solutions of [9] with $z = 4$ and the $D = 11$ solutions of [31] with $z = 3$ are both supersymmetric and generically preserve two supersymmetries. A second aim is to generalise both of these supersymmetric solutions to different values of z . We will construct new supersymmetric solutions using eigenmodes of the Laplacian acting on one-forms on the SE_5 or SE_7 space. If the eigenvalue is μ then we obtain type IIB solutions with $z = 1 + \sqrt{1 + \mu}$ and $D = 11$ solutions with $z = 1 + \frac{1}{2}\sqrt{4 + \mu}$. This gives rise to type IIB solutions with $z \geq 4$ and $D = 11$ solutions with $z \geq 3$, respectively. For the case of S^5 we get solutions with $z = 4, 5, \dots$ while for the case of S^7 we get solutions with $z = 3, 3\frac{1}{2}, 4, \dots$ and both of these preserve 8 supersymmetries.

Our constructions have some similarities with the construction of type IIB solutions in [24] that were based on eigenmodes of the Laplacian acting on scalar functions

on the SE_5 space. Our IIB solutions preserve the same supersymmetry and we show how our solutions can be superposed with those of [24] while maintaining a scaling symmetry. An analogous superposition is possible for the $D = 11$ solutions, which we shall also describe.

2 The type IIB solutions

The ansatz for the type IIB solutions we shall consider is given by

$$\begin{aligned}
ds^2 &= \frac{dr^2}{r^2} + r^2 [2dx^+ dx^- + dx_1^2 + dx_2^2] + ds^2(SE_5) + 2r^2 C dx^+ \\
F_5 &= 4r^3 dx^+ \wedge dx^- \wedge dr \wedge dx_1 \wedge dx_2 + 4Vol(SE_5) \\
&\quad - dx^+ \wedge [*_{CY_3} dC + d(r^4 C) \wedge dx_1 \wedge dx_2]
\end{aligned} \tag{2.1}$$

where SE_5 is an arbitrary five-dimensional Sasaki-Einstein space and the metric $ds^2(SE_5)$ is normalised so that the Ricci tensor is equal to four times the metric (i.e. the same normalisation as that of a unit radius five-sphere). Recall that the metric cone over the SE_5 ,

$$ds^2(CY_3) = dr^2 + r^2 ds^2(SE_5) , \tag{2.2}$$

is Calabi-Yau. The Kähler form on the CY_3 is denoted ω_{ij} and the complex structure is defined¹ by $J_i^j = \omega_{ik} g^{kj}$, where g_{ij} is the Calabi-Yau cone metric. We will define the one-form η , which is dual to the Reeb vector on SE_5 by

$$\eta_i = -J_i^j (d \log r)_j . \tag{2.3}$$

The one-form C is a one-form on the CY_3 cone. When $C = 0$ we have the standard $AdS_5 \times SE_5$ solution of type IIB which, in general, preserves eight supersymmetries (four Poincaré and four superconformal), corresponding to an $N = 1$ SCFT in $d = 4$. More generally, we can deform this solution by choosing $C \neq 0$ provided that dC is co-closed on CY_3 :

$$d *_{CY} dC = 0 . \tag{2.4}$$

With this condition, F_5 is closed and in fact it is also sufficient for the type IIB Einstein equations to be satisfied. As we will show these solutions preserve one

¹While this is standard in the physics literature, often in the maths literature $J_i^j = -\omega_{ik} g^{kj}$.

half of the Poincaré supersymmetries. Note that the solution is invariant under the transformation

$$x^- \rightarrow x^- - \Lambda, \quad C \rightarrow C + d\Lambda \quad (2.5)$$

for some function Λ on the CY cone. Thus, if $dC = 0$, we can remove C , at least locally, by such a transformation.

We will look for solutions where the one-form C has weight λ under the action of $r\partial_r$. Then it is straightforward to check, following [1] and [2] that our solution is invariant under non-relativistic conformal transformations with two spatial dimensions x^1, x^2 and dynamical exponent $z = 2 + \lambda$. For example the scaling symmetry is acting as in (1.1) combined with $r \rightarrow \mu^{-1}r$, $x^- \rightarrow \mu^{2-z}x^-$. Following the analysis of closed and co-closed two forms on cones (such as dC) in appendix A of [33] we consider solutions constructed from a co-closed one-form β on the SE_5 space that is an eigenmode of the Laplacian $\Delta_{SE} = (d^\dagger d + dd^\dagger)_{SE}$:

$$C = r^\lambda \beta, \quad \Delta_{SE} \beta = \mu \beta, \quad d^\dagger \beta = 0. \quad (2.6)$$

It is straightforward to check that dC is co-closed providing that $\mu = \lambda(\lambda + 2)$. For our applications we choose the branch $\lambda = -1 + \sqrt{1 + \mu}$ leading to solutions with

$$z = 1 + \sqrt{1 + \mu}. \quad (2.7)$$

A general result valid for any five-dimensional Einstein space, normalised as we have, is that for co-closed 1-forms $\mu \geq 8$ and $\mu = 8$ holds iff the 1-form is dual to a Killing vector (see section 4.3 of [34]). Thus in general our construction leads to solutions with

$$z \geq 4. \quad (2.8)$$

Since all SE_5 manifolds have at least the Reeb Killing vector, dual to the one-form η , this bound is always saturated. Indeed the solution of [9] with $z = 4$ is in our class. Specifically it can be obtained by setting $C = \sigma r^2 \eta$ (and redefining $x^- \rightarrow -x^-/2$): one can explicitly check that η is co-closed on SE_5 and is an eigenmode of Δ_{SE} with eigenvalue $\mu = 8$. Note that for this solution the two-form dC is proportional to the Kähler-form of the Calabi-Yau cone: $dC = 2\sigma\omega$.

On S^5 the spectrum of Δ_{S^5} acting on one-forms is well known and we have $\mu = (s+1)(s+3)$ for $s = 1, 2, 3, \dots$ (see for example [35] eq (2.20)) leading to $\lambda = s+1$ and hence new classes of solutions with $z = 4, 5, 6, \dots$. Note that these solutions come in families, transforming in the $SO(6)$ irreps **15**, **64**, **175**, \dots . To obtain similar results for $T^{1,1}$ one can consult [36].

We now discuss a construction that can be used when the spectrum of the Laplacian acting on functions is known, but not acting on one-forms. For example, the scalar Laplacian was studied in [40] for the $Y^{p,q}$ metrics [41], but as far as we know it has not been discussed acting on one-forms. Specifically we construct $(1, 1)$ forms dC on the CY cone using scalar functions Φ on the cone as follows. We write

$$C_i = J_i^j \partial_j \Phi \quad (2.9)$$

for some function Φ on CY_3 . A short calculation shows that if

$$\nabla_{CY}^2 \Phi = \alpha \quad (2.10)$$

for some constant α then dC is co-closed. The two-form dC is a $(1, 1)$ form on CY_3 and it is primitive, $J^{ij} dC_{ij} = 0$, if and only if $\alpha = 0$. Observe that the solution of [9] with $z = 4$ fits into this class by taking $\Phi = -\sigma r^2/2$ and $\alpha = -6\sigma$, leading to $C = \sigma r^2 \eta$.

We now consider solutions with $\alpha = 0$, corresponding to harmonic functions² on the CY cone with dC $(1, 1)$ and primitive. We next write

$$\Phi = r^\lambda f \quad (2.11)$$

where f is a function on the SE_5 space satisfying

$$-\nabla_{SE_5}^2 f = kf \quad (2.12)$$

with $k = \lambda(\lambda + 4)$ (see e.g. [37]). For the solutions of interest we choose the branch $\lambda = -2 + \sqrt{4+k}$ leading to $z = \sqrt{4+k}$. For the special case of the five-sphere we can check with the results that we obtained above. The eigenfunctions f on the five-sphere are given by spherical harmonics with $k = l(l+4)$, $l = 1, 2, \dots$ and hence $z = l + 2$. The $l = 1$ harmonic appears to violate the bound (2.8). However, it is straightforward to see that the construction for $l = 1$ leads to $dC = 0$ for which C can be removed by a transformation of the form (2.5). Thus for S^5 we should consider $l \geq 2$ leading to solutions with $z = 4, 5, \dots$, as above. It is worth pointing out that for higher values of l some of the eigenfunctions will also lead to closed C : if we consider the harmonic function on \mathbb{R}^6 given by $x^{i_1} \dots x^{i_l} c_{i_1 \dots i_l}$ where c is symmetric and traceless then, with $J = dx^1 \wedge dx^2 + dx^3 \wedge dx^4 + dx^5 \wedge dx^6$ we see that $dC = 0$ if $J_{[i}^j c_{k]j i_3 \dots i_l} = 0$.

²Note that in general the one-form C defined in (2.9) has a component in the dr direction, unlike in (2.6). However, locally we can remove it by a transformation of the form (2.5). Also, one can directly show that the resulting one-form β is co-closed on the SE_5 space.

2.1 Supersymmetry

We introduce the frame

$$\begin{aligned}
e^+ &= r dx^+ \\
e^- &= r(dx^- + C) \\
e^2 &= r dx_1 \\
e^3 &= r dx_2 \\
e^4 &= \frac{dr}{r} \\
e^m &= e_{SE}^m, \quad m = 5, \dots, 9
\end{aligned} \tag{2.13}$$

where e_{SE}^m is an orthonormal frame for the SE_5 space. We can write

$$F_5 = B_5 + *_{10} B_5 \tag{2.14}$$

$$B_5 = 4e^+ \wedge e^- \wedge e^2 \wedge e^3 \wedge e^4 - r e^+ \wedge dC \wedge e^2 \wedge e^3 \tag{2.15}$$

where we have chosen $\epsilon_{+23456789} = +1$. The Killing spinor equation can be written

$$D_M \epsilon + \frac{i}{16} \not{F} \Gamma_M \epsilon = D_M \epsilon + \frac{i}{2} \not{B} \Gamma_M \epsilon = 0. \tag{2.16}$$

We are using the conventions for type IIB supergravity [42][43] as in [44] and in particular, $\Gamma_{11} = \Gamma_{+23456789}$ with the chiral IIB spinors satisfying $\Gamma_{11} \epsilon = -\epsilon$.

If ϵ are the Killing spinors for the $AdS_5 \times SE_5$ solution, then we find that we must also impose that

$$\begin{aligned}
\Gamma^{+-23} \epsilon &= i \epsilon \\
\Gamma^+ \epsilon &= 0.
\end{aligned} \tag{2.17}$$

The first condition maintains the Poincaré supersymmetries but breaks all of the superconformal supersymmetries (this can be explicitly checked using, for example, the results of [45]). The second condition breaks a further half of these³. Thus when $dC \neq 0$, we preserve two Poincaré supersymmetries for a generic SE_5 and this is increased to eight Poincaré supersymmetries for S^5 .

³That we preserve the Poincaré supersymmetries suggests that we can extend our solutions away from the near horizon limit of the D3-branes. This is indeed the case but we won't expand upon that here.

3 The $D = 11$ solutions

The construction of the $D = 11$ solutions is very similar. We consider the ansatz for $D=11$ supergravity solutions:

$$\begin{aligned} ds^2 &= \frac{d\rho^2}{4\rho^2} + \rho^2 [2dx^+dx^- + dx^2] + ds^2(SE_7) + 2\rho^2 C dx^+ \\ G &= -3\rho^2 dx^+ \wedge dx^- \wedge d\rho \wedge dx + dx^+ \wedge dx \wedge d(\rho^3 C) \end{aligned} \quad (3.1)$$

where SE_7 is a seven-dimensional Sasaki-Einstein space and $ds^2(SE_7)$ is normalised so that the Ricci tensor is equal to six times the metric (this is the normalisation of a unit radius seven-sphere). It is convenient to change coordinates via $\rho = r^2$ to bring the solution to the form

$$\begin{aligned} ds^2 &= \frac{dr^2}{r^2} + r^4 [2dx^+dx^- + dx^2] + ds^2(SE_7) + 2r^4 C dx^+ \\ G &= -6r^5 dx^+ \wedge dx^- \wedge dr \wedge dx + dx^+ \wedge dx \wedge d(r^6 C) . \end{aligned} \quad (3.2)$$

In these coordinates the cone metric

$$ds_{CY}^2 = dr^2 + r^2 ds^2(SE_7) \quad (3.3)$$

is a metric on Calabi-Yau four-fold. We will use the same notation for the CY space as in the previous section.

When the one-form C is zero we have the standard $AdS_4 \times SE_7$ solution of $D = 11$ supergravity that, in general, preserves eight supersymmetries. We again find that all the equations of motion are solved if C is a one-form on CY_4 and the two-form dC is co-closed

$$d *_{CY} dC = 0 . \quad (3.4)$$

The solutions are again invariant under the transformation (2.5). We will consider solutions where the one-form C has weight λ under the action of $r\partial_r$, corresponding to dynamical exponent $z = 2 + \lambda/2$. As before, using the results in appendix A of [33], we consider solutions constructed from a co-closed one-form β on the SE_7 space that is an eigenmode of the Laplacian Δ_{SE} :

$$C = r^\lambda \beta, \quad \Delta_{SE} \beta = \mu \beta, \quad d^\dagger \beta = 0 . \quad (3.5)$$

One can check that dC is co-closed providing that $\mu = \lambda(\lambda + 4)$. For our applications we choose the branch $\lambda = -2 + \sqrt{4 + \mu}$ leading to solutions with

$$z = 1 + \frac{1}{2} \sqrt{4 + \mu} . \quad (3.6)$$

A general result valid for any seven-dimensional Einstein space, normalised as we have, is that for co-closed 1-forms $\mu \geq 12$ and $\mu = 12$ holds iff the 1-form is dual to a Killing vector (see section 4.3 of [34]). Thus in general our construction leads to solutions with

$$z \geq 3 \tag{3.7}$$

and the bound is again saturated for all SE_7 spaces. Observe that the solutions of [31] with $z = 3$ fit into this class. Specifically they are obtained by setting $C = \sigma r^2 \eta$ (after redefining $x \rightarrow x/2$ and $x^- \rightarrow -x^-/8$). On S^7 the spectrum of Δ_{S^7} is well known and we have $\mu = s(s+6) + 5$ for $s = 1, 2, 3 \dots$ (see for example [34] eq (7.2.5)) leading to $\lambda = 1 + s$ and hence new classes of solutions with $z = 3, 3\frac{1}{2}, 4, \dots$. These solutions come in families transforming in the $SO(8)$ irreps **28**, **160_v**, **567_v**, \dots . Results on the spectrum of the Laplacian on some homogeneous SE_7 spaces can be found in [46],[47],[48].

As before we can construct $(1, 1)$ co-closed two-forms dC using scalar functions Φ on CY_4 . We write

$$C_i = J_i^j \partial_j \Phi, \quad \nabla_{CY}^2 \Phi = \alpha \quad . \tag{3.8}$$

and dC is again primitive if and only if $\alpha = 0$. The solutions of [31] with $z = 3$ arise by taking $\Phi = \sigma r^2$ and $\alpha = -8\sigma$ leading to $C = \sigma r^2 \eta$. We now focus on solutions with $\alpha = 0$, corresponding to harmonic functions on the CY cone. We take

$$\Phi = r^\lambda f \tag{3.9}$$

where f is a function on the SE_7 space satisfying

$$-\nabla_{SE_7}^2 f = k f \tag{3.10}$$

with $k = \lambda(\lambda + 6)$. For our applications we choose the branch $\lambda = -3 + \sqrt{9 + k}$ leading to solutions with $z = \frac{1}{2} + \frac{1}{2}\sqrt{9 + k}$. For example, on the seven-sphere the eigenfunctions f are given by spherical harmonics with $k = l(l + 6)$ with $l = 1, 2, \dots$ and hence $z = 2 + l/2$. Excluding the $l = 1$ harmonic, as it can be removed by a transformation of the form (2.5), for S^7 we are left with solutions with $z = 3, 7/2, 4, \dots$, as above.

3.1 Supersymmetry

We introduce a frame

$$\begin{aligned}
e^+ &= r^2 dx^+ \\
e^- &= r^2(dx^- + C) \\
e^2 &= r^2 dx \\
e^3 &= \frac{dr}{r} \\
e^m &= e_{SE_7}^m, \quad m = 4, \dots, 10.
\end{aligned} \tag{3.11}$$

We thus have

$$\begin{aligned}
G &= 6e^+ \wedge e^- \wedge e^2 \wedge e^3 + r^2 e^+ \wedge e^2 \wedge dC \\
*_{11}G &= -6Vol(SE_7) + dx^+ *_{CY} dC
\end{aligned} \tag{3.12}$$

where we have chosen the orientation $\epsilon_{+-23\dots 10} = +1$.

The Killing spinor equation can be written as

$$\nabla_M \epsilon + \frac{1}{288} [\Gamma_M^{N_1 N_2 N_3 N_4} - 8\delta_M^{N_1} \Gamma^{N_2 N_3 N_4}] G_{N_1 N_2 N_3 N_4} \epsilon = 0. \tag{3.13}$$

We are using the conventions for $D = 11$ supergravity [49] as in [50] and in particular $\Gamma_{+-2345678910} = +1$.

If ϵ are the Killing spinors arising for the $AdS_4 \times SE_7$ solution, then we find that we must also impose that

$$\begin{aligned}
\Gamma^{+-2} \epsilon &= -\epsilon \\
\Gamma^+ \epsilon &= 0.
\end{aligned} \tag{3.14}$$

The first condition maintains the Poincaré supersymmetries but breaks all of the superconformal supersymmetries. The second condition breaks a further half of these. Thus when $dC \neq 0$, we preserve two Poincaré supersymmetries for a generic SE_7 and this is increased to eight Poincaré supersymmetries for S^7 .

3.2 Skew-Whiffed Solutions

If $AdS_4 \times SE_7$ is a supersymmetric solution of $D = 11$ supergravity, then if we “skew-whiff” by reversing the sign of the flux (or equivalently changing the orientation of SE_7) then apart from the special case when the SE_7 space is the round S^7 , all supersymmetry is broken [51]. Despite the lack of supersymmetry, such solutions are known to be perturbatively stable [51]. Similarly, if we reverse the sign of the flux in our new solutions (3.2), we will obtain solutions of $D = 11$ supergravity that will generically not preserve any supersymmetry.

4 Further Generalisation

We now discuss a further generalisation of the solutions that we have considered so far, preserving the same amount of supersymmetry, which incorporate the construction of [24]. For type IIB the metric is now given by

$$ds^2 = \frac{dr^2}{r^2} + r^2 [2dx^+ dx^- + dx_1^2 + dx_2^2] + ds^2(SE_5) + r^2 [2C dx^+ + h(dx^+)^2] \quad (4.1)$$

with the five-form unchanged from (2.1). The conditions on the one-form C are as before and we demand that h is a harmonic function on the CY_3 cone:

$$\nabla_{CY}^2 h = 0 . \quad (4.2)$$

Choosing h to have weight λ' under $r\partial_r$ we take

$$h = r^{\lambda'} f' , \quad (4.3)$$

where f' is an eigenfunction of the Laplacian on SE_5 with eigenvalue k'

$$-\nabla_{SE_5}^2 f' = k' f' \quad (4.4)$$

with $k' = \lambda'(\lambda' + 4)$. If we set $C = 0$ and choose the branch $\lambda' = -2 + \sqrt{4 + k'}$ then these are the solutions constructed in section 5 of [24] and have dynamical exponent $z = \frac{1}{2}\sqrt{4 + k'}$. As noted in [24] an application of Lichnerowicz's theorem [52],[53] implies that these solutions have $z \geq 3/2$ with $z = 3/2$ only possible for S^5 . Now if there is a scalar eigenfunction with eigenvalue k' and a one-form eigenmode of the Laplacian on SE_5 with eigenvalue μ that satisfy $z = \frac{1}{2}\sqrt{4 + k'} = 1 + \sqrt{1 + \mu}$ then we can superpose the solution with h as in (4.3) and the one-form C as in (2.6) and have a solution with scaling symmetry with this value of z . For example on S^5 , using the notation as before, we have $k' = l'(l' + 4)$, $l' = 1, 2, \dots$ and $\mu = (s + 1)(s + 3)$, $s = 1, 2, \dots$ and hence we must demand that $l' = 2(s + 2)$, $s = 1, 2, \dots$, giving solutions with $z = 3 + s$.

The story for $D = 11$ is very similar. The metric is now given by

$$ds^2 = \frac{dr^2}{r^2} + r^4 [2dx^+ dx^- + dx^2] + ds^2(SE_7) + r^4 [2C dx^+ + h(dx^+)^2] \quad (4.5)$$

with the four-form unchanged from (3.2). The conditions on the one-form C are as before and we demand that h is a harmonic function on the CY_4 cone:

$$\nabla_{CY}^2 h = 0 . \quad (4.6)$$

Choosing h to have weight λ' under $r\partial_r$ we take

$$h = r^{\lambda'} f' , \tag{4.7}$$

where f' is an eigenfunction of the Laplacian on SE_7 with eigenvalue k'

$$-\nabla_{SE_7}^2 f' = k' f' \tag{4.8}$$

with $k' = \lambda'(\lambda' + 6)$. If we set $C = 0$ and chose the branch $\lambda' = -3 + \sqrt{9 + k'}$ then these solutions have dynamical exponent $z = \frac{1}{4}(1 + \sqrt{9 + k'})$. Lichnerowicz's theorem [52],[53] implies that these solutions have $z \geq 5/4$ with $z = 5/4$ only possible for S^7 . If there is a scalar eigenfunction with eigenvalue k' and a one-form eigenmode of the Laplacian on SE_7 with eigenvalue μ that satisfy $z = \frac{1}{4}(1 + \sqrt{9 + k'}) = 1 + \frac{1}{2}\sqrt{4 + \mu}$ then we can superpose the solution with h as in (4.7) and the one-form C as in (3.5) and have a solution with scaling symmetry with this value of z . For example on S^7 , using the notation as before, we have $k' = l'(l' + 6)$, $l' = 1, 2, \dots$ and $\mu = s(s + 6) + 5$, $s = 1, 2, \dots$ and hence we must demand that $l' = 2(s + 3)$, $s = 1, 2, \dots$, giving solutions with $z = \frac{1}{2}(5 + s)$.

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References

- [1] D. T. Son, "Toward an AdS/cold atoms correspondence: a geometric realization of the Schroedinger symmetry," Phys. Rev. D **78** (2008) 046003 [arXiv:0804.3972 [hep-th]].
- [2] K. Balasubramanian and J. McGreevy, "Gravity duals for non-relativistic CFTs," Phys. Rev. Lett. **101**, 061601 (2008) [arXiv:0804.4053 [hep-th]].
- [3] M. Sakaguchi and K. Yoshida, "Super Schrodinger in Super Conformal," arXiv:0805.2661 [hep-th].
- [4] W. D. Goldberger, "AdS/CFT duality for non-relativistic field theory," arXiv:0806.2867 [hep-th].

- [5] J. L. B. Barbon and C. A. Fuertes, “On the spectrum of nonrelativistic AdS/CFT,” JHEP **0809**, 030 (2008) [arXiv:0806.3244 [hep-th]].
- [6] M. Sakaguchi and K. Yoshida, “More super Schrodinger algebras from $\text{psu}(2,2-4)$,” JHEP **0808**, 049 (2008) [arXiv:0806.3612 [hep-th]].
- [7] W. Y. Wen, “AdS/NRCFT for the (super) Calogero model,” arXiv:0807.0633 [hep-th].
- [8] C. P. Herzog, M. Rangamani and S. F. Ross, “Heating up Galilean holography,” JHEP **0811**, 080 (2008) [arXiv:0807.1099 [hep-th]].
- [9] J. Maldacena, D. Martelli and Y. Tachikawa, “Comments on string theory backgrounds with non-relativistic conformal symmetry,” JHEP **0810**, 072 (2008) [arXiv:0807.1100 [hep-th]].
- [10] A. Adams, K. Balasubramanian and J. McGreevy, “Hot Spacetimes for Cold Atoms,” JHEP **0811**, 059 (2008) [arXiv:0807.1111 [hep-th]].
- [11] Y. Nakayama, “Index for Non-relativistic Superconformal Field Theories,” JHEP **0810**, 083 (2008) [arXiv:0807.3344 [hep-th]].
- [12] D. Minic and M. Pleimling, “Non-relativistic AdS/CFT and Aging/Gravity Duality,” arXiv:0807.3665 [cond-mat.stat-mech].
- [13] J. W. Chen and W. Y. Wen, “Shear Viscosity of a Non-Relativistic Conformal Gas in Two Dimensions,” arXiv:0808.0399 [hep-th].
- [14] A. V. Galajinsky, “Remark on quantum mechanics with conformal Galilean symmetry,” Phys. Rev. D **78**, 087701 (2008) [arXiv:0808.1553 [hep-th]].
- [15] S. Kachru, X. Liu and M. Mulligan, “Gravity Duals of Lifshitz-like Fixed Points,” Phys. Rev. D **78**, 106005 (2008) [arXiv:0808.1725 [hep-th]].
- [16] S. S. Pal, “Null Melvin Twist to Sakai-Sugimoto model,” arXiv:0808.3042 [hep-th].
- [17] S. Sekhar Pal, “Towards Gravity solutions of AdS/CMT,” arXiv:0808.3232 [hep-th].
- [18] S. Pal, “More gravity solutions of AdS/CMT,” arXiv:0809.1756 [hep-th].

- [19] P. Kovtun and D. Nickel, “Black holes and non-relativistic quantum systems,” arXiv:0809.2020 [hep-th].
- [20] C. Duval, M. Hassaine and P. A. Horvathy, “The geometry of Schrödinger symmetry in gravity background/non-relativistic CFT,” arXiv:0809.3128 [hep-th].
- [21] S. S. Lee, “A Non-Fermi Liquid from a Charged Black Hole: A Critical Fermi Ball,” arXiv:0809.3402 [hep-th].
- [22] D. Yamada, “Thermodynamics of Black Holes in Schroedinger Space,” arXiv:0809.4928 [hep-th].
- [23] F. L. Lin and S. Y. Wu, “Non-relativistic Holography and Singular Black Hole,” arXiv:0810.0227 [hep-th].
- [24] S. A. Hartnoll and K. Yoshida, “Families of IIB duals for nonrelativistic CFTs,” arXiv:0810.0298 [hep-th].
- [25] M. Schvellinger, “Kerr-AdS black holes and non-relativistic conformal QM theories in diverse dimensions,” JHEP **0812**, 004 (2008) [arXiv:0810.3011 [hep-th]].
- [26] L. Mazzucato, Y. Oz and S. Theisen, “Non-relativistic Branes,” arXiv:0810.3673 [hep-th].
- [27] M. Rangamani, S. F. Ross, D. T. Son and E. G. Thompson, “Conformal non-relativistic hydrodynamics from gravity,” arXiv:0811.2049 [hep-th].
- [28] A. Akhavan, M. Alishahiha, A. Davody and A. Vahedi, “Non-relativistic CFT and Semi-classical Strings,” arXiv:0811.3067 [hep-th].
- [29] A. Adams, A. Maloney, A. Sinha and S. E. Vazquez, “1/N Effects in Non-Relativistic Gauge-Gravity Duality,” arXiv:0812.0166 [hep-th].
- [30] M. Taylor, “Non-relativistic holography,” arXiv:0812.0530 [hep-th].
- [31] J. P. Gauntlett, S. Kim, O. Varela and D. Waldram, “Consistent supersymmetric Kaluza–Klein truncations with massive modes,” arXiv:0901.0676 [hep-th].
- [32] S. Pal, “Anisotropic gravity solutions in AdS/CMT,” arXiv:0901.0599 [hep-th].
- [33] D. Martelli and J. Sparks, “Symmetry-breaking vacua and baryon condensates in AdS/CFT,” arXiv:0804.3999 [hep-th].

- [34] M. J. Duff, B. E. W. Nilsson and C. N. Pope, “Kaluza-Klein Supergravity,” *Phys. Rept.* **130** (1986) 1.
- [35] H. J. Kim, L. J. Romans and P. van Nieuwenhuizen, “The Mass Spectrum Of Chiral N=2 D=10 Supergravity On S^{*5} ,” *Phys. Rev. D* **32** (1985) 389.
- [36] A. Ceresole, G. Dall’Agata and R. D’Auria, “KK spectroscopy of type IIB supergravity on $AdS(5) \times T(11)$,” *JHEP* **9911** (1999) 009 [arXiv:hep-th/9907216].
- [37] J. P. Gauntlett, D. Martelli, J. Sparks and S. T. Yau, “Obstructions to the existence of Sasaki-Einstein metrics,” *Commun. Math. Phys.* **273** (2007) 803 [arXiv:hep-th/0607080].
- [38] S. S. Gubser, “Einstein manifolds and conformal field theories,” *Phys. Rev. D* **59** (1999) 025006 [arXiv:hep-th/9807164].
- [39] G. W. Gibbons, S. A. Hartnoll and Y. Yasui, “Properties of some five dimensional Einstein metrics,” *Class. Quant. Grav.* **21** (2004) 4697 [arXiv:hep-th/0407030].
- [40] H. Kihara, M. Sakaguchi and Y. Yasui, “Scalar Laplacian on Sasaki-Einstein manifolds $Y(p,q)$,” *Phys. Lett. B* **621** (2005) 288 [arXiv:hep-th/0505259].
- [41] J. P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, “Sasaki-Einstein metrics on $S(2) \times S(3)$,” *Adv. Theor. Math. Phys.* **8** (2004) 711 [arXiv:hep-th/0403002].
- [42] J. H. Schwarz, “Covariant Field Equations Of Chiral N=2 D=10 Supergravity,” *Nucl. Phys. B* **226** (1983) 269.
- [43] P. S. Howe and P. C. West, “The Complete N=2, D=10 Supergravity,” *Nucl. Phys. B* **238** (1984) 181.
- [44] J. P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, “Supersymmetric $AdS(5)$ solutions of type IIB supergravity,” *Class. Quant. Grav.* **23** (2006) 4693 [arXiv:hep-th/0510125].
- [45] H. Lu, C. N. Pope and J. Rahmfeld, “A construction of Killing spinors on S^{*n} ,” *J. Math. Phys.* **40**, 4518 (1999) [arXiv:hep-th/9805151].
- [46] D. Fabbri, P. Fre, L. Gualtieri and P. Termonia, “M-theory on $AdS(4) \times M(111)$: The complete $Osp(2-4) \times SU(3) \times SU(2)$ spectrum from harmonic analysis,” *Nucl. Phys. B* **560** (1999) 617 [arXiv:hep-th/9903036].

- [47] P. Merlatti, “M-theory on $\text{AdS}(4) \times \text{Q}(111)$: The complete $\text{Osp}(2-4) \times \text{SU}(2) \times \text{SU}(2) \times \text{SU}(2)$ spectrum from harmonic analysis,” *Class. Quant. Grav.* **18** (2001) 2797 [arXiv:hep-th/0012159].
- [48] P. Termonia, “The complete $N = 3$ Kaluza-Klein spectrum of 11D supergravity on $\text{AdS}(4) \times \text{N}(010)$,” *Nucl. Phys. B* **577** (2000) 341 [arXiv:hep-th/9909137].
- [49] E. Cremmer, B. Julia and J. Scherk, “Supergravity theory in 11 dimensions,” *Phys. Lett. B* **76** (1978) 409.
- [50] J. P. Gauntlett and S. Pakis, “The geometry of $D = 11$ Killing spinors,” *JHEP* **0304** (2003) 039 [arXiv:hep-th/0212008].
- [51] M. J. Duff, B. E. W. Nilsson and C. N. Pope, “The Criterion For Vacuum Stability In Kaluza-Klein Supergravity,” *Phys. Lett. B* **139** (1984) 154.
- [52] A. Lichnerowicz, “Géométrie des groupes de transformations,” Dunod, Paris, 1958.
- [53] M. Obata, “Certain conditions for a Riemannian manifold to be isometric to a sphere,” *J. Math. Soc. Japan* **14** (1962) 333-340.