Available at: http://publications.ictp.it IC/2008/051

United Nations Educational, Scientific and Cultural Organization
and
International Atomic Energy Agency
THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

THE VACUUM STRUCTURE, SPECIAL RELATIVITY THEORY AND QUANTUM MECHANICS REVISITED: A FIELD THEORY-NO-GEOMETRY APPROACH

The authors dedicate this article to one of the mathematical and physical giants of the XX-th century - academician Prof. Nikolai N. Bogolubov in memory of his 100th Birthday with great appreciation to his brilliant talent and impressive impact to modern nonlinear mathematics and quantum physics

Nikolai N. Bogolubov (Jr.)[1]
The V.A. Steklov Mathematical Institute of RAN, Moscow, Russian Federation
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy,

Anatoliy K. Prykarpatsky[2]
The AGH University of Science and Technology, Kraków 30-059, Poland,
and
The Ivan Franko State Pedagogical University, Drogobych, Lviv region, Ukraine

and

Ufuk Taneri[3]
The Department of Applied Mathematics and Computer Science,
Eastern Mediterranean University EMU, Famagusta, North Cyprus
and
Kyrenia American University GAU, Institute of Graduate Studies, Kyrenia, North Cyprus.

MIRAMARE – TRIESTE
July 2008

[1]nikolai_bogolubov@hotmail.com
[2]pryk.anat@ua.fm; prykanat@cybergal.com
[3]ufuk.taneri@gmail.com

Abstract

The main fundamental principles characterizing the vacuum field structure are formulated and the modeling of the related vacuum medium and charged point particle dynamics by means of devised field theoretic tools are analyzed. The Maxwell electrodynamic theory is revisited and newly derived from the suggested vacuum field structure principles and the classical special relativity theory relationship between the energy and the corresponding point particle mass is revisited and newly obtained. The Lorentz force expression with respect to arbitrary non-inertial reference frames is revisited and discussed in detail, and some new interpretations of relations between the special relativity theory and quantum mechanics are presented. The famous quantum-mechanical Schrödinger type equations for a relativistic point particle in the external potential and magnetic fields within the quasiclassical approximation as the Planck constant $\hbar \to 0$ and the light velocity $c \to \infty$ are obtained.

1. Introduction

It is a generally accepted statement that no one physical theory can present the absolutely true picture of Nature, assuming that there always exist boundaries of its application, which are approved by experiments and new experience data. This statement concerns, evidently, the relativity theory which, as it was pondered by A. Einstein, should logically arise from both the light velocity constance principle with respect to the inertial reference frames and the generalized equivalence principle with respect to gravity and inertial masses. This theory, namely the special relativity theory, proved to be very thoroughly confirmed by many nuclear physics experiments, which simultaneously became the nuclear energetics backgrounds, widely used today worldwide.

Nevertheless, the nature of space-time and surrounding matter objects was and persists to be one of the most intriguing and challenging problems facing mankind and, in particular, natural scientists. As we know, one of the most brilliant inventions in physics of 19-th century was the combining of electricity and magnetism within the Faraday-Maxwell electromagnetism theory. This theory explained the main physical laws of light propagation in space-time and posed new questions concerning the nature of vacuum. Nonetheless, almost all the attempts aiming to unveil the real state of art of the vacuum problem appeared to be unsuccessful in spite of new ideas suggested by Mach, Lorentz, Poincaré, Einstein and some others physicists. Moreover, the non-usual way of treating the space-time devised by Einstein, in reality, favored eclipsing both its nature and the related physical vacuum origin problems [1, 8, 13, 26, 38, 39, 12], reducing them to some physically unmotivated formal mathematical principles and recipes, combined in the well-known special relativity theory (SRT). The SRT appeared to be adapted only to the inertial reference systems and met with hard problems of the electromagnetic Lorentz forces explanation and relationships between inertial and gravity forces. The latter was artificially "dissolved" by means of the well-known "equivalence principle" owing to which the "inertial" mass of a material object was postulated to coincide with its "gravity" mass. In contrast, E. Mach suggested that any motion of a material point in the space-time, both straight-linear and curvi-linear, can also be only relative. In particular, as the inertion law is also relative, the measure of material point inertion, that is its mass, should also be relative and depend on mutual interactions between all material bodies in the Universe. These E. Mach's ideas influenced so strongly A. Einstein that he dreamed of them his whole life, trying to reconcile Mach's relativity principle with his own approach to the general relativity theory. Nonetheless, despite the titanic efforts of A. Einstein, he failed to include the Mach relativity principle into his general relativity theory, since they appeared to be mutually excluding each other.

Simultaneously, the vacuum origin as a problem almost completely disappeared from the Einstein theory being replaced by the geometrization of space-time nature and all related physical phenomena. Meanwhile, the impressive success of 20-th century quantum physics, especially quantum electrodynamics, have demonstrated clearly enough [2, 24, 4, 5, 6, 7, 20, 38] that the vacuum polarization and electron-positron annihilation phenomena make it possible to pose new questions about space-time and vacuum structures, and further to revisit [9, 12, 13, 14, 19, 21, 22, 18, 39] the existing points of view on them.

As well known, the classical mechanics uses the notion of "potential" energy, being a scalar function of spatial variables, very important for formulating dynamical equations, in spite of the fact that it is determined up to arbitrary constant. One can also observe a similar situation in the classical electrodynamic theory, which effectively uses the notions of scalar and potentials related to each other via the well-known Lorentz compatibility gauge constraint [5, 6, 4, 27] and defined up to suitable gauge transformations. These, in some sense, "different" potential functions were later deeply reanalyzed within the classical Einstein relativity theory by many physicists [24, 5, 14, 9, 21, 27, 18, 22, 19, 13, 29, 30, 31, 32, 33] that gave rise to the understanding of their fundamental role in combining two great theories - the electromagnetism and the gravity.

Moreover, new important problems arose owing to the famous Einstein relationship between the internal energy and the velocity dependent mass, belonging to a material particle. But, as it was mentioned by L. Brillouin [9], the relationship between the particle mass and its internal energy takes into account only the kinetic energy, describing no mass pertaining to the potential energy, which makes the classical relativity theory a not completely closed and, physically, not compatible theoretical construction. And as written by P.W. Bridgeman in [28], "... at construction of his general relativity theory A. Einstein did not make use of those lessons, which he had us taught himself, and of his deep penetration, which he demonstrated us in his special relativity theory".

Below we try to unveil some nontrivial aspects of the real space-time and vacuum origin problems, deeply related with the relativity theory and electrodynamics, to derive, from the natural field theory principles, all of the well-known Maxwell electromagnetism and special relativity theories results, to show their relative or only visible coincidence with real physical phenomena and to feature new perspectives facing the modern fundamental physics.

Moreover, having further developed the field approach to the microscopic vacuum structure, previously suggested in [18] and accounted for in [39], we obtained, within the quasi-classical approximation, a new derivation of main quantum mechanical relationships describing evolution of microscopic particle systems, coinciding as $\hbar \to 0$ with those devised at the beginning of the 20-th century by the great physicists Schrödinger, Heisenberg and Dirac.

2. The Maxwell electromagnetism theory: new look and interpretation

We start from the following field theoretical model [39] of the microscopic vacuum structure, considered as some physical reality imbedded into the standard three-dimensional Euclidean space reference system marked with three spatial coordinates $r \in \mathbb{R}^3$, endowed with the standard scalar product $< \cdot, \cdot >$, and parameterized by means of the scalar temporal parameter $t \in \mathbb{R}$. We will describe the physical vacuum matter endowing it with an everywhere enough smooth four-vector potential function $(W, A) : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}^3$, naturally related to light propagation properties.

The material objects, imbedded into the vacuum, we will model (classically here) by means of the scalar charge density function $\rho : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$ and the vector current density $J : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3$, being also everywhere enough smooth functions.

(i) The *first* field theory principle regarding the vacuum we accept is formulated as follows: the four-vector function $(W, A) : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}^3$ satisfies the standard Lorentz type continuity relationship

$$\frac{1}{c}\frac{\partial W}{\partial t} + < \nabla, A >= 0, \tag{2.1}$$

where, by definition, $\nabla := \partial/\partial r$ is the usual gradient operator.

(ii) The *second* field theory principle we accept is a dynamical relationship on the scalar potential component $W : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$:

$$\frac{1}{c^2}\frac{\partial^2 W}{\partial t^2} - \nabla^2 W = \rho, \tag{2.2}$$

assuming the linear law of the small vacuum uniform and isotropic perturbation propagations in the space-time, understood here, evidently, as a first (linear) approximation in the case of weak enough fields.

(iii) The *third* principle is similar to the first one and means simply the continuity condition for the density and current density functions:

$$\frac{\partial \rho}{\partial t} + < \nabla, J >= 0. \tag{2.3}$$

We need to note here that the vacuum field perturbations velocity parameter $c > 0$, used above, coincides with the vacuum light velocity, as we are trying to derive successfully from these first principles the well-known Maxwell electromagnetism field equations, to analyze the related Lorentz forces and special relativity relationships. To do this, we first combine equations (2.1) and (2.2):

$$\frac{1}{c^2}\frac{\partial^2 W}{\partial t^2} = - < \nabla, \frac{1}{c}\frac{\partial A}{\partial t} >=< \nabla, \nabla W > + \rho,$$

whence

$$< \nabla, -\frac{1}{c}\frac{\partial A}{\partial t} - \nabla W >= \rho. \tag{2.4}$$

Having put, by definition,

$$E := -\frac{1}{c}\frac{\partial A}{\partial t} - \nabla W, \tag{2.5}$$

we obtain the first material Maxwell equation

$$< \nabla, E >= \rho \tag{2.6}$$

for the electric field $E : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3$. Having now applied the rotor-operation $\nabla\times$ to expression (2.5) we obtain the first Maxwell field equation

$$\frac{1}{c}\frac{\partial B}{\partial t} - \nabla \times E = 0 \tag{2.7}$$

on the magnetic field vector function $B : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3$, defined as

$$B := \nabla \times A. \tag{2.8}$$

To derive the second Maxwell field equation we will make use of (2.8), (2.1) and (2.5):

$$
\begin{aligned}
\nabla \times B &= \nabla \times (\nabla \times A) = \nabla < \nabla, A > -\nabla^2 A = \\
&= \nabla(-\frac{1}{c}\frac{\partial W}{\partial t}) - \nabla^2 A = \frac{1}{c}\frac{\partial}{\partial t}(-\nabla W - \frac{1}{c}\frac{\partial A}{\partial t} + \frac{1}{c}\frac{\partial A}{\partial t}) - \nabla^2 A = \\
&= \frac{1}{c}\frac{\partial E}{\partial t} + (\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} - \nabla^2 A). \qquad (2.9)
\end{aligned}
$$

We have from (2.5), (2.6) and (2.3) that

$$
< \nabla, \frac{1}{c}\frac{\partial E}{\partial t} >= \frac{1}{c}\frac{\partial \rho}{\partial t} = -\frac{1}{c} < \nabla, J >,
$$

or

$$
< \nabla, -\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} - \nabla(\frac{1}{c}\frac{\partial W}{\partial t}) + \frac{1}{c}J >= 0. \qquad (2.10)
$$

Now making use of (2.1), from (2.10) we obtain that

$$
\begin{aligned}
< & \nabla, -\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} - \nabla(\frac{1}{c}\frac{\partial W}{\partial t}) + \frac{1}{c}J >=< \nabla, -\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} + \nabla < \nabla, A > +\frac{1}{c}J = \\
= & < \nabla, -\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} + \nabla^2 A + \nabla \times (\nabla \times A) + \frac{1}{c}J >= \\
= & < \nabla, -\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} + \nabla^2 A + \frac{1}{c}J >= 0. \qquad (2.11)
\end{aligned}
$$

Thereby, equation (2.11) yields

$$
\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} - \nabla^2 A = \frac{1}{c}(J + \nabla \times S) \qquad (2.12)
$$

for some smooth vector function $S : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3$. Here we need to note that continuity equation (2.3) is defined, concerning the current density vector $J : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3$, up to a vorticity expression, that is $J \simeq J + \nabla \times S$ and equation (2.12) can finally be rewritten down as

$$
\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} - \nabla^2 A = \frac{1}{c}J. \qquad (2.13)
$$

Having substituted (2.13) into (2.9) we obtain the second Maxwell field equation

$$
\nabla \times B - \frac{1}{c}\frac{\partial E}{\partial t} = \frac{1}{c}J. \qquad (2.14)
$$

In addition, from (2.8) one also finds the magnetic no-charge relationship

$$
< \nabla, B >= 0. \qquad (2.15)
$$

Thus, we have derived all the Maxwell electromagnetic field equations from our three main principles (2.1), (2.2) and (2.3). The success of our undertaking will be more impressive if we adapt our results to those following from the well known relativity theory in the case of point charges or masses. Below we will try to demonstrate the corresponding derivations based on some completely new physical conceptions of the vacuum medium first discussed in [18, 39].

Remark 2.1. It is interesting to analyze a partial case of the first field theory vacuum principle (2.1) when the following local conservation law for the scalar potential field function $W : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$ holds:

$$\frac{d}{dt} \int_{\Omega_t} W d^3 r = 0, \tag{2.16}$$

where $\Omega_t \subset \mathbb{R}^3$ is any open domain in space $\mathbb{R}^3$ with the smooth boundary $\partial\Omega_t$ for all $t \in \mathbb{R}$ and $d^3 r$ is the standard volume measure in $\mathbb{R}^3$ in a vicinity of the point $r \in \Omega_t$.

Having calculated expression (2.16) we obtain the following equivalent continuity equation

$$\frac{1}{c}\frac{\partial W}{\partial t} + < \nabla, \frac{v}{c} W >= 0, \tag{2.17}$$

where $\nabla := \nabla_r$ is, as above, the gradient operator and $v := dr/dt$ is the velocity vector of a vacuum medium perturbation at point $r \in \mathbb{R}^3$ carrying the field potential quantity W. Comparing now equations (2.1), (2.17) and using equation (2.3) we can make the suitable identifications:

$$A = \frac{v}{c} W, \qquad J = \rho v, \tag{2.18}$$

well known from the classical electrodynamics and superconductivity theory [2, 8]. Thus, we are faced with a new physical interpretation of the conservative electromagnetic field theory when the vector potential $A : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3$ is completely determined via expression (2.18) by the scalar field potential function $W : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$. It is also evident that all the Maxwell electromagnetism filed equations derived above hold as well in the case (2.18), as it was first demonstrated in [18] (but with some mathematical inaccuracies) and in [39].

Consider now the conservation equation (2.16) jointly with the related integral "vacuum momentum" conservation relationship

$$\frac{d}{dt} \int_{\Omega_t} (\frac{vW}{c^2}) d^3 r = 0, \ \Omega_t|_{t=0} = \Omega_0, \tag{2.19}$$

where, as above, $\Omega_t \subset \mathbb{R}^3$ is for any time $t \in \mathbb{R}$ an open domain with the smooth boundary $\partial\Omega_t$, whose evolution is governed by the equation

$$dr/dt = v(r, t) \tag{2.20}$$

for all $x \in \Omega_t$ and $t \in \mathbb{R}$, as well as by the initial state of the boundary $\partial\Omega_0$. As a result of relation (2.19) one obtains the new continuity equation

$$\frac{d(vW)}{dt} + vW < \nabla, v >= 0. \tag{2.21}$$

Now making use of (2.17) in the equivalent form

$$\frac{dW}{dt} + W < \nabla, v >= 0,$$

we finally obtain a very interesting local conservation relationship

$$dv/dt = 0 \tag{2.22}$$

on the vacuum matter perturbations velocity $v = dr/dt$, which holds for all values of the time parameter $t \in \mathbb{R}$. As it is easy to observe, the obtained relationship completely coincides with the well-known hydrodynamic equation [37] of ideal compressible liquid without any external exertion, that is, any external forces and field "pressure" are equally identical to zero. We received a natural enough result where the propagation velocity of the vacuum field matter is constant and equals exactly $v = c$, that is the light velocity in the vacuum, if to recall the starting wave equation (2.2) owing to which the small vacuum field matter perturbations propagate in the space with the light velocity.

3. Special relativity theory and dynamical field equations

From classical electrodynamics we know that the main dynamical relationship relates the particle mass acceleration to the Lorentz force which strongly depends on the absolute charge velocity.

For the electrodynamics to be independent on the reference system physicists were forced to reject the Galilean transformations and replace them with the artificially postulated Lorentz transformations. This resulted later in the Einstein relativity theory which has partly reconciled the problems concerned with deriving true dynamical equations for a charged point particle.

We will now start from the scalar field vacuum medium function $W : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$ in the conservation condition case (2.16) discussed above. This means, obviously, that the vacuum medium field vector potential $A : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3$, charge and current densities $(\rho, J) : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}^3$ are related owing to expressions (2.18).

Consider now vacuum field medium conservation equations (2.17) and (2.2) at the density $\rho = 0$:

$$\begin{aligned} -\frac{1}{c^2}\frac{\partial^2 W}{\partial t^2} &= \frac{1}{c^2}\frac{\partial}{\partial t}(-\frac{\partial W}{\partial t}) = \frac{1}{c^2}\frac{\partial}{\partial t}(< \nabla, vW >) = \\ &= < \nabla, \frac{\partial}{\partial t}(\frac{Wv}{c^2}) >= - < \nabla, \nabla W > . \end{aligned} \tag{3.1}$$

From relation (3.1) it follows that

$$\frac{\partial}{\partial t}(\frac{Wv}{c^2}) + \nabla W = \nabla \times F, \tag{3.2}$$

where $F : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3$ is some smooth function, which we put, by definition, to be zero owing to the *a priori* assumed vortexless vacuum medium dynamics. So, our dynamical equation on the vacuum medium scalar field function $W : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$ looks like

$$\frac{\partial}{\partial t}(\frac{Wv}{c^2}) + \nabla W = 0. \tag{3.3}$$

Consider now a charged point particle q in the space point $r = R(t) := R_0 + \int_0^t u(t)dt \in \mathbb{R}^3$, depending on time parameter $t \in \mathbb{R}$ and initial point $R_0 \in \mathbb{R}^3$ at $t = 0$. Since the vacuum medium field is described by means of the potential field function $W : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$, which is naturally disturbed by the charged particle q, we will model this fact approximately as the following resulting functional relationship:

$$W(r, t) = \tilde{W}(r, R(t)) \tag{3.4}$$

for some scalar function $\tilde{W} : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$. This function must satisfy equation (2.17), that is

$$< \frac{\partial \tilde{W}}{\partial R}, u > + < \nabla, \tilde{W} v >= 0. \tag{3.5}$$

As we are interested in differential properties of the function $\tilde{W} : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ as $r \to R(t) \in \mathbb{R}^3$, where the charged point particle is located, we obtain from (3.5) that

$$< \frac{\partial \tilde{W}}{\partial R} + \frac{\partial \tilde{W}}{\partial r}, u > |_{r \to R(t)} + \tilde{W} < \nabla, v > |_{r \to R(t)} = 0,$$

giving rise to the relationship

$$\frac{\partial \tilde{W}}{\partial R} = -\frac{\partial \tilde{W}}{\partial r} \tag{3.6}$$

as $r \to R(t)$, since $v|_{r \to R(t)} \to dR(t)/dt := u(t)$ and $< \nabla, v > |_{r \to R(t)} \to < \nabla, u(t) >= 0$ for all $t \in \mathbb{R}$.

Returning now to equation (3.3) we can write, owing to (3.6), that

$$\begin{array}{c} \frac{1}{c^2}\left(\frac{\partial \tilde{W}}{\partial t} v + \tilde{W}\frac{\partial v}{\partial t}\right)\Big|_{r \to R(t)} = \frac{1}{c^2}\left(- < \frac{\partial \tilde{W}}{\partial r}, v > v + \tilde{W}\frac{\partial v}{\partial t}\right)\Big|_{r \to R(t)} = \\ = \frac{1}{c^2}\left(< \frac{\partial \tilde{W}}{\partial R}, u > u + \tilde{W}\frac{du}{dt}\right)\Big|_{r \to R(t)} \Rightarrow \frac{1}{c^2}\frac{d}{dt}(\bar{W}u) = -\frac{\partial \tilde{W}}{\partial r}\Big|_{r \to R(t)} = \frac{\partial \bar{W}}{\partial R}, \end{array} \tag{3.7}$$

where we put, by definition, $\bar{W} := \tilde{W}(r, R(t))|_{r \to R_0}$. Thus, we obtained from (3.7) that the function $\bar{W} : \mathbb{R}^3 \to \mathbb{R}$ satisfies the determining dynamical equation

$$\frac{d}{dt}(-\frac{\bar{W}}{c^2}u) = -\frac{\partial \bar{W}}{\partial R} \tag{3.8}$$

at point $R(t) \in \mathbb{R}^3$, $t \in \mathbb{R}$, of the point charge q location.

Now we need to proceed with our calculations and would like to make the following very important **assumption:** we will interpret the quantity $-\frac{\bar{W}}{c^2}$ as the real "dynamical" mass of our point charge q at point $R(t) \in \mathbb{R}^3$, that is

$$m := -\frac{\bar{W}}{c^2}. \tag{3.9}$$

This, in part, means that the whole observed particle mass m depends only on the vacuum field potential $\bar{W}$ owing to both its gravitational and electromagnetic interactions with long distant and closely ambient it material particles! This statement, evidently, in many points coincides with the well-known Mach principle [9, 18, 39] and formalizes it concerning the real field structure of vacuum. We need here to mention that this idea was also earlier claimed, but not realized practically, by L. Brillouin in [9]. We press here that no assumption about the equivalence of the inertial mass and gravitational mass is made and, moreover, such a kind of statement is completely alien within the theory devised here and in [18, 39].

Using further (3.9) we can rewrite equation (3.8) as

$$\frac{dp}{dt} = -\frac{\partial \bar{W}}{\partial R}, \tag{3.10}$$

where the quantity $p := mu$ has the natural momentum interpretation.

The obtained equation (3.10) is very interesting from the dynamical point of view. Really, from equation (3.10) we obtain that

$$< u, \frac{d}{dt}(mu) >= c^2 < \frac{\partial m}{\partial R}, u >= c^2 \frac{dm}{dt}. \tag{3.11}$$

As a result of (3.11) we easily derive, following [18, 40], the conservative relationship

$$\frac{d}{dt}\left(m\sqrt{1-\frac{u^2}{c^2}} \right) = 0 \tag{3.12}$$

for all $t \in \mathbb{R}$. Really, based on (3.11), we have that

$$m < u, \frac{du}{dt} > + < u, u > \frac{dm}{dt} = c^2 \frac{dm}{dt}, \tag{3.13}$$

or, equivalently,

$$\frac{1}{2} m \frac{du^2}{dt} - (c^2 - u^2)\frac{dm}{dt} = 0. \tag{3.14}$$

As a result of (3.14), we obtain

$$-\frac{1}{2}\frac{1}{(1-\frac{u^2}{c^2})}\frac{d}{dt}(\frac{u^2}{c^2}) + \frac{1}{m}\frac{dm}{dt} = 0, \tag{3.15}$$

giving rise, by means of simple integrating, to the following differential expression:

$$\frac{d}{dt}\ln(\sqrt{1-\frac{u^2}{c^2}}) + \frac{d\ln m}{dt} = \frac{d}{dt}\ln\left(m\sqrt{1-\frac{u^2}{c^2}} \right) = 0. \tag{3.16}$$

The latter is, evidently, equivalent to result (3.12), that is the quantity

$$m\sqrt{1-\frac{u^2}{c^2}} = m_0 \tag{3.17}$$

is constant for all $t \in \mathbb{R}$, giving rise to the well known relativistic expression for the mass of a point particle:

$$m = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}. \tag{3.18}$$

As we can see, the point particle mass m depends, in reality, not on the coordinate $R(t) \in \mathbb{R}^3$ of the point particle q, but on its velocity $u := dR(t)/dt$. Since the field potential $\bar{W} : \mathbb{R}^3 \to \mathbb{R}$ consists of two parts

$$\bar{W} = \bar{W}_0 + \Delta\bar{w}, \tag{3.19}$$

where $\bar{W}_0 : \mathbb{R}^3 \to \mathbb{R}$ is constant and responsible for the external influence of all long distant objects in the Universe upon the point particle q and $\Delta\bar{w} : \mathbb{R}^3 \to \mathbb{R}$ is responsible for the local field potential perturbation by the point charge q and its closest ambient neighborhood. Then, obviously,

$$\Delta m := m - m_0 = -\Delta\bar{w}/c^2 \tag{3.20}$$

is the strictly dynamical mass component belonging to the point particle q. Moreover, since the full momentum $p = mu$ satisfies equation (3.10), one can easily obtain that the quantity

$$\bar{W}^2 - c^2p^2 = E_0^2 \tag{3.21}$$

is not depending on time $t \in \mathbb{R}$, that is, $dE_0/dt = 0$, where $E_0 := m_0c^2$ is the so-called [2, 3, 22, 26] *"internal energy"* of a point particle q. The result (3.21) demonstrates the important property of the energy essence: the point particle q is, in reality, endowed with the only dynamical energy $\Delta E := \Delta mc^2$. Concerning this "internal" particle energy $E_0 = m_0c^2$ we see that it has nothing to do with the real particle energy, since its origin is determined completely owing to the long distant objects of the Universe and can not be used for any physical processes, contrary to the known Einstein theory statements about a "huge" internal energy stored inside the particle mass. Equivalently, the Einstein theory statement about the "equivalence" of the mass and the "internal" energy of particle appears to be senseless, since the main part of the field potential function $\bar{W} : \mathbb{R}^3 \to \mathbb{R}$ at the location point of the point particle q is constant and results owing to the long distant objects in the Universe, which obviously can not be used for so-called "practical applications".

Nonetheless, we have observed above, as a by-product, the well known "relativistic" effect of the particle mass growth depending on the particle velocity in the form (3.18). As it was already mentioned in [18, 39] this "mass growth" is, in reality, completely of dynamical nature and is not a consequence of the Lorentz transformations, as it was stated within the Einstein SRT. Moreover, we can state that all of the so-called "relativistic" effects have nothing to do with the mentioned above Lorentz transformations and with such artificial "effects" as length "shortening" and time "slowing". There is also no reasonable cause to identify the particle mass with its real energy and vice versa. Concerning the interesting physical effect called particles "annihilation" we need to stress here that it has also nothing to do with the transformation of particles masses into energy. The field theoretical explanation of this phenomenon consists in creating their very special bonding state, whose interaction with ambient objects is vanishing. As a result the visible inertial or dynamical mass of this bound state is also zero, exactly what the experiment shows, and nothing else. Inversely, if an intensive enough photon meets such a bound state of two particles, it can break them back into two separate particles, what the experiment shows to happen. Here we recall a similar analogy borrowed from the modern quantum physics of infinite particle systems described by means of the second quantization scenario [23, 42, 40, 41] suggested in 1932 by V. Fock. Within this scenario there are also realized creating-annihilation effects which are present owing to the inter-particle interaction forces. Moreover, as we know from the modern superconductivity theory within this picture one can describe special bound states of particles, so-called "Cooper pairs", whose interaction with each other completely vanishes and whose combined mass strongly differs from the sum of the suitable components and equals the so-called "effective" compound mass, depending strongly on the potential field intensity inside the superconductor matter.

We now proceed to discuss the relation (3.21) derived above, where the conserved quantity E_0 is naturally interpreted as the total energy of a particle moving with velocity $u := dR(t)/dt$ in vacuum endowed with the field potential $\bar{W}$ in a vicinity of the particle q located at point

$R(t) \in \mathbb{R}^3$. We see that the total particle energy E_0 strictly depends on both the field potential $\bar{W}$ and its velocity u, as the particle momentum $p = m_0 u/\sqrt{1-\frac{u^2}{c^2}}$ depends strictly relativistically on its velocity u.

As it was mentioned by L. Brillouin in [9] the Einstein SRT postulates that the total energy E_0 of a moving particle in a potential field U equals $E_0 = m_0 c^2/\sqrt{1-\frac{u^2}{c^2}} + U$. He writes: *"This means that any possibility of existing the particle mass related with the external potential energy is completely excluded... Thereby, the strange situation appears: the internal (particle) energy is endowed with the mass but the external - is not".* (The citation is taken from [9]).

Contrary to this inference from the Einstein SRT, the relation (3.21) obtained above naturally takes into account both the kinetic energy of the particle motion with velocity u and the field potential energy $\bar{W}$ in a vicinity of the particle q located at point $R(t) \in \mathbb{R}^3$.

Moreover, L. Brillouin in [9] writes: "*...Einstein tends by any way to reduce gravity to geometry by means of changing the Newtonian gravitational potential by a tensor potential of second order, realizing the joint description of gravity and geometry; this is achieved owing to the appearance of a huge gap between gravity and electromagnetism.... The (Einstein) article - a genuine mathematical work but its application to the physical reality remains to be open."* Later V. Fock in his famous book [22] tried to rescue the situation that arisen with gravity and electromagnetism but his approach was also based on the Einstein geometrization ideas and no real success was achieved. Evaluating the gravity and electromagnetism theories state of art L. Brillouin in [9] states that "*In general, the necessity of considering a curved space-time Universe is still not proved; the physical meaning of the general relativity is still very vague".*

Having analyzed, from this point of view, the results formulated above we see that all of the Einstein SRT statements were obtained within the classical Euclidean space-time scenario and no four-dimensional space-time geometry, like the invariance of the four-dimensional metric interval with respect to the Lorentz transformations, was involved. We will show below that within the approaches devised above and in [18, 39] we will derive the next very important result of the SRT and Maxwell electromagnetism theory, namely that related to the nature of the Lorentz force acting on a moving in space charged particle. As is well known [9, 22, 19, 18], wishing to make the Lorentz force expression compatible with the postulated relativity principles appeared to be decisive in Einstein's endeavors to construct his SRT and later the general relativity theory.

As a very interesting aspect of the vacuum field theory devised above and in [39, 18] we need here to mention a very close relationship of electromagnetic and gravity fields. Namely, the vacuum field potential, in general, consists of two components: $W := W_g + W_{em}$, where W_g corresponds to the gravity interaction between material particles and W_{em} corresponds to the electromagnetic interaction between their charges, where we have accepted that these two physical vacuum realities are different and independent. Thecorresponding full mass of a particle is given then by expression (3.9) in the form

$$m := -\bar{W}/c^2 = -(\bar{W}_g + \bar{W}_{em})/c^2, \tag{3.22}$$

including both the gravity and electromagnetic interactions between particles. Then, following the derivation of electromagnetic Maxwell equations above, based on the electromagnetic vacuum field potential $W := W_{em}$, we can derive, in the same way, the corresponding Maxwell type gravimagnetic equations on the gravity potential W_g, as follows

$$\begin{gathered} \frac{1}{c^2}\frac{\partial^2 W_g}{\partial t^2} - \nabla^2 W_g = \rho_g, \frac{1}{c^2}\frac{\partial^2 A_g}{\partial t^2} - \nabla^2 A_g = J_g, \\ \nabla \times B_g - \frac{1}{c}\frac{\partial E_g}{\partial t} = \frac{1}{c}J_g, \frac{1}{c}\frac{\partial B_g}{\partial t} - \nabla \times E_g = 0, \\ < \nabla, E_g >= \rho_g, \quad < \nabla, B_g >= 0, \end{gathered} \tag{3.23}$$

where, by definition,

$$E_g := -\nabla W_g - \frac{1}{c}\frac{\partial A_g}{\partial t}, \ B_g := \nabla \times A_g. \tag{3.24}$$

Here ρ_g and J_g denote the gravity particle mass density and mass current density, respectively. The form of governing equations (3.23) is, evidently, strongly motivated by the well known similarity between the Coulomb electrical and Newtonian gravitational forces expressions, and was very deeply previously discussed in [9, 12, 18, 16].

As already shown above, in the conservative case, the following representations

$$J_g = \rho_g v, \ \ A_g = \frac{v}{c}W_g, \tag{3.25}$$

similar to (2.18), hold. As a consequence, we can state that the gravitational waves exist, propagating in vacuum with the same light velocity c as electromagnetic waves, and are described by means of the Maxwell type gravimagnetic equations (3.23). Here we note that similar inferences have been done many years ago in classical oeuvres of famous physicists of the XIX century J.C. Maxwell [15] and O. Heaviside [11], as well as in works of other physicists of the past and present centuries [10, 16, 17, 18].

4. The Lorentz force and the relativity theory principles revisited

It is a well known fact that the Einstein special relativity theory is applicable only for physical processes related to each other by means of the inertial reference systems, moving with constant velocities. In this case one can make use of the Lorentz transformations and calculate the components of suitable four-vectors and the resulting mass growth of particles owing to formula (3.11). A nontrivial problem arises when we wish to analyze these quantities with respect to non-inertial reference systems moving with some nonzero acceleration. Below we will revisit this problem from the vacuum field theory scenario devised above and show that the whole "special" relativity theory emerges as its partial case or by-product and is free of the artificial "inertial reference systems" problems mentioned above.

Really, our vacuum field theory structure is described by the dynamical equation (3.3), which we would like to investigate in a neighborhood of two interacting to each other point particles q_f at point $R_f(t) \in \mathbb{R}^3$ and q at point $R(t) := R_0 + \int_0^t u(t)dt \in \mathbb{R}^3$, respectively, depending on time parameter $t \in \mathbb{R}$ and initial point $R_0 \in \mathbb{R}^3$ at $t = 0$. As was already done in Section 2 we assume that the vacuum potential field function $W : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$ can be represented as $W = \tilde{W}(r; R_f(t), R(t))$ for some function $\tilde{W} : \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ and all $t \in \mathbb{R}$. Then, based on the continuity equation (2.17) we obtain

$$
< \frac{\partial \tilde{W}}{\partial R_f}, u_f > + < \frac{\partial \tilde{W}}{\partial R}, u > + < \frac{\partial \tilde{W}}{\partial r}, v > + \tilde{W} < \nabla, v >= 0. \tag{4.1}
$$

We will now be interested in the potential field function $\tilde{W} : \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ in a vicinity of the relative distance vector $\tilde{R}(t) := R(t) - R_f(t) \in \mathbb{R}^3$, keeping in mind that the interaction between particles q_f and q depends on this relative interparticle distance $\tilde{R}(t) \in \mathbb{R}^3$. From (4.1), as vector $r \to \tilde{R}(t)$, we easily derive that

$$
\left. \frac{\partial \tilde{W}}{\partial R_f} + \frac{\partial \tilde{W}}{\partial R} \right|_{r \to \tilde{R}(t)} = 0, \qquad \left. \frac{\partial \tilde{W}}{\partial R} + \frac{\partial \tilde{W}}{\partial r} \right|_{r \to \tilde{R}(t)} = 0. \tag{4.2}
$$

Combining relations (4.2) with the dynamical field equations (3.3) we obtain that

$$
\begin{aligned}
\frac{1}{c^2} \frac{\partial}{\partial t} \left(\tilde{W} v \right) \Big|_{r \to \tilde{R}(t)} &= \frac{1}{c^2} \left(< \frac{\partial \tilde{W}}{\partial R_f}, u_f > v + < \frac{\partial \tilde{W}}{\partial R}, u > v + \tilde{W} \frac{\partial v}{\partial t} \right) \Big|_{r \to \tilde{R}(t)} = \\
&= \frac{1}{c^2} \left(< \frac{\partial \tilde{W}}{\partial \tilde{R}}, u - u_f > (u - u_f) \right) \Big|_{r \to \tilde{R}(t)} = - \left. \frac{\partial \tilde{W}}{\partial r} \right|_{r \to \tilde{R}(t)} = \left. \frac{\partial \tilde{W}}{\partial R} \right|_{r \to \tilde{R}(t)},
\end{aligned}
$$

whence one derives the new dynamical equation

$$
\frac{d}{dt} \left(-\frac{\bar{W}}{c^2} (u - u_f) \right) = -\frac{\partial \bar{W}}{\partial R} \tag{4.3}
$$

on the resulting function $\bar{W} := \tilde{W}|_{r \to \tilde{R}(t)}$

Equation (4.3) possesses a very important feature of depending on the only relative quantities not depending on the reference system. Moreover, we have not, on the whole, met the necessity to use other transformations of coordinates different from the Galilean transformations. We mention here that the dynamical equation (4.3) was also derived in [18] making use of some not completely true relationships and mathematical manipulations. But the main corollary of [18] and our derivation [39], saying that equation (4.3) fits for all reference systems, both inertial and accelerated, appears to be fundamental and gives rise to new unexpected results in the modern electrodynamics and gravity theory. Below we will proceed with one of very important relativity physics aspects, concerned with the well-known Lorentz force expression measuring the action exerted by external electromagnetic field on a charged point particle q at space point $R(t) \in \mathbb{R}^3$ for any time moment $t \in \mathbb{R}$.

To do this we accept, owing to the vacuum field theory, that the resulting potential field function $\bar{W} : \mathbb{R}^3 \to \mathbb{R}$ can be represented in the vicinity of the charged point particle q as

$$
\bar{W} = \bar{W}_0 + q\varphi, \tag{4.4}
$$

where $\varphi : \mathbb{R}^3 \to \mathbb{R}$ is a suitable local electromagnetic field potential and $\bar{W}_0 : \mathbb{R}^3 \to \mathbb{R}$ is a constant vacuum field potential owing to the particle interaction with the external distant Universe objects.

Then, having substituted (4.4) into the main dynamical field equation (4.3) we obtain that

$$\begin{aligned}
\frac{d}{dt}(-\frac{\bar{W}}{c^2}u) &= \frac{d}{dt}(-\frac{\bar{W}}{c^2}u_f) - \nabla\bar{W} = -\nabla\bar{W} + \frac{\partial}{\partial t}(-\frac{\bar{W}}{c^2}u_f) + < u, \nabla > (-\frac{\bar{W}}{c^2}u_f) = \\
&= -\nabla\bar{W} + \frac{1}{c}\frac{\partial}{\partial t}(-\frac{\bar{W}}{c}u_f) - u \times (u_f \times \nabla\frac{\bar{W}}{c^2}) - < u, u_f > \nabla\bar{W} = \\
&= -\nabla\bar{W}(1 + \frac{< u, u_f >}{c^2}) + \frac{1}{c}\frac{\partial}{\partial t}(-\frac{\bar{W}}{c}u_f) - \frac{1}{c^2}u \times (u_f \times \nabla\bar{W}) = \\
&= -q\nabla\varphi(1 + \frac{< u, u_f >}{c^2}) - \frac{q}{c}\frac{\partial}{\partial t}(\frac{\varphi}{c}u_f) + \frac{q}{c}u \times (\nabla \times \frac{\varphi u_f}{c}) = \\
&= -q\nabla\varphi(1 + \frac{< u, u_f >}{c^2}) - \frac{q}{c}\frac{\partial A}{\partial t} + \frac{q}{c}u \times (\nabla \times A), \quad (4.5)
\end{aligned}$$

where we denoted $u := dR(t)/dt$, $u_f := dR_f(t)/dt$, $\nabla := \partial/\partial R = -\partial/\partial R_f$ and $A := \varphi u_f/c$ to be the related magnetic potential. Since we have already shown that the Lorentz force

$$F := \frac{d}{dt}(-\frac{\bar{W}}{c^2}u) = \frac{d}{dt}\left(\frac{m_0 u}{\sqrt{1-\frac{u^2}{c^2}}}\right)$$

is given by expression (4.5), it can be rewritten in the form

$$\begin{aligned}
F &= \frac{d}{dt}\left(\frac{m_0 u}{\sqrt{1-\frac{u^2}{c^2}}}\right) = qE + \frac{q}{c}u \times B - \frac{q}{c^2}\nabla\varphi < u, u_f > = \\
&= qE + \frac{q}{c}u \times B - \frac{q}{c}\nabla < u, A >, \quad (4.6)
\end{aligned}$$

which was derived also in [18] and where we put, by definition, $E := -\nabla\varphi - \frac{1}{c}\frac{\partial A}{\partial t}$, $B := \nabla \times A$, being respectively the suitable electric and magnetic vector fields.

The resulting expression (4.6) is almost completely equivalent to the well-known [6] classical Lorentz force expression F up to the additional "inertial" term

$$F_c := -\frac{q}{c}\nabla < u, A >, \quad (4.7)$$

which is absent in the Einstein special relativistic theory. Namely, owing to the absence of the term (4.7) the classical relativistic Lorentz force expression in the four-vector form was not invariant with respect to any reference frame transformations, except inertial ones. And, as was noticed in [18, 9], owing only to this crucial fact A. Einstein introduced the Lorentz transformations and related with them visible length shortening and time slowing effects! Moreover, they gave rise to such strange enough and non-adequate notions as non-Euclidean time-spaces [25, 34, 35, 9], black holes [43, 44, 25, 38] and some other singular and nonphysical objects.

Now based on the Lorentz force expression (4.6) we can easily obtain the corresponding energy conservation law for a moving charged point particle q and the ambient magnetic field:

$$\frac{dE_0^2}{dt} = -2c < \nabla_{R_f}\bar{W}, qA > = -\frac{dE_f^2}{dt_f}, \quad (4.8)$$

where we put, by definition, that

$$E_0^2 := \bar{W}^2 - c(p^{'})^2, \qquad E_f := \bar{W}|_{t=t_f} \quad (4.9)$$

where $p' := p - \frac{q}{c}A$ is the *"shifted"* momentum of the charged point particle q in the external magnetic field $B = \nabla \times A$, and took into account that the parameter $R_f := R_f(t)|_{t=t_f} \in \mathbb{R}^3$ and the evolution parameter $t_f = t \in \mathbb{R}$ corresponds to the change of the only magnetic field potential energy E_f. The obtained expressions (4.8) and (4.9) take, evidently, into account the natural balance of energies belonging to both the moving charged point particle q and the ambient magnetic field.

5. Quantum mechanics backgrounds revisited

We will start from relation (3.21) rewritten in the following form:

$$E_0^2 = \bar{W} - p^2c^2, \quad dE_0/dt = 0, \tag{5.1}$$

and make the following canonical quantization replacements:

$$p \to \hat{p} := \frac{\hbar}{i}\nabla, \quad E_0 \to \hat{E}_0 := -\frac{\hbar}{i}\frac{\partial}{\partial t}, \quad \bar{W} \to \bar{W}\circ, \tag{5.2}$$

where $\hat{p} = \frac{\hbar}{i}\nabla$ is the standard spatial translation generator in $\mathbb{R}^3$, $\hat{E}_0 = -\frac{\hbar}{i}\frac{\partial}{\partial t}$ is the standard time translation operator along the real time axis $\mathbb{R}$ and $\bar{W}\circ$ is a usual scalar multiplication operator on the function $\bar{W} : \mathbb{R}^3 \to \mathbb{R}$, all acting in the Hilbert space $\mathcal{H} := L_2(\mathbb{R}^3; \mathbb{C})$ with the standard scalar product $(\cdot, \cdot)$. As an elementary result of this replacement we can easily write, following the quantization recipes similar to those from [3, 6, 4, 5], that the observable squared energy E_0^2 is the average value

$$E_0^2 := (\hat{E}_0\psi, \hat{E}_0\psi) = (\psi, (\bar{W}^2 + \hbar^2c^2\nabla^2)\psi), \tag{5.3}$$

being realized on the one-particle quantum mechanical state vector $\psi \in \mathcal{H}$. Taking into account representation (5.3) and assuming that the Planck constant $\hbar \to 0$, we now will try to factorize the linear differential operator $\bar{W}^2 + \hbar^2c^2\nabla^2$, well defined on a suitable dense linear subset $D(\hat{E}_0) \subset \mathcal{H}$, in the following *a priori* nonnegative canonical form:

$$\bar{W}^2 + \hbar^2c^2\nabla^2 = \hat{P}^+\hat{P}, \tag{5.4}$$

where the sign "+ " means the standard conjugation operation in the Hilbert space $\mathcal{H}$. It is easy to find from (5.4) that

$$\hat{P} := \hat{S}(1 + \hbar^2c^2\bar{W}^{-1} \circ \nabla^2 \circ \bar{W}^{-1})^{1/2}\bar{W}\circ \tag{5.5}$$

with an arbitrary unitary operator $\hat{S} : \mathcal{H} \to \mathcal{H}$, satisfying the usual condition $\hat{S}^+\hat{S} = 1$. As a result, we obtain from (5.3) and (5.4) that

$$(\hat{E}_0\psi, \hat{E}_0\psi) = (\psi, \hat{P}^+\hat{P}\psi) = (\hat{P}\psi, \hat{P}\psi). \tag{5.6}$$

Thereby, based on expression (5.6), we can derive the following Schrödinger type linear evolution equation:

$$\hat{E}_0\psi := i\hbar\frac{\partial\psi}{\partial t} = \hat{U}\hat{P}\psi \tag{5.7}$$

for all $t \in \mathbb{R}$, where $\hat{U} : \mathcal{H} \to \mathcal{H}$ is some unitary operator to be determined later.

Let us now symbolically calculate the operator expression (5.5) up to the symbolic operator accuracy $O(\hbar^4)$:

$$\hat{P} = \hat{S}(1 + \hbar^2 c^2 \bar{W}^{-1} \circ \nabla^2 \circ \bar{W}^{-1})^{1/2} \bar{W} = \hat{S}(1 + \frac{\hbar^2 c^2}{2\bar{W}} \nabla^2 \circ \bar{W}^{-1})\bar{W} + O(\hbar^4). \tag{5.8}$$

Having substituted result (5.8) into (5.7) one obtains (up to the operator accuracy $O(\hbar^4)$) the next evolution equation:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{U}\hat{S}(-\frac{\hbar^2}{2(-\bar{W}/c^2)} \nabla^2 + \bar{W})\psi. \tag{5.9}$$

Taking into account that the energy operator $\hat{E}_0 = -\frac{\hbar}{i}\frac{\partial}{\partial t}$ is formally self-adjoint, from a physical point of view we need to choose in (5.9) that $\hat{U} := \hat{S}^{-1}$, as the operator $\hat{H} := -\frac{\hbar^2}{2(-\bar{W}/c^2)}\nabla^2 + \bar{W}$ is formally self-adjoint.

Recalling now that owing to (3.9) and (3.18) the expression $-\bar{W}/c^2 := m(u) = m_0/\sqrt[2]{1 - \frac{u^2}{c^2}}$ is the dynamical relativistic mass of a particle, whose quantum motion is under study, equation (5.9) can be finally rewritten in the following classical self-adjoint Schrödinger type form:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\ \psi := -\frac{\hbar^2}{2m(u)} \nabla^2 \psi + \bar{W}\psi, \tag{5.10}$$

where $\hat{H} : \mathcal{H} \to \mathcal{H}$ is the corresponding self-adjoint Hamiltonian operator for a relativistic point particle under an external potential field. It is easy to observe that equation (5.10) in the classical non-relativistic case when the velocity $c \to \infty$ reduces to the well-known classical Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}_0\ \psi := -\frac{\hbar^2}{2m_0} \nabla^2 \psi + \bar{W}\psi, \tag{5.11}$$

derived at the beginning of the past century from completely different points of view by the great physicists Schrödinger, Heisenberg and Dirac.

Now we proceed to the similar quantization procedure of expression (4.9) for the case when the magnetic potential is not equal to zero. Making use of the standard quantization scheme (5.2) we obtain, by definition, that the average squared energy value E_0^2 on a suitable quantum state vector $\psi \in \mathcal{H}$ equals

$$E_0^2 := (\hat{E}_0\psi, \hat{E}_0\psi) = (\psi, [\hat{W}^2 - c^2(\frac{\hbar}{i}\nabla + \frac{q}{c}\hat{A})^2]\psi), \tag{5.12}$$

where, as before, $\hat{W} := \bar{W}\circ$ and $\hat{A} := \bar{A}\circ$ are the scalar multiplication operators. The operator expression in the squared brackets on the right-hand side of (5.12) can easily be represented in the following strongly nonnegative form:

$$\hat{W}^2 - c^2(\frac{\hbar}{i}\nabla + \frac{q}{c}\hat{A})^2 := \hat{P}^+\hat{P}, \tag{5.13}$$

where, as above, one can take

$$\hat{P} := [1 - \hat{W}^{-1}(\frac{\hbar}{i}\nabla + \frac{q}{c}\hat{A})^2 \hat{W}^{-1}]^{1/2}\hat{W}. \tag{5.14}$$

Then from equation (5.12) and expression (5.13) we can write down the following "magnetic" Schrödinger type evolution equation:

$$i\hbar\frac{\partial\psi}{\partial t} := \hat{P}\psi, \tag{5.15}$$

which gives rise, under simultaneous conditions $\hbar \to 0$ and $c \to \infty$, if the product $c\hbar = const$, to the following result:

$$i\hbar\frac{\partial\psi}{\partial t} := \hat{H}\psi = [\frac{1}{2m(u)}(\frac{\hbar}{i}\nabla + \frac{q}{c}\hat{A})^2 + W_0 + q\varphi]\psi \tag{5.16}$$

with accuracy $O(\hbar^4)$. Equation (5.16) reduces to the well-known [3] classical "magnetic" Schrödinger type evolution equation with accuracy $O(\hbar^4)$:

$$i\hbar\frac{\partial\psi}{\partial t} := \hat{H}_0\psi = [\frac{1}{2m_0}(\frac{\hbar}{i}\nabla + \frac{q}{c}\hat{A})^2 + W_0 + q\varphi]\psi, \tag{5.17}$$

where, as before, we put $m_0 = -\bar{W}_0/c^2$, since the potential energy relationship $|q\varphi/\bar{W}_0| << 1$.

The results obtained above convey very eloquently that our quasiclassical approach to the description of the vacuum field structure, as the Planck constant $\hbar \to 0$ and the light velocity $c \to \infty$, give rise to the related classical quantum Schrödinger dynamics very naturally, being simultaneously deeply physically motivated. It is also worthy to mention that in all of our derivations we have used no Lorentz invariance in spite of the fact that the dynamical mass $m = m_0/\sqrt[2]{1 - \frac{u^2}{c^2}}$ is expressed by means of a suitable Lorentz factor. It is, evidently, a by-product result of the *second* field theory principle equation (2.2), which is *a priori* Lorentz invariant.

To finish the discussion we make here a plausible claim that the suitable completely relativistic quantum field theoretic analysis of our dynamical vacuum field model via the well-known Dirac type factorization approach [3] will necessary shed new light on the so complicated and sophisticated nature of the quantum world of elementary particles.

Concerning the results described above we could state that the vacuum field theory approach of [18, 39, 9, 12, 19, 32, 20] to fundamental physical phenomena appears to be really a powerful tool in the hands of researchers, who wish to penetrate into the hidden properties of the surrounding Universe. As the microscopical quantum level of describing the vacuum field matter structure is, with no doubt, very important, we see the next challenging steps, both in understanding the backgrounds of quantum processes within the approaches devised in [9, 18, 39] and in this work, and in deriving new physical relationships, which will help us to explain the Nature more deeply and adequately.

Acknowledgments

The authors are cordially thankful to the Abdus Salam International Centre for Theoretical Physics in Trieste, Italy, for the hospitality during their research 2007-2008 scholarships. A.P. is especially grateful to Profs. P.I. Holod (Kyiv, UKMA), J.M. Stakhira (Lviv, NUL), B.M.

Barbashov (Dubna, JINR), Z. Kákol (Kraków, AGH), J. Sławianowski (Warsaw, IPPT), Z. Peradzyński (Warsaw, UW) and M. Błaszak (Poznań, UP) for fruitful discussions, useful comments and remarks. Last but not least thanks go to Prof. O.N. Repchenko for the discussion of some controversial vacuum field theory aspects and to Mrs. Dilys Grilli (Trieste, Publications Office, ICTP) for professional help in preparing the manuscript for publication.

References

[1] Feynman R. P M. . Lectures on gravitation. Notes of California Inst. of Technology, 1971

[2] Feynman R., Leighton R. and Sands M. The Feynman lectures on physics. Electrodynamics, v. 2, Addison-Wesley, Publ. Co., Massachusetts, 1964

[3] Dirac P.A.M. The principles of quantum mechanics. Second edition. Oxford, Clarendon Press, 1935

[4] Bogolubov N. and Shirkov D. Introduction to the theory of quantized fields. Interscience, New York, 1959

[5] Bjorken J.D. and Drell S.D. Relativistic quantum fields. Mc Graw-Hill Book Co., NY, 1965

[6] Akhiezer A.I. and Berestecky V.B. Quantum electrodynamics. "Nauka" Publisher, Moscow, 1969

[7] Schwinger J. Quantum electrodynamics. Dover Publ. Inc. New York, 1958

[8] de Gennes P.-G. Superconductivity of metals and alloys, Benjamin, USA, 1964

[9] Brillouin L. Relativity reexamined. Academic Press Publ., New York and London, 1970

[10] Brillouin L. and Lucas R. Journ. Phys. Radium, 27, 1966, p. 229

[11] Heaviside O. Electromagnetic Theory. New York, 1893, p. 115-118

[12] Carstoiu J. Compt. Rend., 268, 1969, p. 201

[13] Markov M.A. The Mach principle and physical vacuum in general relativity. Problems of Theoretical Physics. Essays dedicated to Nikolai N. Bogolubov on the occasion of his sixtith birthday. "Nauka" Publisher, Moscow, 1969, p. 26-27

[14] Faddeev L.D. Hamiltonian approach to the gravity. Russian Physical Surveys, "Nauka" Publisher, Moscow, 1986

[15] Maxwell J.C. Oeuvres on Electromagnetic Theory. Cambrridge, 1950

[16] Petrov A.Z. Proceedings of the USSR Academy of Sciences, 190, 1970, p. 305

[17] Burghardt R. Acta Phys. Austr., 32, 1970, p. 272-281

[18] Repchenko O. Field physics. Moscow, "Galeria" Publ., 2005

[19] Brans C.H. and Dicke R.H. Mach's principle and a relativistic theory of gravitation. Phys. Rev., 124, 1961, p. 925

[20] Bialynicky-Birula I. Phys Rev., 155, 1967, p. 1414; 166,1968, p. 1505

[21] Logunov A.A. Lectures on relativity theory and gravitation. "Nauka" Publisher, Moscow, 1987

[22] Fock V.A. Theory of space, time and gravity. "Nauka" Publisher, Moscow, 1955

[23] Fock V.A. Konfigurationraum und zweite Quantelung. Zeischrift Phys., Bd. 75, 1932, p. 622-647

[24] Sommerfeld A. Mechanics. v.1, New York, 1952

[25] t'Hooft G. Introduction to general relativity. Institute for Theoretical Physics Utrecht University, Princetonplein 5, 3584 CC Utrecht, the Netherlands, 2002 (www.phys.uu.nl/ thooft/lectures/genrel.pdf)

[26] Taylor E.F. and Wheeler J. A. Space-time physics. Freeman and Company, San Francisco and London, 1966

[27] Pauli W. Theory of relativity. Oxford Publ., 1958

[28] Bridgeman P.W. Reflections of a Scientist. Philosophical Library, New York, 1955

[29] Weinstock R. New approach to special relativity. Am. J. Phys., 33, 1965, p. 640-645.

[30] Lee A.R., Kalotas T.M. Lorentz transformations from the first postulate. Am. J. Phys., 43, 1975, p. 434-437.

[31] Levy-Leblond J.M. One more derivation of the Lorentz transformation. Am. J. Phys., 44, 1976, 271-277.

[32] Mermin N.D. Relativity without light. Am. J. Phys., 52, 1984, p. 119-124.

[33] Sen A. How Galileo could have derived the special theory of relativity. Am. J. Phys., 62, 1994, p. 157-162.

[34] Mermin D.N. It's About Time: Understanding Einstein's Relativity, Princeton, NJ., Princeton University Press, 2005

[35] Barbashov B. M. and Nesterenko V. V. Introduction to the Relativistic String Theory, World Scientific, Singapore, 1990

[36] Collins H. Gravity's Shadow: The Search for Gravitational Waves, Chicago: University of Chicago Press, 2004

[37] Marsden J. and Chorin A. Mathematical foundations of the mechanics of liquid. Springer, New York, 1993

[38] Barbashov B.M., Efimov G.V. and others. (Editors) Selected Problems of Modern Physics. Proceedings of the XII-th International Conference on Selected Problems of Modern Physics. Section 1, Dubna, 2003

[39] Prykarpatsky A.K. and Bogolubov N.N. (Jr.) The field structure of vacuum, Maxwell equations and relativity theory aspects. Preprint ICTP, Trieste, IC/2008/051 (http://publications.ictp.it; arXiv lanl: 0807.3691v.8 [gr-gc] 24.08.2008)

[40] Bogolubov N.N. (Jr.), Prykarpatsky A.K. , Golenia J. and Taneri U. Introductive backgrounds of modern quantum mathematics and application to nonlinear dynamical systems. Intern. Journal of Theor. Physics, v.47, N2, 2008; Preprint ICTP, Trieste, IC/2007/108 (http://publications.ictp.it)

[41] Bogolubov N.N. (Jr.), Prykarpatsky A.K. and Taneri U. Quantum field theory with application to quantum nonlinear optics. World Scientific Publishing Co., 2003

[42] Berezin F.A. The method of second quantization. "Nauka" Publisher, Moscow, 1965

[43] Damour T. General Relativity and Experiment. Proceedings of the XI International Congress on Mathematical Physics, Intern. Press, 1995. Proceedings of the International Conference

[44] Green B. The elegant Universe. Vintage Books Inc., New York, 1999