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NEUTRON TRANSPORT IN FINITE RANDOM MEDIA WITH PURE-TRIPLET SCATTERING

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Abstract

The solution of the one-speed neutron transport equation in a finite slab random medium with pure-triplet anisotropic scattering is studied. The stochastic medium is assumed to consist of two randomly mixed immiscible fluids. The cross section and the scattering kernel are treated as discrete random variables, which obey the same statistics as Markovian processes and exponential chord length statistics. The medium boundaries are considered to have specular reflectivities with angular-dependent externally incident flux. The deterministic solution is obtained by using Pomraning-Eddington approximation. Numerical results are calculated for the average reflectivity and average transmissivity for different values of the single scattering albedo and varying the parameters which characterize the random medium. Compared to the results obtained by Adams et al. in case of isotropic scattering that based on the Monte Carlo technique, it can be seen that we have good comparable data.

Keywords: Neutron transport, Binary random media, Pure-triplet scattering.

INTRODUCTION

There has been increasing interest in formulating linear kinetic theory and particle transport descriptions in a random mixture of two immiscible, nonparticipating materials. The review by Pomraning [1] summarizes the work on the transport of neutral particles in a random mixture of two immiscible, nonparticipating materials and provides an extensive list of references to earlier work. By a random, we mean that the properties of the background material of the medium, with which the particles interact, are only known in a statistical sense.

Due to the importance of this task, many problems [1-13] are formulated and solved. This importance is arisen in many applications. Applications can vary over a wide range from astrophysical clouds in the interstellar medium to pebble bed nuclear reactors [2]. In the case of pebble bed nuclear reactor [3], small spheres of uranium coated in carbon are randomly mixed in the reactor core. The neutron transport in boiling water reactors gives an interesting example of applications. The water, which acts as both a coolant and moderator, is in two-fluid random state (liquid and vapor). Also, the shielding calculations [4] need a statistical transport treatment to obtain an accurate measurement of the shield effectiveness. Another application in the astrophysical setting is the radiative transfer in vortices or jets and through partially cloudy atmosphere which has been attracting a great deal of interest [5], [6]. Here the clouds and clear sky are treated as a two component random mixture. Heat transfer in semiconductors needs a statistical calculations, whereas the semiconductor material consists of two or more materials. In medical applications, the propagation of Laser light in tissues is a question of growing concern [7].

Levermore et al. [8] developed a formalism to treat a certain class of these problems. Pomraning [9] used this formalism to treat the purely scattering rod and planar geometry problems. Two of the most basic assumptions in all of the theoretical analysis done to date in stochastic media transport are that the chord length distributions in the two materials through which the radiation is transported have exponential distributions and that the transport processes are Markovian. The Levermore-Pomraning (L-P) closure [10] for an effective transport equation that can be solved for the mean radiation field depends on these assumptions. The assumption of Markovian processes is exact only in a pure absorbing material [11]. Whenever scattering processes are included, whether through explicit transport terms or through the effective scattering of photons through material absorption and re-emission, the transport of radiation is no longer in a single direction; all directions are coupled. Adams et al. [12] showed that the L.P closure worked well in 1D rods and slabs for pure absorbers. As scattering was added the closure was less accurate.

In this paper, we present the solution of the one-speed neutron transport equation in a stochastic (random) finite slab medium. The discrete random medium is assumed to be consist of two randomly mixed immiscible fluids labeled by 1 and 2. That is, at any point in space and time one or the other component of the mixture is present in its pure state, according to some pre-scribed statistics which we assume to be described as two state homogeneous Markov process. For any given physical realization of the statistics, this flow is described by the deterministic transport equation. The medium boundaries are considered to have specular reflectivities with angular-dependent externally incident flux. Pomraning-Eddington approximation is used to obtain the deterministic solution of the our problem. The formalism obtained by Levermore et al. [8] and Pomraning [9] is used to average the obtained deterministic solution. The effect of anisotropy is very important in studying the neutron transport problems. So, in this work, we include a higher order of anisotropic scattering in finite random medium. This higher order of anisotropic scattering is called pure-triplet scattering. Numerical results are computed for the average reflectivity and transmissivity for different values of the single scattering albedo and varying the parameters that characterize the random medium. Compared to the results available in the literature [12] in case of isotropic scattering, it can be seen that we have good comparable data.

ANALYSIS

The starting point of the analysis is the one-speed neutron transport equation in a planar medium, which is given by [13]

$$\left[\mu\frac{\partial}{\partial z}+\sigma(z)\right]\Psi(z,\mu)=\int_{-1}^{1}\sigma_s(z,\mu'\to\mu)\Psi(z,\mu)\,d\mu' \qquad \textbf{(1)}$$

$$0\le z\le B,\quad -1\le\mu\le 1$$

Here $\Psi(z,\mu)$ is the radiation intensity, with z, and μ representing the spatial, and angular variables, respectively. The quantity $\sigma(z)$ is the cross section function, and $\sigma_s(z,\mu'\to\mu)$ is the scattering kernel. The quantities σ and σ_s in Eq.(1) are treated as discrete random variables, which obey the same statistics.

The key to this analysis is the introduction of the effective variable, namely, the optical depth space variable, defined by [13]

$$\tau(z) = \int_0^z \sigma(z)dz \tag{2}$$

where the optical thickness, L, of the medium is given by

$$L(B) = \int_0^B \sigma(z)dz \tag{3}$$

In terms of t, Eq.(1) becomes

$$\left(\mu\frac{\partial}{\partial\tau}+1\right)I(\tau,\mu) = \int_{-1}^{1}\omega(\tau,\mu'\to\mu)I(\tau,\mu)d\mu' \tag{4}$$

where

$$I(\tau,\mu) \equiv \Psi(z,\mu) \tag{5}$$

and

$$\omega(\tau,\mu'\to\mu) = \sigma_s(z,\mu'\to\mu)/\sigma(z) \tag{6}$$

which is known as the scattering albedo of the medium.

In this paper we consider the one-speed neutron transport equation through a finite plane-parallel homogeneous medium with anisotropic scattering, viz

$$\left(\mu\frac{\partial}{\partial\tau}+1\right)I(\tau,\mu) = \frac{\omega}{2}\int_{-1}^{1}P(\mu,\mu')I(\tau,\mu)d\mu' \tag{7}$$

$$0 \le \tau \le L, \quad -1 \le \mu \le 1$$

subjected to the boundary conditions

$$I(0,\mu) = \Gamma(\mu) + \rho_1^s I(0,-\mu) \tag{8.a}$$

$$I(L,-\mu) = \rho_2^s I(L,\mu) \tag{8.b}$$

where $\Gamma(\mu)$ is the externally-incident flux on the boundary $\tau = 0$, and ρ_i^s, (i = 1 and 2), are the specular reflectivities of the boundaries.

The anisotropic scattering phase function, $P(\mu,\mu')$, can be expanded by terms of Legendre polynomial functions as [22]

$$P(\mu,\mu') = \sum_{n=0}^{\infty} a_n P_n(\mu) P_n(\mu')$$

$$= 1 + a_1\mu\mu' + a_2P_2(\mu)P_2(\mu') + a_3P_3(\mu)P_3(\mu') + \cdots \tag{9}$$

with $P_n(\mu)$ is the Legendre polynomial functions.

Here a_n can be called the anisotropy scattering coefficients. The first term of this expansion is called isotropic scattering ($a_1 = a_2 = a_3 = 0$). The probability of particle scattering is equal for all directions in isotropic scattering. The second term in this expansion corresponds to the linearly anisotropic scattering ($a_1 \neq 0$, $a_2 = a_3 = 0$). The third term corresponds to the quadratic scattering (Rayleigh scattering $a_1 = 0$, $a_2 = 0.5$, $a_3 = 0$). The fourth term corresponds to the triplet scattering ($a_1 = a_2 = 0$, $a_3 \neq 0$). The scattering of neutrons from nucleus is considered as pure-triplet [15].

To proceed, we assume that σ and σ_s obey the same statistics in the sense that $\omega = \sigma_s/\sigma$ is non-stochastic [13]. This means that ω takes the same value inside the two immiscible fluids of the medium. Hence the transport problem described by equations (7) and (8) is only stochastic through the optical depth variable τ and the optical size of the system *L*. The statistics of the problem are entirely described by the joint probability density *P(*τ *;L; z;B)*, defined such that PdτdL is the probability that for a given geometric position z and a given geometric system thickness B, the position z corresponds to an optical depth lying between τ and τ + dτ and the system thickness *B* corresponds to an optical thickness lying between L and *L*+ *dL*. The ensemble-averaged intensity is thus given by [8], [13]

$$\langle I(z,B,\mu)\rangle = \int_0^\infty dL \int_0^\infty d\tau P(\tau,L;z,B) I(\tau,L,\mu) \tag{10}$$

All of the statistical informations are embodied in the joint probability density function *P(*τ*,L;z,B)*. From the transport point of view, this function is assumed to be known. In the special case of binary homogeneous Markov statistics, one has the near separable form [13] denoting the two materials making up the binary mixture by subscripts 1 and 2

$$P(\tau,L;z,B) = \sum_{i=1}^{2} p_i f_i(\tau;z) f_i(L-\tau;B-z) \tag{11}$$

where p_i represents the probability of finding material *i* at any point in the system. In terms of mean slab thickness λ_i of the alternating slabs of the two materials making up the planar system, one has [8]

$$p_i = \frac{\lambda_i}{\lambda_1 + \lambda_2} \tag{12}$$

The function $f_i(\tau;z)$ in Eq.(11) is the probability density function defined such that $f_i d\tau$ is the probability that the planar system has an optical depth lying between τ and $\tau + d\tau$, given that the geometric depth is *z* and given that the point *z* lies in material *i*. For the case of homogeneous binary Markov statistics being discussed, the function $f_i(\tau;z)$ is known in closed analytic form [13]. In view of Eq.(11), Eq.(10) becomes

$$\langle I(z,B,\mu)\rangle = \sum_{i=1}^{2} p_i \int_0^\infty d\tau f_i(\tau;z) \int_0^\infty dL f_i(L-\tau;B-z) I(\tau,L,\mu) \tag{13}$$

Such an average is easily computed for homogeneous binary Markovian statistics. We consider a pure exponential in optical depth space variable, $exp(-k\tau)$, with *k* being a constant, and ask for the ensemble average of this exponential. If we denote this ensemble-average by the function $E_i(k;z)$, we shall have

$$E_i(k;z) = \int_0^\infty f_i(\tau;z)\exp(-k\tau)\,d\tau \tag{14}$$

It was found by Pomraning [13] that

$$E_i(k;z) = \gamma_i \exp(-m^+ z) + (1-\gamma_i)\exp(-m^- z) \tag{15}$$

with $E_i(Nk;0) = 1$, where *N* is the factor times *k*,

$$\gamma_i = \frac{k\sigma_j + (1/\lambda_c) - m^+}{m^- - m^+}, \quad i = 1,2, \quad j = 1,2, \quad i \neq j \tag{16}$$

and

$$2m^{\pm} = (\sigma_2 + \sigma_1)k + \frac{1}{\lambda_c} \mp \sqrt{(\sigma_2 - \sigma_1)^2 k^2 - \frac{2k}{\lambda_c}(p_2 - p_1)(\sigma_2 - \sigma_1) + \frac{1}{\lambda_c^{\,2}}} \tag{17}$$

where

$$\frac{1}{\lambda_c} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \tag{18}$$

The above analysis can be used to evaluate the ensemble-average solution of the problem under consideration.

Deterministic Solution

For pure-triplet scattering, the phase function (9) reduces to

$$P(\mu,\mu') = 1 + a_3 P_3(\mu) P_3(\mu') \qquad \textbf{(19)}$$

Using Eq.(19) in Eq.(7) we get

$$\left(\mu\frac{\partial}{\partial\tau}+1\right)I(\tau,\mu) = \frac{\omega}{2}\left[E(\tau)+a_3P_3(\mu)\int_{-1}^{1}P_3(\mu')I(\tau,\mu)\,d\mu'\right] \qquad \textbf{(20)}$$

For solving the source-free problem Eq.(20) with the boundary conditions (8), we could use the Pomraning-Eddington approximation [16,17]

$$I(\tau,\mu) = \Sigma(\tau,\mu)E(\tau) + O(\tau,\mu)F(\tau) \qquad \textbf{(21)}$$

where $E(\tau)$ is the radiant neutron energy and $F(\tau)$ is the net neutron flux, defined as

$$E(\tau) = \int_{-1}^{1} I(\tau,\mu)\,d\mu, \qquad \textbf{(22)}$$

$$F(\tau) = \int_{-1}^{1} \mu I(\tau,\mu)\,d\mu \qquad \textbf{(23)}$$

and $\Sigma(\tau,\mu)$, $O(\tau,\mu)$ are even and odd functions in μ and are slowly varying functions in τ which are normalized as

$$\int_{-1}^{1} \Sigma(\tau,\mu)\,d\mu = \int_{-1}^{1} \mu O(\tau,\mu)\,d\mu = 1 \qquad \textbf{(24)}$$

Substituting Eq.(21) in Eq.(20), integrating over $\mu \in$ [-1, 1], and using Eq.(24) one gets

$$\frac{dF(\tau)}{d\tau} + \alpha E(\tau) = 0, \qquad \alpha = 1-\omega \qquad \textbf{(25)}$$

In the same way, multiplying of Eq.(20) by μ prior to integration over $\mu \in$ [-1, 1] gives

$$\frac{d}{d\tau}\left[D(\tau)E(\tau)\right] + F(\tau) = 0 \qquad \textbf{(26)}$$

where

$$D(\tau) = \int_{-1}^{1} \mu^2 \Sigma(\tau,\mu)\,d\mu \qquad \textbf{(27)}$$

Substituting Eq.(21) in Eq.(20), and separating the even and odd terms gives two equations determining the even and odd functions, respectively, as

$$\Sigma(\tau,\mu) = \frac{\omega}{2}\left(\frac{1+\alpha\chi\mu^2}{1-k^2\mu^2}\right) \qquad \textbf{(28)}$$

and

$$O(\tau,\mu) = \frac{\omega\mu}{2\alpha}\left(\frac{k^2+\alpha\chi}{1-k^2\mu^2}\right) \qquad \textbf{(29)}$$

where k is the roots of the transcendental (characteristic) equation

$$k = \frac{\omega}{y} \tanh^{-1} k \quad \textbf{(30)}$$

with

$$y = \frac{k^4 + \omega\left[\chi_1\left(k^4/3 + k^2\right) + \chi_2\left(k^4/5 + k^2/3 + 1\right)\right]}{k^4 + \chi_1 k^2 + \chi_2} \quad \textbf{(31)}$$

and

$$\chi = \frac{a_3}{4}(5d - 3)\left(5\mu^2 - 3\right) \quad \textbf{(32.a)}$$

$$\chi_1 = -\frac{3}{4}\alpha a_3 (5d - 3) \quad \textbf{(32.b)}$$

$$\chi_2 = \frac{5}{4}\alpha a_3 (5d - 3) \quad \textbf{(32.c)}$$

where

$$d = \int_{-1}^{1} \mu^3 O(\tau, \mu)\, d\mu \quad \textbf{(33)}$$

whose value can be obtained as

$$d = \frac{1}{k^2} - \frac{\omega}{3(1-\omega)} \quad \textbf{(34)}$$

Since $\Sigma(\tau,\mu)$ is assumed to be a slowly varying function in τ, then $D(\tau)$ is also a slowly varying variable in τ. Therefore, Eqs.(25) and (26) leads to the diffusion-like equation

$$\left(\frac{d^2}{d\tau^2} - k^2\right) E(\tau) = 0 \quad \textbf{(35)}$$

whose solution is given by

$$E(\tau) = A\exp(k\tau) + B\exp(-k\tau) \quad \textbf{(36)}$$

and A & B are constants to be determined. Moreover, we can get

$$F(\tau) = -\frac{\alpha}{k}\left[A\exp(k\tau) - B\exp(-k\tau)\right] \quad \textbf{(37)}$$

Using equations (28), (29), (36), and (37) in Eq.(21), one gets the deterministic solution as

$$I(\tau, \mu) = AG_+(\mu)\exp(k\tau) + BG_-(\mu)\exp(-k\tau) \quad \textbf{(38)}$$

where

$$G_\pm(\mu) = \left(\frac{\omega}{2}\right) \frac{1 \mp \left(k + \frac{\alpha}{k}\chi\right)\mu + \alpha\chi\mu^2}{1 - k^2\mu^2} \quad \textbf{(39)}$$

To determine the constants A and B, a weight function $W(\mu)$ is introduced in order to force the boundary conditions (8) to be fulfilled

$$\int_0^1 d\mu W(\mu)\left[I(0,\mu) - \rho_1^s I(0,-\mu)\right] = I_0 \quad \textbf{(40)}$$

$$\int_0^1 d\mu W(\mu)\left[I(L,-\mu) - \rho_2^s I(L,\mu)\right] = 0 \quad \textbf{(41)}$$

Using Eq.(38) we obtain the two constants A and B as

$$A = \frac{I_0 \beta_{22} \exp(-2kL)}{\beta_{11}\beta_{22}\exp(-2kL) - \beta_{12}\beta_{21}} \quad \textbf{(42)}$$

$$B = \frac{I_0 \beta_{21}}{\beta_{12}\beta_{21} - \beta_{11}\beta_{22}\exp(-2kL)} \quad \textbf{(43)}$$

where

$$\beta_{11} = I_+ - \rho_1^s I_- \quad \textbf{(44.a)}$$

$$\beta_{12} = I_- - \rho_1^s I_+ \quad \textbf{(44.b)}$$

$$\beta_{21} = I_- - \rho_2^s I_+ \quad \textbf{(44.c)}$$

$$\beta_{22} = I_+ - \rho_2^s I_- \quad \textbf{(44.d)}$$

with

$$I_0 = \int_0^1 d\mu W(\mu)\Gamma(\mu) \;, \quad \textbf{(45.a)}$$

$$I_\pm = \int_0^1 d\mu W(\mu) G_\pm(\mu) \quad \textbf{(45.a)}$$

Hence the deterministic values of the reflectivity R_D and transmissivity T_D can be calculated, respectively, from

$$R_D = \int_0^1 \mu I(0,-\mu) d\mu \quad , \quad \textbf{(46)}$$

$$T_D = \int_0^1 \mu I(L,\mu) d\mu \quad \textbf{(47)}$$

to give

$$R_D = A\Upsilon_- + B\Upsilon_+ \quad , \quad \textbf{(48)}$$

$$T_D = A\Upsilon_+ \exp(kL) + B\Upsilon_- \exp(-kL) \quad \textbf{(49)}$$

where

$$\Upsilon_\pm = \int_0^1 \mu G_\pm(\mu) d\mu \quad \textbf{(50)}$$

Average Solution

The above deterministic solution given by (38) with the two equations (42) and (43), can be rewritten as

$$I(\tau,\mu) = \beta_0 \sum_{n=0}^{\infty} \beta_3^n \left\{ \beta_{21} G_-(\mu) \exp\left[-k(2nL+\tau)\right] - \beta_{22} G_+(\mu) \exp\left[-k\left(2(n+1)L-\tau\right)\right] \right\} \quad \textbf{(51)}$$

where

$$\beta_0 = \frac{\mathrm{I}_0}{\beta_{12}\beta_{21}} \quad \text{and} \quad \beta_3 = \frac{\beta_{11}\beta_{22}}{\beta_{12}\beta_{21}} \quad \textbf{(52)}$$

Therefore, the ensemble- averaged solution can be obtained by using Eq.(51) in Eq.(13), to give

$$\left\langle I(z,B,\mu) \right\rangle = \beta_0 \sum_{n=0}^{\infty} \beta_3^n \sum_{i=1}^{2} p_i \left\{ \beta_{21} G_-(\mu) \int_0^\infty d\tau\, f_i(\tau;z) e^{-(2n+1)k\tau} \int_0^\infty d\xi\, f_i(\xi;B-z) e^{-2nk\xi} \right.$$
$$\left. - \beta_{22} G_+(\mu) \int_0^\infty d\tau\, f_i(\tau;z) e^{-(2n+1)k\tau} \int_0^\infty d\xi\, f_i(\xi;B-z) e^{-2(n+1)k\xi} \right\} \quad \textbf{(53)}$$

where $\xi = L - \tau$. In terms of the ensemble-average function defined by (14)-(18), we can write the ensemble- average solution in the form

$$\langle I(z,B,\mu)\rangle = \beta_0 \sum_{n=0}^{\infty} \beta_3{}^n \sum_{i=1}^{2} p_i E_i\left((2n+1)k; z\right)\left\{\beta_{21} G_-(\mu) E_i(2nk; B-z) - \beta_{22} G_+(\mu) E_i\left(2(n+1)k; B-z\right)\right\} \quad \textbf{(54)}$$

Hence, both the average reflectivity $\langle \mathrm{R} \rangle$ and average transmissivity $\langle \mathrm{T} \rangle$ can be calculated, respectively, from

$$\langle R(B)\rangle = \int_0^1 \mu \langle I(0,-\mu)\rangle d\mu \quad , \quad \textbf{(55)}$$

$$\langle T(B)\rangle = \int_0^1 \mu \langle I(L,\mu)\rangle d\mu \quad \textbf{(56)}$$

to give

$$\langle R(B)\rangle = \beta_0 \sum_{n=0}^{\infty} \beta_3{}^n \sum_{i=1}^{2} p_i \left\{\beta_{21} \Upsilon_+ E_i(2nk; B) - \beta_{22} \Upsilon_- E_i\left(2(n+1)k; B\right)\right\} \quad \textbf{(57)}$$

and

$$\langle T(B)\rangle = \beta_0 \left(\beta_{21}\Upsilon_- - \beta_{22}\Upsilon_+\right) \sum_{n=0}^{\infty} \beta_3{}^n \sum_{i=1}^{2} p_i E_i\left((2n+1)k; B\right) \quad \textbf{(58)}$$

NUMERICAL RESULTS

The ensemble-averaged of reflectivity $\langle \mathrm{R} \rangle$ and transmissivity $\langle \mathrm{T} \rangle$ at the boundaries of the considered random medium are computed for different values of the single scattering albedo ω for pure-triplet scattering. The used parameters (λ_i, σ_i, i = 1,2) that characterize the binary discrete stochastic medium are varied according to table (1) for three different cases. These chosen values of λ_i and σ_i are used before in Refs. [9,12] and they correspond to an average cross section $\langle \sigma \rangle$ = 1. We use three different special forms of the weight function for calculations, as [16-18]

$$W_1(\mu) = \mu \quad , \quad \textbf{(59.a)}$$

$$W_2(\mu) = \frac{\mu H(\mu)}{1-k\mu} \quad , \quad \textbf{(59.b)}$$

$$W_3(\mu) = \frac{\sqrt{3}}{2}\mu\left(1+\frac{3}{2}\mu\right) \quad \textbf{(59.c)}$$

where $H(\mu)$ is the Chandrasekhar H- function which is given in an approximate form by [17,18]

$$H(\mu) = 1 - \delta \frac{1-k\mu}{1+k\mu} \quad \textbf{(60)}$$

with

$$\delta = \frac{k/(1+k) + \ln(1+k)}{\ln(1+k) - yk/\omega} \quad \textbf{(61)}$$

and y is given by Eq.(31).

The externally incident flux is assumed to be angular-dependent as

$$\Gamma(\mu) = \mu^{\ell}, \quad \ell = 0,1,2,\cdots \qquad \textbf{(63)}$$

Tables (2) and (3) tabulate the results of average reflectivity $\langle \mathrm{R} \rangle$ and transmissivity $\langle \mathrm{T} \rangle$, respectively, in case of isotropic scattering (a_3 = 0) with ω = 0.9, $\Gamma(\mu)$ = 2, and transparent boundaries ρ^{s}_{i} = 0. The results are calculated via the above three weight functions and compared with the results of Adams et al. [12] based on the Monte Carlo technique .The comparison shows a good agreement.

Further we present, graphically, the data of the average reflectivity $\langle \mathrm{R} \rangle$ and transmissivity $\langle \mathrm{T} \rangle$ in addition to their corresponding deterministic values (R_D & T_D) versus the medium thickness *B* in case of pure-triplet scattering (a_3 = 0.5) for the different cases of the parameters characterize the binary random medium. In these graphs, we have used only the Chandrasekhar weight function $W_2(\mu)$.

CONCLUSIONS

We have treated the neutron transport problem with a higher order of anisotropic scattering, namely, the pure-triplet scattering in a binary random finite medium. The discrete random medium is assumed to be consist of two randomly mixed immiscible fluids labeled by 1 and 2. That is, at any point in space and time one or the other component of the mixture is present in its pure state, according to some prescribed statistics which we assume to be described as two state homogeneous Markov process. The medium is assumed to have specular reflecting boundaries with angular-dependent externally-incident flux. We try to study the effect of randomness in the properties of the medium in the presence of pure-triplet anisotropic scattering on some physical quantities of interest like the average reflectivity $\langle \mathrm{R} \rangle$ and transmissivity $\langle \mathrm{T} \rangle$. Pomraning-Eddington approximation is used at first to obtain the corresponding deterministic solution for this higher order of anisotropic scattering. We have used three different forms for the weight function, that is used to force the assumed boundary conditions to be fulfilled. Then, the formalism obtained by Levermore and Pomraning is used to get the ensemble-average solution. Numerical results are obtained for the average reflectivity $\langle \mathrm{R} \rangle$ and transmissivity $\langle \mathrm{T} \rangle$. One can conclude the following points from the numerical results, as given in tables or in graphical form:

1- The results calculated by using the different three special forms of the weight function are comparable with each other and give a good comparison with the published data [12] (as shown in tables (2) and (3)). So we have used only one weight function in the remaining graphs of calculations. We select the Chandrasekhar weight function $W_2(\mu)$, and the numerical results for the other weight functions can be done in a similar way but we would not do it here.

2- Due to the randomness of the stochastic medium, the average reflectivity $\langle \mathrm{R} \rangle$ and transmissivity $\langle \mathrm{T} \rangle$ depend mainly on the values of the parameters (λ_i, σ_i, i = 1,2) which characterize the considered binary random medium.

3- The randomness of the stochastic medium is more effective for moderate values of the medium thickness *B*. For small and very large values of *B*, the results of the average reflectivity $\langle R \rangle$ and transmissivity $\langle T \rangle$ are very close to the corresponding deterministic ones (R_D & T_D).

Table (1) Parameters λ_i and σ_i

Case	λ_1	λ_2	σ_1	σ_2
1	99/100	11/100	10/99	100/11
2	99/10	11/10	10/99	100/11
3	101/20	101/20	2/101	200/101

Table (2) Comparison data of $\langle R \rangle$ in case of $a_3 = 0$ with $\Gamma(\mu) = 2$, $\rho_i^s = 0$ and $\omega = 0.9$

Case 1					
B	$W_1(\mu)$	$W_2(\mu)$	$W_3(\mu)$	Model*	Exact*
0.1	0.041926	0.040378	0.064567	0.0473	0.0480
1.0	0.235751	0.235345	0.240381	0.2178	0.2563
10.	0.468224	0.468989	0.453980	0.3707	0.4785
Case 2					
0.1	0.037793	0.036213	0.060929	0.0426	0.0426
1.0	0.126961	0.125923	0.141584	0.1255	0.1440
10.	0.419225	0.419779	0.408449	0.2910	0.4344
Case 3					
0.1	0.054324	0.052878	0.075421	0.0669	0.0670
1.0	0.231316	0.230837	0.236994	0.2381	0.2445
10.	0.434306	0.434903	0.422774	0.3272	0.4466

* Ref.[12]

Table (3) Comparison data of $\langle T \rangle$ in case of $a_3 = 0$ with $\Gamma(\mu) = 2$, $\rho_i^s = 0$ and $\omega = 0.9$

Case 1					
B	$W_1(\mu)$	$W_2(\mu)$	$W_3(\mu)$	Model*	Exact*
0.1	0.940331	0.942570	0.901883	0.9344	0.9341
1.0	0.620307	0.621482	0.599323	0.6267	0.5985
10.	0.016973	0.016987	0.016667	0.0237	0.0159
Case 2					
0.1	0.944813	0.947084	0.905888	0.9397	0.9398
1.0	0.783617	0.785370	0.753188	0.7733	0.7666
10.	0.196717	0.196959	0.191968	0.1945	0.1861
Case 3					
0.1	0.927047	0.929188	0.890095	0.9137	0.9136
1.0	0.617059	0.618309	0.595016	0.6086	0.6045
10.	0.108472	0.108654	0.105154	0.1195	0.1037

* Ref.[12]

Figures Captions

1. Figures (1) represent the data of the average reflectivity $\langle R \rangle$ versus the medium thickness B in the case of angular-dependent incident flux $\Gamma(\mu) = \mu^l$, $\ell = 0,1,2, \ldots,$ and $\omega = 0.9$ for:

   (a) $\ell = \rho_l^s = 0,$

   (b) $\ell = 1\ \&\ \rho_l^s = 0.5,$

   (c) $\ell = 2\ \&\ \rho_l^s = 0.25.$

2. Figures (2) represent the results of the average transmissivity $\langle T \rangle$ for the same values as in Figs.(1).

3. Figures (3) show the data of the average reflectivity $\langle R \rangle$ versus the medium thickness B for $\Gamma(\mu) = 1$ and $\rho_l^s = 0.5$ for

   (a) $\omega = 0.5,$

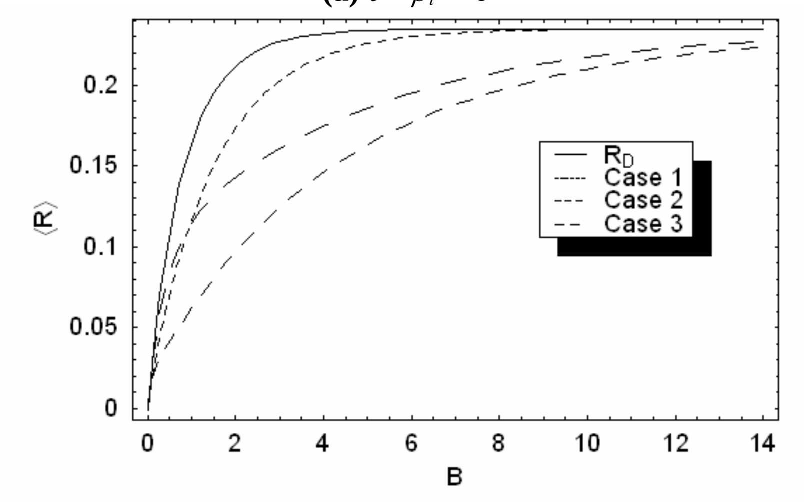
   (b) $\omega = 0.7,$

   (c) $\omega = 0.9.$

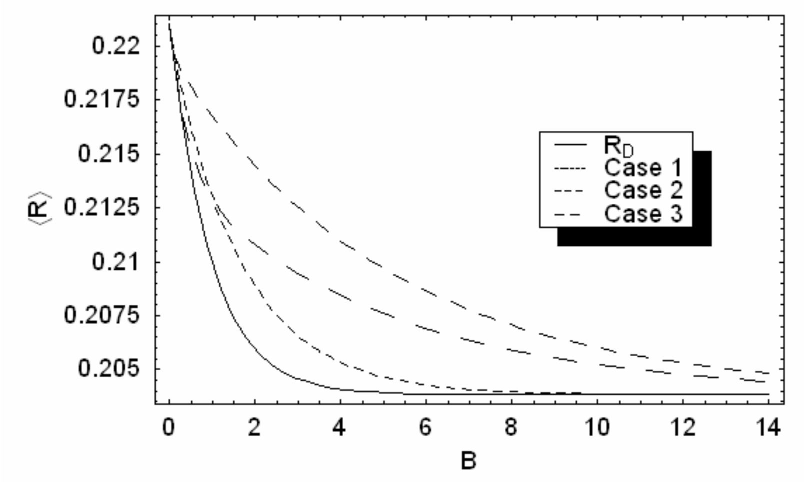
4. Figures (4) represent the results of the average transmissivity $\langle T \rangle$ for the same values as in Figs.(3).

In all graphs, in addition to the different three cases of (λ_i, σ_i, i = 1,2) of the average reflectivity $\langle R \rangle$ and transmissivity $\langle T \rangle$, the corresponding deterministic ones (R_D & T_D) are drawn.

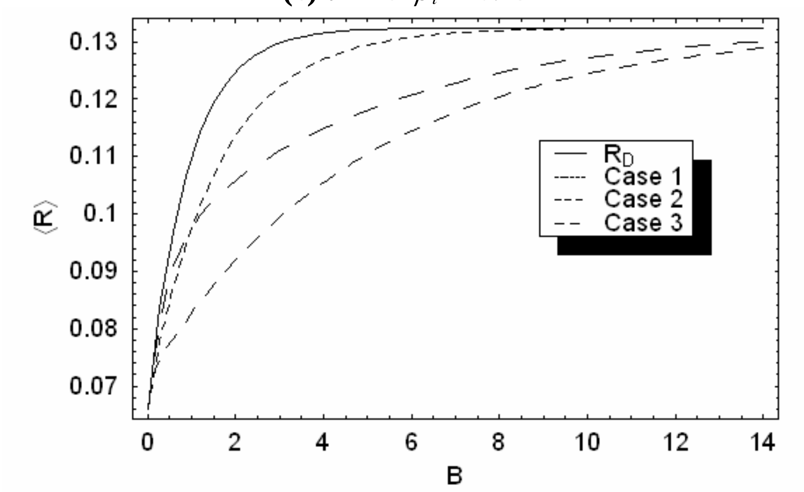
(a) $\ell = \rho_t^s = 0$



(b) $\ell = 1$ & $\rho_t^s = 0.5$

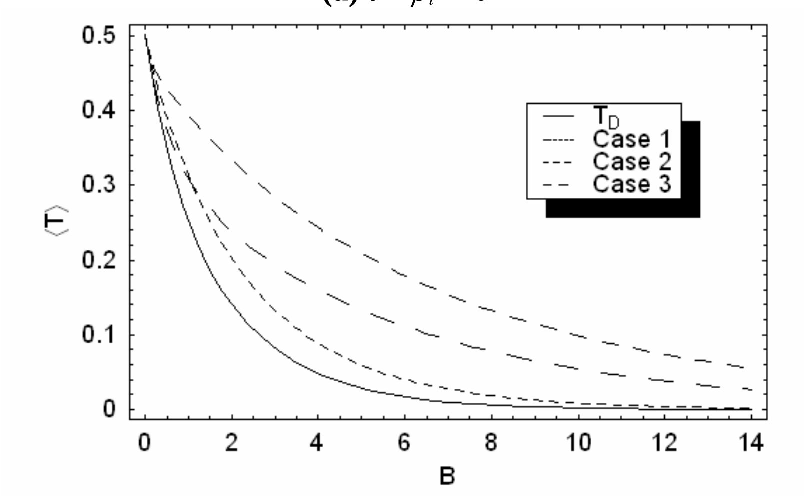


(c) $\ell = 2$ & $\rho_t^s = 0.25$

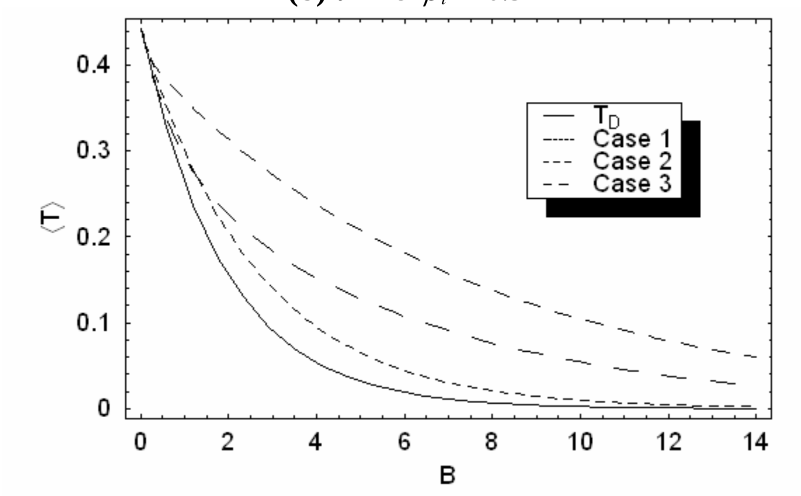


Figs.(1): The average reflectivity $\langle R \rangle$ for $\Gamma(\mu) = \mu^{l}$ & $\omega = 0.9$

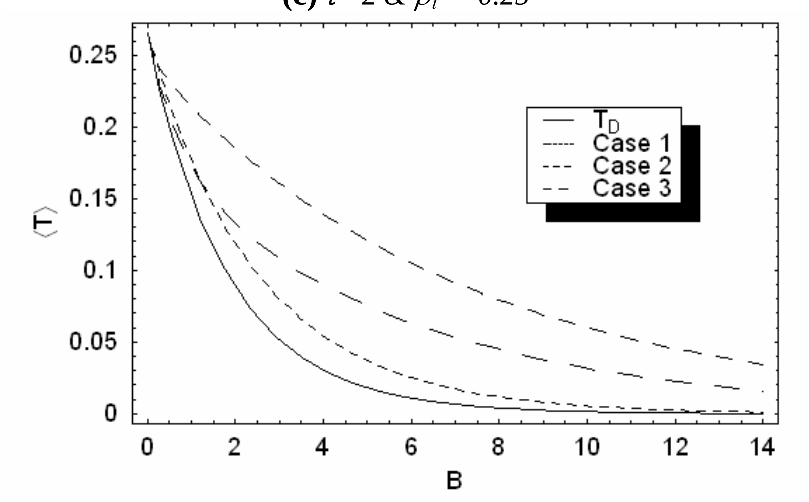
(a) $\ell = \rho_\iota^s = 0$



(b) $\ell = 1$ & $\rho_\iota^s = 0.5$

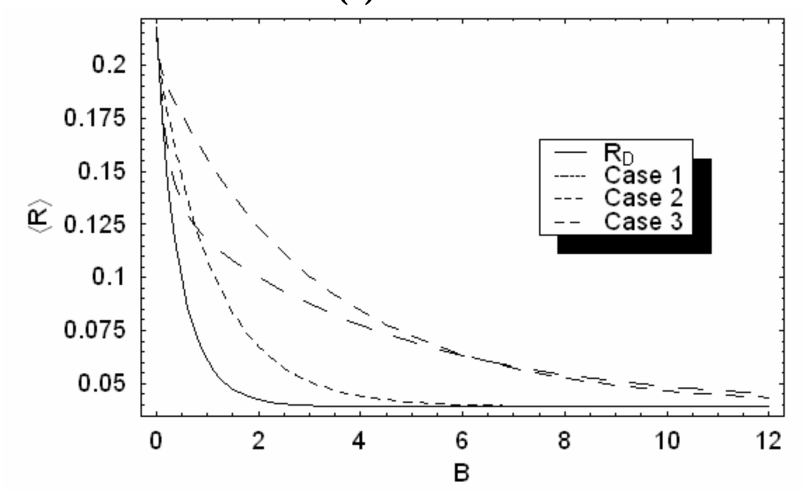


(c) $\ell = 2$ & $\rho_\iota^s = 0.25$

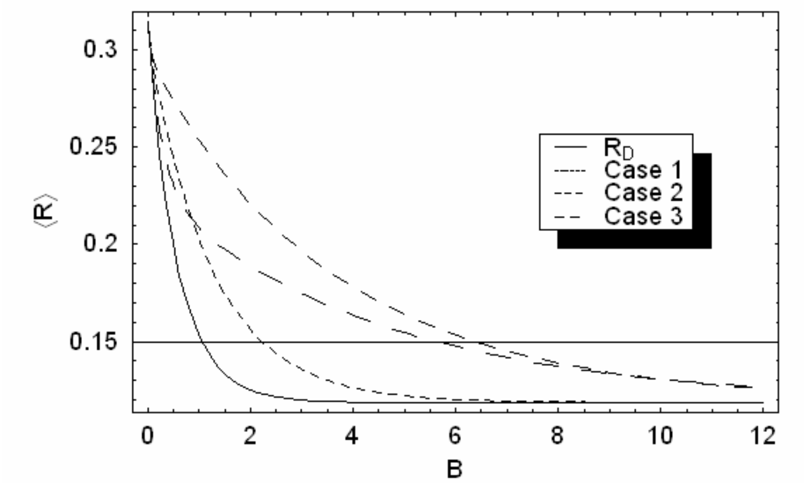


Figs.(2): The average transmissivity $\langle \mathrm{T} \rangle$ for $\Gamma(\mu) = \mu^{l}$ & $\omega = 0.9$

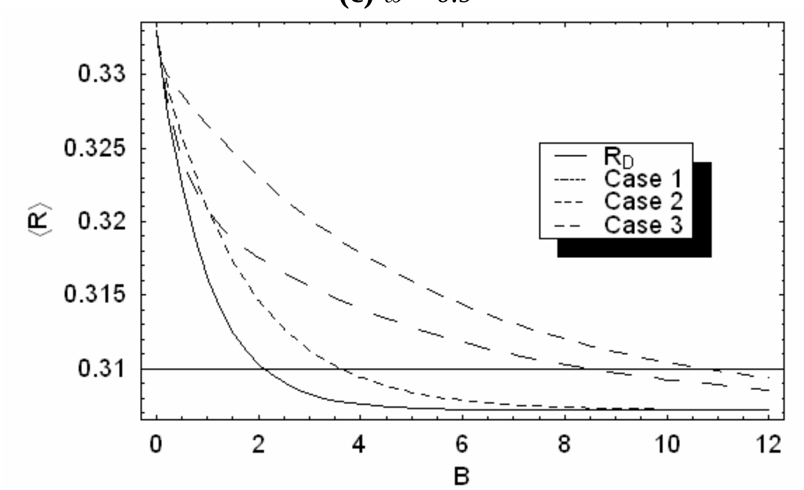
(a) *ω = 0.5*



(b) *ω = 0.7*

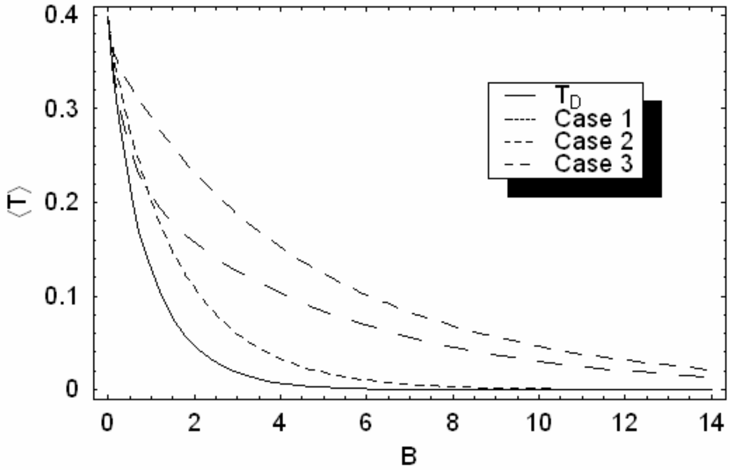


(c) *ω = 0.9*

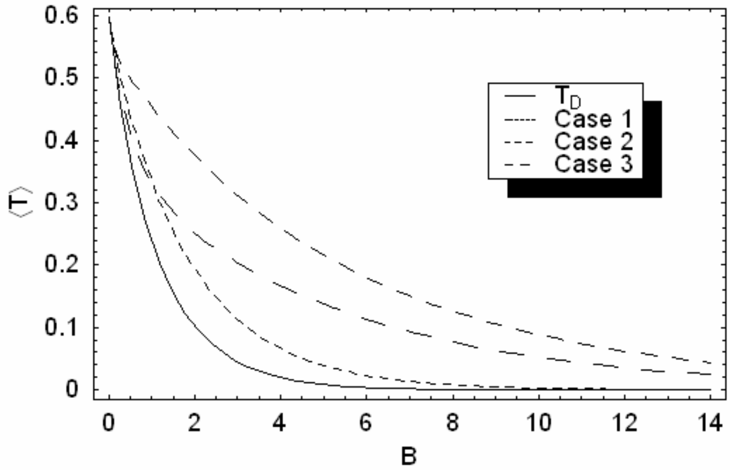


Figs.(3): The average reflectivity $\langle \mathrm{R} \rangle$ for $\Gamma(\mu) = 1$ & $\rho_t^s = 0.5$

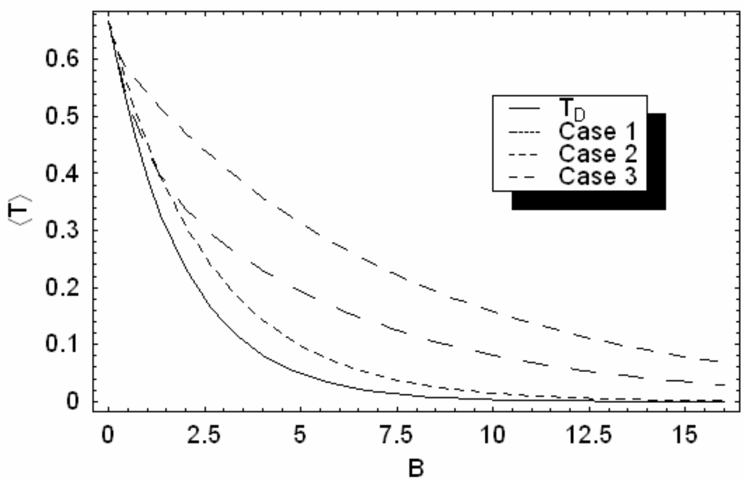
(a) $\omega = 0.5$



(b) $\omega = 0.7$



(c) $\omega = 0.9$



Figs.(4): The average transmissivity $\langle \mathrm{T} \rangle$ for $\Gamma(\mu) = 1$ & $\rho_l^s = 0.5$

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