Spectral distribution of scalar particles created by a moving boundary with Robin boundary condition

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OUTLINE

Outline

1. The Casimir Effect
2. The Dynamical Casimir Effect
   - Introductory Remarks
   - Calculation of the spectrum
3. Summary

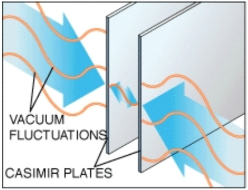
Outline

OUTLINE

The Casimir Effect
The force by the vacuum

- H. B. G. Casimir (1948): force between neutral bodies:

$$\frac{|\mathbf{F}|}{A} = \frac{\hbar c \pi^2}{240\, z^4} \cong \frac{1.3\, mPa}{(z/\mu m)^4} \quad \text{(parallel plates)}$$



Neutral parallel metal plates (at a distance z) attract each other.

- Casimir's method: the force is due to *vacuum energy shift* (caused by plates).

This gives "life" to quantum vacuum fluctuations!

The Casimir Effect

The force by the vacuum

Other quantum vacuum phenomena

- Lamb shift
- Unruh-Davies effect
- Dynamical Casimir Effect

The Casimir Effect

The force by the vacuum

Other quantum vacuum phenomena

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- **Dynamical Casimir Effect**

Outline

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The Dynamical Casimir Effect

A flashing vacuum

- Non-uniformly accelerating boundaries suffer a dissipative force by the vacuum. (Fulling and Davies, 1976.)

Dissipative force on a boundary.

Where does the energy go?

To the field. . .

. . . in the form of quanta!

The Dynamical Casimir Effect
A flashing vacuum

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The Dynamical Casimir Effect

A flashing vacuum

Our goal

Calculate *how many field quanta are created* per unit frequency in a simple dynamical Casimir effect scenario.

Technical details

1. Massless scalar field
2. $D = 1 + 1$
3. Non-relativistic motion: $x = \delta q_0 \cos(\omega_0 t) e^{-|t|/T} \quad (\omega_0 T \gg 1)$
4. Use of Ford-Vilenkin perturbative method
5. Robin boundary condition

The Dynamical Casimir Effect

A flashing vacuum

A quick reminder:

Definition

Robin boundary condition imposes:

$$\left.\frac{\partial\phi}{\partial n}\right|_{bound.} = \frac{1}{\beta}\,\phi\Big|_{bound.} \qquad (\beta \in \mathbb{R}_+)$$

Why do we use them?

1. Continuously interpolates Dirichlet ($\beta = 0$) and Neumann ($\beta \to \infty$) boundary conditions.
2. Simulates penetrable surfaces ($\beta \equiv \omega_P$).

The Dynamical Casimir Effect

A flashing vacuum

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The Dynamical Casimir Effect

A flashing vacuum

Definitions

- The “in-vacuum”: $\hat{a}_{in}(\omega)\,|0_{in}\rangle = 0$

$$t \to -\infty \Rightarrow \hat{\phi}(x,\omega) = \psi_{in}(x,\omega)\hat{a}_{in}(\omega) + \psi^*_{in}(x,-\omega)\hat{a}^\dagger_{in}(-\omega)$$

- The “out-vacuum”: $\hat{a}_{out}(\omega)\,|0_{out}\rangle = 0$

$$t \to +\infty \Rightarrow \hat{\phi}(x,\omega) = \psi_{out}(x,\omega)\hat{a}_{out}(\omega)+\psi^*_{out}(x,-\omega)\hat{a}^\dagger_{out}(-\omega)$$

THE DYNAMICAL CASIMIR EFFECT
A FLASHING VACUUM

Operators evolve in the *Heisenberg* picture:

$t \rightarrow -\infty$

$$|0_{in}\rangle$$
$$\hat{a}_{in}(\omega)\,,\ \hat{a}^{\dagger}_{in}(\omega)$$
$$\hat{N}_{in} = \hat{a}^{\dagger}_{in}(\omega)\hat{a}_{in}(\omega)$$

$t \rightarrow +\infty$

$$|0_{in}\rangle$$
$$\hat{a}_{out}(\omega)\,,\ \hat{a}^{\dagger}_{out}(\omega)$$
$$\hat{N}_{out} = \hat{a}^{\dagger}_{out}(\omega)\hat{a}_{out}(\omega)$$

The Dynamical Casimir Effect
A flashing vacuum

The operator $\hat{a}_{out}$ is related to $\hat{a}_{in}$ and $\hat{a}^{\dagger}_{in}$:

$$\hat{a}_{out} = \alpha\, \hat{a}_{in} + \gamma\, \hat{a}^{\dagger}_{in}$$

(Bogoliubov transformation)

If $\gamma \neq 0$, $\hat{a}_{out}\,|0_{in}\rangle = \gamma \hat{a}^{\dagger}_{in}\,|0_{in}\rangle \neq 0$

So that $\langle 0_{in}|\,\hat{a}^{\dagger}_{out}\hat{a}_{out}\,|0_{in}\rangle \propto |\gamma^2| \neq 0$

This means that
particles are created!

The Dynamical Casimir Effect
A flashing vacuum

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The Dynamical Casimir Effect

A flashing vacuum

After recalling our assumptions ...

1. Massless scalar field
2. $D = 1 + 1$
3. Non-relativistic motion: $x = \delta q_0 \cos(\omega_0 t) e^{-|t|/T} \quad (\omega_0 T \gg 1)$
4. Robin boundary condition on moving boundary

... we can relate $\hat{a}_{in}(\omega)$ and $\hat{a}_{out}(\omega)$ ...

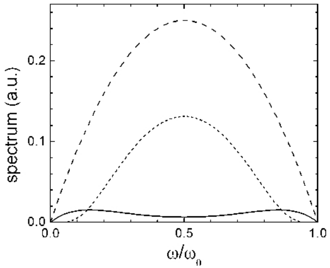
$$\hat{a}_{out}(\omega) = \hat{a}_{in}(\omega) + \frac{2i\sqrt{\omega}}{\sqrt{1+\beta^2\omega^2}} \int \frac{d\omega'}{2\pi} \frac{1+\beta^2\omega\omega'}{\sqrt{1+\beta^2\omega'^2}} \sqrt{|\omega'|} \times$$
$$\times \left[\theta(\omega')\hat{a}_{in}(\omega') - \theta(-\omega')\hat{a}^{\dagger}_{in}(-\omega')\right] \delta Q(\omega - \omega')$$

The Dynamical Casimir Effect

A flashing vacuum

...to finally find the spectrum of emitted particles!

$$\frac{dN}{d\omega}(\omega) = \frac{\delta q_0^2 T}{2\pi}\,\omega(\omega_0-\omega)\,\frac{[1-\beta^2\omega(\omega_0-\omega)]^2}{[1+\beta^2\omega^2][1+\beta^2(\omega_0-\omega)^2]}\,\Theta(\omega_0-\omega)$$



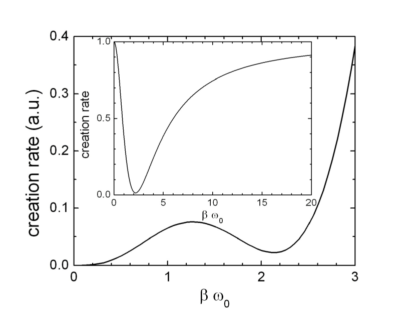
Spectral distribution $dN/d\omega$.

Dashed line: $\beta = 0$ (D)
Solid line: $\beta\omega_0 = 1.7$
Dotted line: $\beta\omega_0 = 5$

The Dynamical Casimir Effect

A flashing vacuum

Total particle creation rate



Total particle creation rate as a function of ω_0.

Inset: Ratio between Robin and Dirichlet BC creation rates.

A remarkable fact!

The inset plot shows a 98.7% reduction in the creation rate for $\beta\omega_0 \cong 2.2$!!

Outline

What I'd Like You to Remember

Vacuum exerts damping on non-uniformly accelerated bodies, generating field quanta.

Robin BC weakens the dynamical Casimir effect in a non-trivial way.

For Further Reading

B. Mintz *et al*
Particle creation by a moving boundary with Robin boundary condition
J. Phys. A: Math. Gen. **39** (2006) 11325-11333

B. Mintz *et al*
Casimir forces for moving boundaries with Robin condition
J. Phys. A: Math. Gen. **39** (2006) 6559-6565