Accounting for the Contributions of Topological Excitations in Phase Transitions Through Dual Actions

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What are topological defects ?

They are finite energy and stable configurations, solutions of the field equations of motion and emerge as a consequence of a SSB process.

Topological defects

Consider SSB: $G \longrightarrow H$, with $H \subset G$

Defect formation during SSB depends on the homotopy groups $\pi_k(G/H)$ of the vacuum manifold $\mathcal{M} = G/H$

If $\pi_k(G/H) \neq I$ then $(2-k)-$ dim defects appear

domain walls $(k=0)$

strings $(k=1)$

Monopoles $(k=2)$

Textures $(k=3)$

Gauge defects

Global defects

Topological Configurations from phase transitions (with SSB):

• GUTs:

$$SU(5),\ SU(10),\ \text{etc} \quad \longrightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \longrightarrow SU(3)_c \times U(1)_{EM}$$

$$(T > T_c^{GUT} \sim 10^{15} GeV) \qquad (T > T_c^{EW} \sim 100 GeV) \qquad (T < T_c^{EW})$$

Applications e.g. in:

- in cosmology (early universe phase transitions)

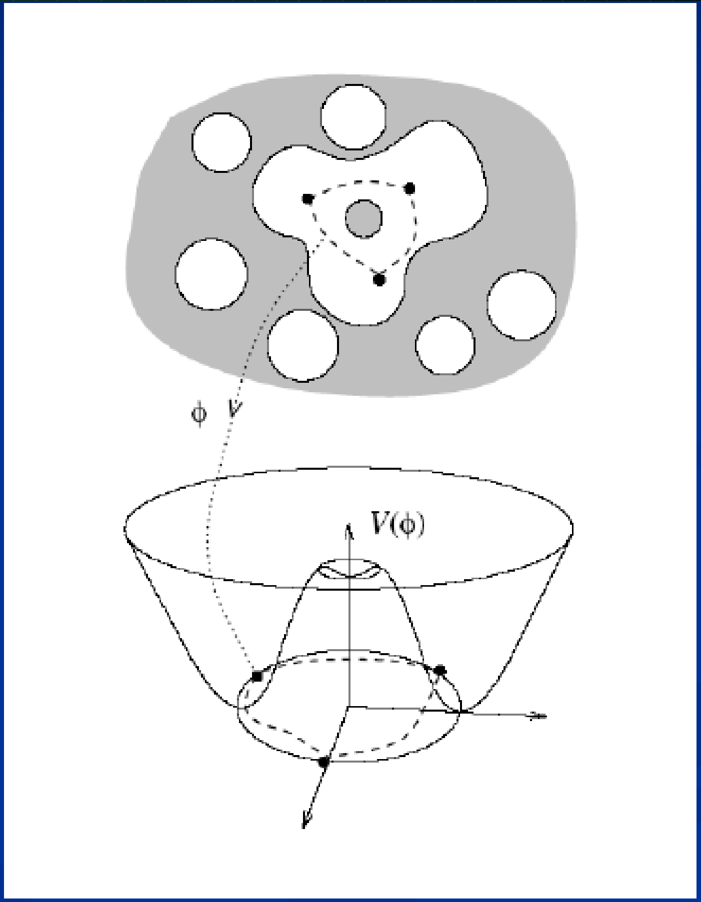
- in condensed matter systems (superfluids, superconductivity, liquid crystals, etc)

- in QCD : confinement ideas

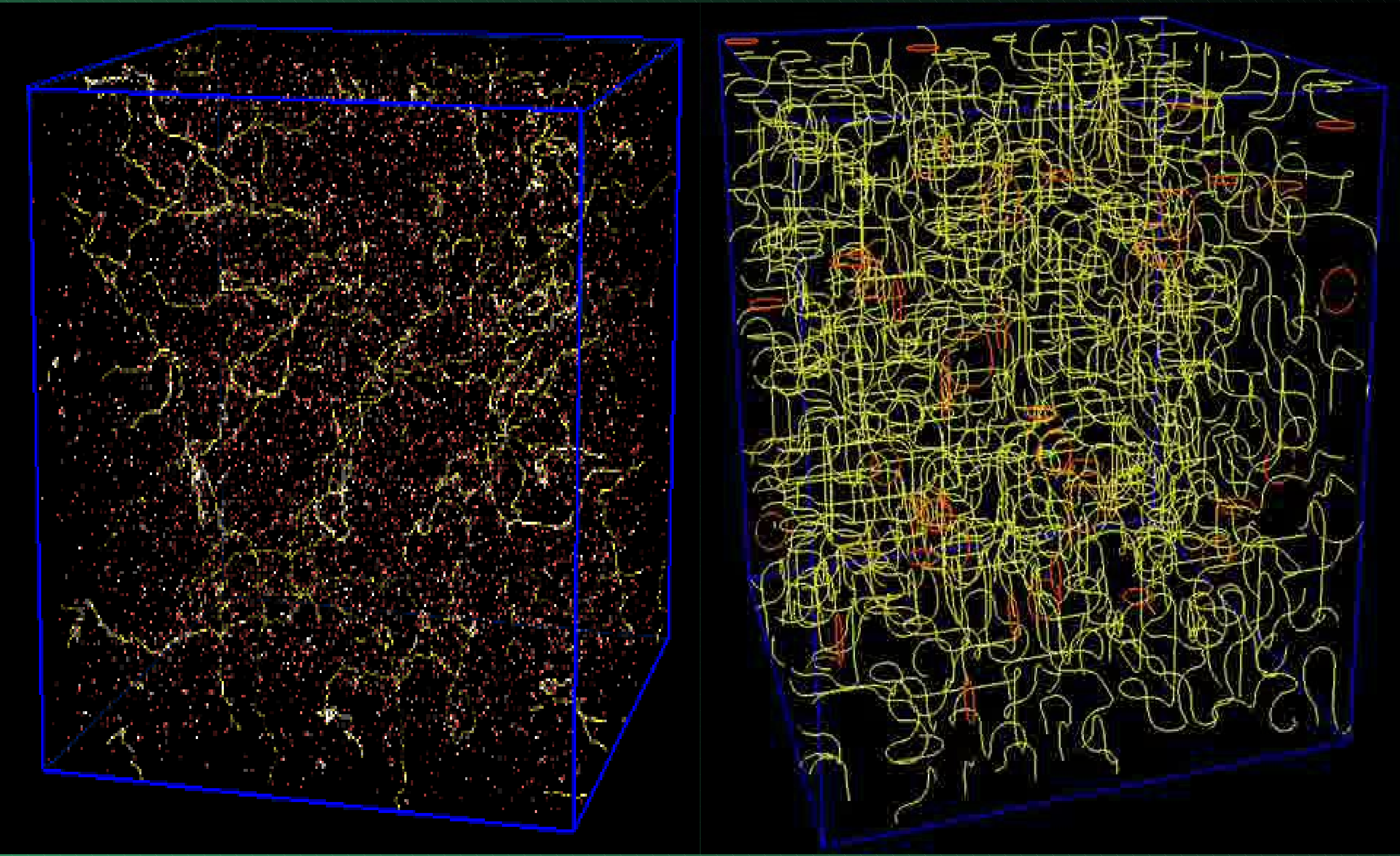
and in many other contexts

Formation of topological defects (Kibble-Zurek mechanism):

- A field breaking a U(1) symmetry will roll to different minima at different points in the universe
- Neighboring field values will try to correlate
- Will often have "trapped winding" with unbroken U(1), producing a core of energy



Cosmic strings have been extensively studied in numerical simulations:



http://www.damtp.cam.ac.uk/user/gr/public/cs_evol.html

3- Deriving the Dual Action: a quantum field theory for strings

The abelian Higgs Model (AHM):

$$\mathcal{L}_H = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left(D_\mu\phi\right)\left(D^\mu\phi\right)^* - V(\phi)$$

$$V(\phi) = \lambda\left(|\phi|^2 - \phi_0{}^2\right)^2$$

There exists a state of unbroken symmetry, $\langle\phi\rangle=0$, corresponding to a local maximum of $V(\phi)$.

In the symmetry breaking case, the minima of $V(\phi)$ lie on the circle $|\phi| = \phi_0$ and the ground state occurs at:

$$\langle\phi\rangle = \phi_0 e^{i\vartheta} \neq 0$$

The Lagrangian is invariant under the local U(1) gauge transformation:

$$A_\mu \to A_\mu + \partial_\mu\theta$$
$$\phi \to e^{-ie\theta}\phi$$

where $\theta(x)$ is an arbitrary coordinate-dependent function

the Nielsen-Olesen string solution:

The symmetry breaking $U(1) \to 1$ with homotopy group $\pi_1 \neq 1$

$\Longrightarrow$ string-like topological excitations:

a unit winding string solution along the z axis (using the cylindrical coordinates r, θ, z)

$$\begin{aligned} \phi_{\rm string} &= \frac{\rho(r)}{\sqrt{2}} e^{i\theta} , \\ A_{\mu,{\rm string}} &= \frac{1}{e} A(r)\, \partial_\mu \theta , \end{aligned}$$

$\rho(r \to 0) = 0$, $A(r \to 0) = 0$

and

$\phi(r \to \infty) \to \rho_v$ and $A(r \to \infty) \to 1$

If we write the field ϕ as

$$\phi = \rho \exp(i\chi)/\sqrt{2}$$

then at spatial infinity ρ goes to the vacuum ρ_v and A_μ becomes a pure gauge

$\Longrightarrow$ for a finite energy configuration:

$$\partial_\mu \chi = e A_\mu \quad \text{at} \quad r \to \infty, \quad \text{so} \quad D_\mu \phi = 0$$

$\Longrightarrow$ by taking some contour C surrounding the symmetry axis

$$\Phi = \oint A_\mu dx^\mu = \oint \partial_\mu \chi dx^\mu = 2\pi/e$$

magnetic flux

$\Longrightarrow$ on the string χ must be singular (ϕ must be single-valued)

$\Longrightarrow \chi(x) = \chi_{\mathrm{reg}}(x) + \chi_{\mathrm{sing}}(x)$

Dual Transformed action

$$Z[\beta] = \int \mathcal{D}A\mathcal{D}\phi\mathcal{D}\phi^* \exp\{-S[A_\mu, \phi, \phi^*] - S_{\mathrm{GF}}\},$$

$\beta = 1/T$

Writing the Higgs field ϕ in the polar parameterization form $\phi = \rho e^{i\chi}/\sqrt{2}$,

$$Z = \int \mathcal{D}A_\mu \mathcal{D}\rho \mathcal{D}\chi \left(\prod_x \rho\right) \exp\{-S[A_\mu, \rho, \chi] - S_{\mathrm{GF}}\},$$

$$S[A_\mu, \rho, \chi] = \int d\tau d^3x \left[\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}\rho^2(\partial_\mu \chi + eA_\mu)^2 - \frac{m_\phi^2}{2}\rho^2 + \frac{\lambda}{4!}\rho^4\right]$$

We want to make explicit the contribution of the nontrivial topological field configuration in the partition function:

$$\chi = \chi_{\mathrm{reg}} + \chi_{\mathrm{sing}}$$

Then:

$$\delta(\partial_\mu C_\mu)$$
$$C_\mu = -i\frac{\kappa}{2}\epsilon_{\mu\nu\lambda\rho}\partial_\nu W_{\lambda\rho} \equiv \kappa V_\mu$$

$$\begin{aligned}\int \mathcal{D}\chi \exp\Big[-\int d^4x \frac{1}{2}\rho^2(\partial_\mu\chi + eA_\mu)^2\Big] &= \int \mathcal{D}\chi_{\mathrm{sing}}\mathcal{D}\chi_{\mathrm{reg}}\mathcal{D}C_\mu\Big(\prod_x \rho^{-4}\Big)\\ &\times \exp\Big\{-\int d^4x\Big[\frac{1}{2\rho^2}C_\mu^2 - iC_\mu(\partial_\mu\chi_{\mathrm{reg}}) - iC_\mu(\partial_\mu\chi_{\mathrm{sing}} + eA_\mu)\Big]\Big\}\\ &= \int \mathcal{D}\chi_{\mathrm{sing}}\Big(\prod_x \rho^{-4}\Big)\mathcal{D}W_{\mu\nu}\\ &\times \exp\Big\{-\int d^4x\Big[\frac{\kappa^2}{2\rho^2}V_\mu^2 + e\kappa A_\mu V_\mu + i\pi\kappa W_{\mu\nu}\omega_{\mu\nu}\Big]\Big\},\end{aligned}$$

κ = arbitrary parameter with mass dimension

$$\omega_{\mu\nu} \equiv \frac{1}{4\pi}\epsilon_{\mu\nu\lambda\rho}\left(\partial_\mu\partial_\nu - \partial_\nu\partial_\mu\right)\chi_{\mathrm{sing}}(x) \quad \equiv \quad \text{vorticity}$$

$$\exp\left(-\frac{1}{4}\int d^4x F_{\mu\nu}^2\right) = \int \mathcal{D}G_{\mu\nu} \exp\left[\int d^4x\left(-\frac{\mu_W^2}{4}G_{\mu\nu}^2 \times -\frac{\mu_W}{2}\tilde{G}_{\mu\nu}F_{\mu\nu}\right)\right],$$

$$e\kappa = \mu_W$$

with

$$\tilde{G}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}G_{\lambda\rho}.$$

$$G_{\mu\nu} = W_{\mu\nu} - \frac{1}{\mu_W}(\partial_\mu B_\nu - \partial_\nu B_\mu)$$

Then:

$$Z = \int \mathcal{D}W_{\mu\nu}\mathcal{D}\chi_{\text{sing}}\mathcal{D}G_{\mu\nu}\delta\left[\epsilon_{\mu\nu\alpha\beta}\partial_\mu\left(G_{\alpha\beta} - \frac{1}{2}W_{\alpha\beta}\right)\right]$$
$$\times \mathcal{D}\rho\left(\prod_x \rho^{-3}\right)\exp\left\{-\int d^4x\left[\frac{\mu_W^2}{4}G_{\mu\nu}^2 + \frac{\mu_W^2}{2e^2\rho^2}V_\mu^2 + \frac{1}{2}(\partial_\mu\rho)^2 - \frac{m_\phi^2}{2}\rho^2 + \frac{\lambda}{4!}\rho^4 + i\pi\frac{\mu_W}{e}W_{\mu\nu}\omega_{\mu\nu}\right]\right\}.$$

Finally:

$$Z = \int \mathcal{D}W_{\mu\nu}\mathcal{D}\chi_{\text{sing}}\mathcal{D}B_\mu\mathcal{D}\rho\Big(\prod_x \rho^{-3}\Big) \times \exp\{-S_{\text{dual}}[W_{\mu\nu}, B_\mu, \rho, \chi_{\text{sing}}] - S_{\text{GF}}\},$$

with

$$S_{\text{dual}} = \int d^4x\Big[\frac{\mu_W^2}{2e^2\rho^2}V_\mu^2 + \frac{1}{4}(\mu_W W_{\mu\nu} - \partial_\mu B_\nu + \partial_\nu B_\mu)^2 + \frac{1}{2}(\partial_\mu\rho)^2 - \frac{m_\phi^2}{2}\rho^2 + \frac{\lambda}{4!}\rho^4 + i\pi\frac{\mu_W}{e}W_{\mu\nu}\omega_{\mu\nu}\Big].$$

(Dirac construction)

$\omega_{\mu\nu} \Longrightarrow$ associated to the surface element of a (tube-like) world sheet of a closed vortex-string:

$$\omega_{\mu\nu}(x) = n\int_S d\sigma_{\mu\nu}(x)\delta^4[x - y(\xi)],$$

element of area on the world sheet swept by the string

$$\int d^4x\, i\pi\frac{\mu_W}{e}W_{\mu\nu}(x)\omega_{\mu\nu}(x) = \frac{i}{2}\int_S d\sigma^{\mu\nu}(y) \times \frac{2\pi\mu_W}{e}W_{\mu\nu}(y).$$

Using:

Marshall–Ramond procedure of quantizing the vortex–strings as nonlocal objects and associate to them a wave function $\Psi[C]$, a functional field

$$+$$

$$D_{\sigma^{\mu\nu}}(x) = \frac{\delta}{\delta\sigma^{\mu\nu}(x)} - i\frac{2\pi\mu_W}{e}W_{\mu\nu}(x)$$

(Y. Nambu, 1976)

$$\int d^4x\, i\pi\frac{\mu_W}{e}W_{\mu\nu}(x)\omega_{\mu\nu}(x)$$

$$S_{\text{string}}(\Psi[C], W_{\mu\nu}) = \oint_C dx_\nu[|D_{\sigma^{\mu\nu}}\Psi[C]|^2 - M_0^4|\Psi[C]|^2],$$

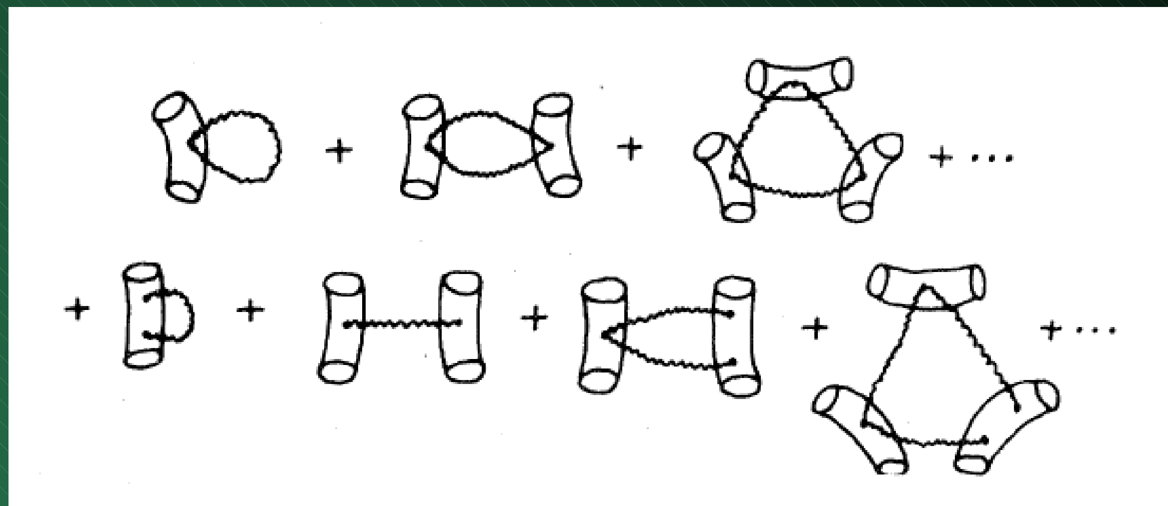
$$M_0^4 \equiv \frac{1}{a^4}(e^{\tau_s a^2} - 6)$$

a ~ string radius

$$\tau_s \sim \pi\rho_c^2$$

The effective potential contributions due to vortex-strings:

When integrationg over the Kalb-Ramond field W (one-loop level):



Defining:

$$\hat{\psi}_C \equiv 4\left(\frac{2\pi}{e}\right)^2 \sum_{C_{x,t}} \frac{1}{a^3 l} |\Psi[C]|^2,$$

$$1/a \sim m_\phi \left(1 - \frac{T^2}{T_c^2}\right)^{1/2}$$

~ 1/(string radius)

where l is the length of a curve C, and $C_{x,t}$ represents a curve passing through a point x in a fixed direction t; also,

ROR, Medeiros Neto, Barci, Linhares, PRD72, 103524 (2005)

$$V_{\text{eff}}^{\text{1-loop}}(\psi_C) = \frac{e^2}{4\pi^2} M_0^4 \psi_C + \frac{3}{2}\frac{1}{\beta}\sum_{n=-\infty}^{+\infty}\int \frac{d^3k}{(2\pi)^3} \ln\left[\omega_n^2 + \mathbf{k}^2 + M_A^2(1+\psi_C)\right]$$

$$M_A^2 = e^2\rho_c^2$$

$$V_{\text{eff,string}}^{(\beta)}(\psi_C) \simeq \left[\frac{e^2}{4\pi^2 a^4}(e^{\tau_s a^2} - 6) + \frac{3e^2\rho_c^2}{16\pi^2 a^2} + \frac{e^2\rho_c^2}{8}T^2\right]\psi_C - \frac{e^3\rho_c^3}{4\pi}(1+\psi_C)^{3/2}T - \frac{3e^4\rho_c^4 \ln(2\Lambda/T)}{32\pi^2}\psi_C^2,$$

$\mathcal{O}(\alpha^{3/2})$

$a = 1/\Lambda$

$\mathcal{O}(\alpha^2)$

String effective mass term: Ms(T). At some T=Ts → Ms(Ts) = 0

$$T_s = \frac{\sqrt{2}}{\pi a^2 \rho_c}\left(6 - e^{\tau_s a^2} - \frac{3a^2\rho_c^2}{4}\right)^{1/2}$$

using:

$$\tau_s = \pi\rho_c^2$$

$$1/a \sim m_\phi\left(1 - \frac{T^2}{T_c^2}\right)^{1/2}$$

$$\rho_c \simeq \sqrt{\frac{6m_\phi^2}{\lambda}}\left(1 - \frac{T^2}{T_c^2}\right)^{1/2}$$

$$T_c = \sqrt{12m_\phi^2/(3e^2 + 2\lambda/3)}$$

$$\frac{T_c - T_s}{T_c} \sim \mathcal{O}\left(\frac{e^{-1/\lambda}}{\lambda^2}\right)[1 + \mathcal{O}(\alpha)],$$

for $e^2 \ll \lambda \ll 1$ $\quad \alpha = e^2/\lambda$

Summary and Outlook:

- *Strings condense at T=Ts*
- *Ts → the Ginzburg temperature Tg*
- *The regime e^2/λ << 1 → like type II superconductor → 2nd order PT*
- *For e^2/λ >> 1 → like type I superconductor, gauge fluctuations are stronger → 1st order PT*
- *Since Ts ~ Tc, the driven mechanism of the 1st order PT is a melting of topological defects*

Monopoles (in the context of the compact AHM) could be added here as external fields

Study of the finite T effects and possible consequences for the confinement picture in the dual superconductor model ?

Same approach could be used in nonabelian gauge field theory (eg Gergi-Glashow model) and study monopole condensation directly (work under way …)