Dark Matter as a non-linear effect of Gravitation

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The rotation curves of stars in disk galaxies are calculated with the Newtonian law of motion applied to a scalar potential derived from the geodesic equation, only, under the slow motion condition, the so-called Nearly Newtonian Gravity (NNG). A nearly Newtonian gravitational potential , $\Phi_{NN} = -\frac{1}{2}c^2(1+g_{44})$, is obtained, characterised by an exact solution of Einsteins equations, with the non-linear effects present in the component g_{44} . This gravitational field lies somewhere between General Relativity and Newtonian Gravity. Therefore, Einsteins equations and the equivalence principle are preserved, but the general covariance is broken.

The resulting curves are remarkably close to the observed rotation curves in spiral galaxies, suggesting that a substantial component of dark matter may be explained by the non-linearity of Einsteins equations.

I. INTRODUCTION

One of the important problem of modern astrophysics is the lack of a proper explanation for the Dark Matter which is some kind of extra matter invisible with respect to electromagnetic spectrum but gravitationally effective and with these features, Dark Matter could be responsible from the discrepancy between the observed rotation curves of stars located outside the nucleus a spiral galaxy.

The theoretical prediction from Newtonian mechanics is shown in figure below (A).

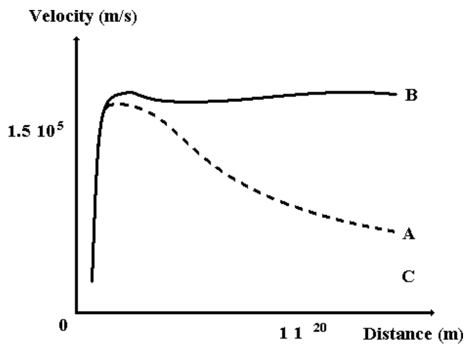

FIG. 1: The discrepancy between the experimental observation (solid line) and the theoretical prediction from newtonian mechanics (dash line).

Outside the galaxy's core, the curve is near flat (B).

II. PROPOSALS OF EXPLANATION FOR ROTATION CURVE PROBLEM

Some proposals have been made to explain the rotation curves problem, based on the conjecture that Newton's gravitational theory does not apply outside the galaxy's core.

- **Dark Matter halos**: Originally called *The missing mass problem* proposed by Fritz Zwicky(1898-1974)[3] in 1937 to explain the discrepancy between theoretical and observed mass in the COMA cluster. Nowadays, the Dark Matter Halo hypothesis is currently the most acclaimed explanation.
- **MOND**: It suggests a modification on second Newton's law which proposes a modified Poisson's equation[4,5]

$$\langle \nabla, \mu\left(\frac{|\nabla\Phi|}{a_0}\right)\rangle = 4\pi G\rho$$

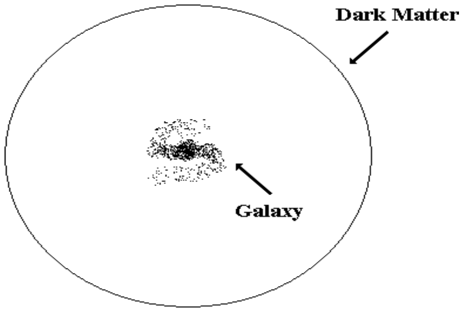

FIG. 2: The hypothesis of dark matter halos .

where $\mu\left(\frac{|\nabla\Phi|}{a_0}\right)$ is a function to be adjusted to the specific type of galaxy and a_0 is an acceleration constant. Although the model can provide a good approximation curve, it does not work very well in the problem of galaxy clusters where the same rotation curve also arises.

- **General Relativity Theory (GRT)**: In principle, it should provide the required correction. The effects of parametric post-newtonian approximation decays with $1/r^3$ [6] and it does not significantly flatten out the Newtonian curve;

These attempts have motivated the emergence of many others gravitational theories, like (1) Add a scalar field to Einstein's equation, in such a way that the scalar-tensor theory corrects the Newtonian limit[7]; (2) Modify the concept of time in general relativity, so that the Newtonian limit of the theory differs from the original Newton's' theory[8,9]; (3) Add a cosmological constant with the appropriate sign[10]; (4) Include higher order curvature terms in the gravitational variational principle[11]; (5) Several brane-world models and variants have been considered, in the hope that the more general brane-world equations of motion may provide the correct velocity curves[12,13,14,15,16,17]. Nevertheless, the problem still unsolved.

III. METHODOLOGY

As a simple model for a disk galaxy, we use a cylinder such that its height h_0 is much smaller than its radius r_0 . The line element produced by such object can be derived from the Weyl[17] cylindrically symmetric metric, expressed in cylindrical coordinates (r, z) as

$$dS^2 = e^{2(\lambda-\sigma)}dr^2 + r^2e^{-2\sigma}d\varphi^2 + e^{2(\lambda-\sigma)}dz^2 - e^{2\sigma}dt^2$$

where $\lambda = \lambda(r,z)$ and $\sigma = \sigma(r,z)$.

The exterior gravitational field outside the cylinder, is given by vacuum Einstein's equations:

$$-\lambda_{,r} + r\sigma_{,r}^2 - r\sigma_{,z}^2 = 0$$

$$-\sigma_{,r} - r\sigma_{,rr} - r\sigma_{,zz} = 0$$

$$\lambda_{,rr} + \lambda_{,zz} + \sigma_{,r}^2 + \sigma_{,z}^2 = 0$$

$$2r\sigma_{,r}\sigma_{,z} = \lambda_{,z}$$

In such non-linear system the superposition of solutions does not apply, so that the dependence of and on z cannot be neglected by use of symmetry arguments. However, when we impose to this geometry the thin disk condition $Z\ \varepsilon\ [-h_0/2, h_0/2]$, for $r\ \varepsilon\ [0, r_0]$, i.e, $h_0 << r_0$.Therefore, the vacuum solution of Einstein's equations for the thin Weyl disk is

$$\sigma(r,z) = \frac{K_0}{2}\ln r + a_0 z + c_0$$

$$\lambda(r,z) = \frac{K_0^2}{2}\ln r - a_0\frac{r^2}{2} + b_0 z + d_0$$

where we have denoted $K_0 = \frac{b}{a} = cte$ and the coefficients c_0 and d_0 are constants.
In the plane z = 0, the nearly Newtonian potential is

$$\Phi_{NN} = -\frac{1}{2}c^2\left(1+g_{44}\right) = -\frac{1}{2}(1-e^{2c_0}r^{K_0})$$

The rotation velocity $v = \omega_0 r$, $\omega_0 =$ constant, is obtained by comparing the radial force $\frac{v^2}{r}\hat{r}$ with $F = -\frac{\partial\Phi_{nn}}{\partial r}\hat{r}$, so that $v = \sqrt{|r\frac{\partial\Phi_{nn}}{\partial r}|}$. In the particular case of a star in the vacuum near the disk, using Φ_{NN} we obtain (G = c=1)

$$v(r) = \sqrt{|\beta_0 M_v K_0 r^{K_0}|}$$

IV. RESULTS

Error bars are the observations[22,23,24], the green triangles are the Newtonian rotation curves and the red points are the rotation curves fitted by NNG.

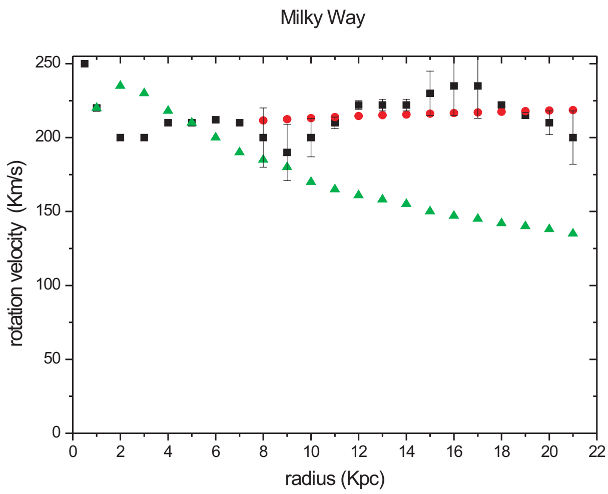

FIG. 3: Via Lactea with $M = 1.99\times10^{41}$Kg, $R = 8,5$Kpc, $K_0 = 0.0682$ and $\beta_0 = 0.0901$

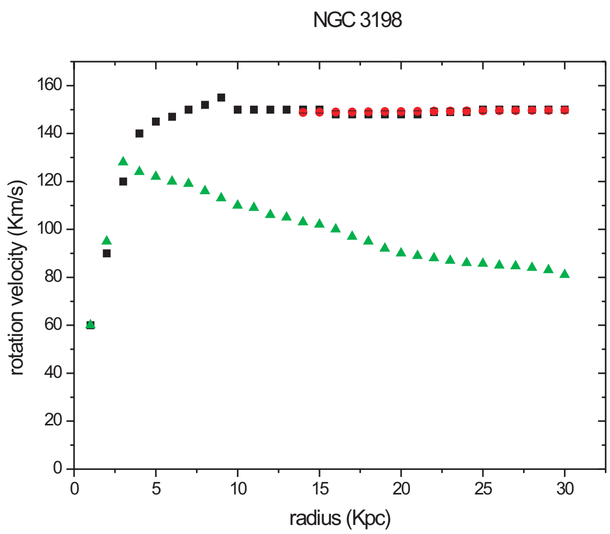

FIG. 4: NGC3198, with $M = 1.19 \times 10^{40}$Kg, $R = 16,6$Kpc, $K_0 = 0.0162$ and $\beta_0 = 0.8224$

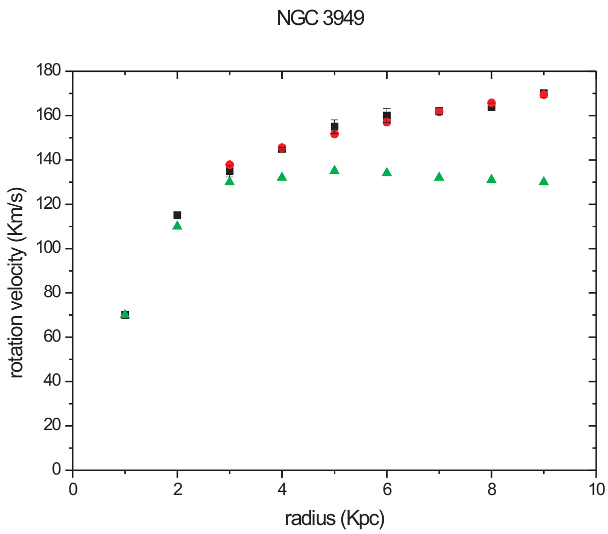

FIG. 5: NGC3949, with $M = 4.97 \times 10^{39}$Kg, $R = 6$Kpc, $K_0 = 0.3766$ and $\beta_0 = 1.1657$

V. CONCLUSIONS

When applying the slow motion condition $v << c$, on the geodesic equation, we obtain a nearly Newtonian equation of motion. We have noted that the symmetry, of the solution of Einsteins equations play a significant role in the velocity curves. This was exemplified by taking the Weyl solution with the thin disk condition to generate the nearly Newtonian potential and the corresponding velocity curves, showing a remarkable approximation to the experimental results.

It's important to note that the calculus employed in this disk galaxy model we consider a particle test (star) orbiting on the edge of the disk (galaxy). This fact explain the form of the curve that we've obtained which not include the initial velocity increase of the rotation curve.

Observing the obtained results, we feel that the slight slope of the curve may improve the test at the cluster scale, with the same value of the ratio b_0/a_0. Therefore, a more realistic galaxy model could be obtained by applying the condition $v << c$ and a similar thin condition to the oblate spheroid[20]. Work on this is still in progress.

VI. REFERENCES

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