2009 International Nuclear Atlantic Conference - INAC 2009
Rio de Janeiro,RJ, Brazil, September27 to October 2, 2009
ASSOCIAÇÃO BRASILEIRA DE ENERGIA NUCLEAR - ABEN
ISBN: 978-85-99141-03-8

PARTICLES GEOMETRY INFLUENCE IN THE THERMAL STRESS LEVEL IN AN SiC REINFORCED ALUMINUM MATRIX COMPOSITE CONSIDERING THE MATERIAL NON-LINEAR BEHAVIOR

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ABSTRACT

An analytical methodology was developed to predict the thermal stress level that occurs in a metallic matrix composite reinforced with SiC particles, when the temperature decreases from 600 ^{o}C to 20 ^{o}C during the fabrication process. This analytical development is based on the Eshelby method, dislocation mechanisms, and the Maxwell-Boltzmann distribution model. The material was assumed to have a linear elastic behavior. The analytical results from this formulation were verified against numerical linear analyses that were performed over a set of random non-uniform distribution of particles that covers a wide range of volumetric ratios. To stick with the analytical hypothesis, particles with round geometry were used. Each stress distribution, represented by the isostress curves at ΔT=-580 ^{o}C, was analyzed with an image analyzer. A statistical procedure was applied to obtain the most probable thermal stress level. Analytical and numerical results compared very well. Plastic deformation as well as particle geometry can alter significantly the stress field in the material. To account for these effects, in this work, several numerical analyses were performed considering the non-linear behavior for the aluminum matrix and distinct particle geometries. Two distinct sets of data with were used. To allow a direct comparison, the first set has the same models (particle form, size and distribution) as used previously. The second set analyze quadrilateral particles and present very tight range of volumetric ratio, closer to what is found in actual SiC composites. A simple and fast algorithm was developed to analyze the new results. The comparison of these results with the previous ones shows, as expected, the strong influence of the elastic-plastic behavior of the aluminum matrix on the composite thermal stress distribution due to its manufacturing process and shows, also, a small influence of the particles geometry and volumetric ratio.

1. INTRODUCTION

The metallic matrix composites reinforced with particles, have important advantages when used as a structural material such as their high strength and good conformability. The properties of the composite depend on the volumetric ratio, the particles size and distribution, besides the matrix microstructure itself. The mechanical properties of composites with long fibers are based on the fiber resistance theory. On the other hand, for composites reinforced with particles instead of fibers, a mathematical model relating strength and particles distribution is hard to be deduced; which is the case of the Aluminum matrix reinforced with SiC particles, used in this work. Thermal stress arises due to the difference between the Aluminum and the SiC thermal expansion coefficients. An analytical was developed [1] to predict the most probable thermal stress in this material due to the fabrication process, when the temperature decreases from 600 ^{o}C to 20 ^{o}C. The methodology assumes a non-linear elastic behavior of the material as basic hypothesis. That work was verified against numerical analyses.

In this work, the same composite was used as previously, Al+SiC, to estimate the most probable thermal stress using numerical analyses, this time, considering the non-linear behavior in the aluminum matrix, once the plastic deformation can alter significantly the stress field in the material. Initially, the same models (particle form, size and distribution) used in [1] were adopted in these new analyses to allow a direct comparison of the material behavior influence on the stress level. A second set of analyses were performed with quadrilateral particles, still using random particles positions and the same material properties. Also, an algorithm, based on the Matlab image processing toolbox, was developed to analyze the new results.

2. THEORETICAL BACKGROUND - LINEAR BEHAVIOR

The analytical methodology to predict the most probable thermal stress in an MMC composite [1] was an adaptation of the Maxwell-Boltzmann statistics [2] and the Shelby’s method, based on the internal stress equilibrium [3, 4, 5, 6, 7]. The main scope of the distribution law is to study how the particles, which form a system, are distributed in their phase space. When one found the probabilities of all possible distributions in the system nature, it is possible to obtain that particular most probable distribution

The Maxwell-Boltzmann distribution law has two terms: one associated with particle distribution and other associated with kinetic energy. In the developed model [1] the second term was replaced by the elastic potential energy, in this case, the elastic stress. With this change it is possible to search for the most probable combination of particle position and elastic stress in the material.

Considering the analytical development done in [1], we have got the equation (1) that represents the stress distribution $n(\sigma)$ in the composite for a random distribution of N particles with a nominal volumetric ratio *f*. The factor *K* is given in eq. (2). In these equations, S is the Eshelby´s tensor, I is the identity matrix, α_M and α_I are tensor´s thermal expansion coefficients, respectively for the matrix and particles, and ΔT is the temperature range experienced by the material. The elastic tensors components of the Aluminum matrix (C_M) and SiC particles (C_I) are given explicitly by equation (3), where E_M and E_I are the matrix and particles elastic modulus and ν_M and ν_I are the matrix and particles Poisson’s ratio. The most probable stress in the material, σ_P, is associated with the maximum eq. (1) and is given by eq. (4).

$$n(\sigma) = 4\pi N \left(\frac{3}{2 C_M K \pi} \right)^{3/2} \sigma^2 \, e^{-\frac{3\sigma^2}{K 2 C_M}} \tag{1}$$

$$K = f\, C_M \{ (S - I) \{ (\mathrm{C_M} - \mathrm{C_I}) [\mathrm{S} - \mathrm{f}\,(\mathrm{S} - \mathrm{I})] - \mathrm{C_M} \}^{-1} \mathrm{C_I} \; (\alpha_I - \alpha_M) \Delta T \}^2 \tag{2}$$

$$C_{Mii} = E_M\,(1 - \nu_M) \,/\, (1 - 2\nu_M)\,(1 + \nu_M) \tag{3.1}$$

$$C_{Mij} = E_M \; \nu_M \,/\, (1 - 2\,\nu_M)\,(1 + \nu_M) \tag{3.2}$$

$$C_{M44} = E_M \,/\, 2\,(1 + \nu_M) \tag{3.3}$$

$$C_{Iii} = E_I (1 - \nu_I) / (1 - 2\nu_I) (1 + \nu_I) \tag{3.4}$$

$$C_{Iij} = E_I \nu_I / (1 - 2 \nu_I) (1 + \nu_I) \tag{3.5}$$

$$C_{I44} = E_I / 2 (1 + \nu_I) \tag{3.6}$$

$$C_{Iij} = E_I \nu_I / (1 - 2 \nu_I) (1 + \nu_I) \tag{3.7}$$

$$\sigma_p^2 = \frac{2f C_M^2 \{(S-I)\{(C_M - C_I)[S - f(S-I)] - C_M\}^{-1} C_I (\alpha_I - \alpha_M)\Delta T\}^2}{3} \tag{4}$$

This formulation is presented in [1] in a very detailed way with some application examples, and a comparison between analytical and numerical results for a MMC composite (Al+SiC) is carried out. About 30 numerical analyses, using finite element methodology, were performed for non-uniform random distributions of round particles (diameter and position), each one with a different volumetric ratio. Each numerical stress distribution analysis, in terms of iso-stress curves at ΔT=-580 °C, was analyzed with an image analyzer and a statistical procedure was applied to obtain the most probable stress level. The analytical and numerical results compared very well.

3. NON-LINEAR NUMERICAL ANALYSES

The present work tries to avoid the restrictions associated with these analytical and numerical analyses: the linear behavior of the material and the indeed regular particles geometry (circular). To go with this approach, the non-linear behavior of the aluminum matrix is adopted as well as the quadrilateral geometry for the particles, which is a more general form. To allow a direct comparison with the previous linear results [1], initially the same 30 models were used adopting the aluminum non-linear behavior and new meshes were generated with a doubled node density. The second set of analyses has 40 models adopting quadrilateral particles, still with random geometry and positioning, and the aluminum non-linear behavior. Two typical round particles random distribution are depicted in figure 1; while figure 2 depicts two typical quadrilateral particles random distribution. The ANSYS uniformly distributed random number generator module was used, along with some other program resources, to get randomness in the particles position and size. The number of particles varies from case to case.

The material properties used in the simulations are (subscripts identifies the material): Young's modulus (GPa), E_{Al} = **73**, E_{SiC} = **45**; transversal modulus (GPa), G_{Al} = 27.4, G_{SiC} = 192; thermal expansion coefficient (C^{-1}), α_{Al} = 23.6 10^{-6}, α_{SiC} = 4.0 10^{-6}; Poisson's ratio, ν_{Al} = 0.33, ν_{SiC} = 0.17; and density (kg/m^3), ρ_{Al} = 2800, ρ_{SiC} = 3200. The stress-strain curve adopted for the Aluminum matrix is presented in table 1.

The thermal loading arises in the cooling stage of the composite manufacturing process: initially the material is at 600 ºC and it is slowly cooled to the room temperature, 20 ºC. In the simulation code, the cooling process takes place within 10 steps. At the end of the process, the analysis is carried out on the stress curves associated with ΔT = -580 ºC.

Table 1: Adopted Aluminum stress-strain curve values (MPa *x* %)

σ - stress	146	175	195	205	215	220	225	228	229	230
ε - strain	0.2	0.45	0.8	1.0	1.5	2.0	3.0	4.0	6.0	8.0

On each analysis, the X displacement is restrained for the nodes on the Y axis (coordinate X = 0); while the Y displacement is restrained for those nodes on the X axis (coordinate Y = 0). This way, the model describes the central region of a hypothetical composite rod.

4. BASIC RESULTS – ISOSTRESS CURVES

Typical results, in terms of the isostress curves, are shown in figure 1 and figure 2, respectively for the first set of analyses, using round particles, and for the second set of analyses, using quadrilateral particles. Each curve in the figures has its specific RGB color to allow the image processing step. In both figures, the isostress curves values are equally spaced. The results referring to the quadrilateral particles were pos-processed in order to generate figures with 9, 20 and 40 isostress curves each. The results presented are those obtained with 20 isostress curves. In the figure 1.a, the stresses range from 20.9 MPa to 333 MPa and in the figure 1.b the stresses range from 18.9 MPa to 318 MPa. For better visualization, the colors were inverted as well as the figure was cropped, eliminating the lateral scale and some unimportant information for better visualization. This crop operation is realized inside the Matlab script to analyze the figures to eliminate unnecessary information generated by ANSYS like the legend, the color scale and the border line around it. In figures 2.a and 2.b the stress scale was preserved as well the 'original' colors.

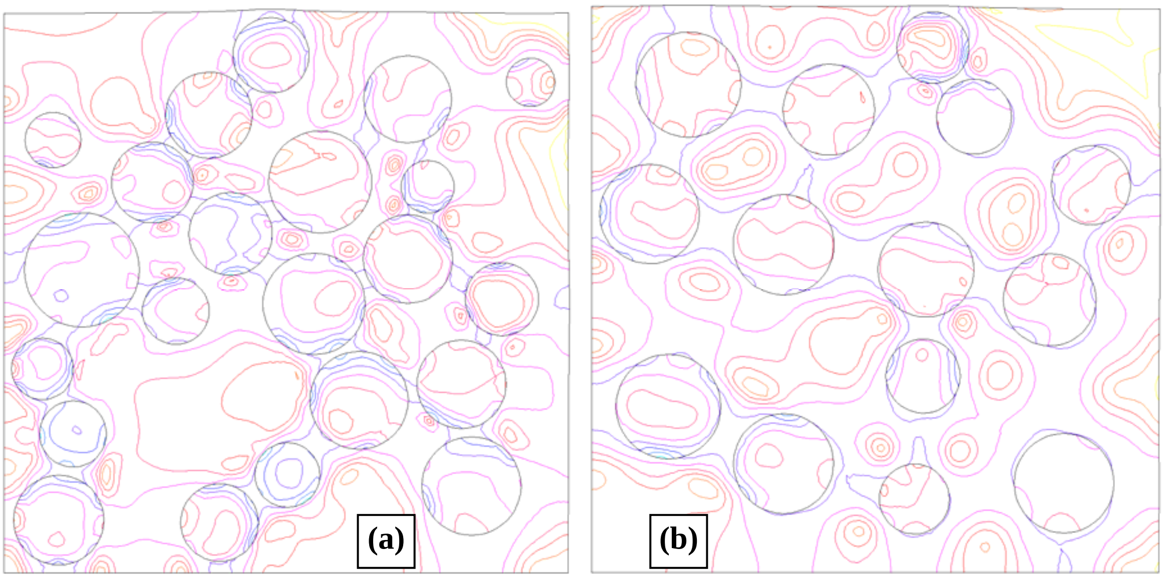

Figure 1. Typical stress distributions, using round particles (9 isostress values each)

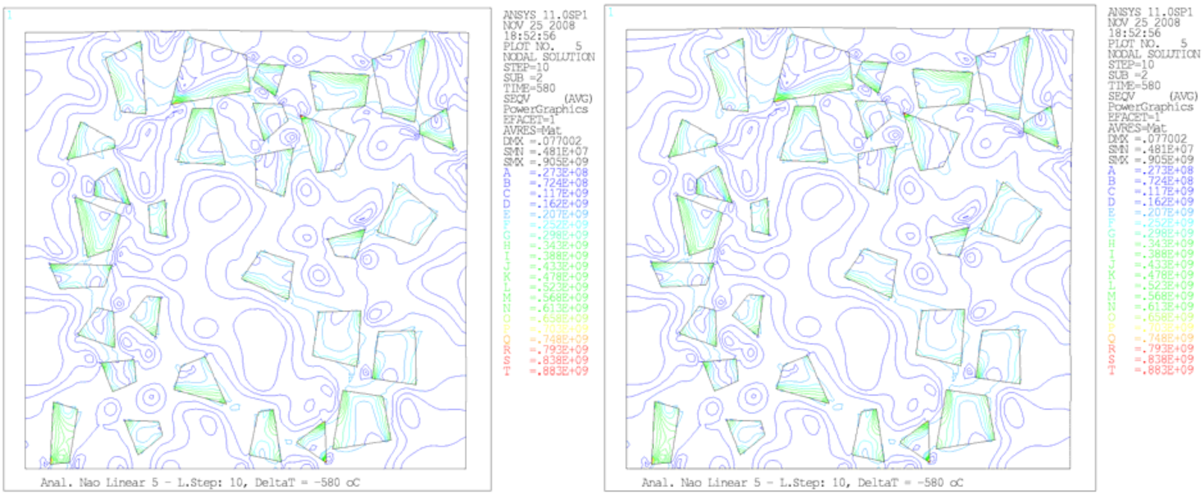

Figure 2. Stress distributions with quadrilateral particles (9 isostress values each) - typical

The algorithm developed to analyze the new (non-linear) results was based on the Matlab image processing toolbox and it gives the percentage of each color present in a given figure. As the isostress curves are equally spaced, for the scope of this work, this percentage is being interpreted as the weight to be applied at each curve/stress to obtain the averaged stress value in the material, for a given analysis. It uses the R(ed), G(reen) and B(lue) values associated with the colors in each one isostress curve (as defined in the ANSYS pos-processor when the figures were generated, in .TIF format). This algorithm was tested against some geometric figures with several curves, each one with known perimeter and area and it worked well with only a few percent of difference between the 'actual' and the calculated values, far less than the involved uncertainties in the process, as those associated with the material properties.

5. RESULTS ANALYSES AND DISCUSSION

Figure 3 shows the maximum and the minimum stress values for each analysis. As shown in that figure, the thermal stresses of round particle composites varies approximately, from 0 MPa until 380 MPa while for the quadrilateral particles the range is from about 0 MPa until 1000 MPa. It is clear, by analyzing the isostress curves, figure 1, that the maximum value occur mainly inside the particles. This is more pronounced in the quadrilateral particles, figure 2. This is quite different from the observed in [1] when linear behavior was adopted.

For each isostress index curve (9 curves/values for round particles analyses and 20 curves/values for the quadrilateral ones) there is a range of values that arises from the own randomness nature of the particles distributions as it occurs in an actual material. There are 30 analyses with round particles and 40 analyses with quadrilateral particles. This randomness can be observed in figure 4 were all curves (09 values each or 40 vales each) were plotted all together. Averaging the percentage for each isostress index curve at each index value there are an average and a standard deviation value and the curves in figure 5 is obtained. This figure 5 shows the averaged percentage for each isostress curve where the 'error' bar is the standard deviation.

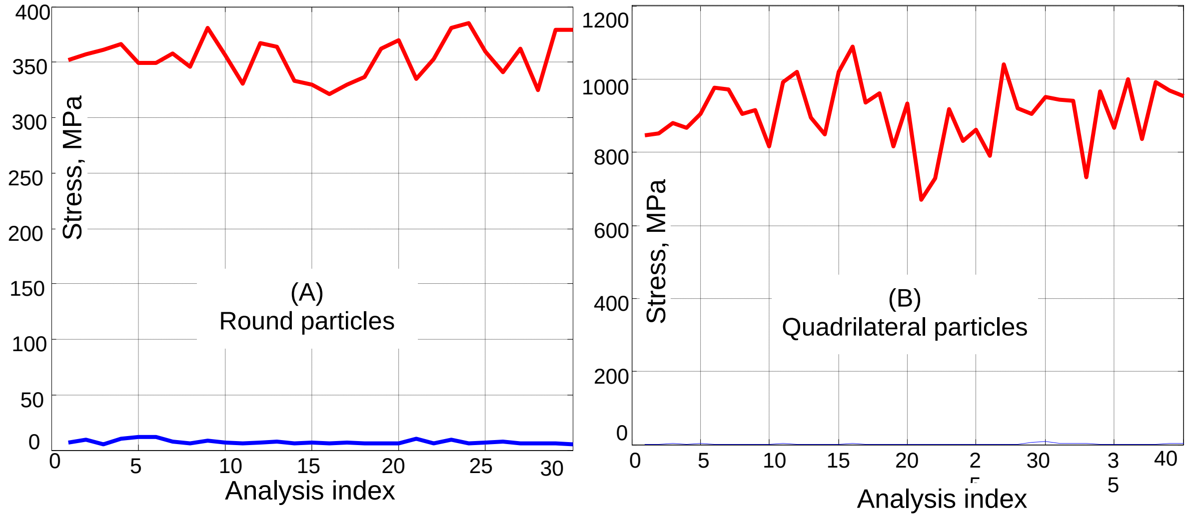

Figure 3. Min & Max stress values (MPa) curves for each analysis.

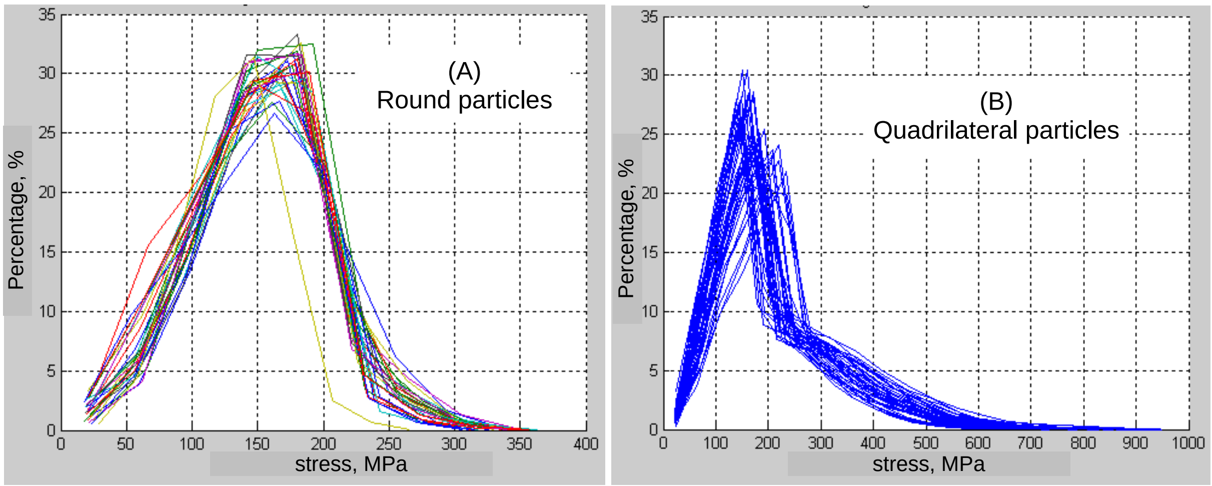

Figure 4. Percentages associated with the isostress curves for each analysis.

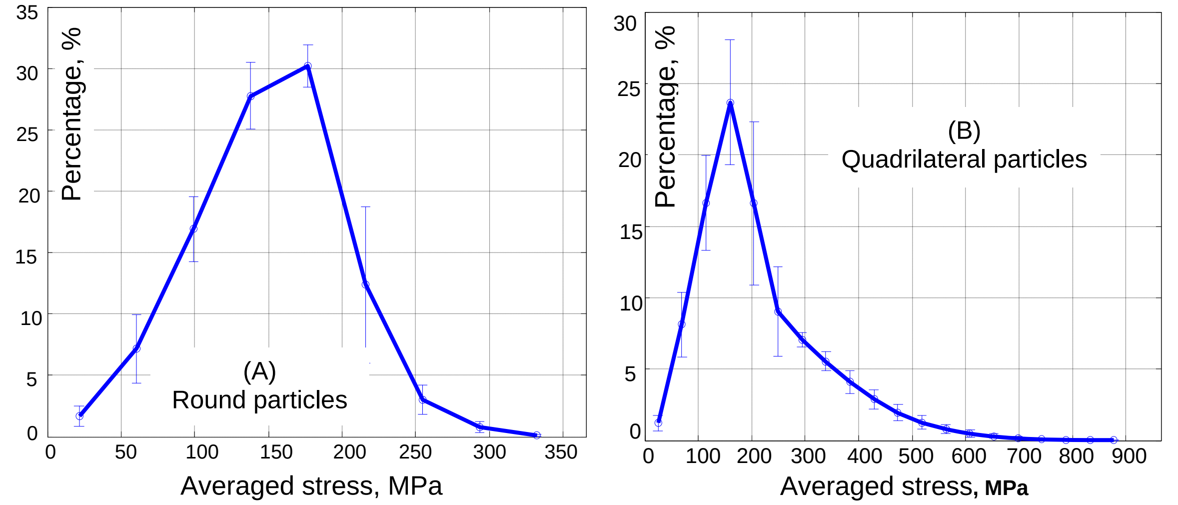

Figure 5. Averaged percentage for each isostress curve and respective error (Std. Dev).

Figure 6 shows the direct comparison among the present (Non-Linear) versus the previous results [1] (Linear), when the volumetric ratio ranges from ~17% to ~35%. This figure shows clearly the strong influence of the Aluminum matrix behavior in the results. In the elastic regime the volumetric ratio has a greater influence on the stress values and when the Aluminum matrix plasticity behavior is taken into account this influence is strongly attenuated

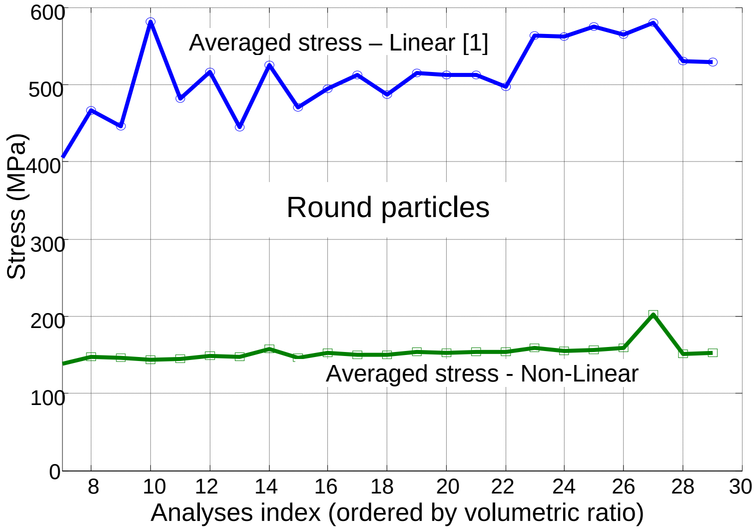

Figure 6. Comparison – Present (Non-Linear) X Previous (Linear) Numerical Results.

Considering the linear behavior, the trend in the thermal stresses is to increase as increases the volumetric ratio (in the analyzed volumetric ratio range of values) with the values oscillating in the range from ~400 MPa to ~580 MPa. This confirms the behavior predicted by the analytical formulation presented elsewhere [1]. The material can not afford those so high stress values as predicted by the analytical results and the correspondent numerical analyses (both considering linear behavior) [1]. So, the actual average thermal stress in the material is much lower than the predicted by the linear analyses, as shown in figure 6. Using the same statistical procedure, as used previously in [1], and from the processed results from the non-linear analyses shown above, the average stress level in the Al+SiC composite, due to the manufacturing process, from 600 ºC to 20 ºC, was evaluated as 171,9 MPa with a standard deviation 12,8 MPa.

6. CONCLUSIONS

As well known, the plastic behavior alters the stress field in a given material, as is the case investigated by this work: the thermal stresses in an MMC (Al+SiC) composite due to its manufacturing process. The briefly presented analytical approach covers the material linear behavior supposing round particles. However, the predicted thermal stresses locally reach values over the Aluminum yielding level which is confirmed by numerical linear analyses. So, to predict a more realistic thermal stress value in this material it is necessary to perform numerical analyses considering the non-linear behavior in the aluminum matrix which was done in this work, Also, a simple and very efficient and fast algorithm, based on the Matlab image processing toolbox was developed to analyze the non linear results in terms of the isostress curves. The comparison of these new results with the previous ones shows the strong influence of the aluminum matrix elastic-plastic behavior on the composite thermal stress distribution due to its manufacturing process (from 600 ºC to 20 ºC). The average stress was evaluated as ~172 MPa with a standard deviation of ~13 MPa. Although this average value is still a high one it is from thermal loading and it should be considered as so.

One can argue about the particle geometry as well as the wide range of volumetric ratios. As mentioned before, these choices were adopted in this work to allow a direct comparison with the previous analytical & numerical work. In a future work, it is planned to make a new set of numerical simulations with quadrilateral particles to verify the influence of the particles geometry using a very tight volumetric ratio near the value in an actual SiC composite.

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