A Solvable Model for Nuclear Shape Phase Transitions

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There has been considerable interest recently in phase transitions that occur between some well-defined nuclear shapes, e.g. the spherical vibrator, the axially deformed rotor and the γ-unstable rotor, which are assigned to the U(5), SU(3) and O(6) symmetries. These shape phase transitions occur through critical points of the IBM phase diagram and correspond to rapid structural changes. The first transition of this type describes transition form the spherical to the γ-unstable phase and has been associated with an E(5) symmetry. Later further critical point symmetries e.g. X(5) and Y(5) have also been proposed for transitions beween other nuclear shape phases.

We proposed [1] the sextic oscillator as a γ-independent potential in the Bohr Hamiltonian

$$-\frac{\mathrm{d}^2\phi}{\mathrm{d}\beta^2} + \left(\frac{(\tau+1)(\tau+2)}{\beta^2} + a^2\beta^6 + 2ab\beta^4 + (b^2 - 4ac)\beta^2 \right) \phi = \epsilon\phi \ ,$$

where $c = \frac{1}{2}(\tau + 2M + \frac{7}{2})$ is a constant. This sextic potential is quasi-exactly solvable, which means that for any non-negative integer value of M, $M+1$ of its solutions can be obtained in an algebraic way. The solutions are written as $\phi_n(\beta) = P_n(\beta^2)(\beta^2)^{c-M-\frac{3}{4}} \exp\left(-\frac{a}{4}\beta^4 - \frac{b}{2}\beta^2\right)$, where P_n is a polynomial of order $n \leq M$. The solutions with $M = 0$ and 1 are sufficient to generate the most widely studied levels, i.e. those with $n \equiv \xi - 1 \leq 1$ and $\tau \leq 3$. The energy eienvalues and E2 transition rates can also be written in closed form. The sextic oscillator has much more flexible structure than potentials considered previously: depending on the parameters this potential has a minimum at $\beta = 0$ or at $\beta > 0$, and might also have a local maximum before reaching its minimum. The Table below displays the comparison of experimental spectroscopic data on ^{134}Ba and the corresponding calculated values using different potentials $V(\beta)$. It is seen that the sextic oscillator allows a better approximation than the other essentially parameter-free potentials. The parameters used in the Table result in a potential that has a local minimum at $\beta > 0$.

	$\frac{E(4^+_{1,2})}{E(2^+_{1,1})}$	$\frac{E(0^+_{2,0})}{E(2^+_{1,1})}$	$\frac{E(6^+_{1,3})}{E(2^+_{1,1})}$	$\frac{B(E2;4^+_{1,2}\to2^+_{1,1})}{B(E2;2^+_{1,1}\to0^+_{1,0})}$	$\frac{B(E2;2^+_{2,0}\to2^+_{1,1})}{B(E2;2^+_{1,1}\to0^+_{1,0})}$	$\frac{B(E2;0^+_{1,3}\to2^+_{1,2})}{B(E2;2^+_{1,1}\to0^+_{1,0})}$
sextic oscillator	2.39	3.68	3.70	1.70	1.03	2.12
E(5)	2.20	3.03	3.59	1.68	0.86	2.21
β^4	2.09	2.39	3.27	1.82	1.41	2.52
^{134}Ba(exp.)	2.31	3.57	3.65	1.56(18)	0.42(12)	

In another application the chain of even Ru isotopes was considered from $A = 98$ to 112 [2]. The parameters were extracted from a fit to the low-lying energy spectrum of each nucleus and were used to plot the corresponding potential. It was found that up to A =102 the potential is essentially an harmonic oscillator, while at A =104 a rather flat potential was seen, in accordance with the expected phase transition and E(5) symmetry there. With increasing A then the minimum got increasingly deeper and moved away from $\beta = 0$.

We discuss the possibility of generalizing the formalism in two ways: first by including dependence on the γ variable allowing for the approximate description of nuclei close to the X(5) symmetry, and second, including higher-lying energy levels in the quasi-exactly solvable formalism.

[1] G. Lévai, J. M. Arias, Phys. Rev. C **69** (2004) 014303.
[2] G. Lévai, AIP Conf. Proc. **726** (2004) 227.