VVER-1000 DOMINANCE RATIO

S.S.Gorodkov, RRC Kurchatov Institute, Moscow, Russia

ABSTRACT

Dominance ratio, or more precisely, its closeness to unity, is important characteristic of large reactor. It allows evaluate beforehand the number of source iterations required in deterministic calculations of power spatial distribution. Or the minimal number of histories to be modeled for achievement of statistical error level desired in large core Monte Carlo calculations. In this work relatively simple approach for dominance ratio evaluation is proposed. It essentially uses core symmetry. Dependence of dominance ratio on neutron flux spatial distribution is demonstrated.

Calculation of nuclear reactor steady state means finding of reactor effective multiplication coefficient and neutron flux distribution in phase space. This distribution $\Psi(\vec{r},\vec{v})$ is the solution of neutron transport equation

$$\Psi = \frac{1}{K_{eff}}\hat{F}\Psi \qquad (1)$$

where $\hat{F}$ is the operator describing transport of neutron from birth to absorption or leakage. It works in specified bounded region and includes boundary conditions.

In codes based on deterministic algorithms this equation is solved in iterations

$$\Psi^{(n)} = \frac{1}{K_{eff}}\hat{F}\Psi^{(n-1)} \qquad (2)$$

beginning from some initial distribution $\Psi^{(0)}$, which is preset either by code author or by code user and is based on some notions of forthcoming solution.

Confidence in iteration process (2) leading to exact solution is based on operator $\hat{F}$ property, which is provable only for some $\hat{F}$ approximations. This property lies in existence of (in)finite set of so-called eigen linearly independent solutions u_i, such as

$$k_i u_i = \frac{1}{K_{eff}}\hat{F}u_i \qquad (3)$$

Having physical meaning everywhere positive solution (3) is denoted $\mathbf{u_0}$. Corresponding to it number $\mathbf{k_0}$ equals obviously to unity and all other $\mathbf{k_i}$ are numbered in order

$$\mathbf{1} > |\mathbf{k_1}| \geq |\mathbf{k_2}| \geq |\mathbf{k_3}| \geq \ldots \qquad (4)$$

Furthermore, any distribution including initial one $\Psi^{(0)}$ may be presented as linear combination of $\mathbf{u_i}$ solutions:

$$\Psi^{(0)} = \mathbf{u_0} + \mathbf{a_1 u_1} + \mathbf{a_2 u_2} + \mathbf{a_3 u_3} + \ldots \qquad (5)$$

After **n** iterations evidently

$$\Psi^{(n)} = \mathbf{u_0} + \mathbf{k_1^n a_1 u_1} + \mathbf{k_2^n a_2 u_2} + \mathbf{k_3^n a_3 u_3} + \ldots \qquad (6)$$

Due to property (4) $\mathbf{a_i}$ amplitudes of all eigen solutions in expansion (6) will decrease with iteration number increase and the faster the greater is difference between $\mathbf{k_i}$ and unity. At sufficient iteration number all these amplitudes, and the difference between $\Psi^{(n)}$ and exact solution $\mathbf{u_0}$, can be made sufficiently small.

Iterations convergency rate will be defined by difference between unity and biggest by module eigenvalue $\mathbf{k_i = k_1}$, which is denoted by ρ and called dominance ratio. Those eigen functions $\mathbf{u_i}$, whose eigen values may concur at dominance ratio determination we will call dominant ones.

ρ is important reactor physical characteristic, essentially influencing many calculation processes. In particular, if only all initial distribution is not concentrated at single point or is not sign changing but is realistic sufficiently uniform in core space function, then $\mathbf{a_1 \approx 1}$, i.e. initial distribution error is of ~100% order. To diminish it 100 times after **n** iterations up to 1%, it is necessary that

$$\mathbf{k_1^n} = \rho^n \approx \mathbf{0.01} \qquad (7)$$

So number of iterations required is

$$\mathbf{n = \ln(0.01) / \ln(\rho) \approx 4.6 / (1 - \rho)} \qquad (8)$$

Dominance ratio allows also, for instance, to evaluate minimal number of histories to be modeled to achieve prescribed statistical error of power distribution in different fuel assemblies of large reactor[1]. Therefore substantial efforts are bent to search of methods of

reliable ρ evaluation. In recent years active part in this search take Monte Carlo specialists [2,3]. Here we present rather different approach to this problem.

Let us consider uniform homogeneous core in the form of parallelepiped with dimensions $-x_C/2<x< x_C/2$, $-y_C/2<y< y_C/2$, $- h_C/2 < z < h_C/2$. Let us take also that neutron transport in this core is described by one group diffusion equation

$$M^2\nabla^2\Psi + \left(\frac{K_\infty}{K_{eff}} - 1\right)\Psi = 0 \qquad (9)$$

which can be transformed to form (1):

$$\Psi = \frac{1}{K_{eff}}\hat{F}\Psi = \frac{1}{K_{eff}}(1 - M^2\nabla^2)^{-1}K_\infty\Psi \qquad (10)$$

Eigen functions u of operator $\frac{1}{K_{eff}}\hat{F}$ will be in this case

$$u_{j,n,m}(x, y, z) \sim \left\{\begin{matrix}\sin(\pi jx/x_C)\\ \cos(\pi jx/x_C)\end{matrix}\right\} \times \left\{\begin{matrix}\sin(\pi ny/y_C)\\ \cos(\pi ny/y_C)\end{matrix}\right\} \times \left\{\begin{matrix}\sin(\pi mz/h_C)\\ \cos(\pi mz/h_C)\end{matrix}\right\} \qquad (11)$$

where **j,n,m = 0**(for cosine only)**,1,2,3. .** Eigen values of these eigen functions are defined by expression

$$k_{j,n,m} = 1 - \frac{\pi^2 m^2 M^2}{h_C^2} - \frac{\pi^2 j^2 M^2}{x_C^2} - \frac{\pi^2 n^2 M^2}{y_C^2} \qquad (12)$$

VVER-1000 core top-bottom reflector influence allows to approximate it in our simplified problem by zero flux condition on extrapolated border. So we will recon further that h_C=370 cm and will keep in solution (12) only proportional to $\cos(\pi mz/h_C)$ eigen functions. In future estimates it will be counted also x_C= y_C=280 cm, so that model core horizontal crossection area will be equal to that of VVER-1000.

In simple cases of reflection (or absorption) on side surface of our model core K_{eff} value will be equal to

$$\frac{K_{eff}}{K_\infty} = 1 - \frac{\pi^2 M^2}{h_C^2} - \frac{\pi^2 (0 \text{ or } 1)^2 M^2}{x_C^2} \qquad (13)$$

and dominance ratio will be defined from expression:

$$1 - \rho \approx \min\left(\frac{3\pi^2 M^2}{h_C^2}; \frac{\pi^2 (1 \text{ or } 3) M^2}{x_C^2} \right) \qquad (14)$$

Keeping in mind that for VVER-1000 $\mathbf{M^2} \sim 50$ cm^2 , we obtain from (14) that in case of core side border reflection $\mathbf{1 - \rho}$ equals to 0.0063 from horizontal component of (11), and in case of absorption – to 0.011 from axial component of (11).

Dominance functions in case of side reflection are

$$\sin(\pi x / x_C), \quad \cos(\pi y / y_C), \quad \cos(\pi z / h_C).$$

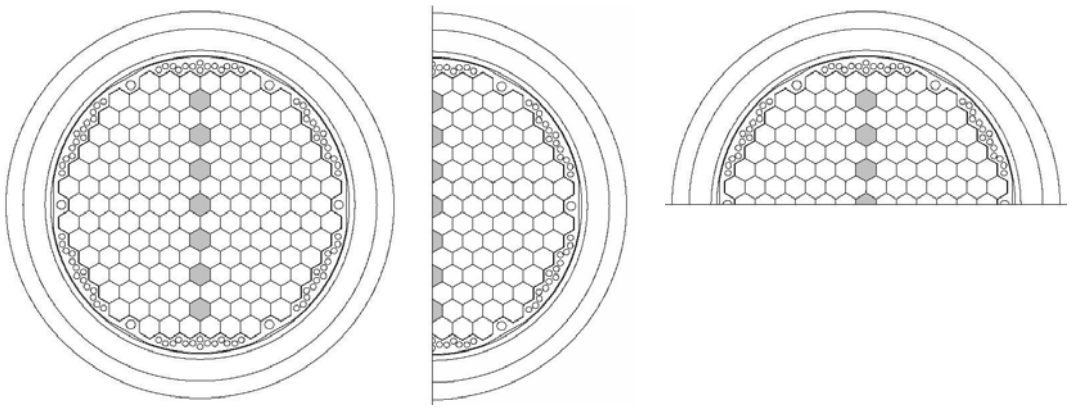
Let us note that these functions are antysymmetric relative to planes **y0z, x0z, x0y** correspondingly. So we make here hypothesis, quite reasonable but probably indefensible. Namely: if reactor core and hence $\mathbf{u_0}$ - physical all over positive solution (3) have mirror symmetry relatively to some plane, then one of dominance functions will be antisymmetric relatively to the same plane.

But if so, then for determination of dominant ratio it is sufficient to calculate $\mathbf{K_{eff}}$ of adequate reactor half with zero flux border condition by core mirror symmetry plane. In principle the task of such kind can be put before many codes, including mesh ones. But the latter are usually problem oriented and corresponding option may be missed in them.

But for universal Monte Carlo codes specification of such border condition will present no difficulties. For instance in rationally prepared input file for MCU code[4] it is enough to change single value with previous border condition – absorption. More correct must be rather unusual border condition – reflection with neutron weight sign change, since at full absorption neutron flux will be zero not on symmetry plane, but on that slightly shifted on linear extrapolation length, which equals approximately to 0.7 of neutron mean free path. So at wish for evaluation of this difference we can slightly move the plane of our interest. It is clear that in VVER-1000 case this difference will be small.

Strictly speaking, VVER-1000 symmetry in **x0y** plane is always rotational one. Mainly that of 120^o, often that of 60^o. Mirror symmetry relatively to **y0z** , or **x0z** planes is possible either in case of all control rod groups are lowered to the same depth, or in case of only some properly placed group being inserted. We still hope that lack of certainly assumed definite mirror symmetry in some situations will lead only to insignificant difference between our estimation of dominance ratio and its true value.

3D full-scale VVER-1000 core model was used with MCU code to model 1 billion of neutron histories in fresh unit number 1 of Rostov NPP on hot zero power level. All control rods were in upper position (variant A), which provided mirror symmetry relatively **y0z** and **x0z** planes. Next three variants were (see fig. 1) with total absorption condition on planes **x=0**(variant B), **y=0**(variant C) and $\mathbf{z=h_C/2}$(variant D, being absent on fig. 1)**.** Calculation results are presented in Table 1.



A B C

Fig. 1. Horizontal crossections of calculation regions in variants A – C. Shadowed are those fuel assemblies where control rods were dropped down in subsequent calculations.

Table 1. Variants A – D calculation results

variant	$\mathbf{K_{eff}}$	$\mathbf{k_i = K_{eff} / K^A_{eff}}$	$\mathbf{1 - k_i}$
A	1.00068	1.00000	
B	0.99584	0.99516	0.0048
C	0.99596	0.99528	0.0047
D	0.98932	0.98865	0.0114

It follows from Table 1, that dominance ratio does not depend strongly on core form, slightly differing from cylinder one. Eigen value of antysymmetric solution along **0z** axis is close to model problem one (**1 - 0.011**). But eigenvalue of planar antisymmetris solution **ρ = 1 - 0.0047** is markedly closer to unity than model one **1 - 0.0068**, received at reflection condition on core side.

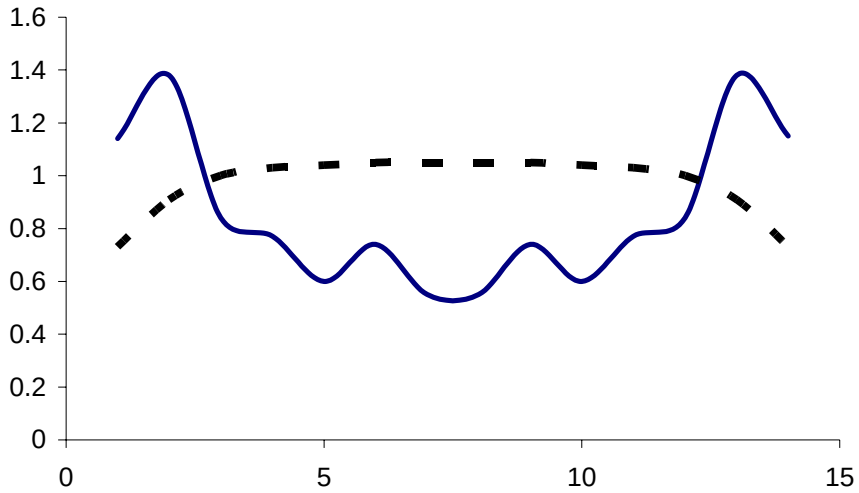


Figure 2. Diametral fuel assemblies power distribution for simplified smoothed core (----) and that of Rostov 1-st unit core on hot zero power level (_____).

To clarify this difference let us look at Figure 2, where power distribution among different fuel assemblies on core diameter is pictured. It is concave in core center. From its

part model solution for side reflection condition with $\rho = 1 - 0.0068$ is core constant. Core side absorption condition will make power distribution convex one and, according to (14), markedly diminish ρ value. To test this assumption we considered simplified better leveled core load (see also Figure 2). Its calculations results see in Table 2. This time evaluated dominance ratio is much closer to model one and slightly bigger, possibly because of leveled power distribution decreases toward sore side.

Table 2. Leveled core calculation results for A and C variants

variant	K_{eff}	$k_i = K_{eff} / K_{eff}^A$	$1 - k_i$
A	1.00944	1.00000	
C	1.00136	0.99200	0.0080

Let us see also how dominance ratio will change after insertion of control rods in seven fuel assemblies shadowed on Figure 1. K_{eff} of whole core will decline. **x0z** and **y0z** symmetries will be conserved. But while K_{eff} of C variant will markedly change also, insertion of control rods on core border of B variant will diminish K_{eff} sufficiently less and dominance ratio will turn out much closer to unity. Which Table 3 demonstrates.

Table 3. Calculation results for A-C variants with control rods inserted.

variant	K_{eff}	$k_i = K_{eff} / K_{eff}^A$	$1 - k_i$
A	0.997145	1.00000	
B	0.995884	0.9987	0.0013
C	0.989214	0.9920	0.0080

We can say in conclusion that according to (8) it follows from $\rho = 1 - 0.0047$ evaluation that number of source iterations necessary for solution ~ 1% accuracy amounts to 1000. Highest possible acceleration of this iteration process with the help of Chebyshev polynomials using dominance ratio obtained can decrease this number to ~ 60. To receive similar statistical error in VVER-1000 Monte Carlo calculations modeling of no less than 1 billion of neutron histories is needed[1].

Let us note that since dominance ratio value is needed for different evaluations, knowledge of its exact value is not of great importance. So approach here presented can be applied even in those cases, where core structure somewhat differs from mirror symmetry.

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LIST OF NOMENCLATURE

$\Psi(\bar{r}, \bar{v})$ - neutron flux distribution in phase space

$\Psi^{(0)}$ - neutron flux initial distribution

$\Psi^{(n)}$ - neutron flux distribution after n source iterations

$\hat{F}$ - neutron transport operator

K_{eff} - effective multiplication coefficient

K_{∞} - effective multiplication coefficient of infinite lattice

M^2 – neutron migration area

u_i - linearly independent solutions of neutron transport equation

u_0 - physical all over positive solution of neutron transport equation

k_i - divided by K_{eff} eigen value, corresponding to u_i solution

ρ - dominance ratio, the value of biggest $k_i = k_1$

x_C - core lateral dimension

h_C - core effective height

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