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EFFECT OF DIFFERENT SIZE DUST GRAINS ON THE PROPERTIES OF SOLITARY WAVES IN SPACE ENVIRONMENTS

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Propagation of nonlinear dust-acoustic (DA) waves in unmagnetized collisionless dusty plasma consisting of dust grains obeys power law dust size distribution and nonthermal ions are investigated. For nonlinear DA waves, a reductive perturbation method was employed to obtain a Korteweg-de Vries (KdV) equation for the first-order potential. The effects of a dust size distribution, dust radius and the non-thermal distribution of ions on the soliton amplitude, width and energy of electrostatic solitary structures are presented.

Keywords: *Dust Acoustic Waves; KdV Equation; Power Law Dust Size Distribution.*

INTRODUCTION

Dusty plasmas or, respectively, complex plasmas are plasmas containing solid or liquid dust particles which are charged. The field of dusty plasmas, which is in fact a multi-disciplinary research topic of recent interest, involves not only numerous physical processes, but also deals with fundamental mechanisms and questions that arise in detailed physics and chemistry leading to particle formation. The heterogeneous and homogeneous nucleation phenomena determine the formation of very small (less than nanometre) sized proto-particles. The formation of proto-particles is followed by agglomeration and coagulation processes, as similarly found in other parts of chemistry such as colloidal chemistry. The presence of massive charged particles in a complex plasma is essential for collective processes [1, 2] and does not only change the charge composition but also introduce new physical processes into the system. It has been shown, both theoretically and experimentally, that the presence of extremely massive and highly charged dust grains modifies the existing plasma wave spectra [3] and even introduces new eigenmodes, i.e. dust acustic (DA) waves [4,5], dust ion acoustic (DIA) waves [6,7], dust-lattice (DL) waves [8,9] etc. As a matter of fact, dust grains reduce the number of free mobile electrons, as some of the latter are absorbed on the grains, and introduce a new timescale of low-frequency dust modes. The charge-to-mass ratios of dust particles are smaller by several orders of magnitude than those of conventional plasma particles, thus leading to various waves of such lower frequency. These electrostatic modes usually exist in an unmagnetized dust plasma. Dust and dust plasmas are quite natural in space. They are present in planetary rings, comet tails, interplanetary and interstellar clouds are found in the vicinity of artificial satellites and space stations etc. [10-13]. Also in laboratories dusty plasma are actively investigated [14, 15]. In the last decade much interest was devoted to studies of the low frequency oscillations in dusty plasmas observed in the several cases mentioned before. Recently, numerous authors have focused their attention on wave propagation not only in mono-sized dust grains but also in multi-size dust grains both in space plasmas and in laboratory experiments [1, 16-20]. Clearly, dust comes in a great variety of sizes, masses and charges. The presence of charged microparticles in plasma can lead to significant modification of the bulk plasma parameters such as the charge distribution and the potential profile. Further, charged microparticles in plasma will significantly depart from the Boltzmann dusty plasma distribution. Their description demands for resorting a new kind of dust distributions. For that we consider in this work so-called fractal-type distributions of power law form. In many space physics situations such a class of distributions could offer some novel and inspiring findings of bulk plasma properties.

For example, for charged dust grains with a characteristic radial parameter a in a given range $[a_{\min}, a_{\max}]$ the differential number density was suggested [3] to be expressed as

$$n_d(a)da = ka^{-\beta}da \qquad (1)$$

Here $n_d(a)da$ denotes the number of charged dust grains with radius between a and $a+da$ that is expected per unit volume, β is the power law index and k normalization constant. The existence of such non-Maxwellain distributions can have profound effects on wave propagation and transport processes [21-23]. Since the distribution is extremely sensitive to the value of β, it is also called the critical exponent. The following values have been frequently assigned according to observations in space plasmas throughout the solar system: β=4.6 for the F-ring of Saturn [20], β=7 and 6 for the G-ring of Saturn [21, 22] and β=3.4 for cometary environments [23]. The constant k is determined via the normalization

$$n_{tot} = \int_{a_{\min}}^{a_{\max}} n_d(a)da, \qquad (2)$$

from which $k = \dfrac{(1-\beta)N_{tot}}{a_{\max}^{(1-\beta)} - a_{\min}^{(1-\beta)}}$. Recently, Duan et al.[24-27] examined the nonlinear effects of a fractal dust grain distribution in plasmas containing Maxwellian ions at two temperatures. More recently, Elwakil et al. [28] investigated a vortex ion dusty plasma and studied the higher-order nonlinearity of DA waves. Observations of space plasmas indicate the presence of substantial nonthermal ion populations. Nonthermal ions from the Earth's bow shock as well as in and around the Earths foreshock have been detected by satellites [29, 30]. The automatic space plasma experiment with a rotating analyzer (ASPERA) on the Phobos satellite has discovered nonthermal ion fluxes from the upper ionosphere of Mars [31]. Closer to the Earth, fast nonthermal ions have been recently observed by the Nozomi satellite in the vicinity of the Moon[32]. The observations revealed that the nonthermal ions possess a partial ring structure in the velocity phase space. The mechanism suggested for its formation is that some of the solar wind ions are deflected in the close vicinity of the Moon [32].

Investigations of small-amplitude dust-acoustic waves (DAWs) usually describe the evolution of the wave by the Korteweg-de Vries (KdV) equation. This equation can be derived from the basic set of fluid equations for the plasma system via the so-called reductive perturbation theory [33].

In this work, our motive is to study the effects of the critical power exponent on the amplitude, width and energy of (DA) solitary waves in an unmagnetized collisionless dusty plasma containing dust grains and non-thermal ions. This paper is organized as follows: In Section 2 we present the basic set of fluid equations governing our plasma model. In Section 3 we derive the KdV equation with lowest-order nonlinearity and dispersion for describing nonlinear DAWs. Finally, discussions and conclusions are given in Sections 4 and 5.

BASIC EQUATIONS

Let us consider a system of an unmagnetized collisionless dusty plasma consisting of charged dust grains and ions, and with the dusty grains much heavier than the ions and having different sizes (i.e. the dust size distribution is given by a power law). We assume that there are N different charged dust grains with masses $m_j \left(\mathrm{j}=1,2,3,...,\mathrm{N} \right)$.

Charge neutrality condition at equilibrium reads $n_{i0} = \sum_{j=1}^{N} n_{d0j} z_{d0j}$, where n_{i0} and n_{d0j} represent the number densities of unperturbed ions and of j-th dust grains, respectively, z_{d0j} is the unperturbed number of charges residing on the j-th dust grain measured in the unit of electron charge e. The dynamics of dust acoustic oscillations (low phase velocity lying between the ion and dust thermal velocities is described in dimensionless variables by:

$$\frac{\partial}{\partial t} n_{dj} + \frac{\partial}{\partial x}(n_{dj} u_{dj}) = 0 , \tag{3a}$$

$$\frac{\partial u_{dj}}{\partial t} + u_{dj} \frac{\partial u_{dj}}{\partial x} - \frac{z_{dj}}{m_{dj}} \frac{\partial \phi}{\partial x} = 0 , \tag{3b}$$

$$\frac{\partial^2 \phi}{\partial x^2} + n_i - \sum_{j=1}^{N} z_{dj} n_{dj} = 0 \tag{3c}$$

With $N_{tot} = \sum_{j=1}^{N} n_{d0j}$, $\overline{z_{d0}} N_{tot} = \sum_{j=1}^{N} n_{d0j} z_{d0j}$ *and* $\bar{a} = \dfrac{\sum_{j=1}^{N} a_j z_{d0j}}{\sum_{j=1}^{N} n_{d0j}}$. Further, the effective temperature T_{eff} can be defined as $\dfrac{1}{T_{eff}} = \dfrac{1}{\overline{z_{d0}} N_{tot}} (\dfrac{n_{i0}}{T_i})$. In the above equations,

$n_{dj}, u_{dj}, z_{dj}, \phi, m_{dj}, n_i, N_{tot}, \overline{z_{d0}},$ *and* $\bar{a}$ are the number density, the velocity, the charge number of the j-th dust grain , the electric potential, the mass of the j-th dust grain, the number density of ions, the total number density of dust grains, the average unperturbed charge number and the average radius of dust particles, respectively. All these quantities are dimensionless and normalized in terms of the following characteristic quantities: the dust density is normalized by $N_{tot}, z_{d0j},$ *and* $\overline{z_{d0}}$. The space coordinate x (in the direction of wave propagation), time t, velocity u_{dj} and electrostatic potential φ are normalized by the effective Debye length, the inverse of effective dust plasma frequency, the effective dust acoustic speed and $\dfrac{T_{eff}}{e}$.

The non-thermal ions density is given by:

$$n_i = [1 + \delta\,\phi + \delta\,\phi^2]\exp(-\phi),\ \delta = \frac{4\delta_1}{1+3\delta_1}. \tag{4}$$

Equations (3a) and (3b) represent the inertia of cold electron and (3c) is the Poisson's equation need to make the self consistent. The ions densities are described by non-thermal distribution given by (4).

DERIVATION OF THE KdV EQUATION

According to the general method of reductive perturbation theory, we introduce the slow stretched co-ordinates:

$$\tau = \varepsilon^{3/2}\,t, \qquad \text{and} \qquad \xi = \varepsilon^{1/2}(x - \lambda t), \tag{5}$$

where ε is a small dimensionless expansion parameter and λ is the wave speed. All physical quantities appearing in (3) are expanded as power series in ε about their equilibrium values as:

$$n_{dj} = n_{d0j} + \varepsilon\, n_{d1j} + \varepsilon^2\, n_{d2j} + \varepsilon^3\, n_{d3j} + \ldots, \tag{6a}$$

$$u_{dj} = \varepsilon u_{1j} + \varepsilon^2 u_{2j} + \varepsilon^3 u_{3j} + \ldots, \tag{6b}$$

$$\phi = \varepsilon\,\phi_1 + \varepsilon^2\,\phi_2 + \varepsilon^3\,\phi_3 + \ldots. \tag{6c}$$

We impose the boundary conditions as $\xi \to \infty,\ n_{dj} = n_{d0}\ ,\ u = 0,\ \phi = 0$.

Substituting (5) and (6) into (3) and equating coefficients of like powers of ε, the lowest-order equations in ε lead to the following results:

$$n_{d1j} = \frac{(\delta-1)}{\sum_{j=1}^{N} z_{dj}}\phi_1 \qquad \text{and} \qquad u_{d1j} = \frac{(\delta-1)\lambda}{n_{d0}\sum_{j=1}^{N} z_{dj}}\phi_1. \tag{7}$$

Poisson's equation gives the linear dispersion relation

$$\lambda^2 = -\frac{n_{d0} z_{dj}\,(\sum_{j=1}^{N} z_{dj})}{m_{dj}\,(\delta-1)}. \tag{8}$$

Considering the coefficients of $O(\varepsilon^2)$ and eliminating the second order perturbed quantities n_{2j}, u_{2j} and ϕ_2 lead to the following KdV equation for the first-order perturbed potential:

$$\frac{\partial}{\partial\tau}\phi_1(\xi,\tau) + A\phi_1(\xi,\tau)\frac{\partial}{\partial\xi}\phi_1(\xi,\tau) + B\frac{\partial^3}{\partial\xi^3}\phi_1(\xi,\tau) = 0, \tag{9}$$

Where $A = (-\lambda^2 + \dfrac{(3(\delta-1)^2\ \lambda^2)}{n_{d0}(\sum_{j=1}^{N} z_{dj})})(2(\delta-1)\ \lambda)^{-1}$ and $B = \dfrac{\lambda}{(1-\delta)}$.

In the case of a continuous power law distribution, we assume $\lambda_{Dd} \gg a$ for all dust grain sizes; then the mass and the charge of dust particles can be expressed as:

$$m_{dj} = k_m a_j^3 \text{ and } z_{dj} = k_z a_j, \tag{10}$$

where $k_m = \frac{4}{3}\pi\rho_d$ *and* $k_z = \frac{4\pi \in_0 V_0}{e}$ are constants, ρ_d is the mass density of the dust grains, V_0 is the electric surface potential, $\in_0$ the vacuum permittivity.
Thus λ^2 and A are given by

$$\lambda^2 = -\frac{(k\,(a_{\min}^{-\beta} - a_{\max}^{-\beta})k_z^2}{\beta(\delta-1)k_m},$$

$$A = -\frac{\lambda}{2(\delta-1)} + \frac{3k\,(\frac{a_{\min}^{-2-\beta}}{2+\beta} - \frac{a_{\max}^{-2-\beta}}{2+\beta})k_z^3}{2(\delta-1)\lambda^3 k_m^2}. \tag{11}$$

The averaged radial grain size and the square of the average velocity $\bar{\lambda}$ of the dust grains are found as

$$\bar{a} = \frac{(\beta-1)(a_{\min}^2 a_{\max}^{\beta} - a_{\min}^{\beta} a_{\max}^2)}{(\beta-2)(a_{\min} a_{\max}^{\beta} - a_{\min}^{\beta} a_{\max})}, \tag{12}$$

$$\bar{\lambda}^2 = -\frac{(\beta-1)(a_{\max}^{\beta} - a_{\min}^{\beta})k_z^2}{\beta(\delta-1)(a_{\min}^{\beta} a_{\max} - a_{\min} a_{\max}^{\beta})k_m}.$$

Note that, if our analytical model which refers to actually multi-sized dust grains is reduced to mono-sized dust grains having the average dust size $\bar{a}$, the coefficient of the nonlinear term in the KdV equation, Eq. (9), can be readily expressed as

$$\bar{A} = \frac{1}{(4(\delta-1))}(2\bar{\lambda} + \frac{3k\,(\beta-1)a_{\min}^{-2-\beta}a_{\max}^{-2-\beta}(a_{\min}^{2(1+\beta)} - a_{\max}^{2(1+\beta)})k_z^3}{(\beta+1)\rho^3(a_{\min}^{\beta}a_{\max} - a_{\min}a_{\max}^{\beta})k_m^2}). \tag{13}$$

In order to solve Eq. (9) for multi-sized grains let us introduce the variable $\eta = \xi - v\,\tau$, η is the coordinate transformed with respect to a frame moving with velocity v .
Integrating Eq. (9) with respect to η and using the boundary conditions as $\xi \to \infty$, $n_{dj} = n_{d0}$, $u = 0$, $\phi = 0$, the hall-mark soliton solution is obtained, i.e.

$$\phi_1 = \phi_m \sec h^2(D\eta), \tag{14}$$

with the peak soliton amplitude ϕ_m and the soliton width D^{-1} given by

$$\phi_m = \frac{3v}{A} \text{ and } D^{-1} = \sqrt{\frac{2B}{v}}. \tag{15}$$

The ratio of the soliton amplitude in multi-sized grain plasmas to that in mono-sized dust plasmas appears to be $P = \frac{\bar{A}}{A}$ and the ratio of soliton widths is $W = \sqrt{\frac{\lambda}{\bar{\lambda}}}$.

Inspecting the structures of Eqs.(7,8,14 and 15), and after some mathematical manipulations, one can easily express the peak soliton velocity and the first-order perturbed velocity of the j-th dust grain as

$$u_m = \frac{(\beta - 2)(\delta - 1)\lambda}{k\,(a_{\min}^{2-\beta} - a_{\max}^{2-\beta})k_z}\phi_m ,$$
$$u_1 = u_m \sec h^2 (D\eta). \qquad (15)$$

The soliton energy E_s is obtained according to the integral [34]

$$E_s = \int_{-\infty}^{+\infty} u_1^2 (\eta) d\eta . \qquad (16)$$

Upon introducing Eq. (15) into Eq. (16) and performing the integration, we find

$$E_s = \frac{4u_m^2}{3D}. \qquad (17)$$

RESULTS AND DISCUSSION

Nonlinear dust-acoustic solitary waves (DAWs) in an unmagnetized, collisionless dusty plasma consisting of different-size dust grains and non-thermal ions have been investigated using an arbitrary power law dust size distribution. To make our result physically relevant, numerical calculations were performed referring to typical dusty plasma parameters as given in Ref. [19]:

n_{i0}	$\sim 10^5 - 10^{10} cm^{-3}$
Ti	$\sim 0.1eV$
n_{d0}	$\sim 10^5 cm^{-3}$
$Q_{dj} = (-eZ_{dj})$	$\sim (-10^4 e) - (-10^5 e)$
$m_d \sim 10^{-15} - 10^{-12}$g	$\sim 10^{\ 9} - 10^{12} m_i$
ρ_d	~ 1 gcm $^{-3}$
a	$\sim 0.1 - 1\ \mu$m

we estimated $k \sim 10^{-12} - 10^{-20}, k_z \sim 10^8 - 10^9, k_m \sim 4$ and in our numerical results we use the values $k = 10^{-20}, k_z = 10^8, k_m = 4 \, and \ \upsilon = 1.7$.

We have devoted some effort to examine the effect of parameters like the power law index β, the energetic population parameter δ_1 and the minimum dust grains radius on the existence of the soliton structure, type, and energy. We note that our dusty plasma model admits the coexistence of both rarefactive and compressive solitons even for multi-size dust grains.

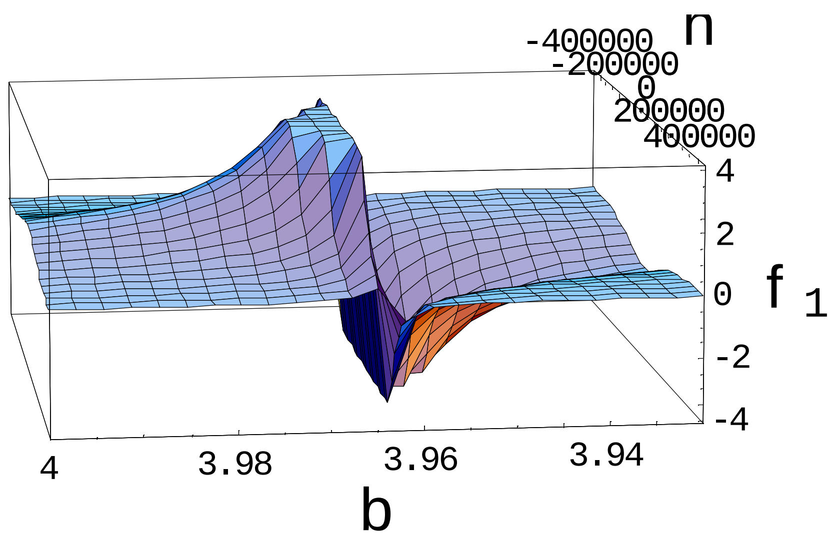

Figure 1: Effect of variations of the power law index β on the amplitude and width of compressive and rarefactive solitons in case of multi-sized.

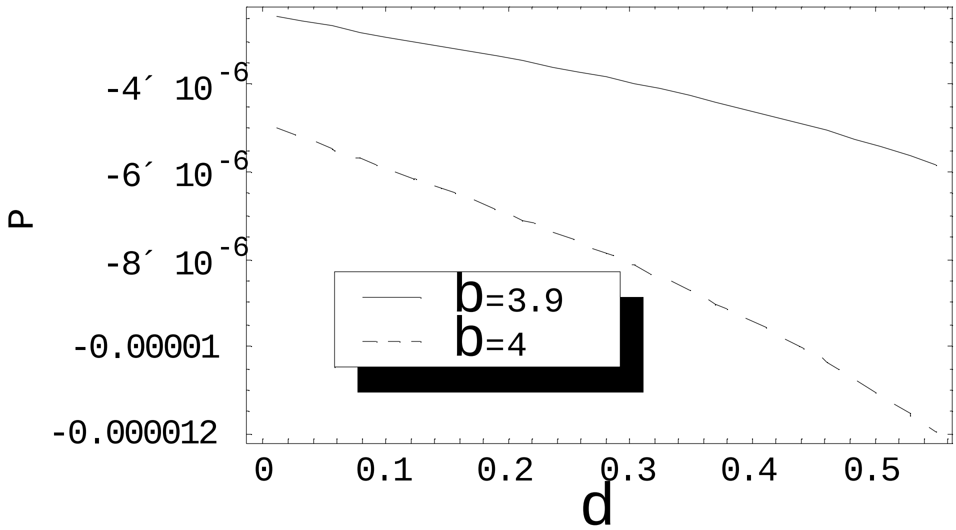

Figure 2: Ratio of the soliton amplitudes in multi-sized and in mono-sized dust plasmas.

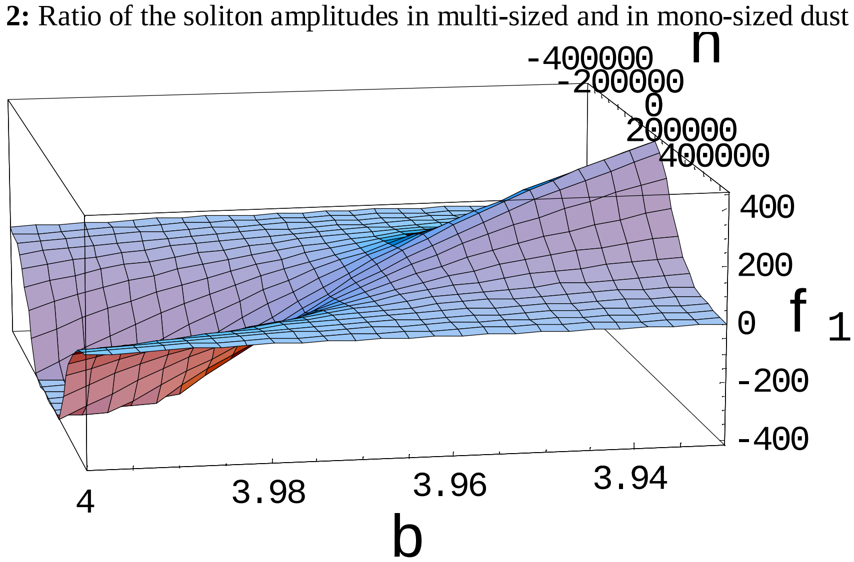

Figure 3: Effect of variations of the power law index β on the amplitude and width of compressive and rarefactive solitons in case of mono-sized dust plasma.

This is confirmed by Fig.(1) showing a well-defined singularity attributable to the effect of the power law index **β** on the formation of rarefactive (β<3.965) and compressive (β>3.965) solitons. Also in this respect, we found that the rarefactive soliton amplitude increases with enhancing the power law index **β** while the compressive soliton amplitude decreases for that. Further, as displayed in Fig.(2), the ratio **P** of the rarefactive soliton amplitude in multi-sized grains to that in mono-sized dust plasmas decreases with the non-thermal ion population parameter δ and reduces as β is enlarged. Note that this ratio of soliton potentials is negative here due to the evaluation in a parameter range, in which the soliton in multi-sized grains propagates as a compressive (rarefactive) solitary wave while the soliton in a mono-sized grain plasma is rarefactive (compressive) as one can infer from Fig. (3). Next, we illustrate in Fig. (4) the relation between P and β for two values of the minimum dust grain radius. It is seen that, for rarefactive solitons in multi-sized dust plasmas, **P** decreases with β and increases with $a_{\min}$. The ratio of rarefactive soliton widths in multi-sized and mono-sized dust plasmas, **W**, enlarges with β and decreases with larger $a_{\min}$ as evident from Fig.(5). Finally the dependance of DA soliton energy on the minimum grain size, $a_{\min}$ and power law index β are displayed in Fig.(6).

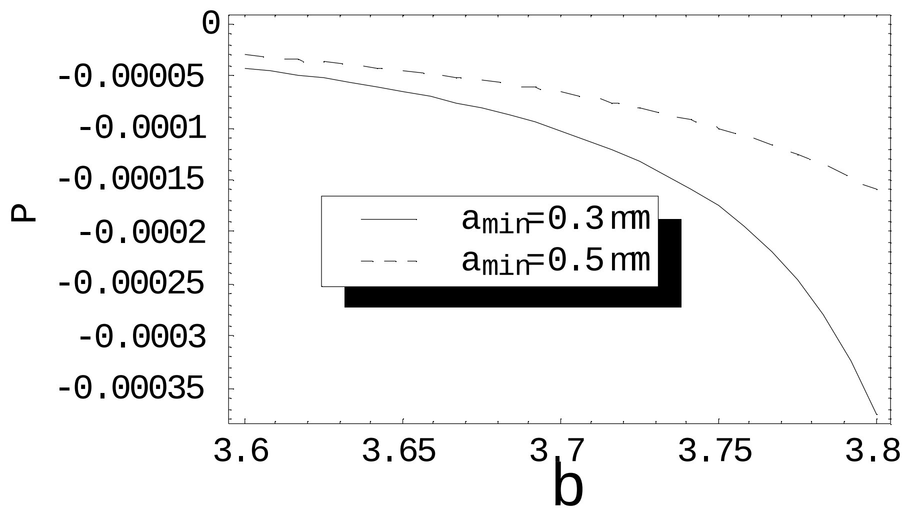

Figure 4: Ratio **P** of the rarefactive soliton amplitude in multi-sized grains to that in mono-sized dust plasmas as a function of the power law index β for different values of the minimum radius.

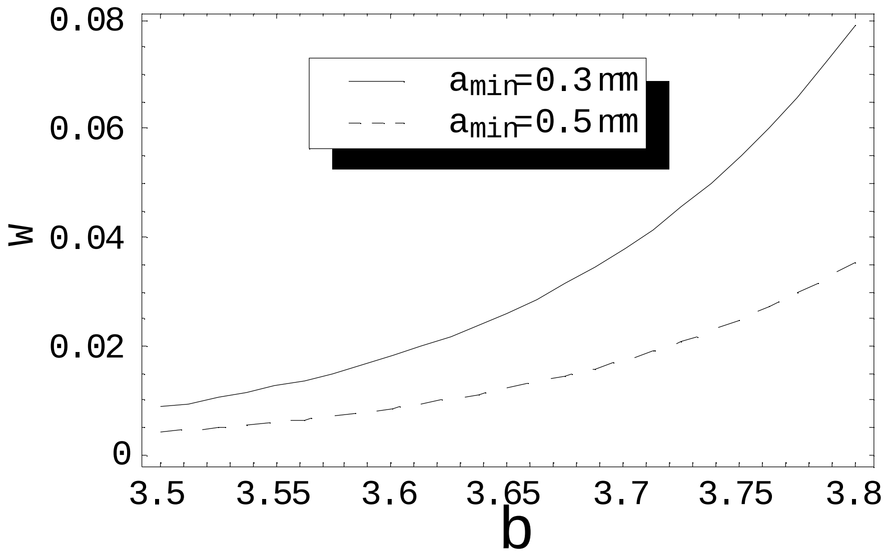

Figure 5: Ratio of the width of rarefactive solitons in multi-sized grains to that in mono-sized dust plasmas versus β for two different minimum grain radii.

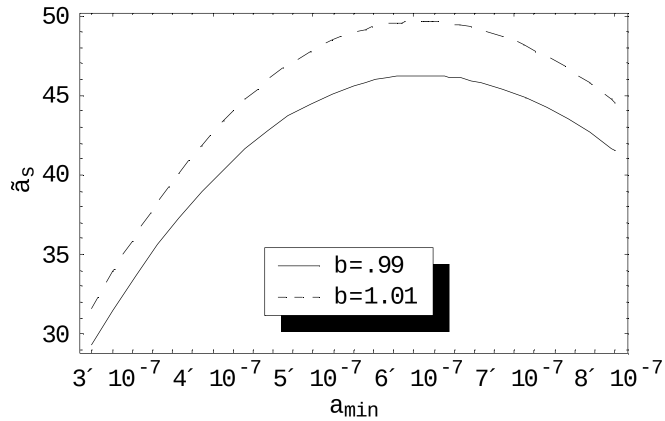

Figure 6: Normalized DA soliton energy depending on the minimum grain size for two different values of β.

These results show that the power law approach with the charactersitic power index β as well as the presence of non-thermal ions as accounted for by the energetic population parameter δ_1 modify significantly the properties of dust-acoustic solitary waves.

CONCLUSION

Though most of the pertinent literature on dusty plasma physics deals with mono-sized dust distributions, in nature we will encounter dusty plasmas containing dust grains of different size and mass; hence a multi-sized dust distribution should form the basis for an adequate examination. For typical dusty plasmas, it was reported from relevant satellite missions that the size of the largest dust grains is, at least, 10 times that of the smallest dust grains, and therefore the velocity of induced dust acoustic (DA) solitons or their propagation distance is 100 times less in the case of largest dust grains than those of the smallest dust grain. Since the sizes of dust grains effect different soliton structures, we focussed on the proper description of an unmagnetized dusty plasma system consisiting of multi-sized dust components obeying a power law distribution determined by the minimum and maximum dust grain radii, and of a non-thermal ion population. The application of reductive perturbation theory to the basic set of dusty fluid equations leads to a KdV equation describing the nonlinear evolution of DA solitary waves. It is emphasized that the amplitude of DA solitons as well as the range of parameters, in which these solitons can exist, are significantly sensitive to the power law index β, the energetic ion population parameter δ_1 and the minimum dust grain radius $a_{\min}$. It was demonstrated that an enhancement of all these parameters can lead to a substantial modification of the DA soliton amplitude and energy.

We inject to note that the analytical model demonstrated here can provide a useful basis for the interpretation of recent observations of solitary waves in dusty plasma environments. For example, the results presented may be applicable to dusty plasmas existing in cometary environments as well as in the F- and G-rings of Saturn.

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