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# SOLITARY WAVES IN SPACE DUSTY PLASMA WITH DUST OF OPPOSITE POLARITY

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The nonlinear propagation of small but finite amplitude dust-acoustic solitary waves ( DAWs) in an unmagnetized, collisionless dusty plasma has been investigated. The fluid model is a generalize to the model of Mamun and Shukla to a more realistic space dusty plasma in different regions of space viz., cometary tails, mesosphere, Jupiter's magnetosphere, etc., by considering a four component dusty plasma consists of charged dusty plasma of opposite polarity, isothermal electrons and vortex like ion distributions in the ambient plasma. A reductive perturbation method was employed to obtain a modified Korteweg-de Vries (mKdV) equation for the first-order potential and a stationary solution is obtained. The effect of the presence of positively charged dust fluid, the specific charge ratio μ, temperature of the positively charged dust fluid, the ratio of constant temperature of free hot ions and the constant temperature of trapped ions and ion temperature are also discussed.

***Keywords:*** *dusty plasma; opposite polarity; dust-acoustic waves; mKdV equation; Solitary solution.*

## INTRODUCTION

The dusty plasma is an ionized gas which contains electrons, ions and small micron or sub-micron sized extremely massive charged dust grains. In fact dust and plasma coexist in a wide variety of cosmic and laboratory environments. They are ubiquitous in different parts of our solar system, namely, inplanetary rings, in circum-solar dust grains, in the interplanetary medium, in cometary comae and tails, in asteroid zones, in mesosphere and magnetosphere, and in interstellar molecular clouds, see for instance [1-5]. Besides these, dust particles have been observed in low temperature plasmas, like those used in plasma processing and plasma crystal. Unique and novel features of dusty plasmas when compared with the usual electron-ion plasmas are the existence of a new very low frequency regime for wave propagation. Also the highly charging of the grains which can fluctuate due to the collection of plasma currents on to the dust surface. It has been shown both theoretically and experimentally that the presence of these extremely massive and highly charged dust grains in a plasma can modify the behavior of the usual waves and instabilities [5-6]. The charging of dust grains occurs due to a variety of processes [6--8]. Actually dust grains of different sizes can acquire different polarities; large grains become negatively charged and onces become small positively charged [9-11]. In fact, positively charged dust particles have been observed in different regions of space, viz. cometary tails [9-11], Jupiter's magnetosphere [12], etc. There are three principal mechanisms by which a dust grain becomes positively charged [13]. These are photoemission in the presence of a flux of ultraviolet (UV) photons, thermionic emission induced by radiative heating, and secondary emission of electrons from the surface of the dust grains. On the other hand, Chow et al. [9] have theoretically shown that due to the size effect on secondary emission insulating dust grains with different sizes in space plasmas can have the opposite polarity (smaller ones being positive and larger ones being negative). This is mainly due to the fact that the excited secondary electrons have shorter (longer) distances to travel to reach the surface of the smaller (larger) dust grains. In other words, there are also direct evidence for the existence of both positively and negatively charged dust particles in the earth's mesosphere [14-16], as well as in cometary tails and comae [2, 17,18]. Because of the involving of the charged dust grains in plasmas, different types of collective processes exist and very rich wave modes can be excited in dusty plasmas such as, DA waves [7,19] dust ion acoustic (DIA) waves [20,21], dust-lattice (DL) waves [22,23]. On another side, Schamel et al [24] reported the existence of a new class of ultra low-frequency nonlinear normal mode called dust-Coulomb waves (DCWs) in a dense charge varying dusty plasma with trapped dust particles. These dust-Coulomb (DC) modes exist in a frequency regime much lower than the DA wave regime. The most well studied of such modes is the so-called (DAW) which arises due to the restoring force provided by the plasma thermal pressure (electrons and ions) while the inertia is due to the dust mass. The inclusion of the dust charge dynamics effects leads to a considerable increase in the richness and variety of the wave motions which can exist in plasma. It also affects the nature of particle wave interaction and the possibility of having a trapped ion distribution in the potential well. The presence of such trapped ions can significantly modify the wave propagation characteristics in collisionless plasmas so one concerting vortex-like ion distributions in phase space instead of the usual Boltzmann law [25]. Some recent theoretical study focused on the effects of ion trapping which is common not only in space plasmas, but also in laboratory experiments [26-28]. Mamun and Shukla [32] have considered dusty plasma model, which consists of positive and negative dust only, and have theoretically investigated the properties of linear and nonlinear electrostatic waves in such dusty plasma. The dusty plasma model of Mamun and Shukla [29] is only valid if a complete depletion of the background electrons and ions is possible, and both positive and negative dust fluids are cold. Recently, El Wakil et al [30] investigated theoretically the higher-order contributions to nonlinear dust-acoustic waves that propagates in a mesospheric dusty plasma with a completely depletion of background (electrons and ions). However, in most space dusty plasma systems a complete depletion of the background electrons and ions is not possible [6, 31-33] and the positive dust component is of finite temperature.

The present work is therefore attempted to generalize the model of Mamun and Shukla [29] to a more realistic space dusty plasma situation, which is particularly relevant to cometary tails [17, 18], upper mesosphere [14-16] , and Jupiter's magnetosphere [17], by considering a four component dusty plasma containing negatively charged cold dust of larger size , positively charged warm adiabatic dust of smaller size , and Boltzmann electrons and ions obeys the vortex like distribution.

This paper is organized as follows; in Section 2, the basic set of equations is introduced and the modified KdV equation was derived. In Section 3, the soliton solutions are obtained. Finally, discussions and conclusions are given in Sections 4 and 5.

## 2. Basic Equations

Let us consider four-component dusty plasma with massive, micron-sized, positively, negatively dust grains, isothermal electron and non-isothermal ion distributions. This study based on the condition that, the negative dust particles are much more massive than positive ones [9]. Here the basic governing equations are:

$$\frac{\partial}{\partial t}n+\frac{\partial}{\partial x}(n\ u)=0\,, \tag{1.a}$$

$$\mu(\frac{\partial u}{\partial t}+u\ \frac{\partial}{\partial x}u)+\frac{\partial \phi}{\partial x}+\frac{\sigma_d}{n}\frac{\partial p}{\partial x}=0\,, \tag{1.b}$$

$$\frac{\partial p}{\partial t}+u\ \frac{\partial p}{\partial x}+3p\ \frac{\partial u}{\partial x}=0\,, \tag{1.c}$$

For positive dust plasma and

$$\frac{\partial}{\partial t}N\ +\frac{\partial}{\partial x}(N\,V\ )=0\,, \tag{2.a}$$

$$(\frac{\partial v}{\partial t}+V\ \frac{\partial}{\partial x}V\ )-\frac{\partial \phi}{\partial x}=0\,, \tag{2.b}$$

For negative dust plasma

Equation (1-2) are supplemented by Poisson's equation

$$\frac{\partial^2}{\partial x^2}\varphi=N\ -\mu_1\,n+\mu_2\ n_e-\mu_3\ n_i\ . \tag{3}$$

In the earlier equations, n, N, u, V , $\phi$, and p are the number density, the velocity of positive (negative) dusty grains, the electric potential and the thermal pressure of the positive dust fluid, respectively. Here n, N are normalized by its equilibrium value $n_0$ and $N_0$, u,and V are normalized by $C_s = \mu\sqrt{v_T}$ ,

[$\mu = Z_n m_p / Z_p m_n, v_T = \left(Z_p k_B T_i / m_p\right)^{1/2}$, $Z_p\left(Z_n\right)$ represents the number of the positive (negative) charges onto the positive (negative) dust grain surface, $m_p(m_n)$ represents the mass of the positive (negative) dust particle, $k_B$ is the Boltzmann constant, p is normalized by $n_0k_BT_p$, $\phi$ is normalized by $k_BT_i/e$, x is the space variable normalized by $\lambda_D = (k_BT_i/4\pi n_0Z_ne^2)^{1/2}$ , t is the time variable normalized by $\omega^{-1}{}_p= (m_2/4\pi N_0Z_n{}^2e^2)^{1/2}$, where $T_p$ is the temperature of the positive dust fluid],

$$\sigma_d=\frac{T_p}{T_iZ_p},\ \mu_1=\frac{n_0Z_p}{N_0Z_n}\ ,\ \mu_2=\frac{n_{e0}}{N_0Z_n}\ ,\ \text{and}\ \mu_3=\frac{n_{i0}}{N_0Z_n}\,.$$

The isothermal electron is given by

$$n_e=\exp[\sigma_i\ \phi]\,, \tag{4.a}$$

where $\sigma_i=\frac{T_e}{T_i}$ and $n_i$ is ion density employing the vortex-like distribution function is given by

$$n_i=1-\phi-\frac{4}{3}b(-\phi)^{\frac{3}{2}}+\frac{\phi^2}{2} \tag{4.b}$$

where b is a constant depending on the temperature parameters of resonant ion ( both free and trapped ), and is given by $b=\pi^{-\frac{1}{2}}(1-\beta),\quad \beta=\frac{T_h}{T_{ht}}$

where $T_h$ and $T_{ht}$ are respectively the constant temperature of free hot ions and the constant temperature of trapped ions. The term $-\frac{4}{3}b\phi^{\frac{3}{2}}$ in the expansion of $n_h$ represents the contribution of resonant ions to the ion density.

To derive the modified KdV equation describing the behavior of the system for longer times and small but finite amplitude DA waves, we introduce the slow stretched coordinates:

$$\tau = \varepsilon^{3/4}\, t, \quad \xi = \varepsilon^{1/4}\,(x - \lambda\, t) \qquad (4.c)$$

where ε is a small dimensionless expansion parameter and λ is the speed of DA waves. All physical quantities appearing in (1-3) are expanded as power series in ε about their equilibrium values as:

$$\begin{aligned}
n &= 1 + \varepsilon\, n_1 + \varepsilon^{\frac{3}{2}}\, n_2 + \varepsilon^2\, n_3 + \ldots\ldots\ldots\ldots, \\
u &= \varepsilon\, u_1 + \varepsilon^{\frac{3}{2}}\, u_2 + \varepsilon^2\, u_3 + \ldots\ldots\ldots\ldots\ldots, \\
N &= 1 + \varepsilon\, N_1 + \varepsilon^{\frac{3}{2}}\, N_2 + \varepsilon^2\, N_3 + \ldots\ldots, \\
V &= \varepsilon\, V_1 + \varepsilon^{\frac{3}{2}}\, V_2 + \varepsilon^2\, V_3 + \ldots\ldots\ldots\ldots\ldots, \\
P &= 1 + \varepsilon\, P_1 + \varepsilon^{\frac{3}{2}}\, P_2 + \varepsilon^2\, P_3 + \ldots\ldots\ldots\ldots, \\
\varphi &= \varepsilon\, \varphi_1 + \varepsilon^{\frac{3}{2}}\, \varphi_2 + \varepsilon^2\, \varphi_3 + \ldots\ldots\ldots\ldots
\end{aligned} \qquad (5)$$

We impose the boundary conditions that as

$$|\xi| \to \infty, \quad n = N = 1, \quad u = V = 0, \ \varphi = 0 \qquad (6)$$

Substituting (4) and (5) into (1-3), and equating coefficients of like powers of ε. Then, from the lowest-order equations in ε, the following results are obtained :

$$n_1 = \frac{\left(\lambda^2\left(\mu_3 + \mu_2\sigma_i\right) - 1\right)\phi_1}{\lambda^2\mu_1}; \quad u = \frac{\left(\lambda^2\left(\mu_3 + \mu_2\sigma_i\right) - 1\right)\phi_1}{\lambda\mu_1}$$

$$P_1 = \frac{3\left(\lambda^2\left(\mu_3 + \mu_2\sigma_i\right) - 1\right)\phi_1}{\lambda^2\mu_1}\ ; N_1 = \frac{-\varphi_1}{\lambda^2}, \quad V = \frac{-\varphi_1}{\lambda}, \qquad (7)$$

Poisson's equation gives the linear dispersion relation

$$-\mu\mu_3\lambda^4 - \mu\mu_2\sigma_i\lambda^4 + \mu\lambda^2 + \mu_1\lambda^2 + 3\mu_3\sigma_d\lambda^2 + 3\mu_2\sigma_d\sigma_i\lambda^2 - 3\sigma_d = 0 \qquad (8)$$

If we consider the coefficients of O ($\varepsilon^2$), we obtain with the aid of (7) the following set of equations :

$$-\lambda\frac{\partial n_2}{\partial\xi}+\frac{\partial u_2}{\partial\xi}+\left\{\frac{\left(\lambda^2\left(\mu_3+\mu_2\sigma_i\right)-1\right)}{\lambda^2\mu_1}\right\}\frac{\partial\varphi_1}{\partial\tau}=0 \qquad (9.a)$$

$$-\lambda\mu\frac{\partial u_2}{\partial\zeta}+\frac{\mu\left(\lambda^2\left(\mu_3+\mu_2\sigma_i\right)-1\right)}{\lambda\mu_1}\frac{\partial\varphi_1}{\partial\tau}+\sigma_d\frac{\partial p_2}{\partial\xi}+\frac{\partial\varphi_2}{\partial\xi}=0, \qquad (9.b)$$

$$-\lambda\frac{\partial p_2}{\partial\xi}+3\frac{\partial u_2}{\partial\xi}+\frac{3\left(\lambda^2\left(\mu_3+\mu_2\sigma_i\right)-1\right)}{\lambda^2\mu_1}\frac{\partial\varphi_1}{\partial\tau}=0 \qquad (9.c)$$

For positive dust plasma and

$$-\frac{1}{\lambda^2}\frac{\partial\varphi}{\partial\tau}-\lambda\frac{\partial N_2}{\partial\xi}+\frac{\partial V_2}{\partial\tau}=0\ , \qquad (10.a)$$

$$-\frac{1}{\lambda}\frac{\partial\varphi_1}{\partial\tau}-\lambda\frac{\partial V_2}{\partial\xi}-\frac{\partial\varphi_2}{\partial\xi}=0 \qquad (10.b)$$

For negative dust plasma and Poisson's equation is

$$\left\{\frac{\partial^2\phi_1}{\partial\zeta^2}+\mu_1 n_2-N_2-\frac{4}{3}b\mu_3\sqrt{-\phi_1}\ \phi_1-\mu_3\phi_2-\mu_2\sigma_i\phi_2\right\}\ =0 \qquad (11)$$

Eliminating the second order perturbed quantities $n_2$, $u_2$, $N_2$, $V_2$ and $\phi_2$ in equations (9-11), we obtain the following mKdV equation for the first-order perturbed potential:

$$\frac{\partial\phi_1}{\partial\tau}+A\sqrt{-\phi_1}\frac{\partial\phi_1}{\partial\xi}+\frac{B}{2}\frac{\partial^3\phi_1}{\partial\xi^3}=0 \qquad (12)$$

where

$$A=\frac{b\lambda^3\mu_3\left(3\sigma_d-\lambda^2\mu\right)}{\lambda^4\mu\left(\mu_3+\mu_2\sigma_i\right)-3\sigma_d};$$

$$B=\frac{\lambda^3\left(\lambda^2\mu-3\sigma_d\right)}{2\left(\lambda^4\mu\left(\mu_3+\mu_2\sigma_i\right)-3\sigma_d\right)}$$

## STATIONARY SOLUTION

Let us introduce the following traveling variable

$$\eta=\xi-\nu\tau. \qquad (13)$$

Here, $\nu$ being the soliton velocity.

Integrating once with respect to the new variable $\eta$ and using the appropriate vanishing boundary conditions for $\phi_1(\eta)$ and their derivatives up to second order as $|\eta|\to\infty$ . we obtain the one-soliton solution of (12) is given by:

$$\phi_1=\phi_{1m}Sech^4(D\eta) \qquad (14)$$

where the soliton amplitude $\phi_{1m}$ and the soliton width $D^{-1}$ are given by

$$\phi_{1m}=-(\frac{15\nu}{8A})^2,\qquad D^{-1}=\sqrt{\frac{8B}{\nu}} \qquad (15)$$

## RESULTS AND DISCUSSION

Nonlinear dust-acoustic solitary waves (DAWs) in an unmagnetized, collisionless dusty plasma consisting of charged dusty plasma of opposite polarity, isothermal electrons and vortex like ion distributions in the ambient plasma have been investigated. To make our result physically relevant, numerical calculations were performed referring to typical dusty plasma parameters as given in Ref. [14-16]. Generally speaking, the present system supports only rarefactive solitons. However, since one of our motivations was to study the effect of some plasma parameters such as $\mu$ , $\sigma_d$, $\sigma_i$ , $\nu$, $\beta$, $\mu_1$, $\mu_2$ and $\mu_3$ on the existence of solitary waves. For example, the dependence of the phase velocity $\lambda$ on $\mu$ and $\mu_3$ for different values of $\sigma_d$ and $\mu_2$ is shown in Fig. (1). the basic properties amplitude and width of the small amplitude electrostatic solitary structures are displayed in Figs. 2-4. It is obvious from Figs. 2-4 that the magnitude of the soliton amplitude and width decrease with the increase of $\sigma_d$ and $\mu_3$, and increase with the increase of $\sigma_i$, $\mu$ and $\mu_2$. On the other hand, it is seen from Figs.4 that the amplitude decreases with the increase of $\beta$ while the width increases with $\nu$. These results show that the parameters $\mu$, $\sigma_d$, $\sigma_i$, $\nu$, $\beta$, $\mu_1$, $\mu_2$ and $\mu_3$ modify significantly the properties of dust-acoustic solitary waves.

## CONCLUSION

We have devoted quite some effort to discuss the proper description of the presence of positively charged dust fluid, the specific charge ratio $\mu$, temperature of the positively charged dust fluid, the ratio of constant temperature of free hot ions and the constant temperature of trapped ions and ion temperature in dusty plasma consisting of charged dusty plasma of opposite polarity, isothermal electrons and vortex like ion distributions in the ambient plasma. The application of the reductive perturbation theory to the basic set of fluid equations leads to a mKdV the equation (12) which describes the nonlinear evolution of the DAWs. It is emphasized that the amplitude of the DAWs as well as the parametric regime where the solitons can exist are sensitive to the positively charged dust fluid parameters. It is interesting to point out that the increase of the temperature of the positively charged dust fluid (the specific charge ratio $\mu$) can lead to the reduction (increasing) of the DAWs soliton amplitude and width. Generally speaking, it is seen that both the soliton amplitude and width significantly affected by some plasma parameters such as $\sigma_i$, $\nu$, $\beta$, $\mu_1$ , $\mu_2$ and $\mu_3$ .

The ranges ($\sigma_i \rightarrow (0.1-0.2), \mu_2 \rightarrow (0.2-0.9), \text{and } \mu_3 \rightarrow (0.3-0.9)$) of the dusty plasma parameters used in this numerical analysis are very wide. Therefore the dusty plasma parameters (viz. $\sigma_i$ , $\mu_2$ and $\mu_3$ ) corresponding to cometary tails [2,21] , upper mesosphere [17-19], and Jupiter's magnetosphere [15] are, certainly, within these ranges. Therefore the present investigation can help us to identify the origin of charge separation as well as dust coagulation in plasma containing positive and negative dust.

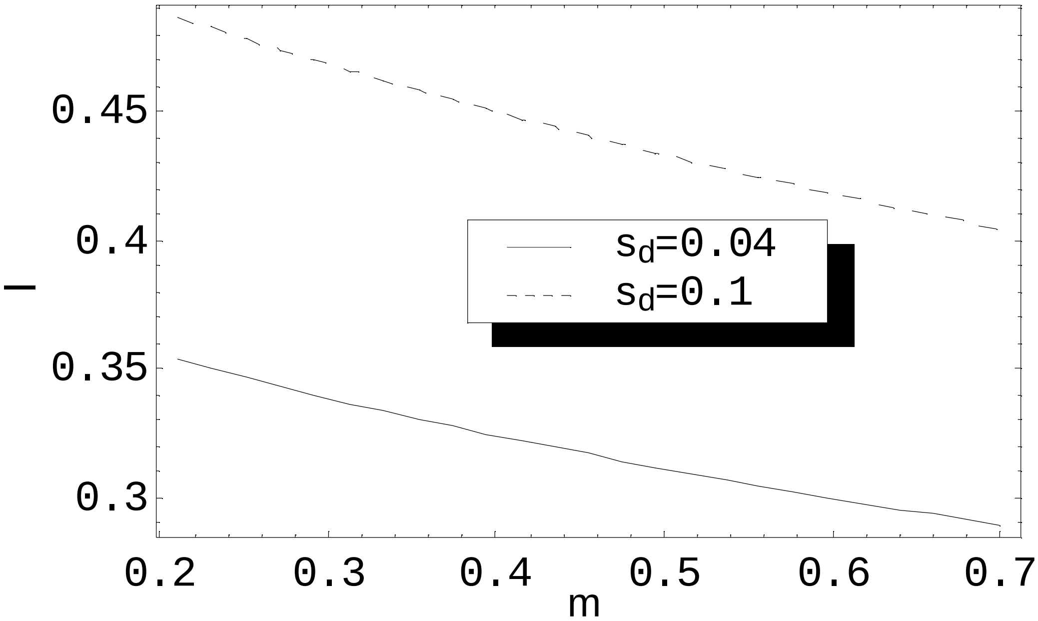


**Fig. 1.a**. Variation of the phase velocity **λ** with respect to **μ** for different values of $\sigma_d$ for $\beta = -0.5$, $\nu = 0.4$, $\sigma_i = 0.2$, $\mu_2 = 0.2$ and $\mu_3 = 0.5$.

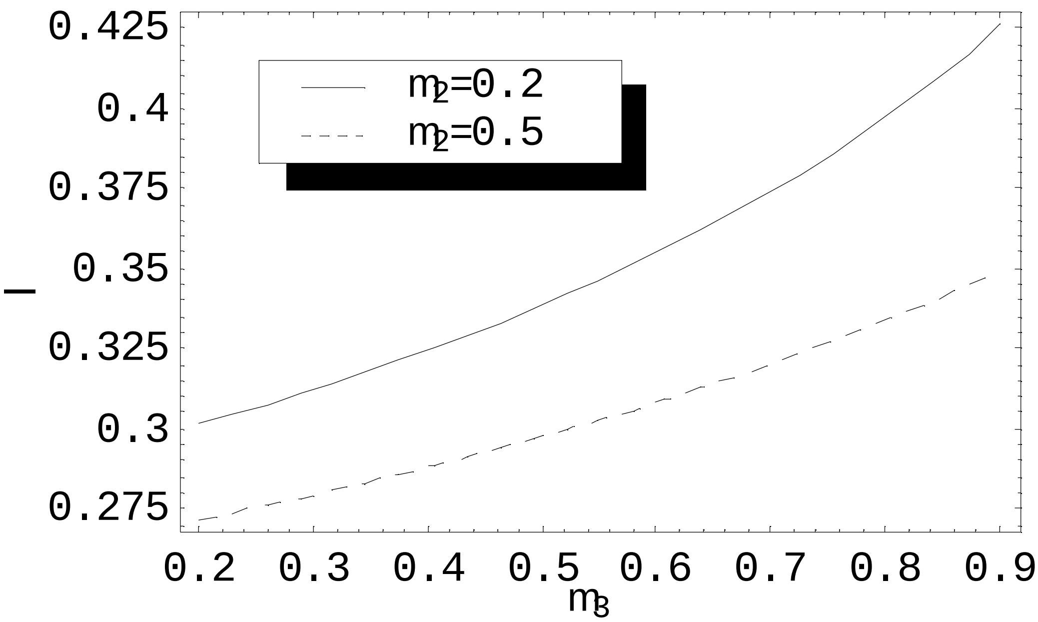


**Fig. 1.b**. Variation of the phase velocity **λ** with respect to **μ₃** for different values of **μ₂** for $\beta = -0.5$, $\nu = 0.4$, $\sigma_i = 0.2$, $\sigma_d = 0.04$ and $\mu = 0.3$.

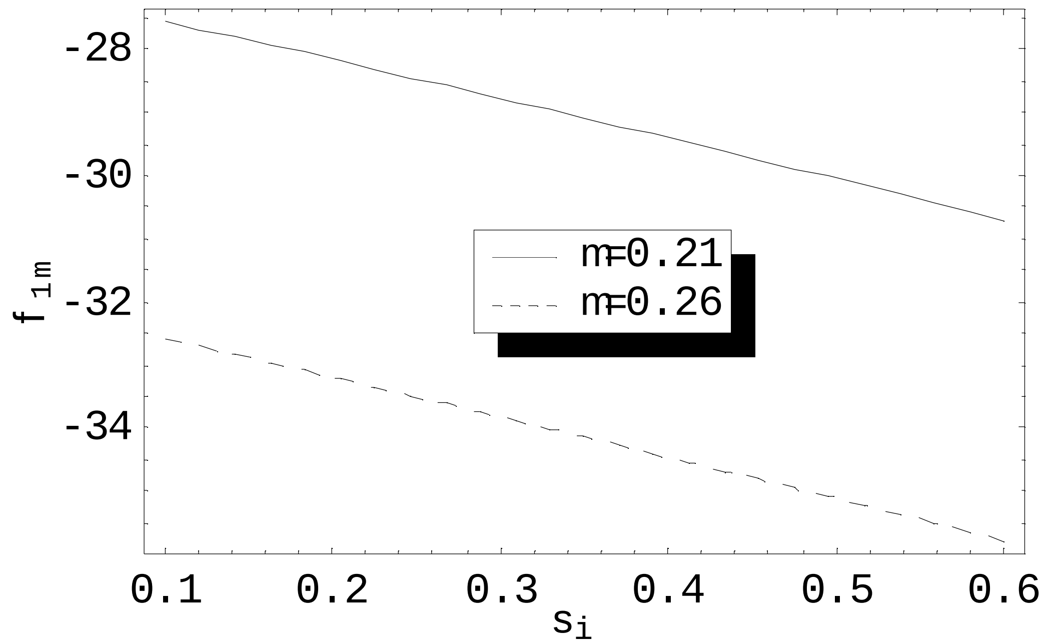


**Fig. 2a**

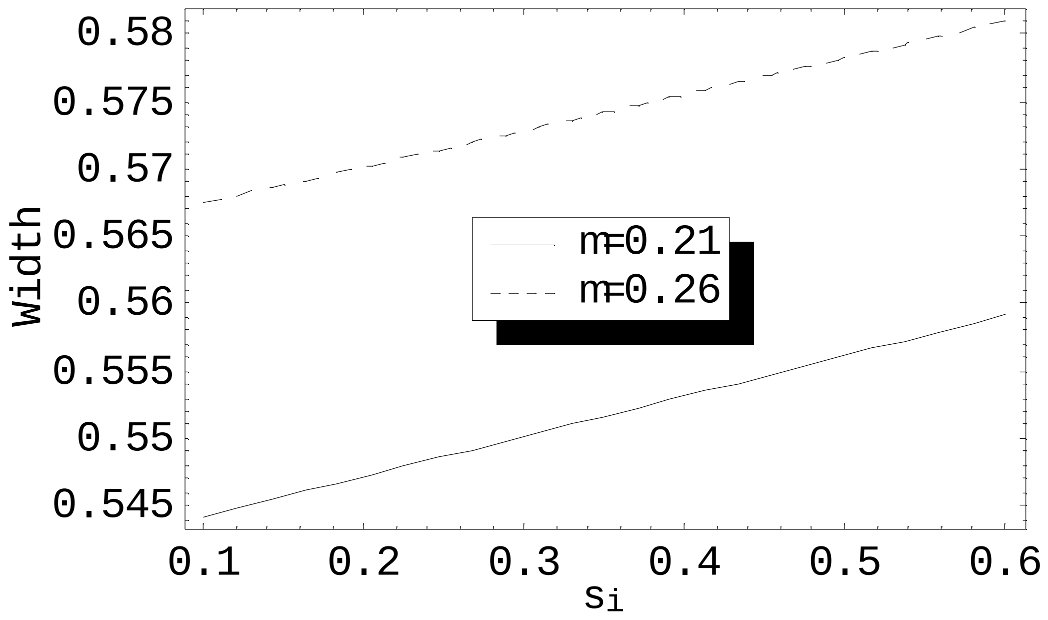


**Fig. 2b**

**Fig.s 2**. The variation of the soliton amplitude and width with respect to $\boldsymbol{\sigma_i}$ for different values of $\boldsymbol{\mu}$ for $\sigma_d = 0.21, \beta = -0.5$, $\nu = 0.4$, $\mu_2 = 0.2$ and $\mu_3 = 0.5$.

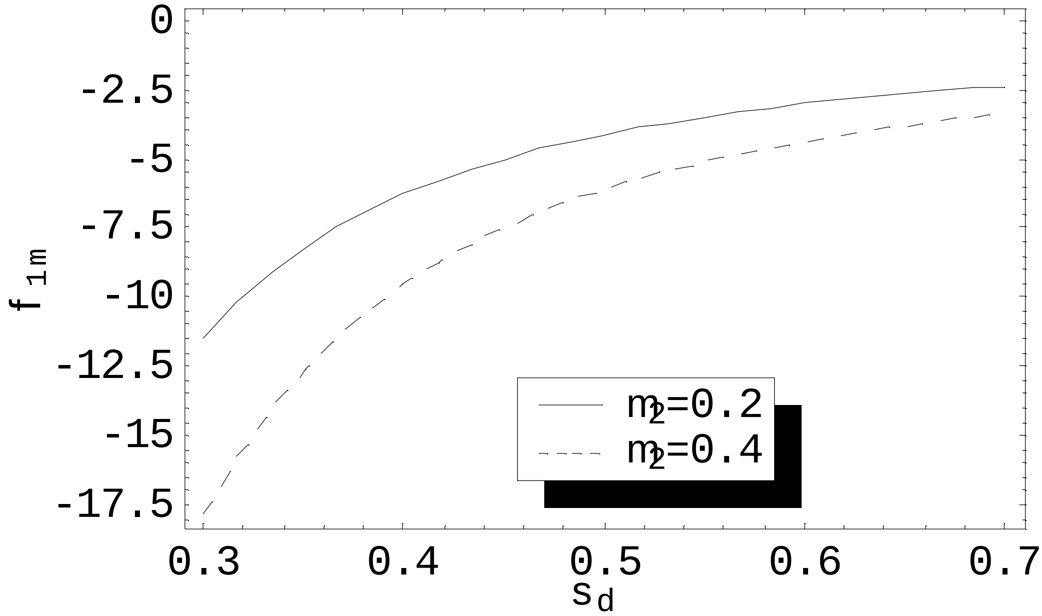


**Fig. 3a**

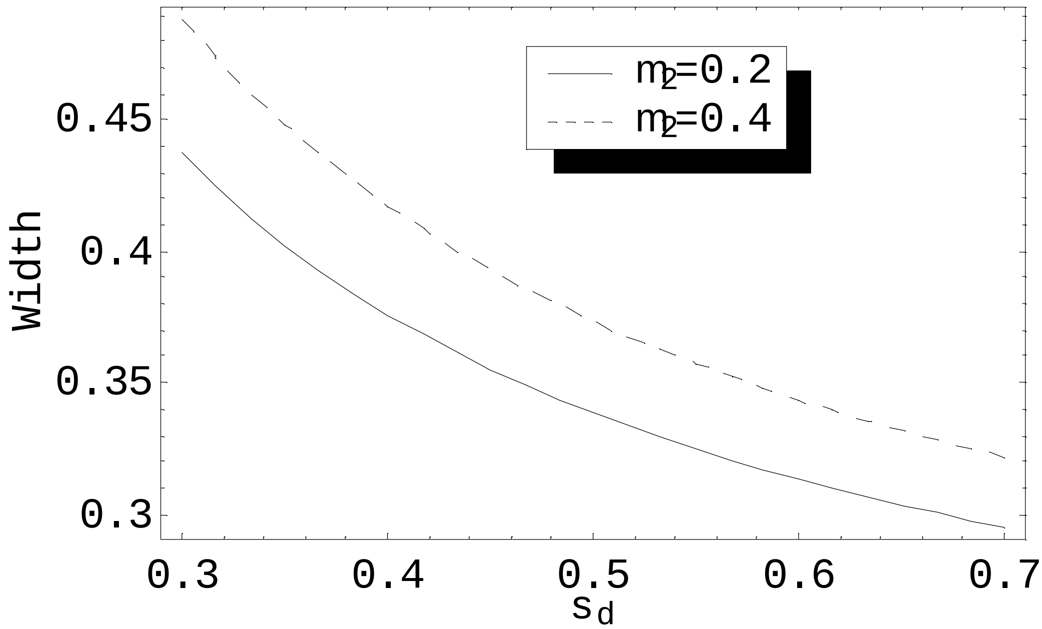


**Fig. 3b**

**Fig.s 3**. The variation of the soliton amplitude and width with respect to $\boldsymbol{\sigma_d}$ for different values of $\boldsymbol{\mu_2}$ for $\sigma_i = 0.04, \beta = -0.5, \ \nu = 0.4, \ \mu_2 = 0.2$ and $\mu_3 = 0.5$.

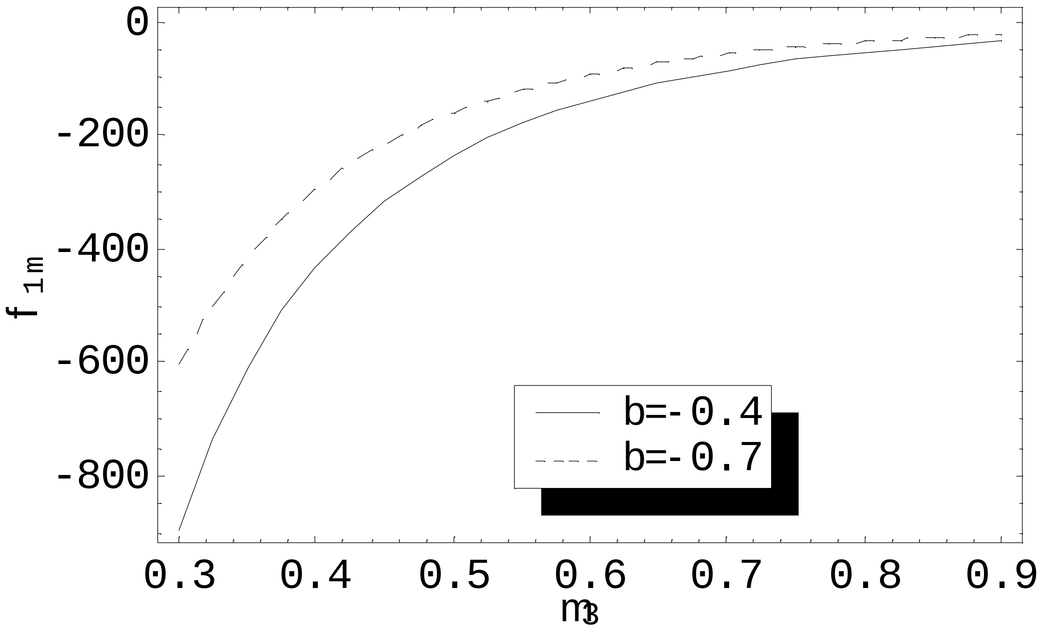


**Fig. 4.a.** The variation of the soliton amplitude with respect to $\mu_3$ for different values of **β** for $\sigma_i = 0.04, \sigma_d = 0.4$, $\nu = 0.3$, $\mu_2 = 0.2$ and $\mu_3 = 0.5$.

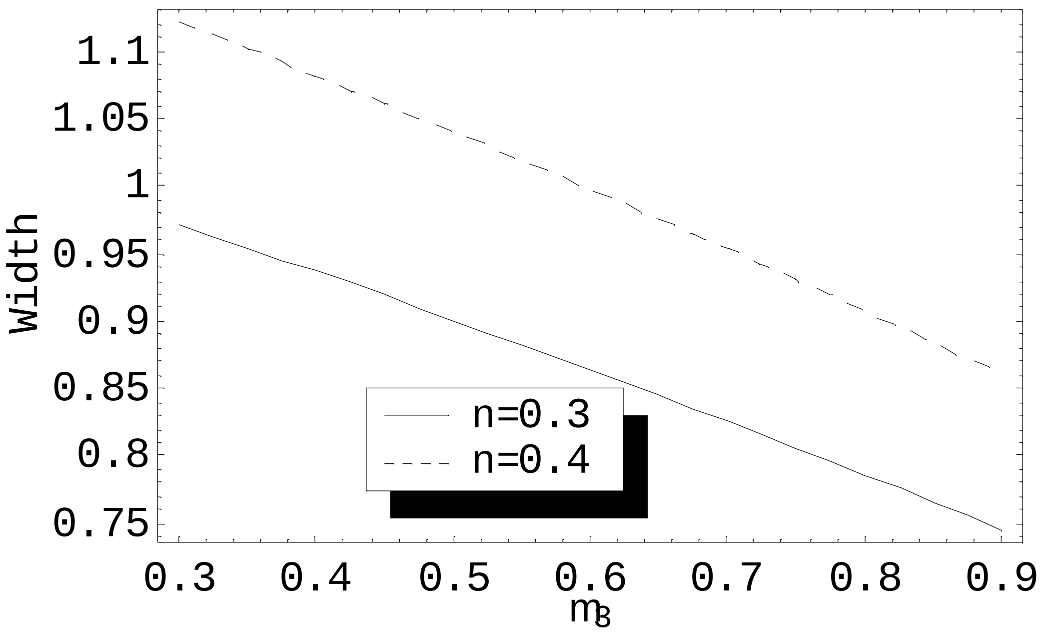


**Fig. 4.b.** The variation of the soliton amplitude with respect to $\mu_3$ for different values of **ν** for $\sigma_i = 0.04, \sigma_d = 0.1$, $\nu = 0.3$, $\mu = 0.2$ and $\mu_2 = 0.5$.

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