Development of Side Coupled Cavities

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Abstract

Side coupled Cavities are good candidates for proton accelerations in the 90-180 MeV range, as it has been first proposed for the CERN LINAC4 project. This is not a new technology used, for example, for the Spallation Neutron Source (SNS). The goal for HIPPI was the development of technical knowledge about it. We summarize here the theoretical and experimental studies. This work is not completed and tuning procedures are under study and must be proven on the prototype.

1 Side Coupled Cavities

We present here a brief summary of side coupled cavities. A Side Coupled Linac is made of a lump chain of resonant cavities, alternatively accelerating and coupling, as shown on the drawing given Figure 1.1. This set of cavities is equivalent to a lumped chain of RLC circuits (figure 1.2).

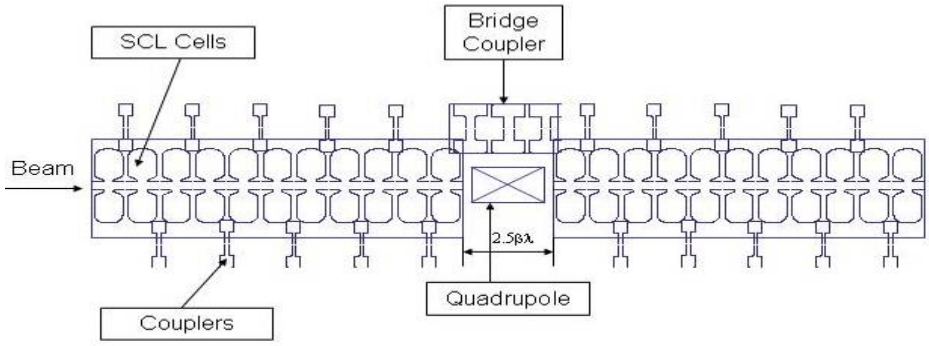

Figure 1.1: Accelerator tank (figure CERN) as initially proposed for the high energy part of LINAC4

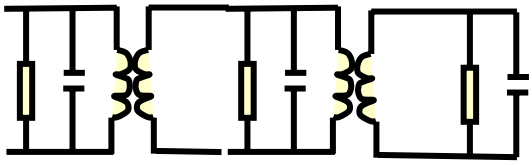

Figure 1.2: Equivalent circuit

	100 MeV	**120 MeV**	**140 MeV**	**160 MeV**
β	0.428	0.462	0.492	0.52
D (cm)	28.83	28.99	28.78	28.86
L (cm)	9.108	9.838	10.481	11.06
g (cm)	2.6	3.1	3.4	3.8
g/L	0.285	0.315	0.324	0.343
Q	20795.1	22120.8	23003	23884.4
ZT2 (MΩ/m)	34.863	37.623	39.771	41.486
T	0.893	0.894	0.897	0.896
Ep/Eo	5.62	5.35	5.374	5.249
Ep (Kil., 3.5MV/m)	0.799	0.761	0.764	0.746
ZT2 (3% Coupling)	25.56	27.61	29.19	30.45

Table 1.1: SCL parameters for linac 4 (courtesy CERN, Eva Benedico-Mora)

If each cavity has the same resonnant frequency ω_0, and considering the classical coupling factor k, the eigenmodes $\vec{X}_i$ and the eigenfrequencies ω_i of such a structure are obtained from the following relations:

$$M = \begin{bmatrix} 1 & k & 0 & 0 \\ k/2 & 1 & k/2 & 0 \\ 0 & \cdots & \cdots & k/2 \\ 0 & 0 & k & 1 \end{bmatrix}$$

$$\lambda_i = \frac{\omega_0^2}{\omega_i^2}, i = 0..N$$

$$X_{in} = \left[\cos\frac{\pi i n}{N}\right]\exp(j\omega_i t) \qquad i = 0..Nth\ cavity$$

$$\omega_i = \frac{\omega_0}{\sqrt{1 + k\cos\frac{\pi i}{N}}}$$

$$\vec{X}_q \perp \vec{X}_p$$

The λ_i values are the eigenvalues of M. The eigenvectors are orthogonal and we suppose now that $\left\|\vec{X}_q\right\| = 1$ (orthonormal basis). For a perfect structure, all the modes are on a coslike curve (figure 1.3 left). For a real structure (ie: different frequencies for each cell), the central mode (the $\pi/2$ mode) is not defined in an unique way (depending on the boundary conditions), and this causes a discontinuity in the curve (the so-called “stop-band”, figure 1.3 right).

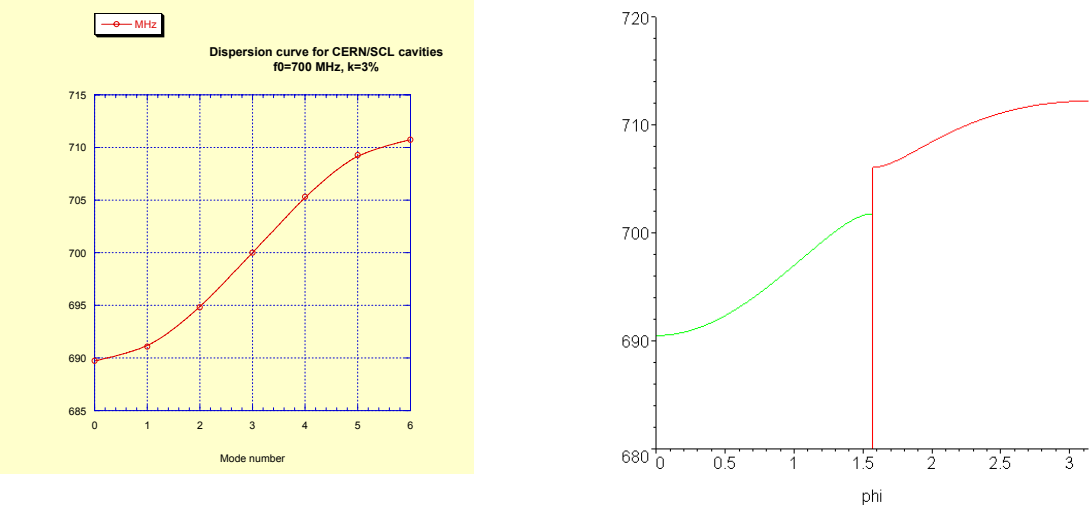

Figure 1.3: Mode frequency for CERN SCL cavities versus phase shift from cell to cell (radians). The accelerating mode is the central one ($\pi/2$ mode). Left: perfect structure (identical cells). Right: imperfect structure (different frequency between accelerating and coupling cells).

2 Design

The Microwave Studio (MWS) software has been used at CERN, as well as HFSS at LPSC to define the cavity structure. The so-called M3 solution has been chosen. Its geometry is 230 mm between cavity axes (accelerating/coupling) and 20mm gap in accelerating cells.

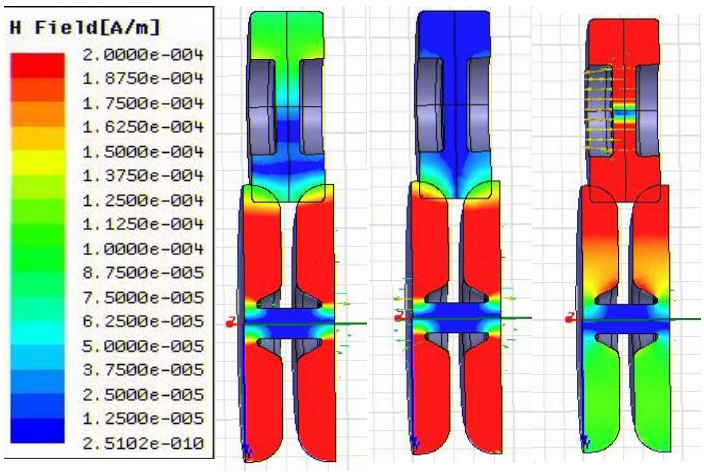

Figure 2.1: Magnetic field for the basic pattern of SCL (2 half accelerating cells + 1 coupling cell), calculated with HFSS

M1	**Q (π/2)**	**k**	ω1/2π MHz	ω2/2π MHz	ω3/2π MHz	ωa/2π MHz	ωc/2π MHz
MWStudio	10155	0,4%	738,6	739,67	741,4	739,67	740,32
HFSS	19956	0,5%	705,8	706,5	711.3	706,5	710.59
M2							
MWStudio	10902	**3%**	721,68	731,33	744.07	731.33	733.92
HFSS	18586	3,4%	697,32	701.13	738.38	701,13	734.05
M3							
HFSS	19128	**3%**	688,71	702,5	710,33	702,5	696,08

Table 2.1: comparison MWS/HFSS simulations. The M3 solution has been chosen.

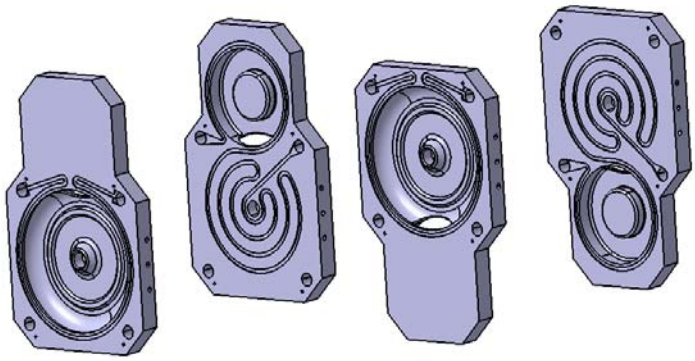

Figure 2.2: SCL assembly. The module is made of elementary parts including half an accelerating cell and half a coupling cell. Here, the cooling system for a 15% duty cuycle operation is represented.

3 Error studies

3.1 Sensitivity do errors

A frequency error (due for example to machining) an a cell error gives a modification of the M matrix, who becomes (I+Λ)M, as follows:

$$(I+\Lambda)M \quad \text{where} \quad \Lambda = \begin{bmatrix} \delta_1 & 0 & \cdots & 0 \\ 0 & \delta_2 & \cdots & \cdots \\ \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & 0 & \delta_N \end{bmatrix}$$

$$\delta_i = 1 - \frac{1}{\left(1+\left(\frac{\Delta f_i}{f_i}\right)\right)^2} \approx -2\frac{\Delta f_i}{f_i}$$

The goal of the study is how to get the mechanical tolerances of the structure. The knowledge of the dependence frequency versus mechanical error is obtained by Superfish simulations.

3.2 Single cell perturbation - First order

The cavity number k est perturbed by an amount δ. The accelerating mode is written versus the δ up to second order.

$$X^*_{\pi/2} = X_{\pi/2} + \delta A_1 + \delta^2 A_2 \equiv X_{\pi/2} + E$$
$$\lambda^*_{\pi/2} = \lambda_{\pi/2} + \delta\alpha_1 + \delta^2\alpha_2$$

The X symbols are the normalized eigenvectors. The star notation is for the perturbed values. The π/2 mode has the « q » index.

We write also :

$$(I+\Lambda) = A$$
$$AMX^* = \lambda^* X^*$$

A is the identity matrix except for the kth term, equal to 1+δ.
At first order, writing the development at δ=0, we get :

$$A'MX^* + AMX'^* = \lambda'^* X^* + \lambda^* X'^*$$
$$A'MX^* + MA_1 = \alpha_1 X + A_1$$

The prime symbols are for the derivative versus δ
A'is zero, except for the kth term, equal to 1.

For the π/2 mode, (eigenvalue 1), we have :

$$\begin{bmatrix} 0 \\ \cdots \\ X_{qk} \\ \cdots \\ 0 \end{bmatrix} + MA_1 = \alpha_1 X_q + A_1$$

Then

$$Y_q = \begin{bmatrix} 0 \\ \cdots \\ X_{qk} \\ \cdots \\ 0 \end{bmatrix}$$

Finally

$$Y_q + MA_1 = \alpha_1 X_q + A_1$$

The A_1 vector is decomposed over the orthonormal eigenvectors X_i, with μ_i components. We get:

$$X_{qk}^2 + \mu_q = \alpha_1 + \mu_q \Rightarrow \alpha_1 = X_{qk}^2$$

$$X_{qk} X_{ik} + \mu_i \lambda_i = \mu_i \Rightarrow \mu_i = \frac{X_{qk} X_{ik}}{1 - \lambda_i} \text{ pour } i \neq q$$

The μ_q coefficient is got by writing the conservation of the norm. So, still at first order :

$$X_q^{*2} = 1 = X_q^2 + 2\delta X_{qk} \mu_q \Rightarrow \mu_q = 0$$

We observe immediatly that μ_i is zero (it is proportional to an alternate sum of cosine terms, over all the modes)

The accelerating mode perturbation is a second order perturbation (at least)

3.3 Single cell perturbation - Second order

The calculation is done now to second order

$$A'' MX^* + 2A' MX'^* + AMX''^* = \lambda''^* X^* + 2\lambda'^* X'^* + \lambda X''^*$$
$$A' MA_1 + AMA_2 = \alpha_2 X + \alpha_1 A_1 + A_2$$

A vaut is the identity matrix for δ=0.

$$Y_q + MA_1 = \alpha_1 X_q + A_1 \Rightarrow MA_1 = \alpha_1 X_q + A_1 - Y_q$$
$$A' MA_1 = \begin{bmatrix} 0 \\ \cdots \\ \alpha_1 X_{qk} + A_{1k} - X_{qk} \\ \cdots \\ 0 \end{bmatrix} \equiv Z_q$$
$$Z_q + MA_2 = \alpha_2 X_q + \alpha_1 A_1 + A_2$$

The A_2 vector is decomposed again among the eigenvectors, with the ε_i cœfficients, leading to:

$$Z_{qk} X_{qk} + \varepsilon_q = \alpha_2 + 0 + \varepsilon_q \Rightarrow \alpha_2 = Z_{qk} X_{qk}$$

$$Z_{qk} X_{ik} + \lambda_i \varepsilon_i = 0 + \alpha_1 \mu_i + \varepsilon_i \Rightarrow \varepsilon_i = \frac{Z_{qk} X_{ik} - \alpha_1 \mu_i}{1 - \lambda_i}$$

The ε_q cœfficient is obtained from the norm:

$$(X_q + \delta A_1 + \delta^2 A_2)^2 = 1$$
$$1 = X_q^2 + 2\delta A_1 X_q + \delta^2 \left(2A_2 X_q + A_1^2\right) = 1 + 0 + \delta^2 \left(2\varepsilon_q + \sum \mu_i^2\right)$$
$$\Rightarrow \varepsilon_q = -\frac{1}{2}\sum \mu_i^2 = -\frac{X_{qk}^2}{2}\sum \frac{X_{ik}^2}{(1-\lambda_i)^2}$$

3.4 Several cavities error

The calculation is the same, to second order. We suppose (for an easier understanding) that two cavities (1 and 2) are perturbed.
The second order terms have been already calculated (previous paragraph), except for the cross terms between cavities 1 and 2.

If the ∂ symbols are used to write the derivation versus the cavity number, we get:

$$\partial_{12} AMX^* + \partial_1 AM\partial_2 X^* + \partial_2 AM\partial_1 X^* + AM\partial_{12} X^* = \partial_{12}\lambda^* X^* + \partial_1 \lambda^* \partial X^* + \lambda^* \partial_{12} X^*$$

We rewrite this équation (q is the accelerating mode) :

$$U_q + M\partial_{12} X_q = \partial_{12}\lambda X_q + V_q + \partial_{12} X_q$$

So, for i≠q :

$$\partial_{12} X_q \equiv \sum_i \zeta_i X_i$$
$$U_q X_i + \zeta_i \lambda_i = 0 + V_q X_i + \zeta_i$$
$$\Rightarrow \zeta_i = \frac{(U_q - V_q)X_i}{1-\lambda_1}$$

And something similar for the q term

3.5 Global sensitivity to errors

The main parameters are the number of cells (N), the coupling factor (k) and the standard deviation σ of the cell frequency error.
The perturbed mode is now known and written:

$$X_{qp}^* = \pm 1 + \sum \delta_i A_{1p} + \sum_{i \neq j} \delta_i \delta_j B_{ijp} + \sum \delta_i^2 C_{ip}$$

The mean value of the electric field can be deduced:

$$< X_q^* >= 1 + C\sigma^2$$

Where C if a function of the C_{ip} coefficients only.

The RMS value is, hence :

$$< X_q^{*2} >= 1 + \sigma^2 \sum A_{ip}^2 + \sigma^4 \sum B_{ijp}^2 + \sigma^4 \sum C_{ip}^2 + 2\sigma^2 C$$

The A terms have a zero contribution (as shown previously).

So, the RMS dispersion of the electrical field is :

$$\sigma_E^2 = \sigma^4 \left[\sum B_{ijp}^2 + \sum C_{ip}^2 \right]$$

3.6 Scaling laws

There are N^2 terms, but their amplitude is in 1/N (normalised vectors). They vary like k^2 (terms $(1-\lambda_i)^2$)
Finally:

$$\sigma_E = \sigma_\delta^2 \frac{\sqrt{N}}{k^2} S$$

If the relative frequency error is $\pm\Delta$ 'width in the following table, with an uniform distribution, we get:

$$\sigma_E = \Delta^2 \frac{4\sqrt{N}}{3k^2} S$$

The S coefficient can now be used by simulation (modelisation by MAPLE software for example). We perform several to check the scaling laws. We get, for our design, S~1.3.

Nb runs	k	Ncells	width	sigma	S
100	0,03	11	1,00E-03	5,60E-03	1,14E+00
100	0,03	11	1,00E-03	5,60E-03	1,14E+00
200	0,03	11	1,00E-03	5,70E-03	1,16E+00
100	0,06	11	1,00E-03	1,40E-03	1,14E+00
100	0,01	11	1,00E-03	4,78E-02	1,08E+00
	0,03	11	2,00E-03	2,19E-02	1,11E+00
	0,03	11	1,00E-04	5,60E-05	1,14E+00
	0,03	5	1,00E-03	2,93E-03	8,84E-01
	0,03	11	1,00E-03	5,56E-03	1,13E+00
	0,03	21	1,00E-03	8,40E-03	1,24E+00
	0,03	23	1,00E-03	9,72E-03	1,37E+00
100	0,03	47	1,00E-03	1,26E-02	1,24E+00

Table 3.1: Estimation of the S parameter by simulation

3.7 Calculation of mechanical tolerances.

The goal for field homogeneity is ±2% full width (3 standard deviations), so, if χ is the frequency sensitivity versus the mechanical error, and τ the mechanical tolerance ($\pm\tau$ is the full mechanical tolerance), then the contribution of this error to the field deviation is (3 standard deviations):

$$3\sigma_E = \frac{4\sqrt{N}}{k^2} S \chi^2 \tau^2$$

The total error is the quadratic sum of the individual values.

3.8 Mechanical tolerances and recommendations

The next table gives the field dispersion obtained by using the standard tolerances that can be achieved by a classical (ie not too expensive) machining. It is done for 2 coupling factors:

k	0,03		
Nacc	44		
Ncoup	40		
S	1,3		
défaut	tolerance	ki (%)	sigma
Diam_AC	0,25	0,27	0,01746203
Gap_AC	0,099	0,74	0,02056937
Epaisseur	0,25	0,44	0,04637378
Angle cône	0,25	0,024	0,00013797
Diam_CC	0,25	0,43	0,0422287
Gap_CC	0,099	2	0,14325877
		3sigmatotal	0,15869753
k	0,05		
Nacc	44		
Ncoup	40		
S	1,3		
défaut	tolerance	ki (%)	sigma
Diam_AC	0,25	0,27	0,00628633
Gap_AC	0,1000002	0,74	0,00755535
Epaisseur	0,25	0,44	0,01669456
Angle cône	0,25	0,024	4,967E-05
Diam_CC	0,25	0,43	0,01520233
Gap_CC	0,099	2	0,05157316
		3sigmatotal	0,0571508

Table 3.2: Field dispersion for the SCL tank

We see that a low coupling factor (3%) leads to 15% dispersion, when a higher value (5%) leads to 6%. This last value is not good enough. In addition, a 5% coupling factor leads to a lower Q-value and too high losses. So the recommendation and the decision has been

- The 3% coupling factor is a good choice
- A tuning ring is mandatory for this structure (leading to complicated mechanical tuning.

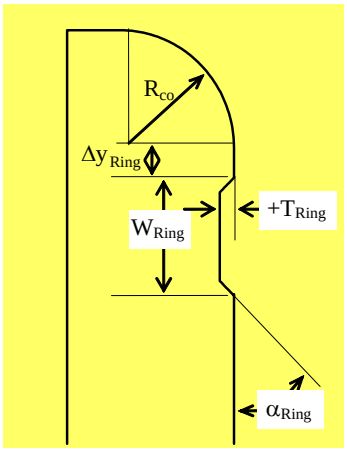

Figure 3.1: Tuning ring at 1GHz (W=20mm, Δy ring=10mm, thickness 5mm)

4 Thermal studies

These studies have been related in detail in CARE/HIPPI Document-05-0011. An example of the proposed solution is given below, for 704 MHz copper cavities. It has been show that solutions exist for a high duty cycle (15%). Nevertheless (see figure 4.1), a rather complicated cooling circuit has to be designed. The brazing might be also an issue. This study has shown a limit around 10-15% duty cycle of such a structure for high duty cycle.

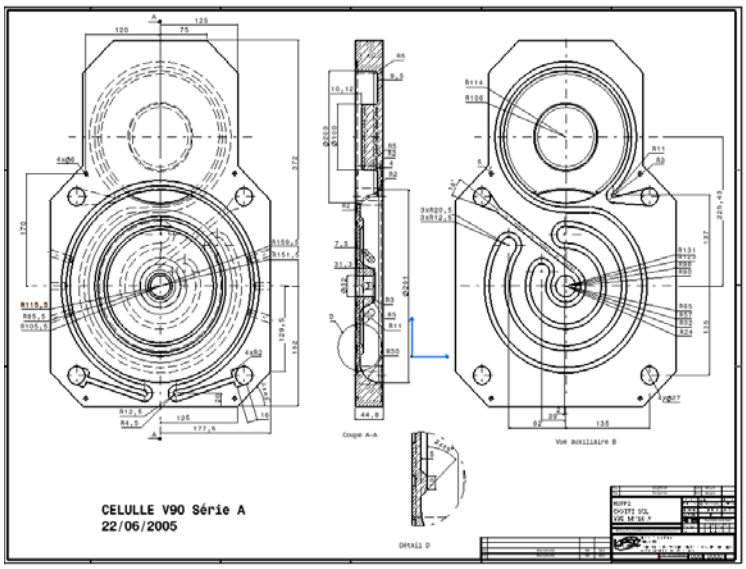

Figure 4.1: cooling system for a 15% duty cycle operation

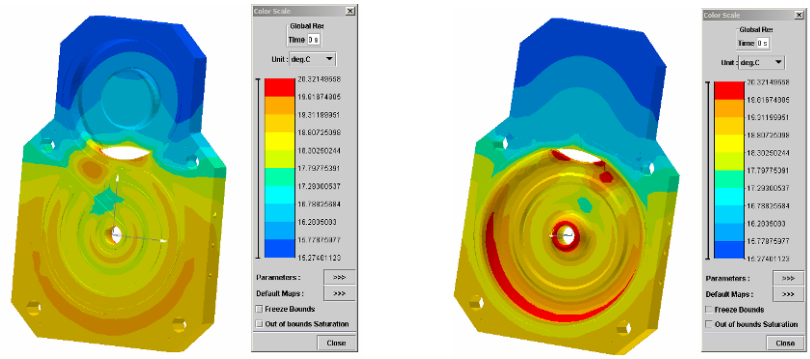

Figure 4.2: Cavity temperatures for a 15% duty cycle operation

5 Prototype and tuning of individual cells

An aluminium prototype has been built to check low-level frequency aspects. Twenty two plates have been manufactured, permitting the construction of a full size tank. The prototype has been scaled to a reduced size around 1 GHz.
The assembly is done by using ordinary screws. The alignment is done by using centering rings between to half accelerating cells. This structure allows prototype from the smallest size (a few elements) up to full size.

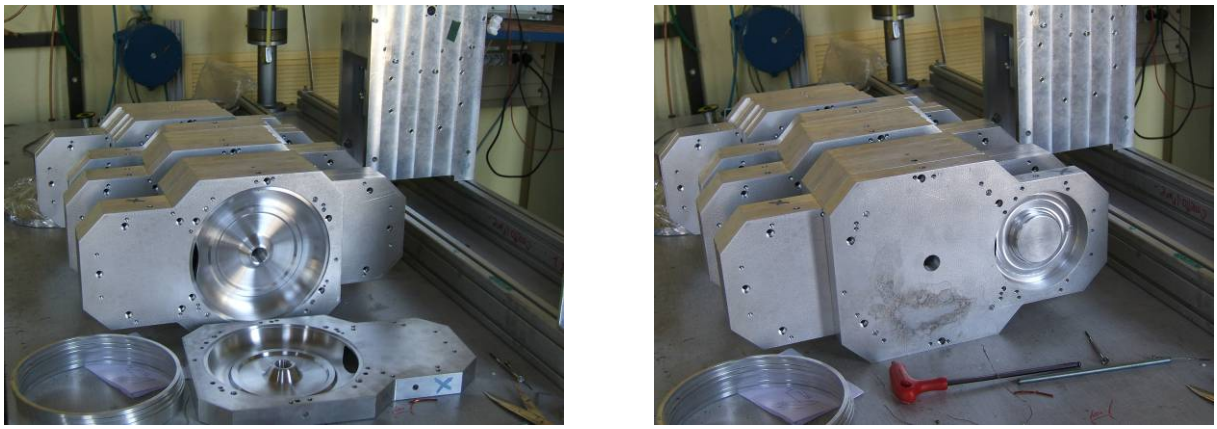

Figure 5.1: view of the prototype (accelerating cells on the left, coupling cells on the right)

5.1 Tuning of individual cells and k factor measurement

The tuning of the accelerating cells can be done only by machining of the tuning ring. The tuning of the coupling cells can be done in two ways: machining of the gap or an adjustable tuner on the cavity axis.

It has been observed that the correct measurement of an individual frequency is not easy: the cavity volume is not well known (even short-circuiting extremity cells), and the perturbation induced by the measurement system itself (antenna for example) is also unknown. Hence, we cannot guarantee that the measured value is the right one. An alternative procedure has been used, based on an idea from Vittorio Vaccaro (Napoli).

The measurement of the coupling cell resonant frequency is presented figure 5.2. Three elementary plates are assembled. The extremity half-cells are short-circuited. A movable tuner is introduced in the accelerating cell, leading to a variation (unknown) of its resonant frequency. As the structure is more complex, the influence of the short-circuited extremities is lower.

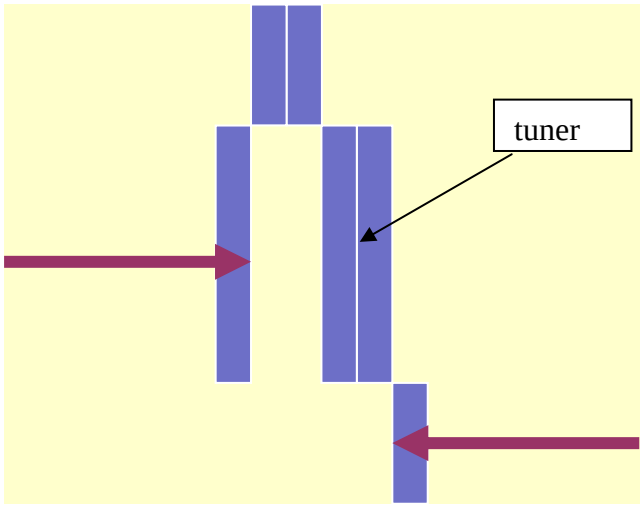

Figure 5.2: Schematic drawing of the frequency measurement set-up.

Let us assume the frequency of the coupling cell is constant, equal to ω_1. Let k be the coupling factor and ω_2 the adjustable frequency of the accelerating cell. The modes of the system are given by:

$$\begin{vmatrix} 1-\frac{\omega_1^2}{\omega^2} & k/2 \\ k/2 & 1-\frac{\omega_2^2}{\omega^2} \end{vmatrix} = 0$$

If F_1 and F_2 are the 2 modes frequencies, measured by a network analyser, rewriting the previous equation leads to:

$$F_1^2 + F_2^2 = \frac{\omega_1^2 + \omega_2^2}{1-k^2} = Y$$

$$F_1^2 F_2^2 = \frac{\omega_1^2 \omega_2^2}{1-k^2} = X$$

Then we get :

$$Y = \frac{\omega_1^2}{1-k^2} + \frac{1}{\omega_1^2} X$$

So, by taking several measurements, corresponding to different positions of the tuner, and without ANY knowledge of the frequency of the accelerating cell, we can get the coupling cell frequency as well as the coupling factor by a simple linear fit. An example is given figure 5.3. It must be noticed that for a coupling factor k=0.03, the ***uncertainty*** due to the fit is around $7*10^{-5}$, ie a 2 per mil uncertainty on k.

The same procedure is used to get the accelerating cell frequency.

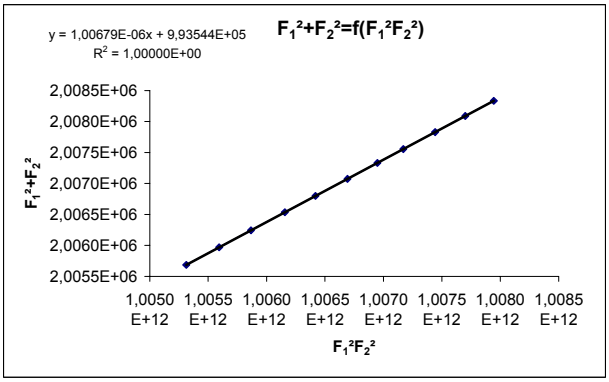

Figure 5.3: Linear fit among the measured eigenfrequencies

After the measurement of all frequencies, all plates are machined. The frequency variation is linear versus the gap in the coupling cells. For the accelerating cells, the required modification of the tuning ring is more delicate to know. Using Superfish, we draw the “tuning curve” given figure 5.4, giving the frequency variation versus the ring machining, from the original frequency (full tuning ring).

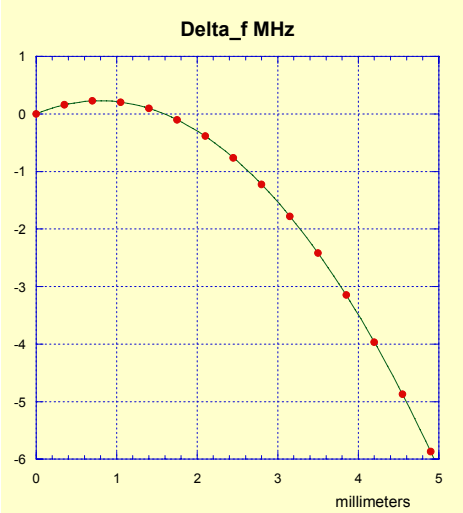

Figure 5.4: Variation of the accelerating frequency with the tuning ring depth frequency, from the initial (measured) value.

Figure 5.5 show the results of the coupling factor measurement for all the plates.

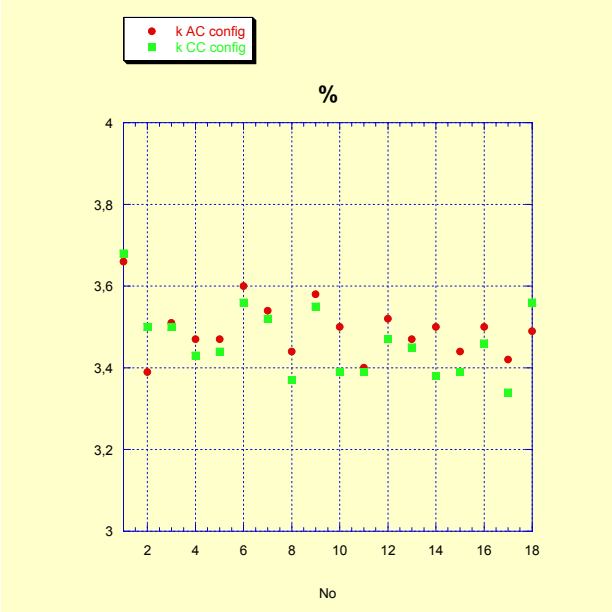

Figure 5.5: Coupling factor. The abscissa is the plate number. The green points are the results obtained when the accelerating cell is varied, the red one when the coupling is varied. The final uncertainty on k might be 0.1% (absolute value).

5.2 Measurement of the second order coupling factor

All the models have supposed the coupling factor from accelerating cell to accelerating cell (or coupling to coupling) to be zero. The same method can be used to measure it and is summarized below (figure 5.6). We use now 3 full cells (3 modes) and we change the frequency of the central cell. We measure now the mode frequencies apart from the central one, named F_2, giving again 2 frequencies F_1 and F_3.

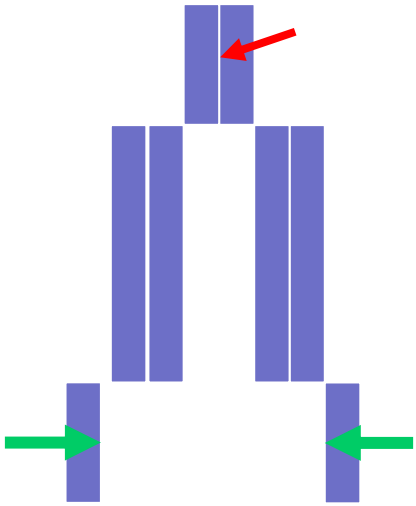

Figure 5.6: Measurement of the second order coupling factor. Four plates are used and end cells are short-circuited. A tuner is put in the central cell to vary the frequency.

The mode frequencies are now solution of the equation:

$$\begin{vmatrix} 1-\frac{\omega_1^2}{\Omega^2} & \frac{1}{2}k_1 & K \\ \frac{1}{2}k_1 & 1-\frac{\omega_2^2}{\Omega^2} & \frac{1}{2}k_2 \\ K & \frac{1}{2}k_2 & 1-\frac{\omega_3^2}{\Omega^2} \end{vmatrix} = 0$$

Let us define (K is the second order coupling factor, k_1 and k_2 the previously measured coupling factors from cell to cell):

$$a = \left(1-\frac{1}{4}k_2^2+\frac{1}{2}k_1 K k_2 - K^2 - \frac{1}{4}k_1^2\right)$$

Hence we get easily:

$$F_1^2 + F_3^2 = F_1^2 F_3^2 \left(\frac{\omega_1^2+\omega_3^2}{\omega_1^2\omega_3^2} - \frac{1}{F_2^2}\right) + \frac{\omega_1^2\omega_3^2}{aF_2^2}$$

It has been observed that the central F_2 remains constant with a very good stability. This has been confirmed by a lot of numerical simulations but not demonstrated (it is not strictly constant). So, again, a linear fit can be used:

$$Y = F_1^2 + F_3^2$$
$$X = F_1^2 F_3^2$$
$$Y = \alpha X + \beta$$

leading to the knowledge of a and, finally of K. The obtained value is K=0.0034. This parameter is important to go further in the system model (it changes significantly the M matrix).

5.3 Tuning results

We present here the values before and after frequency tuning. The frequency goal is 1004.5 MHz. One can see (fig 5.8) that the 9th accelerating cell has a too low frequency.

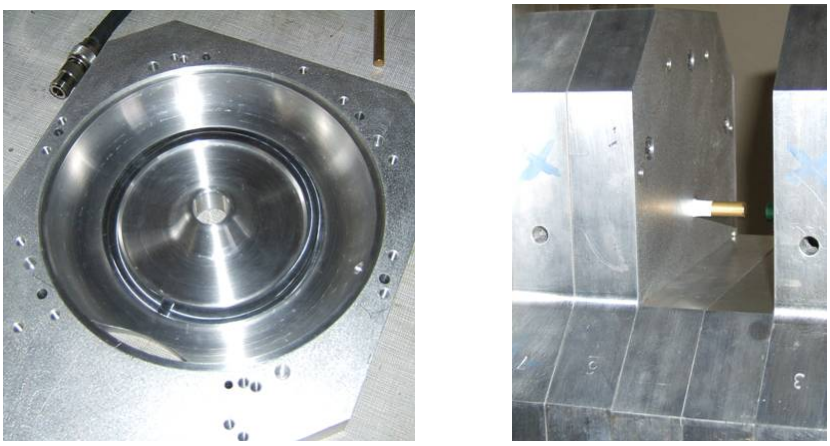

Figure 5.7: Tuning. View of the tuning ring after machining (left) and of the small tuner on the coupling cell (right)

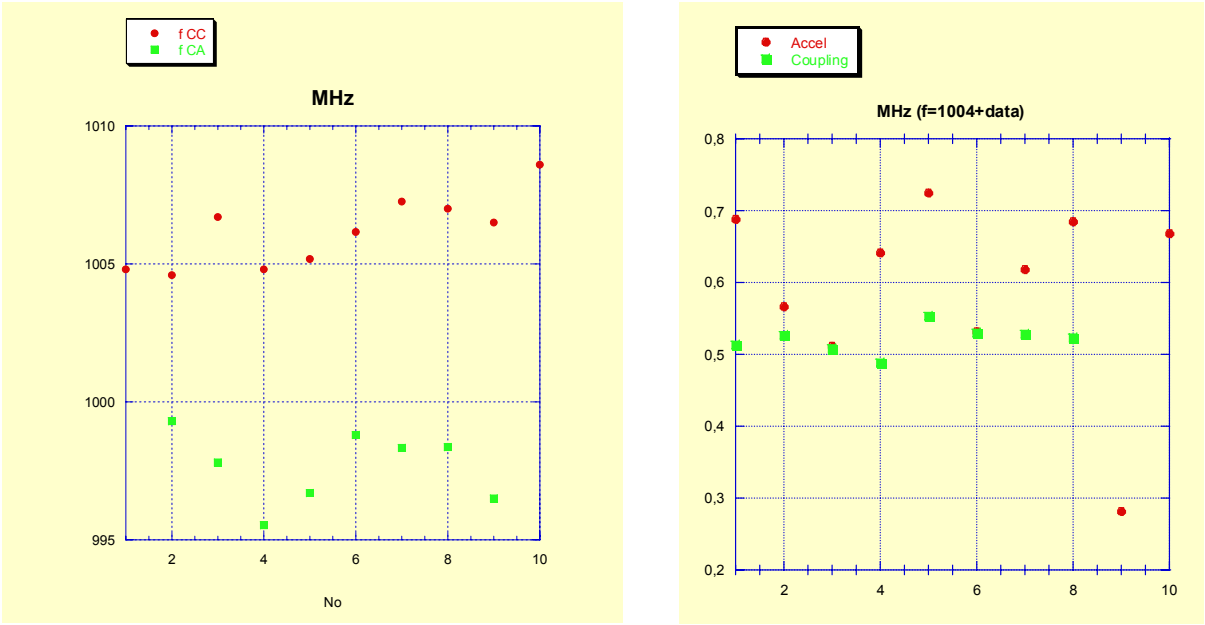

Figure 5.8: Frequencies before (left) and after machining (right). The coupling cell are in green, the accelerating cells in red.

What can we expect from this tuning? By using all the measured parameters (coupling factors, frequencies etc), introduced in a realistic M matrix in a MAPLE code, we can get (figure 5.9) the expected mode frequencies for the whole tank. This very simple tool can be used for predicting the modes of any subprototype and for doing comparisons with experiments.

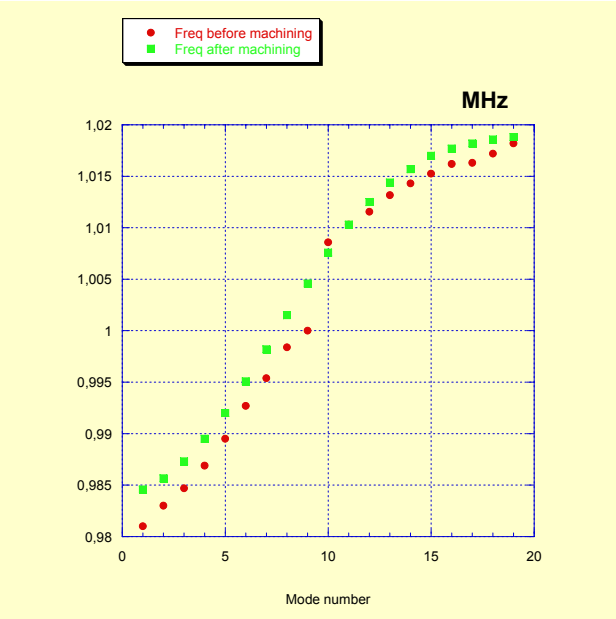

Figure 5.9: Theoretical mode frequencies before and after machining

6 Tuning procedures for coupled cavities

6.1 Introduction

We consider a chain of coupled oscillators like an SCL structure. We show first how the eigenfrequencies and eigenmodes are related to the structure. In a second part we show how, from a measurement of the central mode (close to the ideal $\pi/2$ mode), we can modify the frequency of each oscillator in order to get the nominal frequency for this central mode with a maximum homogeneity. We show that we can achieve the nominal frequency but that the homogeneity is limited by the homogeneity of the coupling coefficients.

6.2 Chain of coupled oscillators

We consider (again) a chain of N oscillators, with N an odd integer. Each oscillator has a frequence ω_i and is coupled to the next one by a coupling factor $k_i/2$ (the factor 2 is needed to have k the classical inductive coupling factor). We consider also the K_i "second order" coupling factors form cell i to cell i+2.

In the most ideal case, such a system is such as $\omega_i = \omega_0$, $k_i = k_0$, $K_i = 0$. The eigenfrequencies are related to the M_0 NxN tridiagonal matrix where (*this matrix is slightly different from the M matrix given before, depending on the boundary conditions on the prototype, but the results are similar*):

$$M_0 = \begin{bmatrix} 1 & k_0/2 & 0 & 0 \\ k_0/2 & 1 & \cdots & \cdots \\ \cdots & \cdots & \cdots & k_0/2 \\ 0 & \cdots & k_0/2 & 1 \end{bmatrix}$$

If λ_n is an eigenvalue of M_0, it is related to the eigenfrequency by the relation $\lambda_n = \left(\frac{\omega_0}{\omega_i}\right)^2$

The $\pi/2$ mode is correspond to the $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \\ 1 \\ \cdots \end{bmatrix}$ eigenvector.

More generally, M_0 has a set of N orthogonal eigenvectors. We normalize them to get an orthonormal basis of eigenvectors written X_i in the following. The normalized vector for the $\pi/2$ mode will be written X^* because it has a specific role in the following.

In the general case, we must consider the matrices:

$$M = \begin{bmatrix} 1 & k_1/2 & K_1 & 0 & \cdots & 0 \\ k_1/2 & 1 & k_2/2 & \cdots & \cdots & \cdots \\ K_1 & k_2/2 & 1 & k_3/2 & K_2 & \cdots \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & K_{N-1} & k_{N-1}/2 & 1 \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \frac{\omega_0^2}{\omega_1^2} & 0 & \cdots & 0 \\ 0 & \frac{\omega_0^2}{\omega_2^2} & \cdots & \cdots \\ \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & 0 & \frac{\omega_0^2}{\omega_N^2} \end{bmatrix}$$

Here ω_0 is the frequency of the $\pi/2$ mode that MUST be achieved. It is introduced in the second matrix in order to have it close to the identity matrix and to be able to use perturbative methods.

The eigenfrequencies and eigenmodes are hence given by the relation

$$\Lambda M X = \frac{\omega_0^2}{\omega^2} X = \lambda X$$

X is any eigenmode, and ω the eigenfrequency. Once again, this expression is independent of ω_0 but this writing will be helpful for programming.

Remark: stop band

Suppose all the K coefficients are zero. Suppose $k_i = k_{i+2} \neq k_{i+1}$. In this case, the k_i series consists of 2 values only. The central mode eigenvalue depends on the value of k_1 and can take 2 values. This difference gives the ***stopband***.

6.3 Tuning procedure

This paragraph is a first analysis. It has to be improved and probably corrected, depending on the experimental results.

The tuning objectives are:

- To get the right frequency ω_0 for the central mode
- To act only on the accelerating cells for a SCL (we will see that, anyway, we can act only on these cells for the $\pi/2$ mode tuning)
- To have the best possible homogeneity for the accelerating cells.
- To have zero in the coupling cells

We measure the central mode (in fact we measure only the mode amplitude in the odd cells in the case of an SCL structure and, in this case, we suppose the field to be zero in the coupling cells). We suppose the normalized (norm equal to 1) central mode not too far from the $\pi/2$ normalized mode.
Let X_m be the measured mode and λ_m the associated eigenvalue (obtained by measurement of the mode frequency).

We have

$$\lambda_m = \left(\frac{\omega_0}{\omega_m}\right)^2$$

with ω_m the measured frequency and ω_0 the frequency needed.

We have:

$$\Lambda M X_m \equiv T X_m = \lambda_m X_m$$

All matrixes being unknown (only the mode and its eigenvalue are known)

Let us consider now a small perturbation E, acting on the accelerating cells only (or not if it appears to be really necessary). E is in fact a set of perturbations. Let us see how such a perturbation acts on eigenvectors. So let us rewrite the problem (the idea being to get corrections):

$$(I+E)TX_m = \lambda_m X_m$$

Let us decompose the mode over the eigenvectors, and rewrite the eigenvalue as a perturbation of the nominal eigenvalue (1 for the $\pi/2$ mode):

$$\begin{cases} X_m = (1+\varepsilon)X^* + \sum \alpha_i X_i \\ \lambda_m = \lambda^* + \delta = 1 + \delta \end{cases}$$

By scalar products, we get:

$$_i = \frac{1}{\lambda^* - \lambda_i} \tilde{X}_i E X^* dx$$

$$\delta = d\lambda^* = \tilde{X}^* E X^* dx$$

We can work now to first order only (perturbative method), leading to (the ~ is transposition):

$$\begin{bmatrix} \frac{1}{\lambda^* - \lambda_1}\tilde{X}_1 \\ \cdots \\ \frac{1}{\lambda^* - \lambda_{N-1}}\tilde{X}_{N-1} \\ \tilde{X}^* \end{bmatrix} \begin{bmatrix} E_1 X_1^* \\ 0 \\ E_3 X_3^* \\ \cdots \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \cdots \\ \alpha_{N-1} \\ \delta \end{bmatrix}$$

Due to the $\pi/2$ mode components, we see we can act only on the accelerating cells as far as first order is concerned. All cells, on the opposite, are concerned by second order.

The correction is now deduced by rewriting this equation and solving it. As the system has more equations than unknown, we proceed by a least squares method, which is obvious for linear system by multiplying by a transpose matrix and inverting the square matrix obtained by this way.

$$A\begin{bmatrix} E_1 \\ 0 \\ \cdots \\ E_N \end{bmatrix} = B \Rightarrow \begin{bmatrix} E_1 \\ \cdots \\ E_N \end{bmatrix} = \left(\tilde{A}A\right)^{-1}\tilde{A}B$$

This approach, once again, is preliminary and must be checked and finalized.

The experimental work started on a reduce set of cell (7) and is based on the bead pull method.

Figure 7.1: view of the 7 cell prototype

A first measurement has shown a field dispersion up to 15%. Further and detailed studies have now to be done. Results are expected for mid-2008.

8 Conclusion

A side coupled cavity has been designed in a CERN-LPSC collaboration to achieve LINAC4 requirements. After RF studies, a complete thermal study has been done, showing that 10-15% is the absolute maximum duty-cycle achievable by such a cavity. Error studies have been developed. They have shown that a tuning ring is mandatory and that a k=3% coupling factor is a good choice (a higher value decreases strongly the quality factor, without getting easily achievable tolerances).
A prototype has been built and each cell has been measured and tuned. A simple and accurate method has been used to get both the resonant frequency and the coupling factor, with a movable tuner and a linear fit. A similar method has been used to get the second order coupling factor.
A large dispersion is observed on k. This is mainly due to the shape of the coupling apertures, which are very sensitive to mechanical errors. A future and realistic design must be very careful to guarantee a constant aperture (the important parameter is more the dispersion of k than its exact value).
Finally, we analyse how to tune the cavity. This has to checked carefully and probably improved or corrected. Results are expected for mid-2008,

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