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Strings

Links between conformal field theory, gauge theory and gravity

présentée par

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Contents

Summary

String theory is a candidate framework for unifying the gauge theories of interacting elementary particles with a quantum theory of gravity. The last years we have made considerable progress in understanding non-perturbative aspects of string theory, and in bringing string theory closer to experiment, via the search for the Standard Model within string theory, but also via phenomenological models inspired by the physics of strings. Despite these advances, many deep problems remain, amongst which a non-perturbative definition of string theory, a better understanding of holography, and the cosmological constant problem.

My research has concentrated on various theoretical aspects of quantum theories of gravity, including holography, black holes physics and cosmology. Through contributions to the field of supersymmetric conformal field theories with continuous spectrum, my co-authors and I have obtained useful insights into holography in backgrounds with linear dilaton profiles, as well as into the embedding of supersymmetric gauge theories in string theory. For instance, our results on conformal field theories with continuous spectrum which serve as building blocks for solutions to string theory enabled us to systematically analyze non-compact examples of mirror symmetry. By introducing boundaries in the conformal field theories, we studied non-perturbative states in these backgrounds. The low-energy degrees of freedom on these D-branes give rise to gauge theories. Using these microscopic tools, we were able to embed an infrared duality for supersymmetric gauge theories in four dimensions into string theory, and to show that it corresponds to a monodromy of boundary states in the bulk conformal field theory moduli space.

In the thesis I have laid bare many more links between conformal field theory, gauge theory and gravity. Most contributions were motivated by string theory, like the analysis of supersymmetry preserving states in compactified gauge theories and their relation to affine algebras, time-dependent aspects of the holographic map between quantum gravity in anti-deSitter space and conformal field theories in the bulk, the direct quantization of strings on black hole backgrounds, the embedding of the no-boundary proposal for a wave-function of the universe in string theory, a non-rational Verlinde formula and the construction of non-geometric solutions to supergravity.

In other developments I have concentrated on theories that quantize gravity directly, which may be a less successful method in unifying all forces in four dimensions, but which provides interesting toy models to analyze generic aspects of theories of quantum gravity. I concentrated for instance on analyzing the degrees of freedom responsible for three-dimensional black hole entropy, and argued that it is hard to reconcile modular invariance and other reasonable requirements on a quantum partition function of gravity with unitarity, outside the context of string theory. I also studied black hole scattering in three dimensions. Likewise, some of my contributions to non-commutative gauge theory, supersymmetric gauge theory, two-dimensional gauge theory, and pair production in two-dimensional anti-deSitter space are fairly independent from the larger framework of string theory.

I believe it is interesting to keep asking hard questions on gauge theory and quantum gravity, inside and outside the fruitful framework of string theory, and to be guided by tough open problems that must lead to further concrete progress, whether incremental or groundbreaking.

Résumé

La théorie de cordes unifie de façon naturelle les théories de jauge, qui décrivent les interactions entre les particules élémentaires, avec une théorie quantique de la gravitation. Ces dernières années ont apporté de grands progrès dans la compréhension des états non-perturbatifs de la théorie, ses aspects holographiques, ainsi que la construction de modèles proches du Modèle Standard. Néanmoins, il reste des défis pour la théorie de cordes, qui incluent une définition non-perturbative, une meilleure compréhension de l'holographie, et le problème de la constante cosmologique.

Ma recherche s'est concentrée sur des aspects formels des théories de gravitation quantique, qui incluent les trous noirs, la dépendance du temps, et l'holographie. Grâce à de nouveaux résultats dans le domaine de la théorie conforme avec spectre continu, mes collaborateurs et moi-même avons avancé dans la compréhension de l'holographie dans des fonds avec dilaton linéaire, ainsi que dans le plongement de théories de jauge supersymétriques dans la théorie de cordes. En particulier, on a étudié des théories conformes supersymétriques avec spectre continu que l'on utilise pour construire des fonds de théories de cordes non-compacts et courbés. Les résultats obtenus nous ont permis de décrire des exemples explicites de symétrie miroir pour des fonds non-compacts. En introduisant des bords dans les théories conformes, on a analysé des états non-perturbatifs de la théorie de cordes, les D-branes. A basse énergie, les degrés de liberté sur les D-branes interagissent par des interactions de jauge. Avec ces outils, on a réussi à plonger une dualité infrarouge de théorie de jauge supersymétrique dans la théorie de cordes, et on a montré que la dualité correspond à une monodromie pour les états de bord dans l'espace de modules de la théorie conforme.

Dans cette thèse, on discute de nombreux autres liens entre la théorie conforme, la théorie de jauge et la gravitation. La plupart des contributions décrites étaient motivées par la théorie de cordes. Des exemples sont l'analyse d'états qui préservent la supersymétrie et leur lien avec les algèbres affines, la dépendance du temps et le dictionnaire holographique, l'analyse directe de la quantification de la gravité en présence d'un trou noir, la réalisation du scenario sans-bord pour la fonction d'onde de l'univers en théorie de cordes, une formule de Verlinde pour les théories conformes non-rationnelles et la construction de solutions non-géometriques à la supergravité.

Dans d'autres travaux, je me suis concentré sur des théories qui quantifient la gravité plus directement, mais qui pourraient avoir moins de succès dans le problème de l'unification des forces en quatre dimensions. Ces théories ont quand-même le potentiel de nous apprendre des aspects communs à toute théorie de gravitation quantique. Par exemple, on a analysé les degrés de liberté responsables de l'entropie d'un trou noir en trois dimensions, et nous avons argumenté sur la difficulté de reconcilier l'invariance modulaire avec l'unitarité en dehors de la théorie de cordes. On a aussi discuté la diffusion de ces trous noirs. D'autres contributions à la théorie de jauge non-commutative, la théorie de jauge supersymétrique, la production de paires dans un espace courbe, et cetera, sont aussi relativement indépendantes du cadre de la théorie de cordes.

Il me semble qu'il reste intéressant d'étudier des questions difficiles sur la théorie de jauge et la gravitation quantique, dans la cadre de la théorie de cordes, et en dehors de ce cadre, et d'être guidé par des problèmes ouverts durs qui doivent mener à un progrès concret par incréments ou par sauts.

Chapter 1

Before entering the labyrinth

A new word is like a fresh seed sewn on the ground of the discussion.
Culture and Value, Ludwig Wittgenstein

A priori

Before entering into the arena where I defend the scientific merits of my work in light of obtaining the degree Habilitation à Diriger des Recherches, I believe it is important to address the question of whether other necessary prerequisites are met[1]. These prerequisites are scientific, administrative and pedagogical. To my understanding, they can loosely be formulated as: having a PhD, a thematic or geographic mobility, leading the way in scientific or administrative projects, having originality and breadth in scientific research, and skills and experience in guiding undergraduate and graduate students. While these conditions may be checked against my curriculum vitae included in appendix A, it may be worthwhile addressing some of them explicitly in this brief introduction.

The themes of my research have been broad. I have published contributions to the fields of non-commutative field theory, supersymmetric field theory, gauge theory in two dimensions, the geometry of supersymmetry, higher derivative effective actions in string theory, the algebra of supersymmetry preserving non-perturbative states, the algebraic structure of non-rational conformal field theories in two dimensions, holography in anti-deSitter space-times, solutions to supersymmetric theories of gravity, string theory cosmology, boundaries in supersymmetric conformal field theories, the relation between non-rational conformal field theories and Calabi-Yau geometries, etcetera. My scientific mobility is therefore tangible. My geographic mobility can be summarized in the diagram:

Leuven, Katholieke Universiteit Leuven →
Brussels, Vrije Universiteit Brussel →
Boston, Massachussets Institute of Technology →
Paris, École Normale Supérieure.

[1] To quote from the original text: L'habilitation à diriger des recherches sanctionne la reconnaissance du haut niveau scientifique du candidat, du caractère original de sa démarche dans un domaine de la science, de son aptitude à maîtriser une stratégie de recherche dans un domaine scientifique ou technologique suffisamment large et de sa capacité à encadrer de jeunes chercheurs.

Amongst the many research directions contained and opened up in my publications, I merely cite the systematic study of higher derivative corrections to the open string effective action by comparing supersymmetric solutions to the effective action and the exact theory, which strongly constrains the effective action and leads to its most accurate determination hitherto. Secondly, I mention the investigation of the application of new results in supersymmetric non-rational conformal field theories to the physics of non-perturbative objects in string theory, linear dilaton holography and gauge theory dynamics. On the administrative front, I note my membership of two European networks, an ANR grant, as well as the fact that I successfully applied for a PIA bi-national grant France-Holland.

The most tangible proof of my experience of guiding graduate students is the large number of publications I co-authored with collaborators who were graduate students at the time of publication. In some cases, I acted as a de facto supervisor or co-supervisor. I moreover guided students in theoretical physics projects at both the Masters 1 and Masters 2 level. And I have teaching experience in experimental labs, exercise sessions, and courses, at the bachelors, masters and postgraduate level. Finally, I successfully defended my PhD thesis on "Strings, links between gauge theory and gravity" in July 2000 at the Vrije Universiteit Brussel, Belgium.

More details on these achievements can be found in my curriculum vitae in appendix A, along with much other data that may be relevant to judging whether I satisfy all the required conditions, beyond the quality of the scientific work summarized in this thesis.

A metro map

Given these prerequisites, I would like to map out the ground we cover in the bulk of my habilitation thesis. In the bibliography to this introductory chapter, I provide a list of my research publications. The papers [p1]-[p9] date from before July 2000 and they were reviewed and expanded upon in my PhD thesis [PhD]. In this habilitation thesis, I will therefore focus on the work I have performed since, which is mostly contained in the publications [p10] -[p42][2]. There are multiple connections between the various papers that I have co-authored, most of which run through string theory, a quantum theory of gravity. I will therefore introduce my thesis with a partial review of the domain of string theory in chapter 2. For the first main line of the thesis, which is developed in chapter 3, I concentrate on the more specific connection between some of my papers, which is the study of the structure of two-dimensional conformal field theories with continuous spectrum, and their interplay with string theory. The dedicated chapter allows for a slightly more detailed treatment of part of my work and it renders tangible the long term coherence of my research. In chapter 4 you will find a selection of other research directions to which I contributed and chapter 5 offers perspectives on future investigations.

The review presented here, which is of considerably smaller scale than the collection of my papers, is partial. I refer to the full original research publications for much more information both on technical details as well as on interconnections with other domains. For a more complete list of references to the original literature in particular, I recommend the collected bibliography present in my articles. My curriculum vitae

[2] For the purposes of this work, it is important to distinguish between the work of others and my own, which leads to an original citation convention.

in appendix A contains the abstracts of my papers. In appendix B I limit myself to presenting a choice of five of my publications[3].

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Special thanks go to all my friends and family.

[3]The precise list of papers can be obtained from the SLAC flagship database SPIRES by typing the command "a j troost" in the search box at the URL http://www.slac.stanford.edu/spires/ . Free electronic copies of all my papers are linked in the search results.

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Chapter 2

Strings as links

Im Kampf zwischen Dir und der Welt, sekundiere der Welt.
Aphorism 52, Franz Kafka

In this chapter, we give a global overview of how our work fits into the large framework of string theory. I review aspects of string theory to the degree necessary to understand motivations for the papers I co-authored. To obtain a complete overview of the subject of string theory, a broader study is necessary.

2.1 Down to earth

As physicists, we are motivated to describe natural phenomena accurately. To a large extent, we have been able to obtain accurate descriptions of processes between elementary constituents of nature via quantum field theories. The electro-weak and strong forces between the three families of leptons and quarks and the vector bosons are coded in a renormalizable quantum field theory with $SU(3) \times SU(2) \times U(1)$ gauge symmetry with three generations of left-handed fermionic matter multiplets in the representations $(1, 2, +\frac{1}{2}) \oplus (1, 1, -1) \oplus (3, 2, -\frac{1}{6}) \oplus (\bar{3}, 1, +\frac{2}{3}) \oplus (\bar{3}, 1, -\frac{1}{3})$, accompanied by their right-handed conjugate counterparts, and by the Higgs boson in the $(1, 2, +\frac{1}{2})$ representation of the gauge group. Gravitational interactions between masses are accurately portrayed by the theory of general relativity.

A contemporary view on physics beyond the Standard Model can be summarized as follows. The actions describing the four forces can be amended to include higher derivative corrections that can parameterize unknown ultraviolet physics, order by order in the energy divided by an ultraviolet cut-off scale. If necessary, new fields can be introduced, which in turn can be coupled to the Standard Model and gravity through local terms in an effective action. This bottom-up view to phenomenology has proven useful in many contexts. It is a worthwhile enterprise to parameterize potential new physics in this fashion, and to attempt to measure the parameters of the low-energy effective Lagrangian.

It is clear however that we would like to have guiding principles to minimize the number of extra fields and parameters that we add to such a low-energy effective Lagrangian. There are already a number of indications that such principles exist. The most important is the low value of the mass of the Higgs boson. It is the only parameter in the Standard Model whose order of magnitude is lower than expectations based on established symmetries. Other indications for new organizing principles include the unification of gauge couplings at large energy scales when supersymmetry is a symmetry of nature and perceived patterns in the Cabibbo-Kobayashi-Maskawa matrix.

There is also a theoretical indication that the bottom-up approach may be too free-roaming. It turns out that it is hard to unify the theory of general relativity and the quantum theory of elementary particles in one framework. One way to formulate the problem is to note that the coupling constant of general relativity has negative mass dimension in four-dimensional space-time, rendering general relativity non-renormalizable. The problem of quantizing gravity turns out to be surprisingly hard to solve. That fact provides us with stricter guidelines as to which low energy effective actions can descend from a unified theory of the Standard Model and gravity at high energy.

A unified theory of quantum gravity and gauge interactions must attempt to provide guidance and predictions for low energy phenomenology. String theory provides such a unified framework, and over the last decades it did supply phenomenologists with good guiding principles for building low energy effective actions with fewer parameters. It provided motivation for supersymmetry, extra dimensions, warped compactifications, localized gravity, little Higgs models, hierarchies in Yukawa couplings and other ideas. String theory has been useful to phenomenologists. Of course, if the scale at which elementary particles behave like strings is around the corner, the phenomenological predictions of string theory are far more direct.

In this thesis we concentrate on a more theoretical approach to the problem of quantum gravity. The breakthroughs and ideas generated by string theory have been many, and they have influenced not only the field of particle phenomenology, but also the domains of quantum chromodynamics, nuclear physics, non-commutative field theory, enumerative geometry, differential geometry, algebra, conformal field theory, condensed matter theory, general relativity and others. And yet, for those who know string theory, the treasure trove seems far from being exhausted. It is fairly clear that many more mathematical structures and physical phenomena remain to be uncovered in string theory and that they are likely to further interplay constructively with other disciplines, both in the theoretical and the phenomenological domain. It is our goal in this thesis to delve more deeply into the theoretical treasure trove.

2.2 Lofty framework

A large part of the research discussed in this thesis can be seen as developing the fruitful interplay between string theory, gravity, gauge theory and conformal field theory. The course of this interdisciplinary domain is harder to follow without understanding the internal logic of the broad framework of string theory. Unfortunately, it is beyond the scope of this thesis to review the full extent of string theory, a theory that generates results that in turn cause streams of thousands of papers within a few years, which generate reviews of hundreds of pages, all with introductory disclaimers of exhaustivity. Under these circumstances, the standard textbooks [1][2][3] and the introductory text [4], as well as the books [5][6] remain the easiest access routes to the field of string theory. In the rest of the introduction we make an effort to provide a rudimentary review of basic facts about string theory that are useful as a gentle introduction to the thesis, and that should allow the reader to interpret technical results that we obtained as partial answers to broader questions.

A stringy skeleton

Perturbative string theory is a first quantized theory in which we consider a two-dimensional worldsheet Σ of a string embedded in a space-time M. The resulting two-dimensional field theory has diffeomorphism and Weyl invariance. A gauge for the worldsheet metric can be chosen such that the metric is conformal to the flat metric. The left-over gauge symmetry which is a combination of Weyl invariance and diffeomorphisms is the conformal group. The conformal group is an infinite dimensional symmetry group of two-dimensional surfaces which preserves angles. A first key feature of string theory that we will frequently refer to in the bulk of our text is the fact that a solution to string theory corresponds to a conformal field theory with zero central charge. After the gauge fixing of the diffeomorphism symmetry, the worldsheet theory contains Fadeev-Popov ghosts. In bosonic string theory, these ghosts correspond to a conformal field theory on the worldsheet of central charge $c = -26$. The other (matter) fields have central charge $c = +26$ such that the total theory is indeed conformally invariant without central charge anomaly.

Quantum perturbative closed string theory is defined as a perturbation series, where each term in the perturbative expansion corresponds to a worldsheet string theory on a Riemann surface Σ of genus g weighted by the string coupling constant g_s to the power $2g - 2$. Tree level corresponds to a spherical worldsheet, one loop level to a torus, and higher loops to Riemann surfaces with handles. One often works with a two-dimensional worldsheet theory that is conformal at one loop order on the worldsheet. The couplings may have to be adjusted at higher order. The perturbation expansion on the worldsheet is controlled by the parameter which is the inverse string tension $\frac{1}{T} = 2\pi\alpha'$ divided by a typical length scale (for instance the radius of curvature) of space-time. The expansion is often referred to as the α' expansion (in contrast to the string coupling constant g_s expansion).

A second key feature of string theory is that the non-linear sigma-model corresponding to the embedding of a two-dimensional worldsheet Σ in a target space M with metric $g_{\mu\nu}$ is conformal at one loop in the α' expansion if the target space metric satisfies Einstein's equation for general relativity. At higher loops in the α' and the string coupling constant expansion, the Einstein equation is corrected by higher derivative terms. We conclude that worldsheet theories that are conformal to all orders in perturbation theory can teach us features of quantum theories of gravity at high curvature.

The possible vibrational modes of the string are many. The least massive excitations are the most pertinent ones for low-energy physics. There may be a negative mass squared excitation, a tachyon, which renders the theory unstable. A standard way to avoid such an excitation is to introduce target space supersymmetry, a symmetry between bosons and fermions, which in turn is associated to worldsheet supersymmetry. A superdiffeomorphism invariant worldsheet theory leads to an extended ghost system of central charge $c = -15$, and matter fields of central charge $c = +15$.

The most interesting excitations are the massless modes. When we study closed strings, these include a graviton of spin 2 which is an excitation of the target space metric $g_{\mu\nu}$ and when we study open strings we typically find a gauge field A_μ in the spectrum, accompanied by a gauge symmetry. The open string endpoints are confined to hypersurfaces in space-times called D-branes (for Dirichlet boundary conditions), and the gauge theories that arise at low energy live on the D-brane hypersurfaces.

There exist other massless excitations which include a two-form anti-symmetric tensor $B_{(2)}$ and anti-symmetric form fields $C_{(p+1)}$. The string is charged under the two-form field $B_{(2)}$ while a Dp-brane of dimension $p+1$ is charged under the anti-symmetric form $C_{(p+1)}$. The anti-symmetric tensor $B_{(2)}$ is called the Neveu-Schwarz-Neveu-Schwarz form and the form fields $C_{(p+1)}$ are called the Ramond-Ramond forms. There is also a scalar field Φ called the dilaton which couples to the worldsheet curvature of the string. The zero-mode of the dilaton corresponds to the string coupling constant g_s. This is because the worldsheet curvature integrated over the surface Σ is equal to twice the genus minus two.

Moreover, there are a host of interesting massless excitations that arise from considering a space-time M which consists for instance of the product of a four-dimensional Minkowski space $M^{3,1}$ and a compact space X, $M = M^{3,1} \times X$. Massless excitations can then arise from particular configurations for string fields on the compact manifold X, which can lead, depending on the geometry and other details of the compact manifold X, to chiral fermions in four dimensions which are interesting for phenomenology.

The fact that the massless excitations of quantum relativistic strings naturally incorporate a graviton, gauge fields and matter fields supports the claim that string theory is a natural candidate to unify the interactions of the Standard Model with general relativity. Over the past decades, steady progress has been made in bringing the low-energy phenomenology of string theory closer to the phenomenology of the Standard Model and in particular its supersymmetric extensions, although a lot of work remains to be done.

Important progress has recently been made on various theoretical fronts as well. Various corners of string theory have been united in a duality web, mostly due to the extra insight we gained into D-branes as non-perturbative states of string theory. We have given a microscopic derivation of supersymmetric black hole entropy. And last but not least, we have gained much insight into dualities between gravitational theories and gauge theories living in one dimension less, which is a manifestation of the holographic nature of quantum gravity. In most concrete examples, it relates perturbative string theory in spaces with negative cosmological constant to large rank gauge theories. Holography has given one more opportunity for string theory to bear on phenomenology, via a reasonably successful gravitational description of hydrodynamics properties of a plasma of quarks and gluons.

On the other hand, a serious challenge to string theory is posed by recent advances in cosmology that have allowed a determination of the slightly positive cosmological constant. A small positive cosmological constant is hard to reconcile naturally with supersymmetry and our understanding of the vacuum in quantum field theories. These simple and puzzling features make the cosmological problem most intriguing.

Global guidelines

The lines of research that we present in this thesis interconnect in both obvious and intricate ways with the broad framework and evolutions that we sketched. They are also inspired by new problems that have emerged as by-products of recent advances. And the research presented here is often triggered by evolutions in other physical or mathematical domains as well, that I haven't taken time to sketch. To render some of these connections more concrete, I indicate a few themes that I believe are interesting and that have motivated the work that follows:

- Can we understand string theory in highly curved non-compact space-times ? When space-time is non-compact, the worldsheet conformal field theory has a continuous spectrum. When the space-time is curved, the conformal field theory will be non-trivially interacting. And when it is highly curved, it will be necessary to resort to exact conformal field theories. Hence our interest in non-trivial conformal field theories with continuous spectrum. One reasonable goal is to understand better the effect of strong (negative) curvatures on geometrical concepts.

- The question of whether alternative theories of quantum gravity are viable. What can we learn from toy models of quantum gravity which are likely to exhibit characteristics common to any quantum theory of gravity ?
- The problem of time-dependence in string theory. Can we naturally construct cosmological solutions to string theory ?
- What can string theory teach us about supersymmetric gauge theories ?
- Does holography apply to other spaces than those with negative cosmological constant ?
- Can we explicitly formulate theories that have characteristics of both gauge theories, and string theories, but that are non-gravitational ?
- Is string theory an efficient tool for identifying interesting problems and research directions in the domains of quantum and conformal field theory ?
- To formulate M-theory, must we abandon geometry and embrace algebra, or vice versa, or neither ?

These are a few lines of questioning that lead to the concrete works that are summarized in the following. Some of these questions obtain a partial answer in this thesis. Most of these problems remain open, and I consider them to be useful sources of inspiration for future research. There are many other very interesting questions that have taken up my time, but for the larger part of this text I shall concentrate on published achievements. I also refer to the final chapter for a discussion of further problems that beg our attention.

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Chapter 3

Continuous conformal field theory

Wenn es Wirklichkeitsinn gibt, muß es auch Möglichkeitsinn geben.
Der Man ohne Eigenschaften, Robert Musil

We have reviewed in chapter 2 that a classical solution to string theory corresponds to a conformal field theory on the sphere. If we wish to study string theory on non-compact spaces with non-trivial curvature, we will need to understand interacting conformal field theories with continuous spectrum. It is the purpose of the present chapter to present some results in non-compact conformal field theory, and to demonstrate how they were inspired by, or gave rise to applications in string theory.

3.1 One motivation

We have gathered a lot of knowledge on string theory on target spaces of the form $M^{3,1} \times X$ where $M^{3,1}$ is four-dimensional Minkowski space, and X is a compact manifold. It is sensible to study such spaces as a first attempt to relate ten-dimensional string theory to four-dimensional phenomenology. Momentum excitations in the compact space X will typically have a mass of the order of one over the length scale associated to X. If that length scale is small (and still large compared to the string scale), those very massive excitations decouple from low-energy physics, and we may be left with a good approximation to the physics in our familiar four dimensions which closely resemble Minkowski space $M^{3,1}$.

It has become increasingly clear that we may also want to study string theory on spaces with a positive or a negative cosmological constant and with more or less than four dimensions. These backgrounds have applications in cosmology, gauge theory as well as elementary particle phenomenology.

Conformal field theories with compact target spaces X have a discrete (possibly infinite) set of conformal primaries. To learn more about string theory on non-compact spaces however, we must study conformal field theories with a continuous spectrum. When the non-compact space has non-trivial curvature, the conformal field theory will be interacting. When the space is highly curved, or at high energy, the higher derivative corrections to general relativity will be important, and we must resort to a conformal field theory description of the target space. At those scales, the usual geometry that particles experience is replaced by a string geometry, or by more algebraic concepts.

3.2 Towards the concrete

In this section, we briefly review a few conformal field theories with continuous spectrum, as well as some facts that we know about them. Our goal is to penetrate the subject, and to present one concrete example that we will be working with.

Conformal field theories[1] on two-dimensional surfaces Σ have a symmetry group generated by maps that preserve angles. The infinitesimal symmetry algebra consists of a left and a right Virasoro algebra. The Virasoro algebra can occasionally grow to an even larger chiral symmetry algebra, like a current or a parafermionic algebra.

A first large class of conformal field theories are rational theories. Rational conformal field theories are those that have a finite number of representations of the chiral algebra in their spectrum. Many beautiful general results have been obtained pertaining to rational conformal field theories, on modular invariant partition functions (i.e. partition functions that are invariant under global diffeomorphisms of the surface Σ), modular transformation properties, fusion rules, simple currents, correlation functions, realizations on the lattice, models with boundary and so on. The algebraic structure of rational conformal field theories is understood to a point where mathematicians can prove beautiful theorems about them.

For non-rational conformal field theories there is much less understanding of the generic algebraic structure. Good progress has been made though in calculating correlation functions of operators on the sphere in particular non-rational conformal field theories (see e.g. [2]-[11]). Those non-trivial non-rational conformal field theories include Liouville theory, which is a theory of a scalar field in two dimensions coupled to the curvature of the surface Σ and with an exponential potential. Other well-studied theories are sine-Liouville theory with a potential of trigonometric form, the conformal field theory with target space the hyperbolic three-space H_3^+ and the gauged Wess-Zumino-Witten model $SL(2,\mathbb{R})/U(1)$ which we will discuss in more detail. The correlation functions of these theories can be computed in terms of the correlation functions of Liouville theory. The relation is highly non-trivial, and it remains useful to think of these as separate conformal field theories. Substantial progress has also been made on computing correlation functions of operators inserted on Σ, or on boundaries of Σ with boundary conditions compatible with conformal symmetry. The correlation functions have largely been solved for by analytic tools, like factorization, decoupling of null vectors, shift equations and an advanced form of the method of separation of variables.

One focus of my work has been to lay bare underlying algebraic structures in non-rational conformal field theory. A useful concrete example to concentrate on is the gauged Wess-Zumino-Witten model $SL(2,\mathbb{R})/U(1)$.

3.3 A concrete continuous model

The gauged Wess-Zumino-Witten model $SL(2,\mathbb{R})/U(1)$ is interesting for various reasons. It has a non-trivially curved target space of infinite volume [12]-[14]. The

[1] See for instance the references [1] for extensive introductions to conformal field theories

Lorentzian continuation of the target space geometry is a two-dimensional black hole. The coset appears naturally in certain string compactifications and it is a conformal field theory that illustrates non-trivial duality relations in the space of conformal field theories.

The conformal field theory can be defined in two steps. We start with an action functional on maps g from a two-dimensional space Σ parameterized by coordinates σ and τ to a group manifold $G = SL(2,\mathbb{R})$:

$$\begin{aligned} g : \Sigma &\rightarrow G = SL(2,\mathbb{R}) \\ (\sigma,\tau) &\mapsto g(\sigma,\tau). \end{aligned} \tag{3.3.1}$$

For a closed two-dimensional space Σ, the action functional is of the form:

$$S = \frac{k}{4\pi}\int_\Sigma Tr(g^{-1}dg * g^{-1}dg) + \frac{k}{24\pi}\int_B Tr(\tilde{g}^{-1}d\tilde{g})^3 \tag{3.3.2}$$

where $g^{-1}dg$ refers to the left-invariant one-form on the group pulled back to the surface Σ via the map g, the space B is a three-manifold with boundary $\Sigma = \partial B$, and the map $\tilde{g}$ is a continuous extension of the map g to B. The operation $*$ is the Hodge star operation on forms. For a generic group manifold G, the model can be shown to be exactly conformal [15]. It gives rise to a two-dimensional conformal field theory with an affine current algebra at level k for both left- and right-movers. In a second step one gauges [16] an anomaly free subgroup $H \subset G$, to obtain a larger class of conformal field theories: coset conformal field theories [17]. We are interested here in the example of the group $G = SL(2,\mathbb{R})$ and we gauge the axial $U(1)$ action $g \rightarrow ghg$ where h is an element of a $U(1)$ subgroup[2].

Classically, we can integrate out the gauge field that arises from gauging the $U(1)$ axial symmetry. After gauging, the three-dimensional target space $SL(2,\mathbb{R})$ becomes a two-dimensional sigma-model with two-dimensional target space parameterized by coordinates $\rho \in [0,\infty[$ and $\theta \in [0,2\pi[$ with metric:

$$ds^2 = k\alpha'(d\rho^2 + \tanh\rho^2 d\theta^2) \tag{3.3.3}$$

where θ is a periodic coordinate satisfying $\theta \equiv \theta + 2\pi$ and $k\alpha'$ is the target space curvature scale[3]. The target space becomes a disk near $\rho = 0$, and it approaches a cylinder near $\rho \rightarrow \infty$ which has given rise to the name cigar conformal field theory. The cigar is semi-infinite, and hollow. To render the quantum theory conformal despite the presence of curvature, there is is a linear coupling to the two-dimensional curvature on Σ coded in the dilaton field Φ given by:

$$e^\Phi = \frac{e^{\Phi_0}}{\cosh\rho}. \tag{3.3.4}$$

It can be thought off as arising from the change in the path integral measure after integrating over the gauge field A, or alternatively, as providing the necessary gradient for the generalized Einstein's equations to be satisfied.

We have skipped over important subtleties in the above derivation of the cigar conformal field theory. In particular, we note that the original model has a target space with an indefinite signature, thus rendering the two-dimensional path integral

[2]Note by contrast that a traditional geometric coset or symmetric space corresponds to modding out a group G by the right action only of a subgroup H.

[3]From the point of view of string theory which we will regain later, the space Σ is the worldsheet of the string and $T = \frac{1}{2\pi\alpha'}$ is the string tension, in accord with the discussion in the previous chapter.

ill-defined. The standard solution to this problem is to think of the target space as being the hyperbolic three-space $H_3^+ = SL(2,\mathbb{C})/SU(2)$. The path integral is then well-defined, but it is clear that the symmetries of the model have changed. In this friction between the euclidean point of view and realizing the symmetries of the model originates a lot of the non-trivial algebraic structure of the cigar conformal field theory. Note that a posteriori, after gauge fixing, the $SL(2,\mathbb{R})/U(1)$ model does have a euclidean target space metric and the path integral is well-behaved.

3.4 Algebraic structures

Torus partition function and supersymmetry

Let us move on to present some concrete results that we obtained on the model at hand. In the work [p16] we discussed the torus partition function of the model. On the basis of the torus path integral for the H_3^+ model [18], we obtained the torus path integral for the coset theory $SL(2,\mathbb{R}/U(1)$ after analytic continuation. The resulting expression still contains an integral over the holonomies of the gauge field on the two-torus. Although the path integral result is formally modular invariant, due to the infinite volume of target space the expression is divergent. After introducing an infrared regulator, the partition function exhibits a divergent piece, which can be shown to be associated to the infinite volume of target space, and a finite piece that contains primary states that correspond to states localized near the tip of the cigar. To perform the integration over the holonomies, it is necessary to introduce an auxiliary plane of integration in which poles in the spectral density give rise to the localized discrete states in the spectrum [4]. The most problematic feature of the derivation is the fact that the infrared regulator breaks modular invariance, which may render the finite piece ambiguous.

The result did provide us with first principle insight into various conjectures and observations in the literature on the cigar conformal field theory, as well as on various aspects of string theory. It had previously been argued on the basis of a duality of conformal field theories that bound states near the tip of the cigar should exist. Also, using properties of singular geometric spaces, as well as on the basis of the unitarity of theories dual to string backgrounds involving the cigar conformal field theory as a building block, a discrete spectrum was expected with a very particular range of $SL(2,\mathbb{R})$ spins. The precise contour shift that appears in the analysis of the torus partition function confirms that the discrete spins fall in the appropriate range.

To discuss further results, we first note that there is a closely related cousin conformal field theory that is obtained by appropriately adding two fermions to the cigar conformal field theory. The resulting model has a chiral algebra which is again much larger than the Virasoro algebra. It is the $N = 2$ superconformal algebra that is not only generated by an energy momentum tensor T but that also contains two spin $3/2$ currents $G^{\pm}$ and a $U(1)_R$ current algebra that rotates the two supercurrents into one another. The $N = 2$ superconformal theory is of interest because it allows to make good use of the model as a building block of supersymmetric string theory. Extended superconformal field theories on the worldsheet can guarantee the stability and supersymmetry of solutions to string theory.

[4] There was a precursor of these technicalities in the derivation of the spectrum for string theory on euclidean AdS_3. However, in that case, the problematic integral that needed analysis in an auxiliary plane arose from integration over the modulus of the torus [11], which is conceptually quite distinct.

With this in mind, two groups generalized the result of [p16] on the torus partition function to apply to the supersymmetric theory [20][p24]. In the work [p24], we went a little further in the analysis, and studied the regulated partition function in more detail. In particular we not only studied the primary states but also the degeneracies of descendent states, and we found a mismatch with the expected number on the basis of the spectrum of primaries [p24]. I believe that worrisome feature is an artifact of the poor choice of regulator.

Clearly, it would be better to have a neat modular invariant volume regulator of the partition function, which has hitherto not been discussed. One of the features of the problem that makes such a regulator hard to identify is the fact that (unlike for instance in the case of a free boson), there is no isometry group of the target space that is as large as target space itself, which would allow factorizing the volume. In this context, I want to mention another work [p27] which treats a technically related issue. It analyzes the problem of Schwinger pair creation in an electric field for a two-dimensional quantum electrodynamics on an AdS_2 space. The isometry group of AdS_2 is $SL(2,\mathbb{R})$ and we were able to exactly calculate Schwinger pair production by factoring the divergence due to the volume of target space, because of the presence of the large isometry group. In that context, the contributions of discrete states can be separated out much more cleanly. A second technically related problem is the definition of the elliptic genus (i.e. the torus partition function with a twist related to the $U(1)_R$ charge). There is a proposal for the elliptic genus of the superconformal cigar model [20], which has been argued for in various non-rigorous ways. Again, the main problem that remains is to understand the modular properties of the proposed elliptic genus, which are anomalous. That fact presumably finds its origin in the non-compactness of target space.

Characters and boundaries

Given our still limited understanding of the modular properties of non-rational conformal field theories, and the precise interconnection between the continuous spectrum and discrete states, we thought it was interesting to study the following problem in more detail. We analyzed the characters corresponding to discrete states in the spectrum as a function of the complex structure parameter τ which is the parameter that specifies the shape of the torus. The torus admits a group $SL(2,\mathbb{Z})$ of non-trivial global diffeomorphisms, which is generated by the generators $T : \tau \to \tau + 1$ and $S : \tau \to -1/\tau$. We studied the behaviour of characters of discrete representations under modular S and T transformations. We found that discrete characters map into discrete plus continuous characters under modular S transformations, while continuous characters map into continuous characters [p23]. We did the analysis on the most elementary bosonic building blocks available (which allows to recuperate the results for the superconformal case). In performing our analysis we payed particular attention to generalizing the existing technical lemmas to all allowed values of the modular parameter τ. We also solved a puzzle posed by the result quoted above: how can the modular S-matrix square to a trivial or charge conjugation matrix if continuous characters are generated from discrete ones, and they in turn generate continuous characters ? We solved the puzzle by showing that an integral over continuous characters can localize on a single continuous character that corresponds to a continuous representation that decomposes into discrete representations [p23].

In rational conformal field theories, there exists a deep relation between modular S-matrices and the algebra of operators in the conformal field theory. The fusion rules, which keep track of when the product of two operators can generate a third, are intimately related to the modular S-matrices via the Verlinde formula. Moreover, when one introduces boundaries for the surface Σ, the Verlinde formula is useful in providing consistent proposals for spectra of boundary operators. These can in turn be coded in formal states called boundary states that live in the space of bulk states. The relation between boundary states and boundaries is most easily viewed as a consequence of the fact that a surface Σ which is an annulus can be read as allowing for time-propagation along the angular direction, or along the radial direction. These two interpretations of time-propagation give rise to a relation between boundary conditions and formal states in the bulk Hilbert space.

It is natural to ask how much of that beautiful story remains intact in non-rational conformal field theories. We did a systematic scan of the relation between modular S-matrices and the fusion rules, as well as between the modular S-matrices and boundary states, and found that an analogue of the rational relations hold, when the modular S-matrices pertain to localized states, or when the boundary operators are well localized in target space [p31]. The analysis gives an indication of the extent to which one can generalize the algebraics of rational conformal field theories to the theories with continuous spectrum.

In any case, the approach of using modular S-matrices to propose consistent boundary states has proven useful in the non-rational case. In [p25] we used that method as well as others to propose consistent boundary states for the $N = 2$ superconformal cigar conformal field theory, which allowed us to verify a conjectured equivalence to the $N = 2$ Liouville conformal field theory in the sector where the space Σ has boundaries. It also allowed us to apply geometrical intuition [10] to the $N = 2$ Liouville conformal field theory.

The geometrical intuition for boundary states came in handy in the work [p40], which zooms in on a context in which one can cleanly isolate a bound state in the spectrum of a non-rational conformal field theory. We showed that by carefully tuning the boundary condition on the cigar conformal field theory on Σ, one can lower the conformal weight of a primary in the continuous part of the spectrum below the gap in the conformal dimension, such that it becomes an isolated bound state in the spectrum. In geometrical terms, one is able to generate an embedded boundary state that is localized in target space, which in turn allows for the localization of boundary vertex operators. Moreover, one can perform the analysis in a way consistent with $N = 2$ superconformal symmetry. A similar phenomenon had been observed in the context of bosonic Liouville theory [22]. We therefore identified a context in which one can very neatly separate bound and unbound states in the spectrum.

A few more results that we can discuss within the context of conformal field theory proper are the following. In the past it had been argued that the cigar conformal field theory, when rendered topological via a twist of the energy momentum tensor with the $U(1)_R$ current, becomes equivalent to bosonic Liouville theory. In the paper [p30] we took a closer look at how the bulk observables of one theory are mapped into the other. To establish a clear map, we needed to conjecture a particular property of the cohomology of the topologically twisted theory, that we were able to prove later using techniques of homological algebra [p32]. We moreover gave detailed arguments and checks as to why the equivalence map extended to the conformal field theories with boundary [p30].

3.5 Applications in string theory

Many of the developments reviewed above in the context and the language of conformal field theory find motivations and applications in string theory, via the fact that we can combine conformal field theories to have a total central charge equal to zero, in such a way that they give rise to a perturbative string theory. It would lead us too far to review all of these applications and motivations in detail. We will attempt to summarize a few of them, using as little technical jargon as possible.

Continuous conformal field theory as a building block

For simplicity we restrict to a particular context in which the above non-rational conformal field theories arise. We recall that string theory exhibits non-perturbative states which have a tension of order $1/g_s$, and other non-perturbative objects with energy density proportional to $1/g_s^2$, where g_s is the string coupling constant. The former are called Dp-branes when they span $p+1$ dimensions, while the latter are $5+1$ dimensional NS5-branes. While strings are electrically charged under the NSNS two-form $B_{(2)}$ via an interaction of the form

$$S_{F1charge} = \int_\Sigma B_{(2)}, \tag{3.5.1}$$

NS5-branes are their electric-magnetic duals, and are therefore magnetically charged under the $B_{(2)}$ two-form. In other words, in ten-dimensional space-time we can define a seven-form Hodge dual to the field strength $H_{(3)} = dB_{(2)}$ of the two-form, which in turn is the derivative of a six-form. The six-form couples to the NS5-brane minimally. The NS5-branes are heavy objects in perturbative string theory and when they are present in the vacuum, they strongly backreact on the space-time geometry, and in particular on the metric $g_{\mu\nu}$, the scalar dilaton Φ and the two-form $B_{(2)}$. Solving for the backreaction, one finds the ten-dimensional metric:

$$\begin{aligned} ds^2 &= -dx^\mu dx_\mu + H dx_i dx^i \\ e^{2\Phi} &= g_s^2 H \\ H_{(3)} &= *_\perp dH. \end{aligned} \tag{3.5.2}$$

where H is a harmonic function in the four-dimensional space transverse to the NS5-branes:

$$H = 1 + \sum_{a=1}^{k} \frac{\alpha'}{|x^i - x_a^i|^2}, \tag{3.5.3}$$

and $*_\perp$ denotes the Hodge star operation on forms in the transverse space. We chose the constant term such that the space-time asymptotes to ten-dimensional Minkowski space. Near the locations $x_a^{i=6,7,8,9}$ of the NS5-brane a throat is generated in the geometry of space-time.

To understand how certain conformal field theories with continuous spectrum arise as building blocks for string theory backgrounds, we first study the case of k NS5-branes which all sit at the origin $x_a^i = 0$ of transverse space. We can zoom in on the throat by taking the limit where the distance r to the NS5-branes is small compared to the string scale $\sqrt{\alpha'}$ to obtain:

$$\begin{aligned} ds^2 &= -dx^\mu dx_\mu + k\alpha' \frac{dr^2}{r^2} + k\alpha' ds^2_{S^3} \\ e^{2\Phi} &= g_s^2 \frac{k\alpha'}{r^2} \\ H_{(3)} &= k d\Omega_{S^3}. \end{aligned} \tag{3.5.4}$$

where $ds^2_{S^3}$ is the metric on the three-sphere that surrounds the NS5-branes and $d\Omega_{S^3}$ is the volume form. We can make the change of variables:

$$\rho = \sqrt{k\alpha'} \log \frac{r}{\sqrt{k\alpha'}} \tag{3.5.5}$$

to obtain the geometry [23]:

$$\begin{aligned} ds^2 &= -dx^\mu dx_\mu + d\rho^2 + k\alpha' ds^2_{S^3} \\ e^{2\Phi} &= g_s^2 e^{-\frac{2\rho}{\sqrt{k\alpha'}}} \\ H_{(3)} &= k d\Omega_{S^3}. \end{aligned} \tag{3.5.6}$$

When we consider a superstring living in the above target space geometry, the worldsheet theory contains the traditional ghosts (for a central charge of $c = -26 + 11 = -15$, as well as six free scalars x^μ, a linear dilaton conformal field theory in the variable ρ, and a Wess-Zumino-Witten conformal field theory on the group manifold $S^3 = SU(2)$ at level k, all of them appropriately accompanied by fermionic partners. The total matter central charge is:

$$c_{matter} = 6 \times (1 + \frac{1}{2}) + 1 \times (1 + \frac{1}{2}) + \frac{6}{k} + 3\frac{k-2}{k} + \frac{3}{2} = 15, \tag{3.5.7}$$

where the first term corresponds to the six free scalars (plus fermions), the next terms to the linear dilaton theory and the last terms to the Wess-Zumino-Witten model. The linear dilaton theory has a central charge which is augmented compared to the free field theory due to the linear dilaton profile. The three-sphere conformal field theory has a central charge slightly smaller than 3 due to the positive curvature of the three-sphere. These two effects neatly compensate eachother to render the total string theory critical (namely of total central charge zero). It is a special feature of the limit of this geometric configuration that it corresponds to an exact conformal field theory. This description therefore incorporates a solution to all higher curvature corrections to the tree level effective action of string theory, and it is therefore valid even at very high curvature. The only restriction on the validity of the solution is that the string coupling must remain small.

We note that when the number k of NS5-branes is small, the solution becomes more highly curved. When we have $k = 2$ NS5-branes, the bosonic conformal field theory corresponding to the three-sphere becomes of zero central charge and it is as if we loose several space-time dimensions. In fact, those dimensions become so small that they allow for very few fluctuations. Note also that the throat near the NS5-branes has a size $\sqrt{k\alpha'}$ which indicates that we cannot squeeze together NS5-branes more densely. When we have a single NS5-brane, the throat becomes of the size of the elementary string excitation in the theory, and the fact that there would be a geometrical throat becomes void.

The above discussion shows on the one hand that we can have a non-trivial (linear dilaton) conformal field theory with continuous spectrum that appears as a building block in string theory. On the other hand it illustrates how conformal field theory concepts can replace geometrical concepts in backgrounds with very high curvature. Let us continue on this path.

Far down the throat of the NS5-branes, the string coupling (or in other words, the exponential of the dilaton field Φ) becomes large. The solution that we found is therefore not a perturbative string theory everywhere in space-time. The physics down the throat could be considerably different from the perturbative physics coded in conformal field theory. It is therefore interesting to modify the above approach to attempt to cut off the strong coupling region in space-time (see [21] for a general discussion). We already saw that a single five-brane does not truly generate at throat, so it is an interesting idea to separate the NS5-branes, but in such a way that the resulting configuration retains some large symmetry. We then take the near NS5-brane limit, and study whether we recognize yet another exact conformal field theory. It turns out that this is possible. Separating k NS5-branes in a transverse plane, and performing a T-duality transformation, we obtain a conformal field theory with the following target space [24]:

$$\mathbb{R}^{5,1} \times (SL(2,\mathbb{R})_k/U(1) \times SU(2)_k/U(1))/\mathbb{Z}_k \qquad (3.5.8)$$

and with central charge:

$$c_{matter} \quad = \quad 9 + (3 + \frac{6}{k}) + (3 - \frac{6}{k}) = 15. \qquad (3.5.9)$$

The two-dimensional coset space $SU(2)/U(1)$ at level k has positive curvature, which diminishes its central charge contribution, while the $SL(2,\mathbb{R})/U(1)$ theory has negative curvature which leads to an increase in its central charge compared to a flat target space theory. Both theories are accompanied by fermions that render the theories $N = 2$ superconformal, and the coset theories are modded out by a $\mathbb{Z}_k$ action. Note also that it is clear in this parameterization that when the original number of NS5-branes was $k = 2$, we loose two space-time dimensions. That is because the unitary supersymmetric $SU(2)/U(1)$ conformal field theory then becomes of central charge $c = 0$ which implies that it is trivial. We are then left with a $\mathbb{R}^{5,1} \times SL(2,\mathbb{R})_2/U(1)/\mathbb{Z}_2$ eight-dimensional target-space. Note that the total central charge of the string theory remains $c_{tot} = 0$, due to the high curvature of the non-compact coset conformal field theory. Despite this fact, these lower-dimensional theories are often called non-critical.

Finally, we have recuperated the conformal field theory $SL(2,\mathbb{R})/U(1)$ conformal field theory as a building block for a superstring theory background. We can now revisit some of the results that were obtained on the conformal field theory, and apply them in string theory.

Bulk spectra applied

There are many interesting aspects that were studied and that remain to be explored for this particular background of string theory. We shall rapidly review some.

It should be clear that the knowledge of the spectrum of the $SL(2,\mathbb{R})/U(1)$ conformal field theory discussed previously completes our knowledge of the spectrum of strings on this background, and in particular it adds to our understanding of the appearance of discrete states in the spectrum. We also argued in [p24] that there is an infinite series of worldsheet instanton corrections to the coset background which can be read off from the supergravity solution. It remains an open problem to check whether a linear sigma-model approach can confirm the coefficient of the first instanton correction (as was done in the article [25] for a simpler case [26]). Also, in [p24] it was argued that an old puzzle regarding space-time supersymmetry of the above background was resolved by the observation that the $\mathbb{Z}_k$ orbifold action renders a purely geometrical approach to the background inadequate. Moreover, an alternative gauged conformal field theory was given, which after T-duality becomes equivalent to the above orbifolded theory, and which does allow for a clean geometric treatment of supersymmetry.

More general backgrounds

The background that we discussed allows for a vast generalization. There is a much studied extension which involves a $\mathbb{R}^{3,1}$ Minkowski factor and a product of compact coset conformal field theories only. Those are the Gepner models, and they have been argued to describe highly curved points in the moduli space of Calabi-Yau compactifications. They gave some of the first examples of equivalences of Calabi-Yau manifolds (mirror symmetry, or T-duality) that were crucial in rendering the physical tools of string theory very useful in enumerative geometry. It is therefore natural to further study the non-compact analogues of Gepner models [p38] which consist of compact and non-compact conformal field theories. In this context, we carefully analyzed the applicability of the Landau-Ginzburg model approach, in which one describes an $N = 2$ superconformal model as the infrared fixed point of a linear sigma-model with given superpotential. Our approach provided handy counting rules for the number of supersymmetry preserving deformations of the models. In our paper [p38] we were able to give a rationale for subtle modifications of the counting rules of the compact case in order for them to apply to the non-compact models. Using those counting rules we were able to provide beautiful examples of mirror symmetry in non-compact Gepner models. We also pointed out a subtle difference with the counting of deformations of non-compact toric Calabi-Yau manifolds, and we gave an example of a model violating a conjecture in the literature that stated that in this class of models one can have at most one Kähler or complex structure modulus.

Boundary spectra applied

We now turn to a review of applications of conformal field theories with continuous spectrum with boundary in the context of string theory. Since the conformal field theories with boundary with a flat target space and with compact coset target spaces were well-known, the construction of the boundary states of the $N = 2$ superconformal non-compact coset target spaces was crucial in getting a more concrete handle on the D-branes in the near-horizon geometry of NS5-branes [27]. We note that it is consistent to first take into account the backreaction of the NS5-branes, which have a mass of the order of $1/g_s^2$, and to only then add D-branes, which have a large mass as well, but of order $1/g_s$. We could therefore construct D-brane boundary states in the background T-dual to k NS5-branes spread on a topologically trivial circle [p28]. It was interesting to track back their target space geometry to the original NS5-brane background, where the branes turn out to trace out beautiful and complicated shapes.

Configurations of D-branes stretching between NS5-branes were crucial in providing some of the most intuitive pictures of the physics of $N = 2$ and $N = 1$ supersymmetric gauge theories in $2, 3$ and 4 dimensions [28][29]. It is gratifying that one can construct those configurations exactly in a double scaling limit. It provides a confirmation of the low-energy spectra on the D-branes, but it also completes the construction of the spectrum to the massive open string states, and gives and explicit handle on the coupling of the D-branes to the massive closed strings as well.

One further application of these types of boundary states is an economic construction of minimal super-Yang-Mills theories in various dimensions [p29]. Recall first that a $D3$-brane in flat ten-dimensional space has massless open string degrees of freedom that at low energy interact via a four-dimensional maximally supersymmetric Yang-Mills theory (namely $N = 4$ super-Yang-Mills with sixteen real supercharges). The D3-brane can move in 6 transverse directions, which provides the 6 scalar degrees of freedom of $\mathcal{N} = 4$ super-Yang-Mills theory that transform in an irreducible representation of the $SU(4)$ R-symmetry group[5]. Let us now come back to the background:

$$\mathbb{R}^{5,1} \times SL(2,\mathbb{R})/\mathbb{Z}_2 \tag{3.5.10}$$

that we constructed previously. Let us put a D3-brane in this background, with all directions along the $\mathbb{R}^{5,1}$. The background breaks half of the supersymmetry of the flat space background, and the D3-brane theory only has 8 real supercharges as a consequence. The D3-brane will attempt to minimize its energy, and therefore also its energy density. It can do this by sitting in the region of strongest coupling, which is at the tip of the cigar. The only massless transverse scalars then correspond to the movement of the D3-brane along the two transverse directions in the Minkowski space. The low-energy theory on the D3-brane is pure $\mathcal{N} = 2$ super-Yang-Mills theory in four dimensions, which indeed has two massless scalar degrees of freedom, and the right amount of supersymmetry.

We can continue along this path and construct minimal super-Yang-Mills theory in four dimensions by considering the background:

$$\mathbb{R}^{3,1} \times SL(2,\mathbb{R})_1, \tag{3.5.11}$$

with N D3-branes at the tip of the cigar. The background is critical since the coset carries central charge $c = 9$ when the level k equals one. Although it is considerably more difficult to make the link between this background and a configuration of NS5-branes and D-branes precise, the background is naturally interpreted as corresponding to two NS5-branes which share 4 space-time directions, and are mutually orthogonal in 4 others, with N D4-branes stretching in between. In any case, the background with the D3-branes provides a very economical construction of $\mathcal{N} = 1$ pure super-Yang-Mills theory, in which one controls not only the massless open strings but also the massive open string degrees of freedom. In principle, the D-brane boundary state also codes the full backreaction of the D-brane on the target space which is very interesting for holographic purposes, although in practice it is extremely hard to extract.

It was also realized [30] that one can easily add flavours to the minimal $\mathcal{N} = 1$ super-Yang-Mills theory by adding space-filling D-branes that cleverly fold onto themselves in supersymmetric fashion in order to avoid supersymmetry breaking as well as a tadpole in a non-dynamical field. That is interesting because it provided a simple controlled context in which to study $\mathcal{N} = 1$ supersymmetric gauge theories with flavours, which exhibit interesting dynamical phenomena. There is for instance a conjecture that $\mathcal{N} = 1$ gauge theories exhibit a duality in their infrared regime. A $U(N_c)$ gauge theory with N_f flavours is infrared dual to a $U(N_f - N_c)$ gauge theory with N_f flavours, and with a new superpotential that depends on the superpotential

[5] An R-symmetry group is a symmetry group that acts differently on the bosons and on the fermions of the theory.

of the initial theory [31] (see also [32]). In [p34][p36] we argued that the infrared duality can be re-interpreted on the worldsheet as corresponding to a monodromy in the space of boundary states under a bulk deformation. Thus, the infrared duality gets a microscopic equivalent in the sense that one is able to embed it in string theory, and control the full open string spectrum under the duality. Although it remains hard to strictly argue the duality beyond the chiral sector, the new microscopic realization may open the way for using worldsheet holomorphy in analyzing the duality. Also, it is conceivable that the control over open string masses can give insight into the precise normalization of the canonical Kähler potential of the low energy gauge theory. That in turn can give a mildly better control over a scenario for supersymmetry breaking using metastable vacua in supersymmetric gauge theories [33].

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Chapter 4

Selected contributions

Since its fishing policy transformed Iceland, the place has become, in effect, a machine for turning cod into Ph.D.'s. But this, of course, creates a new problem: people with Ph.D.'s don't want to fish for a living.

Wall Street on the Tundra – how Iceland went bankrupt, Vanity Fair, Michael Lewis

In this chapter we discuss a selection of contributions to string theory that vary broadly. To discuss them even with a minimum of detail requires us to carve out separate inroads into the landscape of string theory. In the process, we point out a few of the overarching themes that connect various works.

4.1 Three-dimensional black holes

Low-dimensional gravity provides an interesting theoretical playground to study aspects of quantum theories of gravity. And quantum theories of gravity should teach us properties of singular geometries like black holes, or cosmological singularities. Pure gravity in two dimensions does not seem to be sufficiently rich to allow for black hole solutions. In three dimensional gravity with a negative cosmological constant however, black holes exist [1][2]. These BTZ black holes carry mass and angular momentum. Therefore, it is interesting to analyze three-dimensional gravity with negative cosmological constant, and in particular its BTZ black hole solutions.

Microscopics of pure gravity

We have studied BTZ black holes in various contexts. First of all, we have studied them in pure three-dimensional gravity, which is a topological theory. The graviton has no propagating degrees of freedom in three dimensions. In fact, one can rewrite the Einstein-Hilbert action in three dimensions as a Chern-Simons gauge theory. When the cosmological constant is negative, the gauge group is $SO(2,2) \equiv SO(2,1) \times SO(2,1)$. With particular boundary conditions for the metric field at infinity, the theory can be shown to allow for an asymptotic symmetry algebra which consists of a left- and right-moving Virasoro algebra [3]. The symmetry algebra arises from diffeomorphisms that fall off slowly as they reach asymptotic infinity. The central charge of the Virasoro algebras is

$$c = \frac{3}{2}\frac{l}{G_N} \tag{4.1.1}$$

where l is the radius of curvature of the asymptotically AdS_3 space and G_N is the three-dimensional Newton constant [1].

The asymptotic symmetry algebra suggests that unitary quantum gravity on AdS_3 corresponds to a unitary conformal field theory with this central charge. If that theory has a vacuum of conformal dimension zero, then its asymptotic growth of the number of states with high conformal dimension is given by the entropy formula:

$$S = 2\pi\sqrt{\frac{cn_R}{6}} + 2\pi\sqrt{\frac{cn_L}{6}} \tag{4.1.2}$$

where $n_{R,L}$ are the eigenvalues of the left-right Virasoro zero-modes L_0 and $\bar{L}_0$. Semi-classically, these generators measure the energy M and angular momentum J of a given gravity solution via the formulas:

$$\begin{aligned} M &= \frac{1}{l}(L_0 + \bar{L}_0) \\ J &= L_0 - \bar{L}_0. \end{aligned} \tag{4.1.3}$$

For a BTZ black hole of mass M the degeneracy of states counted by the above entropy formula grows precisely as expected from the semi-classical area of the black hole horizon, and the Bekenstein-Hawking entropy formula

$$S = \frac{\text{Area horizon}}{4G_N}. \tag{4.1.4}$$

This fact has been repeatedly observed in various ways, but in a generic theory of quantum gravity it is difficult to show that all the necessary conditions for the above derivation are satisfied. In particular one needs unitarity, a vacuum of conformal dimension zero, and invariance under global diffeomorphisms. Moreover, one would like to have a better understanding of precisely which degrees of freedom the above entropy formula counts.

The degrees of freedom

In the paper [p20] we set out to answer in more detail what the degrees of freedom are that are responsible for the large degeneracy of states at large mass. From the derivation in terms of an asymptotic Virasoro symmetry algebra, it is clear that the degrees of freedom that are counted are located on the boundary of space-time. The gravitational Chern-Simons theory on the boundary is equivalent to a conformal field theory. So, it is these conformal field theory degrees of freedom that are responsible for the large degeneracy. When one takes a closer look at those degrees of freedom, we found that one encounters various problems with applying the above arguments. Namely, the boundary theory needs to be taken or put into euclidean signature if the theory is to be unitary. And if we take a unitary version then it turns out to be the case that the vacuum energy is non-zero (as is the case for instance in a modular invariant partition function for Liouville theory). We showed therefore that there was considerable tension between the demand of modular invariance and unitarity on the one hand, and a realization of the quantum theory of gravity in terms of pure gravity alone [p20]. Recently an interesting attempt was made to reconcile most of these requirements [4] by cleverly constructing a minimal, holomorphically factorized partition function consistent with unitarity and the asymptotic Virasoro

[1] The Einstein-Hilbert action is normalized with a prefactor $\frac{1}{16\pi G_N}$.

algebra. However, a large part of the problem remains intact, in the fact that it is likely that a semi-classical limit of that construction does not exist [5], and that holomorphic factorization may not be compatible with reasonable requirements on a three-dimensional theory of gravity like the absence of naked time-like singularities.

A plausible conclusion is that gravity is thermodynamic, in the sense that the metric may only be a master field for the microscopic degrees of freedom of quantum gravity. A direct quantization of the gravitational field may only work at very high curvature where the degrees of freedom are few, but in the semi-classical regime, one really needs to deal with other statistical mechanical degrees of freedom.

Black hole scattering

Though three-dimensional gravity may not contain the final answer to the quantization of gravity, particularly in four dimensions, it does remain an excellent laboratory for experimenting with properties of quantum gravity. Since three-dimensional gravity is topological, one can imagine gedanken experiments that would be extremely hard to execute in four-dimensional quantum gravity. An example is the scattering of black holes.

Particles in three dimensions are like conical defects in the fabric of space-time [6]. In the Chern-Simons language, particles create a non-trivial holonomy for the gauge field on a contour surrounding them. The scattering of particles becomes a calculation of the influence of the respective holonomies of the particles on eachother's wave-function [7].

In [p21] we analyzed a particularly interesting case of the scattering of excitations in three-dimensional gravity. Since even black holes can be thought off as being associated to a non-trivial Wilson line, we can consider scattering punctures which represent black holes. We studied such a scattering process, in which we considered one black hole to be a fixed target, generating a non-trivial holonomy for the gauge field (representing the gravitational field of the black hole). We then took a second black hole, considered as a probe, which twisted around the first black hole an arbitrary number of times, representing various scattering processes in three-dimensional gravity. We showed that as a result the wave-function of the probe is subject to a projection condition. Where ordinarily the wave-function of the probe is a function on the AdS_3 group manifold, it becomes projected precisely in such a way that it becomes a wave-function on the target BTZ black hole (which is nothing but an orbifold of the AdS_3 group manifold). We thus showed that in a probe approximation where we neglect the effect the gravitational field generated by the second black hole, the probe black hole feels the target black hole via projection of its wave-function. We believe this is an interesting phenomenon. Moreover, it should be possible to compute the full scattering process without making the probe approximation. It would moreover be interesting to see in how far one can compare this scattering to high energy scattering at fixed impact parameter in four-dimensional gravity[2].

BTZ black holes in string theory

String theory is a theory of quantum gravity which admits AdS_3 space-times as well as BTZ black holes as solutions. We can therefore also study BTZ black holes in string theory. String theory contains many more fields than pure gravity. In particular it

[2] I would like to thank E. Rabinovici for posing this problem.

contains infinite towers of massive string states. In string theory, for supersymmetric black holes, it can be understood what the microscopic degrees of freedom are that are responsible for the black hole entropy [8]. At weak string coupling, they correspond to the gauge degrees of freedom on a system of D-branes. The entropy of supersymmetric BTZ black holes (with mass equal to their angular momentum) can thus be understood in string theory microscopically. The essence is an index calculation that can be extrapolated to any value of the string coupling, from the strong gravitational regime to a weak gauge theory regime.

One hopes to go further though, in understanding black holes in string theory, even in the supersymmetric context. There are BTZ black holes in string theory which can be thought off as the near-horizon geometry of a supersymmetric configuration of fundamental strings and NS5-branes. Those objects carry NSNS flux only. Moreover, the BTZ black hole is an orbifold of a group manifold, and both group manifold backgrounds as well as orbifolds have been fairly well-understood in string theory. It is therefore natural to ask how far one can go in quantizing string theory directly on an NSNS BTZ black hole background. Can we show that quantum gravity on this background is well-behaved and unitary, directly ? Presumably that is a tough question to answer. But one can see which problems arise in the process of attempting to answer the question starting from first principles.

The spectrum of string theory backgrounds that include an AdS_3 factor with NSNS flux has been vigorously studied [9]. An essential feature of the spectrum is that it contains strings that wind the angular direction of the AdS_3 space-time and that have a continuous radial momentum [10]. This is possible because the tension of these long strings is compensated by the energy they gain due to the background NSNS flux. The existence of these long strings was crucial to understand the partition function of the theory [11].

In the work [p17] we explicitly showed that these long strings exist in BTZ black hole backgrounds as well. That is to be expected since the BTZ black holes are asymptotically AdS_3. It turns out though that algebraically, they are much harder to study. The underlying reason is that the generator of rotations along the angular direction in a BTZ background has the opposite norm with respect to the Killing metric of $so(2,2)$ from the angular generator in the AdS_3 space-time. That simple fact makes it much harder to analyze the part of the string Hilbert space that corresponds to long strings in black hole backgrounds. Some preliminary steps were taken in [p17], but the problem was not fully solved. The result did resolve some discrepancies between various proposals for spectra of AdS_3 string theory : it was shown that one of those proposals [12] in fact pertains to a massless BTZ black hole by demonstrating that the proposed alternative representations correspond to long strings in a massless BTZ black hole background.

In the work [p18] we took another step in analyzing string theory on BTZ black hole backgrounds from first principles. We concentrated on the simplest BTZ black hole with zero mass and angular momentum, and treated string theory in a particle limit. It is known that the spectrum of strings can often be inferred from the particle spectrum, by creatively adding the oscillator modes. We analyzed the spectrum for strings in the particle limit, and computed particle limits of three-point functions as well. To confirm those calculations, we had to derive new results on Clebsch-Gordan coefficients of $SL(2,\mathbb{R})$ in a light-like basis. We moreover showed that the results have applications to the calculation of Lorentzian AdS/CFT three-point functions.

It should be clear that a lot remains to be done before we understand quantum gravity even in lower-dimensional and supersymmetric black hole backgrounds from first principles.

4.2 Theoretical cosmology in string theory

Realizing standard cosmological scenarios in string theory is hard. String theory is tightly constrained, and one field in which this becomes manifest is string cosmology. That is why it is particularly interesting to confront string theory with present day precision cosmology. Indeed, dark energy makes up seventy-two percent of the energy in our universe today according to recent measurements. Dark energy has a likely interpretation as corresponding to a parameter in the classical equations of general relativity, the cosmological constant. The cosmological constant is then positive. However, when we unify quantum field theory with general relativity as we do in string theory, we are naturally lead to supersymmetric theories. Supersymmetric vacua typically have zero or negative energy density, and therefore a negative cosmological constant. Thus, there is some distance to travel between string theory and present day cosmological scenarios. Various scenarios for obtaining non-supersymmetric vacua with a small positive cosmological constant within string theory have been proposed, but it is fair to say that the proposed mechanisms for obtaining a small positive cosmological constant are not elegant. It may be that we must look for a more systematic understanding of the cosmological constant problem.

In a few works [p37][p26], I have set out to understand some of the basic mechanisms that allow or disallow cosmological scenarios in string theory. These investigations explore the room we have for realizing vacua with a positive cosmological constant in string theory, without immediately worrying about their phenomenological viability. I believe that the cosmological constant problem is sufficiently hard to justify this approach.

We analyzed two-dimensional cosmological space-times in string theory. These space-times are based on the coset conformal field theory $SL(2,\mathbb{R})_k/SO(1,1)$. When the level k of the model is negative, the space-time has an effective number of dimensions which is smaller than two, and behaves like a positive cosmological constant space-time [13]. It should come as no surprise then that the analytic continuation of the model to the Euclidean domain is the coset conformal field theory $SU(2)_{|k|}/U(1)$. Amongst other questions that we asked ourselves, was whether it is possible to generalize the Hartle-Hawking calculation of the norm of the wave-function of the universe without boundary [14] to this string theory context.

The no-boundary proposal for the universe was made in the context of general relativity. If one considers a time-reversal symmetric spatial slice of an dS_n deSitter universe, then it has the topology of a S^{n-1} sphere. One can define a wave-function on the spatial slice by specifying a boundary condition for the metric on the spatial slice, and integrating the euclidean path integral over all metrics and spaces which have the given boundary condition. That defines a functional of the boundary conditions which can be interpreted as the wave-function of the universe at some initial time. It can then be further evolved via a Hamiltonian evolution in Lorentzian signature. Since deSitter space-times are hard to embed and control in string theory, it is difficult to realize the no-boundary proposal in string theory.

The two-dimensional cosmological model above provides an opportunity to analyze the scenario in string theory, but before one can show this, a number of subtle issues need to be addressed. The first is the euclidean geometry of the $SU(2)_{|k|}/U(1)$ coset.

It has a metric of the form:

$$ds^2 = k\alpha'(d\theta^2 + \tan^2\theta d\phi^2) \tag{4.2.1}$$

where $\theta \in [0, \frac{\pi}{2}]$ and $\phi \in [0, 2\pi[$. We see that the space seemingly has a boundary at $\theta = \pi/2$. However, the bulk conformal field theory is entirely well-defined, and needs no specification of boundary conditions. It is possible to prove almost-geometrically that this is the case by performing a T-duality in the angular direction of the disk. One obtains another disk, but where the center and the boundary of the disk are exchanged. The center of the dual disk has a $\mathbb{Z}_k$ orbifold singularity. We can now glue the interior of the original disk to the interior of the T-dual disk and obtain a space without boundary and with no harmful singularities. The resulting space is called a T-fold since it not only allows for gluing patches by diffeomorphisms, but also by T-dualities of string theory. Thus, we have obtained an almost geometric picture of the coset conformal field theory $SU(2)_{|k|}/U(1)$ and it has become manifest that it has no boundary. Similarly, we can think of the two-dimensional cosmology as consisting of two deSitter-like patches, glued by a T-duality. There is no longer a time-like singularity in the cosmology.

We should think of the T-fold cosmology as the analogue of the two-dimensional deSitter space in the Hartle-Hawking no-boundary proposal. We can cut the T-fold cosmology on a time-symmetric spatial slice. The boundary is a slice of the $SU(2)_{|k|}/U(1)$ model. As in the no-boundary proposal, we can define a functional by specifying boundary conditions for all fields in string field theory (including the massless fields like the metric, but also all massive fields), and performing a path integral over all the string field theory configuration space that satisfies that boundary condition. The path integral is horribly complicated. We can attempt to compute the norm of the resulting wave-function in a saddle point approximation by evaluating the string field theory action on the saddle point represented by the $SU(2)_{|k|}/U(1)$ coset model. Again, this is hard, but it is expected that this second quantized quantity is well-approximated by its first quantized counterpart, which is the string theory partition function for the coset model. Since the model is compact, the partition function on a spherical worldsheet is zero, which is a first important difference with the gravity analysis. The first non-zero term is the one-loop torus partition function, which can be computed exactly.

In our work we have constructed strongly curved models that realize the above two-dimensional cosmology and that give rise to a wave-function of the universe with finite norm. One can then normalize the wave-function and attempt to compute vacuum expectation values for observables. That is a doable project since one can marginally perturb the worldsheet conformal field theory, and derive the resulting one-loop partition function with respect to the coefficient of the marginal perturbation. That will allow to compute the vacuum expectation value of marginal perturbations.

We note also that we have crucially made use of a new part of string configuration space which is still poorly understood. It concerns string theory on backgrounds which are generalizations of manifolds. The analogue of standard geometric concepts on these spaces is yet to be developed, and in the context of cosmology, it is particularly interesting to see whether singularity theorems can possibly be evaded using these spaces. We have contributed to this direction of research by showing that one can patch local supergravity solutions together such as to construct a global T-fold that in no frame is equivalent to a geometric background [p41], but we are still far from understanding these classes of backgrounds in generality. In particular, one can imagine generalizing T-folds to G-folds, where G is the full symmetry group of string theory, which is huge. One may be able to define observables in G-folds as BRST invariants in closed string field theory.

The models we presented are useful playgrounds to test ideas about positive cosmological constant space-times in string theory. However, the backgrounds are not realistic. Some work has been done to show that these models can develop large dimensions consistently, but it remains a tough challenge to match these kinds of theoretical models to present day experimental cosmology.

4.3 Algebraic tools in gauge theory

In this section we gather some results in gauge theory that have in common that they were obtained using algebraic tools.

Meta-stability of vacua in two-dimensional gauge theory

From a topological argument describing gauge field configurations at infinity, it is clear that the number of stable vacua in pure two-dimensional gauge theory is equal to the order of the center of the gauge group. When one shows this using algebraic tools, one discovers a slightly finer structure. A choice of vacuum can be represented by a choice of Wilson loop at infinity, in a certain representation μ. The creation of a particle-anti-particle pair in the adjoint representation θ could render the vacuum unstable. It would be energetically favorable to create such a pair if there is any representation ν_i in the tensor product representation $\mu \otimes \theta$ such that $C(\nu_i) \geq C(\mu)$ where $C(\lambda)$ denotes the quadratic Casimir of the representation λ. Using this characterization, vacua μ that are stable under the creation of a single pair can be classified. We call these vacua meta-stable. It turns out to be an interesting problem in algebra to classify these representations. The full list of meta-stable vacua for a gauge group G are given by the level 1 weights of the corresponding affine Kac-Moody algebra [p10]. There are more such weights than the order of the center of the gauge group. That discrepancy between algebraic metastable vacua and the topological argument is resolved by noticing that more than one pair in the adjoint representation can be created. If one classifies the vacua that are stable under the creation of any number of pairs, then one can prove algebraically that those correspond to the miniscule weights of G. Their number is equal to the order of the center of G for all gauge groups G.

Non-commutative Wilson loops

We also worked on an aspect of non-commutative gauge theories [p13]. These theories are gauge theories on a space on which the coordinates do not commute. When we study such a theory on a torus, for particular strengths of the non-commutativity parameter θ, the theory becomes equivalent to an ordinary gauge theory. The rank of the ensuing gauge group of the commutative theory wildly depends on the precise fractional value of the non-commutativity parameter θ. Therefore, if one can identify observables that are smooth in θ, one can find continuity relations between observables in commutative gauge theories of very different rank, thus linking for instance small N and large N behaviour of gauge theories.

We investigated whether it is possible that open Wilson lines in non-commutative gauge theories depend smoothly on N. In order to do that we first explicitly proved how the open Wilson lines transform into closed Wilson lines in an ordinary commutative gauge theory. We then computed certain correlators of Wilson lines both in two-dimensional gauge theory, and in strongly coupled $\mathcal{N} = 4$ super-Yang-Mills theory, at large N. In these theories and that regime, we did find that the original open Wilson lines depend smoothly on θ. It remains very interesting to identify more precisely in how far one can exploit continuity in θ for specific observables to better understand commutative gauge theory dynamics.

Monopoles and affine Lie algebras

There is a close connection between algebra and supersymmetry preserving states in string theory. We worked out one beautiful example of this connection [p14]. We can study four-dimensional gauge theories in string theory, localized on $D3$-branes. When we separate the $D3$-branes, we Higgs the gauge theory. The W-bosons then become massive electrically charged states stretching between the $D3$-branes, while there exist also magnetic monopole solutions, corresponding to $D1$-branes stretching between the $D3$-branes. All of these states can preserve supersymmetry. When we study such supersymmetry preserving states, and in particular the states that have a charge that cannot be further divided, we find that they naturally correspond to the simple roots of the A-type Lie algebra. When we compactify the four-dimensional gauge theory on a circle, the corresponding algebra is the A-type affine algebra.

We were able to extend this analysis to include projections on string theory that correspond to a space-time reflection combined with an orientation reversal on the string worldsheet. When we put fixed four-dimensional orientifold planes into the configuration described above, certain supersymmetric states are projected out while others remain. We were able to show a full correspondence between the affine algebras, including the twisted ones, and the supersymmetric electric and magnetic states in the presence of all available choices of orientifold planes.

As a by-product of the analysis, we introduced a new type of projection on Chan-Paton factors in string theory corresponding to an outer automorphism of the gauge group. We also showed the existence of new magnetic monopole solutions to four-dimensional gauge theories compactified on a circle with twisted boundary conditions.

Integrable systems and supersymmetric four-dimensional gauge theory

In [p15] we showed how elliptic Calogero-Moser integrable models for a general root system arise from a symplectic quotient construction. Our interest in these integrable systems was kindled by their applicability in $\mathcal{N} = 1$ supersymmetric four-dimensional gauge theory. In particular, we argued that the elliptic Calogero-Moser system describes the superpotential of an $\mathcal{N} = 4$ supersymmetric gauge theory with general gauge group and deformed by three mass terms for the chiral adjoint scalars. We checked the proposal by identifying non-perturbative contributions to the superpotential as arising from magnetic monopoles in the gauge theory, and by using soft breaking of a soluble $\mathcal{N} = 2$ gauge theory to $\mathcal{N} = 1$.

4.4 Ramond-Ramond backgrounds in string theory

D-branes are non-perturbative states in string theory that allow us to easily engineer interesting gauge theories. Their backreaction on the space-time geometry includes non-trivial Ramond-Ramond forms. It has proven hard to study string theory on such Ramond-Ramond backgrounds since the corresponding vertex operators introduce cuts on the string worldsheet. A new formalism has been developed to make it more feasible to quantize strings on Ramond-Ramond backgrounds [15]. The formalism is manifestly covariant and supersymmetric, and it includes local couplings that code the Ramond-Ramond background fields. For example, it is useful in formulating string theory on $AdS_5 \times S^5$ with five-form Ramond-Ramond flux, which is the near-horizon limit of the backreacted geometry of $D3$-branes.

T-duality

In [p39] we derived T-duality from worldsheet string theory, in the formalism that includes Ramond-Ramond fluxes and fermion couplings. We were thus able to prove very efficiently T-duality in all of these backgrounds, non-perturbatively on the worldsheet and order by order in the string coupling constant. Our approach is efficient in deriving the T-duality rules, including all form couplings, as well as all fermion couplings. The formalism was later proven useful in analyzing fermionic T-duality [16], which was applied in $AdS_5 \times S^5$ to give insight into the duality between correlation functions and Wilson loop correlators.

Conformal current algebra

When studying string theory on $AdS_3 \times S^3 \times T^4$ with Ramond-Ramond flux, a central ingredient of the worldsheet theory is a sigma-model on the supergroup $PSU(1,1|2)$ whose maximal bosonic subgroup is $SL(2,\mathbb{R}) \times SU(2)$. The sigma-model is conformal with or without a Wess-Zumino term, i.e. with or without the presence of NSNS flux in space-time. To better understand the quantum integrability of the model, as well as the infinite dimensional symmetry algebra for string theory on AdS_3 with Ramond-Ramond flux, we investigated the worldsheet current algebra of supergroup sigma-models [p42].

When the supergroup has zero dual Coxeter number, we showed that a version of the Sugawara construction provides a holomorphic energy momentum tensor from a non-chiral current algebra. We thus gave a constructive proof of the conformality of these models.

We were also able to compute the current algebra as well as the two- and three-point functions to all orders in perturbation theory by using special algebraic properties of the supergroups. Moreover, we recovered the result by doing conformal perturbation theory around the Wess-Zumino-Witten point. Now we would like to investigate the consequences of these results for our original purposes : to better understand integrability of the models, of the coset models, and to construct the space-time Virasoro algebra in the AdS_3 case.

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Chapter 5

Perspectives

C'était la loi des séries en somme, martingale triste dont nous découvrions soudain le secret.

Champs d'honneur, Jean Rouaud

In the course of the text, I have indicated open problems that have guided me in my research. I summarized results that I obtained in the attempts to solve these problems in the preceeding chapters. The questions answered in my research have given rise to new concrete open problems that invite further investigation. And other, broader challenges are put to string theory that require our attention. Let's voyage from the detailed to the broader picture to show that the subjects of string theory, quantum gravity, gauge theory and conformal field theory continue to defy us.

Concrete topics

Let me list a few concrete projects that may be viewed as direct continuations of my research. Many more can be found both in the introductions and conclusions of my papers.

- A modular invariant regularization of the two-dimensional cigar conformal field theory torus partition function has to be attempted, by subtracting a modular invariant term that corresponds to the asymptotic spectrum of operators.
- The description of mirror symmetry for non-compact Gepner models, and the fact that the non-compact Gepner models differ subtly from toric geometries begs for a further clarification of the relation between non-compact Gepner models and Calabi-Yau geometry. We note that non-compact Gepner models sometimes allow for a weakly curved limit, in contrast to their compact counterparts.
- We would like to investigate further the way the tachyon condensation mechanism for bosonic Liouville boundary states is inherited by the supersymmetric theory, in order to clarify the embedding of the infrared duality of supersymmetric gauge theories in string theory. Likewise we want to explicitly compute chiral open string three- and four-point functions in these backgrounds to get a direct handle on the superpotential terms in these models. Moreover, we need

to get better control on the monodromies in boundary states in the full bulk conformal field theory moduli space.

- It would be interesting to compute the scattering of black holes in three-dimensional gravity including the backreaction of the two black holes, by computing the connection in the presence of two point-defects with black hole monodromy in the Chern-Simons formulation of gravity.
- We need to revisit the conformal field theory phenomenon of creation of branes and open string bound states on D-branes in the presence of NS5-branes from a Dirac-Born-Infeld space-time point of view.

Broader goals

We can enlarge our field of vision beyond these detailed problems and formulate a few more encompassing projects. For brevity's sake, let us concentrate on the rapidly developing field of holography and its application to gauge theories.

Recently, a solution to the spectrum of maximally supersymmetric, conformal and planar four-dimensional gauge theory has been conjectured. It is of great interest to prove the conjecture, via the systematic use of the quantum integrability of the theory, which must be proven both on the gauge theory and the string theory side of the correspondence. Moreover, the result begs for a generalization to less supersymmetric, non-conformal and lower-dimensional field theories. A particular direction in which to extend the result would be the AdS_3/CFT_2 correspondence. The dual gauge theory exhibits a non-trivial renormalization group flow in that case, and qualitatively new reasonings are necessary to understand the holographic dictionary. Given a solution to the spectrum, we can also ask to what extent we can prove holography.

Still broader problems related to gravitational and field theory holography are the following. Which field theories allow for a holographic dual ? Can one construct a highly curved dual to super quantum chromodynamics microscopically ? Do condensed matter experiments allow for a gravitational dual description, as is to some extent the case for the quark gluon plasma in the Relativistic Heavy Ion Collider? Can we enlarge the class of solutions to string theory to other backgrounds with Ramond-Ramond flux ?

It should be clear that some of these more ambitious goals are attainable, and that others seem to be hard nuts to crack at the present moment in time. Moreover, many other broader goals govern the evolution of string theory – we concentrated on some questions within the realm of AdS/CFT. Other subfields as for instance the non-perturbative definition of topological string theories via matrix models, the construction of viable cosmological models, the search for Standard Model embeddings and the distinct phenomenological signatures of various string theoretic scenarios, are also thriving.

It remains crucial to keep an eye on the developments of the many subdomains of string theory, and to puzzle together the bigger picture.

Conclusion

All these interesting rapid evolutions cannot distract us from those problems that are most central to our domain. We must attempt to find further phenomenological consequences of string theory, whether through the inspiration of independent phenomenological models, or via the direct measurement of typically stringy physics. For instance, it would be wonderful to have experimental confirmation of supersymmetry by the Large Hadron Collider, since it is such a central building block of superstring theory. In parallel, supersymmetry breaking mechanisms need to be understood better. We must also address the cosmological constant problem within string theory and attempt to understand whether it is a sensible question to pose. We have to find a non-perturbative definition of string theory. The possibility of alternative theories of quantum gravity is also to be investigated, perhaps by exploiting more algebraic approaches to geometry. These fundamental open problems motivate me to attack them head on, and by any realistic means at my disposal.

Appendix A

Curriculum Vitae

Name:	Troost Jan
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Current Position:

Chargé de Recherche CR1, CNRS,
Laboratoire de Physique Théorique, Ecole Normale Supérieure, Paris, France.

Education:

October 1996 - September 2000: graduate studies at the
Institute for Theoretical Physics, Vrije Universiteit Brussel, Belgium.
July 2000: PhD. in Sciences.
Title of PhD. thesis: Strings, links between gauge theory and gravity.
Advisor: Prof. Alexander Sevrin

September 1992 - June 1996: undergraduate studies at the
Departement of Physics, K.U. Leuven, Belgium.
June 1996 : Lic. in Physics.
Title of thesis: Non-linear σ-models in $N = (2, 2)$ superspace.
Advisors: Prof. Alexander Sevrin and Prof. Antoine Van Proeyen

Languages:

Native language: Dutch.
Second languages: French, English.

Assistantships, fellowships and awards:

- January 2009-now: Permanent CNRS researcher CR1 at the École Normale Supérieure in Paris, France.
- January 2005- December 2008: Permanent CNRS researcher CR2 at the École Normale Supérieure in Paris, France.
- April 2004: Classified first in the French Anual National competition (CNRS) for at least three permanent positions in the area of theoretical physics.
- 2004: Individual Marie-Curie Postdoctoral Fellowship funded by the European Commmunity, at Ecole Normale Supérieure, Paris, France.
- 2003-2004: Postdoctoral research associate (funded by CNRS), at Ecole Normale Supérieure, Paris, France.
- 2000-2003: Postdoctoral research associate (funded by DOE and NSF), at MIT, Cambridge, USA.
- 1996-2000: Research assistantship of the Fund for Scientific Research in Flanders (Aspirant FWO).
- 1997: Prize of the Belgian Physical Society (BNV) for best theoretical undergraduate thesis in the years of graduation of 1995 and 1996.
- 1996: Award for the best graduating physicist in the KULeuven class of 1996 (Kanunnik Desmedt Price).
- 1992-1996: Awards for excellence in study, K.U. Leuven (undergraduate degree obtained with highest honors and compliments of the jury).

Teaching experience:

- 2008-2009: Teaching of the "travaux dirigés" Quantum Field Theory (complementary to the course Quantum Field Theory taught by Adel Bilal), as part of the Masters (first year graduate, former DEA) courses, in greater Paris. Composition and grading of half the written exam.
- 2005-2006, 2006-2007, 2007-2008: Teaching of the "travaux dirigés" Quantum Field Theory (complementary to the course Quantum Field Theory taught by Costas Bachas), as part of the Masters (first year graduate, former DEA) courses, in greater Paris. Composition and grading of half the written exam.
- 2006-2007: Teaching of a course "Introduction to Superstring Theory", at first/second year graduate level, in the Doctoral School co-organized by Amsterdam, Brussels and Paris institutions.
- 2004-2005: Occasional substitute teaching (for Boris Pioline) of the "travaux dirigés" Quantum Field Theory, as part of the Masters (first year graduate, former DEA) courses, in greater Paris.

- 1999-2000: Teaching assistant for Analytical Mechanics for second year undergraduate students in mathematics, physics, and chemistry.
- 1998-1999: Assistant for lab of experimental physics for first year undergraduate students in the sciences.
- 2000: Quantum Field Theory for mathematicians, lecture at MIT.

Seminar / workshop / conference organizer:

- Co-organizer of the Paris-Amsterdam meeting on Holography and Black holes, 5 and 6 March 2007.
- Co-organizer of the String Seminar of Greater Paris (ENS-section), 2004-2005, 2005-2006, 2006-2007, 2007-2008, 2008-2009.
- Organizer of numerous informal seminars in the LPTENS during the years 2005-2006, 2006-2007, 2007-2008, 2008-2009.
- Organizer of the String Lunch Seminar at MIT, 2001-2003.
- Co-organizer of joint student seminar Brussels-Leuven 1996-2000.

Seminars:

Numerous invited seminars all over Europe and the USA amongst which:

- Amsterdam University (Netherlands), 2004, Black hole scattering
- University of California at Berkeley (California) 2001, The non-abelian Born-Infeld action
- Free University Brussels (Belgium) 2001, Orientifold planes, magnetic monopoles and affine algebras
- MIT, Cambridge (Massachussets) 2000, The non-abelian Born-Infeld action
- CERN (Switzerland) 2003, Black hole scattering
- University of Chicago (Illinois) 2002, Winding strings and Ads3 black holes
- Rutgers University, New Brunswick (New Jersey) 2002, Winding strings and Ads3 black holes
- London Triangular seminar, Queen Mary/King's College/Imperial College (UK) 2005, D-branes in N=2 Liouville theory and its mirror, 2008, Bound states in N=2 Liouville theory and deep-throat D-branes
- Catholic University of Louvain-la-Neuve (Belgium) 2006, Holography in string theory
- Rencontres Théoriciennes Paris (France) 2004, Black hole scattering
- Brown University Providence (Rhode Island) 2001, The non-abelian Born-Infeld action
- University of Connecticut Storrs (Connecticut) 2001, The non-abelian Born-Infeld action

- University of Wales, Swansea (UK) 2005, D-branes in N=2 Liouville theory and its mirror
- ICTP, Trieste (Italy) 2005, D-branes in N=2 Liouville theory and its mirror
- Utrecht University (Netherlands) 2004, Linear dilaton holography
- ETH, Zurich (Switzerland) 2004, D-branes in N=2 Liouville theory and its mirror

Participation and invited talks in workshops, schools, conferences

Amongst others:

- Participated in Strings 1997 conference, Amsterdam, 16-21 June, 1997
- Participated in Strings 1999 conference, Potsdam, 19 - 24 July 1999
- Participated in Strings 2002 conference, Cambridge, July 15-20 2002
- Participated in Strings 2004, Paris, June 28- July 2 2004
- Invited talk at Corfu Summer Conference, Constituents, Fundamental Forces and Symmetries of the Universe, September 20-26, 2005
- Participated in 23rd International Solvay Conference in Physics "The Quantum Structure of Space and Time", December 1-3, 2005
- Participated in Amsterdam summerworkshop on String theory, Amsterdam, July 3-12, 2006
- Participated in 35th Summer Institute ENS Paris, String theory and supersymmetric gauge theory, 8-19 August 2005
- Participated in 36th Summer Institute ENS Paris, Fundamental interactions and the structure of space-time, 14-25 August, 2006
- Participated in 30 years of Supergravity, Paris, 16-20 October 2006
- Participated in High energy physics in the LHC era, 13-17 November 2006
- Invited talk at the Solvay conference on "Gauge Theories, Strings and Geometry", Brussels, 9-11 May 2006
- Invited talk at the workshop on "String theory, quantization and the orbit method", Reims, 12 June 2007
- Participated in String 2007 conference, Madrid, 25-29 June 2007
- Participated in 37th Summer Institute ENS Paris, On black holes, black rings and modular forms, 13-24 August, 2007
- Pariticpated in A scientific day in honour of Nicolas Sourlas, ENS Paris, 6 June 2008
- Participated in Amsterdam summer workshop on String Theory, Amsterdam 7-11 July 2008.

Administrative tasks

- Administration of the scientific and other content of the LPTENS laboratory website 2007-now.
- Responsable for the PAI program Paris-Amsterdam, France-Netherlands bi-national grant for the years 2006 and 2007.
- Member of two European RTN/TMR networks 2004-2008, and collaborator on various ANR grants.

Vulgarisation of Science:

- The website at the URL: http://tena4.vub.ac.be/beyondstringtheory/ is aimed at a general audience and is meant to explain aspects of string theory to interested laymen with an elementary grasp of physical phenomena. The site draws many thousands of visitors every year, from over a hundred countries.

Training of students

- Director for the internship for the Masters course in Theoretical Physics (former DEA de Physique Théorique) for R. Gicquaud.
- Director for internships for students in the Masters in Physics and the Master in Mathematics course (M1).
- Acting director for the PhD thesis "Non-rational conformal field theories and applications in string theory" by Liguori Jego (student of Elias Kiritsis), 2005-2007.
- Acting director for the PhD thesis by Raphael Benichou (student of Costas Bachas), 2006-2009.
- Numerous research publications with scientists who were PhD students at the time of publication (e.g. with Dan Israel, Ari Pakman, Nikolaos Prezas, ...)

Publications in international journals with peer review

1. Waldemar Schulgin, Jan Troost, *Backreacted T-folds and non-geometric regions in configuration space,* JHEP 0812:098,2008, arXiv:0808.1345 [hep-th].

   Abstract. - We provide the backreaction of the T-fold doubly T-dual to a background with NSNS three-form flux on a three-torus. We extend the backreacted T-fold to include cases with a flux localized in one out of three directions. We analyze the resulting monodromy domain walls and vortices. In these backgrounds, we give an analysis of the action of T-duality on observables like charges and Wilson surfaces. We analyze arguments for the existence of regions in the configuration space of second quantized string theory that cannot be reduced to geometry. Finally, by allowing for space-dependent moduli, we find a supergravity solution which is a T-fold with hyperbolic monodromies.

2. Raphael Benichou, Jan Troost, *Bound states in N=2 Liouville theory with boundary and Deep throat D-branes,* JHEP 0807:125, 2008, arXiv:0805.4766 [hep-th].

   Abstract. - We exhibit bound states in the spectrum of non-compact D-branes in N=2 Liouville conformal field theory. We interpret these states in the study of D-branes in the near-horizon limit of Neveu-Schwarz five-branes spread on a topologically trivial circle. We match semi-classical di-electric and repulsion effects with exact conformal field theory results and describe the fate of D-branes hitting NS5-branes. We also show that the bound states can give rise to massless vector and hyper multiplets in a low-energy gauge theory on D-branes deep inside the throat.

3. Raphael Benichou, Giuseppe Policastro, Jan Troost, *T-duality in Ramond-Ramond backgrounds*, Phys.Lett.B661:192-195,2008, arXiv:0801.1785 [hep-th].

   Abstract. - Using the pure spinor formalism on the world-sheet, we derive the T-duality rules for all target space couplings in an efficient manner. The world-sheet path integral derivation is a proof of the equivalence of the T-dual Ramond-Ramond backgrounds which is valid non-perturbatively in the string length over the curvature radius and to all orders in perturbation theory in the string coupling.

4. Sujay K. Ashok , Raphael Benichou, Jan Troost, *Non-compact Gepner Models, Landau-Ginzburg Orbifolds and Mirror Symmetry*, JHEP 0801:050,2008, arXiv:0710.1990 [hep-th].

   Abstract. - We study non-compact Gepner models that preserve sixteen or eight supercharges in type II string theories. In particular, we develop an orbifolded Landau-Ginzburg description of these models analogous to the Landau-Ginzburg formulation of compact Gepner models. The Landau-Ginzburg description provides an easy and direct access to the geometry of the singularity associated to the non-compact Gepner models. Using these tools, we are able to give an intuitive account of the chiral rings of the models, and of the massless moduli in particular. By studying orbifolds of the singular linear dilaton models, we describe mirror pairs of non-compact Gepner models by suitably adapting the Greene-Plesser construction of mirror pairs for the compact case. For particular models, we take a large level, low curvature limit in which we can analyze corrections to a flat space orbifold approximation of the non-compact Gepner models. This gives rise to a counting of moduli which differs from the toric counting in a subtle way.

5. Costas Kounnas, Nicolaos Toumbas, Jan Troost, *A Wave-function for stringy universes*, JHEP 0708:018,2007, arXiv:0704.1996 [hep-th].

   Abstract. - We define a wave-function for string theory cosmological backgrounds. We give a prescription for computing its norm following an earlier analysis within general relativity. Under Euclidean continuation, the cosmologies we discuss in this paper are described in terms of compact parafermionic worldsheet systems. To define the wave-function we provide a T-fold description of the parafermionic conformal field theory, and of the corresponding string cosmology. In specific examples, we compute the norm of the wave-function and comment on its behavior as a function of moduli.

6. Sujay K. Ashok, Sameer Murthy, Jan Troost, *D-branes in Unoriented Non-critical Strings and Duality in SO(N) and Sp(N) Gauge Theories*, JHEP 0706:047,2007, hep-th/0703148.

Abstract. - We exhibit exact conformal field theory descriptions of SO(N) and Sp(N) pairs of Seiberg-dual gauge theories within string theory. The N=1 gauge theories with flavour are realized as low energy limits of the worldvolume theories on D-branes in unoriented non-critical superstring backgrounds. These unoriented backgrounds are obtained by constructing exact crosscap states in the SL(2,R)/U(1) coset conformal field theory using the modular bootstrap method. Seiberg duality is understood by studying the behaviour of the boundary and crosscap states under monodromy in the closed string parameter space.

7. Pierre Bieliavsky, Charles Jego, Jan Troost, *Open strings in Lie groups and associative products*, Nucl.Phys.B782:94,2007, hep-th/0610329.

   Abstract. - Firstly, we generalize a semi-classical limit of open strings on D-branes in group manifolds. The limit gives rise to rigid open strings, whose dynamics can efficiently be described in terms of a matrix algebra. Alternatively, the dynamics is coded in group theory coefficients whose properties are translated in a diagrammatical language. In the case of compact groups, it is a simplified version of rational boundary conformal field theories, while for non-compact groups, the construction gives rise to new associative products. Secondly, we argue that the intuitive formalism that we provide for the semi-classical limit, extends to the case of quantum groups. The associative product we construct in this way is directly related to the boundary vertex operator algebra of open strings on symmetry preserving branes in WZW models, and generalizations thereof, e.g. to non-compact groups. We treat the groups SU(2) and SL(2,R) explicitly. We also discuss the precise relation of the semi-classical open string dynamics to Berezin quantization and to star product theory.

8. Sameer Murthy, Jan Troost, *D-branes in non-critical superstrings and duality in N=1 gauge theories with flavor*, JHEP 0610:019,2006, hep-th/0606203.

   Abstract. - We study D-branes in flat space times a cigar, and extended in the cigar direction. Some of these branes are new. The branes realize flavor in the four dimensional N=1 gauge theories on the D-branes localized at the tip of the cigar. We study the analytic properties of the boundary conformal field theories on these branes with respect to their defining parameter and find non-trivial monodromies in this parameter. Through this approach, we gain a better understanding of the brane set-ups in ten dimensions involving wrapped NS5-branes. As one application, using the boundary conformal field theory description of the electric and magnetic D-branes, we can understand electric-magnetic (Seiberg) duality in N=1 SQCD microscopically in a string theoretic context.

9. Sujay K. Ashok, Jan Troost, *The Topological Cigar Observables*, JHEP 0608 : 067,2006, hep-th/0604020.

   Abstract. - We study the topologically twisted cigar, namely the SL(2,R)/U(1) superconformal field theory at arbitrary level, and find the BRST cohomology of the topologically twisted N=2 theory. We find a one to one correspondence between the spectrum of the twisted coset and singular vectors in the Wakimoto modules constructed over the SL(2,R) current algebra. The topological cigar cohomology is the crucial ingredient in calculating the closed string spectrum of topological strings on non-compact Gepner models.

10. Charles Jego, Jan Troost, *Notes on the Verlinde formula in non-rational conformal field theories*, Phys. Rev. D74:106002,2006, hep-th/0601085.

Abstract. - We review and extend evidence for the validity of a generalized Verlinde formula in particular non-rational conformal field theories. We identify a subset of representations of the chiral algebra in non-rational conformal field theories that give rise to an analogue of the relation between modular S-matrices and fusion coefficients in rational conformal field theories. To that end we review and extend the Cardy-type brane calculations in bosonic and supersymmetric Liouville theory (and its duals) as well as in the hyperbolic three-plane H3+. We analyze the three-point functions of Liouville theory and of H3+ in detail to directly identify the fusion coefficients from the operator product expansion.

11. Sujay K. Ashok, Sameer Murthy, Jan Troost, *Topological Cigar and the c=1 String : Open and Closed*, JHEP 0602 013, 2006, hep-th/0511239.

    Abstract. - We clarify some aspects of the map between the c=1 string theory at self-dual radius and the topologically twisted cigar at level one. We map the ZZ and FZZT D-branes in the c=1 string theory at self dual radius to the localized and extended branes in the topological theory on the cigar. We show that the open string spectrum on the branes in the two theories are in correspondence with each other, and their two point correlators are equal. We also find a representation of an extended N=2 algebra on the worldsheet which incorporates higher spin currents in terms of asymptotic variables on the cigar.

12. Sujay K. Ashok, Sameer Murthy, Jan Troost, *D-branes in non-critical superstrings and minimal super Yang-Mills in various dimensions*, Nucl. Phys. B749:172-205,2006, hep-th/0504079.

    Abstract. - We construct and analyze D-branes in superstring theories in even dimensions less than ten. The backgrounds under study are supersymmetric $R^{d-1,1} \times SL(2,R)_k/U(1)$ where the level of the supercoset is tuned such as to provide bona fide string theory backgrounds. We provide exact boundary states for D-branes that are localized at the tip of the cigar $SL(2,R)/U(1)$ supercoset conformal field theory. We analyze the spectra of open strings on these D-branes and show explicitly that they are consistent with supersymmetry in $d = 2, 4$ and 6. The low energy theory on the world-volume of the D-brane in each case is pure Yang-Mills theory with minimal supersymmetry. In the case with four macroscopic flat directions $d = 4$, we realize an $\mathcal{N} = 1$ super Yang-Mills theory. We interpret the backreaction for the dilaton as the running of the gauge coupling, and study the relation between R-symmetry breaking in the gauge theory and the backreaction on the RR axion.

13. Dan Israel, Ari Pakman, Jan Troost, *D-branes in Little String Theory*, Nucl. Phys. B722 3, 2005, hep-th/0502073.

    Abstract. - We analyze in detail the D-branes in the near-horizon limit of NS5-branes on a circle, the holographic dual of little string theory in a double scaling limit. We emphasize their geometry in the background of the NS5-branes and show the relation with D-branes in coset models. The exact one-point function giving the coupling of the closed string states with the D-branes and the spectrum of open strings is computed. Using these results, we analyze several aspects of Hanany-Witten setups, using exact CFT analysis. In particular we identify the open string spectrum on the D-branes stretched between NS5-branes which confirms the low-energy analysis in brane constructions, and that allows to go to higher energy scales. As an application we show the emergence of the beta-function of the N=2 gauge theory on D4-branes stretching between NS5-branes from the boundary states describing the D4-branes. We also speculate on the possibility of getting a matrix model description of little string theory from the effective theory on the D1-branes. By considering D3-branes orthogonal to the NS5-branes we find a CFT incarnation of the Hanany-Witten effect of anomalous creation of D-branes. Finally we give a brief description of some non-BPS D-branes.

14. B. Pioline and J. Troost, *Schwinger pair production in AdS2*, JHEP 0503 (2005) 043, hep-th/0501169.

    Abstract. - We analyze the pair production of charged particles in two-dimensional Anti-de Sitter space (AdS2) with a constant, uniform electric field. We compute the production rate both at a semi-classical level, viewing Schwinger pair production as a tunneling event, and at the full quantum level, by extracting the imaginary part of the one-loop amplitude. In contrast to the usual Schwinger pair production in flat space, pair production in AdS_2 requires a sufficiently large electric field $E^2 > M^2 + 1/4$ in order to overcome the confining effect of the AdS geometry – put in another way, the presence of an electric field E raises the Breitenlohner-Freedman bound to $M^2 > -1/4 + E^2$. For E greater than this threshold, the vacuum is unstable to production of charged pairs in the bulk. We expect our results to be helpful in constructing supersymmetric $AdS_2 X S^2$ perturbative string vacua, which enter in the near-horizon limit of extremal charged black holes. Although the generalized Breitenlohner-Freedman bound is obeyed in these cases, production of BPS particles at threshold is possible and relevant for AdS_2 fragmentation.

15. N. Toumbas, J. Troost, *A time-dependent brane in a cosmological background*, JHEP 0411 (2004) 032, hep-th/0410007.

    Abstract. - We study a moving D-brane in a time-dependent background. There is particle production both because of non-trivial cosmological evolution, and by closed string emission from the brane that gradually decelerates due to a gain in mass. The particular model under study is a D0-brane in an SL(2,R)/U(1) cosmology – the techniques used extend to other backgrounds.

16. D. Israel, A. Pakman and J. Troost, *D-branes in N=2 Liouville and its mirror*, Nucl. Phys. B710 (2005) 529, hep-th/0405259.

    Abstract. - We study D-branes in the mirror pair N=2 Liouville / supersymmetric SL(2,R)/U(1) coset superconformal field theories. We build D0, D1 and D2 branes, on the basis of the boundary state construction for the Euclidean AdS(3) conformal field theory. We also construct D0-branes in an orbifold that rotates the angular direction of the cigar. We show how the poles of correlators associated to localized states and bulk interactions naturally decouple in the one-point functions of localized and extended branes. We stress the role played in the analysis of D-brane spectra by primaries in SL(2,R)/U(1) which are descendents of the parent theory.

17. D. Israel, C. Kounnas, A. Pakman and J. Troost, *The partition function of the supersymmetric two-dimensional black hole and little string theory* , JHEP **0406** (2004) 033, hep-th/0403237.

    Abstract. - We compute the partition function of the supersymmetric two-dimensional Euclidean black hole geometry described by the SL(2,R)/U(1) superconformal field theory. We decompose the result in terms of characters of the N=2 superconformal symmetry. We point out puzzling sectors of states besides finding expected discrete and continuous contributions to the partition function. By adding an N=2 minimal model factor of the correct central charge and projecting on integral N=2 charges we compute the partition function of the background dual to little string theory in a double scaling limit. We show the precise correspondence between this theory and the background for NS5-branes on a circle, due to an exact description of the background as a null gauging of SL(2,R) x SU(2). Finally, we discuss the interplay between GSO projection and target space geometry.

18. D. Israel, A. Pakman and J. Troost, *Extended SL(2,R)/U(1) characters, or modular properties of a simple non-rational conformal field theory*, JHEP **0404** (2004) 045, hep-th/0402085.

    Abstract. - We define extended SL(2,R)/U(1) characters which include a sum over winding sectors. By embedding these characters into similarly extended characters of N=2 algebras, we show that they have nice modular transformation properties. We calculate the modular matrices of this simple but non-trivial non-rational conformal field theory explicitly . As a result, we show that discrete SL(2,R) representations mix with continuous SL(2,R) representations under modular transformations in the coset conformal field theory. We comment upon the significance of our results for a general theory of non-rational conformal field theories.

19. J. Troost, *A note on causality in the bulk and stability on the boundary*, Phys. Lett. **B578** (2004) 210, hep-th/0308044.

    Abstract. - By carefully analyzing the radial part of the wave-equation for a scalar field in AdS, we show that for a particular range of boundary conditions on the scalar field, the radial spectrum contains a bound state. Using the AdS/CFT correspondence, we interpret this peculiar phenomenon as being dual to an unstable double trace deformation of the boundary conformal field theory. We thus show how the bulk theory holographically detects whether a boundary perturbation is stable.

20. J. Troost and A. Tsuchiya, *Towards black hole scattering*, Phys. Lett. **B574** (2003) 301, hep-th/0307158.

    Abstract. - We study black holes in three-dimensional Chern-Simons gravity with a negative cosmological constant. In particular, we identify how the Chern-Simons interactions between a scattering particle and a black hole project the particle wavefunction onto a wavefunction in the black hole background. We also analyze the set of space-times that should be allowed in the theory and the way in which boundary conditions affect the spectrum of space-times.

21. J. Troost and A. Tsuchiya, *Three-dimensional black hole entropy*, JHEP **0306** (2003) 029, hep-th/0304211.

    Abstract. - We discuss in detail the properties of gravity with a negative cosmological constant as viewed in Cherns-Simons theory on a line times a disc. We reanalyze the problem of computing the BTZ entropy, and show how the demands of unitarity and modular invariance of the boundary conformal field theory severely constrain proposals in this framework.

22. Y. Satoh and J. Troost, *On time-dependent AdS/CFT*, JHEP **0301** (2003) 027, hep-th/0212089.

    Abstract. - We clarify aspects of the holographic AdS/CFT correspondence that are typical of Lorentzian signature, to lay the foundation for a treatment of time-dependent gravity and conformal field theory phenomena. We provide a derivation of bulk-to-boundary propagators associated to advanced, retarded and Feynman bulk propagators, and provide a better understanding of the boundary conditions satisfied by the bulk fields at the horizon. We interpret the subleading behavior of the wavefunctions in terms of specific vacuum expectation values, and compute two-point functions in our framework. We connect our bulk methods to the closed time path formalism in the boundary field theory.

23. Y. Satoh and J. Troost, *Massless BTZ Black Holes in minisuperspace*, JHEP **0209** (2002) 041, hep-th/0209195.

    Abstract. - We study aspects of the propagation of strings on BTZ black holes. After performing a careful analysis of the global spacetime structure of generic BTZ black holes, and its relation to the geometry of the SL(2,R) group manifold, we focus on the simplest case of the massless BTZ black hole. We study the SL(2,R) Wess-Zumino-Witten model in the worldsheet minisuperspace limit, taking into account special features associated to the Lorentzian signature of spacetime. We analyse the two- and three-point functions in the pointparticle limit. To lay bare the underlying group structure of the correlation functions, we derive new results on Clebsch-Gordan coefficients for SL(2,R) in a parabolic basis. We comment on the application of our results to string theory in singular time-dependent orbifolds, and to a Lorentzian version of the AdS/CFT correspondence.

24. J. Troost, *Winding strings and AdS3 black holes* , JHEP **0209** (2002) 041, hep-th/0206118.

    Abstract. - We start a systematic study of string theory in AdS3 black hole backgrounds. Firstly, we analyse in detail the geodesic structure of the BTZ black hole, including spacelike geodesics. Secondly, we study the spectrum for massive and massless scalar fields, paying particular attention to the connection between SL(2,R) subgroups, the theory of special functions and global properties of the BTZ black holes. We construct classical strings that wind the black holes. Finally, we apply the general formalism to the vacuum black hole background, and formulate the boundary spacetime Virasoro algebra in terms of worldsheet operators. We moreover establish the link between a proposal for a ghost free spectrum for SL(2,R) string propagation and the massless black hole background, thereby claryfing aspects of the AdS3/CFT correspondence.

25. A. Hanany, N. Prezas, J. Troost, *The partition function of the two-dimensional black hole conformal field theory*, JHEP **0204** (2002) 014, hep-th/0202129.

    Abstract. - We compute the partition function of the conformal field theory on the two-dimensional euclidean black hole background using path-integral techniques. We show that the resulting spectrum is consistent with the algebraic expectations for the SL(2,R)/U(1) coset conformal field theory construction. In particular, we find confirmation for the bound on the spin of the discrete representations and we determine the density of the continuous representations. We point out the relevance of the partition function to all string theory backgrounds that include an SL(2,R)/U(1) coset factor.

26. S. P. Kumar, J. Troost, *Geometric construction of elliptic integrable systems and N=1* superpotentials* , JHEP **0201** (2002) 020, hep-th/0112109.

    Abstract. - We show how the elliptic Calogero-Moser integrable systems arise from a symplectic quotient construction, generalising the construction for A(N-1) by Gorsky and Nekrasov to other algebras. This clarifies the role of (twisted) affine Kac-Moody algebras in elliptic Calogero-Moser systems and allows for a natural geometric construction of Lax operators for these systems. We elaborate on the connection of the associated Hamiltonians to superpotentials for N=1* deformations of N=4 supersymmetric gauge theory, and argue how non-perturbative physics generates the elliptic superpotentials. We also discuss the relevance of these systems and the associated quotient construction to open problems in string theory. In an appendix, we use the theory of orbit algebras to show the systematics behind the folding procedures for these integrable models.

27. A. Hanany, J. Troost, *Orientifold planes, affine algebras and magnetic monopoles*, JHEP **0108** (2001) 021, hep-th/0107153.

    Abstract. - We analyze string theory backgrounds that include different kinds of orientifold planes and map out a natural correspondence to (twisted) affine Kac-Moody algebras. The low-energy description of specific BPS states in these backgrounds leads to a construction of explicit twisted magnetic monopole solutions on R3 x S1. These backgrounds yield new low-energy field theories with twisted boundary conditions and the link with affine algebras yields a natural guess for the superpotentials of the corresponding pure N=1, and N=1* gauge theories.

28. Z. Guralnik, J. Troost, *Aspects of Gauge Theory on Commutative and Noncommutative Tori*, JHEP **0105** (2001) 022, hep-th/0103168.

    Abstract. - We study aspects of gauge theory on tori which are a consequences of Morita equivalence. In particular we study the behavior of gauge theory on noncommutative tori for arbitrarily close rational values of the noncommutativity parameter theta. For such values of theta, there are Morita equivalent descriptions in terms of Yang-Mills theories on commutative tori with very different magnetic fluxes and rank. In order for the correlators of open Wilson lines to depend smoothly on theta, the correlators of closed Wilson lines in the commutative Yang-Mills theory must satisfy strong constraints. If exactly satisfied, these constraints give relations between small and large N gauge theories. We verify that these constraints are obeyed at leading order in the 1/N expansion of pure 2-d QCD and of strongly coupled N=4 super Yang-Mills theory.

29. A. Sevrin, J. Troost, W. Troost, *The non-abelian Born-Infeld action at order F^6*, Nucl. Phys. **B603** (2001) 389, hep-th/0101192.

    Abstract. - To gain insight into the non-abelian Born-Infeld (NBI) action, we study coinciding D-branes wrapped on tori, and turn on magnetic fields on their worldvolume. We then compare predictions for the spectrum of open strings stretching between these D-branes, from perturbative string theory and from the effective NBI action. Under some plausible assumptions, we find corrections to the Str-prescription for the NBI action at order F^6. In the process we give a way to classify terms in the NBI action that can be written in terms of field strengths only, in terms of permutation group theory.

30. V. Kac, J. Troost, *The stability of vacua in two-dimensional gauge theory*, Phys. Lett. **B501** (2001) 313, hep-th/0010289.

    Abstract. - We discuss the stability of vacua in two-dimensional gauge theory for any simple, simply connected gauge group. Making use of the representation of a vacuum in terms of a Wilson line at infinity, we determine which vacua are stable against pair production of heavy matter in the adjoint of the gauge group.

By calculating correlators of Wilson loops, we reduce the problem to a problem in representation theory of Lie groups, that we solve in full generality.

31. F. Denef, A. Sevrin, J. Troost, *Non-Abelian Born-Infeld versus String Theory*, Nucl. Phys. **B581** (2000) 135, hep-th/0002180.

    Abstract. - Motivated by the results of Hashimoto and Taylor, we perform a detailed study of the mass spectrum of the non-abelian Born-Infeld theory, defined by the symmetrized trace prescription, on tori with constant magnetic fields turned on. Subsequently, we compare this for several cases to the mass spectrum of intersecting D-branes. Exact agreement is found in only two cases: BPS configurations on the four-torus and coinciding tilted branes. Finally we investigate the fluctuation dynamics of an arbitrarily wrapped Dp-brane with flux.

32. J. Troost, *Constant field strengths on* T^{2n}, Nucl. Phys. **B568** (2000) 180, hep-th/9909187.

    Abstract. - We analyse field strength configurations in U(N) Yang-Mills theory on T^{2n} that are diagonal and constant, extending early work of Van Baal on T^4. The spectrum of fluctuations is determined and the eigenfunctions are given explicitly in terms of theta functions on tori. We show the relevance of the analysis to higher dimensional D-branes and discuss applications of the results in string theory.

33. M. Massar, J. Troost, *The longitudinal fivebrane and tachyon condensation in matrix theory*, Nucl. Phys. **B569** (2000) 417, hep-th/9907128.

    Abstract. - We study a configuration in matrix theory carying longitudinal fivebrane charge, i.e. a D0-D4 bound state. We calculate the one-loop effective potential between a D0-D4 bound state and a D0-anti-D4 bound state and compare our results to a supergravity calculation. Next, we identify the tachyonic fluctuations in the D0-D4 and D0-anti-D4 system. We analyse classically the action for these tachyons and find solutions to the equations of motion corresponding to tachyon condensation.

34. J. Troost, *Higgsed antisymmetric tensors and topological defects*, Phys. Lett. **B458** (1999) 61, hep-th/9903159.

    Abstract. - We find topological defect solutions to the equations of motion of a generalised Higgs model with antisymmetric tensor fields. These solutions are direct higher dimensional analogues of the Nielsen-Olesen vortex solution for a gauge field in four dimensions.

35. M. Massar, J. Troost, *D0-D8-F1 in Massive IIA SUGRA*, Phys. Lett. **B458** (1999) 283, hep-th/9901136.

    Abstract. - We present new supersymmetric solutions of massive IIA supergravity involving D0-branes, D8-branes and strings. For the bosonic fields we use a general ansatz with SO(8) symmetry.

36. M. T. Grisaru, M. Massar, A. Sevrin, J. Troost, *The Quantum Geometry of N=(2,2) Non-Linear Sigma-Models*, Phys. Lett. **B412** (1997) 53, hep-th/9706218.

    Abstract. - We consider a general N=(2,2) non-linear sigma-model in (2,2) superspace. Depending on the details of the complex structures involved, an off-shell description can be given in terms of chiral, twisted chiral and semi-chiral superfields. Using superspace techniques, we derive the conditions the

potential has to satisfy in order to be ultra-violet finite at one loop. We pay particular attention to the effects due to the presence of semi-chiral superfields. A complete description of N=(2,2) strings follows from this.

37. A. Sevrin, J. Troost, *Off-Shell Formulation of N=2 Non-Linear Sigma-Models*, Nucl. Phys. **B492** (1997) 623, hep-th/9610102.

Abstract. - We study d=2, N=(2,2) non-linear sigma-models in (2,2) superspace. By analyzing the most general constraints on a superfield, we show that through an appropriate choice of coordinates, there are no other superfields than chiral, twisted chiral and semi-chiral ones. We study the resulting sigma-models and we speculate on the possibility that all (2,2) non-linear sigma-models can be described using these fields. We apply the results to two examples: the SU(2) x U(1) and the SU(2) x SU(2) WZW model. Pending upon the choice of complex structures, the former can be described in terms of either one semi-chiral multiplet or a chiral and a twisted chiral multiplet. The latter is formulated in terms of one semi-chiral and one twisted chiral multiplet. For both cases we obtain the potential explicitely.

Conference proceedings with peer review

1. J. Bogaerts, A. Sevrin, J. Troost, W. Troost, S. van der Loo, *D-branes and constant electro-magnetic backgrounds* , Proceedings of the workshop "The Quantum Structure of Spacetime and the Geometric Nature of Fundamental Interactions", Berlin, October 2000, hep-th/0101018.

Abstract. - We probe the effective worldvolume theory of a set of coinciding D-branes by switching on constant electro-magnetic fields on them. The comparison of the mass spectrum predicted by string theory with the mass spectrum obtained from the effective action provides insights in the structure of the effective theory.

2. M. T. Grisaru, M. Massar, A. Sevrin, J. Troost, *Some Aspects of N=(2,2), D=2 Supersymmetry*, Fortsch. Phys. **47** (1999) 301, hep-th/9801080.

Abstract. - The off-shell description of N=(2,2) supersymmetric non-linear sigma-models is reviewed. The conditions for ultra-violet finiteness are derived and T-duality is discussed in detail.

3. A. Sevrin, J. Troost, *The Geometry of Supersymmetric Sigma-Models*, Contribution to the proceedings of the workshop Gauge Theories, Applied Supersymmetry and Quantum Gravity, Imperial College, London, 1996, hep-th/9610103.

Abstract. - We review non-linear sigma-models with (2,1) and (2,2) supersymmetry. We focus on off-shell closure of the supersymmetry algebra and give a complete list of (2,2) superfields. We provide evidence to support the conjecture that all N=(2,2) non-linear sigma-models can be described by these fields. This in its turn leads to interesting consequences about the geometry of the target manifolds. One immediate corollary of this conjecture is the existence of a potential for hyper-Kahler manifolds, different from the Kahler potential, which does not only allow for the computation of the metric, but of the three fundamental two-forms as well. Several examples are provided: WZW models on SU(2) x U(1) and SU(2) x SU(2) and four-dimensional special hyper-Kahler manifolds.

Books and chapters of books

1. Constantin Bachas, Jan Troost, *Superstring theories*, Invited contribution to the Elsevier Encyclopedia of Mathematical Physics, physics/0605105.

   Abstract. - We briefly review superstring theories, highlighting the important concepts, developments, and open problems of the subject.

Invited contributions in journals without peer review

1. J. Troost, *Superstrings and co*, Physicalia Mag. **20** (1998) 371

   Abstract. - Key ingredients of super string theory are presented to make the concluding statements about the off-shell description of supersymmetric Wess-Zumino-Witten models more transparent. (Popularized summary of undergraduate thesis awarded with the BNV prize 1997, written for a general audience of physicists.)

In review

1. Sujay K. Ashok, Raphael Benichou, Jan Troost, *Conformal Current Algebra in Two Dimensions*, arXiv:0903.4277 [hep-th].

   Abstract. - We construct a non-chiral current algebra in two dimensions consistent with conformal invariance. We show that the conformal current algebra is realized in non-linear sigma-models on supergroup manifolds with vanishing dual Coxeter number, with or without a Wess-Zumino term. The current algebra is computed using two distinct methods. First we exploit special algebraic properties of supergroups to compute the exact two- and three-point functions of the currents and from them we infer the current algebra. The algebra is also calculated by using conformal perturbation theory about the Wess-Zumino-Witten point and resumming the perturbation series. We also prove that these models realize a non-chiral Kac-Moody algebra and construct an infinite set of commuting operators that is closed under the action of the Kac-Moody generators. The supergroup models that we consider include models with applications to statistical mechanics, condensed matter and string theory. In particular, our results may help to systematically solve and clarify the quantum integrability of PSU(n—n) models and their cosets, which appear prominently in string worldsheet models on anti-deSitter spaces.

Appendix B

A choice of five publications

Orientifold planes, affine algebras and magnetic monopoles

Amihay Hanany, Jan Troost

JHEP **0108:**021,2001.

The partition function of the supersymmetric two-dimensional black hole and little string theory

Dan Israel, Costas Kounnas, Ari Pakman, Jan Troost

JHEP **0406:**033,2004.

D-branes in non-critical superstrings and duality in N=1 gauge theories with flavor

Sameer Murthy, Jan Troost

JHEP **0610:**019,2006.

A wave-function for stringy universes

Costas Kounnas, Nicolaos Toumbas, Jan Troost

JHEP **0708:**018,2007.

Conformal current algebra in two dimensions

Sujay Ashok, Raphael Benichou, Jan Troost

arXiv:0903.4277 [hep-th].