CBPF-NF-010/10

Proton radioactivity: the case for ^{53m}Co proton-emitter isomer

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Abstract - The partial proton emission half-life for ^{53m}Co unstable isomer is re-examined in the framework of a semiempirical model based on tunneling through a Coulomb-plus-centrifugal-plus-overlapping potential barrier within the spherical nucleus approximation. It is shown that the known measured half-life value of 17 s is compatible with a large prolate shape for ^{53m}Co proton emitter and a high angular momentum $\ell = 11$ assigned to the proton transition to the ground-state of ^{52}Fe.

PACS: 23.50.+z

Keywords: proton radioactivity, ground-state and isomeric transitions, half-life systematics

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During the course of a systematic analysis of proton emission half-life data carried out recently by us [1] the case for ^{53m}Co proton emitter appeared quite intriguing for its measured partial proton emission half-life ($T^{\mathsf{e}}_{1/2_{\mathsf{p}}} = 17$ s [2]) resulted apparently fitted to the systematics, however differing from the calculated half-life value by a factor ~ 6. Afterwards, we realized that the reason for such a difference was that the methodology used throughout our analysis [1] does not apply to proton transitions of very high values of angular momentum ℓ, as is the case for the $19/2^-$ high-spin ^{53m}Co proton-emitter isomer [2, 3]. In the present note we show how our previous systematic analysis has been modified in such a way it can now be applied to proton transitions of any ℓ-values. By using only one adjustable parameter the new data analysis resulted, in addition, in a much better global presentation of the partial proton emission half-life data with 90% of the data being reproduced within a factor 4, therefore enabling reliable assignment of angular momentum ℓ for proton transitions to the ground-state of the daughter nucleus as well as information on the degree of deformation for the proton decaying state.

The methodology to systematize the proton emission half-life data obtained till recently for forty-four different neutron-defficient (proton-rich) parent nuclei (from seventy-one measurements comprising ground and isomeric states) has been reported in details in our previous publication [1]. The semiempirical model that has been used is based on Gamow's early ideas of quantum mechanical description of particle tunneling through a potential barrier, where a quite complete analogy to the alpha decay process has been clearly demonstrated. The Coulomb and centrifugal potential barriers are considered in the separation region from the contact configuration of the proton to be emitted with the surface of the daughter nucleus up to the classical, outer turning point. Besides, a narrow, overlapping barrier region is included where the proton drives away from the parent nucleus until the configuration at contact is reached. In addition, the effect of nuclear deformation can be clearly seen when the data are treated by a model in which nuclei are assumed to be spherical. In brief, if lengths are given in fm, masses in u, energies in MeV, and time in s, the half-life has been expressed as

$$\tau = \log_{10} T_{1/2_{\mathsf{p}}} = \tau_0 + \tau_1 + \tau_2 \,, \tag{1}$$

where

$$\tau_0 = -22 + \log_{10}\left[a\,(\mu_0/Q_{\mathsf{p}})^{1/2}\right] \tag{2}$$

is the term related to the frequency of assaults on the barrier,

$$\tau_1 = 0.19(c-a)g\,\sqrt{\mu_0 Q_{\mathsf{p}}}\,H(x,y) \tag{3}$$

is the contribution from the overlapping barrier region, and

$$\tau_2 = 0.27358027 Z_{\mathsf{D}} \left(\frac{\mu_0}{Q_{\mathsf{p}}}\right)^{1/2} \cdot F(x,y) \tag{4}$$

is the one corresponding to the external, separation barrier region. The contact configuration is defined at $c = R_{\mathsf{D}} + r_{\mathsf{p}}$ (R_{D} is the radius of the daughter nucleus, and $r_{\mathsf{p}} = 0.87 \pm 0.02$ fm is the value adopted for the proton radius), and $a = R_{\mathsf{P}} - r_{\mathsf{p}}$ is the difference between the radius of the parent nucleus and the proton radius. The quantity $c - a$ represents, therefore, the extension of the overlapping region, which shows itself very narrow (∼1.7 fm) in proton emission cases. The quantities μ_0 and Q_{p} are the usual reduced mass and Q-value, respectively, for the decaying system, both quantities being corrected for the nuclear screening caused by the surrounding electrons. Differently from our precedent study [1], in the present analysis the values of Q_{p} have been obtained from the measured proton kinetic energy, E_{p}, namely,

$$Q_{\mathsf{p}} = E_{\mathsf{p}} \left(1 + \frac{m_{\mathsf{p}}}{M_{\mathsf{D}}}\right) + S\,, \tag{5}$$

where $m_{\mathsf{p}} = 1.00727646676$ u is the proton mass, M_{D} is the nuclear mass of the daughter nucleus, and S is the electron screening correction as has been detailed in [1]. Z_{D} denotes the atomic number of the daughter nucleus, and g is the adjustable parameter of the model, the value of which should fall in the interval 0–2/3. Parameter g is related to the strength of the potential in the overlapping region, and its value is determined semiempirically from the proton decaying data. The functions $H(x,y)$ and $F(x,y)$ contain the dependence upon angular momentum, ℓ, and they are given by

$$H(x,y) = (x + 2y - 1)^{1/2} \tag{6}$$

$$F(x,y) = \frac{x^{1/2}}{2y} \cdot \ln \frac{x^{1/2} H(x,y) + x + y}{\sqrt{x + y^2}} + \arccos \left[\frac{1}{2}\left(1 - \frac{y-1}{\sqrt{x+y^2}}\right)\right]^{1/2} - \frac{H(x,y)}{2y}\,, \tag{7}$$

where the quantities x and y are defined as

$$x = \frac{20.9008\,\ell(\ell+1)}{\mu_0 c^2 Q_{\mathsf{p}}}\,, \quad y = \frac{1}{2}\frac{Z_{\mathsf{D}} e^2}{c Q_{\mathsf{p}}}\,, \quad e^2 = 1.4399652 \text{ MeV}\cdot\text{fm}. \tag{8}$$

The evaluation of the quantities μ_0, R_{P}, R_{D}, and r_{p} has been done in the same way as reported in [1]. Finally, the input data on E_{p}, ℓ, and $\tau^{\mathsf{e}} = \log_{10} T^{\mathsf{e}}_{1/2_{\mathsf{p}}}$ considered in the present systematics are those taken from the compilations of Refs. [1, 4, 5] where data on new proton emission cases observed during the last four years or so have been included (see table 1).

In order to separate completely the Coulomb (τ_2^{coul}) and centrifugal (τ_2^{cent}) contributions from each other in the external barrier region the "penetrability" function, $F(x,y)$, is written as

$$F(x,y) = F(0,y) + [F(x,y) - F(0,y)] \ , \tag{9}$$

and, after substitution into equation (4), gives

$$\tau_2 = \tau_2^{\mathsf{coul}} + \tau_2^{\mathsf{cent}} \ , \tag{10}$$

where

$$\tau_2^{\mathsf{coul}} = 0.27358027\, Z_{\mathsf{D}} \left(\frac{\mu_0}{Q_{\mathsf{p}}}\right)^{1/2} \cdot F(0,y) \quad \text{and} \tag{11}$$

$$\tau_2^{\mathsf{cent}} = 0.27358027\, Z_{\mathsf{D}} \left(\frac{\mu_0}{Q_{\mathsf{p}}}\right)^{1/2} \cdot \big[F(x,y) - F(0,y)\big] \ , \tag{12}$$

and

$$F(0,y) = \arccos\sqrt{u} - \sqrt{u(1-u)} \ , \quad u = (2y)^{-1} \ , \tag{13}$$

which expressions are valid for all values of $x \geq 0$ and $y > 1/2$. Next, we define a "reduced" half-life, τ_{r}, by subtracting from the half-life all its contributions other than the Coulomb one, i.e.,

$$\tau_{\mathsf{r}} = \tau - (\tau_0 + \tau_1 + \tau_2^{\mathsf{cent}}) \ . \tag{14}$$

This means that τ_{r} is expected proportional to Coulomb quantity $z = Z_{\mathsf{D}}(\mu_0/Q_{\mathsf{p}})^{1/2} \cdot F(0,y)$ and, therefore, one should also expect τ_{r} independent upon angular momentum ℓ.

The best value for the adjustable parameter of the model has been found by minimizing the quantity

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (\tau_i^{\mathsf{c}} - \tau_i^{\mathsf{e}})^2}{n-2}} \ , \tag{15}$$

where τ^{c} and τ^{e} refer to calculated and experimental partial proton decay half-life, respectively, and n is the number of measurements (the case for $^{53\mathsf{m}}$Co should not be considered for the moment). Calculations have indicated that a unique g-value thus obtained ($g \approx 0.14$) was unable to fit the proton decay data altogether with a sufficient degree of reproducibility since it resulted in a somewhat large value for σ, namely, $\sigma \approx 0.522$. Therefore, as has been done in our previous analysis [1] and in the methodology developed by Delion *et al.* [4], the data have been separated into two groups of proton emitter nuclides according to the degree of quadrupole deformation of the parent nuclei (see table 1). This procedure allows one to understand in a quantitative way the influence of nuclear deformation on proton radioactivity. Working in this way we obtained $g = 0.38$ for the group of measurements associated to the largely prolate ($\delta \gtrsim 0.1$) deformed proton emitter nuclides, and $g = 0$ for those of $\delta \lesssim 0.1$, i.e. spherical and nearly spherical and/or oblate shaped nuclides. Such a procedure leads to a much better standard deviation ($\sigma = 0.396$), with 95% of the data being reproduced within a factor 5. Only in two cases (^{144}Tm and $^{177\mathrm{m}}$Tl proton emitters) the calculated and measured half-life values differed from each other more significantly (a factor 8 and 15, respectively).

The overall results of the present systematic analysis are summarized in Fig. 1 where the "reduced" half-life values, τ_r (equation (14)), are plotted against the Coulomb quantity z. As expected, the "reduced" experimental half-life values, $\tau_\mathrm{r}^\mathrm{e}$ (points) are seen well fitted to the unique straight line $\tau_\mathrm{r}^\mathrm{c} = 0.27358z$ irrespective of the ℓ-values (cf. equation (11)).

As concerns the case for $^{53\mathrm{m}}$Co high spin proton emitter isomer we search for the best ℓ-value to be assigned to its proton transition in order to better reproduce the measured half-life value following the present methodology. Calculations have been done for ℓ-values equal to 9, 10, 11, and 12 combined with the two values of g (0 and 0.38) (small circles in Fig. 1). We verified a quite good agreement (within a factor ~ 1.8 only) between calculated and measured half-life values if $\ell = 11$ (instead of the currently adopted $\ell = 9$) is assigned to the transition $^{53\mathrm{m}}\mathrm{Co} \rightarrow {}^{52\mathrm{g}}\mathrm{Fe} + \mathrm{p}$, and the value $g = 0.38$ is the adopted one, as it is clearly shown in Fig. 1. This means that, contrarily to what has been reported in [1], the present analysis indicates that $^{53\mathrm{m}}$Co very probably belongs to the group of largely prolate shaped proton emitter nuclides, otherwise its measured partial proton emission half-life of 17 s would lead, by assuming either $\ell = 9$ or $\ell = 11$, to a "reduced" half-life of 9.4 and 4.9, respectively, in complete disagreement with the predicted value $\tau_\mathrm{r}^\mathrm{c} = 3.5$ (see small open circles in Fig. 1). Another point to remark is that for $\ell = 9$ and the assumption of $g = 0$ (a non-prolate shape for $^{53\mathrm{m}}$Co) the present analysis leads to a partial proton emission half-life of 19 μs, which value is in sharp contradiction with the measured total half-life of 247 ± 12 ms found for $^{53\mathrm{m}}$Co unstable isomer [2]. On the contrary, the assumption of $g = 0.38$ (largely prolate shape) together with the assignment of $\ell = 11$ gives $T^\mathrm{c}_{1/2_\mathrm{p}} = 9.4$ s, i.e. a factor only 1.8 times less than the measured value $T^\mathrm{e}_{1/2_\mathrm{p}} = 17$ s.

Finally, for the sake of comparison, figure 2 shows the potential barriers for two extreme (in half-life values) cases of proton transitions, namely $^{53\mathrm{m}}$Co (part a) and ^{145}Tm (part b). In the former case the centrifugal barrier largely predominates over the Coulomb one, the total potential energy amounting to a value as high as ~ 92 MeV for $\ell = 11$. In the case for ^{145}Tm (figure 2-b) the value $\ell = 5$ makes the Coulomb barrier greater than the centrifugal one, and the relatively low total potential barrier leads to a half-life of a few microseconds for this proton transition.

To conclude, a detailed systematic analysis of all measured partial proton emission half-lives known to date for proton-rich nuclei has been developed in the framework of a semiempirical, one-parameter, quantum mechanical tunneling treatment. The goodness of the fitting procedure to the available proton emission data enabled us to propose the value $\ell = 11$ to be assigned to the proton transition early observed for ^{53m}Co isomer. This ℓ-value is completely compatible with the measured, partial proton emission half-life of 17 s for ^{53m}Co, and this isomer seems likely to exhibit a large prolate quadrupole deformation.

Table 1 – Number of measurements, cases, and nuclear shape and energy state of the parent nuclei in proton transitions to ground-state of the daughter nuclei considered in the present systematic study.

		Number of cases					
		Nuclear shape[a]			Energy state		
Values of ℓ	Number of measurements	large prolate ($\delta \gtrsim 0.1$)	other ($\delta \lesssim 0.1$)	Total	Ground	Isomeric	Total
0	9	1	7	8	6	2	8
2	28	7	7	14	11	3	14
3	5	3	0	3	3	0	3
4	1	1	0	1	0	1	1
5	32	2	16	18	7	11	18
Totals	75	14	30	44	27	17	44

[a] This is defined by the degree of nuclear deformation as given by $\delta = 0.757\beta_2 + 0.273\beta_2^2$, where β_2 is the parameter of quadrupole deformation as tabulated in [6].

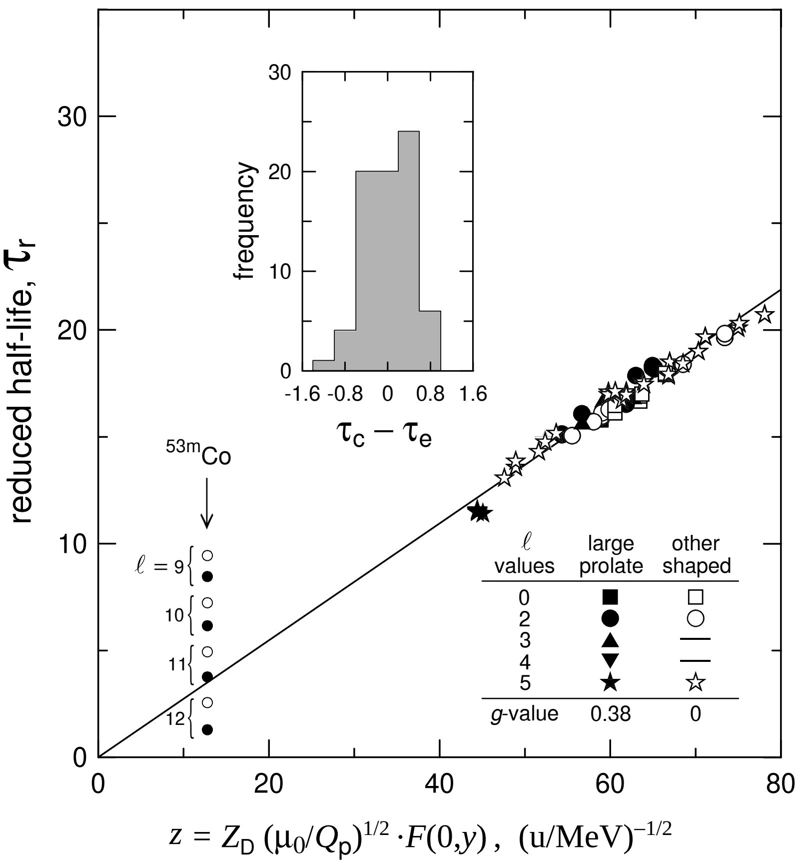

Figure 1: Reduced half-life, τ_{r}, as defined by equation (14) plotted against Coulomb parameter $z = Z_{\mathsf{D}}(\mu_0/Q_{\mathsf{p}})^{1/2} \cdot F(0, y)$. Experimental reduced half-lives (points) are obtained from the measured half-life values compiled in [1, 4, 5]. Different symbols refer to different ℓ-values assigned to the proton transitions and to the best g-values found in the present analysis as indicated. The straight line $\tau_{\mathsf{r}}^{\mathsf{c}} = 0.27358z$ represents the expected trend according to the present calculation model. The data for $^{53\mathsf{m}}$Co unstable isomer ($z = 12.8$) are represented by small circles (open ones correspond to $g = 0$, and full ones to $g = 0.38$). We remark a quite good agreement (ratio of experimental to calculated values amounting to only 1.8) found for $^{53\mathsf{m}}$Co proton emitter isomer when it is supposed to be a largely prolate shaped nuclide and the value $\ell = 11$ is assigned to its proton transition to the ground-state of ^{52}Fe. The inset shows the distribution of the deviation $\Delta\tau = \tau^{\mathsf{c}} - \tau^{\mathsf{e}} = \tau_{\mathsf{r}}^{\mathsf{c}} - \tau_{\mathsf{r}}^{\mathsf{e}}$ where 95% of the experimental data differ from the calculated ones by a factor less than 5.

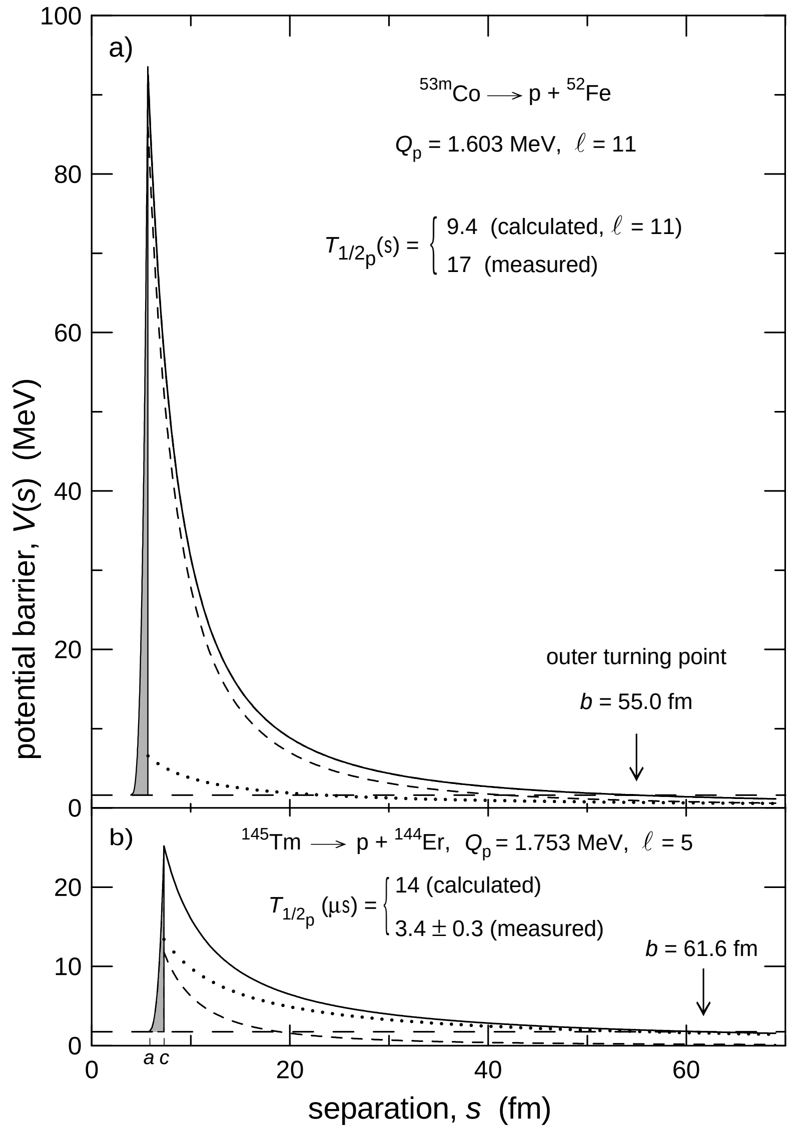

Figure 2: Potential barriers are depicted for proton emission from ^{53m}Co (part a)) and ^{145}Tm (part b)) parent nuclei for comparison. In both cases the dotted line represents the Coulomb potential, short-dashed the centrifugal barrier, full line the Coulomb-plus-centrifugal potential, and long-dashed line the Q_p-value for decay. The overlapping region a–c is emphasized by the shaded area, and the separation region is defined in the interval c–b, where $b = cy[1+(1+x/y^2)^{1/2}]$ is the outer turning point (see text). Note the remarkable differences between each other case. A small deviation (by a factor 1.8 only) between measured and calculated half-life values is obtained by assigning $\ell = 11$ to the proton transition ^{53m}Co $\rightarrow$ ^{52g}Fe.

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