ACCOUNT FOR UNCERTAINTIES OF CONTROL MEASUREMENTS IN THE ASSESSMENT OF DESIGN MARGIN FACTORS.

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ABSTRACT

The paper discusses the feasibility of accounting for uncertainties of control measurements in estimation of design margin factors. The feasibility is also taken into consideration proceeding from the fact how much the processed measured data were corrected by a priori calculated data of measurable parameters. The possibility and feasibility of such data correction is demonstrated by the authors with the help of Bayes theorem famous in mathematical statistics

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The following margin factor is used in design work

$$K_{eng,1}^{calc}(x) = 1 + \frac{\Delta^{calc}(x)}{x}. \quad (1)$$

This formula does not take into account the way of controlling parameter «x» when measurement is made in the course of operation of fuel loading. This approach can lead to the necessity of reducing the power in the course of operation, which is shown in the profile of Figure 1. The profile is plotted with the assumption that the measured and calculated data are independent.

As reflected by Figure 1, the following condition is not satisfied:

$$x^{meas} \le x^{\lim} - \Delta^{meas} \quad (2)$$

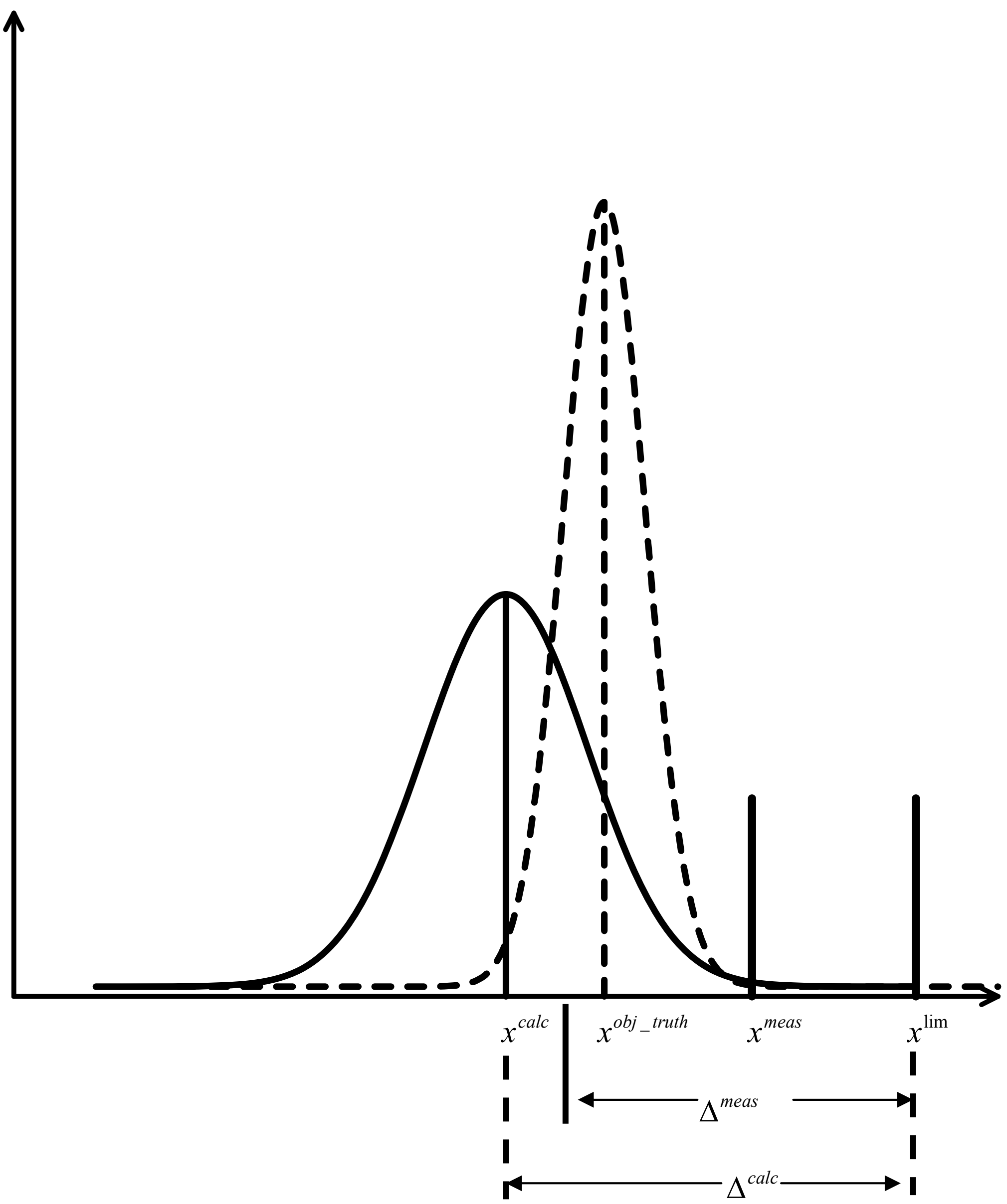

Figure 1

$x^{\lim}$ - the operational limit;
x^{calc} - the calculated value;
x^{obj_truth} - the truth value of parameter;
x^{meas} - the measured value of parameter;
Δ^{calc} - the calculated error;
Δ^{meas} - the measured error.

In order to avoid such condition in design, it is required, to introduce in addition to $K_{eng,1}^{calc}(x)$ another coefficient:

$$K^{calc}_{eng,2}(x) = 1 + \frac{\Delta^{meas}(x)}{x} + \sqrt{\left(\frac{\Delta^{calc}(x)}{x}\right)^2 + \left(\frac{\Delta^{meas}(x)}{x}\right)^2} . \quad (3)$$

Normally, $K^{calc}_{eng,2}(x) > K^{calc}_{eng,1}(x)$, however, $K^{calc}_{eng,2}(x)$ is not introduced. Why do we seldom reduce power after measurement is made?

The matter is the following. Measured data are corrected by most of ICIS systems using preliminary calculated data for these parameters. The following equation is realized in many of ICIS systems:

$$x^{meas} = \alpha \cdot x^{obs} + (1-\alpha) \cdot x^{calc} , \quad (4)$$

where x^{obs} - observed value, $0 < \alpha < 1$.

By the way, equation (4) can be justified using Bayes' theorem famouse in mathematical statistics. The theorem says about the probability of the hypothesis after testing data become available /see the Attachment/.

In case (3), condition (2) will be satisfied (with a rated probability), if $K^{calc}_{eng,2}(x)$ is determine the following way:

$$K^{calc}_{eng,2}(x) = 1 + \frac{\Delta^{meas}(x)}{x} + \alpha \cdot \sqrt{\left(\frac{\Delta^{calc}(x)}{x}\right)^2 + \left(\frac{\Delta^{meas}(x)}{x}\right)^2} . \quad (5)$$

Should the calculated data of magnitude x^{meas}, such as $q^{meas}_l = K^{meas}_V \cdot K^{calc}_k$, be used in calculation of x^{meas}, then

$$K^{calc}_{eng,2}(q_l) = 1 + \frac{\Delta^{meas}(q_l)}{q_l} + \alpha \cdot \sqrt{\left(\frac{\Delta^{calc}(K_V)}{K_V}\right)^2 + \left(\frac{\Delta^{meas}(K_V)}{K_V}\right)^2} . \quad (6)$$

If equation (4) is not clearly distinguished, but there exists a relationship between x^{meas} and x^{calc}, it is possible to determine a coefficient of correlation between those magnitudes $\rho(x^{calc}, x^{meas})$ and use $K^{calc}_{eng,2}(x)$ in the following form

$$K^{calc}_{eng,2}(x) = 1 + \frac{\Delta^{meas}(x)}{x} + \sqrt{\left(\frac{\Delta^{calc}(x)}{x}\right)^2 + \left(\frac{\Delta^{meas}(x)}{x}\right)^2 - 2 \cdot \left(\frac{\Delta^{calc}(x)}{x}\right) \cdot \left(\frac{\Delta^{meas}(x)}{x}\right) \cdot \rho(x^{calc}, x^{meas})}$$

Identically to (6), we have the following equation for q_l

$$K^{calc}_{eng,2}(q_l) = 1 + \frac{\Delta^{meas}(q_l)}{q_l} + \sqrt{\left(\frac{\Delta^{calc}(K_V)}{K_V}\right)^2 + \left(\frac{\Delta^{meas}(K_V)}{K_V}\right)^2 - 2\cdot\left(\frac{\Delta^{calc}(K_V)}{K_V}\right)\cdot\left(\frac{\Delta^{meas}(x)}{x}\right)\cdot\rho(K_V^{calc}, K_V^{meas})}$$

Numerical exercises for obtaining the assessments of $K^{calc}_{eng,1}(q_l)$ and $K^{calc}_{eng,2}(q_l)$ for VVER-1000 prove that those functionals are compared with $\alpha \simeq 0,5$ or $\rho(K_V^{calc}, K_V^{meas}) \simeq 0,7$.

The above considerations and numerical assessments try only to make a qualitative assessment. The decisive role in feasibility and scope of accounting measurement uncertainties in $K^{calc}_{eng}(x)$ is played by economics. It is important know what is more efficient from economical point of view: to ensure at-nominal-power operation due to a certain rise of fuel contribution to the price, or reduce fuel contribution to the price by providing a possibility of low-power operation at the beginning of cycle.

REFERENCES

1. Пустыльник Е.И., *Статистические методы анализа и обработки наблюдений*, «Наука», Москва, 1968

2. Гмурман В.Е.. *Теория вероятностей и математическая статистика*. Высшая школа. Москва, 2001

ATTACHMENT

The formulas presented in this attachment will prove that, as per Bayes’s statistics, if for any random variable x there was it’s a priory value x^{calc} with dispersion $(\sigma^{calc}(x))^2$, and a single measured data of this variable was obtained x^{obs} with dispersion $(\sigma^{obs}(x))^2$, then, the most probable its genuine value will be the following

$$x = \frac{x^{calc}(\sigma^{obs}(x))^2 + x^{obs}(\sigma^{calc}(x))^2}{(\sigma^{calc}(x))^2 + (\sigma^{obs}(x))^2}.$$

Let us assume that a methods for calculation of parameter x has been developed. Such methods has certain uncertainties, and the calculated x can differ from genuine x^t. Let the difference between these magnitudes be characterised by normal distribution law with dispersion $\sigma^{calc}(x)$, e.g. the density of x^t distribution will be expressed by formula

$$P(x^t) = \frac{e^{-\frac{(x^t - x^{calc})^2}{2(\sigma^{calc})^2}}}{\sigma^{calc}\sqrt{2\pi}}.$$

Let us assume that a single measurement of x has been conducted, where $x=A$. Moreover, we are aware that measured data differ from genuine values of x^t, and the differences are distributed according to normal law with dispersion $(\sigma^{obs}(x))^2$. E.e. if x^t is known, the density of probability of detecting A will be the following

$$P_{x^t}(A) = \frac{e^{-\frac{(x^t - A)^2}{2(\sigma^{obs})^2}}}{\sigma^{obs}\sqrt{2\pi}}.$$

$P_{x^t}(A)$ is conventional probability of detecting A on condition that the genuine value is x^t .

Now, let us determine conventional probability $P_A(x^t)$ for obtaining x^t on condition that $x=A$ was registered as measured data. In order to obtain such probability, let us define the probability of joint event as $P(x^t, A)$: x^t will be a genuine value, while measured datum was $x=A$.

This probability can be expressed through conventional probabilities

$$P(x^t, A) = P(x^t) \cdot P_{x^t}(A) = P(A) \cdot P_A(x^t),$$

From this formula, $P_A(x^t) = \frac{P(x^t) \cdot P_{x^t}(A)}{P(A)}$; however, *P(A)* characterizes the probability of A measured datum with any value of x^t , therefore, *P(A)* can be expressed as

$$P(A) = \int d(x^t) P(x^t) P_{x^t}(A).$$

By making the following substitutions: $P(x^t) = \frac{e^{-\frac{(x^t - x^{calc})^2}{2(\sigma^{calc})^2}}}{\sigma^{calc}\sqrt{2\pi}}$ and $P_{x^t}(A) = \frac{e^{-\frac{(x^t - A)^2}{2(\sigma^{obs})^2}}}{\sigma^{obs}\sqrt{2\pi}}$, be obtain

$$P_A(x^t) = -\frac{e^{-\frac{\left[x^t - \frac{x^c(\sigma^{obs})^2 + A(\sigma^c)^2}{(\sigma^{obs})^2 + (\sigma^c)^2}\right]^2}{2\frac{(\sigma^{obs})^2 \cdot (\sigma^{calc})^2}{(\sigma^{obs})^2 + (\sigma^{calc})^2}}}}{\sqrt{2\pi \frac{(\sigma^{obs})^2 \cdot (\sigma^{calc})^2}{(\sigma^{obs})^2 + (\sigma^{calc})^2}}}.$$

This newly obtained normal distribution of x^t is characterized by a new mean value and dispersion.

Thus, the mean value of x^t out of its probable genuine values will be the following

$$\bar{x}^t = \frac{x^{calc}(\sigma^{obs})^2 + A(\sigma^{calc})^2}{(\sigma^{obs})^2 + (\sigma^{calc})^2}$$

It is suggested to adopt this value as a final measured datum.