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Hadronic Vacuum Polarization Contribution to g-2 from the Lattice

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Abstract

We give a short description of the present situation of lattice QCD simulations. We then focus on the computation of the anomalous magnetic moment of the muon using lattice techniques. We demonstrate that by employing improved observables for the muon anomalous magnetic moment, a significant reduction of the lattice error can be obtained. This provides a promising scenario that the accuracy of lattice calculations can match the experimental errors.

Keywords:

1. introduction

The interaction between quarks at large distances becomes strong such that analytical methods as perturbation theory fail to analyze QCD. A method to nevertheless tackle the problem is to formulate QCD on a 4-dimensional, euclidean space-time grid. This setup first of all allows for a rigorous definition of QCD and leads to fundamental theoretical and conceptual investigations. On the other hand, the lattice approach enables theorists to perform large scale numerical simulations.

In the past, lattice physicists had to work with a number of limitations when performing numerical simulations which turn out to be extremely expensive, leading to the need for Petaflop computing and even beyond, a regime of computing power we just reach today. Therefore, for a long time the sea quarks were treated as infinitely heavy, indeed a crude approximation given that the up and down quarks have masses of only O(MeV). In a next step, only the lightest quark doublet, the up and down quarks, were taken into consideration, although their mass values as used in the simulation had been unphysically large.

Nowadays, besides the up and down quarks, also the strange quark is included in the simulations. In addition, these simulations are performed in almost physical conditions, having the quark masses close to their physical values, large lattices with about 3fm linear extent and small values of the lattice spacing such that a controlled continuum limit can be performed. The situation of present days simulation landscape is illustrated in fig. 1, taken from [1]. In the figure, the black cross indicates the physical point.

The fact that presently simulations close to the physical situation can be performed is due to three main developments: *i*) algorithmic breakthroughs which gave a substantial factor $O(> 10)$ of improvement; *ii*) machine development with a computing power of the present BG/P systems which is even outperforming Moore's law, *iii*) conceptual developments, such as the use of improved actions which reduce lattice artefacts and the development of non-perturbative renormalization.

As an example of physical results we can achieve presently, we show in fig. 2 the continuum extrapolated strange baryon spectrum as obtained by the European Twisted Mass Collaboration (ETMC) [2] of which the

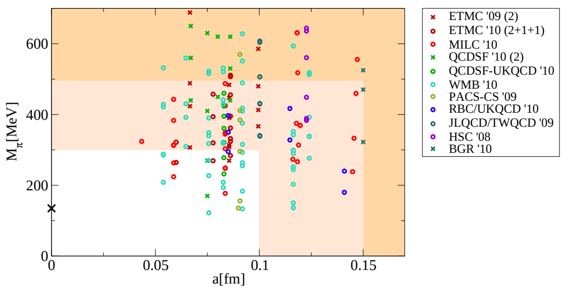

Figure 1: The values of the lattice spacing a and pion masses m_π as employed presently in typical QCD simulations by various collaborations as listed in the legend. The cross denotes the physical point. The figure is taken from [1] where also further details and references to the various collaborations can be found

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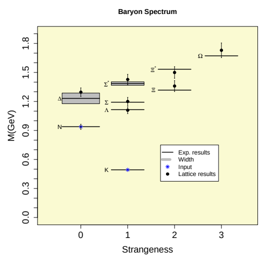

Figure 2: The continuum strange baryon spectrum from the ETM collaboration [2] using $N_f = 2$ flavours of quarks with only mass-degenerate up and down quark masses.

authors are members. The first complete calculation of the baryon spectrum was achieved by the BMW collaboration [3] and nowadays a number of lattice groups are providing calculations of the hadron masses and nucleon structure, see e.g. [4] for a recent review. The baryon spectrum calculation has been considered a benchmark study for lattice QCD for a long time. It is therefore very reassuring that finally this important result can be obtained precisely from ab-initio and non-perturbative lattice simulations.

2. The anomalous magnetic moment of the muon

The progress of lattice calculations discussed in the previous chapter motivates to address more demanding quantities than the baryon masses. A prime example is the anomalous magnetic moment of the muon $a_\mu \equiv (g_\mu - 2)/2$. The reason for looking at this quantity is that here theory and experiment disagree and one finds $a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{th}} = 2.90(91) \times 10^{-9}$ which leads to a larger than 3σ level discrepancy.

Clearly, this is a very interesting result. It means that either in the theoretical calculation something has been neglected or has not properly been included. Or, somewhat much more exciting, the discrepancy points to a breakdown of the standard model of particle interactions and the inconsistency stems from effects of some yet unknown new physics beyond the standard model.

Indeed, calculations show that these new physics effects would lead to a correction to the anomalous magnetic moment of size

$$\delta(a_l^{\mathrm{newphysics}}) = m_{\mathrm{lepton}}^2 / M_{\mathrm{newphysics}}^2 \,. \tag{1}$$

Here m_{lepton} is the mass of one of the leptons and $M_{\mathrm{newphysics}}$ represents the mass (or scale) of a particle originating from the (unknown) new physics beyond the standard model. The formula in eq. (1) shows that in the case of the muon anomalous magnetic moment the effect of new physics would show up about $(m_\mu/m_e)^2 \approx 4 \cdot 10^4$ times stronger than in the case of the electron. In principle, the τ-lepton would be even more suitable to detect these new physics effects, but unfortunately due to the very short lifetime of the τ lepton the experimental measurements of the anomalous magnetic moment of the τ are presently much too imprecise to unveil a possible new physics contribution. This leaves us then with the muon anomalous magnetic moment as the ideal place to look for new physics and indeed a large number of works has been devoted to explore this possibility, see [5].

3. When the lattice enters the game

It has been found that electromagnetic and weak interaction effects can by far not serve as an explanation of this discrepancy [5]. However, the strong interaction can have a large effect since the hadronic contributions a_μ^{had} dominate the uncertainty of the standard model value. The problem is that the strong interactions of quarks and gluons in QCD are intrinsically of non-perturbative nature. Taking these contributions into account by perturbation theory is therefore rather doubtful. Employing additional model assumptions to estimate a_μ^{had} will not provide a fully controlled and reliable calculation of the hadronic contributions and hence an unambiguous and stringent test whether the standard model is correct or must be extended by some new physics cannot be performed.

It is exactly at this point where lattice field theory methods applied to quantum chromodynamics can help – at least in principle. In lattice QCD the theory is formulated on a discrete 4-dimensional euclidean space-time lattice and the theory is approached by means of

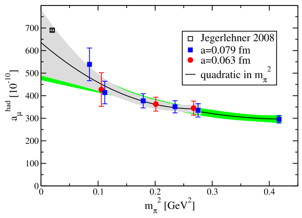

Figure 3: We show an earlier result for the hadronic contribution to the muon anomalous magnetic moment computed by the authors.

numerical simulations. It goes beyond the scope of this article to explain the mathematical concepts of lattice QCD but such numerical simulations allow then to compute physical quantities in a fully non-perturbative fashion without relying on any model assumptions or approximations.

Of course, the discretization itself induces a systematic error which must be removed by making the lattices finer and finer until the continuum of space time points is recovered by some suitable extrapolation process, a procedure which is called the *continuum limit*. In addition, the simulations necessarily demand a finite number of lattice points which can lead to *finite size effects* when the lattice is not large enough in physical units. Finally, often the simulations need to be performed at values of hadron masses that are larger than the ones observed in nature. The reason is that for smaller and smaller hadron masses the computational costs increase rapidly such that one is restricted to values of, say, pion masses that are a factor of about two larger than the ones observed in nature.

All these systematic effects that appear in lattice simulations need to be controlled in a quantitative way. For example, to reach physical values of the pion masses, an extrapolation to the physical point where the pion mass assumes its physical value needs to be performed. This appeared to be very problematic in the past. This is illustrated in fig. 3. The figure shows that the lattice results for $a_\mu^{\rm had}$ are significantly below the experimental number. An extrapolation to the physical point, reconciling the lattice data with experiment becomes in this situation very difficult and even needs some additional model assumptions. This all leads to an error of $a_\mu^{\rm had}$ as obtained from lattice simulations that is about a factor of 10 larger than the phenomenological one [6]. The lattice community have been therefore rather sceptical in the past that lattice QCD can provide a significant contribution to our understanding of the discrepancy in $g_\mu - 2$.

One additional suspicion has been that so-called disconnected (singlet) contributions could be substantial. In all existing lattice calculations these contributions were neglected, however. The reason is simply that these contributions are very noisy and therefore hard to compute reliably. Nevertheless, in ref. [7] a dedicated effort has been undertaken to calculate for the first time these contributions. As a result, it could be established that the dis-connected contributions are in fact small and can be safely neglected. In addition, also the effects of non-zero values of the lattice spacing and the finite volume turned out to be small. Thus the difficulty to reconcile lattice data with the experimental result, shown in fig. 3, is rather puzzling.

A resolution of this puzzle was only given this year by us [7]. We observed that by a suitable redefinition of the lattice observable needed to compute $a_\mu^{\rm had}$ a much smoother and much better controlled approach to the physical point can be achieved.

To illustrate the idea, let us give the definition of $a_\mu^{\rm had}$,

$$a_\mu^{\rm had} = \alpha^2 \int_0^\infty dQ^2 \frac{1}{Q^2} \omega(r) \Pi_R(Q^2) \; . \qquad (2)$$

Here α is the electromagnetic coupling and $\Pi_R(Q^2)$ the renormalized vacuum polarization function, $\Pi_R(Q^2) = \Pi(Q^2) - \Pi(0)$. The functional form of $\omega(r)$ is analytically known and the argument r is given by $r = Q^2/m_\mu^2$ where m_μ denotes the mass of the muon and Q a generic momentum. The key observation is now that on the lattice there is a large freedom to choose a definition of r. The only requirement is that in the limit of reaching a physical pion mass the continuum definition of $r = Q^2/m_\mu^2$ is recovered. Hence, one may define

$$r_{\rm latt} = Q^2 \cdot \frac{H^{\rm phys}}{H} \qquad (3)$$

with possible choices for H

$$\begin{aligned} r_1: &\quad H = 1 &\quad H^{\rm phys} &= 1/m_\mu^2 \\ r_2: &\quad H = m_V^2(m_{\rm PS}) &\quad H^{\rm phys} &= m_\rho^2/m_\mu^2 \\ r_3: &\quad H = f_V^2(m_{\rm PS}) &\quad H^{\rm phys} &= f_\rho^2/m_\mu^2 \; . \end{aligned} \qquad (4)$$

Here, $m_V(m_{\rm PS})$ is the mass of the ρ-meson and $f_V(m_{\rm PS})$ the ρ-meson decay constant as determined on the lattice at unphysical pion masses $m_{\rm PS}$. Furthermore, m_ρ and f_ρ denote the corresponding ρ-meson mass and decay constant at the physical point. All the definitions in eqs. (4) guarantee that indeed the desired definition of r is recovered in the limit of a physical pion mass since then

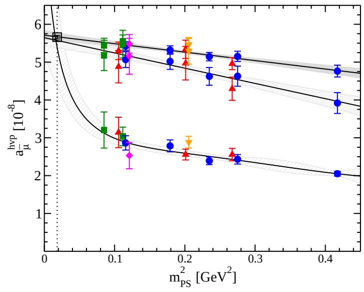

Figure 4: The present day status of computing $a_\mu^{\rm had}$ is represented using the improved observables discussed in the text. The curves correspond to the definitions r_1, r_2 and r_3 in eqs. (4) from bottom to top.

by definition $m_V(m_{\rm PS})$ and $f_V(m_{\rm PS})$ assume their physical values. In fig. 4 we show the results for $a_\mu^{\rm had}$ for all three definitions of r. Clearly, for the definitions r_2 and r_3 the behaviour of the lattice data towards physical pion masses is simply linear and allows for a controlled extrapolation to the physical point. As a result, one finds using definition r_2 in eq. (4) values from the lattice computations and experiment

$$\begin{aligned} a_{\mu,N_f=2}^{\rm had,latt} &= 5.72\,(16)\cdot 10^{-8} \\ a_{\mu,N_f=2}^{\rm had,exp} &= 5.66\,(5)\cdot 10^{-8}\ . \end{aligned} \qquad (5)$$

In the equations above, the index $N_f = 2$ indicates that in the lattice QCD calculations only a mass-degenerate pair of up and down quarks were used. Since the strange and charm quark flavours were neglected the simulations do not correspond to a fully physical situation which also leads to some ambiguity in the experimental extraction of $a_{\mu,N_f=2}^{\rm had,exp}$. This shortcoming needs to be overcome in the future.

4. Conclusion

In conclusion, using the modified and improved definitions of $a_\mu^{\rm had}$ on the lattice it is not only possible to recover the experimental result. As the comparison in eq. (5) shows it is now also possible to come significantly closer to the experimental accuracy. The idea of the improved observables which led to a much reduced error for an important quantity such as $g_\mu - 2$ has therefore led to the promising situation that lattice QCD calculations can match the experimentally obtained errors.

As already mentioned above, the results for $a_\mu^{\rm had}$ here discussed have been achieved for the case of two mass-degenerate quark flavours. What is needed in the future is the inclusion of the strange and the charm quarks to allow for a direct comparison to the experimental results. In addition, newly planned experiments at Fermilab [8] and J-PARC [9] are aiming at an accuracy of below 0.5% for the hadronic contribution to the muon anomalous magnetic moment. To match this accuracy dedicated lattice simulations have to be performed on large enough volumes and as close as possible to the physical point. In addition, explicit effects of isospin breaking and electromagnetism might need to be included. All this is in principle reachable within lattice QCD but, it constitutes a real challenge for the lattice community. An even larger challenge are contributions to $g_\mu - 2$ that appear at higher order of the electromagnetic coupling, most notably the so-called light-by-light contributions.

However, a number of lattice groups [7, 10, 11] are working on this problem presently and it can be expected that the lattice will provide a significant contribution to answer the question, whether the observed discrepancy in the muon anomalous magnetic moment is indeed a sign of new physics.

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