SUSY in Processes with Flavour Violation

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I herewith declare that work presented here is my own, except where otherwise indicated, under the careful supervision of my adviser.

Čestne prehlasujem, že predkladanú prácu som vypracoval samostatne, pod vedením školiteľa a s použitím uvedenej literatúry.

Peter Maták

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Abstract

I this work we present our first results of the calculation of the branching ratio for rare B_s^0 meson di-muon decay. High energy physicists studied flavour changing processes in past decades very intensively. The reason is large sensitivity of such a processes on the contributions of the beyond Standard Model theories, where the amplitudes of flavour changing processes could be enhanced up to several orders by the new particle content. Most of their contributions come from extended higgs sector. As an example of such a theory and we could say the most favourite one, is the Minimal Supersymmetric Standard Model (MSSM). We choose the $B_s^0 \to \mu^+\mu^-$ decay because of its experimental accessibility at LHC. Observation of this process will probably be one of the first signals of new particle physics. We started in our work with short introduction to the idea of supersymmetry, including its motivation in particle physics. Then, in the second chapter, we present the proper calculation of the decay amplitude and branching ration. In all calculations we used $\bar{\text{M}}$S-renormalization scheme.

Contents

Chapter 1

Introduction

In 1970's, the development of quantum field theory as a unification of special relativity and quantum physics together with large progress in experimental high energy physics helped to formulate a successful theory of elementary particles - Standard Model (SM). Quarks and charged leptons are in this theory represented by Dirac fermion fields, with left- and right-handed components entering the Lagrangian density in different ways. The basic renormalizability providing feature of the Standard Model is the gauge symmetry related to group $SU(3)_C \times SU(2)_L \times U(1)_Y$, which is at the scale of about 100 GeV broken spontaneously down to $SU(3)_C \times U(1)_{em}$ due to the presence of a Higgs field. With the spontaneous breakdown of the electroweak part of the gauge symmetry group a mechanism emerges that explains the origin of mass for all fermions as well as vector bosons $W^{\pm}, Z^0$. While masses of the gauge bosons come from the kinetic part of the Higgs doublet, masses of quarks and leptons arise as a direct consequence of the Higgs's nonzero vacuum expectation value in the Yukawa terms. These terms represent the interaction of the scalar field with left- and right-handed parts of the fermion. Standard Model extended to massive neutrinos is in very good agreement with all existing experimental data in accessible energy range $\sim$ 1 TeV and it seems there is no reason to be unsatisfied with it. In spite of that we don't regard the SM as a definite theory of everything but gravity, but only as an effective theory valid below some unknown physical cutoff scale Λ. We believe that approaching this scale in experiment will start discovering the new - beyond Standard Model physics. Whereas in the SM there is no reference to gravity[1], a natural candidate for such Λ would be the Planck scale, $M_P \sim 10^{19}$ GeV. However, now we have a new puzzle connected with the scalar Higgs field. Its scalar potential is of the form

$$V(H, H^\dagger) = m^2 H^\dagger H + \frac{\lambda}{2}(H^\dagger H)^2 \tag{1.1}$$

After electroweak symmetry breakdown due to $m^2 < 0$ we get the Higgs doublet

$$H = \begin{pmatrix} v + \frac{1}{\sqrt{2}}(h^0 + iG^0) \\ G^- \end{pmatrix}. \tag{1.2}$$

The requirement on the minimum of the potential leads to the condition $m^2 = -\lambda v^2$ and we get $\sqrt{2\lambda}v$ for the h^0 mass. However, at one-loop level if we draw the self-energy contributing diagrams for this field (see figure 1.1), we find that due to the Yukawa couplings present in the Lagrangian density, each fermion contributes by huge corrections proportional to Λ^2. However, such mass corrections are not acceptable, because from the measurement of the $M_{W^\pm}, M_{Z^0}$ masses we know that $v \sim 170$ GeV and on the ground of perturbative unitarity the coupling constant $\lambda \lesssim 1$ [11]. Hence the mass of the higgs particle should be of the order of $M_{weak} \sim 100$ GeV. In scientific literature two alternative attitudes to solving this problem are mentioned [12].

[1]Attempts to quantum general relativity as a gauge theory failed on it's nonrenormalizability

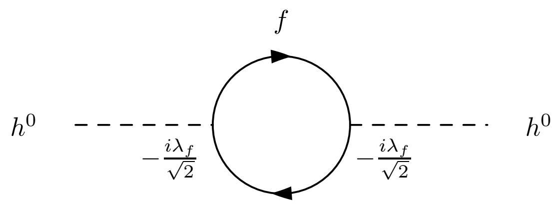


Figure 1.1: Contribution of fermion f to higgs's self-energy [10]

The first one chooses a low physical cutoff scale $\Lambda \sim 1$ TeV, at which the structure of the Higgs boson as a composite particle should emerge. Such an approach is known as technicolor. At present it is disfavoured by the magnitude of flavour-changing neutral interactions [12]. The second attitude is based on the cancelation of contributions diverging in cutoff quadratically with the help of some new symmetry. Motivation for this is the experience in history of building the elements of the Standard Model. For example, considering the positron, or antiparticles in general, helped to protect the self-energies of particles against such Λ^2-terms [10],[11]. In the case of quantum electrodynamics, this cancelation is the consequence of chiral symmetry which is a symmetry of massless QED [10].

1.1 Why Supersymmetry?

Supersymmetry is considered to be a representative of the second approach mentioned above, nevertheless, historically, it appeared to be useful for this purpose only accidentally. Supersymmetric field theories were studied by several mathematical physicists[2] as interesting mathematical objects, based on the generalization of standard Lie algebras of symmetries to graded Lie superalgebra, which besides standard bosonic generators contains also fermionic

[2] Y.A.Golfand, E.P.Likhtman (1971), D.V.Volkov, V.P.Akulov (1972) and J.Wess, B.Zumino (1973)

ones. Structure constants of fermionic generators in such superalgebra are given by a set of its anti-commutators [13]. Under fermionic generators we usually mean Q_α and $\bar{Q}^{\dot{\alpha}}$ that are being transformed under $SL(2,\mathbb{C})$ as two-component Weyl spinors. The dot distinguishes between the two two-dimensional representations of $SL(2,\mathbb{C})$[3]. If such an operator acts on the bosonic state, the resulting state gets spinor transofmation properties and turns into a fermionic state. In the second case, by contraction of spinor indices of Q or $\bar{Q}$ with those of a fermionic state, we get a state with transfomation properties of a bosonic state.

$$Q_\alpha|\text{boson}\rangle = |\text{fermion}\rangle_\alpha \qquad Q^\alpha|\text{fermion}\rangle_\alpha = |\text{boson}\rangle. \tag{1.3}$$

The Haag, Łopuszański and Sohnius extension of the Coleman - Mandula theorem states that if some natural conditions are met (which are satisfied in most realistic theories) then $Q, \bar{Q}$ are invariant under lorentz translations and therefore commute with P_μ as well as the square mass operator $-P_\mu P^\mu$. Considering supersymmetry as an exact symmetry in nature unambiguously leads to adjusting particles into pairs of fermions and bosons with the same mass, so-called supermultiplets [10],[4]. Every fermion of the Standard Model would get its scalar supersymmetric partner, both with the same gauge interactions [4]. From the structure of the lagrangian density needed for supersymmetry to be exact [4],[6] it follows that there are contributions to the higgs's self-energy drawn in the figure 1.2,[10]. The first diagram represents a term proportional to Λ^2, while the second one only contains the logarithm of the cut-off. In case of suitable adjustment of coupling constants $\lambda_{\tilde{f}}$ and λ_f (which is guaranteed by supersymmetry) both quadratically diverging contributions coming from bosonic and fermionic loops cancel each other. This is an example of the cancelation due to symmetry mentioned already in the

[3]For notation see [12], Appendix A

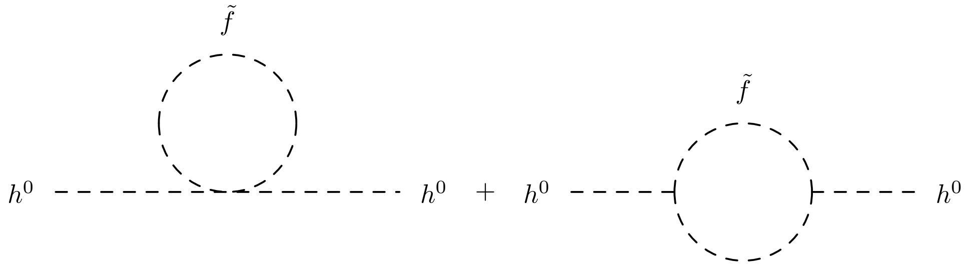


Figure 1.2: Diagrams with $\tilde{f}$ contributing to higgs's self-energy

previous section. Moreover, the quadratically divergent terms will remain absent in all orders of the loop expansion. The reason for this consists in the fact that in renormalizable theories all contributions to fermion self-energy diverge at most logarithmically, which in case of unbroken supersymmetry implies the same for their scalar partners[4][4]. The only disturbing detail resides in the fact that no supersymmetric partners are observed in nature. Thus, supersymmetry has to be broken.

1.2 Hints for physics beyond SM: Why $B_s^0 \to \mu^+\mu^-$?

From the birth of supersymmetry high energy physicists searching for beyond Standard Model physics in various experiments have been trying very hard to find the proof of its existence. A sure proof of such verification would be a discovery of a supersymmetric particle, especially the lightest one, which should be stable if R-parity is conserved[5]. The best candidate for such a particle in the Minimal Supersymmetric Standard Model (MSSM) is the

[4] as well as for higgs boson, which fermionic partner is called higgsino

[5] R-parity is a multiplicative quantum number introduced in MSSM, which is equal $+1$ for particles and -1 for their supersymmetric partners

lightest neutralino mass eigenstate, which corresponds to a mixture of $\tilde{B}^0$ and neutral higgsino [4]. However, no such particle has been found so far. In the absence of a direct supersymmetric signature, from the point of view of experimental accessibility at colliders like the Tevatron or LHC, flavour changing processes have become very interesting. Their advantage is their large sensitivity on new physics. This is related to very small flavour-changing amplitudes in the SM. The Standard Model extensions, as for example the Two-Higgs Doublet Model (2HDM), or Minimal Supersymmetric Standard Model, include new sources of flavour violation, and could increase the rates of these processes strongly (for example [1],[14],[7]). First of all, in such models the amplitude could be enlarged at tree level due to new neutral higgs couplings to down type quarks. Even if these tree level contributions are excluded by experiment one could still consider the 2HDM model already mentioned: the loops in which the charged higgs components play role similar to $W^{\pm}$ still remain and will increase the decay amplitude [1]. In this work we are interested in the MSSM. In the MSSM, the chargino, gluino and u-type squarks appear in loops that generate a change in flavour of the external d-type quark legs. Moreover, in these theories one of the higgs doublets couples to the d-quarks and leptons, while the second one to the u-quarks, and as a result the decay amplitudes involving the higgses depend strongly on the ratio of the higgs vacuum expectation values $v_u/v_d = \tan\beta$. In grand unification theories based on group $SO(10)$, large $\tan\beta \approx \frac{m_t}{m_b}$ is favoured, which allows to increase the Flavour Changing Neutral Current (FCNC) amplitudes by several orders relative to the SM predictions. Good examples are weak B meson decays. In case of a di-muon B_s^0 meson decay the Standard Model branching ratio is equal $\mathcal{B}(B_s^0 \to \mu^+\mu^-) = (3,1 \pm 1,4) \times 10^{-9}$ [2]. However, in the MSSM this decay rate could be enhanced by several orders. So far the latest experimental constrain for this ratio from the DØ detector at Tevatron is $\mathcal{B}(B_s^0 \to \mu^+\mu^-) < 4,3 \times 10^{-8}$ [3], and there still remains room for observing this decay rate above the SM level due to new physics. At the LHC, where the central mass collision energy of the protons will reach the value $\sqrt{s} = 14$ TeV large quantities of B_s^0 mesons will be produced. After observing the muon pair in the detector and calculating its central mass energy, the rare B_s^0 decay could by very easily identified, which allows to establish the precise value of its branching ratio. On the other side, the present non-evidence of such process gives important constrains on parameter spaces of models like MSSM, and that is the reason, why FCNC processes are so intensively studied by theorists.

Chapter 2

Calculation of the amplitude for $B_s^0 \to \mu^+\mu^-$

In following chapter we present our first results of 1-loop analytic calculation of decay amplitude for di-moun B_s^0 decay in the MSSM. In this work we used the notation of [4] for the chiral and gauge supermultiplets, which seemed to us the most standard one. In the Standard Model there are 22 penguin and 4 box diagrams for $B_s^0 \to \mu^+\mu^-$ decay. The body of penguins can contain only neutral particle connected with the loop on one and pair of muons on the other side. The only neutral bosonic particles in the Standard Model are γ, Z^0, h^0 or G^0. It was already noticed in [1] that photon penguin diagrams are zero, because of the summation of the overall momentum with γ_μ between muon external legs - vector current conservation. In order to simplify our analysis in the MSSM, we choose concrete values of the parameters in so-called large $\tan\beta$ region, which is favoured by unification theories ([5],p.25). This choice of parameters allows us to select only diagrams enhanced by higher powers of $\tan\beta$. There are several papers ([1],[2],[7]) analysing this process analytically, but using the effective field theory in which sparticles

v	$\tan\beta$	M_2	μ	m_{A^0}
174 GeV	$50 \propto \frac{m_t}{m_b}$	600 GeV	110 GeV	200 GeV
m_{Q_1}	m_{Q_3}	m_{u_1}	m_{u_3}	A
500 GeV	300 GeV	500 GeV	200 GeV	-300 GeV

are heavier than everything else in the diagrams. this approximation is not good any more, because the new experimental data forced us to take heavy higgs bosons into account. In the MSSM there are 74 1-loop diagrams[1] for $B_s^0 \to \mu^+\mu^-$ and in 13 of them sparticles are involved. According to [1], the largest contribution to decay the amplitude in the large $\tan\beta$ region is obtainned from neutral Higgs boson penguin diagrams. In all four diagrams exist two places, where flavour can by changed. First of all, dominant contribution to decay amplitude comes from external legs represented by $Z_{32}^{1/2}$ (Figure 2.1,(a),(b)), which corresponds to the 1-loop self-energy diagrams for d-type quarks (See 2.2). Another possibility for change of flavour are loops contributing to the vertex of $\bar{b}, s$ and neutral Higgs (Figure 2.1,(c),(d)). Our calculations showed that this effects are approximately 10 times smaller than those with d-quark self-energy flavour changing on external legs.

2.1 d-quark self-energy

For our set of parameters and above all for large $\tan\beta$, the dominant part of the d-type quark self-energy comes from the diagrams 2.2 [1],[2],[8]. In our case with CKM - matrix being still the only source of flavour violation[2] the gluino diagram should be negligible with respect to contribution of the

[1]This result was obtainned by generating 1-loop diagrams in the FeynArts package for Mathematica software

[2]Squark mass matrices are flavour diagonal.

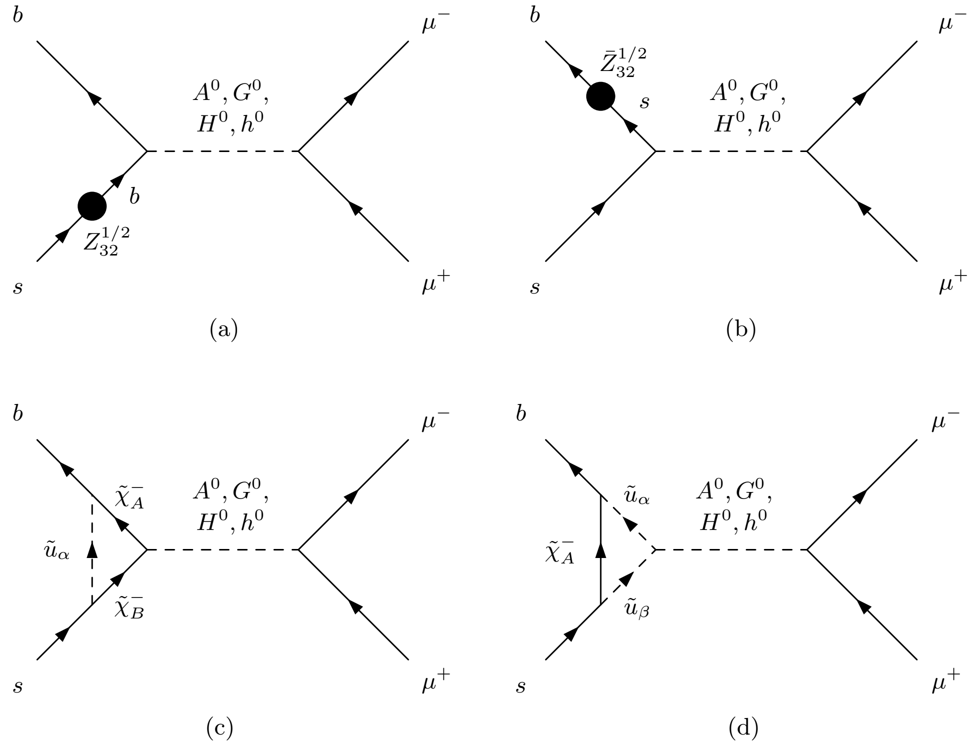


Figure 2.1: Higgs penguin diagrams

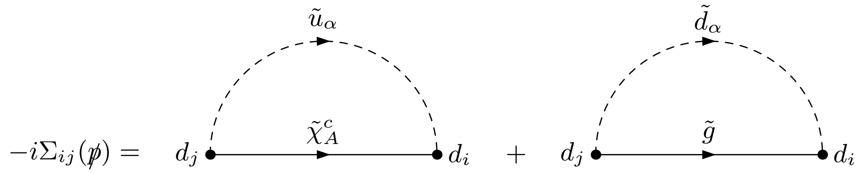


Figure 2.2: d-quark self-energy

chargino diagram to off-diagonal elements $\Sigma_{ij}(\not p)$ matrix. Using Feynman rules from Appendix A we receive the following structure of chargino term.

$$\Sigma_{ij}(\not p) = i\int \frac{d^dk}{(2\pi)^d}(O^A_{i\alpha}P_R + Q^A_{i\alpha}P_L) \times \frac{\not k + m_{\tilde{\chi}_A}}{k^2 - m^2_{\tilde{\chi}_A}}\delta_{AB} \times \tag{2.1}$$
$$\times({O^B}^\dagger_{\beta j}P_L + {Q^B}^\dagger_{\beta j}P_L) \times \frac{1}{(k-p)^2 - m^2_\alpha}\delta_{\alpha\beta}$$

In all expressions of this kind summation through squark $\alpha, \beta \in \{1, \ldots, 6\}$ and chargino $A, B \in \{1, 2\}$ indices is assumed. After the dimensional regularisation procedure with $d = 4 - 2\epsilon$ we get

$$\Sigma_{ij}(\not p) = (\Sigma^R_{ij}\not p + \Sigma^{R,m}_{ij})P_R + (\Sigma^L_{ij}\not p + \Sigma^{L,m}_{ij})P_L + \ldots\infty, \tag{2.2}$$

where

$$\Sigma^R_{ij}(p^2) = iQ^A_{i\alpha}{Q^A}^\dagger_{\alpha j}\frac{1}{16\pi^2}\int_0^1 x\ln C^{\alpha,A}(x)dx \tag{2.3}$$
$$\Sigma^L_{ij}(p^2) = iO^A_{i\alpha}{O^A}^\dagger_{\alpha j}\frac{1}{16\pi^2}\int_0^1 x\ln C^{\alpha,A}(x)dx \tag{2.4}$$
$$\Sigma^{R,m}_{ij}(p^2) = iO^A_{i\alpha}{Q^A}^\dagger_{\alpha j}\frac{m_{\tilde{\chi}_A}}{16\pi^2}\int_0^1 \ln C^{\alpha,A}(x)dx \tag{2.5}$$
$$\Sigma^{L,m}_{ij}(p^2) = iQ^A_{i\alpha}{O^A}^\dagger_{\alpha j}\frac{m_{\tilde{\chi}_A}}{16\pi^2}\int_0^1 \ln C^{\alpha,A}(x)dx \tag{2.6}$$
$$C^{\alpha,A}(x) = (1-x^2)m^2_{\tilde{\chi}_A} + xm^2_\alpha - x(1-x)p^2. \tag{2.7}$$

The infinite part of loop integrals is of the common form

$$\frac{1}{\epsilon} - \gamma + \ln 4\pi\mu^2 \tag{2.8}$$

where μ is the scale. Such terms does not depend on the masses of particles involved in loop, the summation through α and A can by done, which leads to flavour diagonal matrix. The d-quark self-energy $\Sigma(\not{p})$ contributes to $Z_{ij}^{1/2}$-factors for external quark legs ([9],p.220).

$$Z_{ij}^{1/2} = 1 + \frac{1}{2}\frac{\partial \Sigma_{ij}}{\partial \not{p}}\Big|_{\not{p}=m} = 1\delta_{ij} + \frac{1}{2}\delta Z_{ij}^{R} P_R + \frac{1}{2}\delta Z_{ij}^{L} P_L \tag{2.9}$$

$$\bar{Z}_{ij}^{1/2} = \gamma^0 Z_{ij}^{\dagger 1/2}\gamma^0 = 1\delta_{ij} + \frac{1}{2}\delta Z_{ij}^{L\dagger} P_R + \frac{1}{2}\delta Z_{ij}^{R\dagger} P_L \tag{2.10}$$

When calculating the $\not{p}$-derivative of self-energy Σ_{ij}, we get terms of the form

$$\delta Z_{ij}^{R,L} = \left(\Sigma_{ij}^{R,L} + 2m_j^2 \frac{\partial \Sigma_{ij}^{R,L}}{\partial p^2} + 2m_j \frac{\partial \Sigma_{ij}^{R,L,m}}{\partial p^2}\right)\Big|_{p^2=m_j^2}. \tag{2.11}$$

We can see from 2.3-2.7 that p^2-derivatives of corresponding integrals contain fraction with $C^{\alpha,A}(x)$ in the denominator. The masses of b and s quarks are negligibly small compared to the mass of squark or chargino in $C^{\alpha,A}(x)$ and therefore

$$\delta Z_{ij}^{R,L} \approx \Sigma_{ij}^{R,L}\Big|_{m_j^2 \approx 0}, \tag{2.12}$$

which is in good agreement with our numerical results. In case of our process we can write

$$\delta Z_{32}^{R} = i Q_{i\alpha}^{A} {Q^{A}}^{\dagger}_{\alpha j} I^{\alpha,A} \tag{2.13}$$

$$\delta Z_{32}^{L} = i O_{i\alpha}^{A} {O^{A}}^{\dagger}_{\alpha j} I^{\alpha,A} \tag{2.14}$$

$$I^{\alpha,A} = \frac{1}{16\pi^2}\int_0^1 x \ln C^{\alpha,A}(x) dx \tag{2.15}$$

Let us look at the structure of O and Q matrices (for the definition look at appendix A). We can see that

$$Q_{i\alpha}^{A} {Q^{A}}^{\dagger}_{\alpha j} = v_u^{-2}|U_{A2}|^2 m_b m_s \tan^2\beta (\Gamma_{uL}V_{CKM})^*_{\alpha 3}(\Gamma_{uL}V_{CKM})_{\alpha 2} \tag{2.16}$$

and

$$O^A_{i\alpha} {O^A}^\dagger_{\alpha j} = v_u^{-2} |W_{A2}|^2 m_t^2 V_{tb}^* V_{ts} (\Gamma^\dagger_{uR})_{3\alpha} (\Gamma_{uR})_{\alpha 2}. \quad (2.17)$$

Here the U and W matrices mix higgsino and wino in order to obtain charginos which are the mass eigenstates in the MSSM in an analogous way to V_{CKM}. The same holds for $\Gamma_{uR,L} V_{CKM}$ and u-squarks. Considering

$$(\Gamma_{uL})_{\alpha 2} = \delta_{\alpha 2} \qquad (\Gamma_{uR})_{\alpha 2} = \cos\theta_t \delta_{\alpha 3} - \sin\theta_t \delta_{\alpha 6} \quad (2.18)$$

$$(\Gamma_{uL})_{\alpha 3} = -\delta_{\alpha 4} \qquad (\Gamma_{uR})_{\alpha 3} = \cos\theta_t \delta_{\alpha 3} + \sin\theta_t \delta_{\alpha 6}, \quad (2.19)$$

we get

$$\delta Z^R_{32} \approx \frac{|U_{A2}|^2 m_b m_s \tan^2\beta}{16\pi^2 v_u^2} (V_{cb}^* V_{cs} \delta_{\alpha 2} + V_{tb}^* V_{ts} \delta_{\alpha 4}) I^{\alpha, A} \quad (2.20)$$

$$\delta Z^L_{32} \approx \frac{|W_{A2}|^2 m_t^2}{16\pi^2 v_u^2} V_{tb}^* V_{ts} (\cos^2\theta_t \delta_{\alpha 3} - \sin^2\theta_t \delta_{\alpha 6}) I^{\alpha, A} \quad (2.21)$$

or numerically

$$\left| \frac{\delta Z^R_{32}}{\delta Z^L_{32}} \right| \approx 10^{-3} \quad (2.22)$$

Such unexpectable small value of this ratio is caused mainly by m_t^2 in the numerator of δZ^L_{32}, in spite of the $\tan^2\beta$ in δZ^R_{32}, and also by opposite sign of V_{cb} with respect to V_{ts} which almost cancels in the bracket of 2.20. In the expression 2.21 we can also observe strong dependence of δZ^L_{32} on the stop mixing angle θ_t, which was already mentioned in [1] (p.11).

2.2 Neutral Higgs boson penguin diagrams

Now we are ready to write down dominant part of the decay amplitude for $B_s^0 \to \mu^+\mu^-$. According to [1],[2], the pseudoscalar nature of our meson caused that only the following two matrix elements do not vanish

$$\langle 0|\bar{b}\gamma_\mu\gamma_5 s|B_s^0\rangle = -ip_\mu f_{B_s^0} \qquad \langle 0|\bar{b}\gamma_5 s|B_s^0\rangle = if_{B_s^0}\frac{m_{B_s^0}^2}{m_b+m_s}, \tag{2.23}$$

where the decay constant $f_{B_s^0}$ is equal (210 ± 30)MeV [2]. By neutral Higgs boson penguins we understand all (a)-(d) diagrams on the picture 2.1. Comparing first two examples, (a) and (b), it isn't hard to see, that (a) is more than forty times larger than (b), which is due to Yukawa coupling sitting in the quark-quark-higgs vertex. Our calculations also showed that contribution from the other diagrams are negligible compared to (a). Using Feynman rules from appendix A for 2.1(a) we can write

$$\begin{aligned} i\mathcal{M}^{(a)} = \bar{v}_b\big(-iC_d^H(Y_d)_{33}P_L - iC_d^{H*}(Y_d^\dagger)_{33}P_R\big)Z_{32}^{1/2}u_s \times \\ \times\frac{i}{m_{B_s^0}^2-m_H^2}\times i\frac{m_\mu}{v_d}\bar{u}_\mu\big(C_d^H P_L + C_d^{H*}P_R\big)v_\mu \end{aligned} \tag{2.24}$$

After short manipulation using 2.23 and taking $\delta Z_{32}^L >> \delta Z_{32}^R, m_b >> m_s$, $m_H >> m_{B_s^0}$ for all $H\in\{A^0,G^0,H^0,h^0\}$, one can see that

$$i\mathcal{M}^{(a)} \approx -\frac{m_\mu f_{B_s^0} m_{B_s^0}^2}{4v_d^2 m_H^2}|C_d^H|^2\delta Z_{32}^L\bar{u}_\mu(P_L\pm P_R)v_\mu, \tag{2.25}$$

where plus sign between projectors holds for the scalar H^0, h^0 and minus for the pseudoscalar A^0, G^0 Higgs bosons. To get some quantitative results, it is necessary to sum over all the Higgses[3], take squared absolute value of the amplitude and also perform summation through all possible spin orientation of muons. Such manipulation with decay amplitude leads to the final expression

$$\begin{aligned} \sum_{\text{spin}}\big|i\mathcal{M}^{(a)}\big|^2 \approx \frac{(m_\mu f_{B_s^0} m_{B_s^0}^3)^2\tan^4\beta}{32v_u^4}(\delta Z_{32}^L)^2 \times \\ \times\left[\Big(\frac{s_\beta^2}{m_{A^0}^2}+\frac{c_\beta^2}{m_{G^0}^2}\Big)^2+\Big(\frac{c_\alpha^2}{m_{H^0}^2}+\frac{s_\alpha^2}{m_{h^0}^2}\Big)^2\right] \end{aligned} \tag{2.26}$$

[3]The largest contribution comes from A^0 and H^0, which is caused by $\sin\beta$ involved in the C_d^H instead of $\cos\beta$ for G^0, h^0.

Here we used relations for muon external leg factors in the form

$$\sum_{\text{spin}} |\bar{u}_\mu \gamma_5 v_\mu|^2 = 2m_{B_s^0}^2 \tag{2.27}$$

$$\sum_{\text{spin}} |\bar{u}_\mu v_\mu|^2 = 2m_{B_s^0}^2 - 8m_\mu^2 \approx 2m_{B_s^0}^2$$

The branching ration is obtainned in the usual way by multiplying the partial decay width and lifetime of the B_s^0, which is $\tau_{B_s^0} \approx 1.47 \times 10^{-12}$s. In the central mass system we get

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) = \frac{\tau_{B_s^0}}{16\pi m_{B_s^0}} \sum_{\text{spin}} \left|i\mathcal{M}^{(a)}\right|^2 + \mathcal{O}\left(\frac{m_\mu^2}{m_{B_s^0}^2}\right) \approx 3.6 \times 10^{-8} \tag{2.28}$$

This result corresponds to 70% of complete calculation of all diagrams from 2.1.

2.3 Dependence of $\mathcal{B}(B_s^0 \to \mu^+\mu^-)$ on Higgs mass

If we look at the resultant branching ratio from previous section, we can easily see that this value is already experimentally excluded by [3]. However, we could still play some game with our parameters or simply state some constrain to the mass of the A^0 Higgs component. We can see from the equation 2.26 that the square of the amplitude depends on the Higgs mass roughly as $m_{A^0}^{-4}$. The more complete calculation involving all diagrams from 2.1 gives the dependence showed in the figure 2.3. If we use the experimental bound $\mathcal{B}(B_s^0 \to \mu^+\mu^-) < 4,3 \times 10^{-8}$ [3], the restriction to the mass of the will be $m_{A^0} \gtrsim 220$ GeV.

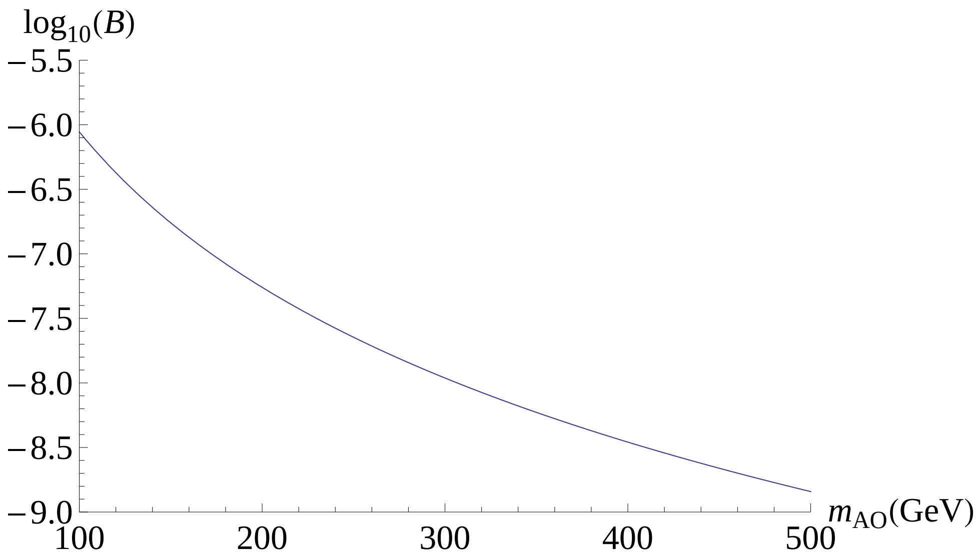


Figure 2.3: Dependence of the $\mathcal{B}(B_s^0 \to \mu^+\mu^-)$ on m_{A^0}

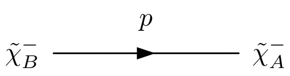
Conclusions

In this work we gave short outline of the motivation of supersymmetry and also showed one important example of its consequences to the amplitude of the flavour changing decay $B_s^0 \to \mu^+\mu^-$. In our computation we used concrete values of supersymmetry breaking parameters. The large value of $\tan\beta \sim 50$ was motivated by $SO(10)$ unification of the top and bottom Yakawa coupling constants. The dominant contribution came from the flavour changing at the s-quark external leg in Higgs penguin diagram (a) in 2.1. The numerical value of the branching ratio gives lower bound to the A^0 mass equal approximately to 220 GeV.

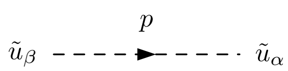
Appendix A

Feynman rules

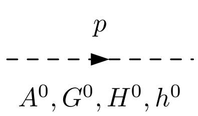
Propagators



$\frac{i}{\not{p} - m_{\tilde{\chi}_A^-}} \delta_{AB}$

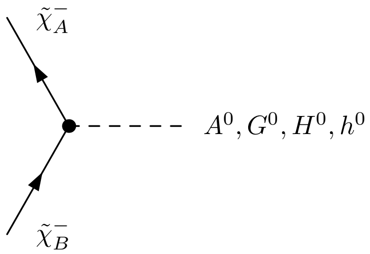


$\frac{i}{p^2 - m_{\tilde{u}_\alpha}^2} \delta_{\alpha\beta}$

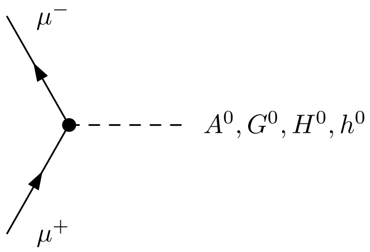


$\frac{i}{p^2 - m^2}$

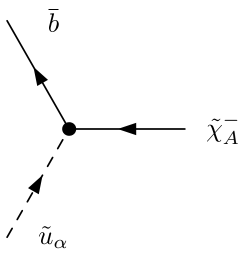
Vertices



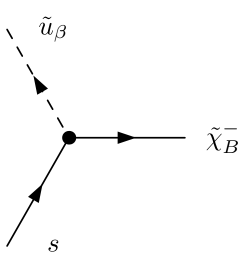
$$-ig_2\Big\{\big(C_d^{H^*}W_{A1}^*U_{B2}^* + C_u^{H^*}W_{A2}^*U_{B1}^*\big)P_L + \big(C_d^H W_{B1}U_{A2} + C_u^H W_{B2}U_{A1}\big)P_R\Big\} = -i(M_{AB}P_L + M_{AB}^\dagger P_R)$$



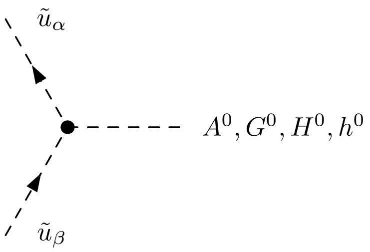
$$-i\tfrac{m_\mu}{v_d}\big(C_d^H P_L + C_d^{H^*}P_R\big)$$



$$i\Big\{\big[W_{A2}(V_{CKM}^\dagger Y_u \Gamma_{uR}^\dagger)_{3\alpha} - g_2 W_{A1}(V_{CKM}^\dagger \Gamma_{uL}^\dagger)_{3\alpha}\big]P_R + U_{A2}^*(Y_d V_{CKM}^\dagger \Gamma_{uL}^\dagger)_{3\alpha}P_L\Big\} = i(O_{3\alpha}^A P_R + Q_{3\alpha}^A P_L)$$



$$i\Big\{\big[W_{B2}^*(\Gamma_{uR}Y_u V_{CKM})_{\beta 2} - g_2 W_{B1}^*(\Gamma_{uL}V_{CKM})_{\beta 2}\big]P_L + U_{B2}(\Gamma_{uL}V_{CKM}Y_d)_{\beta 2}P_R\Big\} = i({O^B}^\dagger_{\beta 2}P_R + {Q^B}^\dagger_{\beta 2}P_L)$$



$$
\begin{aligned}
&i\Big\{(\mu C_d^{H^*} - A C_u^H)(\Gamma_{uR} Y_u \Gamma_{uL}^\dagger)_{\alpha\beta} \\
&+(\mu C_d^H - A C_u^{H^*})(\Gamma_{uL} Y_u \Gamma_{uR}^\dagger)_{\alpha\beta} \\
&-2 v_u \mathrm{Re}(C_u^H)\big(\Gamma_{uL} Y_u^2 \Gamma_{uL}^\dagger + \Gamma_{uR} Y_u^2 \Gamma_{uR}^\dagger\big)_{\alpha\beta} \\
&-\big[\big(\tfrac{g_2^2}{4} - \tfrac{g_1^2}{12}\big)\Gamma_{uL}\Gamma_{uL}^\dagger + \tfrac{g_1^2}{3}\Gamma_{uR}\Gamma_{uR}^\dagger\big]_{\alpha\beta}\times \\
&\times 2\mathrm{Re}\big(v_d C_d^H - v_u C_u^H\big)\delta_{\alpha\beta}\Big\} = i S_{\alpha\beta}
\end{aligned}
$$

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