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NEGATIVITY OF TWO-QUBIT SYSTEM THROUGH SPIN COHERENT STATES

K. Berrada[1]
Laboratoire de Physique Thèorique, Département de Physique, Faculté des Sciences, Université Mohammed V - Agdal, Av. Ibn Battouta, B.P. 1014, Agdal, Rabat, Morocco
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy,

M. El Baz[2], Y. Hassouni[3]
Laboratoire de Physique Thèorique, Département de Physique, Faculté des Sciences, Université Mohammed V - Agdal, Av. Ibn Battouta, B.P. 1014, Agdal, Rabat, Morocco

and

H. Eleuch[4]
Institute for Quantum Studies and Department of Physics, Texas A&M University, TX 77843, USA.

Abstract

Using the negativity, we express and analyze the entanglement of two-qubit nonorthogonal pure states through the spin coherent states. We formulate this measure in terms of the amplitudes of coherent states and we give the conditions for the minimal and the maximal entanglement. We generalize this formalism to the case of a class of mixed states and show that the negativity is also a function of probabilities.

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[1]kberrada@ictp.it
[2]elbaz@fsr.ac.ma
[3]y-hassou@fsr.ac.ma
[4]heleuch@physics.tamu.edu

1 Introduction

Quantum entanglement has been the focus of foundation discussions of quantum mechanics since the time of Schrödinger and the paper of Einstein, Pedolsky, and Rosen [1, 2]. Entanglement is a fundamental notion to distinguish between the quantum and the classical worlds. Recently, it is used as a physical resource to realize various quantum information tasks, such as quantum cryptography [3, 4, 5], quantum teleportation [6], quantum computation [7], dense coding [8, 9], and more recently, metrology [10, 11]. The important problems for entanglement phenomena is to find a method to determine whether a given state of a composite quantum system consisting of two or more subsystems is entangled or not and to choose the best measure quantifying the amount of entanglement. Therefore, the quantification and the characterization of the amount of entanglement have attracted much attention and became one of the most interesting problems in the field of quantum information. For quantifying the entanglement a number of measures, such as the negativity [12, 13, 14, 15], the entanglement of formation [16, 17], the concurrence [18, 19, 20, 21], and the linear entropy [22, 23, 24] have been proposed. When the quantum system is in a pure state, the concept of entanglement is simpler and easily understood while the full characterization of the entanglement properties of mixed states is a difficult and still unsolved mathematical problem [25].

Another concept widely used and applied in quantum information theory is the notion of coherent or quasiclassical states. These states make a very useful tool for the investigation of various problems in physics [26, 27, 28, 29]. Coherent states were first introduced by Schrödinger [30] in the context of the harmonic oscillator, who was interested in finding quantum states which provide a close connection between quantum and classical formulations of a given physical system. Later, the notion of coherent has became very important in quantum optics due to Glauber [31], as eigenstates of the annihilation operator $\hat{a}$ of the harmonic oscillator, while he demonstrated that these states have the interesting property of minimizing the Heisenberg uncertainty relation. Next, the following important coherent states are $SU(2)$ and $SU(1,1)$ coherent states introduced by Peremolov [32, 33] which describe several systems and also have some applications in quantum optics, statistical mechanics, nuclear physics, and condensed matter physics [34, 35, 36].

The aim of the present paper is to measure the degree of entanglement of two-qubit nonorthogonal states which play an important role in the quantum cryptography and quantum information processing. As a measure of entanglement, we use the negativity. We investigate this measure in the framework of coherent states and we give the conditions for the separability and the entanglement.

Our paper is organized as follows. In section 2, we give a review of negativity for two-qubit pure and mixed states. In section 3, we express the negativity in terms of the amplitudes of coherent states and we discuss the degree of the entanglement. In section 4, we give the conclusion.

2 Negativity of an arbitrary state of two-qubit system

In this section we give an outline of the negativity and its quantities for pure and mixed states of two-qubit system.

A general pure state of two-qubit system can be expanded in the standard computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ as

$$|\psi\rangle = A_{00}|00\rangle + A_{01}|01\rangle + A_{10}|10\rangle + A_{11}|11\rangle, \tag{1}$$

where A_{ij} are complex numbers verifying the following normalization condition

$$|A_{00}|^2 + |A_{01}|^2 + |A_{10}|^2 + |A_{11}|^2 = 1.$$

According to the Schrödinger definition, the pure state is separable (factorizable) meaning that it may be written in the following direct product $(a|0\rangle + b|1\rangle)\otimes(c|0\rangle + d|1\rangle)$, otherwise it is entangled or inseparable.

Similarly to the concurrence, the negativity for a two-qubit state $|\psi\rangle$ is usually expressed in terms of the coefficients A_{ij} as [37, 38]

$$N(|\psi\rangle) = 2|A_{00}A_{11} - A_{01}A_{10}|. \tag{2}$$

The negativity is equal to 0 for a separable state and to 1 for a maximally entangled state $\left(i.e., |\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) \text{ or } |\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)\right)$.

In the case of mixed states, the two-qubit quantum state must be represented not by a bracket but by a matrix called density operator (or density matrix) and denoted by ρ in quantum mechanics. It always decomposes ρ into a mixture of the density operators of a set of pure states $|\psi_i\rangle$ as

$$\rho = \sum_i p_i|\psi_i\rangle\langle\psi_i|, \tag{3}$$

where $\{|\psi_i\rangle\}$ is a set (not necessarily orthogonal) of normalized pure states of two-qubit system, and $\{p_i\}$ is a probability distribution.

The mixed state ρ is said to be separable if it can be written as a convex sum of separable pure states, i.e., $\rho = \sum_i p_i\rho_i^1 \otimes \rho_i^2$, where $\rho_i^{1,2}$ is the reduced density operator of qubit $(1,2)$, respectively, given by $\rho_i^{(1,2)} = Tr_{(2,1)}(|\psi_i\rangle\langle\psi_i|)$. The state ρ is entangled if it cannot be represented as a mixture of separable pure states.

Generally, the entanglement of a bipartite system in a pure state is presently well defined and the entanglement measures are widely accepted. In contrast, for measuring the entanglement of mixed states there is no such general method and the most entanglement measures are difficult to calculate analytically because they involve variational expressions. As an alternative, the negativity is a useful measure of the entanglement for mixed states related to the Peres-Horodecki-al criterion [39, 40] (the positive partial transposition criterion). It is defined by

$$N(\rho) = -2\sum_i \mathrm{Max}(0, \lambda_i) \tag{4}$$

where λ_i's are eigenvalues of the partial transpose ρ^T of the density matrix ρ of the bipartite system. Note that for two-qubit mixed states, the partial transpose ρ^T has at most one negative eigenvalue. Similarly to the case of pure states, the negativity given by Eq. (4) ranges from zero for a unentangled state to unity for a maximally entangled state. Recently, G. Vidal and R. F. Werner [41] proved that the negativity is an entanglement monotone (based on convexity) and therefore can be considered as a good measure of entanglement.

In general for a mixed state, negativity and concurrence can differ. As proved by Vestraete et al [42], the negativity of mixed state can never exceed its concurrence and it is always larger than $\sqrt{[1 - C(\rho)]^2 + C^2(\rho)} - [1 - C(\rho)]$ where $C(\rho)$ is the concurrence of the mixed state (see Fig. 1).

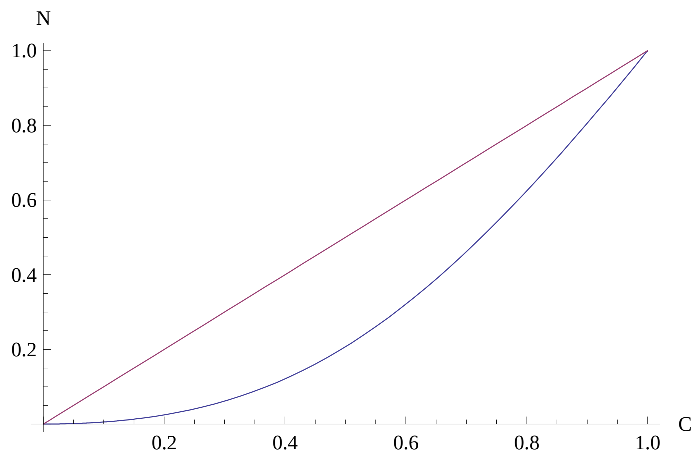

Fig. 1: Negativity versus concurrence.

The upper (respectively lower) bound corresponds to two-qubit mixed states which have the maximal (respectively the minimal) negativity for a fixed concurrence.

3 Negativity and spin coherent states

3.1 The case of pure states

The physical advantage of the coherent states in quantum information theory is due to the fact that they represent robust states which are extensively used and applied for solving different problems in various quantum information processing and transmission tasks, and they are easy to generate experimentally and convenient to use [43].

First, we prove that a qubit presents exactly a $SU(2)$ coherent state (or spin coherent state) of Klauder-Peremolov [33, 34].

The $SU(2)$ coherent state can be expressed as

$$
\begin{aligned}
|Z, j\rangle &= R(Z)|0, j\rangle \\
&= \exp\left(\eta(Z)J_{+} - \eta^{*}(Z)J_{-}\right)|0, j\rangle \\
&= \frac{1}{(1+|Z|^2)^j}\sum_{n=0}^{2j}\binom{2j}{n}^{\frac{1}{2}} Z^n |n, j\rangle
\end{aligned}
\tag{5}
$$

where $R(Z)$ is the rotation operator, and J_- and J_+ are lowering and raising operators of the $su(2)$ Lie algebra, respectively. The generators of $su(2)$ Lie algebra, $J_\pm$ and J_z, satisfy the commutation relations

$$
[J_+, J_-] = 2J_z \qquad ; \qquad [J_z, J_\pm] = J_\pm,
\tag{6}
$$

and acting on an irreducible unitary representation as follows

$$
J_\pm|j, m\rangle = \sqrt{(j \mp m)(j \pm m + 1)}\,|j, m \pm 1\rangle \qquad ; \qquad J_z|j, m\rangle = m|j, m\rangle.
$$

The spin coherent states can be obtained by successively applying the raising operator on the state $|j, -j\rangle$

$$
|Z, j\rangle = \frac{1}{(1+|Z|^2)^j}\sum_{m=-j}^{j}\left[\frac{(2j)!}{(j+m)!(j-m)!}\right]^{\frac{1}{2}} Z^{j+m}|j, m\rangle.
\tag{7}
$$

A change of variable $n = j + m$ will give the formula of Eq. (5).
For a particle with spin $j = \frac{1}{2}$ one gets:

$$
\begin{aligned}
\left| Z, \frac{1}{2}\right\rangle &= \frac{1}{(1+|Z|^2)^{\frac{1}{2}}}\sum_{n=0}^{1}\binom{1}{n}^{\frac{1}{2}} Z^n \left|n, \frac{1}{2}\right\rangle \\
&= \frac{1}{(1+|Z|^2)^{\frac{1}{2}}}\left(\left|0, \frac{1}{2}\right\rangle + Z\left|1, \frac{1}{2}\right\rangle\right)
\end{aligned}
\tag{8}
$$

and for $Z = \tan(\frac{\theta}{2})e^{i\varphi}$, we find that

$$
|Z\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle
\tag{9}
$$

where $|0, \frac{1}{2}\rangle \equiv |0\rangle$ and $|1, \frac{1}{2}\rangle \equiv |1\rangle$ are the basis states.

The above equation represents a qubit state and shows that the treatment and the transmission of the information can be performed by using the spin coherent states.

In general, a two-qubit system prepared in a separable pure state can be expressed as $|\theta_1, \varphi_1\rangle \otimes |\theta_2, \varphi_2\rangle$. Thus, the pure state that represents the simplest extension of separable pure state to an entangled pure state of two-qubit system is given by the unnormalized state [44]

$$
||\psi\rangle = \cos\theta|\theta_1, \varphi_1\rangle \otimes |\theta_2, \varphi_2\rangle + e^{i\phi}\sin\theta|\theta_1^{'}, \varphi_1^{'}\rangle \otimes |\theta_2^{'}, \varphi_2^{'}\rangle
\tag{10}
$$

where $|\theta_1, \varphi_1\rangle$ and $|\theta_1^{'}, \varphi_1^{'}\rangle$ are normalized states of qubit 1 and similarly for $|\theta_2, \varphi_2\rangle$ and $|\theta_2^{'}, \varphi_2^{'}\rangle$ of qubit 2, such that

$$\langle\theta_1,\varphi_1|\theta_1^{'},\varphi_1^{'}\rangle \neq 0 \qquad ; \qquad \langle\theta_2,\varphi_2|\theta_2^{'},\varphi_2^{'}\rangle \neq 0.$$

From Eq. (8), the unnormalized pure state can be expressed as

$$||\psi\rangle = \cos\theta|\alpha\rangle \otimes |\beta\rangle + e^{i\phi}\sin\theta|\alpha^{'}\rangle \otimes |\beta'\rangle \tag{11}$$

where

$$|\alpha\rangle = \frac{1}{\sqrt{(1+|\alpha|^2)}}\left(|0\rangle + \alpha|1\rangle\right)$$
$$|\beta\rangle = \frac{1}{\sqrt{(1+|\beta|^2)}}\left(|0\rangle + \beta|1\rangle\right)$$
$$|\alpha^{'}\rangle = \frac{1}{\sqrt{(1+|\alpha^{'}|^2)}}\left(|0\rangle + \alpha^{'}|1\rangle\right)$$
$$|\beta^{'}\rangle = \frac{1}{\sqrt{(1+|\beta^{'}|^2)}}\left(|0\rangle + \beta^{'}|1\rangle\right)$$

are respectively the states for each qubit.

The norm of this state is

$$\mathcal{N} = \left[1 + \cos\theta\sin\theta e^{i\phi}\langle\alpha^{'}|\alpha\rangle\langle\beta^{'}|\beta\rangle + \cos\theta\sin\theta e^{-i\phi}\langle\alpha|\alpha^{'}\rangle\langle\beta|\beta^{'}\rangle\right]^{-\frac{1}{2}}. \tag{12}$$

Thus the normalized two-qubit pure state may be written as

$$|\psi\rangle = a|\alpha\rangle \otimes |\beta\rangle + b|\alpha^{'}\rangle \otimes |\beta'\rangle, \tag{13}$$

where

$$a = \mathcal{N}\cos\theta$$
$$b = \mathcal{N}e^{i\phi}\sin\theta.$$

Finally, the state (13) can be expressed on the standard computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ as

$$|\psi\rangle = A|00\rangle + B|01\rangle + C|10\rangle + D|11\rangle \tag{14}$$

where

$$A = \lambda + \gamma$$
$$B = \beta\lambda + \beta^{'}\gamma$$
$$C = \alpha\lambda + \alpha^{'}\gamma$$
$$D = \alpha\beta\lambda + \alpha^{'}\beta'\gamma$$

such that

$$\lambda = \frac{a}{\sqrt{(1+|\alpha|^2)(1+|\beta|^2)}} \quad ; \quad \gamma = \frac{b}{\sqrt{(1+|\alpha^{'}|^2)(1+|\beta^{'}|^2)}}.$$

The negativity of the pure state $|\psi\rangle$ is given by

$$N(|\psi\rangle) = 2|AD - BC| \\ = 2\left|\lambda\gamma\left(\alpha - \alpha^{'}\right)\left(\beta - \beta^{'}\right)\right|. \tag{15}$$

The state (13) is separable (i.e., $N(|\psi\rangle) = 0$) in either of the following situations: $\lambda = 0$ or $\gamma = 0$ or $\alpha = \alpha^{'}$ and/or $\beta = \beta^{'}$, and it is maximally entangled (Bell state) when $N(|\psi\rangle) = 1$.

For simplicity we consider the case where $\alpha = \beta$ and $\alpha^{'} = \beta^{'}$. Without loss of generality we suppose that the amplitudes of coherent states are real parameters and we set θ and φ to their values at the extremum of $N(|\psi\rangle)$ $\left(i.e., \frac{\partial^2 N}{\partial\theta\partial\varphi} = 0\right)$ [45]. The negativity is then given by

$$N(|\psi\rangle) = \frac{(\alpha\ -\ \alpha^{'})^2}{2\,\alpha^2\,\alpha^{'2}\ +\ \alpha^{'2}\ +\ 2\,\alpha\,\alpha^{'}\ +\ \alpha^2\ +\ 2}. \tag{16}$$

The maximum of the negativity (i.e., $N(|\psi\rangle) = 1$) corresponds to $\alpha = -\frac{1}{\alpha^{'}}$ (see Fig. 2).

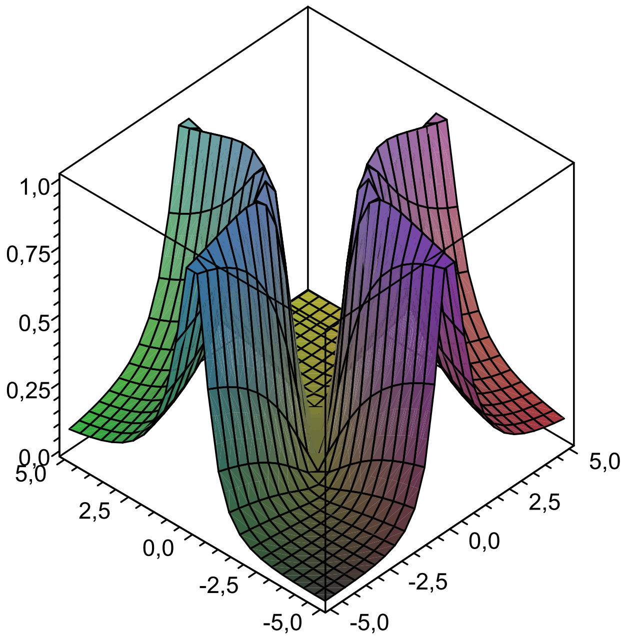

Fig. 2: Negativity as a function of the amplitudes α and $\alpha^{'}$.

3.2 The case of mixed states

In this section we shall extend our results obtained for pure states to the case of a class of mixed states given by a statistical mixture of two pure states of two-qubit system

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \qquad (i = 1, 2) \tag{17}$$

where

$$|\psi_i\rangle = a_i|\alpha_i\rangle \otimes |\beta_i\rangle + b_i|\alpha_i^{'}\rangle \otimes |\beta_i{'}\rangle \tag{18}$$

are the pure states of two-qubit system,
such that

$$a_i = \mathcal{N}_i \cos\theta_i$$
$$b_i = \mathcal{N}_i e^{i\phi_i} \sin\theta_i$$
$$\mathcal{N}_i = \Big[1 + \cos\theta_i \sin\theta_i e^{i\phi_i} \langle\alpha_i^{'}|\alpha_i\rangle\langle\beta_i^{'}|\beta_i\rangle + \cos\theta_i \sin\theta_i e^{-i\phi_i} \langle\alpha_i|\alpha_i^{'}\rangle\langle\beta_i|\beta_i^{'}\rangle\Big]^{-\frac{1}{2}}.$$

In order to obtain the value of negativity, we introduce an orthogonal normalized basis of the Hilbert space, where it is spanned by the nonorthogonal states $|\alpha_i\rangle$ and $|\alpha_i^{'}\rangle$ for qubit 1 and by $|\beta_i\rangle$ and $|\beta_i^{'}\rangle$ for qubit 2 [46]. These states can be chosen as

$$|\alpha_i\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad ; \quad |\alpha_i^{'}\rangle = \begin{pmatrix} \mathcal{N}_1^i \\ \langle\alpha_i|\alpha_i^{'}\rangle \end{pmatrix}$$

$$|\beta_i^{'}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad ; \quad |\beta_i\rangle = \begin{pmatrix} \mathcal{N}_2^i \\ \langle\beta_i^{'}|\beta_i\rangle \end{pmatrix}$$

with

$$\mathcal{N}_1^i = \sqrt{1 - |\langle\alpha_i|\alpha_i^{'}\rangle|^2} \quad \text{and} \quad \mathcal{N}_2^i = \sqrt{1 - |\langle\beta_i^{'}|\beta_i\rangle|^2}.$$

On this basis, the state (18) can be written as

$$|\psi_i\rangle = a_i \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} \mathcal{N}_2^i \\ \langle\beta_i^{'}|\beta_i\rangle \end{pmatrix} + b_i \begin{pmatrix} \mathcal{N}_1^i \\ \langle\alpha_i|\alpha_i^{'}\rangle \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{19}$$
$$= \begin{pmatrix} 0 \\ b_i\mathcal{N}_1^i \\ a_i\mathcal{N}_2^i \\ \mathcal{M}_i \end{pmatrix}$$

where $\mathcal{M}_i = a_i\langle\beta_i^{'}|\beta_i\rangle + b_i\langle\alpha_i|\alpha_i^{'}\rangle$, and the normalization of the state (19) requires that

$$|a_i\mathcal{N}_2^i|^2 + |b_i\mathcal{N}_1^i|^2 + |\mathcal{M}_i|^2 = 1.$$

We consider the case where $|\psi_1\rangle$ or $|\psi_2\rangle$ is separable (*i.e.*, $N(|\psi_1\rangle) = 0$ or $N(|\psi_2\rangle) = 0$), the negativity has the upper and lower bounds expressed by

$$\sqrt{[1 - C(\rho)]^2 + C^2(\rho)} - [1 - C(\rho)] \leq N(\rho) \leq C(\rho) \tag{20}$$

where $C(\rho)$ is the concurrence of two-qubit mixed state given by [45]

$$C(\rho) = 2p_i\left|\lambda_i\gamma_i\left(\alpha_i - \alpha_i^{'}\right)\left(\beta_i - \beta_i^{'}\right)\right| \quad (i = 1 \quad \text{for} \quad N(|\psi_2\rangle) = 0 \quad ; \quad i = 2 \quad \text{for} \quad N(|\psi_1\rangle) = 0). \tag{21}$$

Without loss of generality, we take the simple case where $\alpha_i = \alpha_i^{'}$ and $\beta_i = \beta_i^{'}$ [45]. The concurrence (upper bound of negativity) is simplified as

$$C(\rho) = p_i \frac{(\alpha_i - \alpha_i^{'})^2}{2\,\alpha_i^2\,\alpha_i^{'2} + \alpha_i^{'2} + 2\,\alpha_i\,\alpha_i^{'} + \alpha_i^2 + 2} \tag{22}$$
$$= \frac{p_i}{1 + 2X_i}$$

where

$$X_i = \left(\frac{1 + \alpha_i \alpha_i^{'}}{\alpha_i - \alpha_i^{'}} \right)^2 .$$

Using the expression of concurrence in Eq. (22), we plot the variations of negativity in terms of the amplitudes of coherent states and the probabilities (see Figs. 3, 4, and 5).

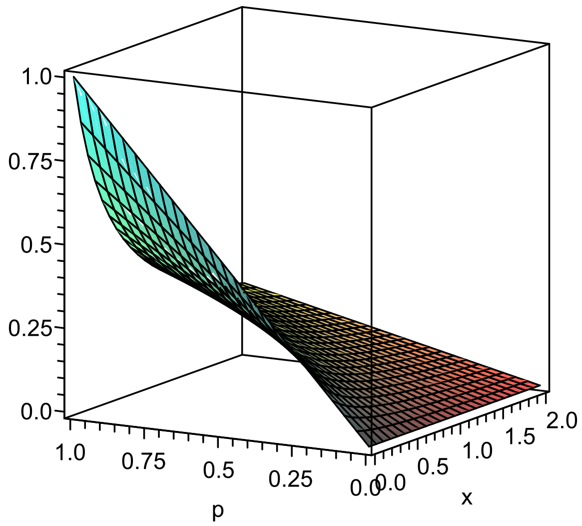

Fig. 3: Upper bound of negativity as a function of X and p.

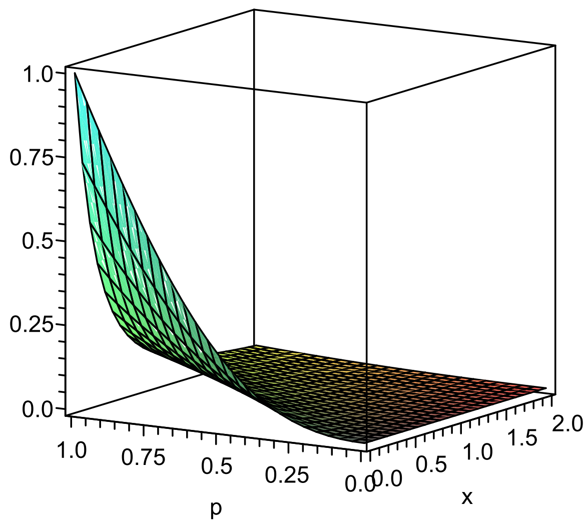

Fig. 4: Lower bound of negativity as a function of X and p.

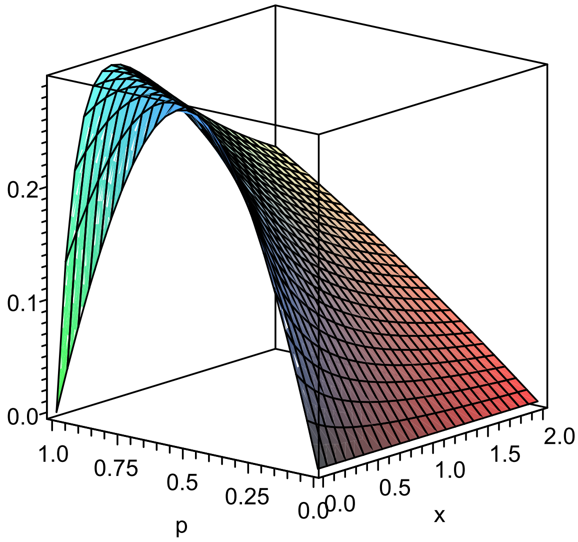

Fig. 5: Negativity between the upper and lower bounds in terms of X and p.

In this way, the mixed state density matrix is written in terms of the amplitudes of coherent states and the probabilities as

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & p_i(b_i\mathcal{N}_i^1)^2 & p_ia_ib_i(\mathcal{N}_i^1)^2 & p_ib_i\mathcal{N}_i^1\mathcal{M}_i \\ 0 & p_ia_ib_i(\mathcal{N}_i^1)^2 & p_i(a_i\mathcal{N}_i^1)^2 & p_ia_i\mathcal{N}_i^1\mathcal{M}_i \\ 0 & p_i\mathcal{N}_i^1\mathcal{M}_i & p_ia_i\mathcal{N}_i^1\mathcal{M}_i & p_i\mathcal{M}_i^2+1-p_i \end{pmatrix} \tag{23}$$
$$(i=1 \quad \text{for} \quad N(|\psi_2\rangle)=0 \quad ; \quad i=2 \quad \text{for} \quad N(|\psi_1\rangle)=0).$$

We notice that from the result obtained in section 3.1, the pure states $|\psi_i\rangle$ $(i=1,2)$ are separable when $a_i=0$, $b_i=0$ or $\alpha_i=\alpha_i^{'}$, and attain their maximum entanglement when $|N(|\psi_i\rangle)\rangle=1$.

The above equation shows that the pure state $|\psi_1\rangle$ (or $|\psi_2\rangle$) and its probability p_1 (or p_2) contain the information about the entanglement of two-qubit mixed state. In this case, the negativity of mixed state is only a function of components of the pure state and its probability.

We now discuss two important cases:

► $\alpha_i=\alpha_i^{'}$, i.e., the pure state $|\psi_i\rangle$ is separable, the partial transpose ρ^T has non-negative eigenvalues which corresponds to separable mixed state $N(\rho)=0$.

► $\alpha_i=-\frac{1}{\alpha_i^{'}}$, i.e., the state $|\psi_i\rangle$ is maximally entangled, the partial transpose ρ^T acquires the form

$$\rho^T = \begin{pmatrix} 0 & 0 & 0 & \frac{p_i}{2} \\ 0 & \frac{p_i}{2} & 0 & 0 \\ 0 & 0 & \frac{p_i}{2} & 0 \\ \frac{p_i}{2} & 0 & 0 & 1-p_i \end{pmatrix} \tag{24}$$

which corresponds to $N(\rho) = \sqrt{(1-p_i)^2 + p_i^2} - (1-p_i)$. Thus, in this case we have a two-qubit mixed state defined as a statistical mixture of a maximally entangled pure state and a separable pure state. These states represent an important class of quantum states [37, 47, 48, 49] which are widely used and applied in quantum information processing and transmission.

4 Conclusion

In conclusion, we have proposed a useful method for determining and quantifying the entanglement of two-qubit system states. This method is based on spin coherent states. These coherent states are useful elements for performing and investigating various quantum information processing and transmission tasks, and the reason for their choice is because they are easy to generate and convenient to use.

In this way, we have shown the relation between spin coherent states and qubits. Relying on this result and by considering the negativity as a measure of entanglement, we have expressed this measure in terms of the amplitudes of coherent states and we have derived the conditions for the minimal and the maximal entanglement in the case of two-qubit system nonorthogonal pure states.

We have generalized this formalism of the entanglement of two-qubit pure states to the case of a class of mixed states. We have studied the behavior of the negativity as a function of the amplitudes and the corresponding probabilities, and also expressed their upper and lower bounds in the language of spin coherent states.

We intend to use and generalize this method to the case of bipartite and multipartite states and consider possible applications in quantum information.

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