

BPS Lorentz-violating vortex solutions.

R. Casana^a, M. M. Ferreira Jr.^a and E. da Hora^b.

^aDepartamento de Física, Universidade Federal do Maranhão, 65085-580, São Luís, Maranhão, Brasil.

^bDepartamento de Física, Universidade Federal da Paraíba, 58051-900, João Pessoa, Paraíba, Brasil.

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The Model.

- Lagrangian density (1, 3):

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}W_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta} + |D_\alpha\phi|^2 - \frac{\lambda^2}{4} \left(|\phi|^2 - v^2 \right)^2, \quad (1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\alpha\phi = \partial_\alpha\phi + ieA_\alpha\phi$.

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- Euler-Lagrange Equations:

$$\partial_\alpha F^{\lambda\alpha} + W^{\lambda\alpha\mu\nu}\partial_\alpha F_{\mu\nu} = ie \left(\phi \overline{D^\lambda\phi} - \overline{\phi} D^\lambda\phi \right), \quad (2)$$

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- We use (U^μ and b^ν arbitrary)

$$W^{\mu\nu\alpha\beta} = \frac{1}{2} \left(\eta^{\mu\alpha} k^{\nu\beta} - \eta^{\nu\alpha} k^{\mu\beta} + \eta^{\nu\beta} k^{\mu\alpha} - \eta^{\mu\beta} k^{\nu\alpha} \right), \quad (4)$$

$$k^{\mu\nu} = \frac{1}{2} (U^\mu b^\nu + U^\nu b^\mu) - \frac{1}{4} \eta^{\mu\nu} U^\alpha b_\alpha, \quad (5)$$

according to Phys. Rev. D **82**, 125006 (2010).

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Vortex solutions.

- Vortex *ansatz*:

$$\phi(r, \theta) = v g(r) e^{in\theta} \quad \text{and} \quad \mathbf{A}(r, \theta) = -\frac{\hat{\theta}}{er} (a(r) - n) , \quad (12)$$

where $n = \pm 1, \pm 2, \pm 3 \dots$

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- Components of Ampère-Maxwell Law (radial one vanishes)

$$\frac{d^2 a}{dr^2} - \frac{1}{r} \frac{da}{dr} = \frac{2e^2 v^2 g^2 a}{(1 + U_z b_z)} \quad (\theta\text{-component}), \quad (13)$$

$$(U_\theta b_z + U_z b_\theta) \left(\frac{d^2 a}{dr^2} - \frac{1}{r} \frac{da}{dr} \right) = 0 \quad (z\text{-component}). \quad (14)$$

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- Many trivial configurations. Ex.: $\mathbf{U} = (U_r, 0, 0)$ and $\mathbf{b} = (0, 0, b_z)$.
- We choose configurations as $U_\theta = b_\theta = 0$ and $U_z b_z = -U_r b_r$.

Energy density and boundary conditions.

- The energy density $\varepsilon(r)$ is (note $U_r b_r < 1$)

$$\varepsilon = \frac{1 - U_r b_r}{2} \left(\frac{1}{er} \frac{da}{dr} \right)^2 + v^2 \left(\left(\frac{dg}{dr} \right)^2 + \frac{g^2 a^2}{r^2} \right) + \frac{\lambda^2 v^4}{4} (1 - g^2)^2 .$$

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- Boundary conditions:

$$g(r \rightarrow \infty) \rightarrow 1 \quad \text{and} \quad a(r \rightarrow \infty) \rightarrow 0 , \quad (15)$$

$$g(r \rightarrow 0) \rightarrow 0 \quad \text{and} \quad a(r \rightarrow 0) \rightarrow n . \quad (16)$$

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- We rewrite the energy density as

$$\begin{aligned} \varepsilon(r) = & \frac{1}{\lambda^2} \left(\frac{1}{r} \frac{da}{dr} \pm \frac{\lambda^2 v^2}{2} (1 - g^2) \right)^2 + v^2 \left(\frac{dg}{dr} \mp \frac{ag}{r} \right)^2 \\ & \mp \frac{v^2}{r} \left(\frac{da}{dr} - \frac{d}{dr} (g^2 a) \right) , \end{aligned} \quad (17)$$

where

$$2e^2 = (1 - U_r b_r) \lambda^2 \quad (18)$$

- BPS Equations:

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- Energy density:

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Total energy:

$$E = \int \varepsilon(r) d^2r = ev^2 |\Phi_B| , \quad (22)$$

where

$$\Phi_B = \int B(r) d^2r = \frac{2\pi n}{e} \quad (23)$$

is the magnetic flux.

- Using the constraint

$$2e^2 = (1 - U_r b_r) \lambda^2 , \quad (24)$$

we fix e and $U_r b_r$ (remember $U_r b_r < 1$) to obtain λ^2 . Then, we numerically solve BPS Equations.

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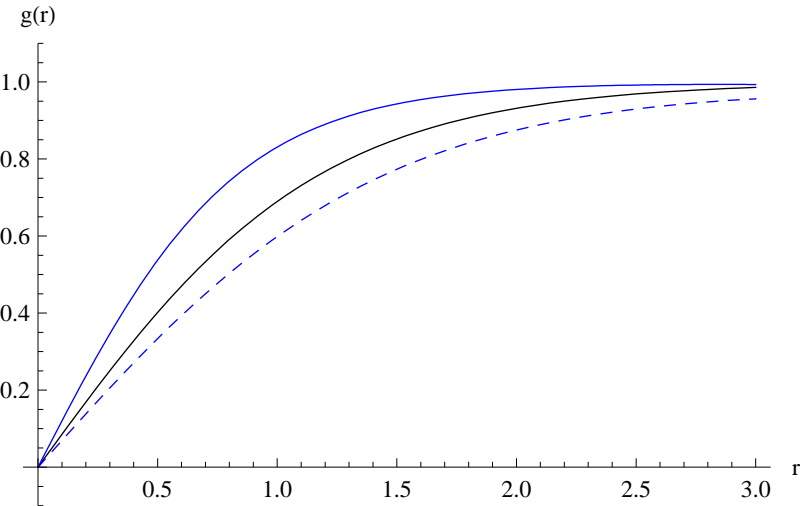
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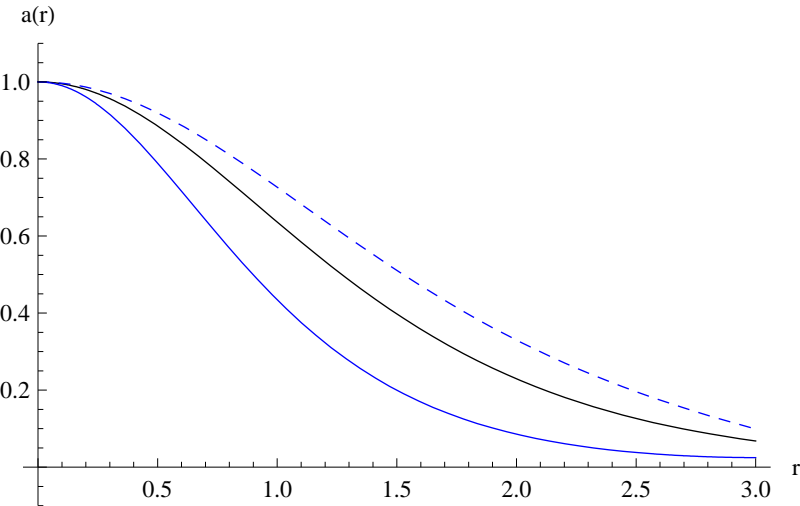
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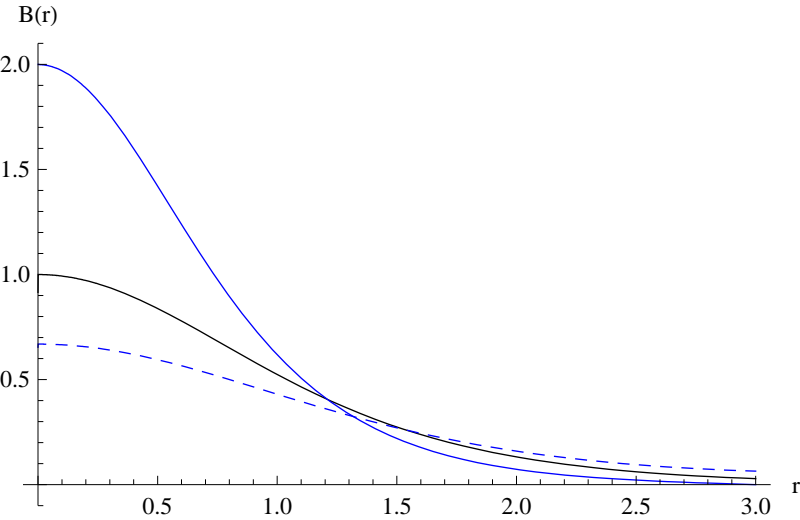
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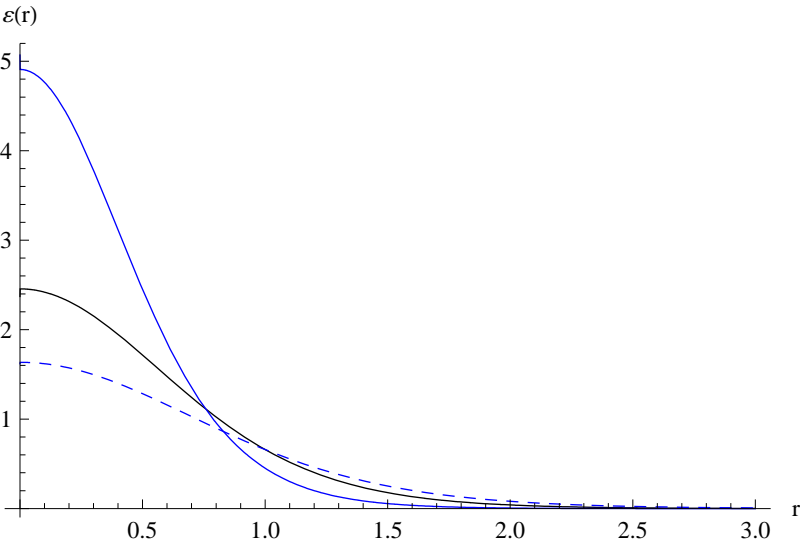
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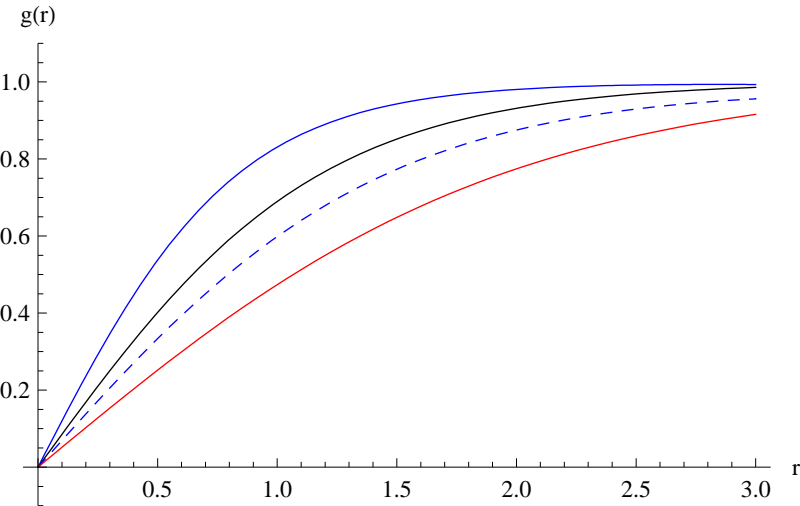


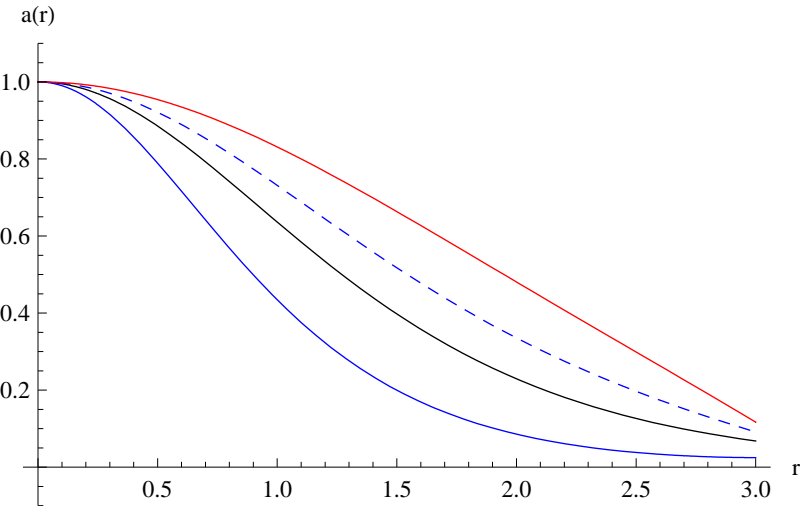
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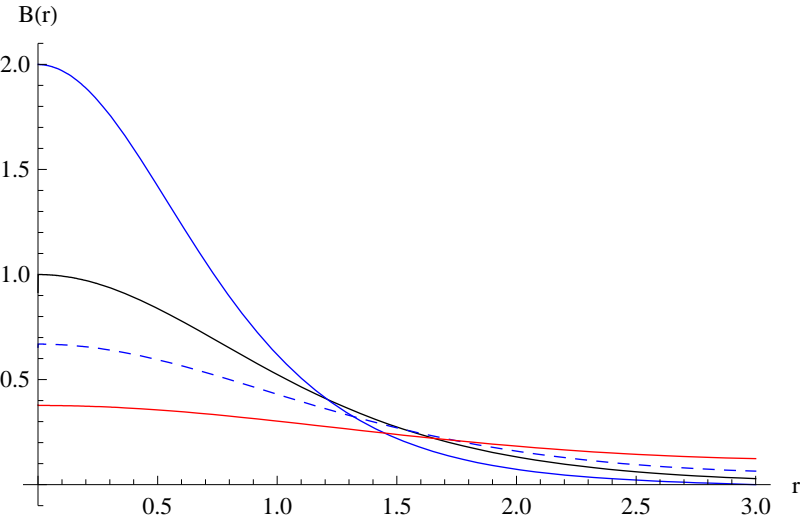
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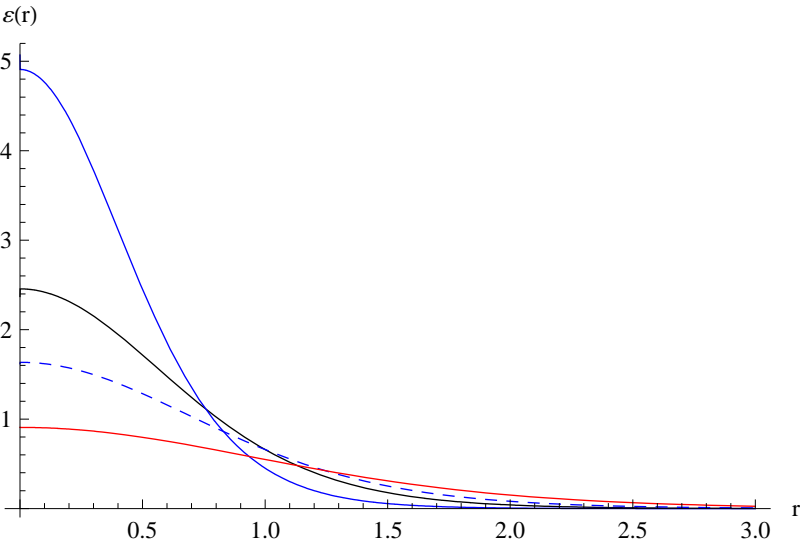
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