2011 8th International Multi-Conference on Systems, Signals & Devices

IDENTIFICATION OF GMS FRICTION MODEL WITHOUT FRICTION FORCE MEASUREMENT

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ABSTRACT

This paper deals with an online identification of the Generalized Maxwell Slip (GMS) friction model for both presliding and sliding regime at the same time. This identification is based on robust adaptive observer without friction force measurement. To apply the observer, a new approach of calculating the filtered friction force from the measurable signals is introduced. Moreover, two approximations are proposed to get the friction model linear over the unknown parameters and an approach of suitable filtering is introduced to guarantee the continuity of the model. Simulation results are presented to prove the efficiency of the approach of identification.

Index Terms— GMS Friction, robust adaptive observer, filtering, identification

1. INTRODUCTION

Friction is a physical phenomenon that occurs in mechanical systems which contains surfaces in contact. The friction phenomenon can be desired in many applications such as brakes and it can be undesired in large class of control systems in which friction can deteriorate control performances by introducing tracking errors or limit cycles and shattering.

In order to compensate the friction undesired phenomena, many models were introduced in literature [1]. The first models introduced were the static models which are simple but they don't represent the entire behavior of this friction. For this raison, other dynamic models [2, 3], which were based on bristle models were introduced but they don't render, in presliding regime, both stiction and presliding. That's why, more recent models are introduced [4, 5] to illustrate the friction microscopic behavior. This models was based on experimental results and the hysteresis behavior was approximated.

More recently, a new model known as GMS model (Generalized Maxwell Slip friction model) was introduced [6]. This model is the most realistic because it illustrates the majority of the friction behaviors. Moreover, they have introduced a reduced friction model which is appropriate for simulation and control purposes [7, 8].

In the control systems, modeling is very important but parameter identification is necessary for the compensation of friction. Many works were done in this field. For example, Coulomb friction model was done [9, 10, 11], and since Coulomb friction represent only a part of friction behavior, other works were done for more complicated friction model like Coulomb and viscous friction [12] and the coulomb and viscous friction with Stribeck effect [13, 14]. Identification of the GMS model is not yet well investigated. In fact, only pre-sliding regime was used for identification [15]. Moreover, this identification was based on off line methods like linear regression , dynamic linear regression and nonlinear regression.

In this work, the identification of the GMS friction model is introduced based on both pre-sliding and sliding regime at the same time. Moreover, the proposed identification approach is on line which is very important in control purpose. This identification is based on a robust adaptive observer which is used to estimate the friction force and the unknown parameters.

This paper is organized as follows : in the second section, the GMS friction model is described and some assumptions are given. An adequate formulation of the friction model to be used with a robust observer is presented. In the third section, numerical results and simulations are presented. The final section is about the conclusion.

2. PROBLEM STATEMENT

Consider a single mass subject to an external force as shown in figure 1. Using the Newton's second law, the dynamic

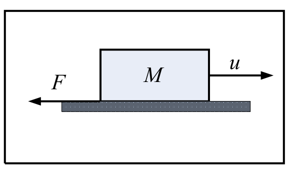


Figure 1. Mass system schema

of the system can be described by the following equation:

$$M\dot{v} = u - F \quad (1)$$

where M is the mass of the system, v is the velocity, u is the external force and F is the frictional force. The frictional force is described by *Generalized Maxwell Slip* model, known as *GMS* model [7]. The GMS model is described by N elasto-slip elements excited by the same input witch is the velocity. The frictional force is then the sum of all elementary forces ([7]).

The GMS model is characterized by two regimes: the first regime is called presliding regime and is defined by the following equation:

$$\frac{dF_i}{dt} = k_i v \tag{2}$$

The model continue to stick until $F_i = \alpha_i s_i(v)$.

The second regime is called sliding regime and is characterized by the following equation:

$$\frac{dF_i}{dt} = sign(v)\alpha_i C\left(1 - \frac{F_i}{\alpha_i s_i(v)}\right) \tag{3}$$

and the model is slipping until the velocity crosses zero.

To switch between the two regimes of the GMS model there is two conditions. In fact, to switch from the presliding regime to the sliding regime, the friction force has to be equal to the quantity $\alpha_i s_i(v)$ and to switch from the sliding regime to the presliding regime, the velocity must cross the value zero.

The total frictional force is then given by

$$F(t) = \sum_{i=1}^{N} F_i(t) + F_v(v) \tag{4}$$

where $F_v(v)$ is the viscous friction.

In a previous work [16], it was supposed that the friction force was known and a new approach for identification of all model parameters based on a robust observer was introduced. In this paper, an other new approach which is based on the same observer but the friction force is considered unknown will be introduced.

It's very difficult to identify the parameters of the GMS model because of the number of the parameters and the nonlinearities in the model. So, to simplify this model, some assumptions are introduced [7] as follow:

Assumption 1 *We suppose that $s_i(v)$ is the same for all elementary models and it will be noted by $S(v)$. This function is the Stribeck function witch describes the static frictional force and is given by*

$$S(v) = \sigma_c sign(v) + (\sigma_s - \sigma_c) sign(v) e^{-\theta_s v^2} \tag{5}$$

where σ_c and σ_s are respectively the Coulomb and the static coefficient. The parameter θ_s is the Stribeck coefficient.

Assumption 2 *To identify the friction model, we have to satisfy the following condition*

$$\sum_{i=1}^{N} \alpha_i = 1 \tag{6}$$

For identification of the GMS unknown parameters, a robust observer is used, for this reason, other assumptions are needed

Assumption 3 *In this paper, we suppose that the model friction is composed by one elementary model, i.e. $N = 1$ so we have $\alpha_1 = 1$.*

Assumption 4 *All model parameters, k, C, σ_c and σ_s are unknown. However, it is assumed that they are constant and bounded.*

Assumption 5 *The velocity $v(t)$ is assumed bounded*

Now based on the assumption 3 and in order to respect the observer conditions, the friction model can be written as follow [16]:

Presliding: In presliding regime , the dynamic of the friction force can be written as follows:

$$\dot{F} = kv \tag{7}$$

if we pose $x = F$ and $\theta_{st} = k$, the equation (7) can be reformulated as follows:

$$\begin{cases} \dot{x} &= \Psi_{st}(u,v)\theta_{st} \\ y &= C_c x \end{cases} \tag{8}$$

where $C_c = 1$ and $\Psi_{st}(u,v) = v$.

Sliding: Based on equations (5) and (3), the dynamic of the friction force is characterized by the following equation:

$$\dot{F} = sign(v)C\left(1 - \frac{F}{S(v)}\right) = \frac{sign(v)}{S(v)} C\left(sign(v)\sigma_c + sign(v)(\sigma_s - \sigma_c)e^{-\theta_s v^2} - F\right) \tag{9}$$

The challenge with the formulation in equation (9) is the nonlinearity over the unknown parameters. However, in order to satisfy the linearity condition of the observer presented in [17], two approximation are introduced:

[A1] the first approximation is to approximate the quantity $\frac{sign(v)}{S(v)}$ by $\frac{1}{g_m}$, where g_m is a constant ($g_m \in [\sigma_c \;\; \sigma_s]$). This approximation is justified because the friction force dynamic (equation (9)) is a first order differential equation with a time constant $\tau(t) = \frac{S(v)}{sign(v)}$ where $v \neq 0$. So the time constant variation is too small when it verifies the following condition: $\tau(t) \in 1/C[\sigma_c \;\; \sigma_s]$ and $\sigma_s - \sigma_c \ll \sigma_c$. The deviation error can be determined by calculating the difference between the equation (9) and the same equation in which the quantity $\frac{sign(v)}{S(v)}$ is replaced by $\frac{1}{g_m}$. This deviation error is given by :

$$F_{d1} = C\left[sign(v) - sign(v)\frac{F}{S(v)} - \frac{S(v)}{g_m} + \frac{F}{g_m}\right] \tag{10}$$

[A2] The second approximation consists on the linearization of the quantity $e^{-\theta_s v^2}$ over a nominal value of θ_s

which is θ_s^N by using the Taylor series. This linearization is given by

$$\begin{aligned} f(\theta_s) &= e^{-\theta_s v^2} = f(\theta_s^N) + \frac{f^{'}(\theta_s^N)}{1!}(\theta_s - \theta_s^N) + F_{d2} \\ &= h_1(u,v) - h_2(u,v)\theta_s + F_{d2} \end{aligned} \quad (11)$$

where $h_1(u,v) = e^{-\theta_s^N v^2} + v^2 e^{-\theta_s^N v^2}\theta_s^N$ and $h_2(u,v) = -v^2 e^{-\theta_s^N v^2}$ are two well known quantities and F_{d2} is the deviation error introduced by the linearization.

Property 1 *The deviation error due to the first approximation **A1** which is F_{d1} is bounded because the friction force is bounded and the Stribeck function $S(v)$ is also bounded. Moreover, the equation given by (10) is bounded when F is bounded.*

Property 2 *The deviation error due to second approximation **A2** which is F_{d2} is bounded because the model parameters and the velocity are bounded.*

The two approximations **A1** and **A2** are used to rewrite the friction force dynamic in sliding regime to get the following equation:

$$\begin{aligned} \dot{F} &= \frac{1}{g_m} C \Big(sign(v)\sigma_c + sign(v)(\sigma_s - \sigma_c)(h_1(u,v) \\ &\quad + h_2(u,v)\theta_s + F_{d2}) - F \Big) + F_{d1} \\ &= -F\frac{C}{g_m} + sign(v)\frac{C\sigma_c}{g_m} + sign(v)h_1(u,v)\frac{C\sigma}{g_m} \\ &\quad + sign(v)h_2(u,v)\frac{C\sigma\theta_s}{g_m} + F_D \end{aligned} \quad (12)$$

where $\sigma = \sigma_s - \sigma_c$ and $F_D = F_{d2}\frac{C\sigma}{g_m}sign(v) + F_{d1}$ is the total deviation error due to the two approximations combined together (**A1** and **A2**).
Based on equation (12) and because of the two approximations **A1** and **A2** applied to the friction dynamic in sliding regime (equation (9)), the equation of the sliding regime is now linear over the unknown parameters. However, due to the two approximations, a bounded perturbation is introduced to the equation of the sliding regime. That's why a robust observer [17] will be used to identify the unknown parameters. In fact, based on the theorem in [17], if the perturbation in the system is bounded and if the condition of the persistence of excitation is verified, the observer will be stable despite there is a bounded perturbation in the system. However, the steady state estimation error will be different from zero because of the bounded perturbation.
Based on equation (12) and by noting

$$\theta_{sl} = \begin{bmatrix} \frac{C}{g_m} & \frac{C\sigma_c}{g_m} & \frac{C\sigma}{g_m} & \frac{C\sigma\theta_s}{g_m} \end{bmatrix}^T \quad (13)$$

$$\Psi_{sl}(u,v) = sign(v)[-sign(v)F,\ 1,\ h_1(u,v),\ h_2(u,v)] \quad (14)$$

if we pose $x = F$, the friction model in the sliding regime can be written as follow :

$$\begin{cases} \dot{x} &= \Psi_{sl}(u,v)\theta_{sl} + F_D \\ y &= C_c x \end{cases} \quad (15)$$

where $C_c = 1$
The two formulations described by the equation (8) and (15) can be combined together using a switching function $Q(t)$ defined by

$$\begin{cases} Q(t) &= 0 \quad \text{if the system is in sliding regime.} \\ Q(t) &= 1 \quad \text{if the system is in presliding regime.} \end{cases} \quad (16)$$

So the formulation of the entire system with the two regimes (sliding and presliding) can be written as follow

$$\begin{cases} \dot{x} &= \psi(u,v)\theta + \omega(t) \\ y &= C_c x \end{cases} \quad (17)$$

where $\theta = [\theta_{sl}^T, \theta_{st}^T]^T$,
$\psi(u,v) = S_{12}(t) = (1 - Q(t))S_1(t) + Q(t)S_2(t)$ and $\omega(t) = (1 - Q(t))F_D$ with $S_1(t) = [\psi_{sl}(u,v) \quad 0]$ and $S_2(t) = [0 \ \ \psi_{st}(u,v)]$.
Because of the switching function $Q(t)$, the two friction regimes can be combined together in one equation which respects the general formulation of the robust observer introduced by Marino and al. [17]. However, to be able to use the observer, the formulation described by the equation (17) should respect an other important condition which is the continuity of the regressor vector $\psi(u,v)$ and the continuity and the bound of the derivate of $\psi(u,v)$. unfortunately, because of the sign operator in the regressor vector and the switching function $Q(t)$, the condition is not respected. Moreover, because of the two approximations **A1** and **A2**, the formulation of the system is not continue so the condition of continuity is not respected.
To solve the problem of continuity, a second order filter is applied to the system. In fact, when this filter is applied to both sides of the system equation, there is no effect on the equation it self but all the discontinuity of signals are eliminated. The filter is chosen of second order because the condition concerns the continuity of regressor vector and the continuity of its derivate.
So to guarantee the condition of continuity, two low pass first order filters in cascade are used. their cut off frequencies are defined by ω_{c1} and ω_{c2}. The transfer function of each filter is defined as follow:

$$H_i(s) = \frac{\omega_{ci}}{s + \omega_{ci}} \quad \text{for} \quad i = 1,2 \quad (18)$$

When these filters are applied, the equation of the friction model can be written as follow:

For the first stage of the filter

$$\begin{cases} \dot{x}_f &= \psi_f(u,v)\theta + \omega_f(t) \\ y_f &= C_c x_f \end{cases} \quad (19)$$

For the second stage of the filter

$$\begin{cases} \dot{x}_F &= \psi_F(u,v)\theta + \omega_F(t) \\ y_F &= C_c x_F \end{cases} \quad (20)$$

where f and F indicate that the corresponding variable is filtered respectively by the first stage and the second stage of the filter.
In this work, the friction force is assumed unknown so an other problem is added. In fact, the friction force is the first element of regressor vector $\psi(u, v)$ and based on equation (20), the regressor vector is filtered to guarantee the continuity condition and since the friction force is unknown, it is not possible to calculate the first element of the regressor vector.
To solve this problem, a special filtering procedure is introduced: the filtered friction force is calculated from a known signals which are the velocity (v) and the input (u). This procedure is detailed in the following lemma:

Lemma 1 *For the system presented in figure 1, the filtered friction force by the first stage of the filter $H_1(s)$ can be obtained from well known signals v and u (see figure 3).*

Proof 1 *the dynamic equation of the system presented by the figure 1 is given by*

$$M\dot{v} = u - F \tag{21}$$

so the friction force is given by

$$-F = M\dot{v} - u \tag{22}$$

If the first order filter is applied (see figure 2), the filtered friction force in the Laplace domain is given by

$$\begin{aligned} -F_f &= \frac{M\omega_{c1}}{s+\omega_{c1}}\dot{v} - \frac{\omega_{c1}}{s+\omega_{c1}}u \\ &= \frac{M\omega_{c1}sv}{s+\omega_{c1}} - \frac{\omega_{c1}}{s+\omega_{c1}}u \\ &= M\omega_{c1}v - \frac{\omega_{c1}u + \omega_{c1}^2 Mv}{s+\omega_{c1}} \end{aligned} \tag{23}$$

if we pose $R(s)$ the state variable in Laplace domain defined by

$$R(s) = \frac{-\omega_{c1}u(s) - \omega_{c1}^2 Mv(s)}{s+\omega_{c1}} \tag{24}$$

then, in time domain, we get

$$\dot{r} + \omega_{c1}r = -\omega_{c1}u - \omega_{c1}^2 Mv \tag{25}$$

and the time derivate of the state variable is given by

$$\dot{r} = -\omega_{c1}r - \omega_{c1}u - \omega_{c1}^2 Mv \tag{26}$$

and if $y_1 = -F_f$ is the output of the system, then the filtered friction force can be given by

$$-F_f = y_1 = r + M\omega_{c1}v \tag{27}$$

So, the state equation of the filtered friction force is obtained from the input of the system u and the velocity v and it is given by

$$\begin{cases} \dot{r} &= -\omega_{c1}r - \omega_{c1}u - \omega_{c1}^2 Mv \\ y_1 &= -F_f = r + M\omega_{c1}v \end{cases} \tag{28}$$

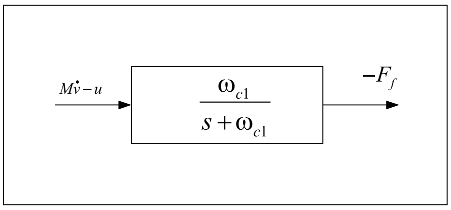


Figure 2. Filtered friction force schema

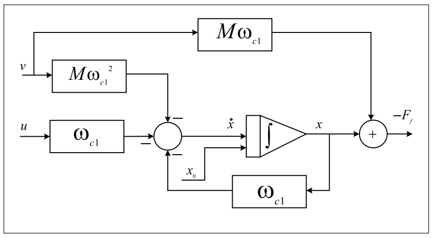


Figure 3. Filtered friction force construction schema

Now, and based on lemma 1, it is possible to determine respectively the first and second stage of the filtered friction force F_f and F_F using known signals as the input u and the velocity v. However, an other problem is raised in the first filtering stage of the regressor vector. In fact, due to the switching function $Q(t)$, there is a commutation between the value of the friction force F and zero. This commutation should be continuous but it's not because the friction force is unknown. Moreover, even if the filtered friction force is build, the continuity condition will be still not verified. The solution of this problem is given in the following lemma:

Lemma 2 *Using appropriate initial conditions, the signal $S_{12f}(t)$ can be obtained either by the approach applied in figure 4 or by the approach presented in figure 5.*

Proof 2 *the detailed proof is given in previous work [16]*

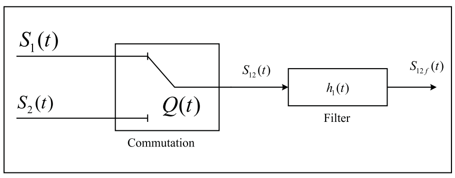


Figure 4. The first filter application approach

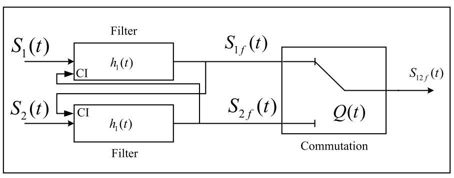


Figure 5. The second filter application approach

Based on the lemma 2, to determine the filtered regressor vector, the second approach (in figure 5) is used for

the first element of the vector which is the filtered friction force by the first stage of the filter. The filtered friction force is constructed using the two signals: the input u and the velocity v (as shown in figure 3). For the other elements of the regressor vector, the first approach (in figure 4) is used.
With the approach of filtering, the robust observer introduced by Marino and al. [17] can be applied because all its conditions are verified. So, the observer can be build as follows

$$\begin{cases} \dot{\hat{x}} &= -KC_c\hat{x} + \psi_F(u,v)\hat{\theta} + Ky_F \\ \dot{\hat{\theta}} &= \Gamma\psi_F(u,v)^T(y_F - C_c\hat{x}) \end{cases} \quad (29)$$

where K and Γ are the observer Gains. The equation of the estimation error dynamic is defined by :

$$\begin{cases} \dot{\tilde{x}} &= -KC_c\tilde{x} + \psi_F(u,y)\tilde{\theta} - \omega_F(t) \\ \dot{\tilde{\theta}} &= -\Gamma\psi_F(u,y)^T C_c\tilde{x} \end{cases} \quad (30)$$

where $\tilde{x} = \hat{x} - x_F$ and $\tilde{\theta} = \hat{\theta} - \theta$.

3. SIMULATION RESULTS

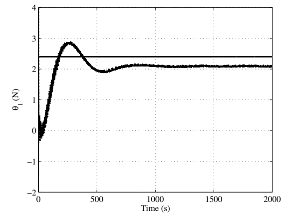
Table 1. Parameters values for the simulation

Parameter	Value	Unit
σ_c	10	N
σ_s	15	N
C	24	N/s
θ_s	10^4	s^2/m^2
M	1	Kg
k	5×10^5	N/m
ω_{c1}	10	rad/s
ω_{c2}	100	rad/s

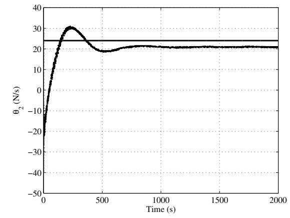
Table 2. Unknown parameters values

Parameter	Real Value	initial Value	Unit
θ_1	2.4	1.2	s^{-1}
θ_2	24	12	N/s
θ_3	12	6	N/s
θ_4	12×10^4	9×10^4	Ns/m^2
θ_5	5×10^5	2.5×10^5	N/m

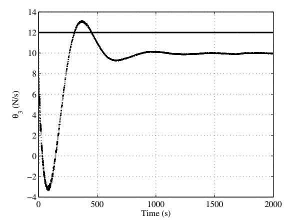
In order to illustrate simulation results and validate the proposed approach to identify the GMS friction model, we have used the parameter simulation values shown in table 1, the unknown parameters with the chosen initial conditions given in table 2 and the input of the system that give the following velocity $v = 0.01sin(0.2\pi t + 0.025)sin(2\pi t)$. Note also, that the initial condition of the state variables and the estimated state variables are taken equal to zero.
For simplicity, the switching functions are assumed ideal. That's means that the instants of commutation from the sliding regime to the presliding regime and vice versa are assumed known. This assumption is not realistic because the switching function is based on the unknown parameters but a deep study of this function will be published promptly.



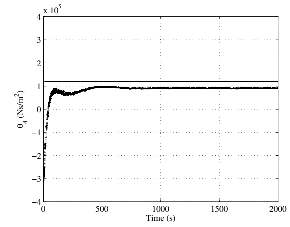
(a): Estimated and real value of θ_1



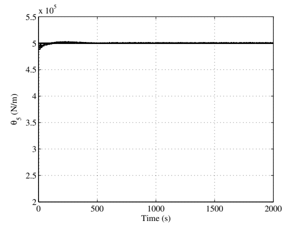
(b): Estimated and real value of θ_2



(c): Estimated and real value of θ_3

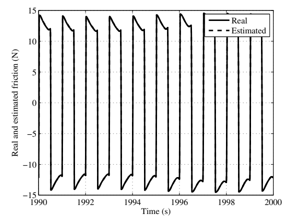


(d): Estimated and real value of θ_4

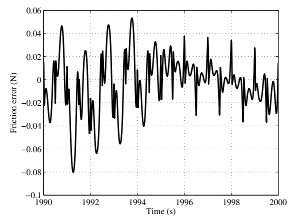


(e): Estimated and real value of θ_5

Figure 6. Estimated and real values of the unknown parameters.



(a)



(b)

Figure 7. (a): Estimated and real friction forces. (b): Friction error

As shown in Figure 6, the estimation values of the parameters converge to the real values but with some deviation. This deviation is due to the perturbation F_D introduced by the two approximations **A1** and **A2**. In fact, as proved by Marino and al. [17], when the system has a bounded perturbation, the estimation errors are attracted in open balls centered in the origin and with a radii affected by the amplitude of the perturbation. However, despite of the deviation of the estimated parameters, the estimated friction force is too close to real force (see Figure 7 (a)) and, in term of estimation error (figure 7 (b)), the amplitude is less then 0.2% $(0.04N)$ of the friction force amplitude $(15N)$. Those results show that the proposed identification approach is very efficient not only for the identification of the unknown parameters of the friction model but also for the friction force.

4. CONCLUSION

In this paper, an online identification approach of the GMS friction model, for both pre-sliding and sliding regime, based on a robust adaptive observer is presented. Two approximations are introduced to avoid nonlinearities in the model over the unknown parameters. Furthermore, as the friction force is assumed unknown, a new approach is introduced to determine the filtered friction force from the measurable signals. Finally, a filtering approach is introduced to discard discontinuities in signals and to access to unknown signals. Simulation results illustrates that the observer is able to identify the unknown parameters and to estimate the friction force even though it was based on an approximated model.

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