Proceedings of the 8th Conference on Nuclear and Particle Physics,
20-24 Nov. 2011, Hurghada, Egypt

ALPHA DECAY OF EVEN-EVEN SUPERHEAVY NUCLEI

M.R. Oudih[1], Y. Hamza[1], M. Fellah[1,2] and N.H. Allal[1,2]

[1]*Laboratoire de Physique Théorique, Faculté de Physique, BP 32 Al-Alia, 16111 Bab-Ezzouar, Algiers, Algeria*
[2]*Centre de Recherche Nucléaire d'Alger, 2 Bd Frantz Fanon, BP399 Alger-Gare, Algiers, Algeria*

Alpha decay properties of even-even superheavy nuclei with 112≤Z≤120 have been investigated using the Hartree-Fock-Bogoliubov approach. The method is based on the SkP Skyrme interaction and the Lipkin-Nogami prescription for treating the pairing correlations. The alpha decay energies are extracted from the binding energies and then used for the calculation of the decay half-lives using a formula similar to that of Viola-Seaborg. The parameters of the formula were obtained through a least square fit to even-even heavy nuclei taken from the tables of Audi-Wapstra and some more recent references. The results are compared with other theoretical evaluations.

Keywords: *Alpha decay, Hartree-Fock-Bogoliubov, Super heavy nuclei*

INTRODUCTION

According to quantum electrodynamics, the concept of the atom as a system composed of a nucleus and the electrons that orbit around it, is valid for very heavy atoms up to atomic number Z = 170. In fact, the limit of the existence of atoms is reached much earlier due to the instability of the nucleus itself. Indeed, among 3000 nuclei known to date, only 287 nuclei exist in nature. The excess of neutrons in a nucleus leads to the reduction of the neutron separation energy, Sn. The limit is then achieved when Sn = 0 (neutron dripline). Similarly, a zero binding energy of the proton (Sp = 0) determines the limit of the existence of proton-rich nuclei (proton dripline). Another limitation to the existence of superheavy nuclei is associated with the maximum number of nucleons which can form the nucleus. Changing the ratio between the number of protons and neutrons in the nucleus leads to the β or α decay.

Alpha decay is one of the most important properties of atomic nuclei. It is become, recently, a particularly powerful tool for the study of nuclei at the limit of stability (drip line), the closed shell nuclei and of heavy and superheavy nuclei. For the latter, the α decay plays a key role since it determines the limit of their existence and allows to identify new elements.

In the present work, the Hartree-Fock-Bogoliubov theory with SkP Skyrme interaction is used to study the decay energy Q_α and half-life of even-even superheavy nuclei considered to be of axial symmetry.

FORMALISM

In the Hartree-Fock-Bogoliubov (HFB) theory, we start with the Hamiltonian

$$\hat{H} = \sum_{\nu\mu} \varepsilon_{\nu\mu}\, a_\nu^+ a_\mu + \frac{1}{4} \sum_{\nu\mu\beta\gamma} v_{\nu\mu\beta\gamma}\, a_\nu^+ a_\mu^+ a_\gamma a_\beta \qquad (1)$$

where

$$\varepsilon_{\nu\mu} = \langle \nu | \hat{T} | \mu \rangle \qquad (2)$$

are the kinetic energy matrix-elements and the quantities

$$v_{\nu\mu\beta\gamma} = \langle \nu\mu | \hat{V} | \beta\gamma \rangle \qquad (3)$$

represent the two body matrix-elements of an effective interaction supposed to be density dependent.

The method is based on the variational approach where the trial wave-function is considered to be a quasiparticle vacuum

$$|\Psi\rangle = \Big(\prod_\nu \alpha_\nu \Big) |0\rangle, \quad \alpha_\mu |\Psi\rangle = 0, \ \forall \mu \qquad (4)$$

The destruction and creation operators α_ν and α_ν^+ of quasiparticles are defined by the canonical Bogoliubov transformation:

$$\begin{pmatrix} \alpha \\ \alpha^+ \end{pmatrix} = \begin{pmatrix} U^+ & V^+ \\ V^T & U^T \end{pmatrix} \begin{pmatrix} a \\ a^+ \end{pmatrix} = W^+ \begin{pmatrix} a \\ a^+ \end{pmatrix} \qquad (5)$$

where the matrices U and V verify the properties

$$U^+ U + V^+ V = I, \quad U U^+ + V^* V^T = I, \quad U^T V + V^T U = 0, \quad U V^+ + V^* U^T = 0.$$

By using the Wick's theorem the energy reads:

$$E(\rho,\kappa) = \frac{\langle \Psi | \hat{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \mathrm{tr}\left(\left(\varepsilon + \frac{1}{2}\Gamma \right) \cdot \rho \right) - \frac{1}{2} \mathrm{tr}\left(\Delta \cdot \kappa^* \right) \qquad (6)$$

which is a functional of the density matrix ρ and the pairing tensor κ defined as:

$$\begin{aligned} \rho_{\nu\mu} &= \langle \Psi | a_\mu^+ a_\nu | \Psi \rangle = \left(V^* \cdot V^T \right)_{\nu\mu} \\ \kappa_{\nu\mu} &= \langle \Psi | a_\mu a_\nu | \Psi \rangle = \left(V^* \cdot U^T \right)_{\nu\mu} = -\left(U \cdot V^+ \right)_{\nu\mu} \end{aligned} \qquad (7)$$

where

$$\Gamma_{\nu\beta} = \sum_{\mu\gamma} v_{\nu\mu\beta\gamma}\ \rho_{\gamma\mu}$$

$$\Delta_{\nu\mu} = \frac{1}{2}\sum_{\beta\gamma} v_{\nu\mu\beta\gamma}\ \kappa_{\beta\gamma}$$

The energy variation with respect to ρ and κ leads to the Hartree-Fock-Bogoliubov equations:

$$\begin{pmatrix} \varepsilon + \Gamma - \lambda & \Delta \\ -\Delta^{*} & -(\varepsilon + \Gamma)^{*} + \lambda \end{pmatrix} \begin{pmatrix} U \\ V \end{pmatrix} = E \begin{pmatrix} U \\ V \end{pmatrix} \tag{8}$$

where E are the quasiparticle energies and λ is the chemical potential introduced as a Lagrange multiplier to ensure an average particle number conservation.
For the Skyrme interaction the total energy of a nucleus is expressed as the volume integral [1]:

$$E[\rho,\kappa] = \int d\vec{r}\ \left(\mathrm{H}(\vec{r}) + V_{pair}(\vec{r}) \right), \tag{9}$$

with

$$\begin{aligned} \mathrm{H}(\vec{r}) = {} & \tfrac{\hbar^2}{2m}\tau + \tfrac{1}{4}t_0\left[\left(2+x_0\right)\rho^2 - \left(1+2x_0\right)\left(\rho_n^2+\rho_p^2\right) \right] \\ & + \tfrac{1}{8}\left[t_1\left(2+x_1\right)+t_2\left(2+x_2\right)\right]\tau\rho + \tfrac{1}{8}\left[t_2\left(2x_2+1\right)-t_1\left(2x_1+1\right)\right]\left(\tau_n\rho_n+\tau_p\rho_p\right) \\ & + \tfrac{1}{32}\left[3t_1\left(2+x_1\right)-t_2\left(2+x_2\right)\right]\left(\vec{\nabla}\rho\right)^2 - \tfrac{1}{32}\left[3t_1\left(2x_1+1\right)+t_2\left(2x_2+1\right)\right]\left[\left(\vec{\nabla}\rho_n\right)^2+\left(\vec{\nabla}\rho_p\right)^2\right] \\ & + \tfrac{1}{24}t_3\rho^{\alpha}\left[\left(2+x_3\right)\rho^2-\left(2x_3+1\right)\left(\rho_n^2+\rho_p^2\right)\right] - \tfrac{1}{16}\left(t_1x_1+t_2x_2\right)\vec{J}^2 + \tfrac{1}{16}\left(t_1-t_2\right)\left(\vec{J}_n^2+\vec{J}_p^2\right) \\ & + \tfrac{1}{2}W_0\left[\vec{J}\cdot\vec{\nabla}\rho + \vec{J}_n\cdot\vec{\nabla}\rho_n + \vec{J}_n\cdot\vec{\nabla}\rho_n\right] \end{aligned}$$

where $\{t_i, x_i, \alpha, W_0\}$ are the force parameters. There are several sets of parameters usually adjusted to the different mass regions or to different observables. The most used sets are, e.g., SkM*, SkP, or Sly4 [2].
The pairing interaction $V_{pair}(\vec{r})$ is considered to be density dependent [3],

$$V_{pair}(\vec{r}) = V_0\left[1-\eta\left(\frac{\rho(\vec{r})}{\rho_0}\right)^{\gamma}\right]\delta\left(\vec{r}_1-\vec{r}_2\right) \tag{10}$$

which has been proposed particularly for use in very neutron rich nuclei. ρ_0 is the saturation density. The pairing strength V_0 is adjusted phenomenologically to reproduce the odd-even staggering of energies in selected chains of nuclei.
In order to correctly treat the pairing correlations in the vicinity of shell closure, a particle-number projection of the wave function has been carried out by the Lipkin-Nogami method [4].

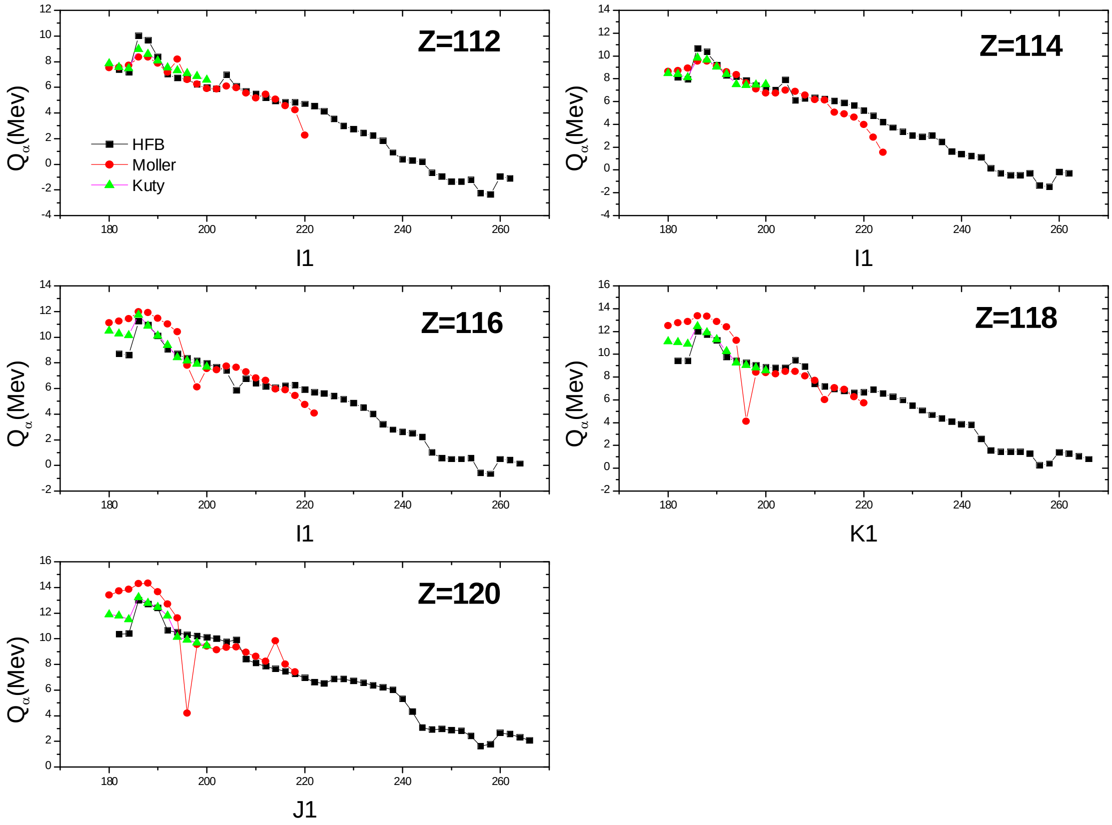

Figure 1. Variation of the α-decay half-lives as a function of neutron number for Z = 112-120.

RESULTS AND DISCUSSION

Among the frequently used interactions in the HFB theory (SkP, SKM* and SLy4), the SkP interaction is the most adapted for the calculation of the binding energy for the neutron rich nuclei. In what follows, the calculations were then carried out by using the SkP parameters.

Initially, the alpha decay energy Q_α of superheavy nuclei with Z = 112–120 toward the neutron dripline is determined by using the binding energy values obtained from the HFB approach:

$$Q_\alpha = BE(A-4, Z-2) + BE(4,2) - BE(A,Z) \qquad (11)$$

In figure 1 we compare our results with Q_α values extracted from the Moller-Nix-Kratz mass tables [5] and mass estimates of Koura, Tachibana, Uno and Yamada (KUTY) [6].

It appears that the results obtained with the three methods show quite similar trends: a sudden increase in the decay energy around N □ 184 or 186 as predicted by many previous calculations. This may indicate the signature of sub-shell closure around this number of neutrons. For N>186 the Q(values for fixed Z decrease with increasing neutron number.

A discrepancy appears between our values and those of Moller for N~ 220 which seems to increase with the number of neutrons.

In a second step, the obtained decay energies are used for the calculation of the decay half-lives using a formula similar to that of Viola-Seaborg [7]:

$$\log_{10} T_\alpha = a\left(Z_d Q_\alpha^{1/2} - Z_d^{2/3}\right) + b \qquad (11)$$

where T_α is in seconds and Q_α in MeV, and Z_d is the atomic number of the daughter nucleus. The parameters of the formula (a=*1.54* et b = 19.30) were obtained through a least square fit to even-even heavy nuclei taken from the tables of Audi-Wapstra [8] and some more recent references [9-11].

The calculated α-decay half-lives are shown in Figure 2 with increasing neutron number. On the figure we compare our results with those calculated with Q_α values deduced from Moller tables using the Viola- Seaborg systematic. We also compare our results with other theoretical evaluations by Chowdhury, et al. [12]. The latter are obtained from the WKB barrier penetration probability using theoretical mass estimates of KUTY.

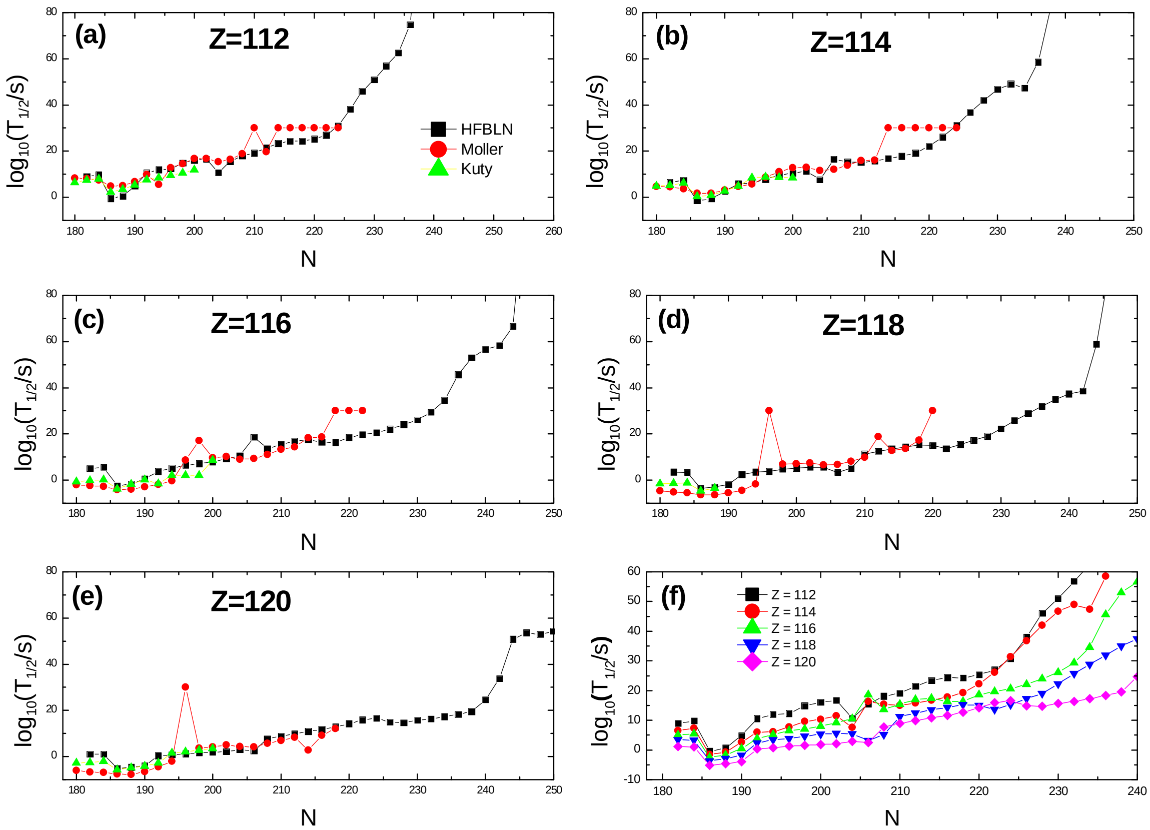

Figure 2. Variation of the α-decay half-lives as a function of neutron number for Z = 112-120

Figures 2.a–2.e show that our results are in good agreement with those of Chowdhury and those calculated with Q_α values deduced from Moller tables with the exception of a few nuclei. It appears that the half-life for fixed Z increases with increasing neutron number. Obviously a sudden fall in T_α appears around N = 184 for the studied elements as a result of the increase in the decay energy.

Finally, one can see from fig2-f that the present results do not indicate an island of increased stability around Z=114 as it is expected by the nuclear shell model and the macroscopic-microscopic model [13-15]. In general the α-decay half lives decrease with increasing Z up to Z=120.

CONCLUSION

In conclusion, the alpha decay properties of 220 even-even superheavy nuclei with $112 \leq Z \leq 120$ in the neutron-rich domain have been investigated using the Hartree-Fock-Bogoliubov approach. The method is based on the SkP Skyrme interaction and the Lipkin-Nogami treatment of the pairing interaction. For most nuclei, the obtained α-decay energy agree well with the theoretical Q values extracted from the Moller-Nix-Kratz and Koura et al. mass tables.

The obtained decay energies were then used to predict the decay half-lives based on a formula similar to that of Viola-Seaborg. The calculated results are in rather good agreement with those obtained from the same formula based on Q values deduced from Moller tables and the decay half-lives obtained in the PCM framework by using Q values from the KUTY mass estimates.

It seems from the present calculation that an island of stability may exist due to a shell effect (shell closures) near N □ 184 with Z = 112–120.

REFERENCES

[1] J. Dobaczewski, W. Nazarewicz, T. R. Werner, J. F. Berger, C. R. Chinn, and J. Dechargé Phys. Rev. C **53**, 2809 (1996).

[2] E. Chabanat, P. Bonche, P. Haensel, J. Meyer and R. Schaeffer, Nucl. Phys. **A635**, 231(1998).

[3] M. Baldo, U. Lombardo, E.E. Saperstein and M.V. Zverev, Phys. Rep. **391**, 261(2004).

[4] M.R. Oudih, M. Fellah, N.H. Allal and N. Benhamouda, Phys. Rev. C **76**, 047307 (2007).

[5] P. Moller, J.R. Nix and K.-L. Kratz, *At. Data Nucl. Data Tables* **66**, 131 (1997).

[6] H. Koura, T. Tachibana, M. Uno and M. Yamada, KUTY mass formula 2005 revised version, *Prog. In Theor. Phys.* **113**, 305 (2005). Available from: http://wwwndc.tokai-sc.jaea.go.jp/nucldata/mass/KTUY05_m246.dat

[7] K.P. Santhosh, R.K. Biju and Antony Joseph, J. Phys. **G**.: Nucl. Part. Phys. **35**, 085102 (2008).

[8] G. Audi, A. -H. Wapstra, and C. Thibault, Nucl. Phys. **A729**, 337 (2003).

[9] P. Roy Chowdhury, C. Smanta and D.N. Basu, Atom. Data Nucl. Data Tables **94**, 781 (2008).

[10] N. Dasgupta-Schubert and M. A. Reyes, Atom. Data Nucl. Data Tables **93**, 907 (2007).

[11] G.Royer and H.F. Zhang, Phys. Rev. C **77,** 037602 (2008).

[12] P. Roy Chowdhury, C. Samanta and D.N. Basu, Atomic Data and Nuclear Data Tables **94**, 781 (2008).

[13] P. Möller and J.R. Nix, J. Phys. **G**.: Nucl. Part. Phys. **20**, 1681 (1994).

[14] I. Muntian, Z. Patyk and A. Sobiczewski, Phys. At. Nucl. **66**, 1015 (2003).

[15] A. Baran, Z. Ljewski, K. Sieja and M. Kowal, Phys. Rev. C **72**, 044310 (2005).