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CALCULATION OF REFLECTANCE AND TRANSMITTANCE OF COATING WITH OPTICALLY ROUGH SURFACES

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A two-flux model for source-free anisotropic scattering rough surfaces is derived using Pomraning-Eddington approximation for the transport equation. Relations have been derived between Kubelka-Munk coefficients (K) and (S) and the transport parameters σ_s, σ_a and g = $<\mu>$. The problem of a collimated source is linked to the solution of the source-free problem. The reflectance and transmittance of surfaces with arbitrary roughness are calculated and compared with the available data.

INTRODUCTION

For ideal surfaces, components of the reflected beam are related to the components of the incident beam by Fresnel reflection equation. The surfaces encountered in engineering applications deviate from ideal as a result of roughness, oxidization and contamination; hence the Radiative properties of these real surfaces differ greatly from those predicted by electromagnetic theory.

In regard to problems of radiative heat transfer; the roughness of surfaces may be divided into two categories: (1) small surface irregularities such that the incident radiation cannot undergo more than a single reflection, (2) deep cavities in which the incident radiation undergoes multi-reflection.

The normally incident radiation from rough surface having small irregularities is reflected partly specularly and partly diffusely [1].

Kubelka-Munk theory (K-M) [2] describes optical characteristics (e.g. reflectance, transmittance and absorbance) by a variety of light scattering media including paints, textiles and papers, and It's widely used in various industrial applications. Moder developments in radiative transfer theory (RTT) enable the derivation of (K-M) parameters from first principles [3].

Kubelka and Munk proposed a theory based on a model of two light fluxes travelling in the forward and backward directions. Subsequently a number of authors refined the theory and compared it with experimental data [4]. Several authors attempted to relate the Kubelka-Munk coefficients to the transport coefficients [5,6]

In this article, the forward and backward solution of the problem to radiation fluxes at the two boundaries of the rough surfaces illuminated by a collimated light are linked to the solution of the source-free problem. The source-free radiative transfer equation is solved using the Pomranning-Erdington approximation which changes it to a two-flux problem. The two-flux problem is linked to the (K-M) model to give relations between the radiative transfer optical coefficient for linear anisotropic scattering and the K-M optical coefficient. Two kinds of reflection coefficient are considered: (i) Fresnel reflection of smooth surfaces of biological tissues, (ii) reflection coefficient of rough surfaces for both collimated and diffused incident radiation.

The reflectance and transmittance of surface with these kinds of reflection coefficients at the boundaries are calculated and compared with available published data.

ANALYSIS

Radiative transfer in an inhomogeneous finite plane parallel medium with anisotropic scattering of rough surface are described by the equation [1]

$$\mu d\psi(z,\mu)/dz + \sigma_t(z)\psi(z,\mu) = \frac{\sigma_s(z)}{2}\int_{-1}^{1} p(\mu,\mu')\psi(z,\mu')d\mu' \qquad 0 \le z \le z_0, -1 \le \mu \le 1 \tag{1}$$

Where $\psi(z,\mu)$ is the radiance at depth z at the angle of propagation specified by its cosine $\mu, \sigma_t(z) = \sigma_a(z) + \sigma_s(z)$, is the total scattering cross-section , $\sigma_s(z)$ is the scattering cross-section and $\sigma_a(z)$ is the absorption cross-section . The phase function which describes the probability of scattering from μ to μ' is $p(\mu,\mu')$ and has the form, for anisotropic scattering approximation,

$$p(\mu,\mu') = 1 + \bar{a}\mu\mu' \tag{2}$$

Where $\bar{a} = 3\bar{g}$, $\bar{g} = \langle\mu\rangle$ is the average cosine of the scattering angle.

The diffused radiance $I_d(z,\mu)$ through the medium is described by the radiative transfer equation

$$\mu dI_d(z,\mu)/dz + \sigma_t(z)I_d(z,\mu) = \frac{\sigma_s(z)}{2}\int_{-1}^{1} p(\mu,\mu')I_d(z,\mu')d\mu' + Q_c(z,\mu) \tag{3}$$

The source due to collimated illumination is

$$Q_c(z,\mu) = \frac{\sigma_s(z)}{2} f_c \exp(-\tau(z)/\mu_0')p(\mu,\mu_0') \tag{4}$$

Where μ_0' is cosine of the refractive angle of the collimated beam and given by Snell's law. The irradiance of the collimated incident on the left boundary just inside the medium is

$$f_c = f_{inc}^c(1-\rho_{1e}^c), \tag{5}$$

f_{inc}^c is the radiance of the external collimated incidence and ρ_{1e}^c is the external specular reflectance of specified rough surface .The optical thickness is

$$\tau(z) = \int_0^z dy\ \sigma_t(y) \tag{6}$$

Equation (3) is solved for the following boundary conditions:

$$I_d(0,\mu) = f_d + \rho_{1i}^d I_d(0,-\mu) \quad , \mu \ge 0 \tag{7}$$

$$I_d(z_0,-\mu)=\rho_{2i}^d\, I_d(z_0,\mu) \qquad ,\mu\geq 0 \tag{8}$$

Where f_d is the diffused incident radiance just inside the left boundary of the medium

$$f_d = f_{inc}^d(1-\rho_{1e}^d) \tag{9}$$

where f_{inc}^d is the external diffused incident radiance on the boundary and ρ_{1e}^d is the external specular reflectivity of the diffused light of some rough surface. ρ_{1i}^d and ρ_{2i}^d are the internal diffused reflectivities of the left and right boundaries of the medium respectively.

The Pomraning-Eddington approximation is used to solve the source-free radiative transfer equation (1) to obtain the forward and backward fluxes at the boundaries $\psi^+(z_0)$ and $\psi^-(0)$ respectively [6,7].

Where:

$$\psi^+(z)=\int_0^1 \psi(z,\mu)\,\mu\, d\mu, \tag{10}$$

$$\psi^-(z)=\int_0^1 \psi(z,-\mu)\,\mu\, d\mu \tag{11}$$

According to the approximation, $\psi(z,\mu)$ is considered as

$$\psi(z,\mu)=E(z)\,\varepsilon(z,\mu)+F(z)\,O(z,\mu) \tag{12}$$

where $E(z)$, $F(z)$, $\varepsilon(z,\mu)$, and $O(z,\mu)$ are defined as

$$\varepsilon(z,\mu) = [\omega(z)/2]\,[1+\bar{a}\,\alpha(z)\mu^2]/(1-\nu^2(z)\mu^2), \tag{13}$$

$$O(z,\mu) = [\omega(z)/2]\,[\nu^2(z)/\alpha(z)+\bar{a}]\,\mu/[1-\nu^2(z)\mu^2], \tag{14}$$

where $\omega(z)$ is the single scattering albedo defined as

$$\omega(z) = \sigma_s(z)/\sigma_t(z), \tag{15a}$$

and

$$\alpha(z) = \sigma_a(z)/\sigma_t(z), \tag{15b}$$

$$\nu^2(z)=\alpha(z)\beta(z)/D\,, \tag{15c}$$

$$\beta(z) = 1-\bar{a}\,\omega(z)/3 \tag{15d}$$

and the slowly varying function D is defined by

$$D=\int_{-1}^1 d\mu\,\mu^2\,\varepsilon(z,\mu) \tag{16}$$

The parameter $\nu(z)$ is defined as the solution of the transcendental equation

$$[2\nu(z)/\omega(z)]\,Y(z) = \mathrm{Ln}\{[1+\nu(z)]/[1-\nu(z)]\} \tag{17a}$$

where

$$Y(z) = [\nu^2(z)+\bar{a}\,\alpha(z)\omega(z)]/[\nu^2(z)+\bar{a}\,\alpha(z)] \tag{17b}$$

The energy fluence rate (the space irradiance) $E(z)[W\ cm^{-2}]$ is defined as

$$E(z)=\int_{-1}^1 d\mu\,\psi(z,\mu)=[\psi^+(z)+\psi^-(z)]/2\chi, \tag{18-a}$$

while the net flux $F(z)[W\ cm^{-2}]$ is defined as

$$F(z)=\int_{-1}^{1} d\mu\,\mu\psi(z,\mu)=[\psi^{+}(z)-\psi^{-}(z)], \qquad (18\text{-b})$$

where the slowly varying function χ is defined by

$$\chi=\int_{0}^{1} d\mu\,\mu\,\varepsilon(z,\mu). \qquad (19)$$

The solution of equation (3) is linked with the solution of source-free problem by the relations

$$F_d^{-}(0)=f_d\,\psi^{-}(0)+Q_1 \qquad (20)$$

$$F_d^{+}(z_0)=f_d\,\psi^{+}(z_0)+Q_2 \qquad (21)$$

Where $F_d^{+}(z_0)$ and $F_d^{-}(0)$ are the forward and backward diffused radiation fluxes at the boundary of the medium with collimated source, and

$$Q_1=\int_{-1}^{1} d\mu\int_{0}^{z_0} dz\,Q_c(z,\mu)\psi(z,-\mu)\;, \qquad (22)$$

and

$$Q_2=\int_{-1}^{1} d\mu\int_{0}^{z_0} dz\,Q_c(z,\mu)\psi(z_0-z,\mu) \qquad (23)$$

Where $\psi(z,\mu)$ is the radiance of the corresponding source-free problem.
The differential (K-M) equations describing the energy balance between diffuse light at the forward and backward direction are

$$d\psi^{+}(z)/dz=-[K(z)+S(z)]\psi^{+}(z)-S(z)\psi^{-}(z) \qquad (24)$$

$$d\psi^{-}(z)/dz=[K(z)+S(z)]\psi^{-}(z)-S(z)\psi^{+}(z) \qquad (25)$$

The forward and backward radiation fluxes $\psi^{+}(z)$ and $\psi^{-}(z)$ have been derived from equations (24) and (25) as:

$$\psi^{+}(z)=c_1\,sinh(Q_0z)+c_2\,cosh(Q_0z) \qquad (26)$$

and

$$\psi^{-}(z)=c_1[a_k\,sinh(Q_0z)+b_k\,cosh(Q_0z)]+c_2[a_k\,cosh(Q_0z)+b_k\,sinh(Q_0z)] \qquad (27)$$

Where $a_k=(K+S)/S\;,\qquad b_k=\sqrt{a_k^{\,2}-1}=Q_0/S \qquad (28)$

Using the boundary conditions of the source-free problem leads to

$$c_1=\left\{0.5\left[\left(\rho_{2i}^{d}-a_k\right)Cosh\left(Q_0z_0\right)-b_kSinh\left(Q_0z_0\right)\right]\right\}\Big/\phi \qquad (29)$$

and

$$c_2=\left\{0.5\left[\left(a_k-\rho_{2i}^{d}\right)Sinh\left(Q_0z_0\right)+b_kCosh\left(Q_0z_0\right)\right]\right\}\Big/\phi \qquad (30)$$

where

$$\phi=\left[a_k\left(1+\rho_{1i}^{d}\rho_{2i}^{d}\right)-\rho_{1i}^{d}-\rho_{2i}^{d}\right]Sinh\left(Q_0z_0\right)+b_k\left(1-\rho_{1i}^{d}\rho_{2i}^{d}\right)Cosh\left(Q_0z_0\right) \qquad (31)$$

From the definition of the reflectivity R and transmitivity T

$$R=\int_{-1}^{1} d\mu\,\mu\,I_{reflected}\left(0,-\mu\right)/\int_{0}^{1} d\mu\,\mu\,I_{incident}\left(0,\mu\right) \qquad (32)$$

$$T=\int_{-1}^{1} d\mu\,\mu\,I_{trancemitted}\left(z_0,\mu\right)/\int_{0}^{1} d\mu\,\mu\,I_{incident}\left(0,\mu\right) \qquad (33)$$

Then

$$R = \{\left(1-\rho_{1i}^{d}\right)F_d^{-}(0) + 0.5 f_{inc}^{d}\,\rho_{1e}^{d} + \mu_0 f_{inc}^{c}\,\rho_{1e}^{c}(\mu_0)\} / \{0.5 f_{inc}^{d} + \mu_0 f_{inc}^{c}\} \qquad (34)$$

$$T = \{\left(1-\rho_{2i}^{d}\right)F_d^{+}(z_0) + \left(1-\rho_{2i}^{c}\left(\mu_0'\right)\right)\mu_0' f_c\, e^{-\sigma_t z_0/\mu_0'}\} / \{0.5 f_{inc}^{d} + \mu_0 f_{inc}^{c}\} \qquad (35)$$

The reflection functions of rough surfaces are considered for collimated and diffused incident radiation respectively.
He XD et al [8] gave expressions for the Bidirectional Reflectance Distribution Function (BRDF) as a sum of specular component and diffuse component respectively as:

$$r = r^c + r^d \qquad (36)$$

Murphy [4] defined the specular and diffused components of the reflectance of rough surface as:

$$r_{ji(e)}^{c} = \frac{r_F(\theta')e^{-g} z\ \Delta}{Cos\,\theta_i\, d\,\omega_i} \qquad (37a)$$

$$r_{ji(e)}^{d} = \frac{r_F(\theta')G\ Z\ D}{\pi Cos\,\theta_i\ Cos\,\theta_r} \quad , \qquad (37b)$$

$$\theta' = Cos^{-1}\left(\left|\hat{k}_r - \hat{k}_i\right| / 2\right) \qquad (38)$$

Where j=1 and 2 refers to the left boundary and the right boundary respectively, i to the internal surface, e to the external surface, and $r_F(\theta')$ is the Fresnel reflection coefficient at the bisecting angle, $\hat{k}_i$ and $\hat{k}_r$ are respectively the unit vectors in the direction of the incident and reflected light. θ_i and θ_r are respectively the polar angles of incidence and reflectance and Δ is

$$\Delta = \begin{cases} 1 & \textbf{in the cone of specular reflection} \\ 0 & \textbf{elsewhere} \end{cases}$$

The geometric factor G is defined as:

$$G = \frac{4\left(1+\cos\theta_i\,\cos\theta_r - \sin\theta_i\,\sin\theta_r\,\cos\phi_r\right)^2}{\left(\cos\theta_i + \cos\theta_r\right)^2}. \qquad (39)$$

The surface roughness function g is given by

$$g = \left[\left(2\pi\sigma/\lambda\right)\left(\cos\theta_i + \cos\theta_r\right)\right]^2 \qquad (40)$$

The distribution function D is given by

$$D \cong \frac{\pi^2\tau^2}{4\lambda^2}\frac{1}{g}\exp\left(\frac{-\upsilon_{xy}^2\tau^2}{4g}\right), \qquad (41)$$

Where

$$\upsilon_{xy} = \frac{2\pi}{\lambda}\left(\sin^2\theta_i - 2\sin\theta_i\,\sin\theta_r\,\cos\phi_r + \sin^2\theta_r\right)^{1/2}. \qquad (42)$$

The effective roughness σ was introduced by He XD et al [8] to allow averaging over only the illuminated (non-shadowed) parts of the surface. Particularly for grazing angles of incidence or reflection, it can be considerably smaller than the rms roughness σ_0. They are related by

$$\sigma=\sigma_0\left(1+z_0{}^2/\sigma_0{}^2\right)^{-1/2} \tag{43}$$

Where z_0 is the root of the equation

$$\sqrt{\frac{\pi}{2}}z = \frac{\sigma_0}{4}(K_i + K_r)\exp\left(-\frac{z^2}{2\sigma_0{}^2}\right) \tag{44}$$

and

$$K_i = \tan(\theta_i) erfc\left(\tau\cot(\theta_i)/2\sigma_0\right), \tag{45a}$$

$$K_r = \tan(\theta_r) erfc\left(\tau\cot(\theta_r)/2\sigma_0\right) \tag{45a}$$

The shadowing function z is given by

$$z = z_i(\theta_i) z_r(\theta_r), \tag{46}$$

Where

$$z_i(\theta_i) = \frac{[1-\frac{1}{2}erfc\left(\tau\cot(\theta_i)/2\sigma_0\right)]}{\Lambda\left(\cot(\theta_i)\right)+1}, \tag{47a}$$

$$z_r(\theta_r) = \frac{[1-\frac{1}{2}erfc\left(\tau\cot(\theta_r)/2\sigma_0\right)]}{\Lambda\left(\cot(\theta_r)\right)+1}, \tag{47b}$$

$$\Lambda\left(\cot(\theta)\right) = \frac{1}{2}[\frac{2\sigma_0}{\sqrt{\pi}\tau\cot(\theta)} - erfc\left(\frac{\tau\cot(\theta)}{2\sigma_0}\right)] \tag{47c}$$

The reflection coefficient ρ is defined as

$$\rho = \int_{-\pi}^{\pi}\int_{0}^{\pi/2} r\,Cos\theta_i\,Cos\theta_r\,Sin\theta_r\,d\theta_r\,d\varphi_r \tag{48}$$

The diffused component of ρ is given by

$$\rho^d = \int_{-\pi}^{\pi}\int_{0}^{\pi/2} r^d\,Cos\theta_i\,Cos\theta_r\,Sin\theta_r\,d\theta_r\,d\varphi_r \tag{49}$$

φ_r is the azimuthal angle of reflection. It is assumed that the azimuthal angle of incidence is $\varphi_i = 0$.

The specular reflectance coefficient is

$$\rho^c = r_F(\theta_i)e^{-g}z \tag{50}$$

For normal incidence, the diffused component of ρ^d is given by

$$\rho^d = \frac{1}{\pi}\int_{0}^{\pi/2} r_F(\theta_r/2)G\,Z\,D\,Sin\theta_r\,d\theta_r \tag{51}$$

and the specular reflectance coefficient is

$$\rho^c = r_F(0)e^{-g} \tag{52}$$

Using equations (49), and (50) for calculating the diffused and specular components of reflection coefficients for rough surface and Substituting these reflection coefficients in equations (34),and (35) to obtain the reflectance and transmittance of the considered rough surface.

NUMERICAL RESULTS

The propagation of light in a multi-layered slab, irradiated at one side by a collimated and /or diffuse incident fluxes are considered. Specular as well as internal diffuse reflection, at the air-tissue, and tissue-air boundary with arbitrary roughness are to be taken into account [9].

The reflectivity R and transmitivity T for human liver and uterus with different thickness as anisotropic scattering media are calculated with $n_i = n_r = 1$, and $n_i = 1, n_r = 1.45$ for liver and 1.38 for uterus [10], and taking into account the roughness of the surface illuminated by normal collimated light of wavelength 635 nm .The calculations are tabulated in tables (1a) and (1b). In tables (2a) and (2b), the same calculations are carried out for muscle tissues for bovine and chicken with $n_i = 1, n_r = 1.37, 1.30$ illuminated by normal collimated light of wavelength 633 nm. The results for the case $n_i = n_r = 1$ are in good agreement with the result prepared by El-Wakil et al [6]. In tables (3a) and (3b), the reflectivity R and transmitivity T are calculated for a skin tissue illuminated by normal collimated light of wavelength 1060 nm.

The calculations are carried out with $n_i = n_r = 1$, and $n_i = 1, n_r \succ 1$ where the refractive indexes of skin tissues are taken into account [11]. The results for the cases where roughness of the surface is considered show clearly the variance in the values of reflectance and transmittance.

Table 1a: The reflectivity R for human liver and uterus with different thicknesses illuminated by normal collimated incidence of unity irradiance at 635 nm wavelength

tissues	n_r	g	z_0 (cm)	σ_a (cm-1)	σ_s (cm-1)	R		
						$n_i = n_r = 1$	$n_i = 1$ Smooth	$n_i = 1$ Rough
liver	1.45	0.68	0.002	2.3	313	0.0280284	0.193351	0.071718
			0.006			0.157014	0.329451	0.201197
			0.010			0.275075	0.379358	0.29968
uterus	1.38	0.69	0.002	0.35	394	0.039394	0.208857	0.0855398
			0.006			0.207165	0.362464	0.245646
			0.010			0.347096	0.431529	0.365095

Table 1b: The transmitivity T for human liver and uterus with different thickness illuminated by normal collimated incidence of unity irradiance at 635 nm wavelength

tissues	n_r	g	z_0 (cm)	σ_a (cm^{-1})	σ_s (cm^{-1})	T		
						$n_i = n_r = 1$	$n_i = 1$ smooth	$n_i = 1$ rough
liver	1.45	0.68	0.002	2.3	313	0.964427	0.776812	0.920065
			0.006			0.817242	0.613784	0.769493
			0.010			0.678918	0.526722	0.647838
uterus	1.38	0.69	0.002	0.35	394	0.959424	0.777902	0.913167
			0.006			0.788705	0.627313	0.749671
			0.010			0.645463	0.553814	0.626473

Table 2a: The reflectivity R and for muscle tissues with different thickness illuminated by normal collimated incidence of unity irradiance at 633 nm wavelength.

tissues	n_r	g	z_0 (cm)	σ_a (cm-1)	σ_s (cm-1)	R		
						$n_i = n_r = 1$	$n_i = 1$ smooth	$n_i = 1$ rough
Bovine	1.37	0.30	0.01	0.4	7.9	0.020529	0.054661	0.024026
			0.10			0.177346	0.221789	0.186867
			0.50			0.432153	0.331814	0.409866
chicken	1.30	0.20	0.01	0.17	4.1	0.013769	0.034012	0.014944
			0.10			0.123109	0.154635	0.128841
			0.50			0.389144	0.352562	0.38004

Table 2b: The transmitivity T and for muscle tissues with different thickness illuminated by normal collimated incidence of unity irradiance at 633 nm wavelength.

tissues	n_r	g	z_0 (cm)	σ_a (cm-1)	σ_s (cm-1)	T		
						$n_i = n_r = 1$	$n_i = 1$ smooth	$n_i = 1$ rough
Bovine	1.37	0.30	0.01	0.4	7.9	0.973518	0.91708	0.969939
			0.10			0.758954	0.671033	0.744089
			0.50			0.255193	0.21181	0.250234
chicken	1.30	0.20	0.01	0.17	4.1	0.983702	0.947376	0.98251
			0.10			0.850493	0.800449	0.843405
			0.50			0.466517	0.42392	0.46166

Table 3a: The reflectivity R for skin tissue of epidermis-air reflection coefficient $\rho_{1i}{}^{d} = 0.4, \rho_{2i}{}^{d} = 0$, for the case $n_i = n_r = 1$, illuminated by normal collimated irradiance $f_{inc}^{c} = 1\ watt / cm^2$ at 633 nm wavelength

tissues	n_r	z_0 (cm)	σ_a (cm-1)	σ_s (cm-1)	R		
					$n_i = n_r = 1$	$n_i = 1$ smooth	$n_i = 1$ rough
Epidermis	1.50	0.0065	10.0	23.33	0.03387	0.085417	0.053743
Upper dermis	1.40	0.0300	03.0	47.67	0.36383	0.31261	0.349958
Blood plexus	1.35	0.0080	02.0	06.00	0.01999	0.041224	0.0199144
Lower dermis	1.40	0.0500	03.0	47.67	0.44306	0.344692	0.421307

Table 3b: The transmitivity T for skin tissue of epidermis-air reflection coefficient $\rho_{1i}{}^{d} = 0.4, \rho_{2i}{}^{d} = 0$, for the case $n_i = n_r = 1$, illuminated by normal collimated irradiance $f^{c}_{inc} = 1\ watt\ / cm^2$ at 633 nm wavelength

tissues	n_r	z_0 (cm)	σ_a (cm-1)	σ_s (cm-1)	T		
					$n_i = n_r = 1$	$n_i = 1$ smooth	$n_i = 1$ rough
Epidermis	1.50	0.0065	10.0	23.33	0.86113	0.787362	0.858552
Upper dermis	1.40	0.0300	03.0	47.67	0.49253	0.44943	0.490735
Blood plexus	1.35	0.0080	02.0	06.00	0.95798	0.915889	0.957905
Lower dermis	1.40	0.0500	03.0	47.67	0.32263	0.285661	0.320839

CONCLUSION

A two-flux model for source-free anisotropic scattering rough surfaces is derived using Pomraning-Eddington approximation for the transport equation.
The problem of the collimated source is linked to the solution of the source-free problem. The reflection coefficients of the rough surfaces are calculated for both diffused and specular cases.

The reflection and transmission coefficients have been calculated for some types of biological tissues to ensure the efficiency of this method. We will applying this method to conducting materials as extension of this work.

The reflectance and transmittance of biological tissues with arbitrary roughness have been calculated and compared with the available data of constant refractive indices.

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