

MATHEMATICAL MODEL OF THREE WINDING AUTOTRANSFORMER

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Annotation

This article deals with the design of mathematical model of three-winding autotransformer for steady state analyses. The article is focused on model simplicity for the purposes of the use in complex transmission systems and authenticity of the model taking into account different types of step-voltage regulator.

Keywords

mathematical model of three-winding autotransformer, step-voltage regulators.

1 INTRODUCTION

Nowadays, there are many programs used for steady state analysis of electric power transmission systems. However, these programs often do not distinguish between transformers and autotransformers.

Autotransformers are almost exclusively used in electric power transmission system and ignoring this fact may cause inaccuracies in simulations of steady states, especially if the tertiary winding is loaded with a compensating element (reactor or capacitor).

In this paper will derive the mathematical model of three-winding autotransformer for two different equivalent elements with consideration of the symmetric system and the symmetric transformer, constant frequency and skip saturation of the transformer core.

2 AUTOTRANSFORMER

Autotransformer does not have the primary and secondary winding in contrast with transformer, but it has the serial and common winding. In no-load state, output voltage corresponds to the common winding induced voltage and input voltage corresponds to sum of the serial winding induced voltage and the common winding induced voltage.

The advantage of autotransformer in comparison with transformer is material saving and smaller dimensions at the same operating parameters (voltage, power). The serial winding of autotransformer has fewer turns than the primary winding of transformer. Furthermore, output current is equal to the sum of input current and current induced in the common winding, so the common winding of autotransformer could be dimensioned to smaller current than nominal output current.

Also, magnetic circuit of transformer is less massive because “With autotransformers we distinguish the passing power which the autotransformer is capable to transfer and the inner power, for which the autotransformer is dimensioned.” [1]. The issue of transformers and autotransformers is solved in detail by ref. [1, 2].

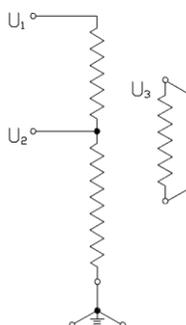


Fig. 1 Three-winding autotransformer

Three-winding autotransformers have windings connected mostly the way as shown on fig. 1. The common winding is connected in a grounded star and the tertiary is delta.

Despite the fact it concerns autotransformer, for compliance with convention, further we will not use term the input/output current (voltage, power) but the primary/secondary current (voltage, power).

3 STEP-VOLTAGE REGULATORS

It is essential to be able to control voltage in electric power transmission system on each voltage level. The use of regulating transformers is the main method of voltage regulation in distribution systems.

There are many voltage regulator types and voltage regulator connections of regulating autotransformers which are described in detail in ref. [2] but in principle we distinguish three basic voltage regulator connections:

- Regulator located at the input of autotransformer
- Regulator located at the output of autotransformer
- Regulator located in the node of autotransformer

3.1 Regulator located at the input

The voltage regulator located at the input of autotransformer regulates voltage by change of number of the serial winding turns. The advantage of this type of connection is that the regulator is dimensioned to the smallest possible current. Great disadvantage is the level of voltage to which insulation of regulator must be dimensioned. Therefore, this connection of voltage regulator is not used in practice with autotransformers.

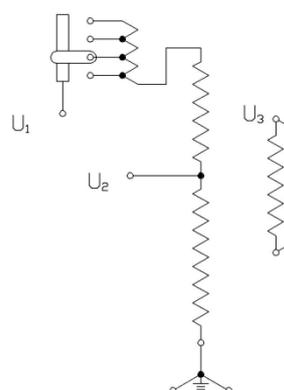


Fig. 2 Regulator located at the input

The ratio between the primary voltage and the secondary voltage may be calculated as follows:

$$p_{12} = \frac{U_1}{U_2} = \frac{N_C + N_S}{N_C} \quad (1)$$

And the ratio between the primary voltage and the tertiary voltage as follows:

$$p_{13} = \frac{U_1}{U_3} = \frac{N_C + N_S}{N_T} \quad (2)$$

We can therefore say that by how many percent the ratio between the primary voltage and the secondary voltage and the ratio between the primary voltage and the tertiary voltage changes, that many percent the count of the common and serial winding turns changes.

3.2 Regulator located at the output

The voltage regulator located at the output of autotransformer regulates voltage either by adding turns to the output (fig. 3a) or by changing proportion between the serial winding turns and the common winding turns (fig. 3b).

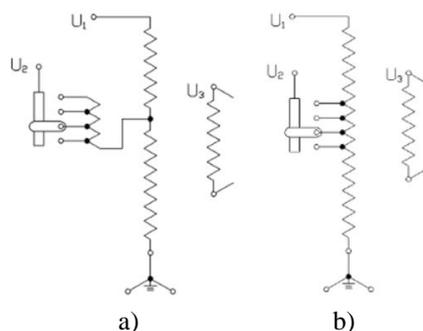


Fig. 3 Regulator located at the output

For both cases is valid that the count of number of the common and serial winding turns is constant and does not change by regulation and so the ratio between the primary voltage and the tertiary voltage is (see equation 2) is constant, too.

3.3 Regulator located in the node

Location of the regulator in the node of autotransformer does not have its equivalent among common types of transformers. Simulating of such autotransformer by the model of common type of transformer may cause mistakes.

Regulator changes the ratio by change of number of the common winding turns.

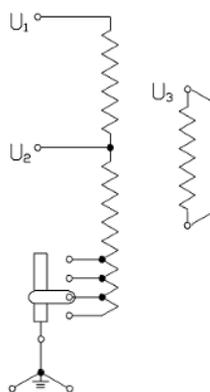


Fig. 4 Regulator located in the node

For the change of number of the common winding turns by k %, the following relation is valid for the ratio between the primary voltage and the secondary voltage:

$$P_{12} = \frac{N_S + (1 + k/100)N_C}{(1 + k/100)N_C} \quad (3)$$

And the ratio between the primary voltage and the tertiary voltage shall be:

$$P_{13} = \frac{N_S + (1 + k/100)N_C}{N_T} \quad (4)$$

We can therefore say that if the regulator is located in the node of autotransformer, not only the ratio between the primary voltage and the secondary voltage is changed by regulation but also the ratio between the primary voltage and the tertiary voltage does change while the change of particular ratio differs.

Percentage change of winding turns is mostly unknown in case of voltage regulators. Change of the secondary voltage k_u is usually identified on the nameplate.

It is possible to prove (and from equation 4 to derive) that in order to reach the step of regulation of the secondary voltage k_u (with constant primary voltage), the step of change of the common winding turns shall be:

$$k = \frac{U_1}{U_1 - U_2} k_u \quad (5)$$

4 EQUIVALENT CIRCUIT

The equivalent circuit of three-winding autotransformer is based on “T- element” and is identical to the equivalent circuit of three-winding transformer.

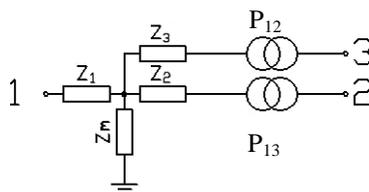


Fig. 4 Equivalent circuit of three-winding autotransformer (T- element)

Real impedances of the serial, common and tertiary windings are replaced by substitute impedances of the primary, secondary and tertiary windings, related to the primary voltage. It is possible to derive the relationship between real impedances and substitute impedances but in this case there is no reason to do so.

Substitute impedances of transformer may be calculated from the system of three equations. The sum of two optional impedances Z_I, Z_J equals to:

$$Z_I + Z_J = \frac{u_{kII}}{100} \frac{U_{N1}^2}{S_N} \quad (6)$$

Also the sum of two optional resistances of windings of the equivalent circuit equals to:

$$R_I + R_J = \Delta P_{KII} \frac{U_{N1}^2}{S_N^2} \quad (7)$$

Magnetizing impedance may be calculated from values measured in no-load test:

$$\frac{1}{Z_M} = Y_M = \frac{i_0}{100} \frac{S_N}{U_{N1}^2} \quad (8)$$

$$G_M = \frac{\Delta P_0}{U_{N1}^2} \quad (9)$$

The ratios are calculated on the basis of the type of regulation according to the equations mentioned in part 3. It is essential to adhere to equation 5 for calculation between the voltage step (stated on the nameplate) and the step of turns in case of regulation.

5 MATHEMATICAL MODEL

In general, mathematical modeling and simulations as well as solving of non-linear models are dealt in ref. [3, 4].

For requirements of modeling of steady state analysis under conditions as described in the introduction, for autotransformer it is enough to derive the square admittance matrix of complex numbers $[Y]$, with size 3×3 which meets the following condition:

$$[I] = [Y][U] \quad (10)$$

Where: $[I]$ – vector of currents flowing into transformer

$[U]$ – vector of terminal voltages

$[Y]$ – admittance matrix

Admittance matrix may be derived by means of one of the following substitute elements.

5.1 π - element

The element is based on the equivalent circuit (fig. 4). Magnetizing admittance is not concentrated in the middle node (like in case of T- element) but divided into particular nodes of transformer.

The next step of transmission from T- element to π - element is the transformation of a star (connection of substitute impedances into a star) into triangle (see fig. 5).

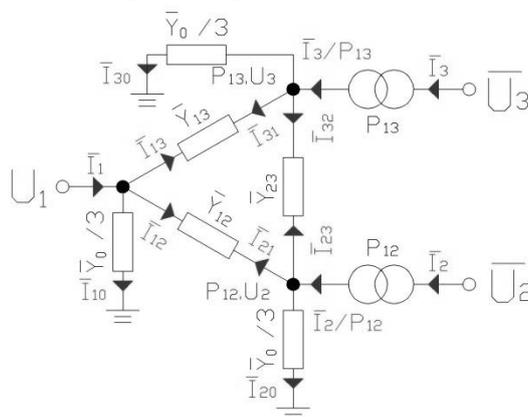


Fig. 5. Diagram of π - element

The following equations are valid for particular nodes:

$$\bar{I}_1 = \bar{I}_{10} + \bar{I}_{12} + \bar{I}_{13} \quad (8)$$

$$\bar{I}_2 / P_{12} = \bar{I}_{20} + \bar{I}_{21} + \bar{I}_{23} \quad (9)$$

$$\bar{I}_3 / P_{13} = \bar{I}_{30} + \bar{I}_{31} + \bar{I}_{32} \quad (10)$$

For the purpose of generalization, if we think for an ideal transformer with ratio $P_{11}=1$ on input 1, then the following is valid for each node A for ground current I_{A0} :

$$\bar{I}_{A0} = P_{1A} \bar{U}_A \frac{\bar{Y}_0}{3} \quad (11)$$

And the following is valid for each pair of nodes A, B:

$$\bar{I}_{AB} = \bar{Y}_{AB}(P_{1A} \bar{U}_A - P_{1B} \bar{U}_B) \quad (12)$$

It is possible to derive the basic equation of π - element of transformer from equations 8 – 12:

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \end{bmatrix} = [\mathbf{Y}] \begin{bmatrix} \bar{U}_1 \\ \bar{U}_2 \\ \bar{U}_3 \end{bmatrix} \quad (13)$$

Where the admittance matrix is:

$$[\mathbf{Y}] = \begin{bmatrix} \bar{Y}_{11} & -P_{12} \bar{Y}_{12} & -P_{13} \bar{Y}_{13} \\ -P_{12} \bar{Y}_{12} & \bar{Y}_{22} & -P_{13} P_{13} \bar{Y}_{23} \\ -P_{13} \bar{Y}_{13} & -P_{13} P_{13} \bar{Y}_{23} & \bar{Y}_{33} \end{bmatrix} \quad (14)$$

And diagonal elements are:

$$\bar{Y}_{11} = \frac{\bar{Y}_0}{3} + \bar{Y}_{12} + \bar{Y}_{13} \quad (15)$$

$$\bar{Y}_{22} = \left(\frac{\bar{Y}_0}{3} + \bar{Y}_{12} + \bar{Y}_{23} \right) P_{12}^2 \quad (16)$$

$$\bar{Y}_{33} = \left(\frac{\bar{Y}_0}{3} + \bar{Y}_{13} + \bar{Y}_{23} \right) P_{13}^2 \quad (17)$$

The advantage of π - element is that it works directly with admittances. It is suitable for the use in case when magnetizing admittance equals zero. The model working with impedance might be insoluble as transverse impedance would equal infinity in that case.

5.2 T- element

T-element is based directly on the equivalent circuit. Under normal conditions the models transform T-element into three independent Γ -elements with expressed middle node. This method tasks calculation itself as each transformer adds a node into the diagram where it is necessary to calculate voltage.

We chose a different approach. For T-element it is easy to derive voltage equations leading to derivation of impedance matrix.

If voltage in the middle node is U_0 , then the following is valid for this voltage

$$\bar{U}_0 = \bar{Z}_M \left(\bar{I}_1 + \frac{\bar{I}_2}{P_{12}} + \frac{\bar{I}_3}{P_{13}} \right) \quad (18)$$

And the following is valid for particular loops of T-element:

$$\bar{U}_1 = \bar{Z}_1 \bar{I}_1 + \bar{Z}_M \left(\bar{I}_1 + \frac{\bar{I}_2}{P_{12}} + \frac{\bar{I}_3}{P_{13}} \right) \quad (19)$$

$$P_{12}\bar{U}_2 = \bar{Z}_2 \frac{\bar{I}_2}{P_{12}} + \bar{Z}_M \left(\bar{I}_1 + \frac{\bar{I}_2}{P_{12}} + \frac{\bar{I}_3}{P_{13}} \right) \quad (20)$$

$$P_{13}\bar{U}_3 = \bar{Z}_3 \frac{\bar{I}_3}{P_{13}} + \bar{Z}_M \left(\bar{I}_1 + \frac{\bar{I}_2}{P_{12}} + \frac{\bar{I}_3}{P_{13}} \right) \quad (21)$$

From equations 19 - 21 it is possible to derive voltage equation of transformer:

$$[\mathbf{U}] = [\mathbf{Z}][\mathbf{I}] \quad (22)$$

Where the impedance matrix is:

$$[\mathbf{Z}] = \begin{bmatrix} \bar{Z}_1 + \bar{Z}_m & \frac{\bar{Z}_M}{P_{12}} & \frac{\bar{Z}_M}{P_{13}} \\ \frac{\bar{Z}_M}{P_{12}} & \bar{Z}_2 + \bar{Z}_m & \frac{\bar{Z}_M}{P_{13}} \\ \frac{\bar{Z}_M}{P_{13}} & \frac{\bar{Z}_M}{P_{12}} & \bar{Z}_3 + \bar{Z}_m \end{bmatrix} \quad (23)$$

With regard to the fact that simulating programs work with the admittance matrix, it may be calculated by inversion of the impedance matrix:

$$[\mathbf{Y}] = [\mathbf{Z}]^{-1} \quad (22)$$

The advantage of this model is higher reliability; however, this model cannot be used if we consider zero magnetizing admittance in transformer.

6 MODEL APPLICATION

Model of autotransformer was programmed in programming language JAVA for demonstration and confirmation of mathematical models. The application allows simulations of steady state analysis of simple electric power transmission systems.

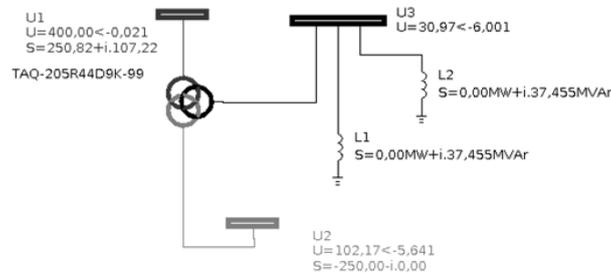


Fig. 5 Print-screen of simulated system

In this application a simple system with autotransformer was simulated and the results were compared with solutions of two other commercially available software.

The first software does not consider autotransformer and uses T-element split into three independent Γ -elements with expressed middle node. The second software uses π - element and considers autotransformer.

A print-screen of application with simulated system is on fig. 5.

The autotransformer was fed by hard source 400 kV (NODE1, or “UZOL1”) during simulation, constant load 250 MW (NODE2, or “UZOL2”) was connected to the secondary terminals and two reactors 32 kV, 40 MVA were connected to the tertiary terminals. Autotransformer parameters are listed in Tab. 1.

Tab. 1 Parameters of autotransformer

Par.	Value	Par.	Value
S_{N1}	350 MVA	ΔP_{K12}	606,2 kW (S_{N1})
S_{N2}	350 MVA	ΔP_{K13}	566,6 kW (S_{N3})
S_{N3}	100 MVA	ΔP_{K23}	578,2 kW (S_{N3})
U_{N1}	400 kV	i_0	0,042 %
U_{N2}	121 kV	ΔP_0	91,7 kW
U_{N3}	34 kV	Step	1,5 %
u_{K12}	13,29 % (S_{N1})	Max	8
u_{K13}	10,24 % (S_{N3})	Min	-8
u_{K23}	5,74 % (S_{N3})	type	In node

The simulations were executed for edge states of voltage regulator (tap: -8 and 8) and for middle state (tap: 0) while values on the secondary and tertiary terminals and values of demanded active and passive power was monitoring. The results are listed in Tab. 3. The results of new model using T-element are labeled as “New T”, the results of new model using π - element are labeled as “New π ”. The results of commercially available software using T-element are labeled as “Com T” and commercially available software using π - element is labeled as “Com π ”.

Tab. 2 The results of simulations

	New T	New π	Com T	Com π
Tap: -8				
U_2 [V]	101814	101812	102300	105734
U_3 [V]	32459	32459	31020	31522
P_1 [MW]	250,87	250,87	250,5	250,87
Q_1 [MVAr]	115,64	115,63	105,4	112,37
Tap: 0				
U_2 [V]	116125	116123	116200	116114
U_3 [V]	30961	30962	31020	30958
P_1 [MW]	250,8	250,8	250,5	250,8
Q_1 [MVAr]	107,39	107,38	105,4	107,6
Tap: 8				
U_2 [kV]	130518	130516	130200	125628
U_3 [kV]	29448	29448	31020	30281
P_1 [MW]	250,74	250,74	250,50	250,47
Q_1 [MVAr]	99,44	99,43	105,4	102,89

The first visible item in Tab. 2 is that differences between the model using T-element and the model using π - element are minor.

Then, the smallest differences are in the middle state when the regulator is set up for zero tap position. Differences range on the level of tenths to hundredths of percent and it is caused by different calculating programs.

Commercially available software using T-element in both edge states of the regulator held the constant tertiary voltage. It is caused by the fact that although the regulator was located in the node of the secondary winding, the software did not consider autotransformer.

Commercially available software using π -element calculated similarly to new designed models. It proved that the tertiary voltage decreases as the secondary voltage increases. However, in comparison with new designed models, voltage changes against the middle position of the regulator are smaller. As the software was different also in value of the secondary voltage (despite the fact that the step of the regulation was defined on the level of 1,5% U_2 and voltage has changed by less than 12 % in the result), we assume that the software does not calculate the step of voltage regulation to the step of turns regulation (see equation 5).

5 CONCLUSION

It is possible to model three winding autotransformer by means of T-element or π -element. Precision of both elements is comparable.

In both cases it is important to take into account whether it concerns transformer or autotransformer and where step-voltage regulator is located.

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