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Focus point gauge mediation in product group unification

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Abstract

In certain models of gauge-mediated supersymmetry breaking with messenger fields in incomplete GUT multiplets, the radiative corrections to the Higgs potential cancel out during renormalization group running. This allows for relatively heavy superpartners and for a 125 GeV Higgs while the fine-tuning remains modest. In this paper, we show that such gauge mediation models with "focus point" behaviour can be naturally embedded into a model of $\mathrm{SU}(5) \times \mathrm{U}(3)$ product group unification.

1 Introduction

If low-energy supersymmetry is realized in nature, the LHC results of the last two years point towards a rather high superpartner mass scale, perhaps in the range of several TeV. However, obtaining an electroweak symmetry breaking scale which is an order of magnitude or more below the superpartner mass scale requires significant fine-tuning. This is the well-known little hierarchy problem of supersymmetry.

Models with non-unified gaugino masses have recently been argued to alleviate the supersymmetric fine-tuning problem in the MSSM [1–7]. For suitable "focus point" ratios of gaugino masses, the radiative corrections to the Higgs potential cancel out during renormalization group running, yielding an electroweak scale which is much smaller than the typical scale of soft SUSY-breaking masses.[1]

High-scale gauge mediation with messenger fields in incomplete GUT multiplets can naturally realize suitable non-universal soft mass ratios [3]. In terms of the messenger indices N_2 and N_3 for pairs of fundamental $\mathrm{SU}(2)_\mathrm{L}$ and $\mathrm{SU}(3)_\mathrm{C}$ messenger multiplets, the favourable models tend to have a ratio around $N_2 : N_3 \approx 5 : 2$ (if the mediation scale is close to the GUT scale, and if $\tan\beta$ is large). Models with such an exotic field content may be obtained from higher-dimensional orbifold GUTs, or from related heterotic string constructions [4]. In the present letter we are proposing an example within the more conventional setting of four-dimensional field theory.

Of course any successful GUT model needs to somehow accommodate incomplete GUT multiplets, in order to solve the doublet-triplet splitting problem in the Higgs sector. Models of product group unification (PGU) [11–13] achieve this by extending the unified gauge group to, for instance, $\mathrm{SU}(5) \times \mathrm{SU}(3)_\mathrm{H} \times \mathrm{U}(1)_\mathrm{H}$. Below the GUT scale, colour $\mathrm{SU}(3)_\mathrm{C}$ is obtained as the diagonal subgroup of $\mathrm{SU}(3)_\mathrm{H}$ and the Georgi-Glashow embedded $\mathrm{SU}(3) \subset \mathrm{SU}(5)$, and hypercharge is similarly a linear combination of $\mathrm{U}(1)_\mathrm{H}$ and $\mathrm{U}(1) \subset$

[1] This is similar in spirit to the original focus point SUSY scenario [8–10], where instead the scalar soft mass contributions to the electroweak scale cancel during the renormalization group evolution.

SU(5). In such models the doublet-triplet splitting problem is easily solved, and also additional incomplete GUT multiplets can be accommodated straightforwardly.

In PGU models the gauge couplings need not unify. In fact, the theory will become non-perturbative immediately above the GUT scale unless the gauge couplings are prevented from unifying. In order to allow for a cutoff scale which is not at $M_{\rm GUT}$ but significantly higher, one needs to ensure that g_2 is the largest among the Standard Model gauge couplings at the GUT scale. This is most easily achieved by adding some additional pairs of $\rm SU(2)_L$ doublets with intermediate-scale masses.

Intriguingly, PGU and focus point gauge mediation are seen to complement each other. On the one hand, for a focus point-like cancellation in gauge mediation, the model needs to contain significantly more weak doublet messengers than colour triplet messengers. On the other hand, in order to maximize the cutoff scale in PGU one needs to deflect the renormalization group running of g_2 relative to the other couplings by adding extra vector-like states, again with significantly more weak doublets than colour triplets.

In the present letter we exploit this observation, by constructing a PGU model with a number of incomplete GUT multiplets with masses below the GUT scale. These will act as gauge mediation messengers, inducing non-unified soft term ratios favourable for naturalness, while at the same time ensuring that the theory remains valid perturbatively at energies above $M_{\rm GUT}$.

Our model predicts the lightest Higgs mass to be compatible with the recent LHC discovery, thanks to large radiative corrections from multi-TeV soft terms, and evades the LHC limits on squark and gluino masses. Nevertheless, the fine-tuning will be modest compared to generic MSSM models with similarly heavy superpartners (although there is still some residual fine-tuning of the order of a percent). The gauge couplings will become non-perturbative at a cutoff scale M_*, which is about an order of magnitude larger than $M_{\rm GUT}$ (but still below the Planck scale).

2 Field content and evolution of couplings

We start by briefly reviewing the main properties of the SU(5) × U(3) PGU model; for more details see e.g. [11–13]. Consider a supersymmetric GUT with gauge group SU(5) × $\mathrm{SU(3)_H \times U(1)_H}$. There are three generations of Standard Model matter fields in the $\mathbf{10} \oplus \overline{\mathbf{5}}$ and a pair of Higgs fields $H, \overline{H}$ in the $\mathbf{5} \oplus \overline{\mathbf{5}}$ of SU(5), all of which are uncharged under $\mathrm{SU(3)_H \times U(1)_H}$. Bi-fundamental fields transforming as $Y = (\mathbf{5}, \overline{\mathbf{3}})$ and $\overline{Y} = (\overline{\mathbf{5}}, \mathbf{3})$ under $\mathrm{SU(5) \times SU(3)_H}$ acquire vacuum expectation values at the GUT breaking scale M_{GUT}, thus breaking $\mathrm{SU(5) \times SU(3)_H \times U(1)_H \to SU(3)_C \times SU(2)_L \times U(1)_Y}$. Here the colour $\mathrm{SU(3)_C}$ emerges as the diagonal subgroup of $\mathrm{SU(3)_H}$ and the Georgi-Glashow embedded SU(3) ⊂ SU(5). Likewise, the hypercharge $\mathrm{U(1)_Y}$ is a linear combination of $\mathrm{U(1)_H}$ and the usual hypercharge generator. Adding a pair $T, \overline{T}$ of Higgs triplet partners in the $\mathbf{3} \oplus \overline{\mathbf{3}}$ of $\mathrm{SU(3)_H}$, the superpotential terms

$$W = H\overline{T}\,\overline{Y} + \overline{H}TY \tag{1}$$

give GUT-scale masses to the triplet components of H and $\overline{H}$. Take the hypercharge generator to be

$$\mathrm{Y} = \mathrm{T}^{24} + \mathrm{Q}\,, \tag{2}$$

where Q is the $\mathrm{U(1)_H}$ charge, and $\mathrm{T}^{24} = \sqrt{\frac{3}{5}}\ \mathrm{diag}\,(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2})$. The requirement of leaving hypercharge unbroken then fixes the GUT-normalized $\mathrm{U(1)_H}$ charge of Y to be $\mathrm{Q}[Y] = -\frac{1}{3}$.

The model also contains a singlet S (which is needed to give a VEV to Y and $\overline{Y}$) and an $\mathrm{SU(3)_H}$ adjoint X (which is needed to give masses to the $\mathrm{SU(3)_C}$ octet contained in $Y\overline{Y}$). The particle content is summarized in Table 1. The superpotential is

$$W = H\overline{T}\,\overline{Y} + \overline{H}TY + S(Y\overline{Y} + T\overline{T} - v^2) + YX\overline{Y} + TX\overline{T} + \text{(MSSM Yukawa couplings)}\,. \tag{3}$$

These are all the renormalizable terms allowed by a certain discrete R-symmetry, which also serves to forbid dangerous dimension-5 operators.

field	SU(5)	$SU(3)_H$	$U(1)_H$
$3 \times \mathbf{10}$	$\mathbf{10}$	$\mathbf{1}$	0
$3 \times \overline{\mathbf{5}}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	0
H	$\mathbf{5}$	$\mathbf{1}$	0
$\overline{H}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	0
T	$\mathbf{1}$	$\mathbf{3}$	1/3
$\overline{T}$	$\mathbf{1}$	$\overline{\mathbf{3}}$	−1/3
Y	$\mathbf{5}$	$\overline{\mathbf{3}}$	−1/3
$\overline{Y}$	$\overline{\mathbf{5}}$	$\mathbf{3}$	1/3
S	$\mathbf{1}$	$\mathbf{1}$	0
X	$\mathbf{1}$	$\mathbf{8}$	0

Table 1: Field content of a minimal model with SU(5) × U(3) product group unification.
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To extend this to a model of messenger gauge mediation, we add N_5 pairs of messengers Φ_I, $\overline{\Phi}_I$ in the $\mathbf{5} \oplus \overline{\mathbf{5}}$ of SU(5), along with $N_{3\mathrm{H}}$ pairs of additional fields Ψ_i, $\overline{\Psi}_i$ in the $\mathbf{3} \oplus \overline{\mathbf{3}}$ of $SU(3)_H$, where $N_5 > N_{3\mathrm{H}}$. The $U(1)_H$ charge assignments are arbitrary so far; we take $Q[\Phi_I] = 0$ and $Q[\Psi_i] = \frac{1}{3}$ for simplicity, to match the charges of H and T. The superpotential operators

$$W = \lambda_{Ij}\, \Phi_I \overline{\Psi}_j\, \overline{Y} + \lambda'_{Ij} \overline{\Phi}_I \Psi_j Y \tag{4}$$

will decouple $N_{3\mathrm{H}}$ triplets at the scale M_{GUT}, leaving $N_2 \equiv N_5$ doublet pairs and $N_3 \equiv N_5 - N_{3\mathrm{H}}$ triplet pairs massless. If $Z = M + F\theta^2$ is a SUSY-breaking spurion, the coupling

$$W = \kappa_{IJ}\, Z \Phi_I \overline{\Phi}_J \tag{5}$$

will eventually give supersymmetric masses M to all remaining messengers. We assume that $M \ll M_{\mathrm{GUT}}$, such that the GUT-scale massive triplets can be neglected for SUSY breaking mediation, and $F \ll M^2$ (a possible dynamical origin of M and F is sketched later in section 4). This defines a simple model of messenger gauge mediation, with the somewhat unusual property that the light messenger fields do not come in complete GUT multiplets. Table 2 summarizes the messenger field content.

field	SU(5)	$SU(3)_H$	$U(1)_H$
$N_5 \times \Phi$	$\mathbf{5}$	$\mathbf{1}$	0
$N_5 \times \overline{\Phi}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	0
$N_{3H} \times \Psi$	$\mathbf{1}$	$\mathbf{3}$	1/3
$N_{3H} \times \overline{\Psi}$	$\mathbf{1}$	$\overline{\mathbf{3}}$	−1/3

Table 2: Messenger fields and their charges.

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Below the GUT-breaking scale, the messenger couplings become

$$W = \sum_{i=1}^{N_2} \kappa_{2i}\, Z\Phi_{2i}\overline{\Phi}_{2i} + \sum_{a=1}^{N_3} \kappa_{3a}\, Z\Phi_{3a}\overline{\Phi}_{3a} \ , \tag{6}$$

where Φ_{2i} and Φ_{3a} denote the remaining $SU(2)_L$ doublet and $SU(3)_C$ triplet messengers. Here, we have diagonalized the Yukawa interactions (i.e. the mass matrix of the remaining messenger fields) at leading order without loss of generality. It should be noted that the gauge-mediated MSSM soft masses are independent of the Yukawa couplings $\kappa_{2,3}$ at leading order, since we are assuming that the messengers couple to a single spurion and contributions from the GUT-scale messengers are suppressed.

We can now investigate the evolution of the gauge couplings, using one-loop running and step-function decoupling for a rough estimate. We need to consider three distinct regimes. Below M, the field content is that of the MSSM, and the couplings run as usual. Between M and $M_{\rm GUT}$, the one-loop β function coefficients are

$$b_1 = \frac{3}{5}\left(11 + N_2 + \frac{2}{3}N_3\right) , \qquad b_2 = 1 + N_2 \, , \qquad b_3 = -3 + N_3 \, . \tag{7}$$

Finally, above $M_{\rm GUT}$ the β function coefficients are

$$\begin{aligned} b_{1\rm H} &= \frac{3}{5}\left(4 + \frac{2}{3}N_{3\rm H}\right) = \frac{12}{5} + \frac{2}{5}\left(N_2 - N_3\right) \, , \\ b_{3\rm H} &= N_{3\rm H} = N_2 - N_3 \, , \\ b_5 &= N_5 - 5 = N_2 - 5 \, . \end{aligned} \tag{8}$$

Furthermore, at $M_{\rm GUT}$ the couplings satisfy the tree-level matching conditions

$$\frac{1}{\alpha_1} = \frac{1}{\alpha_{1\rm H}} + \frac{1}{\alpha_5}\,, \qquad \frac{1}{\alpha_2} = \frac{1}{\alpha_5}\,, \qquad \frac{1}{\alpha_3} = \frac{1}{\alpha_{3\rm H}} + \frac{1}{\alpha_5}\,. \tag{9}$$

From Eq. (9) it is evident that a unified gauge coupling, $\alpha_1 = \alpha_2 = \alpha_3$ at $M_{\rm GUT}$, would correspond to strongly coupled $\mathrm{SU}(3)_{\rm H}$ and $\mathrm{U}(1)_{\rm H}$ groups at the GUT-breaking scale. Conversely, if gauge coupling unification is sacrificed by allowing for nonzero N_5 and $N_{3\rm H}$, Eqs. (7), (8) and (9) can be used to estimate the scale at which the theory becomes strongly coupled in the UV.

3 A concrete model

When including a large number of charged fields, it is difficult to construct a model which remains perturbative all the way to the Planck scale. However, in our model a somewhat lower cutoff scale $M_* < M_{\rm Planck}$ is actually preferable for a number of reasons. First, a cutoff scale around $M_* = 10^{17}$ GeV would be of the right order to explain the lack of $m_s - m_\mu$ unification, as the Yukawa couplings are corrected by higher-dimensional operators such as $W = \mathbf{10}\,\overline{\mathbf{5}}\,\overline{H}\,\overline{Y}\,Y/(M_*)^2$.[2] And second, a sub-Planckian cutoff allows for a solution of the Polonyi problem (which is generally a concern for high-scale gauge mediation, as it is for gravity mediation; see e.g. [14]) using the mechanism of adiabatic suppression [15].

We therefore choose the cutoff scale to be $M_* = 10^{17}$ GeV. Moreover, we take the GUT-breaking scale to be $M_{\rm GUT} = 10^{16}$ GeV, and the messenger scale to be $M = 10^{14}$ GeV — note that a large separation between the messenger scale and the GUT-breaking scale is preferred, because we will neglect any contributions to the soft terms from GUT-scale massive triplet messengers.[3]

[2] At the same order, the term $\overline{Y}\,Y/(M_*)^2$ can appear in the gauge kinetic functions, which slightly modifies the matching conditions for the gauge coupling constants Eq. (9). For $M_* \simeq 10^{17}$ GeV and $\mathcal{O}(1)$ coefficients, however, the correction to the matching conditions is negligibly small.

[3] Here, we have tacitly rescaled the spurion to absorb the typical size of the κ in Eq. (6), so that the messenger scale is given by M.

In our model $\tan\beta$ is large, and μ is smaller than the soft SUSY-breaking terms, because μ and B_μ are only generated by subdominant gravity-mediated effects. In terms of the running parameters at the soft mass scale $M_{\rm IR}$, large $\tan\beta$ implies

$$m_Z^2 = -2\left(|\mu|^2 + m_{H_u}^2\right)\Big|_{M_{\rm IR}}\,, \tag{10}$$

and thus to obtain a realistic electroweak scale, the contributions to $m_{H_u}^2$ from the various soft terms have to approximately cancel out in the renormalization group evolution. This will at most happen for a few select choices of messenger indices N_2 and N_3. Numerically solving the two-loop renormalization group equations between $M_{\rm UV} = 10^{14}$ GeV and $M_{\rm IR} = 5\times 10^3$ GeV yields

$$\begin{aligned} m_{H_u}^2\Big|_{M_{\rm IR}} =& \Big(-0.79\,M_3^2 + 0.20\,M_2^2 - 0.01\,M_1\,M_3 - 0.06\,M_2\,M_3 \\ & - 0.02\,m_{d_3}^2 - 0.32\,m_{u_3}^2 - 0.29\,m_{Q_3}^2 + 0.04\,m_{H_d}^2 + 0.70\,m_{H_u}^2\Big)\Big|_{M_{\rm UV}}\,. \end{aligned} \tag{11}$$

Here we have omitted terms with coefficients < 0.01 (although they are internally kept in the following calculations). We have also neglected any terms involving the A-parameters at the messenger scale, since these are expected to be small in gauge mediation. The standard one-loop messenger gauge mediation expressions for the gaugino masses at $M_{\rm UV} = M$ are

$$\begin{aligned} M_1 &= \frac{g_1^2}{16\pi^2}\frac{F}{M}\left(\frac{3}{5}N_2 + \frac{2}{5}N_3\right)\,, \\ M_2 &= \frac{g_2^2}{16\pi^2}\frac{F}{M}\,N_2\,, \\ M_3 &= \frac{g_3^2}{16\pi^2}\frac{F}{M}\,N_3\,, \end{aligned} \tag{12}$$

while the scalar soft masses are

$$
\begin{aligned}
m_{Q_3}^2 &= 2\left(\frac{F}{M}\right)^2 \left[\left(\frac{g_3^2}{16\pi^2}\right)^2 \cdot \frac{4}{3}N_3 + \left(\frac{g_2^2}{16\pi^2}\right)^2 \cdot \frac{3}{4}N_2 + \left(\frac{g_1^2}{16\pi^2}\right)^2 \cdot \frac{1}{60}\left(\frac{2}{5}N_3 + \frac{3}{5}N_2\right)\right] , \\
m_{u_3}^2 &= 2\left(\frac{F}{M}\right)^2 \left[\left(\frac{g_3^2}{16\pi^2}\right)^2 \cdot \frac{4}{3}N_3 + \left(\frac{g_1^2}{16\pi^2}\right)^2 \cdot \frac{4}{15}\left(\frac{2}{5}N_3 + \frac{3}{5}N_2\right)\right] , \\
m_{d_3}^2 &= 2\left(\frac{F}{M}\right)^2 \left[\left(\frac{g_3^2}{16\pi^2}\right)^2 \cdot \frac{4}{3}N_3 + \left(\frac{g_1^2}{16\pi^2}\right)^2 \cdot \frac{1}{15}\left(\frac{2}{5}N_3 + \frac{3}{5}N_2\right)\right] , \\
m_{H_u}^2 &= m_{H_d}^2 = 2\left(\frac{F}{M}\right)^2 \left[\left(\frac{g_2^2}{16\pi^2}\right)^2 \cdot \frac{3}{4}N_2 + \left(\frac{g_1^2}{16\pi^2}\right)^2 \cdot \frac{3}{20}\left(\frac{2}{5}N_3 + \frac{3}{5}N_2\right)\right] .
\end{aligned}
\tag{13}
$$

Crucially, as mentioned before and as usual in minimal gauge mediation, the soft terms only depend on the ratio F/M and on the messenger field content. In particular there is no dependence on the unknown Yukawa couplings κ in Eqs. (5) or (6).[4]

Substituting Eqs. (12) and (13) into Eq. (11) we obtain

$$
m_{H_u}^2\Big|_{M_{\rm IR}} = \left(\frac{1}{16\pi^2}\frac{F}{M}\right)^2 \left(-0.215\,N_3^2 - 0.449\,N_3 + 0.044\,N_2^2 + 0.150\,N_2 - 0.015\,N_3\,N_2\right) . \tag{14}
$$

Suitable combinations for (N_2, N_3) to ensure small negative $m_{H_u}^2$ at $M_{\rm IR}$ are given by $(N_2, N_3) = (17, 7)$ or $(N_2, N_3) = (22, 9)$. (Note that the ratio $N_3 : N_2$ is always around $2 : 5$ for a "gaugino focus point", for N_2 and N_3 sufficiently large. Here, both 7/17 and 9/22 are close to 0.41.)

The one-loop running of the gauge couplings is shown in Fig. 1 for $(N_2, N_3) = (22, 9)$ and in Fig. 2 for $(N_2, N_3) = (17, 7)$. Above the GUT-breaking scale the couplings are seen to be quite large, and the reliability of the one-loop approximation should be questioned. We have therefore used two-loop running in this regime, in the (conservative) limit where any Yukawa couplings can be neglected.

[4] Strictly speaking, the above soft masses are valid only for degenerate couplings $\kappa_{2,i} = \kappa_{3,a}$, whereupon all the messengers decouple at the same messenger scale. However, our results are not significantly altered even if the κ are not all equal, as long as they are of a similar order of magnitude.

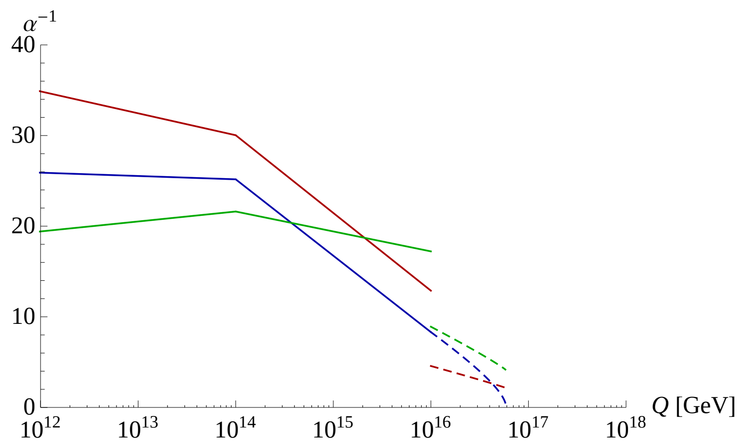

Figure 1: Evolution of the gauge couplings for $(N_2, N_3) = (22, 9)$. Solid red curve: α_1^{-1}, solid blue curve: α_2^{-1}, solid green curve: α_3^{-1}. Dashed red curve: $\alpha_{1\mathrm{H}}^{-1}$, dashed blue curve: α_5^{-1}, dashed green curve: $\alpha_{3\mathrm{H}}^{-1}$. Two-loop running is used above the matching scale $M_{\mathrm{GUT}} = 10^{16}$ GeV, but the effect of Yukawa couplings is neglected.

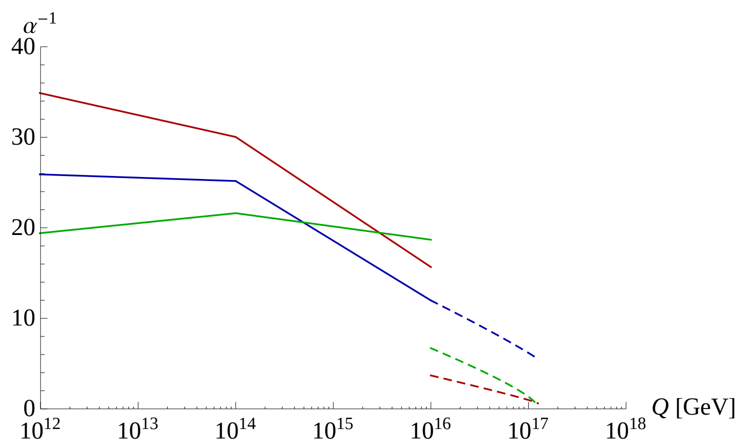

Figure 2: Evolution of the gauge couplings for $(N_2, N_3) = (17, 7)$. The colour code is the same as in Fig. 1.

In the $(N_2, N_3) = (22, 9)$ model, the two-loop correction is quite significant, and leads to the theory becoming perturbatively unreliable at scales below the previously assumed cutoff M_*. We therefore discard this possibility: the matter content of the model is too large to guarantee perturbative control an order of magnitude above the GUT-breaking scale. Note, however, that sizeable Yukawa couplings might still change this behaviour.

On the other hand, in the $(N_2, N_3) = (17, 7)$ model a Landau pole is reached only around $M_* = 10^{17}$ GeV. We therefore choose this model to compute a benchmark parameter point using SOFTSUSY [16]. Set $F = (4 \times 10^9\,\mathrm{GeV})^2$, which implies that μ and $\sqrt{B_\mu}$ should be $\mathcal{O}(F/M_*) \sim 200$ GeV. For practical purposes, they are fixed by requiring realistic electroweak symmetry breaking. In our benchmark point they come out slightly high: $\mu(M) = 232$ GeV and $\sqrt{B_\mu}(M) = 614$ GeV. For scalar and gaugino soft masses, we use only the gauge-mediated contributions from Eqs. (12) and (13), disregarding possible "gravity-mediated" corrections $\mathcal{O}(F/M_*)$.

The low-energy sparticle spectrum is shown in Table 3. Note that the lightest Higgs mass is on the low end of what is compatible with the LHC observation, when taking a theory uncertainty of 3 GeV into account. The reason is that, even with multi-TeV stops, a large Higgs mass is difficult to obtain if the A-terms are only generated radiatively. The superpartners are seen to be too heavy to be produced at the LHC as expected, with the exception of some relatively light higgsino-like charginos and neutralinos $\chi_1^\pm$, χ_1^0, and χ_2^0. This is similar to previous models of high-scale gauge mediation in which the μ term is induced by subdominant gravity mediation effects, see e.g. [4]. Light higgsinos will be difficult to see at the LHC, but could be discovered at a future linear collider.

To estimate the fine-tuning, we take the usual definition

$$\Delta = \max_{\text{parameters } a} \frac{\partial \log m_Z}{\partial \log a}\,, \qquad\qquad \text{fine-tuning} = \frac{1}{\Delta}\,. \tag{15}$$

There are only two independent dimensionful parameters a which enter m_Z (at large $\tan\beta$). Namely,

$$m_Z^2 = -2\left(|\mu|^2 + m_{H_u}^2\right)\Big|_{M_{\mathrm{IR}}} = 0.42\, m_{\mathrm{GMSB}}^2 - 1.62\,|\mu|^2\Big|_{M_{\mathrm{UV}}} \tag{16}$$

particle	mass [GeV]
h_0	123
χ_1^0	207
$\chi_1^\pm$	208
χ_2^0	209
χ_3^0	2900
χ_4^0	6900
$\chi_2^\pm$	6900
H_0	3000

particle	mass [GeV]
A_0	3000
$H^\pm$	3000
$\tilde{g}$	7000
$\tilde{\tau}_1$	2000
other sleptons	$3200-6000$
$\tilde{t}_1$	5000
$\tilde{t}_2$	7500
other squarks	$6400-8600$

Table 3: Mass spectrum for $(N_2, N_3) = (17, 7)$, with $F = (4 \times 10^9 \text{ GeV})^2$, $M = 10^{14}$ GeV, $\mu = 230$ GeV, and $\sqrt{B_\mu} = 614$ GeV. For these parameters, $\tan\beta = 49$.

where

$$m_{\text{GMSB}} \equiv \frac{1}{16\pi^2}\frac{F}{M} \approx 1 \text{ TeV}\,. \tag{17}$$

While this relation is not precise enough to predict m_Z accurately, it does serve to illustrate that the residual fine-tuning is of the order $(m_Z/m_{\text{GMSB}})^2$ (because the coefficient of m_{GMSB} is $\mathcal{O}(1)$). For the present benchmark point, this implies that $1/\Delta \sim \mathcal{O}(1\,\%)$, which is a considerable improvement over generic models with similarly heavy superpartners. It should be pointed out, however, that the sensitivity of the focus point cancellation to the dimensionless Standard Model couplings (which are usually not included into the fine-tuning definition) is likely very high.

Finally, if the Z superfield of the previous section is identified with the goldstino multiplet, then the LSP is a 4 GeV gravitino. If furthermore R-parity is conserved, within standard cosmology, the decays of the χ_1^0 NLSP will spoil the successful prediction of light element abundances during Big Bang Nucleosynthesis. In the current model, however, the relic abundance of the NSLP is rather low, $\Omega_\chi h^2 \lesssim 10^{-2}$, due to the large annihilation cross section of the Higgsino. Therefore the BBN problem can solved by slightly lowering the SUSY breaking, such as to lower the gravitino mass to $\lesssim 1$ GeV which reduces the higgsino lifetime to $\mathcal{O}(10^2)$ seconds [17, 18].[5] On the other hand, the actual goldstino direction of the hidden sector may have other components besides Z (as in the model of the following section), in which case the gravitino could also be heavier than χ_1^0.[6]

4 The origin of the SUSY breaking spurion

So far, we have simply assumed that the messengers couple to a SUSY-breaking spurion field Z whose vacuum expectation value is

$$\langle Z \rangle = M + F\theta^2 \ . \tag{18}$$

In this section, we sketch a possible dynamical origin of F and M. We consider the model of cascade SUSY breaking which was developed in Refs. [19, 20]. This model contains a primary SUSY-breaking field Z_0 and a secondary SUSY-breaking field Z_1, the latter of which will be identified with Z.

In the cascade SUSY breaking model, the Kähler potential and superpotential are

$$K = Z_0^\dagger Z_0 + Z_1^\dagger Z_1 - \frac{c_0^2}{4\Lambda^2}(Z_0^\dagger Z_0)^2 + \frac{c_1^2}{\Lambda^2} Z_0^\dagger Z_0 Z_1^\dagger Z_1 + \dots \ , \tag{19}$$

$$W = \Lambda^2 Z_0 + \frac{h}{3} Z_1^3 + \kappa \, Z_1 \Phi \overline{\Phi} \, , \tag{20}$$

where Λ denotes a dimensionful parameter while $c_{0,1}$, h and κ are dimensionless coefficients. In the following, we take h and κ to be real and positive by rotating the phases of the fields appropriately. We are assuming that the higher dimensional Kähler potential terms are generated radiatively by integrating out certain fields in a generalized

[5]Note however that the gravitino cannot be made arbitrarily light. To obtain an MSSM spectrum similar to the one of Table 3, the messenger scale also would need to be lowered accordingly, keeping F/M constant. But a too low messenger scale would result in a too large g_5 coupling at M_{GUT}, which would then again blow up very quickly.

[6] In the Higgsino LSP case, the thermal relic abundance is too small to account for the observed dark matter density, so one would need non-thermal sources for the Higgsino or another dark matter candidate.

O'Raifeartaigh model [19, 20]. It can be shown that $c_0^2 > 0$ when the quartic Z_0 term is perturbatively generated by integrating out fields with R-charges 0 or 2 [21]. Similarly, for $c_1^2 > 0$ one needs Z_1 to couple to fields with R-charges other than 0 or 2 [19, 20].

In this model, there is an R-symmetric but SUSY-breaking vacuum at

$$Z_0 = 0\,, \tag{21}$$

$$F_{Z_0} = \Lambda^2\,, \tag{22}$$

around which the primary SUSY-breaking field obtains a mass

$$m_{Z_0}^2 = c_0^2 \Lambda^2\ . \tag{23}$$

Once Z_0 breaks SUSY, the secondary SUSY-breaking field Z_1 obtains a soft SUSY-breaking mass term. The scalar potential for Z_1 is given by

$$V(Z_1) \simeq -m_{Z_1}^2 |Z_1|^2 + |h Z_1^2|^2, \qquad m_{Z_1}^2 = c_1^2 \frac{|F_{Z_0}|^2}{\Lambda^2} = c_1^2 \Lambda^2\ . \tag{24}$$

Therefore, for $m_{Z_1}^2 > 0$, the secondary SUSY-breaking field obtains a non-vanishing expectation value,

$$\langle Z_1 \rangle \simeq \frac{m_{Z_1}}{\sqrt{2}h} \simeq \frac{c_1}{\sqrt{2}h}\Lambda\ , \tag{25}$$

which breaks SUSY by

$$F_{Z_1} = h\,\langle Z_1^* \rangle^2 \simeq \frac{m_{Z_1}^2}{2h} = \frac{c_1^2}{2h}\Lambda^2\ . \tag{26}$$

In this way, secondary SUSY breaking is initiated by spontaneous R-symmetry breaking which is, in turn, triggered by fundamental SUSY breaking.

Through the coupling between the messengers and Z_1 in Eq. (19), the secondary SUSY breaking field Z_1 plays the role of the spurion in the previous sections, i.e.

$$M = \kappa\,\langle Z_1 \rangle \simeq \frac{\kappa c_1}{\sqrt{2}h}\Lambda\ , \quad F = \kappa\, F_{Z_1} \simeq \frac{\kappa c_1^2}{2h}\Lambda^2\ . \tag{27}$$

In the following, we assume that $c_1^2 \simeq 2h$ and $\kappa \simeq 1$, in order to obtain a gravitino mass which is as low as possible for a given F, i.e. $F \simeq F_{Z_1} \simeq F_{Z_0}$, while keeping the messenger mass as high as possible.[7] Under these assumptions, the parameters used in the previous section,

$$M \simeq 10^{14}\,\mathrm{GeV}\ , \quad F \simeq (4 \times 10^9\,\mathrm{GeV})^2\ , \tag{28}$$

are obtained by choosing

$$\Lambda \simeq 4 \times 10^9\,\mathrm{GeV}\ , \quad c_1 \simeq 5 \times 10^{-5}\ . \tag{29}$$

Finally, let us comment on the origins of the μ and B_μ term. As mentioned earlier, one may consider "gravity-mediated" contributions,

$$K = c_H \frac{Z^\dagger}{M_*} H_u H_d + c_B \frac{Z^\dagger Z}{M_*^2} H_u H_d + \text{ h.c. }, \tag{30}$$

which leads to

$$\mu = \sqrt{3} c_H \frac{M_{\mathrm{Planck}}}{M_*} m_{3/2}\ , \tag{31}$$

$$\sqrt{B_\mu} = \sqrt{3} c_B^{1/2} \frac{M_{\mathrm{Planck}}}{M_*} m_{3/2}\ . \tag{32}$$

Thus, with the coefficients c_H and c_B of order unity and $M_* \simeq 10^{17}\,\mathrm{GeV}$, one obtains values for μ and B_μ of the order of the weak scale as desired.

It should be noted that one of the phases of $c_{H,B}$ cannot be eliminated by field redefinitions, which leads to CP-violating processes and gives rise to, for instance, an electron electric dipole moment (EDM). In fact, if the relative phase between μ and B_μ is of order unity, the predicted electron EDM slightly exceeds the current limit for the mass spectrum in Table 3. Avoiding the bound requires some amount of tuning between c_H and c_B.

Similarly, completely generic gravity-mediated contributions to soft masses and A-terms would lead to unacceptably large flavour changing neutral currents. While the soft

[7] As mentioned above, another possibility would be to have a much heavier gravitino mass and a Higgsino LSP. In this case, parameter choices such as $c_1 \ll h$ and $\kappa \ll 1$ are also allowed.

terms are dominated by the flavour-universal gauge-mediated contributions, subdominant flavour-violating corrections are still potentially dangerous, see e.g. [23,24]. Keeping them under control also requires some tuning or an underlying symmetry. However, a detailed analysis of the flavour and CP problems in our model is beyond the scope of this work.

5 Conclusions

We have shown that focus point gauge mediation can naturally be embedded into a model of SU(5) $\times$ U(3) product group unification. For the cutoff of the product group unification model to be substantially higher than the GUT-breaking scale, we have added to the MSSM a number of vector-like weak doublets at intermediate energies. These act as gauge mediation messengers, leading to the non-unified soft term mass ratios which are required for a focus point-like cancellation between the radiative corrections to the Higgs potential, and consequently a little hierarchy between the soft mass scale and the electroweak scale. In the example we constructed, perturbativity can be maintained until about an order of magnitude above the GUT-breaking scale. The superpartners, except for some higgsino-like neutralinos and charginos, are however predicted to be very heavy and out of LHC reach. The resulting Higgs mass is compatible with 125 GeV, while the fine-tuning is still comparatively modest.

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