EXPONENTIAL AND CRITICAL EXPERIMENTS

VOL. III
The following States are Members of the International Atomic Energy Agency:

<table>
<thead>
<tr>
<th>Afghanistan</th>
<th>Ivory Coast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albania</td>
<td>Japan</td>
</tr>
<tr>
<td>Algeria</td>
<td>Republic of Korea</td>
</tr>
<tr>
<td>Argentina</td>
<td>Lebanon</td>
</tr>
<tr>
<td>Australia</td>
<td>Liberia</td>
</tr>
<tr>
<td>Austria</td>
<td>Libya</td>
</tr>
<tr>
<td>Belgium</td>
<td>Luxembourg</td>
</tr>
<tr>
<td>Brazil</td>
<td>Mali</td>
</tr>
<tr>
<td>Bulgaria</td>
<td>Mexico</td>
</tr>
<tr>
<td>Burma</td>
<td>Monaco</td>
</tr>
<tr>
<td>Byelorussian Soviet Republic</td>
<td>Morocco</td>
</tr>
<tr>
<td>Cambodia</td>
<td>Nethelands</td>
</tr>
<tr>
<td>Canada</td>
<td>New Zealand</td>
</tr>
<tr>
<td>Ceylon</td>
<td>Nicaragua</td>
</tr>
<tr>
<td>Chile</td>
<td>Norway</td>
</tr>
<tr>
<td>China</td>
<td>Pakistan</td>
</tr>
<tr>
<td>Colombia</td>
<td>Paraguay</td>
</tr>
<tr>
<td>Congo (Leopoldville)</td>
<td>Peru</td>
</tr>
<tr>
<td>Cuba</td>
<td>Philippines</td>
</tr>
<tr>
<td>Czechoslovak Socialist Republic</td>
<td>Poland</td>
</tr>
<tr>
<td>Denmark</td>
<td>Portugal</td>
</tr>
<tr>
<td>Dominican Republic</td>
<td>Romania</td>
</tr>
<tr>
<td>Ecuador</td>
<td>Saudi Arabia</td>
</tr>
<tr>
<td>El Salvador</td>
<td>Senegal</td>
</tr>
<tr>
<td>Ethiopia</td>
<td>South Africa</td>
</tr>
<tr>
<td>Finland</td>
<td>Spain</td>
</tr>
<tr>
<td>France</td>
<td>Sudan</td>
</tr>
<tr>
<td>Federal Republic of Germany</td>
<td>Sweden</td>
</tr>
<tr>
<td>Gabon</td>
<td>Switzerland</td>
</tr>
<tr>
<td>Ghana</td>
<td>Syria</td>
</tr>
<tr>
<td>Greece</td>
<td>Thailand</td>
</tr>
<tr>
<td>Guatemala</td>
<td>Tunisia</td>
</tr>
<tr>
<td>Haiti</td>
<td>Turkey</td>
</tr>
<tr>
<td>Holy See</td>
<td>Ukrainian Soviet Socialist Republic</td>
</tr>
<tr>
<td>Honduras</td>
<td>United Arab Republic</td>
</tr>
<tr>
<td>Hungary</td>
<td>United Kingdom of Great Britain and Northern Ireland</td>
</tr>
<tr>
<td>Iceland</td>
<td>United States of America</td>
</tr>
<tr>
<td>India</td>
<td>Uruguay</td>
</tr>
<tr>
<td>Indonesia</td>
<td>Venezuela</td>
</tr>
<tr>
<td>Iran</td>
<td>Viet-Nam</td>
</tr>
<tr>
<td>Iraq</td>
<td>Yugoslavia</td>
</tr>
<tr>
<td>Israel</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td></td>
</tr>
</tbody>
</table>

The Agency's Statute was approved on 23 October 1956 by the Conference on the Statute of the IAEA held at United Nations Headquarters, New York; it entered into force on 29 July 1957. The Headquarters of the Agency are situated in Vienna. Its principal objective is "to accelerate and enlarge the contribution of atomic energy to peace, health and prosperity throughout the world".

Printed by the IAEA in Austria
April 1964
EXPONENTIAL AND CRITICAL EXPERIMENTS

PROCEEDINGS OF THE SYMPOSIUM ON EXPONENTIAL AND CRITICAL EXPERIMENTS HELD BY THE INTERNATIONAL ATOMIC ENERGY AGENCY IN AMSTERDAM, NETHERLANDS, 2 - 6 SEPTEMBER 1963

In three volumes

VOL. III

INTERNATIONAL ATOMIC ENERGY AGENCY VIENNA, 1964
EXPONENTIAL AND CRITICAL EXPERIMENTS
IAEA, VIENNA, 1964
STI/PUB/79
FOREWORD

In September 1963, the International Atomic Energy Agency organized the Symposium on Exponential and Critical Experiments in Amsterdam, Netherlands, at the invitation of the Government of the Netherlands. The Symposium enabled scientists from Member States to discuss the results of such experiments which provide the physics data necessary for the design of power reactors.

Great advances made in recent years in this field have provided scientists with highly sophisticated and reliable experimental and theoretical methods. This trend is reflected in the presentation, at the Symposium, of many new experimental techniques resulting in more detailed and accurate information and a reduction of costs. Both the number of experimental parameters and their range of variation have been extended, and a closer degree of simulation of the actual power reactor has been achieved, for example, by means of high temperature critical assemblies. Basic types of lattices have continued to be the objective of many investigations, and extensive theoretical analyses have been carried out to provide a more thorough understanding of the neutron physics involved.

Twenty nine countries and 3 international organizations were represented by 198 participants. Seventy one papers were presented. These numbers alone show the wide interest which the topic commands in the field of reactor design.

We hope that this publication, which includes the papers presented at the Symposium and a record of the discussions, will prove useful as a work of reference to scientists working in this field.

The Agency expresses its gratitude to the Government of the Netherlands for its generous assistance and the hospitality offered during the meeting. It thanks all participants for their contributions, especially the chairmen of the sessions and panel members.
EDITORIAL NOTE

The papers and discussions incorporated in the proceedings published by the International Atomic Energy Agency are edited by the Agency's editorial staff to the extent considered necessary for the reader's assistance. The views expressed and the general style adopted remain, however, the responsibility of the named authors or participants.

For the sake of speed of publication the present Proceedings have been printed by composition typing and photo-offset lithography. Within the limitations imposed by this method, every effort has been made to maintain a high editorial standard; in particular, the units and symbols employed are to the fullest practicable extent those standardized or recommended by the competent international scientific bodies.

The affiliations of authors are those given at the time of nomination.

The use in these Proceedings of particular designations of countries or territories does not imply any judgment by the Agency as to the legal status of such countries or territories, of their authorities and institutions or of the delimitation of their boundaries.

The mention of specific companies or of their products or brand-names does not imply any endorsement or recommendation on the part of the International Atomic Energy Agency.
CONTENTS OF VOL. III

VII. ASSEMBLIES WITH GRAPHITE

Reactor physics development for advanced gas-cooled reactors (SM-42/49) .............................................. 3
  J. Moore
Discussion .......................................................... 25
Concept of a flexible exponential experiment for high-temperature, gas-cooled reactor systems (SM-42/10) ............ 27
  H. Grüm
Discussion .......................................................... 32
High-temperature gas-cooled reactor critical experiment and its application (SM-42/37) .................................. 35
  R.G. Bardes, J.R. Brown, M.K. Drake, P.U. Fischer,
  D.C. Pound, J.B. Sampson and H.B. Stewart
Discussion .......................................................... 60
On the subcritical assembly for high-temperature use (SM-42/3) ................................................................. 63
  Y. Sakurai, T. Sekiya, T. Suita, H. Hishida, H. Hamada and
  K. Nagashima
Discussion .......................................................... 69
Étude d'un ensemble sous-critique par la méthode de la caractéristique de fréquence (SM-42/68) ......................... 71
  I.I. Purica, N. Seferian et E. Răcătăianu
Some measurements of thermal-neutron spectra (SM-42/18) ................................................................. 87
  M.J. Poole, P. Schofield and R.N. Sinclair
Discussion .......................................................... 110
Subcritical experimental work at the Delft reactor institute (SM-42/57) ......................................................... 113
  H.R. Kleijn, A.W. Van der Heijden and H. Van Dam
Discussion .......................................................... 124
Use of an exponential assembly to investigate boundary conditions in neutron diffusion (SM-42/1) .................... 127
  C.G. James and P.J. Grant
Discussion .......................................................... 137

VIII. SPECIAL TECHNIQUES (THEORETICAL AND EXPERIMENTAL)

Mesure des sections efficaces effectives d'échantillons fissiles par une méthode d'oscillation dans les assemblages critiques (SM-42/64) ......................................................... 141
  R. Vidal, O. Trédiakoff et M. Robin
Discussion .......................................................... 158
Technique des neutrons pulsés appliquée aux assemblages critiques et sous-critiques (SM-42/63) ......................... 163
  M. Sagot, H. Tellier, R. Jacquemart et J. Kremser
Discussion .......................................................... 189
Measurements of \( \partial \rho/\partial t \) in a water-moderated reactor by an oscillating moderator-level technique (SM-42/74) ................................................................. 191
  D. Babala
Discussion .......................................................... 202
The impulse response of an exponential assembly (SM-42/43) ................................................................. 205
  R.E. Uhrig
Discussion .......................................................... 239
Use of the neutron die-away technique to test control rod effectiveness theories (SM-42/44) ......................... 241
  R.B. Perez, G. de Saussure and E.G. Silver
Pulsed and static neutron measurements in light-water and heavy-water exponentials (SM-42/45) ........................................... 259
T. F. Parkinson, R. B. Perez, D. N. Fry, R. H. Hartley, S. P. D. Smith and N. J. Diaz

Medida de la laplaciana en una experiencia exponencial (SM-42/48) .... 283
E. Rodríguez Mayquez

Discussion .............................................. 287

The evaluation of buckling and diffusion coefficients from two-region experiments (SM-42/47) ........................................... 289
R. Persson

Discussion .............................................. 302

Measurement of anisotropy of diffusion constant in media with empty channels (SM-42/5) .............................................. 305
M. Čopič, T. Kalin, G. Pregl and F. Žerdel

Discussion .............................................. 318

Determination of neutron temperature using boron filters (SM-42/60) .... 321
T. Rzeszot and E. Warda

Some experimental characteristics of the graphite, water-moderated critical assembly for the Second Polish Research Reactor (SM-42/61) 333

Исследование физических свойств активной зоны BBЭР на критических сборках (SM-42/89) 355
Г.Я.Андрианов, В.А.Боищенский, А.Н.Камышан, Л.В.Комиссаров, В.А.Кузьминцев, Г.Л.Лунин, В.Н.Семенов и В.И.Хализов

IX. ORGANIC-MODERATED OR COOLED ASSEMBLIES

The EXPO exponential facility (SM-42/53) .............................................. 383
H. Neltrup and P. L. Øigaard

Discussion .............................................. 399

R.O.S.P.O. organic-moderated critical facility (SM-42/77) 401
G. Bitelli, S. Grifoni, R. Martinelli and E. Santandrea

Critical analysis of the progressive substitution method for material buckling measurements (SM-42/51) 423
G. Casini and J. Mégier

Discussion .............................................. 441

An application of heterogeneous reactor theory to substitution experiments (SM-42/52) 443
G. Biaessler

Discussion .............................................. 451

Panel Discussion .............................................. 457

List of Chairmen of Sessions and Secretariat of the Symposium 467
List of Participants .............................................. 468
Author Index .............................................. 478
VII

ASSEMBLIES WITH GRAPHITE
REACTOR PHYSICS DEVELOPMENT FOR ADVANCED GAS-COOLED REACTORS

J. MOORE
UNITED KINGDOM ATOMIC ENERGY AUTHORITY, WINDSCALE, UNITED KINGDOM

Abstract — Résumé — Аннотация — Resumen

REACTOR PHYSICS DEVELOPMENT FOR ADVANCED GAS-COOLED REACTORS. The paper is a general one reviewing the detailed experimental and theoretical work that has been done during design, development and commissioning of the Windscale AGR. and in general support of development of Advanced Gas-Cooled Reactors for civil power production. The main purpose of the paper is to describe the considerable amount of work that has been necessary to develop adequate theoretical methods for calculating (a) flux distribution and reactivity balance in a complex reactor core, (b) power distribution in complex fuel geometries, and (c) the effect of irradiation on fuel cycles and power distribution. As an introduction reference is made to experimental information and theoretical methods available from work on uranium-Magnox systems and the experimental information obtained from the British Industries Collaborative Experimental Programme (BICEP) both of which provided the starting point for development of theoretical methods applicable to AGRs.

The approach to critical facility APEX and the zero energy reactor HERO have been used with regular lattice arrangements and with combinations of perturbers such as control rods in them to determine lattice parameters for AGR and to check the theoretical methods developed for dealing with heterogeneous reactor cores. The theoretical methods that have been developed and used to date are known as hetrecontrol and FTD2. Experiments were designed to check detailed features of these methods and measurements on a number of different sized “reactor” cores in APEX and HERO were analysed to determine a self-consistent set of lattice constants that fitted the experimental results. These purely empirical constants were the used in hetrecontrol and FTD2 to successfully plan commissioning and choice of the loading pattern for the Windscale AGR.

Reference is made to experimental techniques that have been tested and those developed for the particular problems encountered. The methods examined for measurement of reactivity effects in APEX, HERO and AGR and for determining fine structure data and power distribution in the complex fuel assemblies are of particular interest.

Current and future theoretical work is concentrated primarily on development of an alternative method to hetrecontrol and FTD2 for dealing with reactor cores after considerable burn-up of the fuel. The experimental programme on HERO is designed to test these methods with complex cores including plutonium bearing fuel. Additional information on the effect of plutonium will be derived from operation of AGR and physics measurements on fuel after irradiation.

RECHERCHES EN PHYSIQUE DES RÉACTEURS, POUR DES RÉACTEURS PERFECTIONNÉS REFROIDIS PAR UN GAZ. Le mémoire relate les recherches expérimentales et théoriques auxquelles on a procédé lors de l'étude, de la réalisation et de la mise en service du réacteur perfectionné refroidi par un gaz (AGR) de Windscale et, d'une façon générale, pour la mise au point d'un filière de ce type en vue de la production d'énergie électrique industrielle. Il décrit l'important volume de travail qui a été nécessaire en vue d'élaborer les méthodes théoriques voulues pour calculer: a) la répartition du flux et l'équilibre de la réactivité dans un cœur complexe; b) la répartition de la puissance dans des géométries de combustible complexes; c) les effets de l'irradiation sur le cycle du combustible et la répartition de la puissance. A titre d'introduction, le mémoire résume la documentation expérimentale et les méthodes théoriques qui sont le résultat des recherches sur la filière à uranium gainé de magnox et décrit la documentation expérimentale obtenue par le programme commun des industries britanniques (BICEP); toutes ces données ont servi de point de départ pour l'élaboration de méthodes théoriques applicables à l'AGR.

On s'est servi de l'ensemble critique APEX et du réacteur HERO de puissance zéro avec des configurations de réseau régulières et diverses combinaisons de perturbateurs (notamment des barres de commande) pour calculer les paramètres de réseau de l'AGR et vérifier les méthodes théoriques élaborées pour étudier des cœurs de réacteurs hétérogènes. Ces méthodes théoriques, utilisées jusqu'à ce jour, sont connues sous les noms
д'«hetrecontrol» et de «FTD2». Les expériences avaient pour but de vérifier dans le détail les caractéristiques de ces méthodes; on a analysé les mesures faites sur plusieurs cœurs de «réacteur» et de différentes dimensions dans les installations APEX et HERO pour déterminer une série cohérente de constantes de réseau concordant avec les résultats des expériences. A ces constantes purement empiriques, on a appliqué ensuite les méthodes «hetrecontrol» et «FTD2» pour préparer la mise en service sans accord d'AGR et le choix du régime de chargement de ce réacteur.

Le mémoire énumère les techniques expérimentales qui ont été essayées et celles qui ont été élaborées pour résoudre certains problèmes qui se présentaient. Particulièrement intéressantes sont les méthodes ayant pour but de mesurer les effets sur la réactivité dans les installations APEX, HERO et AGR, et de déterminer les données relatives à la structure fine ainsi que la répartition de la puissance dans les assemblages complexes.

Les recherches théoriques actuelles et futures sont axées principalement sur la mise au point d'une méthode capable de remplacer «hetrecontrol» et «FTD2» pour les études sur des cœurs de réacteur après qu'une bonne partie du combustible a «brûlé». Le programme d'expériences avec l'installation HERO a pour but de vérifier ces méthodes au moyen de cœurs complexes contenant du plutonium. On compte obtenir des renseignements supplémentaires sur l'effet du plutonium au cours du fonctionnement d'AGR et à la suite de mesures de physique sur le combustible irradié.

РАЗРАБОТКА МЕТОДОВ В ОБЛАСТИ РЕАКТОРНОЙ ФИЗИКИ ДЛЯ УСОВЕРШЕНСТВОВАННОГО РЕАКТОРА С ГАЗОВЫМ ОХЛАЖДЕНИЕМ. Рассматривается тщательная экспериментальная и теоретическая работа, проведенная при конструировании, строительстве и вводе в эксплуатацию Уиндскейлского усовершенствованного реактора с газовым охлаждением и при оказании общей помощи при разработке усовершенствованного реактора с газовым охлаждением (AGR) для производства энергии для гражданских целей.

Дается описание значительного объема работ, которые были необходимы для разработки пригодных теоретических методов расчета: 1) распределения потока и баланса реактивности в сложной активной зоне реактора; 2) распределения энергии в топливе со сложной геометрией и 3) влияния облучения на топливные циклы и распределение энергии. В качестве введения делается ссылка на экспериментальные данные и теоретические методы, полученные в результате работ над ураново-магноксювными системами, и на экспериментальные данные, полученные в результате Совместной экспериментальной программы Британской промышленности (BICEP), которые явились отправным моментом в разработке теоретических методов, применяемых к усовершенствованным реакторам с газовым охлаждением.

Для определения параметров решетки для реактора AGR и проверки теоретических методов, разработанных для гетерогенных активных зон реакторов, использовались критическая установка APEX и реактор нулевой мощности HERO с обычными расположениями решеток и комбинациями изменяющихся работы реактора элементов, например регулирующих стержней. Теоретические методы, разработанные и применявшиеся до настоящего времени, известны как "гетероконтрол", FTD2. Эксперименты имели целью подробно проверить особенности этих методов, и для определения согласованного между собой ряда констант решетки, соответствующих результатам экспериментов, были проанализированы результаты измерений, выполненных на ряде "реакторных" активных зон различного размера в установках APEX и HERO. Эти чисто эмпирические константы были затем использованы в методах гетероконтрола и FTD2 для успешного планирования ввода в эксплуатацию и выбора вида нагрузки для Уиндскейлского AGR.

Дается ссылка на экспериментальные методы, которые были проверены и/или специально разработаны для решения встретившихся проблем. Особый интерес представляют методы, использовавшиеся для измерения эффектов реактивности в реакторах APEX, HERO и AGR и для определения данных точной структуры и распределения энергии в сложных топливных сборках.

Осуществляемые в настоящее время теоретические работы сконцентрированы, главным образом, на разработке альтернативного метода в отношении "гетероконтрола" и FTD2 для расчета активных зон реактора после значительного выгорания топлива. На этом же будут сконцентрированы работы и в будущем. Задачей программы экспериментов на установке HERO является испытание этих методов на сложных активных зонах, включая активные зоны с топливом, производяющим плутоний. Дополнительные данные о влиянии плутония будут получены благодаря эксплуатации реактора AGR и физическим измерениям облученного топлива.

PROGRESOS DE LA FÍSICA DE LOS REACTORES DE TIPO AVANZADO REFRIGERADOS POR GAS. La memoria describe los trabajos experimentales y teóricos que se han ejecutado durante el diseño, el desarrollo y la puesta en marcha del reactor AGR de Windscale y para facilitar el desarrollo de nuevos tipos de reactores
refrigerados por gas para la producción de energía nucleoeléctrica con fines civiles. La principal finalidad de la memoria es describir el considerable trabajo que entrañó el desarrollo de métodos teóricos adecuados para calcular: a) la distribución del flujo y el balance de la reactividad en un cuerpo complejo, b) la distribución de la potencia en geometrías complejas del combustible, y c) el efecto de la irradiación sobre los ciclos del combustible y la distribución de la potencia. A modo de introducción se menciona la información experimental y los métodos teóricos que constituye el resultado de los trabajos con sistemas uranio-magnox, y los datos experimentales comunicados por el British Industries Collaborative Experimental Program (BICEP), en los que se basó el desarrollo de los métodos teóricos que se han aplicado a los reactores AGR.

Con el fin de determinar los parámetros del reticulado del AGR y comprobar los métodos teóricos establecidos para cuerpos de reactor heterogéneos, se ha empleado el conjunto crítico APEX y el reactor HERO de energía nula, tanto con reticulados normales como con combinaciones de perturbadores tales como barras de control. Los métodos teóricos desarrollados y empleados hasta ahora se conocen por el nombre de «heterocontrol» y «FTD2». Se prepararon experimentos para comprobar algunos detalles de las características de estos métodos y se han analizado mediciones efectuadas en las instalaciones APEX y HERO con varios cuerpos de reactor de diversos tamaños con el fin de determinar series coherentes de constantes reticulares que concuerden con los resultados experimentales. Seguidamente, a estas constantes puramente empíricas se aplicaron los métodos «heterocontrol» y «FTD2» para planear la puesta en marcha y elegir el esquema de carga del reactor AGR de Windscale.

La memoria menciona las técnicas experimentales comprobadas y las que se han desarrollado para resolver los problemas particulares que se presentaron. Reviste particular interés el examen de los métodos de medición de los efectos de la reactividad en APEX, HERO y AGR, y para determinar los datos relativos a la estructura fina y a la distribución de la potencia en los conjuntos combustibles complejos.

Los trabajos se concentran principalmente en el desarrollo de un método capaz de sustituir al «heterocontrol» y al «FTD2», para los cuerpos de reactor cuando el combustible haya alcanzado un grado de combustión considerable. La finalidad del programa experimental con la instalación HERO es precisamente comprobar estos métodos con cuerpos complejos, inclusive con combustible que contenga plutonio. La explotación del reactor AGR y las mediciones físicas que se realicen con el combustible después de la irradiación permitirán obtener datos adicionales sobre el efecto del plutonio.

INTRODUCTION

After the successful commissioning of the first Calder Hall reactor and launching of the United Kingdom Civil power programme based on the use of metal fuel canned in magnesium alloy, the Atomic Energy Authority turned its attention to development of improved types of gas-cooled reactor that would utilize uranium oxide fuel operating at higher temperatures. The potentials for advancement in maximum fuel rating from about 4 to 20 MW/t and for increasing can surface temperature by at least 250°C were sufficiently promising for the Authority to decide to provide facilities and plan the development programme to obtain information that would be required for a generation of large advanced gas-cooled reactors (A.G.R.s) in addition to demonstrating the A.G.R. concept by building a prototype. This paper indicates the scale of effort that has been required for the reactor physics content in this development programme and describes the choice of experimental facilities and the sequence in the development of reactor physics theory. The objective of the programme has been to obtain information required not only for design but also for efficient operation of a large-scale power production programme.

Development of methods for physics calculations for use in the design of advanced gas-cooled reactors commenced in 1957 as an integral part of the advanced gas-cooled reactor design study. This early work clearly indicated that the system had good potential for economic power production and
led to the decision to build the Windscale prototype on a tight time scale with a target date for commencement of commissioning in April 1961. Development of theoretical methods and definition of the required experimental programme were linked initially to this target, but since in practice commissioning experiments did not commence until February 1962 in HERO and June 1962 in A.G.R., there proved to be time for experimental work to be aimed more at the requirements for a large civil power programme than solely for the commissioning of the Windscale A.G.R.

DEVELOPMENT OF THEORY

At the start of the project there was a considerable amount of experimental information on single rods of natural uranium metal in graphite lattices which had been correlated and found to be adequate for design purposes for the Calder and later uranium-magnox type civil reactors giving an accuracy on initial reactivity of within ±0.5%. All these reactors have simple fuel geometry and fixed enrichment (natural uranium) and operate to a relatively low burn-up of about 4000 MWd/t. The method of calculation adopted for design of A.G.R.s was to extend this empirical method derived for the uranium-magnox reactors and to modify the extension as new experimental information became available. An experimental programme was initiated in parallel with this theoretical work to provide data for use in the correlation and with a longer term requirement to obtain data for operation as well as for design.

The methods of calculation developed for the uranium-magnox systems had to be extended to allow for lower fuel density, the presence of oxygen in the fuel, the presence of the stainless-steel can acting as an absorber in association with fuel and considerable sub-division of the fuel with the eventual choice of clusters of 21 elements, each of 0.4-in diameter. Since the system required enriched fuel, the choice of enrichment introduced an additional design variable, and the required longer fuel burn-up called for estimation of bigger changes in reactivity than had been experienced previously in the United Kingdom in graphite-moderated systems. The fuel content in each channel would be a larger absorber than would a channel of fuel in the uranium-magnox reactors and would represent a bigger perturbation in heterogeneous reactor calculations.

Two parallel lines of experimental work were planned, namely,

(i) A survey of different fuel geometries and lattice pitches with regular lattices in exponential and fine-structure facilities to obtain lattice constants for uniform lattices; and

(ii) Detailed investigation of the specific geometry chosen for the Windscale A.G.R., including the effect of perturbations in the reactor lattice and the effect of irradiation, and to provide data for development of theoretical methods for the planning of fuel cycles and reactor operation.

The survey work which was carried out at A.E.E. Winfrith under the British Industries Collaboration Experimental Programme (BICEP) agreement has been completed [1]. The first phase of the other work, which is that leading up to and including commissioning of the Windscale A.G.R.,
has been finished; and longer-term work aimed at allowing for plutonium build-up, calculation of power distribution and optimization of fuel cycles is in progress.

Theoretical work can be conveniently considered from the following three points of view:

(i) In aid of design of the W.A.G.R. including choice of enrichment [2]
This work comprises correlation of experimental data obtained from early uranium-magnox and the BICEP work and modified in the later stages to include information arising from the Windscale experimental facilities for deduction of lattice constants and also development of the hetrecontrol method for calculating reactivity balance for the initial core. The work is now complete.

(ii) More basic methods of calculating lattice parameters, in particular development of the Spectrox method [3]
This approach shows potential for development to the stage at which it can be used for deduction of suitable parameters to describe reactor channels in complex core calculations.

(iii) Development of methods for computing reactivity and power distribution in operating reactors for use in optimizing operation and planning fuel cycles.

Tables I and II indicate the time scale for the experimental programme and for development of reactor physics theory.

Regular lattices

The design theory in current use for calculation of the reproduction constant $K_e$ of an infinite lattice is that developed from the simple four-factor formula

$$K_e = \epsilon p f \eta$$

with experimentally determined values for buckling, migration asymmetry and resonance integral (giving resonance escape probability $p$) together with calculated values of migration area and thermal utilization factor $f$. This gives a value of $\epsilon \eta$ which is used in calculation of reactivity of a finite system with the same lattice. Such experiments have been made with a wide range of fuel and graphite geometries to obtain a large number of experimental results, which have been correlated to give relationships between geometry, enrichment and lattice parameters.

This type of approach was first used to obtain working formulae for design of the uranium-magnox reactors. These formulae were modified to allow for the changes in advancing from simple rods of natural uranium metal to the more complex A.G.R. fuel, and the modified formulae were correlated with additional experimental results as they became available. The first analysis was made with results obtained from experiments with $\text{UO}_2$ fuel in heavy water for deduction of the effect of changing from single rod and from uranium to $\text{UO}_2$ to fuel clusters, which was followed by correlation with experimental results obtained from BICEP. The range of BICEP experiments was made with fuel that differed from the final fuel used in the W.A.G.R. in that (i) aluminium cladding was used, (ii) there were no axial
<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.G.R. PHYSICS EXPERIMENTS</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Design, construct and commission</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experiments with perturbed lattices</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Development of techniques for measurement of subcritical reactivity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bare builds with A.G.R. fuel</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2. HERO</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Design, construct and commission</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A.G.R. commissioning experiments</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Installation of heating equipment etc.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experiments for testing MAGPIE measurement of temperature coefficients etc.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3. A.G.R.</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Commissioning experiments</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temperature coefficient, xenon, long-term reactivity change</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control and kinetics experiments</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4. Fine-structure power distribution</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat transfer design experiments</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flux distribution, A.G.R. fuel</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Techniques for power distribution, fast-fission factor and conversion factor</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fine-structure experiments for design of irradiation experiments</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power distribution from irradiated fuel (La Pr method)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table II

**DEVELOPMENT OF REACTOR PHYSICS THEORY.**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel enrichment</td>
<td>Frozen APEX</td>
<td>HERO A.G.R.</td>
<td>On power critical</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U metal fuel</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UO₂ fuel</td>
<td>APEX</td>
<td>HERO</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2. Experimental lattice data</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>BICEP experiments</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>APEX Al-clad fuel</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A.G.R. fuel</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3. Lattice physics correlation theory</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Design theory: metal fuel</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modification for UO₂</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlation with D₂O-UO₂ lattice experiments</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Re-correlation with introduction of BICEP and APEX data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4. Reactor calculations</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple methods</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hétrecontrol</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FTDS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAGPIE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5. Basic methods</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast effects and</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fine-structure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spectrox - homogenized model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MINX - irradiation changes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PIII - heterogeneous model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transmission shadow method - heterogeneous</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
gaps between fuel elements, and (iii) the maximum U$^{235}$ content in the fuel was 1.8%, compared with the final choice of 2.5% for the W. A. G. R.

The main points in the development of this method for calculation of lattice constants are as follows:

(a) Simplification of fuel geometry by the assumption that the fuel is a single homogeneous rod of reduced density and with a radius equal to that of the circle that circumscribes the bundle of fuel elements. The lattice properties are deduced in relation to this rod, and the model is used for calculation of the fast-fission factor $\epsilon$. In the model no allowance is made for axial gaps, and the graphite sleeve is assumed to be part of the bulk moderator.

(b) The total neutron utilization factor per neutron absorbed in U$^{235}$ was deduced by (i) computation of thermal events with a Maxwellian distribution for thermal neutrons with temperatures slightly greater than the moderator temperature, (ii) use of simple diffusion theory and correlation with fine-structure measurements for calculation of the graphite disadvantage factor, (iii) assumption of $l/E$ relationship for high-energy events, and (iv) addition of epithermal sub-resonance events.

(c) Use of exponential experiments for determination of $K_\infty$ and thermal-fission factor $\eta_{235}$ and also resonance escape probability $p$ from experiments with the same fuel geometry and different lattice pitches.

(d) Diffusion and slowing-down areas were computed for the total cell volume, and corrections were applied for streaming.

Comparison of the method with the results of the BICEP experiments showed good agreement; for example, a standard deviation of 0.45% in $K_\infty$ was obtained by analysis of 29 experiments.

The choice of the pitch circles for the two rings of elements in the A.G.R. fuel design [4] is dictated by difference in heat ratings between the rings resulting from fine-structure effects and by heat transfer and gas flow considerations. In practice, the choice was made on the rather rough assumption that there was an $I_0$ distribution across the cluster, giving the mean flux as a function of relative position of the two rings of fuel elements. Information from the BICEP experiments became available at a later date and has been used as a check of more sophisticated methods of deduction of power distribution.

Experiments with A.G.R. fuel

Early experiments in APEX and HERO showed discrepancies with the correlation theory and the BICEP results.

Material buckling and migration area asymmetry were measured on APEX by approach-to-critical experiments with two heights of fuel, two and then three fuel assemblies high and with no graphite reflector around the stack. The experiments indicated that migration area asymmetry deduced by the correlation theory was overestimated by about 7%.

Measurements of reactivity of simple critical cores in HERO and by the approach technique in APEX with and without a reflector were not consistent and indicated disagreement with theory of about 2% in reactivity.
In an attempt to explain these discrepancies, exponential experiments were made with three lattice pitches and with horizontal and vertical builds for determination of migration area asymmetry and resonance escape probability. The fuel used for these experiments was that supplied for normal operation of the HERO reactor, which was identical to W.A.G.R. fuel but of lower enrichment (1.8% $^{235}$U). By the use of resonance escape probability determined from these experiments the discrepancy between calculated reactivity and experiment for the APEX bare builds could be reconciled.

Various adjustments of the constants used in the correlation methods can be made to give reasonable empirical modifications to the method in explanation of particular situations. However, since these methods are now considered to be adequate for design purposes, which is to calculate initial reactivity to within 2%, no further general improvements are being sought, and available effort is being more profitably employed in the development of basic methods for regular lattice calculations and methods for dealing with the more complex problems arising in large cores. Some correlation work is continuing to improve the understanding of effects at core-reflector boundaries for assistance in interpretation of small-core experiments.

Reactor cores

The problem of calculating flux distribution in A.G.R.s was recognized towards the end of 1958 when it was realized that the four steel test loops running through the reactor would have a major effect on flux and power distribution. Calculation methods had to be developed as design of the Windscale A.G.R. progressed. Estimates had to be made of change in reactivity of the complex core to permit choice of enrichment in September 1960 and later choice of numbers of control rods, absorbers etc.

In early work, reactivity was calculated by simple addition of the effect of single perturbations calculated according to the theory of CODD and RENNIE [5], but plans were quickly laid for the development of a more elaborate method, hetrecontrol [2], in which the homogeneous Codd-Rennie equations were solved for arbitrary arrangement of up to 48 singularities. In the A.G.R. lattice in which singularities such as loop tubes and control rods are inserted into fuel channels it is necessary to account for the effect of moderation in the singularities, which is small on reactivity but can be very large on flux distribution. A two-group diffusion calculation was devised to account for this effect and was incorporated into hetrecontrol in the middle of 1961. Hetrecontrol was then used for the carrying out of reactivity calculations on the complex core, which led to choice of numbers of control rods, absorbers etc. and rough planning of the initial loading pattern for the reactor.

The first tests on the effect of perturbations for checking hetrecontrol, and to some extent the correlation theories were made by means of experiments in APEX with single-channel perturbations and then with more than one channel for determination of interaction effects. The method was then used together with the FTD2 method [6] for the analysis of small-core experiments in HERO and for the planning of initial loading of the W.A.G.R. During loading of the W.A.G.R. each step was checked with hetrecontrol.
and FTD2 calculations, which led to minor modifications as loading to full size proceeded.

Since the generalized Codd-Rennie approach, and hence hetrecontrol, does not deal with the situation arising when there is fuel of varying irradiations present, it was recognized that this work on hetrecontrol was only of value for the design and choice of initial loading and that an alternative method allowing for burn-up effects would be required for planning of the fuel cycle and dealing with operational problems.

The FTD2 method has been developed for the planning of fuel cycle and day-to-day operation of the Calder reactors. The method consists essentially in a two-group diffusion calculation with each channel in the reactor as a mesh point in the programme. Lattice constants for the individual channels are chosen to be those that would pertain in an infinite lattice, and experimental values are used to describe singularities. This particular programme has worked very well for the Calder reactors; but it was clear that it would not deal adequately with A.G.R., because fuel channels represented bigger perturbations and bigger changes would arise from the longer irradiation expected in A.G.R. However, after some modification FTD2 has been used to date with reasonable success, in particular for the calculation of flux distribution.

In 1961 work was started on a more complex programme designed to deal with irradiation effects in the complex A.G.R. lattice with a view to development of the programme for later use in the planning of fuel cycle and optimization of operation of civil A.G.R.s. The programme, MAGPIE, has been devised to treat each reactor channel as a singularity, and it can be written to deal with as many neutron groups as required. Initially a programme with five groups is being written with a view to dealing with water systems as well as A.G.R.s. Each channel in the programme is represented by parameters that define the value of the channel as a neutron source and as a neutron sink. Initially the source-sink constants will be deduced empirically from experimental measurements in HERO. As work proceeds, lattice parameters arising from development of basic lattice calculations will be compared with the empirical values arising from the HERO programme; and it is hoped that eventually the empirical values obtained can be reconciled with values derived from basic theory.

The development of the MAGPIE method, or an alternative should MAGPIE prove to be inadequate, requires a considerable amount of experimental work with different arrangements of perturbations in reactor cores. The main source of experimental information will be from such experiments in HERO, which will include experiments with a number of channels of plutonium bearing fuel and with one or more channels of irradiated fuel transferred from the W.A.G.R. The methods developed will also be applied in the planning of fuel cycle and operation of the W.A.G.R., which will assist in their testing.

Basic methods

The first serious attempt to develop basic methods of calculation founded on basic data and not on integral experiments arose from the need for estimation of the effect of beryllium canning material on temperature coeffi-
cients in the W.A.G.R. Although the use of beryllium was abandoned, the development of basic methods of calculation has continued. These methods, assisted by improved measurement of basic nuclear cross-sections, are now developed to the stage at which they can be used for many situations; and they show promise of becoming better than correlation methods for analysis of uniform lattices. As development of the methods proceeds, it is clear that the nature of experimental work will tend more towards experiments designed to test uncertainties in theory than towards series of experiments directly associated with specific reactor geometries.

The Spectrox method has been used with some success for calculation of flux ratios between fuel and moderator as functions of neutron energy in uranium oxide cluster lattices. The method uses one-group diffusion theory in the moderator, collision probability in the fuel region and a generalized heavy gas model for neutron thermalization. More recently, developments combining Spectrox and collision probability theory in a fuel cluster indicate that it should be possible to calculate flux distribution and power distribution within the fuel cluster and to allow for changes arising during irradiation.

This work is now beginning to have impact upon the A.G.R. development programme as an alternative to the use of correlation methods for design purposes, and it is hoped that in the future the method will prove to be suitable for the derivation of descriptions of fuel and absorber channels for use in power distribution and burn-up calculations on complex reactor cores. However, it is prudent to plan this part of the development programme with the intention of basing description of channels on experimental results obtained in complex cores in HERO and to compare such descriptions with basic theory as it becomes practicable to do so.

EXPERIMENTAL TECHNIQUES AND FACILITIES

Lattice constants

The basic lattice constants determined experimentally are material buckling $B_m^2$, migration area asymmetry $(M_z/M_r)^2$ and resonance integral from which resonance escape probability $p$ may be deduced. In the BICEP experiments these constants were obtained from conventional exponential experiments, but because of the small critical size resulting from the high reactivity of the W.A.G.R. lattice it was decided that an approach-to-critical facility APEX was more suitable for experiments with W.A.G.R. fuel. The advantage of the approach method is that measurement of stack widths and large relaxation lengths with small stacks can be avoided, with a consequent reduction in error on the values of buckling and migration area asymmetry. Doubts as to the applicability of exponential results to reactor lattices and the use of infinite-medium diffusion coefficients derived from small exponential geometries are reduced to some extent by the use of the larger approach-to-critical core.

APEX was built as a bare rectangular system with the 10.75-in W.A.G.R. pitch for the lattice-constant measurements. The graphite and associated fuel were loaded together during the approach to critical, which simplified interpretation of the results; and building was continued until $K_{eff}$ was 0.99.
The experiment consisted essentially of the determination of the extrapolated dimensions and critical size of the system. The experiment was repeated with another height of fuel and graphite to enable deduction of material buckling and migration area asymmetry. With this technique the buckling was obtained to within 0.5% and the migration asymmetry to within 1%.

In cylindrical form with a radial reflector APEX (Fig. 1) was used extensively to provide ad hoc information relating to the commissioning of
the W.A.G.R., for example (i) the effects of absorbers and vacancies and interactions between them, (ii) fuel rating in loops, (iii) investigation of a method of reducing reactivity variations in power cycling experiments, and (iv) a comparison of the source-jerk and pulsed-neutron techniques for measuring subcritical reactivity [7].

Power distribution

Knowledge of the distribution of power generation within reactor fuel is necessary to permit choice of initial design, and it continues to have operational importance throughout the life of the fuel. The spacing of fuel elements in a fuel cluster to achieve maximum can temperature in all elements requires detailed understanding of heat transfer and fine-structure flux distribution.

Power distribution measurements have been made in a fine-structure stack (Fig. 2) with discs of manganese inserted between fuel pellets to give reaction rates at different positions within the fuel cluster, and manganese foils and wires located in the moderator and on the surface of and between fuel elements have been used to give reaction rates at these positions. Fuel foils and fuel pellets have also been used for determination of local power generation rates.

The fine-structure facility used for this work consisted of a graphite stack 86 in high with a 7×5 triangular lattice pitch of 10.75 in and containing 31 channels. Radial measurements were made in a plane 26 in from the top of the stack. The central graphite brick and associated graphite spacers

Fig. 2
Fine-structure stack
could be withdrawn as a unit and had removable, sliding sections of graphite into which manganese foils could be inserted. The stack was driven by a 15-kc antimony-beryllium source consisting of 24 antimony bars, 8 in long and 1 in in diameter, canned in stainless steel and irradiated in a Calder reactor. The neutron source could be switched on and off by these antimony bars, being pushed along eight source tubes which passed under a 1-in thick slab of beryllium located at the base of the stack. This arrangement gave a flux in the measuring plane of the stack of about $10^5$ n/cm$^2$s initially and decaying to about $10^4$ n/cm$^2$s before renewal of the antimony sources.

The graphite can be built in many different ways to give a variety of lattice pitches, and it can be operated at temperatures up to 400°C.

Reactivity changes and temperature coefficients

A just-critical system is very sensitive to small changes in reactivity, which can be measured either in terms of flux doubling time on movement of calibrated absorbers, such as control rods, or changes in air pressure required to hold the system critical. Also, since the available flux is likely to be higher than in a subcritical system, flux distributions with various absorber patterns can be measured very readily. With these two aims largely in mind, we have built a zero-energy critical system HERO (Fig. 3) at Windscale. The size of the core is the same as that of the W.A.G.R, and the present build uses the same triangular lattice pitch of 10.75 in.

The core of HERO can be heated to 500°C by circulation of hot CO$_2$, and internal ducting permits a separate CO$_2$ circuit to be used to control the temperature of fuel in the central 19 channels at a different temperature from that of the rest of the core. With these features it is possible to measure temperature coefficients and to separate total coefficient into its components resulting from fuel and moderator.

The reactor has two degrees of containment, a stainless-steel vessel and a concrete shield designed to permit the use of plutonium-bearing fuels at temperatures of 500°C in the reactor core.

Irradiated fuel

Until recently reactor physics measurements could only be made with fuel before irradiation, and the subsequent changes resulting from irradiation had to be allowed for by calculation. A limited amount of information on reactivity could be obtained on irradiated fuel by oscillator or substitution measurements, but more detailed measurements involving the distribution of foils within the fuel would be extremely difficult. An alternative method has been developed at Windscale to determine the distributions of power generation and burn-up applicable to the reactor conditions by measurements on the fuel after discharge from the reactor. The method which is non-destructive consists in movement of the irradiated fuel past a collimating slit through which the relative intensities of particular fission products are measured with a gamma spectrometer. The fission products chosen are Ba/La$^{140}$ and Ce/Pr$^{144}$. The former has a half-life of 12.8 d and hence represents the conditions that existed about a month before dis-
charge, whereas the latter with a half-life of 285 d represents average conditions for a considerable portion of the life of the fuel.

Measurements to date have shown the method to have a high resolving power (Fig. 4).
Data processing

The reactor physics experiments described in this section require a considerable amount of β- and λ-counting of monitors comprising discs, disc sections and wires. Large-window G-M counters are used for β-counting of the smaller foils, β-scintillation phosphors for wires and large foils; and scintillation techniques, for all λ-counting.

The large number of foils involved are handled by fully automatic counting units, and the output from the counting units is punched directly on to eight-hole tape, which is analysed by a small computer. Corrections for paralysis, background and decay from a common zero time are made automatically by the computer. In addition, when required, calibration factors and corrections for macroscopic flux can be fed into the computer to yield a fully corrected and processed set of counting results.

EXPERIMENTAL PROGRAMME

The overall experimental programme is shown in Table I.

APEX

Until A.G.R. fuel became available towards the end of 1961, experiments were made in APEX with UO₂ fuel clusters containing 1.8% U²³⁵ and with aluminium cans. These experiments were designed to develop methods for the measurement of shut-down reactivity for later use in HERO and A.G.R. and to test current theories that were being used for the planning of loading of A.G.R., in particular for prediction of the effect of perturbations in the lattice.

Two methods, the source-jerk and the pulsatron methods [7], were developed and later used successfully during the commissioning of A.G.R. [8] for measuring reactivity when subcritical and changes in reactivity arising from changes in loading pattern, such as the addition of absorbers and removal or insertion of additional channels of fuel. Although these methods had been used previously, there had not been any thorough investigation of the errors arising from variation of the position of the neutron source and detection units in different-sized cores. Fig. 5 indicates the agreement between this and other methods used for the calibration of control rods in a particular-sized core in A.G.R.

Perturbation measurements showed that the A.G.R. control rods would hold more reactivity than predicted. Experimental determination of the reactivity held by empty channels, steel bars and replicas of some irradiation experiments and of the interaction between two absorbers permitted useful checks of current theory. However, this type of experiment can be made more accurately in a critical facility, and analysis of results from such a facility would be simpler.

The final experiments in APEX were those with A.G.R. fuel in two unreflected builds with different heights to yield migration area asymmetry and material buckling. Unfortunately, at the time, the programme for utili-
zation of A.G.R. fuel did not permit similar experiments to be made with different lattice pitches to yield resonance integral and so permit deduction of resonance escape probability.

APEX as an approach-to-critical facility ceased operation in the first half of 1962. Since then it has been re-arranged to become a fine-structure stack and a thermal column, which are driven by neutrons produced from a small linear accelerator.

A.G.R. commissioning

The first stage of A.G.R. commissioning was a number of experiments made with different-sized cores of up to 43 channels in HERO and the analysis of these together with the results of the bare builds in APEX. The analysis consisted of use of current methods of calculation for empirical determination of a set of lattice constants that fitted all the results. These experiments showed that the methods used underestimated critical size and the reactivity held by channels of fuel near the edge of the core. The latter error, arising from inadequate theoretical treatment of the reflector, was compensated for by an artificial increase in the diffusion coefficient of the reflector.

After empirical values of extrapolation length for lattice constants were deduced, the next step was the obtaining of experimental values of extrapo-
ulation length for perturbers by analysis of experiments with the particular perturbers — control rods, loop tubes etc. — in small cores in HERO.

Hetrecontrol with the new constants (Tables 1 and 2 in [7]) was shown to be adequate by an experiment of 92 channels containing 13 absorbers in HERO, where there was agreement to 0.5 mile of reactivity and reasonable agreement on flux distribution.

During loading to full size, periodic checks of the above method were made, and they led to minor changes in the planned loading pattern as loading progressed.

A comprehensive series of measurements was made with various techniques for calibration of the control rods (Fig. 5). Calibration by critical and subcritical techniques not requiring the use of air as a poison was particularly successful and leads to the possibility of saving commissioning time on new reactors by suitable integration of construction and commissioning programmes. Measurements of subcritical reactivity were made with the source-jerk and pulsatron methods in addition to measuring changes in flux that occurred when rods were dropped into the reactor and when they were run out at a fixed speed. The reactivity held by the rods in different axial positions was measured by the normal method of balancing of the reactor with air pressure and in addition by deduction of the change in reactivity as a function of movement about the balance point from flux halving and doubling times. The latter measurements were made with the control rods at a number of axial positions, and the reactor was held near critical for each of these positions, both by change of air pressure and by alteration of the absorber loading pattern.

Other aspects of reactor commissioning and experimental techniques employed during commissioning are not particularly relevant to this paper and have been described elsewhere [8].

The main points arising from the reactor physics aspects of commissioning are that current theory did not adequately allow for effects at the core-reflector boundary, control rods were more effective than predicted, and, as expected, flux distribution in the core was particularly sensitive to changes in absorber pattern.

HERO

After completion of the A.G.R. commissioning experiments made in HERO early in 1962 the reactor was closed down for installation of the heating circuits and of additional instrumentation and other equipment required for operation of the reactor as a hot facility capable of containing plutonium-bearing fuels. This work was completed in March 1963, and the experimental physics programme commenced towards the end of March.

Initial experiments in the reactor were designed for investigation of the effect of a reflector around small cores and the heterogeneous character of small cores, in particular the effect of addition of channels of fuel at the periphery of the core. These experiments were made with stainless-steel clad fuel containing 1.8% U\(^{235}\) which gave a critical size of 56 channels. Before these experiments the experiments with APEX unreflected had been
satisfactorily accounted for with the value of resonance escape probability deduced from exponential experiments. However, the small core experiments made with A.G.R. fuel in 1962 could not be fully explained with this new value for resonance escape probability, and it was clear that the discrepancy resulted from either reflector effects or inadequate treatment of the effect of channels at the periphery of the small core. The new experiments with the 56-channel core demonstrated the need for use of heterogeneous theory for calculation of the reactivity of such cores.

The next phase of the HERO programme which is currently being carried out is the making of a series of small and intermediate-sized cores in HERO containing perturbations and the use of the results of measurements in them to test the programme MAGPIE. On completion of these experiments the heated circuits will be used to make temperature-coefficient measurements with the 1.8% U\textsuperscript{235} stainless-steel clad fuel.

Final experiments to be made with no plutonium in the core consist in the determination of the reactivity of 2.5% U\textsuperscript{235} A.G.R. fuel by substitution of this fuel in place of 1.8% U\textsuperscript{235} HERO fuel in a medium-sized core. These experiments are designed to yield experience of substitution experiments before they are made with plutonium-bearing fuel.

The above programme should be completed early in 1964 when fuel containing plutonium will become available for experiments in the reactor. The planned programme for the following two years consists in a series of experiments with critical cores containing up to 19 channels of PuO\textsubscript{2}-UO\textsubscript{2} fuels designed to provide data for the MAGPIE programme and to test the programme. To supplement the experiments with PuO\textsubscript{2}-UO\textsubscript{2} fuels, arrangements are being made for initial acceptance of one and if necessary more channels of fuel after irradiation in A.G.R. Measurements with this irradiated fuel will be designed to check the single-channel concept applied to a channel with varying burn-up and hence plutonium content along the length of the channel. The types of experiment in this part of the programme are

(i) heterogeneous cores for test of MAGPIE etc.,
(ii) substitution experiments to obtain basic lattice parameters, and
(iii) study of individual fuel stringers with perturbation theory.

The above experiments will include measurements at different temperatures and measurement of the temperature coefficient with plutonium fuels.

In addition to the main reactor physics programme for HERO, it is anticipated that the reactor will be required from time to time to make ad hoc measurements in aid of design of irradiation experiments and to yield data on new designs of fuel elements. Experiments may also be required for a better understanding of phenomena observed in the W.A.G.R.

Fine structure

Survey experiments carried out as part of the BICEP programme have provided considerable data on the effect of changing cluster geometry, which has been of value in the formulation of the correlation methods of calculation and in provision of experimental data to check calculation methods. The programme on the Windscale fine-structure stack has been primarily for determination of the detailed power distribution in both the axial and
the radial directions for fuels of immediate interest to the Windscale A.G.R. To date these fuels have been the 1.8% U\textsuperscript{235} HERO fuel, the 2.5% U\textsuperscript{235} A.G.R. fuel and a number of other fuels supplied for irradiation experiments in the W.A.G.R. The programme has included experiments with mixed lattices designed for determination of the minimum amount of fuel that can be used for sufficiently accurate flux distribution measurements for the assessment of power distribution. Early experiments showed that the size of stack chosen, 5X7 channels, gave a sufficiently large region of constant spectrum as measured by cadmium ratio in the region of the centre channel in the stack.

Radial measurements showed that the outer-to-inner ratio of 1.10 was considerably lower than had been assumed for design purposes. However, errors in estimated heat transfer performance more than compensated so that the actual measured temperature of outer ring elements was in fact about 25°C higher than that of inner ring elements in high-flux regions in A.G.R. Measurements of flux gradient across a fuel element and at the surface of the element have demonstrated that the radial flux distribution within a fuel pellet can be represented adequately by the super-position of an I\textsubscript{0} distribution on linear gradient components and the variation round an element by a symmetrical cosine distribution.

Measurements in the axial direction have shown appreciable flux peaking near axial discontinuities such as those caused by insulating pellets in the centre position in fuel elements and by the presence of support grids between the ends of fuel elements. In the current A.G.R. fuel this peaking would produce a maximum temperature rise of about 8°C resulting from a flux peak of 20-25%. Heterogeneous methods developed for the calculation of flux distribution, including axial distribution, are beginning to predict sufficiently accurately the flux and power distribution, which should reduce the amount of experimental work of this type required in the future. However, it is expected that experiments will be needed to check chosen designs before insertion of experiments or new fuel in the W.A.G.R. or in civil reactors.

Apart from ad hoc measurements on specific designs of fuel, it is anticipated that future requirements for experiments will be the establishment of theory developed for calculation of the effect of burn-up on power distribution within fuel assemblies. Fine-structure measurements will be made in the fine-structure stack and in HERO, with the PuO\textsubscript{2}-UO\textsubscript{2} fuel mixtures to be used in the HERO programme. In addition, considerable experimental information will be obtained by deduction of power distribution before discharge and averaged over the life of fuel assemblies by analysis of them after discharge with the lanthanum-cerium technique.

Reactor kinetics

A very thorough theoretical study of the kinetic performance of the uranium-magnox type reactors has been made, and the theory developed has been checked by a series of experiments at different irradiation levels in the Calder reactors. The mathematical techniques developed are directly applicable to the A.G.R. system, so development for A.G.R.s in the reactor kinetics field has been fairly straight-forward.
The main effects in comparison of the kinetics behaviour of A.G.R.s and uranium-magnox reactors are that increased rating and higher burn-up tend to cause shortened response times, but these effects are compensated for by the stabilizing effect of the larger difference between mean fuel temperature and coolant inlet temperature, which increases the $^{235}\text{U}$ negative temperature coefficient. A further important point is that fuel enrichment in A.G.R.s increases the $^{235}\text{U}$ temperature coefficient contribution, thus offsetting the increase in positive moderator coefficient arising from longer burn-up; for example, an average fuel irradiation of 10 000 MWd/t(e) in an A.G.R. causes a moderator temperature coefficient similar to that at 3000 MWd/t(e) in a uranium-magnox reactor. The re-entrant core design adopted in the Windscale A.G.R. holds the bulk of the moderator at the inlet coolant temperature, which has a significant stabilizing influence by reducing the change in moderator temperature following power changes.

The programme of reactor kinetics investigations to be made on the Windscale A.G.R. has been designed to provide data for checking methods of calculation applicable to civil A.G.R.s as well as to the W.A.G.R. Experimental data is required on reactor stability and control to assist in the choice of numbers of control zones in a reactor and the response required from automatic control systems. Operational transients resulting from start-up and load changing and also the consequences of fault conditions, such as failure of the main blowers and loss of coolant, need to be studied. In a civil A.G.R. design it is likely that, even with stainless-steel canned fuel and a re-entrant core design, some form of zonal control would be required to deal with azimuthal oscillations; but it is unlikely that as many zones of control will be required as in the civil uranium-magnox stations. The need for automatic control is not obvious and will depend to some extent on whether a re-entrant core design is chosen.

Information on temperature coefficients in the unirradiated reactor was obtained during commissioning and early operation of the W.A.G.R. Fuel and moderator temperature coefficients were deduced from measurements made before and after step changes were made in coolant temperature and in reactor power. We made an independent measurement of fuel temperature coefficient by allowing fuel temperature to rise fairly rapidly with little change in the bulk moderator temperature by operation at low power with no coolant flow. The effect of burn-up on temperature coefficient will be determined at intervals by means of step and ramp changes in reactivity and coolant conditions. In addition, data relevant to fault conditions will be obtained from suitably designed transient experiments.

STAFF AND COSTS

Figure 6 shows the professional staff engaged in the work described, excluding that on BICEP and on the development of basic methods of calculation at A.E.E. Winfrith.

Experimental work for the A.G.R. project is done by three experimental groups which are concerned with A.G.R., HERO and the subcritical facilities plus instrumentation and technique development. Experimental results are analysed by the experimental groups concerned usually to the
stage at which data is fully corrected and compared with existing theory. Effort shown in Fig. 6, except for A.G.R., is that associated with planning, operation and analysis and includes operational and maintenance effort. The group concerned with the subcritical facilities is also responsible for development of special instrumentation requirements in the Reactor Development Laboratories as a whole, which is not included. On A.G.R. there are two distinct groups, an Operations Group responsible for operation of the reactor and a Research Group responsible for planning utilization of the reactor and analysing results of experiments and information arising from the reactor. Until recently the reactor physics content of work on A.G.R. had been primarily concerned with physics measurements during commissioning, but emphasis is now changing to analysis of reactor behaviour, determination of long-term reactivity changes and experiments to investigate the kinetic response of this type of system.

Development of theory for design and operation is carried out jointly by two groups, one located with the Experimental Groups at Windscale and the other in Central Technical Services, located at Risley, where it is closely associated with the Reactor Design Office and liaison groups responsible for advising and negotiating with the electricity generating boards and industry. These theoretical groups have been responsible for developing the correlation and heterocontrol methods of calculation and for reactor kinetics calculations. In addition, they are developing new theories for design and operation, in particular for fuel-cycle and power-distribution calculations for operating reactors. The theoretical groups carry out any second-stage processing of experimental data that may be required and tend to formulate the problems that are the basis of the future experimental programme.

The capital costs of the main facilities located at Windscale are HERO, £1.2 million; APEX, £75 000; and Fine structure, £20 000.
ADVANCED GAS-COOLED REACTORS

REFERENCES


DISCUSSION

A. P. MARKS: How close to critical did you go with your APEX facility, and what do you think was gained by only going so far?

J. MOORE: We operated up to about 0.98 of critical, and carried out some experiments to 0.99. The reason why we decided on an approach-to-critical instead of a critical facility was financial: in an approach stack the instrumentation is considerably simpler than in a critical system, and I think one can make experiments with the former somewhat more quickly than with the latter. On balance, however, I should have preferred a critical system to an approach.

F. EBERSOLDT: I understand you have two large tubes for heating the reactor up to 500°C. What actual method do you use for heating?

J. MOORE: The reactor gas, carbon dioxide, is passed through an electrical heater and circulated through the reactor core. A countercurrent flow heat exchanger is used to reduce the operating temperature of the gas circulators.

A.V. CAMPISE: You mention that you plan to make temperature coefficient measurements, and I understood from your answer to the last question that you are blowing hot gas through the core. To what accuracy do you believe you will be able to determine temperature coefficients using this technique?

J. MOORE: There are many measurements of temperature throughout the reactor. Each measurement will be accurate to 1°C or better. The accuracy of temperature coefficient measurement for the moderator and for the fuel is ± 0.05 mN/°C over 50°C.
CONCEPT OF A FLEXIBLE EXPONENTIAL EXPERIMENT FOR HIGH-TEMPERATURE, GAS-COOLED REACTOR SYSTEMS

H. GRÜMM
ÖSTERREICHISCHE STUDIENGESELLSCHAFT FÜR ATOMENERGIE G. m. b. H.
SEIBERSDORF, AUSTRIA

Abstract — Résumé — Аннотация — Resumen

CONCEPT OF A FLEXIBLE EXPONENTIAL EXPERIMENT FOR HIGH TEMPERATURE GAS-COOLED REACTOR SYSTEMS. In HTGR-systems various atom-ratios of fissionable material, fertile material and moderator are used. The principle of a multipurpose exponential experiment is discussed, which covers this wide range and allows the performance of a variety of exponential experiments of different material compositions using only one type of basic element. The proposed system is especially designed for pebble-bed systems but can easily be modified to simulate systems with cylindrical fuel elements.

The fundamental idea is to use coated particles with different diameters (no high quality demands) for fissionable or fertile materials. Therefore, separation of particle mixtures is possible simply by screening.

The moderator elements are graphite sleeves with movable covers, which can be put one into the other forming a set of coaxial cylinders. The central cylinder of each set is filled with the desired mixture of spherical fissionable and fertile particles and graphite powder. Various lattices can be simulated by changing the size of the graphite cylinders. The critical arrangement will be set up in a stagnant helium atmosphere and heated up to high temperatures.

PROJET D'UN ENSEMBLE EXPONENTIEL D'UN GRANDE SOUPLESSE D'EXPLOITATION, POUR L'ÉTUDE DES RÉACTEURS À HAUTE TEMPÉRATURE REFROIDIS PAR UN GAZ. Dans les réacteurs à haute température refroidis par un gaz, les rapports des nombres d'atomes de produits fissiles, de matières fertiles et de ralentisseur varient. L'auteur étudie le principe d'un ensemble exponentiel à fins multiples qui permettrait de couvrir cette gamme étendue de rapports et de procéder à une série d'expériences exponentielles avec différentes compositions de matières, en n'utilisant qu'un seul type d'éléments de base. Le projet d'ensemble est spécialement conçu pour des réacteurs à éléments sphériques, mais peut être facilement modifié pour simuler des réacteurs à éléments cylindriques.

Le principe consiste à utiliser des particules enrobées, de diamètres différents (une haute précision n'est pas nécessaire) pour les produits fissiles et les matières fertiles. La séparation des particules mélangées peut donc se faire par simple tamisage.

Les éléments ralentisseurs sont des manchons de graphite à couvercles amovibles; ils peuvent s'emboîter les uns dans les autres et former ainsi un jeu de cylindres coaxiaux. Le cylindre central de chaque jeu est rempli du mélange choisi de particules sphériques fissiles et fertiles et de poudre de graphite. On peut simuler divers réseaux en changeant la dimension des cylindres de graphite. La configuration critique sera placée dans une atmosphère d'hélium et chauffée à haute température.

КОНЦЕПЦИЯ ЛЕГКО ПРИСПОСАБЛИВАЕМОГО ЭКСПОНЕНЦИАЛЬНОГО ОПЫТА ДЛЯ СИСТЕМ HTGR. В системах HTGR применяются различные атомные отношения расщепляющегося и воспроизводящего материалов и замедлителя. Обсуждается принцип многоцелевого экспоненциального опыта охватывающего большую зону и дающего возможность проведения разнообразных экспоненциальных опытов с различной структурой материала при использовании только одного типа основных элементов. Предлагаемая система специально предназначается для систем с агломерированным слоем, однако она может легко быть приспособлена для имитации систем с цилиндрическими топливными элементами.

Основная идея состоит в использовании покрытых частиц с различными диаметрами (не требуются высокого качества) в качестве расщепляющегося и соответственно воспроизводящего материала. Поэтому разделение смесей частиц возможно с помощью простого метода экранирования.

Элементы замедлителя представляют собой графитовые муфты с перемещающимися по-
The use of coated particles facilitates the setup of an extremely versatile, though not too expensive, exponential experiment. The same fertile and fissile materials can be used for measurements on HTGR (high-temperature, gas-cooled reactor) cores with very different C-U\textsuperscript{235}-Th ratios. The Austrian Institut für Reaktorentwicklung has worked out a proposal using this possibility, and the implementation of the experiment is planned for the coming years.

2. FUEL

The basic idea is the use of coated particles of fertile and fissile material, the diameters of these two sorts of particle being considerably different. Particle diameters of below 100 μm for the fissile material (20% or 93% U\textsuperscript{235}) and of 300 μm or more for the fertile material (Th\textsuperscript{232}), for example, could meet this requirement. By the addition of graphite powder, particles of both kinds may be mixed to form fuel inserts with a certain U-Th ratio. After completion of the measurements on the core with this particular atomic ratio, fertile and fissile materials may be separated simply by sieving. They may be used for fuel inserts of a different composition.

At the end of the series of measurements, fertile and fissile materials can be recovered separately, with almost no loss. The only requirements the coated particles will have to meet are low contamination of the surface.

La idea fundamental consiste en utilizar partículas revestidas, con diferentes diámetros (sin tolerancias muy estrictas) para el material fisionable y el material fértil, respectivamente. De esa manera, basta un simple tamizado para separar las partículas mezcladas.

Los elementos moderadores están formados por manguitos de grafito con tapas desmontables que pueden introducirse uno en otro formando un juego de cilindros coaxiales. El cilindro central de cada juego se rellena con la mezcla deseada de partículas esféricas fisionables y fértil y con grafito en polvo. Modificando las dimensiones de los cilindros de grafito, es posible simular distintos retículos. La disposición crítica se rodeará de una atmósfera de helio en reposo y se calentará a temperaturas elevadas.

1. GENERAL REMARKS

The use of coated particles facilitates the setup of an extremely versatile, though not too expensive, exponential experiment. The same fertile and fissile materials can be used for measurements on HTGR (high-temperature, gas-cooled reactor) cores with very different C-U\textsuperscript{235}-Th ratios. The Austrian Institut für Reaktorentwicklung has worked out a proposal using this possibility, and the implementation of the experiment is planned for the coming years.

2. FUEL

The basic idea is the use of coated particles of fertile and fissile material, the diameters of these two sorts of particle being considerably different. Particle diameters of below 100 μm for the fissile material (20% or 93% U\textsuperscript{235}) and of 300 μm or more for the fertile material (Th\textsuperscript{232}), for example, could meet this requirement. By the addition of graphite powder, particles of both kinds may be mixed to form fuel inserts with a certain U-Th ratio. After completion of the measurements on the core with this particular atomic ratio, fertile and fissile materials may be separated simply by sieving. They may be used for fuel inserts of a different composition.

At the end of the series of measurements, fertile and fissile materials can be recovered separately, with almost no loss. The only requirements the coated particles will have to meet are low contamination of the surface.
and markedly different diameters. Thus the fabrication costs can be kept down.

3. MODERATOR

In the core of the experiment very different fuel-element shapes can be simulated. In the case of fuel balls, for example, it is suitable to take graphite cylinders, as shown in Fig. 1. Such hollow cylinders of appropriate

dimensions may be fitted into one another to yield elements of equal outside dimensions containing fuel inserts of different size (Fig. 1, left), or they may be used in the assembly of elements of different size with equal inserts (Fig. 1, right) - all in all, six combinations.

These elements may be arranged in a subcritical core. A wide variety of average graphite densities, or void ratios, that is, can be covered by appropriate arrangement of them.

A core consisting of cylindrical fuel elements can, of course, be built in a similar way. Furthermore, it is feasible to simulate multi-region cores, extremely inhomogeneous cores etc.

4. SIZE AND MATERIAL REQUIREMENTS

Preliminary calculations of $B_2$, $L_2$ and $k_e$ have been carried out to yield an idea as to the versatility of the experiment and the material requirements. The results are shown in Figs. 2-4. As one sees, the design of the experi-
Fig. 2
Results of preliminary calculations

Fig. 3
Results of preliminary calculations
5. MECHANICAL DESIGN

The exponential experiment should yield results over a wide temperature range. It is, however, impossible to simulate the temperature differences existing between the fuel and moderator of a real reactor. The heating of the core requires special design as indicated in Fig. 5.

The core (1) is built on a graphite mounting plate so it can be removed as a whole from the reflector (2). The top reflector is a self-supporting plate and may be removed as well. This is to facilitate a rearrangement of the core. The graphite reflector contains silicium carbide rods for the heating, and it is covered by a thick thermal-insulation layer. Cooling coils (8) are provided for faster cooling after completion of the measurements. This results in better accessibility of the core. The thermal design is such that core and reflector can be kept at constant temperatures up to 500°C. A problem arises from the necessary use of a protective gas. For avoidance of high cost the use of nitrogen is being considered so that the requirements as to the leak-tightness of the outer containment are not too severe. The nitrogen would be blown into the bottom part of the containment and exhausted at the top, together with the air passing through the concrete.
shield and around the outside of the containment. The question whether this solution will meet safety requirements is yet to be answered.

Another problem arises from the fact that the core is not accessible for measurements when at high temperatures. One has to allow for several channels passing through thermal insulation and reflector. Investigations are being carried out to establish the experiment's range of potentiality as far as the measuring technique is concerned. Special techniques, corresponding to this particular design, are to be developed.

ACKNOWLEDGEMENTS

The author wishes to thank K. Donat and O. Eder for the calculations and the design work.

DISCUSSION

W. C. REDMAN: I should like some additional information on the nature of the coated pellets to which you refer. Is the core of the pellet pure fuel
material or is it an oxide or carbide? What type of coating surrounds it and do you have any information on its resistance to wear or abrasion?

H. GRÜMM: We shall probably use carbides of uranium or thorium coated with about 50μm of pyrolitic carbon. No specially high demands are made because there is no load on the system. We think the abrasive resistance of our normal coatings is good enough for our purposes.
HIGH-TEMPERATURE GAS-COOLED REACTOR CRITICAL EXPERIMENT AND ITS APPLICATION* 

GENERAL DYNAMICS CORPORATION, SAN DIEGO, CALIF., UNITED STATES OF AMERICA

Abstract — Résumé — Аннотация — Resumen

HIGH-TEMPERATURE GAS-COOLED REACTOR CRITICAL EXPERIMENT AND ITS APPLICATION TO THORIUM ABSORPTION RATES. In developing the concept of the HTGR and its first prototype at Peach Bottom, General Atomic made the decision that a critical experiment was required to provide adequately certain necessary input data for the nuclear analysis. The specific needs of the nuclear design theory for input data relating to thorium absorptions led to an experimental design consisting of a central lattice-type critical assembly with surrounding buffer and driver regions. This type of assembly, in which the spectrum of interest can be established in the relatively small central lattice having a desired geometry, provides a useful tool for obtaining a variety of input data for nuclear analysis surveys of new concepts. The particular advantages of this approach over that of constructing a mock-up assembly will be discussed, as well as the role of the theory in determining what experiments are most useful and how these experiments are then used in verifying design techniques.

Two relatively new techniques were developed for use in the lattice assembly. These were a reactivity oscillation technique for determining the thorium Doppler coefficient, and an activation technique for determining both the resonance integral of thorium dispersed in graphite and its temperature dependence (activation Doppler coefficient).

The Doppler coefficient measurement by reactivity oscillation utilized the entire central fuel element in a technique which permitted heating this fuel element to 800°F and accurately subtracting experimentally the thermal-base effects, that is, those effects not contributing to the thorium resonance capture. Comparison of results with theory for a range of conditions shows excellent agreement.

The measurement of the thorium resonance integral and its temperature dependence will be described. The technique developed for measuring resonance capture makes use of gold as the standard and vanadium as the material giving the 1/v absorption rate. This technique is dictated by the fact that the thorium is dispersed in graphite and the usual cadmium-ratio technique is difficult to apply. Comparison of experimental and theoretical results shows excellent agreement over a range of variables. In addition, the results of both activation and reactivity measurements of Doppler coefficient are in agreement, a fact which is felt to be significant in view of the disparity between results from these two techniques in the literature.

* This work was supported by the United States Atomic Energy Commission under Contract AT(04-3)-314. 
Les auteurs ont mis au point deux méthodes relativement nouvelles qui peuvent être utilisées avec l'assemblage décrit ci-dessus: une méthode d'oscillation de la réactivité pour déterminer le coefficient Doppler pour le thorium; une méthode d'activation pour déterminer à la fois l'intégrale de résonance pour le thorium dispersé dans le graphite et ses variations en fonction de la température (coefficient Doppler d'activation).

Pour mesurer le coefficient Doppler par oscillation de la réactivité, on se sert de la totalité de la cartouche centrale au cours d'une opération qui permet de la porter à une température pouvant atteindre 425°C et d'éliminer expérimentalement les effets qui ne contribuent pas à la capture de neutrons de résonance par le thorium. Pour toute une gamme d'expériences, les résultats obtenus concordent bien avec les résultats théoriques.

Le mémoire décrit la mesure de l'intégrale de résonance pour le thorium et ses variations en fonction de la température. Dans le procédé que l'on a mis au point pour mesurer la capture de résonance, on utilise l'or comme étalon et le vanadium comme matière donnant le taux d'absorption en 1/v. Ce procédé a été choisi parce que le thorium est dispersé dans le graphite et qu'il est difficile d'appliquer le procédé habituel du rapport cadmium. Les résultats empiriques concordent bien avec les résultats théoriques dans une large gamme de variables. En outre, les résultats des mesures du coefficient Doppler par les deux méthodes (oscillation de la réactivité et activation) concordent. Les auteurs estiment que ce fait mérite d'être relevé, car dans les ouvrages publiés jusqu'ici ces deux procédés donnaient des résultats différents.

КРИТИЧЕСКИЙ ОПЫТ, ПОСТАВЛЕННЫЙ НА ВЫСОКОТЕМПЕРАТУРНОМ РЕАКТОРЕ С ГАЗОВЫМ ОХЛАЖДЕНИЕМ, И ПРИМЕНЕНИЕ ЕГО ДЛЯ ОПРЕДЕЛЕНИЯ СТЕПЕНИ ПОГЛОЩЕНИЯ ТОРИЯ. При разработке идеи реактора HTGR и его первого прототипа в Пич-Боттоме пришли к решению о необходимости обеспечить соответствующие исходные данные для проведения ядерного анализа. Конкретные потребности теории ядерного проектирования на исходных данных относительно поглощения тория привели к созданию экспериментального проекта, состоящего из критической сборки типа сборки с центральной решеткой с окружающим аммортизатором и передвижными активными зонами. Сборка такого типа, в которой в сравнительно небольшой центральной решетке с жёсткой геометрией может быть установлен представляемый интерес спектр, является полезной установкой для получения разнообразных исходных данных в целях проведения ядерного анализа новых идей. Обсуждаются конкретные преимущества этого метода по сравнению со строительством сборки-модели, а также роль теории в определении, какие опыты являются наиболее полезными и как эти опыты затем используются при проверке методов проектирования.

Были разработаны два сравнительно новых метода для использования в сборке решеток — метод измерения колебаний реактивности для определения коэффициента Доплера для тория и метод для определения реактивности закону 1/v, —ванадий. Этот метод обусловливается тем фактом, что торий диспергируется в графите, и точно определить опытным путем тепловые эффекты, т.е. те эффекты, которые не оказывают влияния на величину резонансного захвата тория. Сравнение результатов с теорией для ряда условий свидетельствует о прекрасном согласовании и применимости этого метода по сравнению с другими методами проектирования.

Дается описание измерения резонансного интеграла тория и его зависимости от температуры. При этом методе для измерения резонансного захвата в качестве стандарта используется золото, а в качестве материка, дающего величину поглощения, подчиняющуюся закону 1/v, —ванадий. Этот метод обусловливается тем фактом, что торий диспергируется в графите и трудно применять обычный метод кадмевого отношения. Сравнение экспериментальных и теоретических результатов свидетельствует о прекрасном согласии в определении реактивности во всем диапазоне переменных. Кроме того, согласуются результаты измерения коэффициента Доплера как методом активации, так и методом определения реактивности, —факт, который, как показывает, является важным ввиду наличия разнообразия между результатами применения этих двух методов, имеющимися в литературе.

EXPERIMENTO CRÍTICO EFECTUADO EN UN REACTOR DE ELEVADA TEMPERATURA REFRIGERADO POR GAS Y SU APLICACIÓN PARA CALCULAR LOS ÍNDICES DE ABSORCIÓN DEL TORIO. Al definir los principios teóricos del reactor de elevada temperatura refrigerado por gas, y de su primer prototipo en Peach Bottom, la General Atomic decidió efectuar un experimento crítico con miras a reunir ciertos datos de entrada requeridos para el análisis nuclear. Debido a las necesidades específicas de la teoría de las construcciones nucleares, en lo que atañe a los datos de entrada acerca de la absorción del torio, los autores elaboraron un sistema experimental formado por un conjunto crítico con reticulado central rodeado por una región amortiguadora y otra
HIGH-TEMPERATURE GAS-COOLED REACTOR 37

activadora. Este tipo de conjunto, en el que el espectro que se ha de analizar puede limitarse al reticulado central relativamente pequeño, y cuya geometría puede determinarse a voluntad, permite obtener diversos datos de entrada para los estudios relativos a análisis nucleares de instalaciones nuevas. Los autores describen las ventajas de este método en comparación con el de una maqueta y la función de la teoría, consistente en determinar cuáles son los experimentos más útiles y la manera en que deben utilizarse para comprobar los proyectos.

Los autores elaboraron dos técnicas relativamente nuevas para su utilización en un conjunto dotado de reticulado. Una se basa en la técnica de oscilación de la reactividad para determinar el coeficiente Doppler correspondiente al torio, y la otra en la activación para determinar la integral de resonancia del torio dispersado en el grafito y su variación en función de la temperatura (coeficiente Doppler de activación).

Para medir el coeficiente Doppler por oscilación de la reactividad, se utilizó todo el elemento combustible situado en el centro, calentando este elemento hasta 800°F y procediendo a una sustracción experimental exacta de los efectos que no contribuyen a la captura por resonancia en el torio. Los resultados obtenidos concuerdan satisfactoriamente con los valores teóricos en diversas condiciones.

Los autores describen la medición de la integral de resonancia en el torio y su variación en función de la temperatura. En la técnica que han elaborado para medir la captura por resonancia, se utiliza oro como patrón y vanadio como el material que cumple la ley 1/v. La elección de esta técnica obedece al hecho de que el torio se dispersa en el grafito y a la dificultad de aplicar el método basado en la razón cádmica. Los resultados experimentales concuerdan con los teóricos en diversas condiciones. Asimismo, concuerdan entre sí las mediciones del efecto Doppler efectuadas por activación y la realizadas por determinación de la reactividad, lo que se considera significativo debido a que en las obras publicadas subsiste cierta divergencia en los resultados obtenidos con ambas técnicas.

I. INTRODUCTION

An important part of the research and development leading to the design of the 40-MW(e) Peach Bottom High-temperature, Gas-cooled Reactor (HTGR) involved the nuclear design and physics evaluation of the reactor. Although much of the information had to be developed from analytical work, some supporting work was required from a critical-experiment programme. Because of the kind of information which was needed for the nuclear-design effort, both the type of critical experiment and the experimental programme were, perhaps, somewhat unusual. In order to supply basic information on the resonance-absorption characteristics of thorium, the Doppler coefficient of thorium, control-rod effectiveness and certain lattice parameters, we utilized a lattice-cell experiment. In addition, as a later part of the programme, a small-scale, clean-geometry critical experiment was assembled for observation of the accuracy of the nuclear-design calculational techniques and cross-section data used in the calculation of physics parameters for semi-homogeneous, graphite-moderated reactors with U\textsuperscript{235} as the fuel and thorium as the fertile material.

Two relatively new experimental techniques were developed for measuring resonance integrals and Doppler coefficients in the lattice assembly. These were (1) a reactivity oscillation technique for evaluating thorium Doppler coefficients and (2) an activation technique for determining both the resonance integral of thorium dispersed in graphite and its temperature dependence. Doppler coefficients for fuel elements containing several different thorium concentrations were measured by oscillation of cold and hot fuel elements having otherwise identical characteristics in the central region of the test-lattice cell. Comparison of results with theory for the range of conditions examined showed excellent agreement.
The activation measurement of the thorium resonance integral and its temperature dependence made use of the fact that vanadium is approximately a $1/\nu$ absorber and could be used in evaluation of the $1/\nu$ part of the thorium activation, which was then subtracted from the total thorium activation to yield the activation resonance integral. This technique was necessary because of the fact that the thorium in the HTGR fuel element is dispersed in graphite, making the usual cadmium-ratio technique difficult to apply. Comparison of experimental and theoretical results showed excellent agreement in the range of variables tested. In addition, the results of both activation and reactivity measurements of Doppler coefficient were in agreement, a fact which is felt to be significant in view of disagreements in results reported by previous investigators [1].

Experiments on the small, clean-geometry assembly were used for a test of calculational procedures for evaluation of overall temperature coefficients. Reactivity-coefficient measurements afforded a method of checking cross-section data.

II. PHILOSOPHY OF THE HTGR CRITICAL-EXPERIMENT PROGRAMME

The 40-MW(e) Peach Bottom HTGR employs a graphite moderator and semihomogeneous fuel elements to achieve the following characteristics: high temperatures, high thermal efficiencies, long fuel-burnup times and good neutron efficiency. The reactor core, which is 7 1/2 ft long and approximately 9 ft in diameter, is made up of 804 fuel elements. The 3 1/2-in OD fuel elements are composed of a central graphite spine surrounded by an annular fuel body, which in turn is enclosed in a graphite tube. The fuel bodies, which contain approximately one-third of all the graphite in the core, consist of a dispersion of enriched $^{235}$U and thorium in graphite, the average carbon:thorium:$^{235}$U atom ratio being 1824:6.7:1.0.

Because of the objectives of the reactor and its design details, the evaluation of some of the reactor physics characteristics involves some very sophisticated analytical procedures. For example, the average graphite temperature in the core is approximately 1200°K, while the reflector graphite temperature is approximately 600°K. Because of the heavy fuel loading in the core and the higher moderator temperature, the cold neutrons which are returned by the reflector have a relatively short diffusion length in the core graphite, thereby creating a strong power peak at the core-reflector interface. This effect is difficult to simulate correctly in a critical experiment and is also difficult to evaluate from few-group diffusion-theory calculations.

In addition, the HTGR has as a design objective a long fuel residence time and, therefore, a large fuel burnup fraction. In the Peach Bottom reactor it is expected that the average fuel burnup will be approximately 0.6 fissions/initial fissile atom. Because of the large fuel burnup and the production of $^{233}$U fuel, the characteristics of the core will change appreciably during its burnup history.

Finally, because of the fact that approximately one-third of the moderator is intimately mixed with the fuel, a large part of the moderator temperature coefficient will be prompt, and its contribution to the overall power
coefficient will be very important. The moderator coefficient and the Doppler coefficient of thorium are the only two important contributors to the temperature-coefficient characteristics of the reactor. Furthermore, the temperature-coefficient characteristics change quite radically with fuel burnup owing to the fact that the variation of the U\(^{233}\) cross-section with energy in the neighbourhood of 1 eV is quite different from that of the U\(^{235}\) cross-section. Since a small fraction of the thermal neutrons falls in this energy range at operating temperatures, the temperature coefficient and therefore the kinetic characteristics of the reactor change throughout the reactor lifetime.

From the previous discussion it can be seen that some of the important physics characteristics are related to high-temperature operation and neutronic changes that occur throughout the operating history of the reactor. For this reason it appeared that the experimental data available from a full-scale critical experiment would not be adequate to specify the nuclear design of the HTGR. Therefore, a detailed analytical programme was undertaken to develop methods for calculation in great detail of the neutron reaction rates in various parts of the reactor core, taking into account the change in neutron spectrum from point to point throughout the reactor core and reflector. In addition, methods were developed for calculation of the changes in core composition and the changes in reaction rates throughout the reactor operating history. Thus, the important problems which were presented to the experimental physicists were related to the adequacy of theoretical calculational techniques and data to be used in the calculations.

As one result of the above considerations, it was concluded that a critical-experiment programme should be undertaken for measurement of reaction rates and the temperature derivatives of these reaction rates in a HTGR lattice. Although the detailed temperature characteristics of the HTGR could not be duplicated in the critical experiments, the calculational methods and data could nevertheless be checked by experiments covering a fairly broad range of temperatures and compositions. On the basis of the agreement or lack of agreement between experiment and calculations, the areas where differential cross-section data and calculational techniques might be inadequate could then be identified. In some cases where conflicting results occurred, the experimental data might even assist in the correct choice of the best differential cross-sections. As a result of the above reasoning, it was also concluded that some experiments on an HTGR-type, small-scale critical assembly should be performed to test the overall calculational techniques and cross-section data.

Two types of critical experiment were therefore conducted in support of the HTGR nuclear-design programme. The first was the test-lattice experiment [2], where detailed measurements of reaction rates were examined in a lattice having a cold-neutron spectrum characteristic of the HTGR. This programme provided a method for checking the resonance integral of thorium, the Doppler coefficient of thorium, the detailed flux distribution in the lattice and control-rod effectiveness within a cell. The second experiment [3] was designed as a gross test of the calculational procedures and data. A small critical experiment having a clean geometry and a composition similar to that of the HTGR was constructed. This assembly had approximately one-sixth the volume of the HTGR core and was surrounded
on all sides by a 2-ft graphite reflector. Owing to the small core size and the large reflector area, this experiment provided a severe test of the calculational methods. Experiments with this facility encompassed reactivity-coefficient measurements, neutron-flux distributions, effectiveness of groups of control rods and a measurement of the overall temperature coefficient.

III. HTGR CRITICAL-ASSEMBLY FACILITY

A general view of the HTGR critical assembly is shown in Fig. 1. The assembly was built in two halves so that it could be separated at the mid-plane. Gross control of the reactor was achieved by the separation of the two halves, and fine control was accomplished by means of control rods that could be driven into the active lattice region. Figure 2 shows typical fuel elements used in the critical assembly. Most of the fuel elements consisted of rectangular graphite blocks containing fuel compacts inserted in two parallel holes. Compacts of either U\(^{235}\) and graphite or thorium and graphite could be loaded into the blocks for achievement of the composition desired. At the right of the figure are the parts of an HTGR mock-up fuel element used in the central lattice-cell region of the test-lattice reactor. Included in the figure are the graphite sleeve, an annular fuel body and a section of the graphite spine.

Fig. 1
Overall view of Peach Bottom HTGR critical assembly

Figure 3 shows the face of one of the halves of the 6-ft long test-lattice reactor. The 16-in-diameter test-lattice cell, consisting of a hexagonal array of 19 HTGR fuel elements, is shown at the centre of the assembly. Reactivity measurements were obtained in the central fuel-element position.
Fig. 2
Fuel elements used in HTGR critical assembly

Fig. 3
Peach Bottom HTGR critical assembly: view at plane of separation
of the lattice cell. Figure 4 is a schematic drawing of the assembly. The 19 cylindrical fuel elements at the centre of the reactor had essentially the same material composition and dimensions as did the fuel elements in the Peach Bottom HTGR. This test-cell region was surrounded by a buffer-lattice region roughly 3-ft-square consisting of critical-assembly fuel blocks containing $^{235}U$ and thorium. The composition of the buffer region was chosen such that the reaction rates in the region were approximately the same as those in the test-lattice region. The loading was adjusted by means of boron/stainless-steel poison strips to obtain a convenient amount of excess reactivity and to achieve a flat neutron flux over the test-cell diameter and much of the buffer-lattice region. The buffer lattice was surrounded by a driver lattice ~10-in thick and consisting of fuel blocks containing $^{235}U$ and graphite. The $^{235}U$ composition was chosen primarily to assure that the entire assembly could be made critical for all test programmes contemplated. The driver lattice was surrounded by a graphite reflector ~1 ft thick (the critical assembly was unreflected on the ends).

![Fig. 4](image)

Core cross-section in the transverse plane perpendicular to the axis of symmetry
The reflector is not shown; the bridge bars are made of the same material as the adjacent blocks of the region.

Figure 5 is a view of the face of the one-sixth-scale mock-up of the HTGR. The mock-up assembly had a core about 5 ft in diameter and 4-ft long, which could be separated at the vertical mid-plane. The core was completely reflected by about 2 ft of graphite and contained 19 square holes having approximately the same fractional void volume as that required for control rods in the Peach Bottom HTGR.
The fuel elements were all rectangular fuel blocks containing both $^{235}\text{U}$ and thorium, the carbon: thorium: $^{235}\text{U}$ atom ratio being approximately the same as that specified for the Peach Bottom reactor. The assembly contained no rhodium poison or boron burnable poison but did have some boron/stainless-steel strips to shim the reactivity. In most of the experiments the inventory of the boron/stainless-steel strips was chosen so that the assembly was just critical at room temperature with all control rods removed.

IV. CRITICAL-EXPERIMENT MEASUREMENTS IN THE TEST-LATTICE ASSEMBLY

In this section the utility of the test-lattice assembly will be illustrated by a review of some of the important experiments which were performed during the first part of the HTGR critical-assembly programme. In particular, the activation measurements leading to the evaluation of the thorium resonance integral will be described. The measurement of the Doppler coefficient by pile oscillator and activation techniques will also be described.

Until 1962 the differential cross-section data that were available for thorium [4] led to an infinitely dilute resonance integral of 96 b. Resonance-integral measurements [5] indicated values ranging from 60 to 110 b. Clearly, more definitive experimental information was required to check the dif-
ferential data and resolve the discrepancies in the integral measurements. Experiments were therefore undertaken to measure the thorium resonance integral under the following conditions:

1. Infinitely dilute sample, 1/E spectrum;
2. Infinitely dilute sample, HTGR spectrum; and
3. HTGR composition and geometry, HTGR spectrum.

The usual activation technique, involving the irradiation of cadmium-covered samples, was difficult to apply to the HTGR measurements, since the HTGR fuel element contains moderator material intimately mixed with thorium. Therefore, an alternative technique, which involved activation measurements of both thorium and vanadium, was developed. Previous measurements [6] on vanadium had shown that it is very nearly a 1/v absorber and leads to a radioactive isotope having a convenient half-life of 3.77 min. Hence, by renormalizing of the vanadium absorption rate to that of an isotope having the same thermal absorption cross-section as thorium, the 1/v absorption rate of thorium arising from the epithermal flux could be evaluated and subtracted from the overall thorium absorption rate — leading to the resonance-absorption component for the thorium sample. This procedure was identified as the vanadium-subtraction technique [7].

The relative absorption rates for thorium and vanadium were determined by activation measurements in a well-thermalized Maxwellian flux obtained in the thermal column of the TRIGA Mark F reactor [8] at General Atomic. The relative absorption rates for the two samples could be calculated quite accurately for this condition, since the thermal-neutron absorption cross-sections are well known for the two nuclides.

A similar calibration was carried out for some standard gold foils so that the thorium resonance integral could be measured relative to that of gold, which is well known. The resonance activation of the gold and the thorium were then measured in a particular region of the General Atomic TRIGA Mark F reactor where the epithermal flux was calculated to be approximately 1/E.

The spectrum appropriate to the Peach Bottom HTGR was developed in the central lattice region of the HTGR critical assembly. Two types of fuel geometry, annular and cylindrical, were used in the measurements. The measurements made in the annular geometry were of primary interest, because that configuration corresponded to the design geometry of the Peach Bottom HTGR. Measurements were also made in thorium-graphite cylindrical pellets corresponding to the pellets used in the buffer region of the test-lattice reactor.

In most cases, thin metallic foils were used in the activation determination. However, in some cases the activation of the thorium within the thorium-graphite compact was directly determined. Major factors in quantitative measurements of resonance absorption are the type, size, geometry, thickness and preparation of the various foils. There were two important factors to be considered in the foil selection. First, the foils had to be sufficiently thin that there would be practically no self-shielding effect in the resonance-energy region but sufficiently thick that they could be handled. Secondly, there had to be a uniform distribution of material throughout the foils so that any small amount of self-shielding that was present would be constant. Table I gives typical characteristics of the thin foils employed.
TABLE 1

FOIL CHARACTERISTICS

<table>
<thead>
<tr>
<th>Material</th>
<th>$\sigma_{238}$ m/s (b)</th>
<th>Thickness (mg/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Au$^{197}$</td>
<td>98.86</td>
<td>0.17</td>
</tr>
<tr>
<td>V</td>
<td>4.5</td>
<td>7.6</td>
</tr>
<tr>
<td>Th$^{232}$</td>
<td>7.45</td>
<td>3.3</td>
</tr>
</tbody>
</table>

The foil position, in relation to the medium in which the activation was being made, was very important in the HTGR spectrum measurements. In most cases, the geometry of the thorium-containing region was a cylindrical annulus. Pie-shaped foils, as shown in Fig. 6, were used for correct averaging of the radial variation of the activation rate over a sector of the annulus. Each foil covered one-sixth of the cross-sectional area of the annulus.

![Diagram](image)
Two sets of thorium, gold and vanadium foils were used in reactor positions having the same flux environment.

Figure 7 shows the detailed arrangement of the foils in the compacts. It was important to eliminate gaps between the compacts to prevent any streaming down the foil-compact interface. The uranium was omitted from the compacts adjacent to the thorium foil for avoidance of contamination of the foils with fission-product activity. The perturbation caused by omitting the uranium in this compact was shown to be a 1% correction in the measured resonance reaction rates, since the flux dip caused by the uranium was less than a few percent and was mostly thermal. The thorium foils were positioned between the two thorium-graphite compacts. The vanadium and gold foils were positioned between one of the thorium-graphite compacts and the thorium-uranium graphite compact. These foils were enclosed in aluminium foil to prevent their becoming contaminated by either activated thorium or uranium fission products. This assembly of foils and compacts then became part of the loaded central fuel element of the test-lattice region.

The foils activated in the TRIGA thermal column were arranged so as to duplicate, as nearly as possible, the activation assembly in the test-lattice region. To ensure that each part of the activation assembly received
the same exposure, the assembly was positioned on a small turntable driven by a 30-rpm motor.

The 1/É spectrum measurements were made by activation of foil detectors inside a hollow dummy element in the TRIGA Mark F reactor. The foils were positioned along the centreline of a dummy element at the vertical midplane of the core adjacent to the reflector. Three sets of foils were used, each set being composed of one of each type of foil (gold, vanadium and thorium). The foils were again separated by aluminium spacers to prevent cross-contamination and were packaged in aluminium foil. Duplicate sets of foils were exposed in the thermal column. The foil packages for the thermal-column exposure were positioned on the rotating turntable for the irradiation.

Several factors had to be considered in the selection of the length of irradiation time and the power level at which the irradiations were to be made. These factors were primarily associated with the achievement of sufficient activation of the materials involved so that good counting statistics could be obtained. For elimination of any errors associated with counting-equipment drift or foil decay uncertainties, two activations were made simultaneously, i.e. the activation in the spectrum of interest and the activation in the Maxwellian spectrum.

In the case of the HTGR spectrum measurements, both the critical assembly and the TRIGA Mark F reactor (with thermal column) were involved. The simultaneous start-up and shutdown of these facilities was accomplished by means of telephonic communications. The uncertainty in absolute time between the shutdown time of the two reactors was of the order of 1 s or less. This uncertainty was insignificant compared with the shortest half-life of any of the detector foils (vanadium, with a half-life of 225 s). In both reactors the relative power level was maintained constant to better than 0.1% during the irradiations.

When the foils were removed from their respective irradiation positions, the most important factor was minimizing the delay in starting the count of the 225-s half-life activity of the vanadium foils. The foils of a particular material from both irradiations were counted concurrently; e.g. the vanadium foils from the critical assembly were counted at the same time as were the vanadium foils from the thermal column. This system of counting eliminated possible errors in decay corrections or drifts in the counting equipment.

Two separate counting systems were used to measure the induced activity of the foils. The beta activity of the 22-min Th²³³ was measured with a gas-flow proportional counter. The gamma activity of the vanadium and the gold (0.4-MeV gamma emission line) was measured with a 1 1/2-in NaI(Tl) well-type scintillation crystal.

A separate and independent method for measuring the activation rates of thorium was also developed. This method was based on the direct measurement of the activation rate of thorium dispersed in a thorium-graphite compact. Basically, the method involved the activation of a thorium-graphite compact, together with foils of vanadium and gold, in both the critical assembly and the TRIGA thermal column, as described previously. This method was used as a supplement to foil measurements, and in all cases where it was applied thorium foil detectors were also used. The gold and
vanadium activations were used in the interpretation of both the foil and the compact activations.

This method involved a milling device, shown in Fig. 8. The irradiated compact was clamped to the centre of a simple milling head that could be slowly rotated by a motor-driven, worm-gear drive. A small, high-speed milling tool was positioned to cut the flat end of the compact. As the motor slowly (2 rpm) rotated the compact, the high-speed milling tool cut a thin layer from the compact surface. The thorium-graphite powder removed in the cutting was collected on a filter paper, which was inserted into a holder in a short suction tube. From this point on, the collected sample was treated basically the same as the foil detectors.

The significant point about this method of measurement is that the activation detector was essentially the same material as that for which the resonance absorption rate was being studied.

The measured activities, as obtained from the two counting systems, were corrected for weights and decay times in the usual manner. From the appropriate activation ratios for vanadium, gold and thorium, it was possible to calculate the resonance integral for thorium in a particular spectrum. The basic principle behind the vanadium-subtraction method and detailed results obtained with it have been described by SAMPSON [9]. Table II summarizes the results of these measurements for several geometric configurations and thorium loadings.

It is noted that the experiments led to an infinitely dilute resonance integral of 79 b, instead of the 96 b previously obtained from the differential cross-section data. As a result of this discrepancy, the resonance parameters of thorium in the neighbourhood of 20 eV were adjusted to yield agreement. It can be seen from the comparison of the calculated and measured
results that this correction does not lead to good agreement for the other samples. Subsequent differential cross-section measurements [10] and interpretations [11] have been reported, but it appears that additional work on thorium is required.

In addition to the measurements of the thorium resonance integrals, a programme was undertaken for measurement of the temperature response of the resonance integral, or the Doppler coefficient, for various fuel-element compositions and geometries [12]. These Doppler-coefficient measurements were performed primarily by a reactivity oscillation technique, where the reactivity swing resulting from the alternate substitution of a cold and a hot fuel element in the test lattice was measured. In addition, activation measurements of the neutron absorption rates in hot and cold thorium samples were made for one particular fuel-element design.

The component of the temperature coefficient of a power reactor arising from the Doppler coefficient can be shown to be [1]

\[ \frac{1}{k} \frac{dk}{dT}_{\text{Doppler}} = \frac{1}{\rho} \frac{dp}{dT} = \alpha \ln \rho, \]

where \( \alpha = (1/I)(dI/dT) \) is defined as the Doppler coefficient. It was, then, the magnitude of the Doppler coefficient \( \alpha \) which was the objective of the oscillation experiment in the test-lattice assembly. It can be shown [13] from perturbation theory that the component of the reactivity swing resulting
from the change in thorium absorption rate, i.e. $\Delta \rho_{\text{res}}$, is related to the resonance absorption $I_{\text{eff}}$ approximately by

$$(1/\rho_{\text{res}})(\Delta \rho_{\text{res}}/\Delta T) = (1/I_{\text{eff}})(\Delta I_{\text{eff}}/\Delta T).$$

Detailed calculations for the test-lattice assembly showed that the error in this relation would be less than about 1% for cases of interest.

For evaluation of the Doppler coefficient, it was thus necessary to measure both the incremental change in reactivity $\Delta \rho_{\text{res}}$, resulting from the temperature change, and the total reactivity value $\rho_{\text{res}}$ of the sample. It was, of course, important to obtain only the components of reactivity $\rho_{\text{res}}$ and the reactivity swing $\Delta \rho_{\text{res}}$ which were contributed by the change in the resonance absorption of the thorium.

The reactivity $\rho_{\text{res}}$ was measured simply by observation of the difference in reactivity between the thorium-loaded, HTGR-type fuel element and a fuel element containing the same materials but having the thorium replaced by boron so that the $1/\nu$ absorption was the same in both cases. Under these conditions,

$$\rho_{\text{res}} = \rho - \rho_{1/\nu},$$

where $\rho$ is the total reactivity effect of the thorium and $\rho_{1/\nu}$ is the $1/\nu$ contribution as deduced from the fuel element containing boron.

The total reactivity swing between the hot and cold fuel elements includes components resulting from the change in resonance absorption and the $1/\nu$ absorption. Furthermore, the presence of the hot graphite moderator tends to re-thermalize the neutrons in the immediate neighbourhood of the sample and consequently changes the reaction rates of all the thermal absorbers locally. Hence,

$$\Delta \rho = \rho(T) - \rho(T_0)$$

$$= \rho_{\text{res}}(T) + \rho_{\text{th}}(T) - \rho_{\text{res}}(T_0) - \rho_{\text{th}}(T_0),$$

where $T$ and $T_0$ refer to the hot and cold samples and $\rho_{\text{res}}$ and $\rho_{\text{th}}$ refer to the reactivity changes arising from changes in resonance absorption and thermal spectrum absorptions, respectively. It is important to remember that the reactivity effect $\rho_{\text{th}}$ may include changes both in absorption rates in the sample and in absorption rates in surrounding fuel, thorium and other absorber materials. This is particularly true when the sample is at an elevated temperature relative to the rest of the lattice.

Rewriting the above equation as

$$\Delta \rho = [\rho_{\text{res}}(T) - \rho_{\text{res}}(T_0)] - [\rho_{\text{th}}(T) - \rho_{\text{th}}(T_0)]$$

$$= \Delta \rho_{\text{res}} - \Delta \rho_{\text{th}},$$

we see that the resonance term can be obtained from the total reactivity swing by subtraction of the thermal-base component $\Delta \rho_{\text{th}}$. In order to obtain
an accurate value of $\Delta \rho_{\text{th}}$, it is therefore necessary (1) to minimize the component $\Delta \rho_{\text{th}}$, and (2) to evaluate carefully the $\Delta \rho_{\text{th}}$, using samples where the thorium is again replaced by an equivalent $1/\nu$ absorber.

The physical nature of the thermal-base effect was analysed theoretically [12], and it was shown that the effect nearly vanishes in a spatially uniform neutron-flux distribution. This can be seen from rather simple considerations. If the fuel and all absorber cross-sections have a $1/\nu$ variation in the thermal-spectrum region, if the fuel and absorber atoms are all located together relative to neutron-flux variations, and, finally, if the $k_{\infty}$ of the lattice is just unity, then a local change in the thermal-neutron spectrum would affect all local reaction rates in the same way - and therefore the overall reactivity would be unchanged.

Thus, for minimizing of the thermal-base effect, the radial flux distribution in the assembly was flattened by the introduction of poison in the void space between fuel elements. No attempt was made to flatten the flux in the axial direction. In addition to flattening the radial flux, careful attention was given to the $1/\nu$ poison distribution in the sample itself to ensure that local changes in the neutron flux would not affect reaction rates in the fuel and other absorbers in different ways. By this means, the thermal-base effect was reduced to about 20% of the total reactivity swing.

Since the change in reactivity caused by replacing a cold fuel element with a hot fuel element was typically 0.01 dollar or less and the sensitivity to which the reactivity differences could be measured was 0.0001 dollar, it was desirable to make repetitive measurements of these reactivity changes to improve the statistical accuracy of the measurement. The method used in these measurements might be more appropriately described as a periodic substitution procedure rather than an oscillation procedure. The purpose was to interchange periodically the hot and cold central fuel tubes and to record the reactivity difference associated with a measured temperature difference.

The oscillator mechanism used for these measurements consisted of a double-acting pneumatic cylinder with a piston with an 8-ft stroke [12]. This device was equipped with hydraulic snubbers which contacted stops at both ends of its travel. By means of stainless-steel cables through pulleys attached to the walls of the room, this mechanism moved a 14-ft long oscillator element horizontally through the core. The oscillated fuel element consisted of two 6-ft long graphite fuel tubes separated by a 2-ft long section of Transite (thermal insulation) and coupled to the graphite by threaded aluminium sleeves. All sections of the oscillator sample had a common diameter of 3.57 in. Full travel of the pneumatic cylinder resulted in the exchange of one 6-ft long fuel tube in the core for the other, an action requiring about 1 s.

A 6-ft long cylindrical oven was positioned horizontally at one end of the core so that when one end of the oscillator element (the cold fuel tube) was in the core, the other end (the hot fuel tube) was in the oven. When the hot end was in the core, the cold end rested in a graphite trough situated horizontally off the other end of the core. Figure 9 shows the location of the oven.

A thin air gap and strips of Transite provided sufficient thermal insulation around the hot fuel element so that heat transfer to the surrounding
six fuel tubes resulted in a temperature rise of less than 10°F/min when the hot element was at 800°F. Over a period of several hours, the Transite insulation between the hot and cold fuel tubes was sufficient to limit the temperature rise to less than 10°F at the end of the cold fuel tube. Thermocouples monitored the temperature of the entire 14-ft oscillator element. The temperature distribution over the hot fuel tube of the oscillator element was fairly uniform. In the longitudinal direction, the maximum variation from end to centre was about 40°F at the highest temperature (800°F). The radial distribution was essentially uniform.

During the measurements of the Doppler coefficient, the oscillator mechanism was usually programmed to move every 30 s, each movement requiring about 1 s, so that each end of the oscillator element alternately had a dwell time of about 29 s in the core.

Rather than record the variations of the neutron flux as the oscillator element was moved, we automatically maintained the flux level constant by means of a servo-driven control rod located in the reflector. This servo-rod was a thin boral strip 3-ft long, having a total worth of about 6 cents. The servo-system was actuated by an error signal between the average current output of two ion chambers (gamma-compensated) located in the reflector and a fixed current supply. A servo-amplifier operating at high gain drove the servo-rod positioning motor. The speed of travel of the servo-rod was such that it changed reactivity at the rate of 3 cents/s over the most effective portion of its position.

Doppler-coefficient measurements were made for several thorium loadings and for three different sizes of annuli by the reactivity-swing method. Figures 10 and 11 show, respectively, the dependence of reactivity on temperature and the Doppler coefficient as a function of temperature for three fuel compacts measured. Table III gives the measured and calculated Doppler
Dependence of reactivity changes on temperature change, with a radially flat flux.

Doppler coefficient for small HTGR annular fuel elements of several different C/Th ratios

Measured \((1/p_{\text{res}})(\Delta p_{\text{res}}/\Delta T)\)

- C/Th = 31.5
- C/Th = 48.7
- C/Th = 55.7
- Calculated
coefficients averaged over the temperature range 300°K to 700°K for several fuel-element compositions.

An overall experimental uncertainty of ±10% has been estimated for the individual measurements of \(1/(l)(dl/dT)\). The uncertainty in the average Doppler coefficient, over the temperature range 300°K to 700°K, is probably smaller than ±10%. This estimate of probable accuracy is based partly on statistical uncertainties and partly on observations of the experimental reproducibility.

As a check on the reactivity method of determining the Doppler coefficient, an activation technique was used for one measurement [12]. The result of a direct activation measurement of \(1/(l_{eff})(dl_{eff}/dT)\) was \((3.7 \pm 0.8) \times 10^{-4}/°C\) averaged over the range 300°K to 700°K. This is to be compared with values of \((3.6 \pm 0.4) \times 10^{-4}/°C\) and \(3.4 \times 10^{-4}/°C\), respectively, obtained over the same temperature range for the reactivity oscillation method of measurement and the theoretical determination. The agreement is quite satisfactory and indicates that there are no serious systematic errors in the measurement by the reactivity method.

The activation method of measurement utilized the I/\(\nu\) subtraction technique that was developed for measurements of the resonance integral of thorium, as previously described. The measurements of the Doppler coefficient were made in a small vacuum-insulated oven at the centre of the critical assembly, where temperatures up to 1000°F were obtained. The oven was designed as a thin aluminium shell, 4 3/4-in long and 3 3/4-in in outside diameter, so that it could be located in the central fuel-element position of the test-lattice region. The oven contained a 4-in length of fuel element. Heating was accomplished by Nichrome wire in the graphite sleeve. The oven was evacuated to a pressure below 20 μm. The shell temperature was found to be 130°F with the oven operating at 1000°F. Internal thermocouples showed less than a 5°F variation from centre to edge at a centre temperature of 1000°F.

The activation technique used to measure the Doppler coefficient was based on activation of thorium and vanadium detectors in three positions, namely, in a fuel element heated in the oven, in a cool fuel element outside the oven and in a Maxwellian spectrum. Gold-197 was activated in the latter
two of these positions and used as the standard. The detectors exposed inside and outside the oven were placed at the same positions in their respective fuel elements. The Maxwellian flux was developed in the thermal column on the side of the TRIGA [8] reactor. The activation measurements in the Maxwellian flux made it possible to eliminate the counting-geometry factor. Measurements of the vanadium activation rates in the hot and cold samples permitted separation of the change in thorium absorption rates arising from changes in the thermal-neutron absorption. By subtraction of this component from the total changes in thorium absorption rate, the Doppler coefficient was obtained.

V. CRITICAL-EXPERIMENT MEASUREMENTS IN THE CLEAN-GEOMETRY ASSEMBLY

The test-lattice experiments were directed primarily toward the development of fundamental information such as the evaluation of effective resonance integrals and Doppler coefficients for thorium in HTGR fuel compacts. In contrast, the experiments on the one-sixth-scale mock-up were designed to test the accuracy of the calculational procedures for clean, reflected reactors having a composition in the range of the HTGR [3].

<table>
<thead>
<tr>
<th>TABLE IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMPARISON OF CHARACTERISTICS OF THE ONE-SIXTH-SCALE MOCK-UP ASSEMBLY AND THE PEACH BOTTOM HTGR</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Peach Bottom</th>
<th>Critical assembly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core length (in)</td>
<td>90</td>
<td>48</td>
</tr>
<tr>
<td>Core diameter (in)</td>
<td>109</td>
<td>58.7</td>
</tr>
<tr>
<td>Core volume (l)</td>
<td>14,000</td>
<td>2,130</td>
</tr>
<tr>
<td>Reflector thickness (in)</td>
<td>~24</td>
<td>24</td>
</tr>
<tr>
<td>U²³⁵ inventory (kg)</td>
<td>220</td>
<td>19.7</td>
</tr>
<tr>
<td>C:Th:U²³⁵ atom ratio</td>
<td>1824/6.7/1</td>
<td>2775/12/1</td>
</tr>
<tr>
<td>Boron:U²³⁵ weight ratio</td>
<td>~0.005</td>
<td>~0.003</td>
</tr>
</tbody>
</table>

Table IV shows a comparison of the characteristics of the small-scale mock-up assembly and the Peach Bottom reactor. It can be seen from the C: Th: U²³⁵ atom ratios for the two reactors that the critical assembly was more lightly loaded than was the Peach Bottom reactor. The lighter loading and the greater effectiveness of the reflector in the critical assembly resulted in a much softer spectrum than was calculated for the Peach Bottom reactor.
reactor, and consequently the experiment was not a good test of epithermal-reaction characteristics. On the other hand, the small core and strong reflectors resulted in strong variations in both the epithermal- and thermal-neutron spectra along radial and axial traverses. Hence, the experiment was a severe test of the multigroup, multidimensional diffusion-theory programmes for calculation of the multiplication constant and neutron-flux distributions.

The critical assembly corresponded roughly to the cold, clean, unrodded configuration of the Peach Bottom HTGR. The multiplication of the HTGR under the same conditions would be about $k_{\text{eff}} = 1.12$. The difference in multiplication constants resulted primarily from the much larger leakage from the smaller critical-assembly core. In addition, there was no rhodium present in the critical assembly.

Table V shows the results of diffusion-theory calculations of the critical assembly with various calculational models. Included in the table are similar results for the Peach Bottom reactor. It can be seen that the critical-assembly results were very sensitive to both the number of energy groups and number of dimensions used in the calculations. In the first calculation four epithermal groups and six thermal groups were used in a two-dimensional $R-Z$ diffusion-theory calculation. The epithermal-group constants were computed from the multigroup code GAM-I [14] and the thermal-groups constants from the thermalization code GATHER-I [15]. The two-dimensional, 10-group diffusion-theory code GAMBLE [16] was used for the eigenvalue calculations. The calculated multiplication constant for the critical assembly differed from the measured value by 0.3% in $k$. The Peach Bottom multiplication refers to the beginning-of-life condition with the reactor at operating temperature and all control rods removed.

By the use of four energy groups, i.e. three epithermal and one thermal group, an error of 2.3% in $k$ was introduced in the critical-experiment calculation. The Peach Bottom multiplication was only altered 0.3% by this change in group structure. This smaller discrepancy is ascribed to the larger core dimensions and the smaller importance of spatial variations in the neutron energy spectrum for the power reactor.

With the one-dimensional code FEVER [17] and four energy groups, the total discrepancy was about 4.0% in $k$ for the critical experiment, showing the importance of the geometric effects in the smaller reactor. A small difference existed in the Peach Bottom multiplication, probably because of the manner in which loading non-uniformities in the axial direction were homogenized in the one-dimensional model.

In general, the results showed the importance of adequate energy and geometric detail in calculational methods used for small, reflector-dominated reactors. However, the calculations indicated that the one-dimensional, four-group calculational model could be used with some confidence in static calculations of the Peach Bottom-type reactors. However, in burnup calculations, where composition changes occur in both the radial and the axial directions during fuel burnup, a two-dimensional model would be required.

Distributed reactivity coefficients of boron and $^{235}\text{U}$ were measured in the critical assembly and compared with calculated values. The results are shown in Table VI. The calculated values indicated errors of about
TABLE V
RESULTS OF DIFFUSION-THEORY CALCULATIONS
FOR CRITICAL ASSEMBLY
AND PEACH BOTTOM REACTOR

<table>
<thead>
<tr>
<th>Calculational model</th>
<th>Eigenvalue, ( k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of dimensions</td>
<td>No. of groups</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

TABLE VI
COMPARISON OF MEASURED
AND CALCULATED DISTRIBUTED
REACTIVITY COEFFICIENTS

<table>
<thead>
<tr>
<th>Element</th>
<th>Measured ((\xi/g))</th>
<th>Calculated ((\xi/g))</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0.132</td>
<td>0.136</td>
</tr>
<tr>
<td>U(^{235})</td>
<td>0.00190</td>
<td>0.00199</td>
</tr>
</tbody>
</table>

3% and 5% for boron and fuel, respectively. It would have been desirable
to have a harder spectrum for a more rigorous test of the epithermal cross-
section for U\(^{235}\). The discrepancy in the calculations is probably too small
to indicate a significant uncertainty in the cross-section.

The spatial distribution of the neutron flux was measured by the activ-
ation of bare and cadmium-covered manganese foils and was also used as
a basis for checking of calculations. Figure 12 shows the calculated manga-
nese absorption rate as a function of radius, indicated by the solid lines.
The crosses indicate measurements along a vertical traverse in the central
plane of the core, while the circles represent measurements along a hori-
zontal traverse. The deviations between the calculations and measurements
result primarily from the slightly different radial distances to the side re-
fl ector in the two directions. In general, the agreement between the cal-
culated and measured reaction rates was very satisfying in view of the im-
portant variation of the neutron energy spectrum with radius.

The overall temperature coefficient of the mock-up assembly was also
measured by uniform heating of the core and reflector in small steps up to
500°F and measurement of the reactivity change from room temperature at
each step.
Comparison of horizontal and vertical activation traverses for bare and cadmium-covered manganese foils, as calculated and as measured.

Fig. 12

The temperature coefficient was calculated by evaluation of the effective group cross-sections at various temperatures within the range of interest, with the slowing-down code GAM-I and the thermalization code GATHER-I. The GATHER-I calculations incorporated the use of scattering kernels as given by the SUMMIT [18] code for carbon and oxygen at the various temperatures calculated. The criticality of the system at each temperature was obtained by use of the two-dimensional diffusion-theory code GAMBLE, in which four slowing-down and six thermal energy groups were used.

The calculated multiplication of the system as a function of core temperature is compared with the multiplication measured with the assembly at various temperatures in Fig. 13. The calculated multiplication of the system is very close to the measured multiplication at all temperatures. The largest discrepancy occurred when the core was initially being heated. It is felt that the discrepancy in these initial values was produced by moisture which had been absorbed in the graphite before the assembly was con-
structured. During the process of heating the assembly, the moisture was driven off, and the discrepancy was nearly eliminated by the time a temperature of 350°F was reached.

At all temperatures the Doppler coefficient was calculated to be the largest contributor to the overall negative temperature coefficient, representing more than 90% of the net value. Other contributions arose from leakage effects, changes in thermal utilization and changes in eta.

The results of the critical-experiment programme on the small-scale mock-up assembly provided evidence that the calculational methods used in the analysis of the Peach Bottom reactor design are apparently adequate when properly applied.

REFERENCES

DISCUSSION

F. FEINER: What is the conversion ratio of the HTGR? I should also like to ask Mr. Moore what the conversion factor is for the AGR.

P. U. FISCHER: The conversion ratio in the Peach Bottom reactor is between 0.4 and 0.5. This figure is, of course, not typical for HTGRs. The Peach Bottom reactor is a small prototype plant, and therefore has a large leakage. The conversion ratio of large HTGRs is much closer to 1.0.

J. MOORE: Regarding the AGR, we have not yet made an accurate measurement, but the conversion ratio will be about 0.77.

W.C. REDMAN: Dr. Engelder has described to us a technique using dysprosium for measuring the epicadmium and sub-cadmium contributions to uranium activity*, whilst you have used vanadium in your method for thorium. I wonder whether you would care to comment on the relative merits of dysprosium and vanadium as a 1/v detector.

P. U. FISCHER: Vanadium gave the most convenient foils for our purpose because it is a 1/v absorber over a wide energy range and because of the 4-min half-life, which permitted very quick counting of the foils.

T.C. ENGELDER: I should like to reply to Mr. Redman's question regarding the relative merits of vanadium, the reference material in the Bardes method of measuring resonance absorption in fertile material, and dysprosium, the reference material in our thermal activation technique for the same type of measurement. We chose dysprosium for the following

* BARRETT, L.G., CLARK, R.H., ENGELDER, T.C. and SNIDOW, N.L., "Role of critical exponential and small lattice experiments in design of spectral shift control reactor", these Proceedings II.
reasons: (1) it has a high activation cross-section, (2) it has a convenient half-life, and (3) the experimental correction for epicadmium activation is very small, i.e. it has a high cadmium ratio in our lattices. These advantages outweigh the disadvantage of the non-$1/v$ behaviour, which requires a calculated correction in our technique of at most 5 to 10% in $\rho_{28}$ itself. We feel this correction can be computed to an accuracy of about 10%, so the error introduced in $\rho_{28}$ is at most 1%. There are various other differences between the two techniques, so that I would hesitate to say which material is superior for the Bardes method. However, both techniques are moving in the right direction in so far as they are avoiding cadmium perturbations which have caused difficulties in the standard techniques for cadmium ratio measurements.

F. FEINER: We have found Ni$^{64}$ to be the truest simulator of $1/v$ absorption, although the low isotopic abundance and activation cross-section make it difficult to use at low power levels. For epithermal spectra the manganese resonance absorption is troublesome, and nickel or vanadium are preferable. I think Mr. Fischer's experimentalist colleagues would probably say that the 4-min half-life of vanadium is a little on the short side, but on the other hand you can get much higher specific activities than from the nickel.
ON THE SUBCRITICAL ASSEMBLY FOR HIGH-TEMPERATURE USE

Y. SAKURAI, T. SEKIYA AND T. SUITA
OSAKA UNIVERSITY
AND
H. HISHIDA, H. HAMADA AND K. NAGASHIMA
SUMITOMO ATOMIC ENERGY INDUSTRIES GROUP,
JAPAN

Abstract — Résumé — Аннотация — Resumen

THE SUBCRITICAL ASSEMBLY FOR HIGH-TEMPERATURE USE. The subcritical assemblies are of the graphite-moderated, natural-uranium fuelled type. The special design of this assembly is characterized by its high-temperature portion which can be put into the heart of the core assemblies of a lower-temperature portion and enables information to be obtained both for the future high-temperature gas-cooled reactor and the direct energy conversion from high-temperature gas medium to electricity. Furthermore, it is intended to be used for the training of undergraduate students.

The assembly was piled up into the volume of $2 \times 2 \times 3$ m consisting of graphite blocks of 10 cm square bar. The high-temperature portion of 1 m$^3$ is energized by Joule's heating and can be maintained at the temperature up to 2000°C.

The fuel of UO$_2$ pellets is used for the low-temperature portion, and the UC$_2$ pellets are prepared for the high-temperature part. The fuels are inserted into the graphite sheath of square rods.

The Am-Be sources of 5 c are put under the pedestal located at the bottom of the assemblies; on the other hand, the pulsed neutrons are injected at the arbitrary point inside the assembly. The target at the end of the extension tube of the accelerator is bombarded by the deuteron beam.

The terminal bus of Joule's heating is cooled by water and helium gas is used for the cooling of overall wall surface of the high-temperature portion. These high-temperature gas loops are designed to be applicable to the research of direct conversion of nuclear energy. Concerning the control systems, these assemblies are equipped with control and measuring systems comparable to those of small-scale nuclear reactors.

The experimental approach to find out the buckling constant of the system is being studied and the best arrangement of the pulsed neutron source and the detector locations has been examined. Furthermore, the unique method of measuring the neutron flux density at high temperature has been developed. Finally, it is intended to study the influence of high temperature upon the reactor parameters.
данный сборка была смонтирована в объеме $2 \times 2 \times 3$ м из графитовых блоков в виде квадратных стержней длиной 10 см. Высокотемпературная область объемом 1 м³ образуется с помощью джоулева тепла, в ней может поддерживаться температура до 2000°С.

Для низкотемпературной области применяется топливо в виде таблеток $\text{UO}_2$, для высокотемпературной части изготовляются таблетки из $\text{UC}_2$. Топливо помещается внутри графитовой оболочки квадратных стержней.

Источники Am–Be мощностью 5 кюри устанавливаются под подставкой, расположенной в нижней части сборки. С другой стороны, импульсные нейтроны инжектируются в произвольной точке внутри сборки. Мишень в конце расширительной трубы ускорителя бомбардируется пучком дейтронов.

Выходная шина системы джоулева подогрева охлаждается водой, а для охлаждения всей поверхности стенок высокотемпературной области применяются гелий. Эти высокотемпературные газовые петли проектируются таким образом, что их можно использовать для исследования вопросов непосредственного превращения ядерной энергии в электрическую. Сборки оснащаются системами управления и контрольно-измерительными системами подобно небольшим атомным реакторам.

В настоящее время изучается экспериментальный метод определения лапласиана системы и исследуется наилучшая схема расположения источника импульсных нейтронов и детекторов. Далее, разрабатывается уникальный метод измерения плотности потоков нейтронов при высокой температуре. Наконец, предполагается изучить вопрос о влиянии высокой температуры на параметры реактора.

CONJUNTO SUBCRÍTICO PARA TEMPERATURA ELEVADA. El autor estudió un conjunto subcrítico del tipo alimentado por uranio natural y moderado por grafito. Este conjunto se caracteriza por la existencia de una zona de elevada temperatura que se puede crear en un cuerpo de menor temperatura y que permite obtener informaciones aplicables a un futuro reactor de temperatura elevada refrigerado con gas, así como datos sobre la conversión directa de la energía nuclear en electricidad en un medio gaseoso a temperatura elevada. Además, el conjunto se utiliza para el adiestramiento de los estudiantes universitarios.

El conjunto está constituido por barras cuadradas de grafito de 10 cm de lado apiladas hasta formar un volumen de $2 \times 2 \times 3$ m. La zona de alta temperatura, que tiene 1 m³, se calienta por efecto Joule y puede mantenerse a temperaturas hasta de 2000°C.

En la zona de baja temperatura, se usa combustible de pastillas de $\text{UO}_2$, mientras que para la zona de temperatura elevada, se han preparado pastillas de $\text{UC}_2$. El combustible está revestido de vainas de grafito, de sección cuadrada.

Las fuentes de Am–Be, de 5 c se instalan debajo del pedestal ubicado en la base del conjunto, mientras que el flujo neutónico pulsante se inyecta en un punto arbitrario. El blanco, situado en el extremo del tubo de extensión del acelerador, es bombeado por el haz de deuterones.

Las barras de conexión para el calentamiento por efecto Joule se enfrian con agua y la superficie total de separación de la zona de alta temperatura se enfria con helio. Este circuito de gas refrigerante está diseñado de manera que pueda servir para investigar la conversión directa de la energía nuclear en electricidad. El conjunto está provisto de dispositivos de control y medición semejantes a los de un pequeño reactor nuclear.

Se estudió la manera de determinar experimentalmente el laplaciano del sistema y se procura encontrar la mejor ubicación posible para la fuente neutróica pulsante y para los detectores. Además, se está per-
feccionando un método exclusivo de medición de la densidad del flujo neutrórico a temperatura elevada. Por último, el autor se propone investigar la influencia de esta temperatura elevada sobre los parámetros del reactor.

1. INTRODUCTION

The subcritical assemblies under construction by the members of Nuclear Engineering Department of Osaka University and Sumitomo Atomic Energy Industries are surveyed. The special design of these assemblies is characterized by a high-temperature section which can be put into the centre of the low-temperature core assembly and their capability of providing information both for the future high-temperature, gas-cooled reactor and for direct energy conversion from a high-temperature gas medium to electricity. Furthermore, they are intended for use for the training of undergraduate students.

2. GENERAL DESCRIPTION

The subcritical assemblies are of the heterogeneous, graphite-moderated, natural uranium type.

The assemblies consist of graphite bars, each $10 \times 10$ cm in cross-section, placed horizontally to form a rectangular parallelepiped approximately $200 \times 200$ cm in cross-section and $300$ cm in height. The high-temperature core of $1 \text{m}^3$ can be inserted into the centre of the whole assembly.

The pedestal consists of solid graphite bars stacked approximately $50$ cm high. The Am-Be neutron sources of $5 \text{c}$ are located on this pedestal, and the cadmium shutter is arranged between the core and the pedestal. A pulsed-neutron generator is located above the assembly for pulsed-neutron experiments.

3. ROOM-TEMPERATURE ASSEMBLY

The fuel for room-temperature use consists of $\text{UO}_2$ pellets. The cylindrical pellets, each $2.4 \text{ cm}$ in diameter and $2.5 \text{ cm}$ in height, are contained in a graphite sheath of $3.6 \text{ cm}$ in outer diameter, $2.46 \text{ cm}$ in inner diameter and $100 \text{ cm}$ in length. The graphite blocks, each $10 \times 10 \text{ cm}$ in cross-section and $100 \text{ cm}$ in length, in which the sheath is inserted, are piled up into a lattice form.

The minimum lattice distance is $10 \text{ cm}$. If the fuel rod is replaced by the graphite plug alternately, then the lattice distance of $10 \times 2\frac{1}{2}$ cm is obtained.

The reactor constants for this case are as follows:

\[
\begin{align*}
\eta &= 1.33 \\
f &= 0.909 \\
p &= 0.880 \\
\epsilon &= 1 \\
k &= 1.064 \\
L^2 &= 2500
\end{align*}
\]

If we further increase the number of graphite plugs instead of fuel-filled sheaths, we can easily obtain the various lattice constants with the selection of fuel-to-moderator ratios.
The moderator and the reflector consist of the same material as described above. The piled-up assembly is covered with a cadmium plate so as to ensure the boundary condition necessary for the calculations. Fig. 1 shows the structure of the whole assembly. In addition to the above described assembly, the equipment includes control and instrumentation systems comparable to those of a small-scale nuclear reactor.

The control system is used for student training and is equipped with a neutron detecting device and an automatic control. Three control rods are horizontally movable, and the scrambling action relies on gravity.

4. HIGH-TEMPERATURE ASSEMBLY

The high-temperature assembly consists of a 1 m³ graphite block, and high temperature is maintained by the thermal insulating layer, a carbon block with 8.7 cm thickness, covered with stainless-steel plate. The outer surface of the insulating layer is cooled by the circulating helium gas, the whole is enclosed in the aluminium vessel.

Four graphite electrodes for Joule's heating are inserted horizontally through the assembly. The maximum power to this assembly is 130 kW, while 25 kW is needed to maintain the temperature at 2000°C.

Heat removal from this high-temperature assembly is partly by water cooling in the terminals of the electrodes and partly by helium gas cooling for the overall system.
UO₂ pellets are employed for the fuel of the high-temperature assembly. The dimensions of the UC₂ fuel are the same as those of the UO₂ fuel for the room-temperature assembly. The guiding hole is arranged for the high-temperature fission chambers and foil holders.

Fig. 2 illustrates the outline of this high-temperature assembly. The high-temperature core assembly is designed for independent operation or for insertion into the low-temperature assembly.

5. NEUTRON SOURCE

The accelerator, which is movable horizontally on the guiding rail above the whole system, injects the deuteron beam into the assembly.

The total neutron yield is about $10^7 \text{n/s}$; the rating of the accelerator is 300 kV, 0.1 mA, of the high-frequency Cockcroft type. The neutron sources of Am-Be emit neutrons at a rate of about $10^7 \text{n/s}$ for four capsules, each having 1.25 c.

6. THE THERMAL-NEUTRON FLUX FROM A PULSED SOURCE

The aim of the following calculation is to avoid the ambiguity inherent in the diffusion approximation applied to a small-sized medium including a pulsed-neutron source, which is actually the case for the high-temperature core assembly. The time variation and the spatial distribution of thermal-neutron angular flux are calculated through the $P_3$ approximation of the one-velocity, time-dependent Boltzman transport equation; and a set of decay parameters is obtained as the eigenvalues under Marshak's free boundary condition, while non-asymptotic spatial distribution of thermal-neutron flux gives a measure of the deviation from the diffusion approximation. The chronological origin in the following solution is set arbitrarily at the moment when any thermal neutrons are produced through the slowing-down process; and for avoidance of the complexity of multi-group treatment, the problem is solved at first for a non-multiplying medium.
Let \( N(r, \Omega, t) \) be the space-time-angular distribution of a thermal-neutron flux; then the time-dependent, one-velocity kinetic equation is written as follows:

\[
\frac{\partial N(r, \Omega, t)}{\partial t} + \nabla_m \cdot \text{grad } N(r, \Omega, t) + \left( \frac{V_m}{\ell_{\text{tot}}} \right) N(r, \Omega, t) = \int d\Omega' \left( \frac{V_m}{\ell_s} \right) N(r, \Omega', t) f(\Omega' \rightarrow \Omega),
\]

where \( \ell_{\text{tot}} \) and \( \ell_s \) are the total and the scattering mean-free-paths; and \( f(\Omega' \rightarrow \Omega) \) is the probability for a thermal neutron initially at the solid angle \( \Omega' \) to come into \( \Omega \) through a single scattering. Assuming \( f(\Omega' \rightarrow \Omega) = 1/4 \) and with the application of Marshak's boundary condition, we reduce Eq. (1) as follows:

\[
\left\{ \frac{2 + \frac{1}{\xi v_1^2} \left( -1 + \frac{3}{\nu_1^2} (1-\alpha) \right) \left( \frac{5}{8} A_p^2 + \frac{5}{8} C_q^2 + \frac{5}{4} B_{1,p,q}^2 \right)}{4 \alpha - 1 + \frac{15}{\xi v_1^2} \left( 1 + \frac{3}{\nu_1^2} (\alpha - 1) \right) B_{1,p,q} \tanh B_{1,p,q} b} \right\} + \frac{4 \alpha - 1}{\xi v_2^2} B_{1,p,q} \tanh B_{1,p,q} b
\]

\[
= \frac{2 + \frac{3}{\nu_2^2} \left( -1 + \frac{3}{\nu_2^2} (1 - \alpha) \right) \left( \frac{5}{8} A_p^2 + \frac{5}{8} C_q^2 + \frac{5}{4} B_{1,p,q}^2 \right)}{4 \alpha - 1 + \frac{15}{\xi v_2^2} \left( 1 + \frac{3}{\nu_2^2} (\alpha - 1) \right) B_{2,p,q} \tanh B_{2,p,q} b} + \frac{4 \alpha - 1}{\xi v_2^2} B_{2,p,q} \tanh B_{2,p,q} b,
\]

where

\[
\xi \equiv \left( \frac{1}{\ell_{\text{tot}}} - \frac{\lambda}{v_m} \right) \quad \alpha \equiv \frac{1}{\xi} \ell_s
\]

\[
v_{1,2} \equiv \frac{(10 - \frac{55}{9} \alpha) + \left[ (10 - \frac{55}{9} \alpha)^2 - \frac{420}{9} (1-\alpha) \right]^{1/2}}{2}
\]

\[
B_{1,p,q}^2 = \left( \frac{\pi r_p^2}{2 a_0} \right)^2 + \left( \frac{\pi r_q^2}{2 a_0} \right)^2 + \xi^2 v_j^2
\]

\( p, q : 1, 2, 3, \ldots \)

\( j : 1 \) or \( 2 \)

\( \lambda \) is a decay parameter which is given by the above characteristic equation.

Through a brief analysis of the quantities \( \xi^2 v_{1,2}^2 \) and \( B_{1,p,q}^2 \), it follows for \( \lambda > \nu \Sigma_a \) with \( B_{1,p,q}^2 < 0 \) that all we have to do is to find a set of as many zeros of Eq. (2) as desired for \( p \) and \( q \) by means of a computer. If any complex zero of \( \lambda \) exists then, it can easily be verified that its complex conjugate exists in a set of complex zeros and the resultant thermal-neutron angular flux is real (Table I).

The result is applicable to a multiplying medium with the equations governing the slowing-down process. The overall solution for the small-
sized multiplying medium with the problem of the distortion of fast-neutron spectrum resulting from escape through boundary is being sought.

**BIBLIOGRAPHY**


**DISCUSSION**

G.H. KINCHIN: Does the helium circulate through your high temperature chamber when it is hot, and what is the permissible impurity level in the helium?

Y. SAKURAI: The helium flowing through the loop does not directly touch the high-temperature graphite blocks. The helium in contact with the high-temperature portion is enclosed, whilst that surrounding the core flows through the external loop. I have no information about the helium impurity level.

R.E. UHRIG: I understand that you expect to use this assembly for training undergraduate students. Could you elaborate on this?

Y. SAKURAI: For training purposes we shall use only the room-temperature portion of the assembly. The high-temperature portion is for research.

O.H. CRITCHLEY: The purpose of the experiments must be to promote the development of a power-producing reactor. In view of the very high temperatures of operation of such a reactor system, the possibility of a breach in the pressure circuit cannot be overlooked. Have any safety measures been developed so as to limit or prevent the exothermic reaction that would occur when the air which would replace the helium coolant came into contact with the intensely hot graphite core?

Y. SAKURAI: As I said, we have two helium gas systems: the one which comes into direct contact with the high-temperature fuel is enclosed, while the other, which flows through the external loop, surrounds the vessel of the high-temperature portion. If the loop were breached no great danger would ensue.

We are not considering this work in direct relation to a power reactor at the moment.
ÉTUDE D'UN ENSEMBLE SOUS-CRITIQUE PAR LA MÉTHODE DE LA CARACTÉRISTIQUE DE FRÉQUENCE*

I. I. PURICA, N. SEFERIAN ET E. RĂCĂTĂIANU
INSTITUT DE PHYSIQUE ATOMIQUE, BUCAREST, ROUMANIE

Abstract — Résumé — Аннотация — Resumen

STUDY OF A SUBCRITICAL ASSEMBLY BY THE FREQUENCY CHARACTERISTIC METHOD. The possibilities offered by the frequency characteristic method for the study of a subcritical assembly are compared with the possibilities offered by other methods. The preliminary experimental results obtained with a graphite and enriched-uranium assembly are interpreted in the light of Galanin's lattice theory. Nelkin's theory was used to interpret the neutron spectrum data obtained.

ÉTUDE D'UN ENSEMBLE SOUS-CRITIQUE PAR LA MÉTHODE DE LA CARACTÉRISTIQUE DE FRÉQUENCE. On compare les possibilités offertes par la méthode de la caractéristique de fréquence pour l'étude d'un ensemble sous-critique avec les possibilités offertes par d'autres méthodes. Les résultats expérimentaux préliminaires, obtenus par l'étude d'un réacteur à graphite et uranium enrichi, sont interprétés par la théorie des réseaux exposée par Galanin. On a étudié les informations que l'on peut obtenir sur le spectre de neutrons en utilisant la théorie de Nelkine.

ИЗУЧЕНИЕ ПОДКРИТИЧЕСКОЙ СБОРКИ МЕТОДОМ ХАРАКТЕРИСТИЧЕСКОЙ ЧАСТОТЫ. Сравниваются возможности, предоставляемые методом характеристической частоты для изучения подкритической сборки, с возможностями, предоставляемыми другими методами. Предварительные экспериментальные результаты, полученные при изучении графитового реактора с обогащенным ураном, были интерпретированы при помощи разработанной Галаниным теории решеток. На основе теории Нелькина изучена информация, полученная на спектре нейтронов.

ESTUDIO DE UN CONJUNTO SUBCRÍTICO POR EL MÉTODO DE LA CARACTERÍSTICA DE FRECUENCIA. Los autores comparan las posibilidades que presenta el método de la característica de frecuencia, para el estudio de un conjunto subcrítico, con las que ofrecen otros métodos. Con arreglo a la teoría de los reticulados, expuesta por Galanin, interpretan los resultados experimentales preliminares, obtenidos mediante el estudio de un reactor de grafito y uranio enriquecido. Examinan las informaciones que sobre la base de la teoría de M. Nelkin pueden obtenerse acerca del espectro neutónico.

A. LA THÉORIE DE LA MÉTHODE DE LA CARACTÉRISTIQUE DE FRÉQUENCE POUR UN MILIEU MULTIPLECTEUR HÉTÉROGÈNE**

1. Pour l'étude des caractéristiques nucléaires des modérateurs ou des réseaux multiplicateurs, on emploie soit des méthodes statiques, soit la méthode de la source pulsée.

Dans [1] nous avons démontré qu'on peut obtenir des informations identiques, lorsqu'on emploie la méthode de la caractéristique de fréquence au lieu de la méthode de la source pulsée.

La fonction qui donne la variation temporelle du flux des neutrons thermiques, lorsque le milieu est excité par une source pulsée, est simple-

* Ce mémoire n'a pu être présenté au colloque par suite de circonstances indépendantes de la volonté des auteurs.

** Le chapitre A est du à I. I. Purica.
ment la transformée de Fourier de la fonction décrivant la variation de l'amplitude de l'oscillation du flux neutronique avec la fréquence, lorsque le milieu est excité par une source modulée sinusoidalement.

La méthode utilisée par RADEVSKI et HOROVITZ [2] est un cas particulier de la méthode que nous utilisons, caractérisé par certaines conditions restrictives. Nous ferons ici une analyse des informations que l'on peut obtenir par l'emploi de la méthode de la caractéristique de fréquence pour l'étude d'un ensemble sous-critique hétérogène.

2. Dans le réseau multiplicateur d'un ensemble sous-critique, le flux des neutrons thermiques $\phi(\vec{r}, E, t)$ dans l'élément de volume $d\vec{r}$, l'intervalle d'énergie $dE$, au moment $t$, satisfait l'équation:

$$\frac{1}{\nu(E)} \frac{\partial \phi(\vec{r}, E, t)}{\partial t} = D(E) \nabla^2 \phi(\vec{r}, E, t)$$

$$- [\Sigma_a(E) + \int_{E_0}^{E} \Sigma_s(E' \rightarrow E')dE'] \phi(\vec{r}, E, t)$$

$$+ \int_{E_0}^{E} \Sigma_s(E' \rightarrow E') \phi(\vec{r}, E', t)dE' + Q(\vec{r}, E, t) \tag{1}$$

où $D(E)$ est le coefficient de diffusion, $\Sigma_a(E)$ la section macroscopique d'absorption dans le modérateur, $\Sigma_s(E)$ la section de diffusion, $\Sigma_s(E \rightarrow E')dE'/\Sigma_s(E)$ la probabilité pour un neutron, ayant l'énergie $E$, d'être diffusé dans l'intervalle de l'énergie $(E', E' + dE')$.

L'énergie $E$ est mesurée en unité $kT$, $T$ étant la température du modérateur. $Q(\vec{r}, E, t)$, la source de neutrons thermiques se compose des deux termes suivants: $Q_f(\vec{r}, E, t)$, la source de neutrons thermiques apparaissant à cause de la fission de l'uranium (source de fission) et $Q_{ex}(\vec{r}, E, t)$ la source de neutrons injectés dans le milieu.

L'équation intégral différentielle (1) peut être réduite à un système d'équations algébriques en faisant les transformations suivantes:

a) Développons en série le flux $\phi(\vec{r}, E, t)$, dans le système de base de fonctions propres $\chi_n(\vec{r})$ du laplacien: $\nabla^2 \chi_n(\vec{r}) + B_n^2 \chi_n(\vec{r}) = 0$. $B_n^2$ est le paramètre géométrique du réseau. Les coefficients du développement seront

$$\varphi_n(E, t) = \int_{V_R} d\vec{r} \phi(\vec{r}, E, t)\chi_n(\vec{r}).$$

b) Nous allons calculer la transformée de Laplace de l'équation satisfaite par $\varphi_n(E, t)$. L'image de $\varphi_n(E, t)$ sera

$$\tilde{\varphi}_n(E, s) = \int_0^{\infty} dt e^{-st} \varphi_n(E, t).$$

c) Enfin, nous développons la fonction $\tilde{\varphi}_n(E, s)$ dans une série de polynômes
de Laguerre $L^{(1)}_i(E)$, pondérée par la fonction $M(E) = E \exp -E$, dont les coefficients auront la forme

$$\varphi_{ln}(s) = \int dE \, L^{(1)}_{i}(E) \varphi_n(E, s).$$

Après ces transformations on obtient

$$\sum_{i=1}^{\infty} A_{ki,n} \varphi_{i,n}(s) = q_{k,n}(s)$$

où

$$A_{ki,n} = (D)_{ki} B^2_{n} + (\Sigma_{a})_{ki} - \gamma_{ki} + (1/v)_{ki} s,$$

$$q_{k,n}(s) = \int_{V}^{0} ds \int_{0}^{\infty} dE \chi_{k}(F) e^{-st} L^{(1)}_{k}(E) Q(F, E, t).$$

( ($k = 0, 1, 2, \ldots \infty$ et $n = 1, 2, 3, \ldots \infty$).

Les valeurs des coefficients $C(E)$ de l'équation (1) après la transformation (C) ont été désignées par:

$$(C)_{ki} = \int dE \, L^{(1)}_{k}(E) C(E) L^{(1)}_{i}(E) M(E)$$

et

$$\gamma_{ki} = \int dE \int dF \, [L^{(1)}_{k}(E) \Sigma_{s}(E' \rightarrow E) M(E') L^{(1)}_{i}(E')]$$

$$- L^{(1)}_{k}(E) \Sigma_{d}(E \rightarrow E') M(E) L^{(1)}_{i}(E')]].$$

Le flux neutronique devient ainsi:

$$\phi(F, E, t) = \sum_{n=1}^{\infty} \sum_{i=0}^{\infty} L^{(1)}_{i}(E) M(E) \chi_{n}(F) \int_{0}^{t} \varphi_{ln}(s) e^{st} ds.$$
Nous allons analyser spécialement le terme représentant la source de neutrons thermiques pour le cas d'un ensemble sous-critique hétérogène.

3. Pour exprimer la source de fission nous utiliserons les notations suivantes:

- \( \eta \): nombre de neutrons de fission produits par la capture d'un neutron thermique dans une barre combustible,
- \( \Sigma_a^b(E) \): section macroscopique d'absorption dans la barre combustible,
- \( \gamma_k \): coordonnée de l'axe de la barre. On considérera des barres cylindriques,
- \( \varphi_0 \): rayon de la barre qui est petit par rapport au rayon de la cellule,
- \( K(\vec{r}_k - \vec{r}, E, t - t') \): noyau de l'équation représentant la probabilité pour qu'un neutron de fission, produit dans la barre \( k \), au moment \( t' \), soit thermalisé jusqu'à l'énergie \( E \), dans le point \( r \), au moment \( t \).

La source de fission sera décrite par l'expression:

\[
Q^F(\vec{r}', E, t) = \sum_{k=1}^{N} \int d\vec{r}' \int dE' \int dt' \eta \Sigma_a^b(E) \phi_b(\vec{r}_k + \vec{r}', E, t') K(\vec{r}_k - \vec{r}', E, t - t')
\]

\[
- \sum_{k=1}^{N} \int d\vec{r}' \Gamma^b(E) \phi_b(\vec{r}_k + \vec{r}', E, t) \delta(\vec{r}' - \vec{r}_k).
\]

Le premier terme représente les neutrons thermiques produits à la suite des fissions tandis que le deuxième donne les neutrons absorbés dans les barres (dont le nombre total est \( N \)). On a noté par \( V_b \) le volume d'une barre et \( r' = r - \vec{r}_k \).

Introduisons, d'après GALANIN [6], «la constante thermique» pour la barre combustible:

\[
\Gamma(E) = V_b \Sigma^b(E) / Q_b(E)
\]

\( Q_b(E) \) est défini par

\[
Q_b(E) = \frac{\phi(\vec{r}_k, E)}{\int_{V_b} d\vec{r} \phi(\vec{r}_k + \vec{r}, E)}.
\]

\( \phi(\vec{r}, E) \) est le flux neutronique à la surface de la barre \( (\vec{r} = \vec{r}_k + \vec{r}_0) \). Etant donné le petit rayon de la barre nous pouvons considérer \( \vec{r} = \vec{r}_k \). On a supposé de même en (9) que la cellule est infinie axialement et que la distribution radiale du flux conserve sa forme dans le temps. Donc, on peut considérer \( Q_b \) comme une fonction de l'énergie \( E \) seulement.

Dans ce cas:
CARACTÉRISTIQUES DE FRÉQUENCE

\[ \int_{V_b} d\vec{r}' \sum_{a} \Sigma_a(E) \phi(\vec{r}_k + \vec{r}', E, t) = \Gamma(E) \phi(\vec{r}_k, E, t). \] (10)

Après avoir introduit (10) en (7), on fait la transformation de Laplace. Tenant compte que nous avons affaire à un produit de convolution, on a

\[ \tilde{Q}^p(\vec{r}, E, s) = \sum_{k=1}^{N} \left\{ \int dE' \eta \Gamma(E') \tilde{\phi}(\vec{r}_k, E', E, s) \tilde{R}(\vec{r}_k - \vec{r}, E, s) \right. \\
- \left. \Gamma(E) \tilde{\phi}(\vec{r}_k, E, s) \delta(\vec{r} - \vec{r}_k) \right\}. \] (11)

GALANIN a démontré [7] que l'on peut homogénéiser le réseau d'un réacteur hétérogène avec une bonne précision si le nombre de barres dépasse 20, et si le réseau est symétrique. Nous ferons ici l'homogénéisation en passant de la somme par rapport à \( k \) à l'intégrale étendue sur tout le volume de l'ensemble sous-critique:

\[ V_c \sum_{k} f(\vec{r}_k) \rightarrow \int f(\vec{r}) d\vec{r}. \] (12)

\( V_c \) est le volume d'une cellule et \( V_R \) celui de l'ensemble sous-critique.

En même temps il est nécessaire de modifier l'expression de la section d'absorption dans le modérateur:

\[ \Sigma_a(E) \rightarrow \frac{V_m \Sigma_a(E)}{V_c Q_m(E)}. \] (13)

\( Q_m(E) \) est défini de la même manière qu'en (9). Une analyse plus détaillée du processus d'homogénéisation pour des régimes variables fera l'objet d'une autre communication.

Pour le réseau homogénéisé (11) devient

\[ \tilde{Q}^p(\vec{r}, E, s) = \eta \int dE' \frac{\Gamma(E')}{V_c} \int d\vec{r}' \delta(\vec{r}_k, E', E, s) \tilde{R}(\vec{r}_k - \vec{r}, E, s) \]
- \[ \frac{\Gamma(E)}{V_c} \int d\vec{r}_k \delta(\vec{r} - \vec{r}_k) \phi(\vec{r}_k, E, s). \] (14)

Nous développons la transformée de Laplace de la source d'après les fonctions propres \( \chi_j(\vec{r}) \) du laplacien. Les coefficients seront donnés par:

\[ \tilde{q}^m(E, s) = \int dE' \frac{\Gamma(E')}{V_c} \int d\vec{r} \chi_j(\vec{r}) \sum_{i=1}^{N} \chi_i(\vec{r}_k) \phi_i(E, s) \tilde{R}(\vec{r}_k - \vec{r}, E, s) \]
- \[ \frac{\Gamma(E)}{V_c} \phi_n(E, s). \] (15)
Si le noyau de ralentissement $K$ est fonction seulement de $|\vec{r}_k - \vec{r}|$, nous pouvons appliquer le deuxième théorème des réacteurs nucléaires [8]:

$$
\int d\vec{r}_k x_l(\vec{r}_k) K(|\vec{r}_k - \vec{r}|, E, s) = x_l(\vec{r}) \tilde{\mathcal{K}}(B^2_k, E, s).
$$

$\tilde{\mathcal{K}}(B^2_k, E, s)$ est la transformée de Fourier du noyau de ralentissement $K(|\vec{r}_k - \vec{r}|, E, s)$, par rapport à la variable spatiale. Les fonctions $x_l(\vec{r})$ étant orthonormales, nous obtenons de (15):

$$
\tilde{q}^F(E, s) = \int dE \frac{\Gamma(E')}{V_c} \tilde{\varphi}_n(E', s) \tilde{\mathcal{K}}(B^2_n, E, s) - \frac{\Gamma(E)}{V_c} \tilde{\varphi}_n(E, s).
$$

(16)

Représentant $\tilde{\varphi}_n(E, s)$ par une série de polynômes de Laguerre:

$$
\tilde{\varphi}_n(E, s) = \sum_l \phi_{l,n}(s) L_l^{(1)}(E) M(E)
$$

et utilisant (4), la source de fission prend la forme:

$$
\tilde{q}^F_k(s) = \sum_l \phi_{l,n}(s) \left[ \tilde{\mathcal{K}}(B^2_n, s) \frac{\Gamma(\lambda_l)}{V_c} \eta - \frac{\Gamma(\lambda_k)}{V_c} \right].
$$

(17)

En (17) on a utilisé les notations:

$$
\tilde{\mathcal{K}}(B^2_n, s) = \int dE L_k^{(1)}(E) \tilde{\mathcal{K}}(B^2_n, E, s),
$$

(18)

$$
(\Gamma)_{\lambda_l} = \int dE' \Gamma(E') L_k^{(1)}(E') M(E'),
$$

(19)

$$
(\Gamma)_{\lambda_k} = \int dE L_k^{(1)}(E) \Gamma(E) L_k^{(1)}(E) M(E).
$$

(20)

4. Nous utiliserons l'expression (17) de la source de fission dans le système (2). Si nous tenons compte de la définition (8) de $\Gamma(E)$, remplaçons la section d'absorption dans le modérateur par (13), et écrivons

$$
\tilde{\Sigma}_a(E) = \frac{V_m}{V_c} \tilde{\Sigma}_a(E),
$$

(21)

$$
\tilde{\Sigma}_b(E) = \frac{V_m}{V_c} \tilde{\Sigma}_b(E) + \tilde{\Sigma}_b^b(E).
$$

Le système d'équations (2) peut alors être écrit sous la forme:

$$
\sum_{l=0}^{\infty} \left[ (D)_{kl} B^0_n + (\bar{B})_{kl} - \gamma_{kl} - \eta(\bar{B})_{kl} \tilde{\mathcal{K}}(B^2_n, s) + \frac{1}{V_c} \frac{l}{k} \frac{1}{s} \right] \phi_{l,n}(s) = \tilde{q}^F_k(n, s). (21')
$$
La transformée de Fourier du noyau de ralentissement $\mathcal{X}(B_n^2, E, s)$ dépend du modérateur utilisé dans le réseau hétérogène. Pour le graphite on peut considérer avec une bonne approximation que le processus de ralentissement est décrit par la théorie de l'âge de Fermi. Lorsque nous négligeons le temps de ralentissement par rapport au temps de diffusion, $\mathcal{X}$ sera donné par

$$\mathcal{X}(B_n^2, E) = p e^{-\tau(E)\delta(E - E_0)},$$

(22)

où

- $E_0$: l'énergie à laquelle les neutrons thermalisés pénètrent dans le spectre thermique.
- $p$: la probabilité d'éviter la capture de résonance. Celle-ci n'est pas influencée sensiblement par les variations du spectre thermique parce que les résonances se trouvent à des énergies plus grandes.

Nous introduisons pour le coefficient d’utilisation des neutrons thermiques la définition suivante:

$$f_{ki} = \frac{\sum_k \bar{h}_{ki}}{\sum_k \bar{h}_{ki}}.$$  
(23)

En première approximation, $k = i = 0$, on suppose que la distribution énergétique du flux neutronique est $M(E)$. En employant l'expression (22) de $\mathcal{X}(B_n^2, E)$ et comme $\gamma_{00} = 0$, on a

$$\left\{ W_{00} \frac{S}{v_0} + D_0 B_n^2 + (\bar{E}_{00}^h) [1 - \eta f_{k0} \mathcal{X}_0(B_n^2)] + W_{00} \frac{S}{v_0} \right\} \phi_0(s) = q_0^{ex}(s).$$

(24)

La valeur propre du système en première approximation en résulte immédiatement:

$$\lambda_{0,n} = \frac{v_{0a}}{W_{00}} \left\{ D_0 B_n^2 + (\bar{E}_{00}^h) [1 - \eta f_{k0} \exp -\tau(E_0)B_n^2] \right\}.$$  
(26)

Donc

$$\varphi_{0,n} = \frac{q_0^{ex}(s) v_0/W_{00}}{s + \lambda_{0,n}}.$$  
(27)
La connaissance de l'expression de la source neutronique extérieure \(q^{\text{ex}}_{1,n}(s)\) permet de trouver le flux \(\phi(\vec{r}, E, t)\) par l'utilisation de (6).

5. La source extérieure, dans la méthode de la caractéristique de fréquence, peut être obtenue par la modulation sinusoïdale d'un faisceau de neutrons thermiques. On peut obtenir facilement un tel faisceau, en utilisant la colonne thermique d'un réacteur nucléaire.

L'expression analytique de la source extérieure, pour une pulsation donnée \(\omega\), est en général:

\[
q^{\text{ex}}_{1,n}(s) = \mathcal{R}(\vec{r}) \mathcal{M}(E)(1 - a e^{j\omega t})
\]

où a est le degré de modulation.

Les conditions expérimentales imposent à la source une configuration spécifique. Nous allons analyser ici l'influence de celle-ci sur le terme variable de (28).

Le faisceau de neutrons, après avoir passé par le dispositif de modulation est diffusé par une cible située à l'intérieur de l'ensemble sous-critique.

Si \(x\) est la distance entre la cible et le modulateur, les neutrons diffusés dans tout l'espace par la cible seront:

\[
q^{\text{ex}}_0 \int_{-1}^{1} d\mu \int_{0}^{\infty} dE' \mathcal{M}(E') \mathcal{\Sigma}(E'\rightarrow E, \mu)[1 - a e^{j\omega(t - x)}].
\]

\(\mathcal{\Sigma}(E'\rightarrow E, \mu)\) est le noyau de diffusion de la cible, \(\mu\) le cosinus de l'angle de diffusion, \(x_1 - x_0\) l'épaisseur de la cible, et \(\int d\mu \mathcal{\Sigma}(E'\rightarrow E, \mu) = \mathcal{\Sigma}(E'\rightarrow E)\).

Nous rapportons tout au système de coordonnées de l'ensemble sous-critique dans lequel les points de la cible sont caractérisés par \(\vec{r}\):

\[
q^{\text{ex}}_{\vec{r}, E, t} = q^{\text{ex}}_0 R(\vec{r}) \int_{0}^{\infty} dE' \mathcal{M}(E') \mathcal{\Sigma}(E'\rightarrow E)[1 - a e^{j\omega(t - x)}].
\]

Le système d'équations étant linéaire nous retenons seulement la composante alternative de la source extérieure et pour \(a = 1\).

Le terme alternatif \(q^{\text{ex}}_{k,n}(s)\) de (24) devient, en tenant compte de (4):

\[
q^{\text{ex}}_{k,n}(s) = \int_{\mathcal{R}} \int_{0}^{\infty} dt \chi_{k}^{n}(\vec{r}) e^{-\gamma t} R(\vec{r}) e^{j\omega t}
\]

\[
\int_{0}^{\infty} dE \int_{0}^{\infty} dE' \mathcal{\Sigma}(E'\rightarrow E) \mathcal{M}(E') L_{k}^{(1)}(E)e^{-j\omega(t - \gamma_0)}/\gamma_0.
\]

En considérant la source ponctuelle \(R(\vec{r}) = \delta(\vec{r} - \vec{r}_0)\), en posant \(\Omega = \omega(x - \gamma_0)/\gamma_0\), et en utilisant le principe de la balance détaillée, (30) peut être mise sous la forme
CARACTÉRISTIQUE DE FRÉQUENCE

\[ \Theta_{k,n}^s(s) = \frac{\Theta_{k,n}^s(\vec{r})}{s - j\omega} \mathcal{F}_k(j\Omega), \]

où l'on a désigné:

\[ \mathcal{F}_k(j\omega) = \int dE M(E)L_k^{(1)}(E)\Sigma(E, j\Omega) \]

\[ \Sigma(E, j\Omega) = \int dE' \Sigma(E \rightarrow E') \exp(-j\Omega/\sqrt{E'}) \]

Nous sommes intéressés à la manière dont \( \mathcal{F}_k(j\Omega) \) dépend de la valeur \( \omega \).

Si nous développons en série l'exponentielle dans l'expression de \( (E, j\omega) \), nous obtenons une série

\[ \mathcal{F}_k(j\Omega) = \sum_{m=0}^{\infty} \frac{(-j\Omega)^m}{m!} J_{k,m}, \]

dont les coefficients

\[ J_{k,m} = \int dE \int dE' (E)L_k^{(1)}(E)\Sigma(E \rightarrow E')(1/E')^m, \]

peuvent être calculés si nous employons pour \( \Sigma(E \rightarrow E') \) l'approximation du gaz lourd en utilisant les résultats obtenus par HURWITZ, NELKIN, HABETLER [9].

Les calculs montrent que pour le domaine de \( \Omega \) qui nous intéresse, la variation de l'amplitude de \( F_k(j\Omega) \) est petite, de sorte que l'on peut la considérer comme une correction.

Cela a été d'ailleurs vérifié aussi par l'expérience.

Nous pouvons donc considérer en première approximation \( \mathcal{F}_k(j\Omega) \sim \text{const} \).

En utilisant l'expression de la source extérieure (31) dans l'équation (24) on peut résoudre (24) par rapport à \( \varphi_{in} \) et obtenir le flux neutronique \( \phi(\vec{r}, E, t) \).

Nous avons montré [1] que lorsque les conditions de séparabilité des valeurs propres \( \lambda_{i,n} \) sont réalisées, le carré de l'amplitude de la composante alternative du flux peut être mise sous la forme:

\[ |\tilde{\psi}|^2 = a_1 + \frac{a_2}{\omega^2 + \lambda^2}, \]

où \( \lambda \) est la valeur propre \( \lambda_{i,n} \) ayant la valeur minimum. Le terme \( a_1 \) représente la contribution des autres valeurs propres dans le domaine \( \Delta\omega \) où \( \lambda \) est séparable.

On peut ainsi déterminer les valeurs propres \( \lambda \). En déterminant \( \lambda \) en fonction du paramètre géométrique \( B^2 \) on obtient des informations sur un
ensemble sous-critique qui sont de la même puissance que celles obtenues maintenant par l'emploi de la méthode de la source pulsée.

B. LES RÉSULTATS EXPÉRIMENTAUX OBTENUS AVEC UN ENSEMBLE SOUS-CRITIQUE A GRAPHITE ET URANIUM ENRICHI

1. La vérification des possibilités offertes par la méthode de la caractéristique de fréquence pour l'étude des réseaux hétérogènes a été faite avec l'ensemble sous-critique à uranium enrichi 10%, modéré par graphite, appartenant à l'Institut de physique atomique de l'Académie de la République Populaire Roumaine [10].

L'ensemble sous-critique est constitué de cellules parallélépipédiques en graphite, dont la base est carrée (9,5X9,5 cm) et la hauteur variable. Sur l'axe de la cellule se trouve un tuyau d'aluminium (ϕ 1,5 cm), à l'intérieur duquel on peut introduire soit une barre de combustible, soit un absorbant. Les cellules peuvent être assemblées de sorte que l'on peut obtenir des configurations géométriques différentes.

Les barres d'uranium utilisées sont du même type que celles qu'on emploie dans le réacteur VVR-S. Elles sont cylindriques, avec un diamètre de 0,6 cm et une hauteur de 50 cm, et sont gainées par 2 mm d'aluminium. Les barres peuvent être assemblées en longueur pour obtenir des configurations à trois hauteurs différentes.

Pour réaliser les conditions à la limite (l'annulation du flux neutronique sur la surface extérieure), pour chaque configuration réalisée, la surface extérieure a été recouverte par du cadmium (1 mm d'épaisseur).

L'ensemble sous-critique est situé à l'intérieur d'une protection d'acier et d'eau (fig. 1. a). L'axe du canal horizontal de la colonne thermique du réacteur VVR-S est parallèle à ses surfaces latérales.

Le faisceau neutronique de la colonne thermique coupe la cible diffuseuse, située sur l'axe vertical du prisme, sur une surface utile de 4X4 cm. La cible est en graphite et a une épaisseur de 2,5 cm.


2. La modulation du faisceau a été faite à l'aide d'un disque formé par deux plaques d'acier entre lesquelles se trouve 1 mm du cadmium. Les extrémités du disque ont été usinées d'après une courbe calculée d'avance, ainsi que la surface libre, résultant de l'intersection du disque avec le faisceau neutronique, soit une fonction sinusoidale de l'angle de rotation du disque. Pendant une rotation complète, le disque peut produire quatre périodes. La pulsation du flux neutronique dans le faisceau (ωs⁻¹) dépend du nombre de tours du disque (N tours/min) par la relation:

\[ \omega = 2\pi \left( \frac{4N}{60} \right) \]

On a vérifié pour quelques configurations la variation sinusoidale du nombre total des neutrons diffusés par la cible, pour les positions du disque
Ensemble sous-critique.

de 5 en 5 degrés. Les points expérimentaux se sont situés bien sur une sinusoïde. La figure 1.b représente le disque DM et la courbe de variation des neutrons dans le faisceau avec l'angle de rotation $\phi$.

Le disque est entraîné par un moteur à courant continu dont le nombre de tours peut varier entre 200 tours/min et 4000 tours/min, permettant ainsi l'exploration d'un domaine de fréquence entre 13 et 280 Hz.

Le moteur est alimenté par un groupe convertisseur. La stabilisation du nombre de tours a été assurée par une bonne stabilisation de la tension.

On a effectué la mesure du nombre de tours N avec une précision maximum par le dénombrement de tours (pendant la durée $\theta$ d'une expérience) à l'aide d'un dispositif optique FT (fig. 2). Les impulsions produites par FT ont été amplifiées par $A_1$, discriminées par $D_1$ et dénombrées par $N_\omega$.
Le rapport entre le nombre total des impulsions reçues par $N \omega$ et le temps de la mesure $\theta$ nous donne la valeur moyenne du nombre de tours pendant $\theta$. Due à la faible stabilisation, la précision pour $\omega$ a été entre 0.5 - 1.5%.

La mesure de la valeur moyenne et de l'amplitude du flux neutronique à l'intérieur de l'ensemble sous-critique a été faite par la méthode décrite par Raievski et Horowitz. Dans ce but on a utilisé le schéma représenté dans la figure 2. Le détecteur est un scintillateur à bore et un photomultiplicateur $K$. Les impulsions de $K$ sont amplifiées par $A_2$, discriminées par $D_2$ et puis sont reçues simultanément par un compteur $N$ qui compte le nombre total des impulsions, à l'aide d'un distributeur $P$, par quatre compteurs $N_1$, $N_2$, $N_3$, et $N_4$. Le distributeur est commandé par la position du disque $DM$ à l'aide du dispositif optique $FT$, ainsi que chaque compteur $N_k$ compte le nombre total des impulsions pour le même quart de période, pendant toute la durée de la mesure.

Le carré de l'amplitude de l'oscillation du flux neutronique sera

$$A^2(\omega) = K(\alpha^2 + \beta^2),$$

avec

$$\alpha = -N_1 - N_2 + N_3 + N_4,$$
$$\beta = -N_1 + N_2 + N_3 - N_4.$$  \hspace{1cm} (2)

K est fonction de l'efficacité du compteur et de la durée de la mesure.

La valeur moyenne est donnée par $KN = K \sum_{k=1}^{4} N_k$. La vérification du fonctionnement correct du schéma a été faite en contrôlant la réalisation de l'égalité entre le nombre des impulsions enregistrées par $N$ et la somme des impulsions enregistrées par les compteurs $N_k$.

Le schéma utilisé permet ainsi la mesure simultanée de la valeur moyenne de l'amplitude de l'oscillation et de la pulsation $\omega$.

Par la variation de la pulsation $\omega$ on peut déterminer la caractéristique $A^2(\omega)$.

3. Pour obtenir la variation de $\lambda$ avec le facteur géométrique $B^2$ du réseau, on a utilisé six configurations différentes, dont les caractéristiques sont données dans le tableau I.

Pour chaque configuration, on a mesuré la courbe donnant l'amplitude du flux en fonction de la pulsation pour l'ensemble sous-critique sans les barreaux d'uranium et avec les barreaux d'uranium.

Pour chaque point de ces courbes on a utilisé un nombre de $10^5$ impulsions tel que l'erreur statistique soit au-dessous de 1%.

Le flux neutronique dans le faisceau de la colonne thermique a été maintenu constant avec la précision du réglage de la puissance du réacteur VVR-S qui est de 0.5%.

Dans la figure 3 on peut voir quelques courbes $A^2(\omega)$ pour l'ensemble sous-critique sans uranium et avec uranium, pour la même configuration.

La dispersion des points est plus grande que celle qui est donnée par l'erreur statistique et elle est produite par la modulation du spectre des
CARACTÉRISTIQUES DES CONFIGURATIONS

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Dimensions (cm)</th>
<th>Nombre de cellules*</th>
<th>$B^2$ (cm$^{-2}$)</th>
<th>$\Sigma$(gt)</th>
<th>$\lambda_m$ (t=1)</th>
<th>$\lambda_u$ (t=1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a b h</td>
<td>a b h</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_1$</td>
<td>57 47,5 124</td>
<td>28</td>
<td>$8,04 \cdot 10^{-3}$</td>
<td>448</td>
<td>1268</td>
<td>2068</td>
</tr>
<tr>
<td>$C_2$</td>
<td>57 66,5 124</td>
<td>40</td>
<td>$5,89 \cdot 10^{-3}$</td>
<td>640</td>
<td>998</td>
<td>1630</td>
</tr>
<tr>
<td>$C_3$</td>
<td>76 66,5 124</td>
<td>54</td>
<td>$4,56 \cdot 10^{-3}$</td>
<td>846</td>
<td>835</td>
<td>1170</td>
</tr>
<tr>
<td>$C_4$</td>
<td>76 85,5 124</td>
<td>70</td>
<td>$3,66 \cdot 10^{-3}$</td>
<td>1097</td>
<td>654</td>
<td>1025</td>
</tr>
<tr>
<td>$C_5$</td>
<td>95 85,5 124</td>
<td>88</td>
<td>$3,07 \cdot 10^{-3}$</td>
<td>1410</td>
<td>-</td>
<td>793</td>
</tr>
<tr>
<td>$C_6$</td>
<td>114 85,5 124</td>
<td>106</td>
<td>$2,74 \cdot 10^{-3}$</td>
<td>1696</td>
<td>663</td>
<td>-</td>
</tr>
</tbody>
</table>

* A cause de la position du détecteur deux cellules sont restées sans barres d'uranium.

Figure 3
Quelques courbes $A^2$($u$).
neutrons. Les courbes $A^2(\omega)$ ont été approximées avec l'expression analytique

$$A^2(\omega) = \alpha + \beta/(\lambda^2 + \omega^2).$$

(3)

On peut voir dans le tableau I les valeurs de $\lambda$ déduites de ces courbes pour l'ensemble sous-critique sans barres d'uranium $\lambda_m$, et avec des barres d'uranium $\lambda_u$.

La valeur du rapport $\alpha/\beta$ n'a pas dépasse quelques %, pour l'ensemble sous-critique sans barres d'uranium, ce qui démontre que l'influence des harmoniques supérieurs est négligeable.

Dans la figure 4 sont représentés les courbes $\lambda_m(B^2)$ et $\lambda_u(B^2)$. Les valeurs expérimentales concordent bien avec des courbes ayant l'expression analytique:

$$\lambda_m(B^2) = 250 + 129 \cdot 10^3 B^2$$

(4)

$$\lambda_u(B^2) = 914 + 165 \cdot 10^3 B^2 - 1645 \exp(-320 B^2).$$

(5)

4. La comparaison des résultats expérimentaux obtenus avec les résultats théoriques (Chapitre A de la présente communication), démontre en première approximation que par la méthode de la caractéristique de fréquence, on peut obtenir des résultats similaires à ceux qui sont obtenus
CARACTÉRISTIQUE DE FRÉQUENCE

par la méthode de la source pulsée (par exemple, les résultats obtenus par CAMPBELL et STELSON [11]).

En comparant l'expression de $\lambda_m(B^2)$ à la droite que l'on devrait obtenir pour un prisme de graphite (fig. 4), on constate que la section d'absorption est plus grande et que le coefficient de diffusion est plus petit. Cette différence provient de l'influence des fuites neutroniques par les tuyaux d'aluminium qui sont vides lorsque l'ensemble est sans uranium.

En comparant les différents termes de $\lambda_u(B^2)$ avec l'expression (26) du chapitre A et tenant compte du fait que les expériences ont été faites à une température $T_e$ de $\sim 30^\circ C$, on a: $\tau = 320 \text{ cm}^2$; $B_0^2 = 1,20 \cdot 10^{-3} \text{ cm}^{-2}$.

$$\frac{v_0}{W_{00}} \sqrt{\frac{T_e}{T_0}} (\Sigma^b)_{00} = 914 \text{ s}^{-1}$$

$$\frac{v_0}{W_{00}} \sqrt{\frac{T_e}{T_0}} D_0 = 165 \cdot 10^3 \text{ cm}^2 \text{ s}^{-1}$$

$$\eta_{pf00} \frac{v_0}{W_{00}} \sqrt{\frac{T_e}{T_0}} (\Sigma^b)_{00} = 1645 \text{ s}^{-1}.$$ (8)

De ces relations on obtient immédiatement $(\Sigma^b)_{00} = 4,12 \cdot 10^{-3} \text{ cm}^{-1}$; $D_0 = 0,928 \text{ cm}$ et $\eta_{pf00} = 1,80 = k_\omega$.

Ces valeurs concordent très bien avec les valeurs calculées pour l'ensemble sous-critique en partant des caractéristiques nucléaires du graphite et des barres d'uranium.

Il faut remarquer quand même que la valeur de $\tau$ est beaucoup plus petite par rapport à la valeur de $\tau$ en graphite ($\tau \approx 360 \text{ cm}^2$).

D'ailleurs dans l'expérience de Campbell et Stelson on a obtenu pour l'eau $\tau = 21,2 \text{ cm}^2$ dans une interprétation similaire des résultats obtenus par la méthode de la source pulsée.

Étant donné que l'erreur dans l'approximation de points expérimentaux par une courbe de la forme (38) est plus grande ($>8\%$) pour $\lambda_u$ que pour $\lambda_m$ ($<1\%$) et étant donné la section d'absorption très grande des barreaux d'uranium la première approximation n'est plus satisfaisante pour interpréter les résultats expérimentaux.

Par une amélioration de l'appareillage expérimental, pour amener la précision jusqu'à celle que la méthode permet d'obtenir, on pourrait séparer les deux valeurs propres obtenues en deuxième approximation et obtenir ainsi des informations sur la modification du spectre maxwellien dans les réseaux multiplicatifs.

REMERCIEMENTS

Nous tenons à remercier ici M. l'Académicien HORI\HULUBEI pour les encouragements reçus pendant l'accomplissement de ce travail.
RÉFÉRENCES


SOME MEASUREMENTS OF THERMAL-NEUTRON SPECTRA *

M.J. POOLE, P. SCHOFIELD AND R. N. SINCLAIR
UNITED KINGDOM ATOMIC ENERGY AUTHORITY,
RESEARCH GROUP, ATOMIC ENERGY RESEARCH ESTABLISHMENT,
HARWELL, ENGLAND

Abstract — Résumé — Аннотация — Resumen

SOME MEASUREMENTS OF THERMAL-NEUTRON SPECTRA. Complementary programmes to determine moderator scattering law and to test its validity through spectrum measurements have been initiated at Harwell. The scattering-law experiments have been largely carried out at Chalk River, while the data processing is done at the Argonne National Laboratory and the analysis and necessary extrapolation from the measurements performed at Harwell. The spectrum measurements fall naturally into two parts. Using time-of-flight spectroscopy a wide range of measurements has been made of thermal neutron spectra in homogeneous poisoned moderators. This work parallels and extends the earlier work of the author and of Beyster et al. and serves to check the validity of energy transfer cross-sections $\sigma(E'\rightarrow E)$ derived from the scattering law in use. However such an experiment is completely insensitive to the angular dependence of scattering and to that part of the scattering cross-section involving no energy change of the scattered neutron, both of which are important in any spatially dependent problem. Accordingly other experiments have been undertaken in which spatial or thermal discontinuities were deliberately introduced to make the spectrum depend on the complete scattering law. The first such is the so-called "two block experiment" in which thermal neutrons are allowed to diffuse from a block of graphite at room temperature into a second block whose temperature may be raised to 400°C. Neutron spectra are measured at various positions near to the temperature discontinuity by extracting a beam of neutrons from each position and passing this into a chopper time-of-flight spectrometer. As a preliminary analysis "rethermalization cross-sections" have been derived from the experiment which may be compared with those of Bennet et al. who performed a similar experiment using energy sensitive detectors. In order to obtain a more detailed comparison, multigroup diffusion-theory calculations are being carried out, using the Chalk River scattering-law data, in an attempt to predict the shape of the energy spectrum at positions in both blocks.

A second experiment introduces a spatial discontinuity instead of thermal. A disc of cadmium sufficiently thick to be black to all neutrons below 0.4 eV is fixed centrally in a tank of water and spectra have been measured near to the disc as a function both of position and of direction of travel of the neutrons (relative to the plane of the cadmium). Finally may be mentioned spectrum measurements in a graphite-uranium lattice which provide a check on spectrum calculations in reactor geometry. Many spectrum experiments can be carried out in either critical or subcritical geometry, and in some discussion is given of the relative merits of those two methods of approach.

MESURE DE SPECTRES DE NEUTRONS THERMIQUES. Les auteurs ont entrepris à Harwell l’exécution de travaux complémentaires pour déterminer la loi de diffusion dans le ralentisseur et pour vérifier la validité de cette loi en procédant à des mesures du spectre. La plupart des expériences sur la loi de diffusion ont été faites à Chalk River (Canada); le traitement des données se fait au Laboratoire national d’Argonne (Etats-Unis), et l’analyse ainsi que l’extrapolation nécessaire des résultats des mesures se font à Harwell. Les mesures du spectre se divisent naturellement en deux groupes. En utilisant la spectroscopic du temps de vol, les auteurs ont fait un grand nombre de mesures de spectres de neutrons thérmiques dans des ralentisseurs empoisonnés homogènes. Ces travaux complètent ceux qui ont déjà été effectués par les auteurs et par Beyster et al.; ils servent à vérifier les valeurs des sections efficaces du transfert d’énergie $\sigma(E'\rightarrow E)$ déduites de la loi de diffusion en usage. Toutefois, cette expérience n’est nullement sensible aux variations angulaires de la diffusion ni à la partie de la section efficace de diffusion qui ne comporte pas de changement de l’énergie du neutron diffusé; or, ces deux facteurs ont leur importance lorsque l’espace intervient dans le problème. C’est pourquoi les auteurs ont fait d’autres expériences où ils ont introduit intentionnellement des solutions de continuité spatiales.

* Including work by D. H. DAY, D. B. GAYTHER, R. H. JONES and R. C. F. McLATCHIE.
ou thermiques, afin que le spectre varie selon la loi de diffusion dans sa totalité. La première de ces expériences est celle dite «expérience des deux blocs» où on laisse les neutrons thermiques se diffuser d'un bloc de graphite à la température ambiante dans un deuxième bloc dont la température peut être portée à 400°C. On mesure les spectres de neutrons dans diverses positions au voisinage de la solution de continuité de la température en prélevant un faisceau de neutrons dans chaque position et en le dirigeant vers un spectromètre de temps de vol à sélecteur. De cette expérience, une analyse préliminaire permet de déduire des «sections efficaces de rethermalisation» comparables à celles de Bennet et al. qui ont fait une expérience analogue en utilisant des détecteurs sensibles à l'énergie. Afin d'obtenir une comparaison plus détaillée, les auteurs ont procédé à des calculs selon une théorie de la diffusion à plusieurs groupes, en utilisant les données obtenues à Chalk River. Ces calculs devraient permettre de prévoir la forme du spectre de l'énergie dans certaines positions dans les deux blocs.

Dans la deuxième expérience, les auteurs ont introduit une solution de continuité spatiale et non plus une solution de continuité thermique. Ils ont placé un disque de cadmium suffisamment épais pour être imperméable à tous les neutrons au-dessous de 0,4 eV au centre d'un réservoir rempli d'eau, et mesuré les spectres au voisinage du disque tant en fonction de la position qu'en fonction du sens de déplacement des neutrons (par rapport au plan du disque de cadmium). Enfin, il convient de mentionner des mesures de spectres dans un réseau uranium-graphite qui permettent de vérifier les calculs de spectre dans la géométrie du réacteur. On peut effectuer de nombreuses expériences sur les spectres soit en géométrie critique soit en géométrie sous-critique. Les auteurs examinent les mérites respectifs de ces deux méthodes.

**NEKOTORYE IZMERENIYA SPECTROV TEPLOVYXH NEUTRONOV.** В Харуэлле начато осуществление дополнительных программ в целях определения закона рассеяния в замедлителе и проверки его действия в процессе измерений спектров. Опыты с законом рассеяния в основном проводились в Чок-Ривер, в то время как обработка данных производится в Аргонской национальной лаборатории, а анализ и необходимая экстраполяция на основе результатов измерений—в Харуэлле. Спектральные измерения обычно распадаются на две части. С помощью спектроскопических методов определения времени пролета на широком диапазоне проведены измерения спектров тепловых нейтронов в отравленных гомогенных замедлителях. Эта работа осуществляется параллельно и в продолжение более ранней работы автора, служит для проверки пригодности поперечных сечений перехода энергии α(E'-E), выводимых из применяемого закона рассеяния. Однако такой опыт абсолютно нечувствителен к угловой зависимости рассеяния и соответствующей части поперечных сечений рассеяния, не вызывающей обмен энергий с нейтроном рассеяния, причем оба эти момента играют важную роль в решении любой проблемы пространственной зависимости. Соответственно осуществлялись другие опыты, в которых нарочито вводились разрывы пространственной и тепловой непрерывности, чтобы поставить спектр в зависимость от полного закона рассеяния. Первым из таких опытов является так называемый "опыт с двумя блоками", в котором тепловые нейтроны диффундируют из графитового блока при комнатной температуре во второй блок, температура которого может подниматься до 400°C. Спектры нейтронов измеряются в различных положениях около разрыва температурной непрерывности путем выделения пучка нейтронов из каждого положения и пропускания его через спектрометр по времени пролета с селектором нейтронов. В качестве предварительного анализа из опыта, который может быть сравнен с аналогичными опытом Беннета и др., которые применяли дедекторы, чувствительные к энергии, выводится "поперечные сечения ретермализации". В целях получения более подробных сравнительных данных в настоящее время проводятся расчеты по многогрупповой теории диффузии с использованием полученных в Чок-Ривер данных по закону рассеяния с попыткой предугадать форму энергетического спектра в положениях в обоих блоках.

Во втором опыте вместо разрыва тепловой непрерывности вводится разрыв пространственной непрерывности. Кадмевый диск достаточной толщины, чтобы считаться "черным" для всех нейтронов с энергией меньше 0,4 эв, закрепляется в центре в баке с водой, и спектры измеряются около диска как функция обоях положений и направления перемещения нейтронов (относительно плоскости кадмевого диска). Наконец, можно упомянуть о спектральных измерениях в графито-урановой решетке, которая обеспечивает проверку расчетов спектров при геометрии реактора. Многие опыты по снятию спектров могут проводиться либо при критической, либо при подкритической геометрии. Обсуждаются относительные методы на основе этих двух методов подход к решению проблемы.

**ALGUNAS MEDICIONES DE ESPECTROS DE NEUTRONES TÉRMICOS.** Los autores han ejecutado programas complementarios con objeto de determinar la ley de dispersión del moderador y comprobar su validez por
THERMAL-NEUTRON SPECTRA

mediante de mediciones espectrales. La mayor parte de los experimentos sobre la ley de dispersión se realizaron en Chalk River y la elaboración de los datos, en el Laboratorio Nacional de Argonne; por otra parte, su análisis y las extrapolaciones necesarias se llevaron a cabo en Harwell. Las mediciones espectrales se dividen naturalmente en dos categorías. Mediante la espectroscopía de tiempo de vuelo, los autores efectuaron amplias mediciones de los espectros de neutrones térmicos en moderadores homogéneos envenenados. Estos trabajos, que complementan los realizados anteriormente por los autores, y Beyster y colaboradores, sirven para comprobar la validez de las secciones eficaces de transmisión de energía \( a(E^+ - E) \) deducidas de la ley de dispersión. Sin embargo, los resultados experimentales son totalmente independientes del ángulo de dispersión y de aquella parte de la sección eficaz de dispersión que no implique cambio de la energía del neutrón dispersado; ambos fenómenos son importantes en un problema en el que intervienen un factor espacial. Por ello, los autores efectuaron otros experimentos en los que introdujeron deliberadamente discontinuidades espaciales y térmicas a fin de que el espectro se rige por la ley de dispersión completa. En el primero de estos experimentos, que se denomina "experimento de dos bloques", los neutrones térmicos se dispersan a partir de un bloque de grafito a temperatura ambiente en un segundo bloque cuya temperatura puede llegar hasta los 400°C. Los espectros neutrónicos se midieron en diferentes posiciones cerca de la discontinuidad térmica extrayendo un haz neutrónico en cada posición y haciéndolo pasar por un espectrómetro de tiempo de vuelo dotado de selector. Este experimento, que puede compararse con los realizados por Bennet y colaboradores, quienes realizaron estudios análogos utilizando detectores sensibles a la energía, permitió deducir las "secciones eficaces de retermalización" con carácter preliminar. A fin de establecer comparaciones más detalladas, los autores están efectuando cálculos basados en la teoría de difusión de muchos grupos, utilizando los datos sobre la ley de dispersión obtenidos en Chalk River. Tratarán de predecir la forma del espectro energético en distintas posiciones en ambos bloques.

En un segundo experimento, han introducido una discontinuidad espacial en lugar de una térmica. En el centro de un tanque de agua fijaron un disco de cadmio de espesor suficiente para interceptar todos los neutrones de energía inferior a 0,4 eV y midieron los espectros cerca del disco en función de la posición y de la dirección de la trayectoria de los neutrones (respecto del plano del disco). Por último, mencionan las mediciones espectrales efectuadas en un reticulado de grafito-uranio, que permiten comprobar los cálculos en condiciones geométricas propias de los reactores. Pueden efectuarse muchos experimentos sobre el espectro en una geometría crítica o subcrítica; en la memoria se examinan las ventajas relativas de los dos métodos utilizados.

1. INTRODUCTION

The interpretation of critical and exponential experiments requires some knowledge of the neutron spectrum in the lattice under consideration. As long as interest was confined to large, well moderated systems (as, e.g., many graphite and heavy-water reactors are) and as long as calculation of the cold clean properties of the reactor were the outstanding problem, then it was possible to proceed by assuming that this spectrum consisted of a Maxwellian component more or less in thermal equilibrium with the moderator joined in some arbitrary manner to a "1/E" tail, which in turn was joined to the original fission spectrum. However, as soon as attention is turned to undermoderated systems, effects of spatial discontinuities or the behaviour of the temperature coefficient during the life of the reactor, then a more detailed and precise knowledge of the neutron spectrum is needed.

The energy and spatial variation of thermal neutrons in a reactor system can, in principle, be computed, provided that all the relevant cross-sections are known. Apart from the total cross-section for the nuclei present, the detailed scattering properties of the medium must also be known. The scattering cross-sections, which depend on the state of chemical binding of the atoms of the moderating material present, enter the transport equation as \( \Sigma(E, E', \mathcal{Q}, \mathcal{Q}') \) where \( E \) and \( E' \) are the initial and final neutron energies and...
\( \Omega, \Omega' \) is the cosine of the angle between their directions of motion \( \Omega \) and \( \Omega' \). It is only in the past few years that, on the one hand, data on these cross-sections have become available and, on the other, computer codes capable of producing adequate solutions of the transport equation for thermal neutrons in reactor systems have been developed.

The scattering cross-section can be expressed rigorously in terms of the scattering law \( S(\alpha \beta) \) \cite{1}, which for a given moderator at a given temperature depends only on the momentum change and the energy change of a neutron undergoing a collision. Even with this simplification, it is still not possible to measure with sufficient accuracy the scattering law for the complete range of variables; and it is necessary to rely heavily on theoretical methods of extrapolation. These methods are based on certain assumptions about the general form of the scattering law. Thus, it is extremely important that measurements of spectra be made to test the adequacy of the approximations used in the computation of the scattering law. Three classes of experiment may be distinguished:

(a) Experiments in large homogeneous systems, i.e. in systems where the loss of neutrons from any volume element by diffusion is very small compared with loss by capture and can be treated as a small correction. Such experiments are sensitive only to the energy-transfer kernel \( \Sigma(E \rightarrow E') \), the isotropic part of \( \Sigma(E, E', \Omega, \Omega') \). They are not sensitive to the energy dependence of the total scattering cross-section, which only enters through the leakage correction, or to the angular dependence of the scattering.

(b) Experiments in which the behaviour of spectra near plane boundaries is investigated. The purpose here is the introduction of spatial variation of the spectrum in a simple manner so that difficulties in handling the transport equation do not obscure the interpretation of the experiment. The spatial variation introduces a dependence of the spectrum on the energy dependence of the total cross-section and on the angular dependence of the scattering law. The former is known to be of predominant importance in the calculation of thermal utilization, while as yet very little is known about the importance of anisotropic scattering.

(c) Experiments designed to test some aspect of transport theory relevant to a particular reactor or to test the operation of a computer code written to calculate spectra in that reactor.

Measurement of spectra in homogeneous media have already been reported by one of the authors \cite{2} by BEYSTER \textit{et al.} \cite{3} and by BURKART \cite{4}. A further series of such experiments is reported later in this paper. Typical experiments of type (b), also reported in this paper, are the "two-block" experiment and that on the leakage spectrum from a water surface. In the former, two blocks of graphite maintained at different temperatures are mounted adjacent to one another with the cold block against one of the irradiation panels of the LIDO reactor. By means of a chopper spectrometer the neutron spectrum in the blocks is measured as a function of position relative to the interface between them. The latter experiment measures the spectrum of neutrons in water incident to a "black" boundary in the water as a function of the angle between the direction of travel of the neutrons and the plane of the "black" absorber.
In conclusion, some discussion is given on experiments that have been carried out in the United Kingdom for measurement of spectra in subcritical and critical systems.

2. THE SCATTERING LAW

The scattering laws used in calculations of spectra reported here are based on the incoherent Gaussian approximation. The details of the approximation have been described by EGGLESTAFF [5] and methods of computation of $\Sigma(E, E', q, \Omega')$ by EGGLESTAFF and SCHOFIELD [1]. With this approximation the cross-sections can be computed from a frequency distribution function $p(\beta)$, which is characteristic of the moderator at a given temperature, and can, in principle, be determined experimentally. Two computer programmes exist which evaluate the required cross-sections, given $p(\beta)$. The programme LEAP written by McLatchie evaluates the scattering law, and the programme PIXSE written by MacDougall evaluates the "isotropic cross-sections" $\Sigma(E' \rightarrow E)$ and also produces the total cross-section $\Sigma(E)$ and group-averaged cross-sections for conventional multigroup calculations.

We must summarize here the models which have been used in calculations reported in this paper.

For each model two parameters $M$ and $T$ are quoted. These are weighted averages of $p(\beta)$ which give some indication of the moderating properties of the model [6]. $T$ is two thirds of the mean kinetic energy of the atoms and determines the mean energy loss per collision for high-energy neutrons, while $M$ is related to the rate of energy exchange for neutrons nearly in thermal equilibrium with the moderator. Explicitly,

$$
\overline{T} = \int_0^\infty \{ p(\beta) \cdot \beta/2 \coth \beta/2 \} \, d\beta 
$$

$$
\overline{M} = \int_0^\infty \{ p(\beta) \cdot (\beta/4 \sinh \beta/2) \cdot (\beta/2)^2 K_2(\beta/2) \} \, d\beta, 
$$

where $K_2(x)$ is the modified Bessel function. For a monatomic gas (corresponding to $p(\beta) = \delta(\beta)$), $T = M = 1$.

(a) Water

Apart from calculations with the free-gas model with $A = 1$, spectra calculated with two realistic models for water are presented in this paper. The first of these is given by a $p(\beta)$ due to Egelstaff (see Fig. 1). For $\beta < 8$ this is obtained directly from experiment and includes those frequencies arising from hindered rotations of the molecule. The remainder of the continuous curve represents the band of vibrational frequencies observed at around 0.2 eV, and the delta function at $\beta = 20$ represents the highest vibrational bands. The other model is that of NELKIN [7], in which the hindered rotation as well as the vibrational bands are represented by $\delta$-functions.

For the Egelstaff model one finds $T = 6.19$, $M = 0.18$, while for Nelkin $T = 4.64$, $M = 0.32$. Thus Nelkin water is a better moderator than Egelstaff water, mainly because of the high weight Egelstaff ascribes to the high-frequency mode.
Fig. 1

The frequency distribution for room-temperature water

The continuous curve and delta function is the Egelstaff model, and the dashed delta functions correspond to the Nelkin model. $\beta$ is the frequency in units of $k_BT/\hbar$.

(b) Graphite

The frequency distribution used for the graphite calculations is shown in Fig. 2. This is derived from experimental measurements at Chalk River for room-temperature graphite. For this model $T = 2.03, M = 0.46$. These values may be compared with those from the distribution of Yoshimori and Kitano, which has been used extensively in the United States for calculations of spectra [8]. These are $T = 2.35, M = 0.31$.

(c) Heavy water

No calculations for heavy water have yet been performed with any but the monatomic gas model.

3. CALCULATION OF SPECTRA IN HOMOGENEOUS MEDIA

Calculation of the spectrum in a homogeneous medium involves solution of the neutron balance equation.

$$[\Sigma_d(E) + \Sigma_a(E)] \phi(E) = \int_0^1 \Sigma(E' \to E) \phi(E')dE' + S(E) \tag{3.1}$$

for any set of values of $\Sigma(E' \to E)$ and $\Sigma_d(E)$. ($\Sigma_d(E)$ is the macroscopic capture cross-section.) As in practice the experiments are rarely a sufficiently good approximation to infinite media for leakage to be ignored en-
The frequency distribution for room-temperature graphite entirely; this is taken into account by the addition of a term \( D(E) \nabla^2 \phi/\phi \) (here \( d(E) \) is the diffusion coefficient) to the LHS to give

\[
\left[ \Sigma_a(E) + D(E) \left( \nabla^2 \phi/\phi \right) + \Sigma_t(E) \right] \phi(E) = \int_0^\infty \Sigma(E' \rightarrow E) \phi(E') dE' + S(E). \tag{3.2}
\]

Finally, if the source is of high-energy neutrons, we can write for \( E < E_m \)

\[
\left[ \Sigma_a(E) + D(E) \left( \nabla^2 \phi/\phi \right) + \Sigma_t(E) \right] \phi(E) = \int_0^{E_m} \Sigma(E' \rightarrow E) \phi(E') dE' + \int_{E_m}^{\infty} \Sigma(E' \rightarrow E) \phi(E') dE', \tag{3.3}
\]

where \( E_m \) is an energy high enough that up-scattering can be ignored and \( \phi(E') \) is assumed to be known from simple slowing-down theory, from an experiment or from the CORNGOLD asymptotic expansion [9].

The ratio \( \nabla^2 \phi/\phi \) is assumed to be independent of energy, and its value at the point of measurement is determined from experimental flux plots. \( \phi'(E) \) has a form which lies between \( 1/E \) and \( (1/E)^{0.9} \). It is sufficiently accurate to use \( 1/E \), except in the most highly poisoned cases when the appropriate power for the \( 1/E \) term was obtained from the experimental spectra.

A programme (called SIMPH) has been written for the IBM STRETCH computer for the solution of equation (3.3) by an iteration method [10]. Values of \( \Sigma(E' \rightarrow E) \) are obtained by the use of the programmes LEAP and PIXSE, and \( \Sigma_t(E) \) is taken from BNL 325. Where the absorber is introduced in the form of thin sheets, corrections to \( \Sigma_a(E) \) are necessary for self-shielding and for fine structure in the assembly. The self-shielding correction was calculated with the assumption of isotropic flux incident on the absorber sheet and ignoring scattering in the absorber. In all cases the correction lay between 0.9 and 1.0. Fine structure in periodic slab geometry was ob-
tained from the programme RIPPLE [11]; but, as for the cases presented it produced a correction of only 2% in $E_a(E)$, its effect was ignored.

4. SPECTRA IN POISONED HOMOGENEOUS MEDIA

(a) Experimental method

Neutron spectra in homogeneously poisoned moderators have been measured by time-of-flight with the method first described by one of the authors [2]. Fig. 3 shows the experimental arrangement. The poisoned moderator is mounted adjacent to (or in some cases immediately above) the target of the electron linac, which irradiates it with pulses of fast neutrons $2 \mu s$ in duration, at a rate of either 12 or 50 pulses/s. A beam of thermal neutrons emerges from a re-entrant hole penetrating the moderator and, after definition by a collimator, enters a flight tube. The velocity spectrum of neutrons in this beam is then determined by measurement of their flight time to a bank of boron trifluoride counters placed either 11 m or 60 m from the source of the beam. The length-of-flight tube in use is selected so that the decay time in the moderator (which determines the effective pulse duration for the thermal neutrons) is small compared with the flight time. A correction is necessary for the mean delay of neutrons emerging from the system.

The position described for the re-entrant hole relative to the target ensures that the neutron current at the end of this hole is normal to the axis of the flight tube. The relation between the spectrum of neutrons in the extracted beam and the flux in the medium is to a first approximation given by

$$\psi(O, \mu, E) = \frac{1}{2} \phi(O, E) \left( 1 + \frac{\lambda(E)}{\phi(E)} \frac{\partial \phi}{\partial z} \right)_{O,E},$$

(4.1)

where $\psi(O, \mu, E)$ is the spectrum of neutrons in a beam extracted along the
z-axis from position $z = 0$ at an angle $\cos^{-1} \mu$ to the direction of the neutron current.

For determination of the leakage from the system, a flux plot is made with manganese foil detectors. The results of such a plot are shown in Fig. 4. (Here the x-direction is the line joining the end of the probe tube to the source, and the neutron beam emerges in the z-direction.) From this plot a value of $\nabla^2 \phi / \phi$ is obtained. Strictly speaking, this only refers to the mean energy at which neutrons are absorbed in the detector; however, provided the dimensions of the system are always large compared with $\lambda_0(E)$, little error is introduced by the assumption that $\nabla^2 \phi / \phi$ is independent of energy.

Spectra in poisoned water, heavy water and graphite have all been measured as a function of absorber concentration and, in the case of water, of
temperature (see Table I). Solutions of a cadmium salt or of a mixture of boric acid with borax were used in the case of water and heavy water. These were each contained in a tank of suitable size fitted with a re-entrant probe tube penetrating to the centre and closed at this point by a thin polythene or aluminium window. In the case of graphite moderator, a well defined homogeneous mixture of moderator and poison is difficult to produce, so graphite slabs 1.9-cm thick interleaved with thin plates of absorber were used. This absorber consisted either of copper sheets or of cadmium-plated steel. The stack was mounted with the planes of the slabs horizontal, and the re-entrant hole used for extraction of a neutron beam entered with its axis parallel to these planes had a height equal to that of three layers and a width of 4.4 cm. The whole stack comprised a 57-cm cube.

(b) Results

Spectra have been measured in water poisoned with boron and cadmium. For each case the spectrum to be expected has been calculated by the method of section 3 with both the perfect gas-scattering law and a scattering law derived from the Chalk River experiments [12]. Some calculated spectra involving the use of the Nelkin model are also available*. Figs. 5 to 7 show the measured and calculated spectra. It is immediately obvious that either of the bound hydrogen models yields a better representation of the spectrum than does the free-gas model. Figs. 8 and 9 give the results of experiments in graphite poisoned with copper and cadmium respectively. On the same graphs are plotted the results of calculations with the scattering kernel given by EGELSTAFF [16]. The agreement is satisfactory particularly in view of the conclusions to be presented in section 6.

CORNGOLD [9] has shown that in an infinite medium with a 1/V absorber the spectrum is asymptotic at high energies to a series of the form

\[ E \phi(E) = 1 - AE^{1} + BE^{1} - CE^{3/2} + \ldots, \]

where A has shown the value 3\(\alpha(\Theta)/\Theta^{2}\) and B is \[6T(\sigma a/\sigma_{e})^{2} + T\] for water. In this expression (3/2) \(T\) is the mean vibrational energy of the moderator atoms and \(\sigma_{e}\) is the free-atom scattering cross-section. Thus it is possible to determine \(T/\Theta\) from measurements of the neutron spectrum by making a polynomial fit to the epithermal region \((E > 0.3 \text{ eV})\). This has been done in the case of water to give \((T/\Theta) = (3.9 \pm 0.4)\). \(T/\Theta\) can also be deduced from the scattering law or rather from the phonon spectrum used to derive the scattering law. For the Egelstaff model \((T/\Theta) = 6.19\) and for the Nelkin model \((T/\Theta) = 4.64\), while on the gas model \((T/\Theta) = 1\). Fig. 10 shows the measured epithermal spectra, together with curves corresponding to \((T/\Theta) = 1, 4.7\) and 3.9. The curve derived from the scattering-law parameters contains the first five terms of the polynomial. The most accurate value of \((T/\Theta)\) that could be derived from the data was obtained by making a best fit to the results at sufficiently high energies and using only the first three terms of the polynomial. The curve at \((T/\Theta) = 4.7\) corresponds to a variation of twice the quoted error. The curves show that significant chemical binding effects extend to 5.0 eV in room-temperature water.

* The authors are indebted to R. Beyster, General Atomics, for these spectra.
<table>
<thead>
<tr>
<th>Moderator</th>
<th>Moderator temperature (T°K)</th>
<th>Absorber</th>
<th>No. of absorbing atoms/moderating atom</th>
<th>Relaxation time at 0.0253 eV (μs)</th>
<th>Flight-path length at 0.0253 eV (m)</th>
<th>Channel width at 0.0253 eV (μs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂O</td>
<td>298</td>
<td>Cd</td>
<td>1.98 x 10⁻³</td>
<td>13</td>
<td>11.54</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>Cd</td>
<td>1.98 x 10⁻³</td>
<td>13</td>
<td>11.54</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>363</td>
<td>Cd</td>
<td>1.98 x 10⁻³</td>
<td>13</td>
<td>11.54</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>298</td>
<td>Cd</td>
<td>2.25 x 10⁻³</td>
<td>11</td>
<td>11.54</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>298</td>
<td>Cd</td>
<td>2.51 x 10⁻³</td>
<td>10</td>
<td>11.54</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>298</td>
<td>Cd</td>
<td>4.61 x 10⁻³</td>
<td>6</td>
<td>11.54</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>298</td>
<td>Cd</td>
<td>4.76 x 10⁻³</td>
<td>5</td>
<td>11.54</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>298</td>
<td>B</td>
<td>9.12 x 10⁻³</td>
<td>10</td>
<td>11.54</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>328</td>
<td>B</td>
<td>9.12 x 10⁻³</td>
<td>10</td>
<td>11.54</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>361</td>
<td>B</td>
<td>9.12 x 10⁻³</td>
<td>10</td>
<td>11.54</td>
<td>100</td>
</tr>
<tr>
<td>D₂O</td>
<td>293</td>
<td>B</td>
<td>3.22 x 10⁻⁸</td>
<td>55</td>
<td>60.32</td>
<td>800</td>
</tr>
<tr>
<td>C</td>
<td>290</td>
<td>Cu</td>
<td>2.82 x 10⁻⁸</td>
<td>280</td>
<td>58.14</td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>290</td>
<td>Cu</td>
<td>5.63 x 10⁻⁸</td>
<td>180</td>
<td>58.14</td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>290</td>
<td>Cu</td>
<td>8.81 x 10⁻⁸</td>
<td>130</td>
<td>58.14</td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>290</td>
<td>Cu</td>
<td>1.16 x 10⁻¹</td>
<td>110</td>
<td>11.54</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>290</td>
<td>Cu</td>
<td>1.44 x 10⁻¹</td>
<td>70</td>
<td>58.14</td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>290</td>
<td>Cd</td>
<td>4.90 x 10⁻⁵</td>
<td>280</td>
<td>58.14</td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>290</td>
<td>Cd</td>
<td>9.39 x 10⁻⁵</td>
<td>180</td>
<td>58.14</td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>290</td>
<td>Cd</td>
<td>1.42 x 10⁻⁴</td>
<td>130</td>
<td>58.14</td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>290</td>
<td>Cd</td>
<td>1.84 x 10⁻⁴</td>
<td>110</td>
<td>58.14</td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>290</td>
<td>Cd</td>
<td>4.85 x 10⁻⁴</td>
<td>40</td>
<td>11.54</td>
<td>100</td>
</tr>
</tbody>
</table>

* atoms/D₂O mol
Experimental and calculated spectra for highly poisoned water at various temperatures

Free gas
Nelkin model
Scattering law

Moderator: H₂O
Poison: Boron
C₀ 7x10 b/H atom
Tank size: 23-in diameter
Window: 0.007-in steel

5. SPECTRA NEAR PLANE BOUNDARIES

(a) The two-block experiment

The two-block experiment is a measurement of the neutron spectrum near to a temperature discontinuity in graphite. Ideally, the problem is that of two semi-infinite non-absorbing half-spaces at temperatures T₁ and T₂ separated by a plane boundary, with a uniform plane source of neutrons
in thermal equilibrium at a large distance from the boundary in one medium. The neutron spectrum is to be determined as a function of position on both sides of the boundary. Under these conditions the total flux is independent of position [14]. In practice, of course, the blocks must be finite in extent and, further, have to be separated by some form of thermal insulator at the interface (see Fig. 11). Provided the blocks are sufficiently large, the total flux should have a cosine distribution in the direction parallel to the edges of the boundary (they-and z-directions of Fig. 9, the origin being at the centre of the interface). If this is the case, and it has been verified experimentally, then the lateral leakage can be allowed for as an effective absorption cross-section $D(E) \{B_E^2 + B_p^2\}$ and the problem reduced to one-space dimension.
A computer programme has been written to solve this problem in the diffusion approximation. At the present stage of the calculation, the effect of the thermal insulation is neglected.

The apparatus designed to achieve the desired conditions is shown in Fig. 11. A block of graphite A, 143 cm X 145 cm X 71 cm, was positioned against one of the irradiation panels of the LIDO reactor. This block remained at room temperature and provided the region at temperature $T_1$. Separated from this by a distance of 4 cm was a second block 143 cm X 145 cm X 142 cm, which was thermally insulated and could be heated to the chosen temperature $T_2$. Between the two stacks were two layers of crumpled aluminium foil and a channel for cooling air. Preliminary tests showed that the temperature of the hot block was uniform to within 5°C with this arrangement. The
Fig. 8
Measured and calculated spectra in graphite poisoned with a \( \frac{1}{v} \) absorber (copper)

Moderator: 22.5-in graphite cube
Poison: Copper (\( \frac{1}{v} \) absorber)
Temperature: 290°K
--- --- Scattering law prediction

Fig. 9
Spectra in graphite poisoned with a resonant absorber (cadmium)

Moderator: 22.5-in graphite cube
Poison: Cadmium
Temperature: 290°K
--- --- --- Scattering law prediction
Fig. 10
The epithermal spectrum in water

\( \langle \sigma_0 \rangle = 1.01 \text{ b/H atom} \)

- dashed, \( \bar{T} = 4.7 \)
- dot-dashed, free gas, \( \bar{T} = 1 \)
- solid, polynomial fit to experiment, \( \bar{T} = 3.9 \pm 0.4 \)

Fig. 11
The two-block experiment
Fig. 12
Neutron spectra near to the join between graphite at 250°C and 35°C

(a) In 250°C, graphite 32 cm from the interface
(b) 35°C
(c) In 35°C, graphite 2 cm from the interface

The solid lines are Maxwellian distributions with maxima corresponding to the following temperatures:
(a) 250°C  (b) 147°C  (c) 76°C

amount of aluminium in the gap amounted to only 1.35 g/cm². On the outer surface, both blocks were covered with boron glass tiles, which gave well defined boundary conditions for the neutron distribution in the blocks.

Near this thermal barrier a wide slot was cut in each block, penetrating from one side to the centre. This slot was filled by a series of narrow graphite stringers, and by withdrawal of one of these a channel could be opened leading to any desired position on the x-axis of the block. This produced a beam of neutrons emerging parallel to the z-axis, which, after additional collimation, passed into a chopper spectrometer. The spectro-
meter was of conventional design, had a resolution ~5 μs/m and could be positioned to receive the beam from any of the possible channel positions (up to x = ±33 cm).

Chopper measurements were supplemented by foil activities with manganese gold alloy and lutetium–manganese mixture foils. (The former gave the spatial flux distribution.) This was shown to be well represented by a cosine term in each of the y- and z-directions and to be exponential along the axis with the same relaxation length in each block within the limits of experimental error. From the lutetium–manganese activation ratio it was possible to derive the temperature of the neutrons T_n (assuming a Maxwellian velocity distribution), and T_n is plotted as a function of x in Fig. 13.

Fig. 13

The variation of the effective neutron temperature near to a temperature discontinuity in graphite

+ From lutetium and manganese activation
\( \times \) From neutron energy spectra measured by time-of-flight.
\( \circ \) The indicated spreads result partly from inaccuracies of measurement and partly from the fact that the spectra are not exactly Maxwellian distributions.

Chopper runs have been made for \( T_2 = 100^\circ C, 175^\circ C \) and \( 250^\circ C \). Typical spectra are shown in Fig. 12. Up to the highest value of \( T_2 \) used, departures from the Maxwellian shape are small, and it has therefore been possible to characterize the spectra by a neutron temperature \( T_n \). Values of \( T_n \) obtained from all the chopper runs are shown in Fig. 13.

Preliminary analysis of these results has been made by means of diffusion theory and postulating two overlapping thermal groups with Maxwellian distribution of velocities corresponding to the temperature of each stack. If \( \Sigma(2 \rightarrow 1) \) is the rethermalization cross-section for neutrons of temperature \( T_2 \) diffusing in graphite at temperatures \( T_1 \) and \( \Sigma(1 \rightarrow 2) \) that for neutrons of temperature \( T_1 \) in graphite of temperature \( T_2 \), the following equations can be written for neutrons diffusing in the block at \( T_1 \):
THERMAL-NEUTRON SPECTRA

\[ D \nabla^2 \phi_1 - \Sigma_a \phi_1 + \Sigma_{(2\rightarrow1)} \phi_2 = 0 \]

\[ D \nabla^2 \phi_2 - [ \Sigma_a + \Sigma_{(1\rightarrow)} ] \phi_2 = 0, \]

where \( \phi_1 \) and \( \phi_2 \) are the fluxes of neutrons in groups 1 and 2, \( D \) is the diffusion coefficient and \( \Sigma_a \) the macroscopic absorption cross-section for the graphite. Similar equations involving \( \Sigma(1\rightarrow2) \) may be written for the hot block. A neutron temperature \( T_n \) is defined equal to the flux weighted mean of \( T_1 \) and \( T_2 \). This is a function only of the unknowns \( \Sigma(1\rightarrow2) \) and \( \Sigma(2\rightarrow1) \) which can thus be derived from the measured values of \( T_n \). Table II gives

### Table II

<table>
<thead>
<tr>
<th>( T_1 ) (°K)</th>
<th>( T_2 ) (°K)</th>
<th>( \Sigma(1\rightarrow2) \times 10^3 \text{cm}^{-1} )</th>
<th>( \Sigma(2\rightarrow1) \times 10^3 \text{cm}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>303</td>
<td>372</td>
<td>19 ± 5</td>
<td>12 ± 5</td>
</tr>
<tr>
<td>296</td>
<td>405</td>
<td>18 ± 5</td>
<td>30 ± 10</td>
</tr>
<tr>
<td>293</td>
<td>473</td>
<td>23 ± 8</td>
<td>25 ± 5</td>
</tr>
<tr>
<td>301</td>
<td>523</td>
<td>19 ± 6</td>
<td>27 ± 5</td>
</tr>
<tr>
<td>305</td>
<td>609</td>
<td>21 ± 5</td>
<td>30 ± 5</td>
</tr>
<tr>
<td>308</td>
<td>448</td>
<td>18 ± 3</td>
<td>-</td>
</tr>
<tr>
<td>308</td>
<td>523</td>
<td>26 ± 6</td>
<td>-</td>
</tr>
</tbody>
</table>

the rethermalization cross-sections for cases where sufficient data is already available.

(b) Spectrum incident on a plane absorbing interface

With the technique already described for homogeneous systems, thermal-neutron spectra have been measured near to a sheet of cadmium immersed in water, both as a function of the angle between the direction of motion of the neutron and the plane of the surface and as a function of the normal distance from the absorber. Fig. 14 illustrates the experimental arrangement; 14(a) is the ideal geometry required, while 14(b) shows how this is realized in practice. A cadmium disc 2-mm thick and 12 cm in radius forms the perfectly absorbing surface. This is immersed in a cubical tank of 56-cm side, and a beam of neutrons is extracted through a hole in the cadmium by means of a probe tube and aluminium window. The walls of the probe tube are lined with cadmium to minimize the perturbing effect of the hole in the cadmium disc. The cadmium disc may be considered completely opaque to all neutrons having less than 0.2-eV energy; thus, up to this
energy the required conditions of the experiment are satisfied. Over 2.0 eV the cadmium is completely transparent and will have no effect on the neutron distribution. This effect is utilized for normalization of the observed spectra to each other without the use of a separate flux monitor. (To be independent of small but troublesome variations in source position caused by beam drift in the accelerator this would have to be located at the probe tube window and would destroy the conditions required for the experiment.) At intermediate energies the cadmium is partially transparent and this part of the spectrum was discarded.

Neutron spectra have been measured at $x = 0$ with $\phi = 90^\circ$, $60^\circ$, $45^\circ$ and $30^\circ$ and at $\phi = 90^\circ$ for various values of $x$ up to 4 cm. These spectra are shown in Figs. 15 and 16. A separate experiment has been made for $\phi = 0$. The spectrum of neutrons grazing the surface of the water was observed, and this was very close to a Maxwellian distribution in equilibrium with the water. As this spectrum was not normalized in intensity relative to those shown in Fig. 15, it has not been included in the figure. Considering first the spectra measured for different angles at $x = 0$, one can see that the mean energy of the thermal group is in general greater than $\frac{3}{2}kT_m$ ($T_m =$ moderator temperature).

Now, if all scattering in the moderator is assumed to be isotropic, it is simple to show that
Fig. 15

The angular dependence of spectra at cadmium surface in water
Angle between neutron direction and plane of cadmium disc ($\phi$)

- $\bullet$ 90°C
- $\times$ 60°C
- $\triangle$ 45°C
- $\circ$ 30°C

$\psi(0, -\mu, E) = \frac{1}{\mu} \psi(0, -1, E) + \frac{1}{\mu l(E)} (1 - \frac{1}{\mu}) \int_0^1 \psi(x, -1, E)e^{-x/\mu l(E)} \, dx$, (5.2)

from which we can derive

$\psi(0, 0, E) = \psi(0, -1, E) - l(E) \frac{\partial}{\partial x} \psi(x, -1, E) \bigg|_{x=0}$, (5.3)

where $\psi(x, \mu, E)dEdu$ is the number of neutrons between E and E+dE having angles to the normal between $\cos^{-1} \mu$ and $\cos^{-1}(\mu + du)$ passing through unit area at a distance x from the origin, and l(E) is the scattering mean-free path in water. Spectra derived from (5.3) are compared with the measured spectra in Fig. 17. It can at once be seen that (5.3) is not satisfied by the experimentally measured spectra. The only assumption made in the deviation of (5.3) was that of isotropic scattering. From this disagreement we may deduce that angular effects in the scattering law are in fact of importance in the determination of the spectrum at a surface.
Fig. 16
The spatial dependence of spectra near to a cadmium surface in water
Neutron flux as function of normal distance (x) from cadmium disc (φ = 90°)

Fig. 17
The spectra measured in a water surface $\Psi(0, 0, E)$ and the spectrum predicted
with the assumption of isotropic scattering.
6. SPECTRA IN REACTOR LATTICES

The third class of experiment is intended to verify a particular reactor physics calculation, usually the spectrum in a reactor. Such experiments were among the earliest time-of-flight spectrum measurements. Both subcritical assemblies and zero-energy criticaals have been used for this purpose, and it is useful to consider when either is the most appropriate. Discussion will be confined to the specific case of chopper measurements with a beam extracted from the assembly. A critical assembly will always reproduce the conditions in a reactor more exactly than will a subcritical; and, other things being equal, there would be no advantage in using a subcritical if a critical assembly were equally easily available. However, there is always difficulty in extracting a representative beam from a heterogeneous system, and the additional complication necessary in a critical experiment further limits the positions and directions in which this can be done. In addition, a critical experiment is usually considerably more expensive than is a subcritical, with the consequence that, once built, it will not be used for spectrum measurements only. This leads to undesirable compromises in the design, and the flux in such a zero-energy reactor is often really inadequate for chopper measurement. Thus, where possible, the use of a subcritical assembly reserved for spectrum measurements is to be preferred.

The spectrum in any region of a heterogeneous system depends both on the absorption and on the net gain or loss of neutrons by diffusion in that region. This latter may at least qualitatively be divided into local diffusion caused by the fine structure of the flux distribution and leakage from the assembly as a whole. Only the latter will differ between a critical and subcritical system. Thus the use of a subcritical assembly for spectrum measurements is permissible in large systems where the spectrum is not significantly affected by overall leakage, provided that the subcritical assembly is itself sufficiently large that any additional leakage introduced does not modify the spectrum. The large, well moderated graphite and heavy-water reactors can all be investigated by this method. On the other hand, if one considers the highly undermoderated enriched systems (of which HTGR is an example), then the size of a subcritical would approach that of a critical experiment before a characteristic spectrum was obtained, and most of its advantages would be lost.

Experiments using both critical and subcritical assemblies have been made, and the results are available in the literature. COATES et al. [15] has measured spectra in a subcritical section of a Calder Hall lattice (graphite-natural uranium). A neutron beam was extracted from a position in the moderator half way between two fuel rods and the spectrum measured with a chopper. This assembly was designed specifically for spectrum measurements; it can be loaded with fuel on several pitches between 5 in and 11 in; beams can be extracted from various positions in the lattice cell, and it is possible to heat the whole assembly up to 350°C. MACDOUGAL [16] has compared the spectra obtained with theoretical spectra calculated by means of the Winfrith DSN code. Scattering data for graphite (obtained from the LEAP and PIXSE codes) correspond either to a scattering law proposed by Egelstaff or to the monatomic gas scattering. In all cases Macdougal
finds closer agreement of the experiment with the spectra calculated by means of the realistic scattering law.

Chopper measurements have also been made of the neutron spectrum in the ZENITH reactor [17]. In this case the core was undermoderated and enriched, and consequently measurements had to be made on the critical system because of the design of the reactor. This involved accepting quite stringent limitations on the positions from which a beam could be extracted (the spectrum observed was averaged over the end of a fuel rod) and, because of the relatively low flux at which the reactor operated, on the resolution of the chopper. Nevertheless, results were obtained which Wikner (unpublished) has shown to be in substantial agreement with spectra calculated with a bound atom model to predict the scattering law for graphite. MACDOUGAL, in the paper quoted and in a later paper [18] also demonstrates the sensitivity of reactor parameters to the scattering law and concludes that in the case of the Calder lattice the choice of scattering law is only just significant, the use of a monatomic gas law giving only 0.5% change in reactivity and 20% change in the bulk moderator temperature coefficient. The residual discrepancies between calculated and theoretical spectra are therefore unimportant to the reactor designer. Similar conclusions are reached from the available measurements on the ZENITH cases. However, no such analysis exists for other moderators, and this conclusion must, at least for the time being, be taken as specific to graphite.

REFERENCES


DISCUSSION

F. FEINER: Could you comment on the possibility of making position-dependent spectrum measurements?
M.J. POOLE: A proper discussion of the question of position-dependent spectrum measurements would be a lengthy one. In a complex fuel element such as that for the SGHW, for example, such measurements would be impossible without perturbing the system to an extent rendering the results meaningless. At the other extreme, measurements as a function of position in the moderator of a widely spaced rod or slab lattice are certainly possible. Measurements are also possible along the axis of a cylindrical fuel bar, but as this is a rather special direction in the lattice it may not be easy to relate this single measurement to, for example, the direction-averaged flux spectrum. Takahashi has recently been considering the theory of this very problem in slab lattices with some success. One thing is clear: it is very important that theoretical calculations of what is actually measured should be made to provide a proper basis for comparison with experiment.

B. VITTOZ: Since the phonon spectrum is determined, how do you go about expressing the neutron-phonon scattering cross-section?

M.J. POOLE: The neutron cross-sections are deduced from the phonon spectrum by the method of Schofield and Egelstaff, which makes use of the fact that the most important part of the scattering law is given by the double Fourier transform of the self-correlation function of Van Hove. This is represented as a Gaussian distribution, whose width is a function of time, and the second differential of this width with respect to time is the velocity correlation function for the atoms of the system. Recalculation is carried out by use of the programmes STEP and PIXSE, which together yield the scattering matrix.

B. VITTOZ: What effect do the dimensions of the vessel exert on the experimental or theoretical results?

M.J. POOLE: The dimensions of the vessel are important in determining the leakage of the system, and it is essential that the leakage is properly included in the computations when comparing experimental with theoretical spectra. The method by which this is done is described in the paper, and it will be noted that the source position has as much influence as the actual dimensions of the system.

J. WALKER: Does the heterogeneous character of the poison used in the graphite experiment raise problems in the spectral measurements or in their analysis?

M.J. POOLE: Yes. There are two effects. The first is the self-shielding in the absorber sheets, which may amount to about 10% at particular energies. This has been dealt with so far by a simple calculation which makes no allowance for the scattering in the absorber. The second effect is due to fine flux structure in the 2-cm-spaced slab lattice. This has been calculated by the RIPPLE programme (a one-energy group collision probability method), and for the cases so far considered this has indicated a correction of less than 2% to the effective absorption cross-section used in the SIMPH programme. The effect has therefore been neglected.

W. C. REDMAN: You have indicated that because of flux level or power level limitations in ZENITH you had to average the spectrum over a typical lattice cell. I do not find the curves reproduced in the paper, and so I should like to ask what was the flux at the measuring position, or, alternatively, what was the power level in the core?
M. J. POOLE: The flux was about $10^8$ n/cm$^2$ s. When the chopper was designed and the experiments performed we thought we would have difficulty in getting any spectrum at all unless we averaged over the whole 3-in cell, because $10^8$ is quite a low flux for choppers. The count rate actually proved to be higher than anticipated and the experiments took less time; I think it would be possible now to look over about half the diameter of the cell and get much more of the fuel effect, but of course there are limitations of time.

W. C. REDMAN: I am impressed to learn that the effect of the resonances in the plutonium isotopes was visible; perhaps you are understating your ability to measure good spectra in critical assemblies.

D. BRETON: How long does it take to make a spectral measurement with a flux of $10^8$ n/cm$^2$ s?

M. J. POOLE: A run would take about 12 h; it's difficult to generalize without specifying the accuracy and resolution required.
SUBCRITICAL EXPERIMENTAL WORK AT THE DELFT REACTOR INSTITUTE

H R KLEIN, A W VAN DER HEIDEN AND H. VAN DAM
REACTOR INSTITUTE, DELFT, THE NETHERLANDS

Abstract — Résumé — Аннотация — Resumen

SUBCRITICAL EXPERIMENTAL WORK AT THE DELFT REACTOR INSTITUTE. After a brief discussion of the programme of the reactor physics division of the Reactor Institute, water and graphite-moderated subcritical assemblies are described. The subcritical experimental theory is summarized by giving the basic equations. Further corrections are mentioned, which are to be added to the experimentally determined neutron distribution for material buckling determinations. Special emphasis is placed on the fact that when the neutron source used is placed in a pedestal the assembly sees the source in a fictitious position. Methods are given to determine this fictitious source position.

TRAVAUX CONCERNANT LES EXPERIENCES SOUS-CRITIQUES A L'INSTITUT DES REACTEURS DE DELFT. Après avoir brièvement examiné le programme de la Division de physique des réacteurs, l'auteur décrit les assemblages sous-critiques ralentis à l'eau et au graphite du Reactor Institut. Il résume la théorie des expériences sous-critiques en donnant les équations fondamentales. Il indique d'autres corrections qu'il faut apporter à la distribution des neutrons établie par voie expérimentale pour déterminer le laplacien matière. Il insiste sur le fait que, lorsque la source de neutrons utilisée se trouve placée dans un support, elle est vue de l'assemblage dans une position fictive. Il indique par quelle méthode on peut déterminer cette position fictive de la source.

ПОДКРИТИЧЕСКИЕ ЭКСПЕРИМЕНТАЛЬНЫЕ РАБОТЫ В РЕАКТОРНОМ ИНСТИТУТЕ В ДЕЛФТЕ. После краткого обсуждения программы отделов физики реакторов описываются подкритические сборки с графитовым замедлителем. В результате выведения основных уравнений обобщается теория подкритических экспериментов. Упоминаются новые исправления, которые необходимо внести в данные о распределении нейтронов для определения материального лапласиана, полученные опытным путем. Особый упор делается на тот факт, что при размещении используемого источника нейтронов на пьедестале со стороны сборки источник представляется в неправильном положении. Приводятся методы по определению такого неправильного положения источника.

EXPERIMENTOS SUBCRÍTICOS EN EL INSTITUTO DE REACTORES DE DELFT. Después de un breve examen del programa de la División de física de reactores, el autor describe los conjuntos subcríticos moderados por agua y grafito del Reactor Institut. Resume la teoría de los experimentos subcríticos, indicando las ecuaciones fundamentales. Menciona asimismo otras correcciones que deben introducirse en la distribución neutróica medida experimentalmente para determinar el laplaciano material. Insiste en el hecho de que cuando la fuente neutónica utilizada está colocada en un soporte, el conjunto se encuentra en una posición ficticia con respecto a la fuente. Expone métodos adecuados para determinar esta posición ficticia de la fuente.

THE REACTOR INSTITUTE AT DELFT

Before a discussion of some of the results obtained from our experimental work with subcritical assemblies, it seems appropriate to mention just a few facts about the Reactor Institute at Delft.

This Institute is a governmental interacademic laboratory headed by three directors (chemistry, physics and general'services) and supervised for the Ministry of Education, Arts and Sciences by the Board of Regents.
of the Delft Technological University. The objectives of the establishment are to promote education and to perform research work in the field of atomic energy in the widest possible sense. A staff consisting of approximately 80 employees, 25 of whom have an academic degree, operates the Institute and executes a scientific programme. The scientific directors and the head of the reactor division also lecture at the Delft University (Fig. 1).

In addition to a neutron generator, a 1200-c Co source and, from an experimental point of view, a very flexible 1-MW pool-type research reactor, which went critical on 25 April 1963, the facilities include well
equipped laboratories for low- and medium-level chemistry, biology, solid state-, nuclear- and reactor physics, while plans are in an advanced state for extension of these facilities by a 1.5-MeV electron generator.

Because of the interacademic status of the laboratory, scientists from universities and research establishments can work as guests at the Reactor Institute whenever possible and wherever their work requires the facilities mentioned. In addition to our regular Dutch students, at the present time a number of foreign students are performing their doctoral work at the Institute.

1. PROGRAMME OF THE REACTOR PHYSICS DIVISION

In the past three years the reactor physics division has devoted much of its time to the design and construction of the Institute's nuclear reactor. Starting with the first critical experiment, most of the experimental work involved the initial calibration of the reactor and the determination of the physics parameters necessary for proper operation of the facility.

In addition to the engineering physics work, more fundamental reactor physics experiments have been carried out with a graphite-moderated and a light-water-moderated subcritical assembly, both fuelled with natural uranium.

These experiments were not performed with a prototype of a particular power reactor in mind but merely for educational purposes. Many of the experimental and theoretical studies were handled by students from the physics department of the Delft University who were majoring in reactor physics under the direction of staff members of the Institute. In all this work special emphasis was placed on a proper understanding and interpretation of the phenomena observed.

In this programme Fermi lifetimes, diffusion lengths, material bucklings, cadmium ratios, multiplication factors etc. have been measured in various configurations of the subcritical assemblies.

Since much of the experimental reactor physics work involves the determination of neutron flux levels with foil and wire activation methods, considerable effort was put into the improvement of the measuring techniques. At present these techniques have been developed to such an extent that we are able to determine relative neutron fluxes with errors less than 1% by very careful calibration of foils and instruments and upon introduction of the various necessary correction factors.

Besides macroscopic neutron flux distributions, the determination of microscopic distributions in connection with measurements concerning disadvantage factors and resonance parameters received attention. Absolute flux levels have been determined with 4π(β-γ)-coincidence techniques.

2. DESCRIPTION OF THE R.I. SUBCRITICAL ASSEMBLIES

Since the early days of the thermal-nuclear reactor, exponential assemblies have proved to be useful tools for the experimental study of multiplying characteristics of moderator and fissionable material mixtures.

They are smaller than critical assemblies and consequently require less material, a fact which was particularly important in the initial work
of Fermi and his associates because of the scarcity of high-quality graphite and uranium. Another advantage is that no complicated and expensive control mechanisms and instrumentation systems are required and that because of the flux levels on which the experiments mostly operate no problems arise with respect to radiation protection. The subcriticals have by nature inherent disadvantages. The applicability of the results to actual core design is only acceptable when the multiplication rates are on the order of 15 ($k_{\text{eff}}$ approximately 0.95). However, an indication of the neutron behaviour can already be obtained when the regeneration factor is on the order of 5. In particular, when the experiments are performed with small assemblies the results should be corrected for source location, higher harmonics and edge effects. These properties therefore make subcritical assemblies particularly attractive for training reactor physics students.

As it could be expected that light-water and graphite-moderated reactors would play an important role in power production in the near future, it was decided a few years ago to make subcritical assemblies of this nature available for educational purposes (Fig. 2).

![Fig. 2](image)

Side view of the laboratories and reactor building of the Reactor Institute at Delft

Nuclear-grade graphite was obtained in France from the Pechiney Company, and natural uranium was leased to us by the United States Atomic Energy Commission. The central workshop of the Delft University machined the graphite to proper dimensions and manufactured aluminium cannings for the fuel elements. The graphite assembly (Fig. 3) is rectangularly shaped, the sides being 240 cm long. It is placed on top of an 80-cm high pedestal of solid graphite in which the neutron source can be located at various positions. The assembly has 144 vertical fuel channels in a square
array. As the rise in multiplication resulting from the filling of the outer channels is almost compensated by the increase of leakage (less reflector material), only 100 fuel channels have been loaded during the experiments with 2000 kg of natural uranium in all. Horizontal and vertical slots through the assembly are provided to permit flux mapping, for which graphite stringers are used. The top and sides of the facility are not covered by any neutron absorbing material to act as a mathematically determined boundary, since the influence of neutron backscatter from the walls of the laboratory is negligible. With a moderator-to-fuel ratio of 83 and a 20-cm pitch of the fuel elements the effective multiplication factor was measured to be 0.76. At present this assembly is modified in such a way that the fuel channels will run horizontally and that the pedestal can be located either parallel or perpendicular to the fuel elements. Also the pitch will then be changed to 14 cm, and with small alterations 24 cm can be obtained.

In the new pedestal the source will be rotated in a vertical plane with a variable radius for more thorough study of higher harmonics in the flux distribution. In the water-moderated assembly (Fig. 4) the same fuel is used. In a tank which is connected to a mixed-bed de-ionizing system for maintenance of proper water conditions, a bottom and top grid plate keep the fuel elements (250 to 350) in the right position. By changing of the grid plates other water-to-fuel ratios can be obtained. The water subcritical assembly is also placed on top of a graphite pedestal in which the neutron source is placed. The effective multiplication factor of this assembly with a moderator-to-fuel ratio of 2.2 was measured to be 0.81. In future experiments this assembly will be used for studies in relation to super-heater cores, where special emphasis will be placed on the determination of
spectrum changes from variations in void fractions. Preliminary measurements showed that the neutron flux obtained with the available neutron source was too low to yield reliable results. Therefore, the assembly will now be placed on top of the thermal column of the reactor.

The natural uranium metal is available in cylindrically shaped slugs (Fig. 5) having a hole through the axis. The outside diameter is 28.5 mm, and the internal hole diameter is 15.0 mm. Five 205-mm long slugs are canned in concentric aluminium tubes with wall thicknesses of 0.75 mm. The central holes allow us to perform a number of instructive experiments. For instance, by insertion of various moderating materials the effect of cooling can be determined. Experiments of this nature have also been executed with the graphite assembly.

The neutron source consists of a homogeneous mixture of 79.24 g of plutonium and 39.29 g of beryllium enclosed in a tantalum housing. This 5-c source supplied by the Monsanto Company emits $8.3 \times 10^6$ neutrons per second. For improvement of the standards of the experiments a check was made on the anisotropy of the neutron emission with a special graphite pile. The anisotropy turned out to be less than 1%.

3. THEORY OF SUBCRITICAL ASSEMBLIES

Although the theory of subcritical assemblies is described in the standard literature, the basic equations will be summarized here. For simplicity the discussions will be limited to parallelepiped configurations.
The neutron source is placed in the plane \( z = 0 \) and may be either an operating reactor or a conventional neutron source based on an \((\alpha, n)\)- or \((\gamma, n)\)-nuclear reaction.

Since the assembly is supposed to be uniform, the balance equation for the thermal neutrons can be expressed as

\[
D_t \nabla^2 \phi(t,r) - \int \phi(t,r') \int \nabla' K(r, r') \phi(t,r') + \int \nabla' H(r, r') S(r') = 0, \tag{1}
\]

where \( S(r') \) is the source strength and \( H \) and \( K \) are the diffusion kernels for source neutrons and neutrons resulting from fission. In general, \( H \) and \( K \) are not similar since the source spectrum and the fission spectrum differ.

If \( \phi(r) \) satisfies the wave equation \( \nabla^2 \phi + B^2 \phi = 0 \), equation (1) can, for the geometry under consideration, be transformed into

\[
\partial^2 \phi / \partial x^2 + \partial^2 \phi / \partial y^2 + \partial^2 \phi / \partial z^2 + (\alpha^2 + \beta^2 - \gamma^2) \phi = 0, \tag{2}
\]

where the influence of the source neutrons has been neglected, which is correct only when the system is observed a number of absorbing mean-free-paths from the plane \( z = 0 \). The boundary conditions for equation (2) are \( \phi(\pm a/2, y, z) = \phi(x, \pm b/2, z) = \phi(x, y, c) = 0 \), where \( \phi \) is symmetrical with respect to the axis \((0, 0, z)\); \( a, b \) and \( c \) are the dimensions of the assembly including the extrapolation lengths.
It follows that the solution for equation (2) is

\[ \phi(x, y, z) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \cos \left( \frac{\pi mx}{a} \right) \cos \left( \frac{\pi ny}{b} \right) e^{-\gamma_{mn} z} \left[ 1 - e^{-2\gamma_{mn} (c-z)} \right], \]  

(3)

where \( m \) and \( n \) are odd integers indicating the order of the harmonic. The factor \( a_{mn} \) is a constant which depends on the harmonic, the source strength and the leakage. Using an isotropic point source emitting 5 thermal neutrons per second we obtain \( a_{mn} = 2S/abD\gamma_{mn} \), where \( D \) is the diffusion coefficient.

In this case, the complete equation for the neutron flux distribution through the assembly is given by

\[ \phi(x, y, z) = \frac{2So}{abD} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\gamma_{mn}} \cos \left( \frac{\pi mx}{a} \right) \cos \left( \frac{\pi ny}{b} \right) e^{-\gamma_{mn} z} \left[ 1 - e^{-2\gamma_{mn} (c-z)} \right], \]  

(4)

where \( \alpha \) is somewhat smaller than 1 and is added to account for the leakage of neutrons from the pedestal.

From equation (4) the following conclusion can be drawn:

1. As long as \( z \) is comparatively smaller than \( c \), the factor \( (1 - e^{-2\gamma_{mn} (c-z)}) \) approaches unity, in which case the flux distribution for each mode decreases exponentially along the \( z \)-axis. Near the top of the assembly the edge effect influences the exponential nature.

2. For all values of \( m \) and \( n \) the relation

\[(m\pi/a)^2 + (n\pi/b)^2 - \gamma_{mn}^2 = B^2\]

holds. Since \( B^2 \) (material buckling) depends only on the material composition and geometry of the unit cell, which is constant throughout the assembly, it follows that \( \gamma_{mn} \) increases in value for the higher harmonics, as a result of which the influence of 5-th and higher modes is hardly noticeable even for small values of \( z \). Further, the effect of the 1, 3; 3, 1 and 3, 3 harmonics also decreases rapidly for higher values of \( z \).

4. CORRECTIONS TO MEASUREMENTS

For determination of the material buckling \( B^2 \) for subcritical lattices of moderate size from \( \gamma_{11} \), it is necessary to correct the measured axial neutron flux distribution for higher harmonics near the source location and the edge effect near the top. The neutron distribution is plotted against \( z \) on semi-logarithmic paper, and the fundamental mode is obtained by calculation of the slope of the line resulting from a least-squares fit to the "corrected" points. Since both correction factors contain \( \gamma_{mn} \), an iteration process is required, which can be eliminated by the use of Uhrig’s method [3].

Up to now it has been assumed that the plane \( z = 0 \) is located on one of the sides of the assembly. In many cases, however, (as in ours) the neutron source is positioned in a pedestal, and consequently moderator material is present between the source and the assembly. When the corrections
mentioned at the beginning of this section were made, it could readily be seen that the corrected points did not fall on a straight line in the case of the water-moderated assembly (Fig. 6). This effect was not as noticeable for the graphite subcritical assembly. On the basis of these phenomena, we concluded that the assembly sees the neutron source in a position which differs from the true position; hence the source is situated at a fictitious position as a result of different diffusion properties of the two media: the pedestal and the assembly. When the fictitious source position had been determined, the measured points of the neutron distribution could be corrected for proper determination of $\gamma_{11}$ by the use of the fictitious distances to the source in the iteration calculations.

5. DETERMINATION OF THE FICTITIOUS SOURCE POSITION

(a) By calculation

The neutron flux distribution for specific constant values of $z$ is given by the relation

$$\phi(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} K_{mn} \cos \left( \frac{\pi mx}{a} \right) \cos \left( \frac{\pi ny}{b} \right).$$  (5)
When sufficient experimentally determined data of $\phi(x, y)$ for various values of $z$ are available, $K_{mn}(z)$ can be calculated from

$$K_{mn} = \frac{1}{ab} \int_{a/2}^{a} \int_{b/2}^{b} \phi(x, y) \cos \frac{\pi mx}{a} \cos \frac{\pi ny}{b} \, dx \, dy. \quad (6)$$

In our graphite assembly the axial distribution can only be measured in the planes $y = 0$ and $x = 0$. The application of Fourier analysis to the flux distribution in the plane $y = 0$ results in

$$\sum_{n} K_{mn} = (1/a) \int_{-a/2}^{a/2} \phi(x, 0) \cos (\pi mx/a) \, dx = I_m.$$ 

With Simpson's rule the integrals can be numerically determined with good accuracy when 12 points of $\phi(x)$ are available. The result is a set of equations:

\begin{align*}
I_1 &= K_{11} + K_{13} + K_{15} + K_{17} + \ldots \\
I_3 &= K_{31} + K_{33} + \ldots \\
I_5 &= + K_{51} + K_{55} + \ldots 
\end{align*}

in which $K_{mn} = K_{nm}$ for reasons of symmetry. When the influence of the 3, 3 mode and higher harmonics is neglected, it follows that $K_{11} = I_1 - I_3$, which can be used to determine $B_{m}^0$. Also in this case

\begin{align*}
I_1 &= (C/\gamma_{11}) e^{\gamma_{11} z} + (C/\gamma_{13}) e^{\gamma_{13} z} \quad (7) \\
I_3 &= (C/\gamma_{13}) e^{\gamma_{13} z} + (C/\gamma_{33}) e^{\gamma_{33} z} \quad (8)
\end{align*}

The calculation of $z$ from equations (7) and (8) gives the distance between the point where the flux is measured and the fictitious source position. Applying this method to our experimental data obtained from the graphite assembly we found the results listed in Table I.

Although a certain discrepancy exists between the various values for $z_a - z_f$ (because of inaccuracies in experimentally determined points), it can be seen that the neutrons in the graphite assembly act as though the neutron source were located approximately 16 cm higher in the pedestal than it really is.
b. Graphical method

With the assumption of the validity of diffusion theory in both the pedestal and the assembly the following relations hold: in the pedestal

\[
\frac{\phi_{13}}{\phi_{11}}_g = \left(\frac{L_{13}}{L_{11}}\right) \exp \left\{ \left(\frac{1}{L_{11}} - \frac{1}{L_{13}}\right) z \right\},
\]

where \( L \) is the diffusion length.

Experimentally we found this relation to be

\[
\frac{\phi_{13}}{\phi_{11}}_g = 0.555 \, e^{-2.01 \, z}.
\]

In the graphite subcritical assembly

\[
\frac{\phi_{13}}{\phi_{11}}_{g+u} = \left(\frac{\gamma_{11}}{\gamma_{13}}\right) \exp \left\{ \left(\frac{1}{\gamma_{11}} - \frac{1}{\gamma_{13}}\right) z \right\},
\]

which turned out to be

\[
\frac{\phi_{13}}{\phi_{11}}_{g+u} = 0.405 \, e^{-2.44 \, z}.
\]

Comparing equations (9a) and (10a), one can see that the 1,3 mode penetrates farther along the z-axis in graphite than in the graphite/natural uranium assembly. This means that the harmonic corrections in both media have to be treated separately under the condition that the ratio of \( \phi_{13}/\phi_{11} \) is the same on both sides of the interface. A semi-logarithmic plot of both distributions results in two straight lines with different slopes. The distribution of the ratio \( \phi_{13}/\phi_{11} \) can now be extrapolated to the point where the value \( \gamma_{11}/\gamma_{13} \) is reached. The fictitious source position is located at this point (Fig. 7).

In general, the fictitious source position can be determined by extrapolation of the ratio of \( \phi_{nm}/\phi_{11} \) in the assembly to the point where it reaches the value \( \gamma_{11}/\gamma_{nm} \).

With the use of the 3,3 harmonic in this construction the results were found to have the expected accuracy.
REFERENCES


DISCUSSION

R. PERSSON: I do not fully understand why you cannot have a straight line without taking these source positions into account. Is it because of the higher harmonics?

H.R. KLEIJN: The point is that the diffusion properties of the pedestal and the uranium-graphite assembly differ. This effect has not been taken into account in the elementary equations, where it is supposed that the z=zero value is at the edge of the assembly. Normally, then, one assumes that the source is located at the bottom of the assembly rather than in the pedestal. In our case about 40 cm of moderating material is introduced between the beginning of the assembly and the source, and that accounts for the effect in question.

R. PERSSON: But you have to measure far away from boundaries in order to define a buckling, and you have to be free from the high harmonics. I can't see why the position of the source comes in, because you need only source neutrons.

H.R. KLEIJN: That is so, but only when you are working with large assemblies. We are a very small institute, however, with a very limited budget, so that our assembly is necessarily rather small. The size of the
assembly is 2.4 m² and the diffusion length is about 50 cm in graphite. We made the correction because of the small dimensions.

R. PERSSON: Are you trying to distribute your source in order to avoid the high harmonics?

H.R. KLEIJN: Yes, this has been done. We mention in the paper that we are rearranging our assembly so as to rotate the neutron source on various radii, to study the effect of higher harmonics more carefully. The trouble was that we originally started with polonium–beryllium sources and the slots in the assembly were dimensioned for these sources; later on we got this plutonium–beryllium source, which did not fit the original holes. We are aware of the consequences of this; we have done some appropriate calculations, and there are indeed positions where the influence of these harmonics is reduced.

F. EBERSOLDT: I should like to comment on your expression for the flux. Unfortunately, you did not explain how you obtained the coefficients for your expansion. I think you assumed that in the plane z = 0, the neutron current density in the direction of z is equal to the source divided by 2. This hypothesis is only justified if the environment is symmetrical with respect to z = 0, and in your case the source is at the end of the column. This is not important for determining material buckling, which depends only on the slope of the flux curve, but these coefficients are important for determining the absolute flux value.

H.R. KLEIJN: You are perfectly right; we did make the assumption to which you refer, and the value of 2 is, in fact, not necessarily correct, especially in our case, where the source is located in the pedestal. You have to divide by more than 2 in order to account for the leakage. However, I did not mention this, because it had no significance for our purposes.
USE OF AN EXPONENTIAL ASSEMBLY TO INVESTIGATE BOUNDARY CONDITIONS IN NEUTRON DIFFUSION*

C.G. JAMES AND P.J. GRANT
IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY, LONDON, UNITED KINGDOM

Abstract — Résumé — Аннотация — Resumen

USE OF AN EXPONENTIAL ASSEMBLY TO INVESTIGATE BOUNDARY CONDITIONS IN NEUTRON DIFFUSION. It has been established that valid measurements of control-rod effectiveness can be made in an exponential assembly and the method has been used to investigate the influence of environment on control-rod worth. Measurements in unfuelled and fuelled systems have been used to deduce thermal and epithermal extrapolation distances for the rods and these values are compared with theoretical predictions.

ÉTUDE DES CONDITIONS AUX LIMITES DANS LA DIFFUSION DES NEUTRONS, AU MOYEN D'UN ENSEMBLE EXPONENTIEL. Il a été établi que l'on pouvait faire des mesures valables de l'efficacité des barres de commande dans un ensemble exponentiel. Les auteurs ont employé cette méthode pour étudier l'influence du milieu sur l'antiréactivité des barres de commande. De mesures faites dans des ensembles avec et sans combustible, ils ont déduit les longueurs extrapolées thermiques et épithermiques pour les barres et les ont comparées aux valeurs théoriques.

ИСПОЛЬЗОВАНИЕ ЭКСПОНЕНЦИАЛЬНОЙ СБОРКИ ДЛЯ ИЗУЧЕНИЯ ГРАНИЧНЫХ УСЛОВИЙ ПРИ ДИФФУЗИИ НЕЙТРОНОВ. Установлено, что в экспонциальной сборке могут проводиться достоверные измерения эффективности управляющих стержней. Этот метод используется для изучения влияния окружающей среды на компенсирующую способность управляющих стержней. Измерения в системах без топлива и в системах с топливом применяются в целях установления размеров тепловой и надтепловой экстраполяции для данных стержней. Эти значения сравниваются с теоретически предложенными.

UTILIZACIÓN DE UN CONJUNTO EXPONENCIAL PARA INVESTIGAR LAS CONDICIONES EN LOS LÍMITES EN LA DIFUSIÓN NEUTRÓNICA. Se ha establecido que es posible efectuar determinaciones válidas de la eficacia de las barras de control en un conjunto exponential y los autores recurrieron a este método para estudiar la influencia del ambiente sobre el valor de las barras de control. Aplicaron los resultados de mediciones efectuadas en sistemas con y sin combustible a fin de deducir las distancias de extrapolación térmica y epitérmica para las barras y luego compararon estos valores con los calculados teóricamente.

INTRODUCTION

The use of a subcritical assembly for the determination of the nuclear parameters of a reactor system is a well established technique which has covered a wide range of lattice parameters. There is no reason in principle why a subcritical assembly should not be used for many measurements which are now carried out in critical systems. Difficulties may arise in practice from the smaller size of the exponential assembly or in the interpretation of the results.

* A review of work by C.G. James, D.A. Meneley and C.E. Till on the uranium-graphite subcritical assembly at Imperial College, London.
It is the purpose of this paper to describe some investigations of boundary conditions in neutron diffusion problems which have been carried out on the graphite and natural uranium subcritical assembly at Imperial College. These have been concerned with the measurement of control rod effectiveness and the influence of the environment of the control rod upon its worth. From experiments in both unfuelled and fuelled systems it is possible to deduce extrapolation distances for thermal and epithermal neutrons and to compare these values with those predicted by theory.

DETERMINATION OF CONTROL ROD EFFECTIVENESS

The calculation of the reactivity effect of a control rod in a nuclear reactor depends on the evaluation of the boundary conditions for neutron absorption at the surface of the rod. Since the rod is normally a strong absorber of neutrons, diffusion theory is not applicable within it. However, more rigorous calculations based on neutron transport theory are either impossible or extremely difficult except in a few relatively simple cases. It is customary, therefore, to use diffusion theory for the calculations and to apply boundary conditions which are conveniently expressed in terms of linear extrapolation distances at the absorber boundary. These extrapolation distances have not been calculated exactly. What is commonly done is to apply the estimates of KUSHNERIUK and McKAY [1], who have solved the transport equations by a variational method. However, this method was evolved for a comparatively simple case, and the results of the computations are now being used a long way out of their original context. Very few experimental measurements of control rod effectiveness under conditions subject to rigorous analysis have been published. In general, these results show the difficulty of making accurate predictions of the effect of the control rod. For example, in the usual two-group treatment, absorption of fast neutrons is neglected, and this tends to result in underestimation of the effectiveness of the rod. However, it is difficult to evaluate a meaningful extrapolation length for fast neutrons. Measurements of control rod effectiveness have usually been carried out in critical systems. These tend to be expensive, and there is normally a great demand on experimental time for other work. The demand for experimental time on a subcritical assembly is very much less, and accordingly it is possible to investigate control rod effects in very much greater detail.

The present work was undertaken with two main objects. The first was to investigate and, if possible, substantiate the validity of control rod measurements in an exponential pile. The second was to carry out a series of control rod measurements in which the environment of the control rod was changed to reveal the effects on the neutron boundary conditions.

Basis of the method [2]

In terms of two-group neutron diffusion theory, the neutron balance in an exponential assembly may be described by the equation

\[
K_n = \frac{1 + \mu^2 L_{SR}^2 - \gamma^2 L_{SZ}^2}{1 + \mu^2 L_{SR}^2 - \gamma^2 L_{SZ}^2},
\]
where $L_R^2$ and $L_S^2$ are the anisotropic diffusion areas and $L_{SR}^2$ and $L_{S2}^2$ are the slowing-down areas. The radial geometric buckling $\mu^2$ is determined by the radial boundary conditions in the stack. For an unperturbed rectangular assembly the fundamental radial buckling is

$$\mu^2 = (\pi/a)^2 + (\pi/b)^2,$$

where $a$ and $b$ are the extrapolated radial dimensions. The axial buckling $-\gamma^2$ does not depend on the axial dimensions of the stack or the axial boundary conditions but assumes the value required to maintain the neutron balance.

When a control rod is inserted into the exponential stack in a direction perpendicular to the source plane, the radial buckling $\mu^2$ is changed and $\gamma^2$ takes on a new value to satisfy the neutron balance condition. From measurements of the change of $\gamma^2$ it is therefore possible to deduce the corresponding values for the change of $\mu^2$ brought about by the insertion of the control rod. For conversion of the measurements of buckling change into information about the properties of the control rod it is necessary to examine the validity of certain basic assumptions of the method and then to develop the appropriate theoretical description for a circular control rod in a square assembly. A number of comparisons with the normal critical assembly will now be considered.

1. **The axial flux distribution**

   The major difference here is that in the exponential assembly we have a negative axial buckling. Introduction of the control rod changes the axial buckling but not the functional form of the flux distribution. There is no fundamental difference between negative and positive buckling, and therefore the validity of the experimental method is not in doubt from this cause.

2. **The radial flux distribution**

   As with the critical assembly, the introduction of a control rod drastically alters the shape of the radial flux distribution in both fundamental and higher flux modes. It is therefore necessary to ensure that higher harmonic contributions to the measured flux distribution have been eliminated. In the present work measurements have shown that harmonics is small provided readings are taken more than 1 m from the top of the source pedestal.

3. **Perturbation in neutron energy spectrum near the rod**

   Since the absorption of the control rod varies with neutron energy, there is a change in neutron energy spectrum in its neighbourhood; and this will give rise to some change in the multiplying properties of the lattice. This change is part of the effectiveness of the rod; but if it is to be included in the measurement, it is necessary that the neutron spectrum perturbation should not extend over the whole volume of the exponential assembly. In other words, for analysis of the results in terms of lattice properties it is necessary that the neutron spectrum should regain its infinite lattice value...
within the bounds of the assembly. In the present work measurements of the cadmium ratio showed that this condition was in fact satisfied.

(4) The change in the flux shape near the outer boundary

A significant part of the effectiveness of a control rod stems from the increased outward leakage of neutrons arising from the changed flux shape. Near the outer boundary the neutron energy spectrum differs from that in the infinite lattice. Changes in flux shape might alter the extent of the departure from the equilibrium spectrum, and this effect would be expected to differ in a small assembly from its value in a large assembly, for example, an operating reactor. However, the difference will be a second order one only, and we are satisfied that it is negligible in the present case.

Two-group fluxes for a control rod in the centre of a square subcritical assembly

The solution of the two-group equations for a square subcritical assembly may be carried out in terms of cylindrical polar co-ordinates. The general solution for an assembly of height h is

\[
\phi_1 = \sum_{n=0}^{m} \sum_{m=0}^{n} \left\{ A_n J_{4n}(\mu_m r) + C_n Y_{4n}(\mu_m r) + E_n I_{4n}(\nu_m r) + F_n K_{4n}(\nu_m r) \right\} \\
\times \cos 4n \theta \sinh \gamma_m (h - z)
\]

\[
\phi_2 = \sum_{n=0}^{m} \sum_{m=0}^{n} \left\{ S_1 A_n J_{4n}(\mu_m r) + C_n Y_{4n}(\mu_m r) \right\} + \sum_{n=0}^{m} \sum_{m=0}^{n} \left\{ S_2 E_n I_{4n}(\nu_m r) + F_n K_{4n}(\nu_m r) \right\} \\
\times \cos 4n \theta \sinh \gamma_m (h - z),
\]

where the Z-axis lies along the centre line of the pile normal to the source plane and is a four-fold axis of symmetry. \( \phi_1 \) and \( \phi_2 \) are the fast and thermal fluxes. \( \mu_n \) and \( \nu_n \) are the two roots for the radial buckling.

For an assembly made up of a lattice which is asymmetrical between the Z and R directions we have the following expressions for \( \mu^2 \) and \( \nu^2 \):

\[
\mu^2 = -\frac{1}{2} \left\{ (1 - \gamma^2 L^2_{Z2})/L^2_R + (1 - \gamma^2 L^2_{SZ})/L^2_{SR} \right\} \\
+ \frac{1}{2} \left\{ (1 - \gamma^2 L^2_{Z2})/L^2_R + (1 - \gamma^2 L^2_{SZ})/L^2_{SR} \right\}^2 \\
+ 4 \left\{ [K_m^* - (1 - \gamma^2 L^2_{Z2})(1 - \gamma^2 L^2_{SZ})]/L^2_R L^2_{SR} \right\}^{1/2}
\]

\[
\nu^2 = \mu^2 + (1 - \gamma^2 L^2_{Z2})/L^2_R + (1 - \gamma^2 L^2_{SZ})/L^2_{SR}
\]
The terms in $\mu$ and in $\nu$ represent the asymptotic and non-asymptotic solutions respectively. $S_1$ and $S_2$ are coupling constants given by

$$S_1 = p \Sigma_1/\Sigma_2 (1 - \gamma_m^2 L^2_2 + \mu_m^2 L^2_R)$$

$$S_2 = p \Sigma_1/\Sigma_2 (1 - \nu_m^2 L^2_2 - \nu_R^2 L^2_R).$$

In the present experiments we took care to measure only the fundamental mode of the flux distribution, so the analysis can be limited to this mode and the subscript $n$ may be omitted. In practice it is found that the irregular parts of the solution are negligible for $n$ greater than zero, so the coefficients $C_n$ and $F_n$ are zero for $n \neq 0$. Similarly, the regular parts of the solution are found to be negligible beyond the first two terms of the series.

In order to evaluate these solutions we must apply the appropriate boundary conditions for our experimental system. There are two internal boundary conditions at the surface of the control rod represented by extrapolation distances for fast- and thermal-neutron fluxes. Therefore, for evaluation of the six constants in the flux expressions four conditions must be laid down at the external boundary. We may obtain these by putting the fast and thermal fluxes equal to zero in two points on the outer boundary $R = a/2 \cos \theta$. For convenience $\theta = \pi/8$ and $\theta = \pi/4$ are chosen. Applying these boundary conditions, we obtain the following transcendental equation for $\mu$:

$$Y_0 (\mu R_0)/J_0 (\mu R_0) = [AY_0 (\mu c) + B\mu Y_1 (\mu c)]/[A J_0 (\mu c) + B\mu J_1 (\mu c)] ,$$

where

$$A = \left[ \lambda^2 - \frac{S_1}{S_2} \lambda_1 + \frac{1}{\nu} \frac{K_0 (\nu c)}{K_1 (\nu c)} \left( 1 - \frac{S_1}{S_2} \right) \right] ,$$

$$B = \left[ \left( 1 - \frac{S_1}{S_2} \right) \lambda_1 \lambda_2 + \left( \lambda_1 - \frac{S_1}{S_2} \lambda_2 \right) \frac{1}{\nu} \frac{K_0 (\nu c)}{K_1 (\nu c)} \right] ,$$

$\lambda_1$ and $\lambda_2$ are the extrapolation lengths for fast and thermal neutrons respectively at the control rod boundary $R = c$, and where $R_0 = a/(2 \cos \pi/8)$.

The approximations have been made that $K_0 (\nu R_0)/I_0 (\nu R_0) \approx 0$ and that fourth-order Bessel functions of small argument may be neglected. By fitting the expressions at this value of $R_0$, we find that fourth-order Bessel terms which would otherwise be present are there equal to zero. When the absorber is effective only for thermal neutrons, $\lambda_1$ is infinite and

$$A = \frac{S_1}{S_2} ,$$

$$B = - \left[ \left( 1 - \frac{S_1}{S_2} \right) \lambda_2 + \frac{1}{\nu} \frac{K_0 (\nu c)}{K_1 (\nu c)} \right] .$$
If the absorber is equally effective for the fast and thermal groups, $A = 1$, $B = \lambda$. The appropriate equation can be solved for one unknown extrapolation length in terms of the radial buckling or for the buckling in terms of known extrapolation lengths.

Apparatus and experimental method

The experiments were carried out in a graphite and natural uranium pile 8 ft square and 7 ft 3 in high. This is constructed from blocks of reactor-grade graphite, 8 X 8 in by 29 in long with a 4.25-in diameter hole on the long axis. The blocks are arranged in a 12 X 12 lattice, three blocks high; and each channel contains a fuel element made up of 1-in diameter uranium metal rods supported in aluminium tubes of thickness 0.0625 in. The air channels may be almost completely blocked by means of additional graphite sleeves. Access for measuring devices is given by 1.25-in square holes on an 8 X 8-in lattice. These are formed by cutting away of 0.625-in square from the long edges of the blocks. The central measuring hole in the stack is replaced by a cylindrical channel 3.91-in in diameter to accommodate the control rod. The mean graphite density is 1.75 $g/cm^3$.

Neutrons are supplied from four antimony beryllium sources which are situated in a graphite pedestal beneath the fuel-moderator lattice. The sources are initially of 25 c of antimony-124 and provide a maximum thermal-neutron flux of about $10^5$ neutrons per cm$^2$/s at the interface between the pedestal and the lattice. The general arrangement is shown schematically in Fig. 1.

![Fig. 1](image)

Arrangement of stack and control rod

Neutron flux measurements were made by means of boron-trifluoride counters feeding into a conventional system of head-amplifier and main-amplifier, discriminator and fast scaler. The standard deviation of indi-
individual counting measurements was never greater than about 0.3%. Measurements were taken in a region of the stack where harmonic distortions were negligible. Outer boundary extrapolation lengths were fixed by least-squares analysis of cosine flux plots, followed by an empirical straight-line fit to the values as a function of the lattice mean-free-path. The axial relaxation length was found by least-squares fitting to a hyperbolic sine function, with the extrapolated height of the assembly held constant.

Experimental results

Measurements were made with five boron steel rods with a nominal content of 4% of boron by weight. All were in the form of circular tubes of 3/16-in wall thickness. Two other measurements were made with cadmium rods formed by 0.018-in thick cadmium sheet wrapped round a circular metal former. In addition to these measurements with rods which were black to thermal neutrons, measurements were made with mild steel tubes as the control element in order to ascertain the effectiveness of material which does not absorb all incident thermal neutrons.

Since the disturbance of thermal-neutron flux in the exponential stack is largest for the largest boron steel rod, the flux spectrum in this case was examined experimentally for verification that the necessary condition for meaningful measurements was being satisfied. Measurements of the flux distribution in a region sufficiently far from the neutron sources to be representative of the lattice spectrum were made along a horizontal diagonal of the pile with the boron trifluoride counter bare and covered with a 0.018-in thick sheath of cadmium. The ratio of counts above the cadmium cut-off to counts below the cut-off was found and compared with the values for the unperturbed pile. These measurements are shown in Fig. 2. The disturbance of the neutron spectrum near the rod is well illustrated, and it can also be seen that the ratio returns to the unperturbed value. In both cases there is a marked hardening of the neutron flux near the edge of the pile. The main condition for the validity of the results has therefore been fully met.

Figure 3 shows the changes in radial buckling $\delta \mu^2$, which have been deduced from the change in the axial buckling, plotted as a function of control rod diameter. The theoretical curves have been constructed for extrapolation lengths calculated by the methods of Kushneriuk and McKay. The two-group curve assumes no absorption of fast neutrons while the one-group curve assumes that $\lambda_1$ equals $\lambda_2$. From Fig. 3 a number of points emerge:

(a) One-group theory greatly over-estimates the effectiveness (as would be expected) while two-group calculation with no allowance for absorption in the fast group under-estimates the effectiveness by some 20% to 30% in the case of the boron steel rods. This discrepancy is in line with the results obtained with large power reactors and shows that the method gives meaningful results and that there are no substantial scale effects.

(b) While the exact relationship between experimental and theoretical curves depends on the correctness of the detailed parameters used in the calculation, the effect of epithermal absorption in boron steel is clearly demonstrated by the generally lower level of the points for cadmium control rods.
(c) From the separation between the theoretical, the cadmium and the boron steel curves it is in principle possible to investigate the boundary conditions for the fast-neutron group. We will discuss later some further experiments specifically designed to elucidate this point.
TABLE I

COMPARISON OF EXPERIMENTAL AND CALCULATED EXTRAPOLATION LENGTHS FOR MILD STEEL RODS

<table>
<thead>
<tr>
<th>Rod size</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer diameter (in)</td>
<td>1.50</td>
<td>2.25</td>
<td>3.00</td>
</tr>
<tr>
<td>Inner diameter (in)</td>
<td>1.125</td>
<td>1.75</td>
<td>2.50</td>
</tr>
<tr>
<td>Channel diameter (in)</td>
<td>2.94</td>
<td>3.91</td>
<td>3.91</td>
</tr>
<tr>
<td>( \lambda_1 ) (experimental) (cm)</td>
<td>23.5 ± 3.3</td>
<td>18.9 ± 1.9</td>
<td>14.5 ± 1.3</td>
</tr>
<tr>
<td>( \lambda_2 ) (calculated) (cm)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Kushneriuk and McKay</td>
<td>26.7</td>
<td>18.4</td>
<td>13.1</td>
</tr>
<tr>
<td>(2) Diffusion theory</td>
<td>26.0</td>
<td>17.8</td>
<td>13.0</td>
</tr>
</tbody>
</table>

The experimental results for mild steel rods are given in Table I. On the assumption of zero absorption in the fast group, experimental values of the thermal extrapolation lengths have been derived from the measured change in buckling. Two values of the thermal extrapolation length have been calculated for each case and are also shown in the table. These were found by:

(1) The use of the method of Kushneriuk and McKay applied to rods which both scatter and absorb. This assumed that the rods consisted of iron, density 7.95 g/cm³ and microscopic absorption scattering and cross-section of 2.53 and 11.1 b, respectively. Since these authors do not treat the case of a hollow cylinder, the tubes were taken as solid rods of the same outside diameter but with the density reduced to represent the same weight of material per unit length.

(2) Simple diffusion theory was applied for calculation of the flux distribution in a hollow cylinder. If a constant flux is assumed in the air gap surrounding the rod, this leads to the expression

\[
\lambda_2 = \frac{D_{2R} c}{K_s D_b} \left\{ \frac{I_0 (\kappa_b) K_1 (\kappa_g) + K_0 (\kappa_b) I_1 (\kappa_g)}{I_1 (\kappa_b) K_1 (\kappa_g) - K_1 (\kappa_b) I_1 (\kappa_g)} \right\}
\]

where the subscript \( s \) refers to steel, \( D_s \) being the thermal-neutron diffusion coefficient and \( \kappa_s \) the inverse diffusion length, \( b \) is the outer radius and \( g \) the inner radius of the tube, \( D_{2R} \) is the radial value of the thermal-neutron diffusion coefficient in the surrounding medium.

The main feature of these results is that there is remarkably good agreement between theory and experiment. Thermal-neutron absorption appears to account satisfactorily for the observed effects. The fast-neutron effect in the mild steel rods is thus greatly reduced by comparison with the boron steel rods, where it amounted to an average of about 20%. This is not unreasonable since, although both materials are essentially \( 1/V \) absorbers, much of the volume of the boron steel is screened from the thermal flux by virtue of the very high absorption cross-section. This reduces the contri-
duction of thermal-neutron absorption to the total effect and emphasizes the fast contribution, while the thermal-neutron flux depression in mild steel is quite small.

Experiments for investigation of the fast-neutron absorption effect [3]

For determination of the fast-neutron absorption by the control rod two further series of measurements have been made. In the first series all the fuel was removed from the exponential stack, and the experiments were carried out on black absorbers in a purely diffusing medium. We investigated three lattices without fuel and having different degrees of voidage to find the correct boundary conditions for thermal neutrons at the surface of the control rod in a medium which contains large voids. The results of these experiments are shown in Fig. 4 where the ratio of experimental to theoretical extrapolation length is plotted as a function of rod size. Theoretical extrapolation lengths were calculated by the method of Kushneriuk and McKay with the mean-free-path for the homogenized surrounding medium calculated according to the method of GRANT [4]. It will be seen that the case where the moderator is almost solid agrees well with calculations. The lattices with large channels show less control rod effectiveness than predicted. However, from these experiments it is possible to derive empirical correction factors to be applied to the theoretical extrapolation lengths for thermal neutrons.

Following the experiments in the unfuelled lattices, the previous experiments with fuelled lattices were repeated and extended. Two lattices, one with 4.25-in coolant channels and one with very little voidage, were
investigated with three types of control rod: cadmium, boron steel and boron carbide. By the use of the empirically determined method for computing the thermal extrapolation lengths it was possible to separate out the effect of epithermal neutrons and thus to determine the epithermal group extrapolation lengths of any type of rod. For a theoretical interpretation of these results a numerical integration of the Fermi age equation in the neighbourhood of the control rod was carried out with, for each energy integral, the appropriate extrapolation length calculated according to the method of Kushneriuk and McKay. After determination of the energy spectrum of neutrons in the neighbourhood of the control rod an effective fast group extrapolation length was obtained by averaging over this energy spectrum. The details of this method will be published elsewhere [5]. It was found that the use of the empirical thermal extrapolation length together with this method of calculation of the epithermal extrapolation length fitted the experimental results within the errors of the measurement.

From these experiments the following conclusions may be drawn:

(1) A subcritical assembly provides a convenient and accurate method for the study of control rod behaviour and for the investigation, for example, of the effects of control rod environment, sheath materials, moderating cores in control rods etc. If necessary, these experiments could be carried out at temperatures comparable with those existing in power reactors;

(2) The chief requirement now for accurate prediction of control rod effects in graphite-uranium lattices is for an adequate description of the effect on the thermal-neutron group of the large coolant channels in the surrounding medium.

REFERENCES


DISCUSSION

R. Persson: Have you found any streaming effects in the channel where you put the control rod?

P. J. Grant: None that we could determine exactly. I presume you are thinking of direct streaming of neutrons from the pedestal. We have not investigated this possibility very closely, but in connection with other experiments on neutron diffusion in general, we put cadmium disks at the bottom of empty channels at the pedestal interface to try and prevent this type of effect, and we also partially filled the channels with graphite to see whether this made any difference. Our impression was that there was some slight direct streaming from the pedestal, but we were not able to measure it.
VIII

SPECIAL TECHNIQUES (THEORETICAL AND EXPERIMENTAL)
Abstract — Résumé — Аннотация — Resumen

USE OF AN OSCILLATION TECHNIQUE TO MEASURE EFFECTIVE CROSS-SECTIONS OF FISSIONABLE SAMPLES IN CRITICAL ASSEMBLIES. The authors describe the technique used to measure the effective absorption and neutron-yield cross-sections of a fissionable sample. These two values are determined by analysing the signals due to the variation in reactivity (overall signal) and the local perturbation in the flux (local signal) produced by the oscillating sample. These signals are standardized by means of a set of samples containing quantities of fissionable material (U\textsuperscript{235}) and an absorber, boron, which are well known. The measurements are made for different neutron spectra characterized by lattice parameters which constitute the central zone within which the sample moves.

This technique is used to study the effective cross-sections of uranium-plutonium alloys for different heavy-water and graphite lattices in the MINERVE and MARIUS critical assemblies. The same experiments are carried out on fuel samples of different irradiations in order to determine the evolution of effective cross-sections as a function of the spectrum and the irradiations.

MESURE DES SECTIONS EFFICACES EFFECTIVES D’ÉCHANTILLONS FISSILES PAR UNE MÉTHODE D’OSCILLATION DANS LES ASSEMBLAGES CRITIQUES. On décrit la méthode utilisée pour mesurer les sections efficaces effectives d’absorption et de production de neutrons d’un échantillon fissile. Ces deux grandeurs sont déterminées en analysant les signaux dus à la variation de réactivité (signal global) et à la perturbation locale de flux (signal local) produits par l’échantillon oscillant. Ces signaux sont étalonnés à l’aide d’un jeu d’échantillons dont les teneurs en matériau fissile (\textsuperscript{235}U) et en absorbeur (bor) sont bien connues. Les mesures sont réalisées pour différents spectres de neutrons caractérisés par les paramètres du réseau constituant la zone centrale à l’intérieur de laquelle se déplace l’échantillon.

A l’aide de cette méthode on étudie les sections efficaces effectives d’alliage uranium-plutonium pour différents réseaux à eau lourde et à graphite dans les assemblages critiques MINERVE et MARIUS. Les mêmes expériences sont effectuées sur des échantillons de combustibles irradiés à différents taux, de façon à déterminer l’évolution des sections efficaces en fonction du spectre et du taux d’irradiation.

ИЗМЕРЕНИЕ ЭФФЕКТИВНЫХ СЕЧЕНИЙ ДЕЛЯЩИХСЯ ОБРАЗЦОВ МЕТОДОМ ОСЦИЛЛЯЦИИ В КРИТИЧЕСКИХ СБОРКАХ. Дается описание метода измерения эффективных сечений поглощения и производства нейтронов делящегося образца. Эти две величины определены при анализе вибрационных сигналов, поступающих при изменении реактивности (глобальный сигнал) и при местном нарушении потока (местный сигнал). Сигналы калибруются с помощью набора образцов, в которых содержание делящегося вещества, урана-235, и поглотителя, бора, хорошо известно. Проведены измерения для различных спектров нейтронов, характеризуемых параметрами решетки, представляющей собой центральную зону, внутри которой перемещается образец. С помощью такого метода проводится исследование эффективных сечений урано-плутониевого сплава для различных решеток на тяжелой воде и графите в таких критических сборках, как MINERVA, MARIUS. Те же опыты были проведены с образцами топлива, облученного при различной интенсивности, с целью определить эволюцию эффективных сечений в зависимости от спектра и интенсивности облучения.

MEDICIÓN DE LAS SECCIONES EFICACES EFECTIVAS DE MUESTRAS-FISIONABLES EN CONJUNTOS CRÍTICOS POR UN MÉTODO DE OSCILACIÓN. Los autores describen el método empleado para medir las secciones efficaces efectivas de absorción y de producción de neutrones de una muestra fisionable. Ambos
valores se determinan analizando las señales originadas por la variación de reactividad (señal global) y por la perturbación local de flujo (señal local) producidas por la muestra oscilante. Las señales se calibran con ayuda de un juego de muestras cuyo contenido en material fisiable, $^{235}\text{U}$, y boro (absorbedor), es bien conocido. Las mediciones se efectúan para diversos espectros neuéntricos caracterizados por los parámetros del reticulo que constituye la zona central en cuyo interior se desplaza la muestra.

Con este método, se han estudiado las secciones eficas efectivas de aleaciones de uranio-plutonio para diversos reticulos de agua pesada y de grafito en los conjuntos críticos MINERVE y MARIUS.

Los mismos experimentos se han efectuado con muestras de combustibles sometidas a distintas irradiaciones, con el fin de determinar la evolución de las secciones eficas en función del espectro y del grado de irradiaión.

INTRODUCTION

L'évolution de la réactivité d'un réacteur de puissance en fonction du taux d'irradiation du combustible est un équilibre entre la consommation de matiére fissile, la production d'autres éléments fissiles et l'accumulation des produits de fission.

Pour les réacteurs à uranium naturel ou faiblement enrichi, les effets, de signe contraire, se compensent pour donner un bilan légèrement positif au début. Pour un certain taux d'irradiation dépendant des paramètres du réseau et des conditions d'irradiation, le bilan s'annule puis devient négatif.

Le taux d'irradiation correspondant au changement de signe de la réactivité qui définit la durée de vie de la charge de combustible intervient ainsi directement dans le prix de revient de l'énergie produite. La connaissance des sections efficaces du combustible pour différents taux d'irradiation qui permettent de définir le bilan de réactivité, présente un grand intérêt du point de vue économique pour les réacteurs à uranium naturel.

Ce bilan, qui est la différence de deux effets, ne pourra être valablement calculé que si l'on connaît les sections efficaces effectives avec une très grande précision.

Plusieurs auteurs, indépendamment des études théoriques, ont mis au point des méthodes permettant de mesurer les sections efficaces de fission et d'absorption des matériaux fissiles.


Nous exposons ici une méthode originale qui évite ces deux difficultés et donne ainsi des résultats directement exploitables pour l'étude de l'évolution des combustibles en fonction du taux d'irradiation.

Au sein d'un réseau donné, les propriétés nucléaires d'un combustible sont caractérisées essentiellement par sa section efficace effective d'absorption $\bar{\Sigma}_a$ et sa section efficace effective de production de neutrons ($\nu \bar{\Sigma}_f$).

Les neutrons absorbés dans le combustible sont constitués essentiellement de neutrons du domaine thermique ou proche épithermique alors que les neutrons produits sont rapides.

Il est donc théoriquement possible de distinguer grâce à un dispositif d'observation judicieusement choisi les modifications de $\bar{\Sigma}_a$ et de $\nu \bar{\Sigma}_f$ subies par une portion du combustible consituant le réseau étudié. Les dimensions
de la portion du combustible perturbée ainsi que l'importance relative de ces perturbations doivent être maintenues aussi faibles que possible afin de les observer sans pour cela perturber à son tour le spectre caractéristique du réseau dans lequel on travaille.

Les expériences réalisées sur les ensembels critiques MINERVE et MARIUS ont confirmé les possibilités théoriques et permis de dégager peu à peu une procédure expérimentale satisfaisante.

1. PRINCIPE DE LA MÉTHODE

1.1. Phénomènes fondamentaux

Considérons, pour simplifier l'exposé, un réacteur sphérique homogène critique de rayon $R$. Imaginons que l'on fasse subir à une portion sphérique de rayon $p < R$ de ce réacteur une variation que nous supposerons périodique de sa section d'absorption effective $\Sigma_a$. Le calcul montre qu'en un point situé à une distance $r$ du centre, la densité de neutrons subit une variation également périodique $\delta n$ liée à la variation $\delta \Sigma_a$ par la fonction de transfert $H_a(r, p)$ c'est-à-dire que

$$\frac{\delta n}{n} (r, p) = \delta \Sigma_a(p) x H_a(r, p)$$

Il en est de même pour une variation de $(\nu \Sigma_f)$ et l'on a dans ce cas.

$$\frac{\delta n}{n} (r, p)_f = \delta \nu \Sigma_f(p) x H_f(r, p).$$

L'utilisation de la variable de Laplace $p$ et des fonctions de transfert permet d'étendre aisément à des fonctions du temps quelconques les résultats numériques de calculs effectués pour des fonctions périodiques.

Si la zone perturbée comprend l'ensemble du réacteur $(p = R)$, la perturbation $\delta n/n$ ne dépend plus de $r$ et les fonction $H_a$ et $H_f$ sont tout simplement proportionnelles à la fonction de transfert du réacteur $H(p)$ reliant une variation de réactivité $\delta K(p)$ à la variation $\delta n(p)/n$ correspondante. On a donc, dans ce cas,

$$\frac{\delta n}{n} (p) = \delta K(p) x H(p),$$

où $\delta K(p)$ est une combinaison linéaire à coefficients réels et constants

$$\delta K(p) = A \delta \Sigma_a(p) + B \delta \nu \Sigma_f(p).$$

Si la zone perturbée est de rayon très faible $(p < R)$ on vérifie que $H_a$ et $H_f$ peuvent se mettre sous la forme

$$H_a(r, p) = A H(p) + a(r),$$

$$H_f(r, p) = B H(p) + b(r).$$
où a(r) et b(r) sont des fonctions réelles de r qui tendent très rapidement vers zéro lorsque r augmente.

Le fait important sur lequel est basée la méthode de mesure utilisée est que:

\[ \Delta(r) = \begin{vmatrix} A & a(r) \\ B & b(r) \end{vmatrix} \neq 0 \]  

et plus précisément que la valeur de ce déterminant est une fonction rapidement croissante lorsque le point r se rapproche de la zone perturbée. La perturbation de la densité de neutrons \( \delta n \) peut donc être en tout point considérée comme la somme de deux effets:

- l'un relié à \( \delta \hat{E}_a \) et \( B \delta \hat{E}_f \) par la fonction de transfert du réacteur et indépendant de la position; nous l'appellerons perturbation globale.
- l'autre proportionnel à \( a(r) \delta \hat{E}_a \) et \( b(r) \delta \nu \hat{E}_f \), diminuant rapidement lorsqu'on s'éloigne de la zone perturbée, plus rapidement pour l'effet d'absorption que pour l'effet de fission: nous l'appellerons perturbation locale.

La relation fondamentale sur laquelle s'appuie la méthode décrite est donc:

\[ H_\ell(r, p) = BH(p) + b(r) \]  

\[ \delta K(p) = A \delta \hat{E}_a(p) + B \delta \nu \hat{E}_f(p) \]  

Les perturbations \( \delta \hat{E}_a \) et \( \delta \nu \hat{E}_f \) que l'on cherche à mesurer sont les différences entre le \( \hat{E}_a \) et le \( \nu \hat{E}_f \) d'un combustible de référence (par exemple, l'uranium naturel) et ceux d'un combustible irradié ou reconstitué de même géométrie. Les effets dus à une modification de la géométrie sont par principe exclus de la méthode puisqu'ils introduisent une modification des caractéristiques du réseau et pas seulement des propriétés nucléaires du combustible.

Afin de préserver le spectre du réseau de référence, cette comparaison s'effectuera en imprimant un mouvement périodique à un canal de combustible pris dans son ensemble, une petite portion de ce canal ayant été remplacée par une portion géométriquement identique de combustible à étudier.

Le phénomène observé est alors constitué par une superposition de phénomènes analogues à celui étudié ci-dessus mais décalés dans le temps
MÉTHODE D'Oscillation

1.2. Séparation des signaux local et global

Les expériences effectuées ont permis de vérifier les conclusions du paragraphe ci-dessus mais elles ont également montré qu'il est très difficile d'obtenir une stabilité suffisante de la puissance du réacteur. On peut dans certaines conditions avoir un signal global très intense et un signal local faible ce qui rend la séparation délicate puisque le détecteur local mesure la somme de deux signaux.

L'utilisation d'un pilote automatique de hautes performances, spécialement conçu pour ces mesures, permet d'éliminer la quasi-totalité du signal global dans le réacteur et en particulier au niveau du détecteur local tout en offrant un moyen commode de mesurer la variation de réactivité qui lui correspond. On obtient ainsi la séparation des perturbations locale et globale par élimination de la perturbation globale, les grandeurs mesurées devenant:

\[ \delta K(p) = A \delta \dot{L}_a + B \delta \nu \dot{L}_f \]  
(10)

\[ \frac{\delta n}{n} (r, p) = a(r) \delta \dot{L}_a + b(r) \delta \nu \dot{L}_f. \]  
(11)

La stabilité de puissance du réacteur est obtenue du même coup.

1.3. Etalonnage des signaux - Principe d'équivalence

Les valeurs des coefficients \( A, B, a(r), b(r) \) dépendent de façon compliquée des caractéristiques du réacteur, de la géométrie de l'échantillon oscillé et de son mouvement ainsi que de la position et de la forme du détecteur local.

Il faut donc étalonner les signaux observés à l'aide d'un jeu d'échantillons étalons, tous de géométrie identique mais correspondant à des écarts \( \delta \dot{L}_a \) et \( \delta \nu \dot{L}_f \) bien définis par rapport au combustible de référence qui fournit le point zéro.

Ces échantillons étalons seront obtenus en pratique en ajoutant au combustible de référence des quantités connues d'absorbeur (bore) et de matériau fissile (uranium 235).

Chaque échantillon, défini par la valeur des deux signaux qui lui sont associés, est représenté dans le plan par un point dont les axes de coordonnées primaires correspondent respectivement au signal global et au signal local observés, exprimés en unités arbitraires. On pourra définir des
systèmes de coordonnées secondaires, pas nécessairement rectilignes, si les variations sont grandes, par exemple, un axe correspondant à l'adjonction progressive d'un matériau donné. Pour des variations suffisamment petites, les phénomènes seront linéaires, donc additifs, et à chaque point du plan, on pourra faire correspondre des coordonnées obtenues par projection parallèlement à deux axes distincts quelconques. En général, on utilise les axes 235U et bore.

Deux échantillons correspondant à un même point du plan seront appelés équivalents. Cette équivalence n'est bien entendu valable que pour le réseau et le dispositif expérimental particulier considérés. La section efficace effective du bore étant indépendante du spectre de neutrons présent dans l'échantillon, et les sections efficaces effectives de l'235U en dépendant relativement peu, le choix de ces deux matériaux pour constituer le système de référence habituel paraît logique.

L'emploi d'étalons secondaires n'est cependant pas exclu et pourra même devenir indispensable pour l'étude de problèmes particuliers tel que l'effet d'ombre dû au 240Pu.

On considère en général une répartition homogène des matériaux d'addition mais l'étude d'échantillons irradiés fera apparaître des effets de répartition qu'il sera nécessaire d'étudier à l'aide d'étalons appropriés.

Avec les nouveaux systèmes de coordonnées les équations de base s'écrivent:
- Axes de coordonnées primaires: G et L
  \[ G = \delta K = A \delta L_a + B \delta \nu \ell_f \]  
  \[ L = \delta n = a \delta L_a + b \delta \nu \ell_f \]  
- Systèmes de référence habituel: 235U et bore
  \[ G = B_g \mu + U_g m \]  
  \[ L = B_l \mu + U_l m \]

\[ \mu = \text{masse d'absorbeur de référence: bore} \]
\[ m = \text{masse de matière fissile de référence: } 235\text{U}. \]

2. RÉALISATION EXPÉRIMENTALE

Pour arriver à une interprétation simple des mesures et éviter des corrections délicates à évaluer, on cherche à réaliser des conditions expérimentales telles que les perturbations apportées au spectre neutronique à l'endroit de la mesure par l'ensemble de détection ou par l'échantillon à mesurer, soient aussi faibles que possible tout en restant compatibles avec la précision souhaitée.
2.1. Intérêt de la méthode d'oscillation

Les équations que nous avons adoptées pour exposer la méthode utilisent la représentation de Laplace car pour cette expérience, les fonctions périodiques possèdent des avantages particuliers.

En effet, les fluctuations de réactivité dues à la nature aléatoire des phénomènes nucléaires et en particulier du nombre de neutrons produits par fission constituent un bruit à distribution spectrale sensiblement uniforme, donc un bruit «blanc». La distribution spectrale du bruit observé lorsqu'on cherche à mesurer la densité de neutrons s'obtient en multipliant le bruit «origine» par le carré du module de la fonction de transfert du réacteur, compte tenu de la bande passante du dispositif de mesures.

Lorsque le réacteur ne comporte pas de pilote automatique, les mesures statiques sont très délicates si l'on cherche une grande sensibilité puisque le module de la fonction de transfert du réacteur tend vers 1\( ^{\infty} \) quand la fréquence tend vers zéro. Si l'on considère qu'au bruit «blanc» d'origine neutronique viennent s'ajouter les fluctuations de réactivité dues aux influences extérieures (pression atmosphérique) dont le spectre comporte une nette prédominance de fréquences très basses, il est pratiquement impossible de faire des mesures statiques, les dérives de puissance étant inévitables.

On est naturellement conduit, dans ce cas, à limiter la bande passante du système de mesure à un domaine aussi étroit que possible entourant une fréquence choisie dans le domaine où le module de la fonction de transfert d'un réacteur typique varie très lentement, d'où le mouvement périodique.

Lorsque le réacteur comporte un pilote automatique convenablement adapté, des mesures statiques seraient a priori tout aussi justifiées, sous réserve de limiter soigneusement la bande passante du système de mesure au voisinage de la fréquence zéro, c'est-à-dire d'accroître suffisamment la durée d'une mesure. L'existence du bruit d'origine extérieure, prépondérant aux très basses fréquences (dérives) conduit à préférer la méthode d'oscillation qui permet de choisir la bande passante du système de mesure dans un domaine où ce bruit a perdu de son intensité.

Le choix de la fréquence d'oscillation résulte d'un compromis entre la condition ci-dessus et la nécessité d'obtenir pour le pilote automatique, un gain en boucle ouverte élevé afin d'avoir une compensation efficace de la perturbation globale. On est ainsi conduit à choisir une fréquence assez basse.

2.2. Conditions d'oscillation

L'échantillon à mesurer est placé à l'intérieur d'un train continu de cartouches aussi voisines que possible de façon à former une cellule identique à celles du réseau. Sa longueur est telle que, pour l'amplitude complète de l'oscillation, le canal reste entièrement chargé de combustible. Dans ces conditions, pendant l'oscillation, le réseau n'est absolument pas perturbé, seul l'effet de l'échantillon intervient sur la réactivité par la différence de ses caractéristiques avec celles du train.

Compte-tenu des possibilités de l'empilement et de l'oscillateur mécanique, on choisit la position moyenne d'oscillation telle qu'elle produise, pour une amplitude donnée, le maximum d'effet sur la réactivité.
2.3. Mesure du signal global. Pilote automatique

La mesure du signal de réactivité $\Delta k$ s'effectue par l'intermédiaire d'un pilote automatique. Il est constitué d'une chambre d'ionisation placée à la périphérie du cœur, donc insensible à la perturbation locale et d'un amplificateur d'asservissement commandant une barre rotative absorbante, conçue de façon que sa réponse en réactivité soit une fonction linéaire de sa position angulaire sur une certaine fraction de sa rotation.

Ce dispositif facilite considérablement le travail expérimental, la pile étant parfaitement stable pendant les mesures et aucune chambre ne sera influencée par la perturbation globale.

La barre comprend deux parties:
- Un stator fixe, constitué par un tube d'aluminium recouvert intérieurement de cadmium sur une fraction de sa circonférence,
- Un rotor mobile, constitué par un noyau cylindrique d'aluminium recouvert de cadmium sur une partie de sa périphérie.

Les variations de position de barre rotative sont asservies aux variations de puissance du réacteur. La chambre d'ionisation engendre sur un préamplificateur une tension proportionnelle à la puissance neutronique. En pied de la résistance de charge de la chambre est injectée une contre-tension destinée à annuler la composante continue. Ce signal résultant attaque un ensemble amplificateur de tension muni d'un réseau correcteur destiné à améliorer la réponse du servo-mécanisme.

Le signal sortant de cette première partie de la chaine électronique, est envoyé sur un amplificateur de puissance alimentant le moteur commandant la rotation de la barre. Ce dernier amplificateur est contre-réactionné en tension par un potentiomètre linéaire solidaire de l'axe du moteur, délivrant une tension proportionnelle à la variation de position angulaire de la barre et constitue ainsi un asservissement de position de la barre de pilotage.

Le moteur de commande est un moteur à courant continu « servalco » présentant une très faible inertie et un couple élevé.

Un potentiomètre linéaire, calé sur l'arbre de la barre donne une tension proportionnelle à la position angulaire. Ce signal est envoyé sur l'ensemble analyseur.

En analysant le fondamental $6\theta$ de la position angulaire on obtient une grandeur proportionnelle à la variation de réactivité produite par l'oscillation de l'échantillon.

La réponse est aussi intégrée pendant la durée de la mesure de façon à déterminer la position moyenne d'oscillation afin de tenir compte des dérives éventuelles de réactivité.

En raison de l'action du pilote, le mouvement de la barre est pratiquement en phase avec le déplacement du train.

2.4. Mesure du signal local – Chambre annulaire

La mesure du signal local s'effectue à l'aide d'une chambre d'ionisation annulaire compensée d'un type spécial. Placée dans le canal d'oscillation, elle est traversée par le train. L'échantillon se rapprochant périodiquement de la chambre, crée ainsi un signal local périodique.
Cette chambre, chargée en bore, donne un courant proportionnel à la densité locale de neutrons (abstraction faite de leur répartition en énergie). Son volume actif entourant entièrement l'échantillon, elle reçoit la plus grande partie de la perturbation locale.

Il faut cependant chercher un compromis entre la sensibilité du détecteur et la perturbation qu'il crée. En effet, une grande sensibilité nécessite un dépôt de bore important, ce qui entraîne une dépression du flux et crée ainsi une déformation du spectre au voisinage de l'échantillon. De même, les matériaux placés entre l'échantillon et le dépôt de bore (gainage, parois d'aluminium, isolants) ralentissent les neutrons de fission, augmentant ainsi la sensibilité de la chambre à la variation locale du flux rapide, ce qui réduit l'efficacité de la séparation fission-absorption.

Afin d'uniformiser l'absorption le long du canal, celui-ci est entièrement garni de chambres fictives de même géométrie.

En déplaçant la chambre par rapport au train, on peut obtenir un signal à la fréquence de l'oscillation, à la fréquence double ou un signal qui est la résultante des deux. En général on place la chambre annulaire au point correspondant à la position moyenne d'oscillation. Ainsi, à chaque période, l'échantillon traverse deux fois la chambre, créant une perturbation locale à la fréquence double de la fréquence d'oscillation.

L'utilisation de l'harmonique deux permet aussi d'éliminer presque complètement le signal local produit par les neutrons retardés et éventuellement les γ émis par le train.

La chambre d'ionisation compensée aux γ est constituée d'un assemblage de tubes d'aluminium concentriques comportant: deux enveloppes extérieure et intérieure, une électrode HT, et deux électrodes «signal». Le diamètre intérieur et la longueur sont choisis en fonction de la géométrie des échantillons.

Les deux électrodes collectant les particules ionisées sont usinées à partir d'un tube découpé en deux parties par deux hélices ayant un pas de 10 cm et distantes de 5 cm. L'une est recouverte d'un dépôt de bore d'environ 0,2 mg/cm². On réalise ainsi, en associant à chaque hélice une partie de l'électrode HT lui faisant face, deux zones actives d'environ 6 mm d'épaisseur, de volume et de géométrie identiques occupant des positions homologues par rapport à l'axe de la chambre.

L'électrode recouverte de bore collecte les particules ionisées par les α créés par la réaction (n, α) sur le bore et par les γ; l'autre collecte uniquement les particules ionisées par les γ.

Les signaux recueillis sur chaque électrode sont appliqués à un préamplificateur à deux voies. Sur la voie neutrons, est placé en pied de la résistance de charge, une contre-tension fixe destinée à annuler la composante continue. Ensuite, les signaux correspondants sont envoyés sur chacune des deux voies d'un amplificateur symétrique. On recueille en sortie leur différence, c'est-à-dire uniquement le signal dû aux neutrons, si les deux voies et les deux électrodes sont identiques.

En fait, pour assurer aussi parfaitement que possible la compensation γ on ajuste les gains de chaque voie de façon à obtenir un signal nul à la sortie de l'amplificateur pour un flux de γ sans neutrons qui peut être produit par exemple par une source γ intense.
2.5. Analyseur des signaux

La variation de la position angulaire de la barre rotative, signal global, et la variation locale du flux, signal local, sont analysées par un ensemble analogique.

Le principe de l'appareil est de réaliser le développement en série de Fourier des signaux qui lui sont appliqués à l'entrée. L'ensemble effectue la multiplication du signal par les fonctions sinus et cosinus et l'intégration du produit sur un nombre de cycles déterminé.

La multiplication se fait par l'intermédiaire d'un potentiomètre sinus-cosinus entraîné par l'arbre de l'oscillateur à la vitesse d'un ou deux tours par période suivant qu'il analyse le fondamental ou l'harmonique deux. L'intégrateur analogique est constitué par un amplificateur contre-réactionné à très grand gain et faible dérive.

Le signal de recopie de la barre est aussi intégré pendant toute la mesure, avec une constante de temps appropriée, avant multiplication par le potentiomètre sinus-cosinus. On obtient ainsi la position moyenne de la barre permettant de corriger les dérives éventuelles.

Les signaux, après intégration, sont lus sur un voltmètre digital.

2.6. Étalons de mesure

Les signaux locaux et globaux recueillis à l'analyseur doivent être étalonnés en quantité d'absorbeur et de matière fissile. Pour cela, on utilise une série de billettes étalons qui ont les mêmes dimensions et sont placées dans les mêmes gaines que les billettes à mesurer.

On dispose des jeux suivants:
- Étalons de matières fissiles:
  - U enrichi à 0,69; 0,76; 0,80; 0,83 et 0,86%.
  - Cette série permet de tracer la droite d'étalonnage en fonction de la masse de $^{235}$U.
- Étalons d'absorbeurs:
  - U naturel contenant 3, 6, 9 et 12 ppm de bore
  - $^{235}$U enrichi à 0,76; 0,80 et 0,86 contenant 3 et 9 ppm de bore.
  - Cette série permet de tracer la droite d'étalonnage en ppm de bore.
-Étalons d'uranium naturel:
  - On dispose aussi d'un jeu de billettes d'uranium naturel possédant de légères différences de masse afin de tracer la courbe utilisée pour faire la correction de masse.

3. EXPÉRIENCES SUR LES RÉSEAUX ÀEAU LOURDE

Les mesures sur les réseaux à eau lourde sont effectuées dans la zone centrale du réacteur MINERVE [3].

3.1. Description du réacteur MINERVE

Le réacteur MINERVE est une pile piscine de faible puissance. Cet assemblage critique est essentiellement utilisé pour mesurer, par la mé-
MÉTHODE D'OSCILLATION

Méthode d'oscillation, les caractéristiques neutroniques des matériaux, en particulier les sections efficaces d'absorption et de fission et les intégrales de résonance.

La principale originalité du réacteur réside dans le bloc supportant les éléments combustibles. Celui-ci est composé de quatre grilles, pouvant se déplacer sur une table suivant les deux diagonales, découvrant ainsi au centre un carré dont le côté peut varier de 0 à 75 cm. Ce dispositif permet de réserver à l'intérieur du cœur une zone centrale de section carrée, à axe vertical, dans laquelle on peut introduire les portions de réseaux que l'on veut étudier.

La zone nourricière, modérée à l'eau légère, est constituée par une couronne d'éléments combustibles type MTR enrichis à 90% en $^{235}$U. L'ensemble est entouré, sur les faces latérales, par un réflecteur de graphite gainé d'aluminium.

3.2. Conditions expérimentales

a) Réseau de base

Les réseaux étudiés dans MINERVE sont ceux de la filière uranium naturel-eau lourde.

On place au centre du cœur une cuve étanche de section carrée, de 75 cm de côté et de 90 cm de hauteur, contenant le réseau à étudier (fig. 1).

On peut réaliser des réseaux à uranium naturel, ou à uranium faiblement enrichi ou appauvri, dont on fait varier le pas. Pour le pas 160 mm, la cuve contient 25 cellules, pour le pas 110 mm, elle en contient 49.
La cellule centrale constitue le canal d'oscillation autour duquel est placée la chambre annulaire.

b) Spectre dans la zone centrale

Le point le plus important dans cette expérience est de réaliser dans la zone centrale un spectre aussi identique que possible à celui qui existe au centre d'un réseau complet.

Des mesures de comparaison entre AQUILON, expérience critique à eau lourde, et la zone centrale de MINERVE ont été effectuées avec des détecteurs par activation. On mesure dans les deux réseaux ayant des paramètres identiques les indices de spectre en particulier les rapports Pu/U et Lu/Mn.

Une seconde perturbation à craindre est celle produite par la chambre annulaire. Les mêmes mesures d'indices de spectre ont été réalisées en la remplaçant par un boîtier de volume identique mais rempli d'eau lourde.

**TABLEAU 1**

**VALEURS DES INDICES DE SPECTRE**

<table>
<thead>
<tr>
<th>MINERVE</th>
<th>AQUILON</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avec chambre annulaire</td>
<td>Avec boîtier d'eau lourde</td>
</tr>
<tr>
<td>Dans l'élément combustible</td>
<td>1,196</td>
</tr>
<tr>
<td>Dans le modérateur</td>
<td>1,085</td>
</tr>
</tbody>
</table>

On indique dans le tableau I les valeurs des indices de spectre obtenus avec les rapports Pu/U dans la cellule centrale correspondant au réseau ayant un pas de 160 mm.

Si on désigne par R l'indice de spectre, il est relié aux rapports des activités des détecteurs U et Pu par la relation:

\[ R = \frac{A_x}{A_0}, \]

où \( A_1 \) et \( A_0 \) sont les rapports des activités dans le spectre inconnu et dans le spectre thermique de référence. La précision est de \( \pm 0,01 \).

Ces résultats montrent qu'au centre de la cavité centrale, on a un spectre pratiquement identique à celui d'AQUILON, et que la perturbation due à la chambre annulaire est négligeable dans le train d'oscillation.

3.3. Sensibilités et précisions

Les échantillons mesurés sont des billettes de 29,2 mm de diamètre et de 10 cm de longueur. L'amplitude d'oscillation est de \( \pm 15 \) cm. Le point bas d'oscillation coïncide avec le maximum de flux.
Dans ces conditions, on obtient un signal global important tout en restant dans une zone où le spectre n'est pas perturbé.

Pour les conditions d'oscillation ainsi définies, on obtient les sensibilités suivantes

- signal global - 1 g de U₅ correspond à $\Delta k/k = 3 \cdot 10^{-5}$,
  1 mg de bore correspond à $\Delta k/k = -0.1 \cdot 10^{-5}$

- signal local - 1 g de U₅ correspond à $\Delta \phi/\phi = -2 \cdot 10^{-3}$,
  1 mg de bore correspond à $\Delta \phi/\phi = -5 \cdot 10^{-5}$

et les précisions de mesures suivantes

- signal global - $\Delta k/k = \pm 10^{-7}$
- signal local - $\Delta \phi/\phi = \pm 10^{-5}$

La précision atteinte sur les équivalences correspond à $\pm 4$ mg d¹²³⁵U et $\pm 0.15$ mg de bore.

L'échantillon de référence, en uranium naturel, pesant environ 1235 g, on voit qu'on peut déterminer les différences des sections efficaces effective à 1% près.

3.4. Programme expérimental

Par cette méthode on étudie actuellement la section efficace effective de fission et d'absorption d'alliage uranium-plutonium dans les réseaux à eau lourde. Les valeurs ainsi obtenues seront contrôlées par des mesures de laplacien par la méthode de substitution sur un réseau complet dans AQUILON.

Les mesures d'oscillation donnent, par rapport à l'uranium naturel, les écarts $\Delta \sigma_a$ et $\Delta \sigma_f$ des sections efficaces d'absorption et de fission de l'échantillon étudié. Les mesures de laplacien dans Aquilon sur des réseaux d'uranium naturel, appauvri et enrichi, permettent de calculer à partir des écarts des sections efficaces, la valeur du laplacien d'un réseau constitué d'éléments de composition identique à l'échantillon d'oscillation.

Les mesures actuellement en cours sur des alliages d'uranium et de plutonium permettront de vérifier si le laplacien calculé est identique au laplacien mesuré.

D'autres études sont en cours pour mesurer la variation des sections efficaces en fonction du taux d'irradiation.

Des échantillons d'alliage Pu-Al et d'uranium enrichi à 1,6% en ²³⁵U sont en cours d'irradiation sur le réacteur à eau lourde EL3. Ils seront mesurés dans MINERVE ce qui permettra, après les analyses chimique et isotopique, de déduire des mesures les sections efficaces des différents matériaux fissiles formés et la valeur globale des produits de fission. On peut aussi espérer, si la correspondance oscillation-substitution est satisfaisante, calculer les variations du laplacien d'un réseau en fonction du taux d'irradiation.
4. EXPÉRIENCES SUR LES RÉSEAUX À GRAPHITE

Les mesures sur les réseaux à graphite sont effectuées dans la zone centrale du réacteur MARIUS [4].

4.1. Description du réacteur MARIUS

Le réacteur MARIUS est une expérience critique destinée aux études sur les réseaux de la filière uranium naturel graphite à froid. C'est un empilement horizontal à pas carré reposant sur un berceau métallique à l'intérieur d'une fosse creusée dans le sol. Les barres de l'empilement sont creusées d'un canal central destiné à recevoir le combustible.

La particularité de MARIUS réside dans le fait que l'on peut faire varier le diamètre de ce canal et la distance entre canaux. Pour cela, la région centrale de l'empilement est percée de canaux de grand diamètre, que l'on peut réduire en introduisant un jeu de chemises convenable. D'autre part, il est possible d'extraire de la fosse cette région centrale et de la remplacer par une nouvelle région constituée des mêmes barres et de planches de graphite intercalaires, convenablement disposées afin d'obtenir des pas différents.

4.2. Conditions expérimentales

Le réseau environnant est constitué de cellules dont le diamètre est de 70 mm, chargé de cartouches d'uranium naturel dont le diamètre est de 29,2 mm. On fait les mesures dans les pas de 224 mm et de $224\sqrt{2} = 316,8$ mm. On oscille dans un canal voisin du centre un train d'uranium naturel (fig. 2). Les échantillons sont des billettes de 30 cm de longueur et de 29,2 mm de diamètre.

La position moyenne d'oscillation, située au quart de la longueur extrapolée, H, coïncide avec le milieu de la chambre annulaire. L'amplitude d'oscillation est égale à H/8. Ces conditions correspondent à un signal de réactivité pour lequel le fondamental est maximum et qui ne contient pas d'harmoniques pairs. Le signal local est à la fréquence double du mouvement.

Comme pour les réseaux à eau lourde, on a vérifié que la chambre annulaire ne perturbait pas le spectre au voisinage de l'échantillon.

4.3. Sensibilités et précisions

Dans les conditions d'oscillation, on obtient les sensibilités suivantes, pour le pas de 224 mm:

- signal global - 1 g de U₅ correspond à $\Delta k/k = 0,6 \cdot 10^{-5}$,
  1 mg de bore correspond à $\Delta k/k = -2 \cdot 10^{-7}$,

- signal local - 1 g de U₅ correspond à $\Delta \phi/\phi = -2,5 \cdot 10^{-4}$,
  1 mg de bore correspond à $\Delta \phi/\phi = -4 \cdot 10^{-5}$,
et les précisions de mesures suivantes:

- signal global \(-\Delta k/k = \pm 10^{-7}\),
- signal local \(-\Delta \phi/\phi = \pm 10^{-5}\).

La précision atteinte sur les équivalences correspond à \(\pm 20\) mg de \(^{235}\)U et à \(\pm 0,3\) mg de bore.

L'échantillon de référence ayant une masse d'environ 3710 g, on voit que l'on peut déterminer les différences des sections efficaces effectives, comme pour MINERVE à 1% près.

4.4. Programme expérimental

Comme pour les réseaux à eau lourde, on détermine les sections effectives d'alliage uranium-plutonium pour différents paramètres de réseaux.

Avec les valeurs obtenues, on calcule le laplacien et on vérifie qu'il est identique à celui mesuré directement par la méthode de substitution.

Des échantillons d'uranium naturel irradiés dans les réacteurs de puissance sont mesurés dans les mêmes conditions avant de pratiquer les analyses chimiques.

5. INTERPRÉTATION DES MESURES

Pour obtenir une interprétation satisfaisante des signaux, il faut remarquer que ceux-ci traduisent la différence entre l'échantillon à mesurer
et le train de référence. L'échantillon de référence est une billette d'uranium naturel qui, dans les conditions idéales, donne un signal nul. Pour tous les autres échantillons enrichis, ou empoisonnés, les signaux seront rapportés à la billette d'uranium naturel. Ils seront donc proportionnels à la masse de bore ajouté et s'il s'agit d'échantillons enrichis, aux différences de masse d'\(^{235}\text{U}\) et d'\(^{238}\text{U}\).

Puisqu'on veut étalonner les axes en masse d'\(^{235}\text{U}\), il est important de corriger les signaux de chaque échantillon pour les ramener à la valeur qu'ils auraient s'ils avaient pour le même enrichissement une masse telle que le nombre de noyaux d'\(^{238}\text{U}\) soit identique à celui de la billette d'uranium naturel prise comme référence. Les signaux seront ainsi proportionnels uniquement à la différence de masse d'\(^{235}\text{U}\).

5.1. Représentation graphique des résultats

Chaque billette étalon, après correction de masse est caractérisée par la masse \(m\) d'\(^{235}\text{U}\) qu'elle contient en excès par rapport à la billette d'uranium naturel et par la masse \(\mu\) de bore.

Les mesures donnent, pour chaque échantillon, les deux quantités: \(G\), signal global et \(L\), signal local.

Si on tient compte des signaux \(G_0\) et \(L_0\) produits par le train de référence, les résultats de chaque billette peuvent s'exprimer par les relations dérivées de (14) et (15)

\[
G = G_0 + B_g \mu + U_g m,
\]

\[
L = L_0 + B_f \mu + U_f m,
\]

A l'intérieur des marges d'erreur dont sont affectées les mesures, les résultats montrent que les coefficients \(B_g\), \(B_f\) et \(U_g\), \(U_f\) sont constants, les relations précédentes représentant donc des droites.

Ainsi, sur un diagramme où l'on porte \(G\) en ordonnée et \(L\) en abscisse, on peut définir un autre système d'axes linéaires correspondant, l'un à l'enrichissement exprimé en grammes d'\(^{235}\text{U}\), l'autre à l'empoisonnement exprimé en grammes de bore.

En portant sur le diagramme \((G, L)\) les résultats des mesures faites sur une billette inconnue, on déduit directement ses coordonnées dans l'autre système, ce qui donne \(m\), la masse d'\(^{235}\text{U}\) équivalente et \(\mu\), la masse de bore équivalente.

Ces valeurs représentent les différences entre la billette considérée et la billette de référence contenant la même masse d'uranium naturel.

Cette représentation graphique a été plus particulièrement étudiée dans un réseau uranium naturel-graphite au pas de 244 mm (fig. 3).

5.2. Calcul des sections efficaces effectives

Les résultats des mesures après lecture sur les diagrammes sont exprimés en g d'\(^{235}\text{U}\) et en g de bore. Ces unités découlent naturellement de la technique de mesure. Elles ne sont cependant pas utilisables directement
MÉTHODE D'OSCILLATION

Fig. 3

Diagramme de $G$ en fonction de $L$ dans MARIUS (diamètre: 29.2 mm, pas: 224 mm).

dans les calculs d'évolution des combustibles. Il faut ensuite les traduire en barns de sections efficaces effectives.

Ce nouvel étalement des axes permet de placer l'axe fission sur les diagrammes.

Une droite parallèle à l'axe des fissions peut être graduée en valeur de $\tilde{\eta}$. Si $M$ désigne le point du diagramme déterminé expérimentalement, l'intersection du segment $OM$ et de la droite donne la valeur du $\tilde{\eta}$ du matériau qui, ajouté à une quantité d'uranium naturel égale à celle de la billette de référence, reconstitue la billette considérée.

Pour un échantillon d'uranium naturel contenant environ 0,05% de Pu dans le réseau au pas 224 mm, on a trouvé pour le Pu

$$\frac{\sigma_f}{\sigma_a} = 0,666 \pm 0,004.$$ 

Les tables de Cogné établies pour le modèle graphite avec les paramètres de spectre du réseau correspondant donnent

$$\frac{\sigma_f}{\sigma_a} = 0,673.$$
Les valeurs approchées trouvées pour $\eta$ sont les suivantes:

- pas 224 mm: $\eta = 1,844$,
- pas 316,8 mm: $\eta = 1,874$.

CONCLUSION

Les études concernant la méthode que nous avons mise au point ne sont pas encore terminées. Il reste vraisemblablement certaines conditions d'utilisation à préciser ainsi que ses limites d'application. Enfin, les expériences en cours montreront les améliorations à lui apporter, en particulier en ce qui concerne l'appareillage, afin d'atteindre la sensibilité maximum.

Les résultats obtenus et les précisions atteintes montrent que la méthode est très satisfaisante et qu'il est possible de l'appliquer pour mettre en évidence des effets plus fins concernant en particulier les répartitions radiales des absorbents et matériaux fissiles dans les billettes irradiées.

Cette méthode est aussi d'un grand intérêt pour l'étude de l'évolution de la réactivité des réacteurs de puissance. Il est en effet impossible de faire des mesures de laplacien sur des réseaux irradiés. Par contre, avec les méthodes d'oscillation, on peut définir pour un échantillon irradié un échantillon reconstitué qui lui est équivalent et calculer ou mesurer le laplacien de ce réseau reconstitué.

Ces mesures seront bientôt appliquées sur des expériences critiques chaudes afin d'obtenir la gamme complète des sections efficaces effectives pour différents réseaux et différentes températures.

RÉFÉRENCES


DISCUSSION

G. BLAESSER: We have been carrying out fairly extensive studies at Ispra to investigate different methods of oscillation. On the whole, our conclusions on the possibilities of determining the absorption and the production of the sample separately are similar to yours*. There are, however, slight differences. While our results show your Eq.(2) to be correct, we found

* See EURATOM Document EUR/221e.
that the expression corresponding to Eq. (1) can be considered only as an approximation which is valid when the detector is several mean paths away from the sample. Where this is not the case, the detector response depends not only on the total effective absorption cross-section (integrated over all energies) but also on the details of the cross-section as a function of energy.

I should like to make one further remark on the measurement of larger samples. Larger samples can be measured. The important thing is to ensure that the ratio of the thermal to the epithermal flux is not altered appreciably. The measurement then gives one the difference in effective (self-shielded) cross-sections or total absorption (and production) rather than the difference in the spectral averaged microscopic cross-sections. Even a slight modification in the geometry can be permitted.

R. VIDAL: On the whole I would agree with you. I should, however, like to point out that we try to keep the samples as small as possible so as to have a minimum of perturbations and that our objective is to determine an equivalence between an unknown sample and a uranium-boron sample. If the samples and thus the perturbations are sufficiently small, we can determine equivalent samples. We preferred to use small samples, as can be seen in the diagram shown in the paper, where the $^{235}\text{U}$ and the boron axes are practically straight lines. The use of larger samples increases perturbation, e.g. absorption perturbation, and this produces curved axes. Of course, it would be possible to plot curved axes but that would involve making a number of corrections. Moreover, the method is sufficiently sensitive as it is, so that it is not really necessary to use very large samples.

A. CAMPISE: What kind of stable period can you obtain with your critical assembly?

R. VIDAL: Two factors can limit the sensitivity of the assembly - pile fluctuations, or reactivity drifts caused by pressure or temperature changes. The measurements are carried out with MINERVA, which is a swimming-pool reactor. The pressure coefficient is therefore zero and the temperature drifts are less than 0.01°C for the duration of one measurement. As the temperature coefficient is near zero, the only factor limiting sensitivity for oscillation periods of around 15 s is neutron fluctuation which, at a $\Delta K/K$ of $10^{-8}$, is well below our limit of sensitivity.

A. CAMPISE: The form of your equations seems to imply that the fission-neutron importance and the absorption-neutron importance are identical, which is not true. Would you care to comment on this?

R. VIDAL: In my equations I defined two coefficients, $A$ and $B$, which are quite different from one another, they are even of opposite sign. The magnitude of these coefficients no longer interests me once I can determine them with standards.

A. CAMPISE: You have a way, then, of differentiating between the fission importance and the absorption importance?

R. VIDAL: Certainly. The difference in importance between the absorbed thermal neutrons and the fission neutrons produced constitutes the basis of our method.

C.E. COHN: On this question of the stability of the reactor with respect to drifts, I might point out that Bennett and Long at Argonne have done a series of studies involving reactivity measurement with an automatic control rod. They have developed a method for treating the data by which
the effect of extraneous reactivity drifts can be eliminated providing these
drifts vary no more rapidly than the second power of the time.

There are one or two questions I should like to ask. How did you make
sure that the motion of the regulating rod would not perturb the local effect
and that the purely local effect would not affect the regulating rod? How
would the experiment be affected if these conditions were not perfectly
fulfilled?

R. VIDAL: As you can see from Fig. 2, the rotating rod is on the peri-
phery of the reference zone while the chamber for measuring the local signal
is in the centre. There is therefore a distance of about two metres between
the two. As the rod has a total reactivity effect of $\Delta K/K = 2 \times 10^{-4}$, no ap-
preciable perturbation can be detected on the local signal. Similarly, since
the chamber controlling the mechanism of the rotating rod is on the outer
periphery, i.e. three metres from the sample, the local perturbation of the
sample is attenuated before it reaches it.

If, however, reactivity compensation is not perfect, the local signal
still cannot be affected for the following reason. The local chamber is in the
middle of the total oscillation run so that for each period traversed twice
by the sample it emits a signal at a frequency double the value of the os-
cillation frequency. One thus measures the local signal on the second har-
monic; the total signal is given by the fundamental.

U. FARINELLI: From the point of view of studying the long-term be-
haviour of specific reactors, it would be extremely useful if one could
measure the effective cross-sections of fuel in a spectrum as close as pos-
sible to that of the pile at the same burn-up. Do you plan to carry out such
experiments, using irradiated fuel in the reference lattice as well as in the
sample?

R. VIDAL: Obviously, our data are not as complete as they would be
if we carried out our experiments in an irradiated-fuel lattice. There is a
plan to carry out experiments with the new graphite critical assembly
CESAR. This would involve placing a number of irradiated fuel cells near
the oscillation channel. It may be that some solution will soon be found to
the problems posed by the activity of these cells but, in our opinion, ex-
tremely interesting data can be obtained with measurements made in a non-
irradiated lattice at high temperatures and varying CO$_2$ pressures. Using
this technique it is possible to determine the U$^{235}$ and boron fuel equivalent
of the irradiated fuel, build a reference lattice with this fuel equivalent, and
then perform the same measurements. An experiment of this type is at
present being carried out with U-Pu alloys. Measurements are being ob-
tained in a natural uranium reference lattice and will be performed over
again in a lattice made up of U-Pu alloy fuel elements.

F. STORRER: The accuracy you quote for the effective cross-sections
is quite remarkable. While I can see that projection on to the boron and
uranium axes will yield a reasonably high degree of accuracy, it seems to
me that the degree of accuracy will be much lower when the projection is
made on to the absorption and production axes, since the latter form an angle
of nearly 180°. It will be more difficult to discriminate between absorption
and production than between boron equivalent and uranium equivalent. In
other words, determinations of effective absorption and production cross-
sections will probably not be very accurate.
R. VIDAL: The diagram shown in our paper is based on measurements made a year ago. At that time the annular chamber was insulated with plexiglass and lined with graphite to facilitate the movement of the fuel element. The moderating materials slowed up the fission neutrons, thus increasing their effect on the local detector.

Measurements are now made with an improved version of the chamber in which most of the moderating materials have been eliminated. This makes it possible to reduce the angle between the absorption and the production axes.

I should like to emphasize once again, however, that, from the point of view of studying the reactivity pattern of power reactors, it is just as important to obtain $^{235}\text{U}$ and boron equivalences as it is to obtain effective neutron production and absorption cross-sections.
APPLICATION OF THE PULSED-NEUTRON TECHNIQUE TO CRITICAL AND SUBCRITICAL ASSEMBLIES.

The pulsed-neutron technique was used in different critical experiments. This technique is particularly interesting since it can be used without altering the reactor structures. Its application requires only a time selector and pulsed sources which, because of their low density, are introduced on a small scale into the reactor or its reflector. The technique can be used to measure the neutron lifetime and calibrate control rods quickly and in perfectly safe conditions.

Two different RUBÉOLE cores were studied in a critical experiment with a beryllium-oxide moderator and slightly enriched uranium-oxide fuel, and the negative importance function was measured in different arrays: (a) Reduction in the volume of the non-reflected core; (b) Introduction of cadmium control rods; (c) Unloading of peripheral fuel; (d) Fuel replacement.

Other experiments reported (ALIZÉ: light water and uranium enriched to 90%; AQUILON: heavy water and natural uranium; MARIUS: graphite and natural uranium) show that the technique is applicable to natural-uranium reactors.

Using a non-portable neutron generator, the same technique was employed to measure graphite diffusion parameters. The buckling range was between 7 m$^{-2}$ and 155 m$^{-2}$ and the following results were obtained:

- Diffusion coefficient $D_0 = (2.19 \pm 0.03) \times 10^5$ cm$^2$ s$^{-1}$
- Cooling coefficient $C = (37.9 \pm 4) \times 10^5$ cm$^4$ s$^{-1}$
- Graphite anisotropy $\frac{D_n}{D_{||}} = 1.017 \pm 0.008$

These results are compared with those obtained by other authors.
МЕТОД ИМПУЛЬСНЫХ НЕЙТРОНОВ, ПРИМЕНЯЕМЫЙ К КРИТИЧЕСКИМ И ПОД-КРИТИЧЕСКИМ СБОРКАМ. Использовался метод пульсирующих источников нейтронов в различных критических опытах. Этот метод весьма интересен тем, что его применение не требует никакого изменения в структуре реакторов: достаточно иметь временный селектор и пульсирующие источники, которые ввиду их небольшого объема легко вводятся в реактор или его отражатель. Метод позволяет производить измерение периода жизни нейтронов и быстрое калибрование поглощающих стержней в великолепных условиях безопасности.

Нами также исследованы две различные активные зоны Рубеолы, критический опыт с замедлением при помощи окиси бериллия, содержащей слегка обогащенный окись урана, а также измерены значительные отрицательные реактивности, введенные при: а) уменьшении размера зоны без отражателя; б) наличии кадмиевых поглощающих стержней; в) разгрузке периферического топлива; г) замене топлива.

Другие опыты были проведены на установках ALIZE (обычная вода и обогащенный до 90% уран), AQUILON (тяжелая вода и природный уран), MARIUS (графит и природный уран). Опыты показывают, что метод применим к реакторам на природном уране.

Тот же метод, но на этот раз с использованием непортативного генератора нейтронов, был применен для измерения параметров диффузии графита. Диапазон лапласианов покрывает от $7 \text{м}^{-2}$ до $155 \text{м}^{-2}$; получены следующие результаты:

- коэффициент диффузии $D_0 = (2,19 \pm 0,03) \times 10^5 \text{см}^2/\text{сек}$;
- коэффициент охлаждения $C = (37,9 \pm 4) \times 10^5 \text{см}^4/\text{сек}^2$;
- анизотропия графита $D_\perp/D_\parallel = 1,017 \pm 0,008$.

Эти результаты сравниваются с результатами, полученными другими авторами.

TÉCNICA DE NEUTRONES PULSANTES APLICADA A LOS CONJUNTOS CRÍTICOS Y SUBCRÍTICOS. Los autores han aplicado el método de las fuentes neutrónicas pulsantes a diversos experimentos críticos. Ese método presenta particular interés porque su ejecución no exige modificar la estructura de los reactores: basta disponer de un selector de tiempo y de fuentes pulsantes que, debido a su pequeño volumen, son fáciles de introducir en el reactor o su reflector. El método permite medir la vida de los neutrones y calibrar rápidamente las barras de absorción en excelentes condiciones de seguridad.

También han estudiado dos cuerpos diferentes de RUBEOLÉ, experimento crítico moderado con óxido de berilio cargado con óxido de uranio ligeramente enriquecido, midiendo considerables reactividades negativas introducidas bajo diversas formas: a) disminución del volumen del cuerpo sin reflector, b) introducción de barras absorbadoras de cadmio, c) descarga de combustible periférico, y d) sustitución de combustible.

La memoria informa sobre otros experimentos realizados con ALIZÉ: agua ligera y uranio enriquecido al 90%; AQUILON; agua pesada y uranio natural; MARIUS: grafite y uranio natural. Muestran que el método es aplicable a los reactores de uranio natural.

Con un generador neutrónico no portátil, se ha aplicado la misma técnica a la medición de los parámetros de difusión del grafite. La gama de laplacianos abarca de $7 \text{м}^{-2}$ a $155 \text{м}^{-2}$. Los resultados obtenidos son los siguientes:

- Coeficiente de difusión $D_0 = (2,19 \pm 0,03) \times 10^5 \text{см}^2/\text{сек}$
- Coeficiente de enfriamiento $C = (37,9 \pm 4) \times 10^5 \text{см}^4/\text{сек}^2$
- Anisotropía del grafite $D_\perp/D_\parallel = 1,017 \pm 0,008$.

Los resultados se comparan con los publicados por otros autores.

INTRODUCTION

Le principe de la technique qui utilise des sources pulsées de neutrons pour obtenir certains paramètres liés à la physique des réacteurs est suffisamment connu pour qu'il ne soit pas nécessaire de l'exposer ici. L'excellent article de BECKURTS [1] fait le point de l'état et des promesses de cette technique jusqu'au mois d'octobre 1960. Depuis cette date, le nombre de résultats publiés par de nombreuses équipes montre bien l'intérêt que présentent les sources pulsées de neutrons pour l'étude des réacteurs et des modérateurs. Nous utilisons cette méthode à Saclay depuis environ
deux années dans plusieurs expériences critiques très différentes les unes des autres, et il s'est avéré qu'elle était, dans tous les cas, très facilement mise en œuvre et permettait d'obtenir rapidement des résultats intéressants. Très souvent ceux-ci ont été comparés aux valeurs calculées ou aux résultats obtenus par une autre technique expérimentale.

Nous donnons, dans la partie A, des exemples des principales expériences que nous avons réalisées sur les milieux reproducteurs qui sont essentiellement des mesures du rapport de la fraction effective des neutrons retardés au temps de vie des neutrons prompts dans la pile critique $\beta_{\text{eff}}/\tau$ et des mesures de réactivités négatives. Nous avons, par ailleurs, réalisé un programme expérimental de mesures des paramètres de diffusion du graphite dont nous traitons dans la partie B de ce rapport.

A. ÉTUDES SUR EXPÉRIENCES CRITIQUES

1. Rappel de la méthode

De brèves bouffées de neutrons sont envoyées à intervalles réguliers dans le milieu multiplicateur dont on étudie la réponse impulsionnelle. Au bout d'un temps suffisamment long après la fin d'une bouffée, afin que les neutrons émis par la source soient ralentis, l'expression du flux, en géométrie cylindrique, obtenue à partir des équations de la diffusion à deux groupes, est la suivante:

$$\varphi = \sum_{l,m,n} B_{l,m,n} \left[ e^{-\alpha_{l,m,n} t} + C_{l,m,n} e^{-\alpha_{l,m,n} t} \right],$$

où $B_{l,m,n}$ est indépendant du temps,

$$\alpha_{l,m,n} = \frac{\beta_{\text{eff}} \left[ 1 - \frac{\Delta k_{l,m,n} (1 - \beta_{\text{eff}})}{\beta_{\text{eff}}} \right]}{\tau_{l,m,n}} - \lambda,$$

$$\alpha'_{l,m,n} = \frac{\lambda \Delta k_{l,m,n}}{\beta_{\text{eff}} - \Delta k_{l,m,n}},$$

$$C_{l,m,n} = \frac{\lambda}{\beta_{\text{eff}} / \tau_{l,m,n}} \left( \frac{\beta_{\text{eff}}}{\beta_{\text{eff}} - \Delta k_{l,m,n}} \right)^2,$$

et où $l,m \geq 1; n \geq 0$.

Dans ces expressions, $\tau_{l,m,n}$ est le temps de vie des neutrons de l'harmonique considéré, soit

$$\tau_{l,m,n} = \frac{\tau_1}{1 + \frac{\tau_1}{L_{B_{l,m,n}}^2} + \frac{\tau_2}{1 + \frac{\tau_2}{L_{B_{l,m,n}}^2}},$$

où $\tau_1$ et $\tau_2$ sont respectivement le temps de ralentissement et le temps de vie à l'état thermique des neutrons en milieu infini, où $1/\lambda$ est le temps de vie moyen des émetteurs de neutrons retardés, et où $\Delta k_{l,m,n} = (k_{\text{eff}})_{l,m,n} - 1$.

Le second terme de l'expression du flux correspond aux neutrons retardés et peut être considéré comme constant pendant la décroissance du
premier terme. L'accumulation du flux résiduel correspondant à ce second terme crée donc une composante continue qui vient s'ajouter à celle due à la source constante de fissions spontanées existant dans le milieu. Ces deux composantes continues sont très faibles lorsque le facteur de multiplication $k_{\text{eff}}$ est très inférieur à l'unité, mais elles deviennent importantes et gênantes lorsque l'on se trouve très près de la criticalité: la première est alors atténuée en espaçant les bouffées de neutrons. Les harmoniques d'ordre supérieur décroissent plus rapidement que le terme fondamental, et les positions des sources et des détecteurs sont correctement choisies pour éviter l'excitation et la détection de ces harmoniques: la variation du flux, peu de temps après l'envoi de la bouffée de neutrons, peut donc être simplifiée:

$$\varphi = Be^{\alpha t} + F$$

avec

$$\alpha = \frac{\beta_{\text{eff}}}{\tau} \left[ 1 - \frac{\Delta k (1 - \beta_{\text{eff}})}{\beta_{\text{eff}}} \right] - \lambda.$$

Nous ne faisons pas l'approximation qui consiste à négliger le terme $\lambda$ lorsqu'il s'agit d'un réacteur proche de la criticalité dont le temps de vie est assez grand (de l'ordre de 1 ms). Ce terme est cependant toujours négligeable lorsque $k_{\text{eff}}$ est nettement inférieur à l'unité. La mesure de $\alpha$ permet donc de déterminer l'un des deux paramètres $\beta_{\text{eff}}/\tau$ ou $\Delta k$ lorsque l'autre est connu.

2. Appareillage expérimental

La facilité avec laquelle nous avons pu travailler sur différentes expériences critiques tient essentiellement au fait que le matériel utilisé est très facilement transportable et ne nécessite aucun aménagement spécial autour des réacteurs.

Les bouffées de neutrons sont émises par des sources de faible volume dont le nombre varie entre 1 et 4. Chaque source est consituée par une enveloppe cylindrique en aluminium dont le diamètre extérieur est de 9,8 cm et dont la longueur est de 45,4 cm, qui contient un transformateur d'impulsion alimentant une ampoule à vide renfermant la cible de tritium et la source de deutérium. Un champ magnétique convenable est entretenu par des aimants permanents entourant l'ampoule. Des impulsions de 3 kV sont envoyées à l'entrée du transformateur qui délivre à l'ampoule des impulsions de 150 kV. Le deutérium présent dans l'ampoule est alors ionisé, et les deutons accélérés vers la cible où ils produisent les neutrons par réaction $(d, t)$.

Les sources sont synchronisées et déclenchées par un dispositif commun. La fréquence maxima de répétition est de 10 bouffées/s, les fréquences utilisées allant de 10 à 0,1 bouffées/s, suivant l'écart à la criticalité du réacteur. Chaque source délivre environ $4 \cdot 10^6$ neutrons par bouffée dont la durée est de 3 à 4 µs et est capable de fournir plus de 100 000 bouffées; certaines ont en effet donné 400 000 bouffées de neutrons sans que leur inten-
sité baisse sensiblement. Les câbles d'alimentation de la source sont à basse tension et peuvent être de grande longueur: nous utilisons en général 30 m de câble entre les sources et l'électronique associée, ce qui permet de garder celle-ci en salle de contrôle.

Le détecteur de neutrons est un compteur BF₃, sauf sur PROSERPINE où des chambres à fission de faible volume ont été utilisées, suivi d'un analyseur en temps qui était au début un analyseur ELA à tubes, puis un analyseur TMC transistorisé à 256 canaux. Les nombreux avantages de ce dernier peuvent être rapidement énoncés: le temps de résolution de son électronique est très faible, beaucoup plus faible que celui du compteur BF₃; son transport est très facile et il est très robuste, ce qui est indispensable à son déplacement d'un montage expérimental à un autre; enfin nous pensons qu'il est nécessaire de disposer d'un grand nombre de canaux d'analyse afin de pouvoir déterminer avec précision la composante continue du flux (que l'on peut obtenir également avec le 1er canal de largeur variable du sélecteur) et d'ajuster a posteriori, au cours du dépouillement, le retard à l'analyse par la suppression d'un certain nombre des premiers canaux. Il présente cependant un inconvénient majeur lorsque l'on travaille sur un milieu à court temps de vie (réacteur à H₂O et ²³⁵U, modérateur hydrogéné). Le temps de mise en mémoire des informations recueillies dans chaque canal demande une durée de 10 μs, ce qui représente une perte d'information trop importante.

De notre point de vue, l'analyseur en temps souhaitable dans le domaine des neutrons thermiques devrait avoir un temps mort, après chaque canal, de l'ordre de 2 μs et posséder une double entrée, de manière à pouvoir travailler simultanément avec deux détecteurs sur environ 200 canaux chacun. Cette double entrée serait surtout utile à l'étude des modérateurs, car elle permettrait une étude plus rapide des harmoniques et du spectre neutronique.

Les pertes de comptage de l'ensemble BF₃ - TMC sont dues uniquement au BF₃ dont le temps de résolution est mesuré par la méthode des sources additionnelles. Il ne semble pas possible de réduire ce temps de résolution au delà de 3 μs; comme nous nous imposons des corrections de pertes de comptage faibles dans les canaux les plus chargés (inférieures à 5%), ce temps de résolution encore assez élevé, nous oblige parfois à utiliser des détecteurs peu sensibles, ce qui n'est pas souhaitable. Les scintillateurs à LiI délivrent des impulsions légèrement plus courtes, de l'ordre de 2 μs, mais leur sensibilité au rayonnement γ, plus grande que celle des compteurs BF₃, nous amène à préférer ces derniers. Il est évident qu'un détecteur de neutrons thermiques ayant une sensibilité de l'ordre de 10 à 20 coups/neutron, peu sensible au rayonnement γ, et possédant un temps de résolution de l'ordre 0,1 μs serait très utile car il réduirait très sensiblement la durée des expériences.

Le diagramme de la figure 1 schématisé le montage électronique généralement utilisé. Une mesure de décroissance nécessite entre 2000 et 8000 bouffées de neutrons, et la plupart du temps sa durée est de l'ordre de 45 min. Elle est plus grande lorsque l'on est très éloigné de la criticalité ou au contraire très proche. Dans ce dernier cas (entre 50 et 300 ·10⁶) cette durée est de 2 à 3 h, imposée par la fréquence de répétition très faible (0,3 à 0,1 bouffée/s).
3. Expériences critiques étudiées

La description des cinq expériences critiques étudiées a déjà été publiée dans les rapports signalés en [2, 3, 4, 5, 6].

ALIZÉ II est un réacteur à eau légère et uranium enrichi à 90%; le combustible est un alliage U-Al sous forme de plaques placées dans des boîtes. Les variations de réactivité sont obtenues en général par changement de la concentration du bore dans le modérateur. Les sources sont placées à l’extérieur de la cuve, les compteurs dans l’eau, contre le cœur (fig. 2).

AQUILON est modéré à l’eau lourde dans laquelle plongent les barres d’uranium ou d’oxyde d’uranium naturel. La réactivité est définie par l’ajustement du niveau d’eau lourde. Les sources sont au dessus du niveau d’eau lourde, les compteurs sont dans les canaux horizontaux du réflecteur de graphite inférieur (fig. 2).

MARIUS est constitué par un empilement de graphite à canaux horizontaux chargés de cartouches d’uranium naturel. La réactivité est liée au nombre de canaux chargés. Les sources sont au bout des canaux centraux, le détecteur est dans un canal vide du réflecteur radial (fig. 2).

PROSERPINE est une expérience critique homogène dont la cuve qui reçoit la solution d’uranium 235 dans l’eau est entourée d’un réflecteur d’oxyde de béryllium suivi d’un réflecteur de graphite. La réactivité est définie par l’ajustement du niveau de la solution. Les sources sont à l’extérieur du réflecteur de graphite, ce qui diminue leur efficacité, et les chambres à fission sont contre la cuve (fig. 3).

RUBÉOLE est un massif d’oxyde de béryllium à canaux verticaux sans réflecteur radial, placé entre deux réflecteurs empoisonnés au cadmium. Le niveau de séparation du cœur et du réflecteur supérieur est variable,
Figure 1
Positions des sources et compteurs dans ALIZÉ, AQUILON et MARIUS.

Figure 2
Positions des sources et compteurs dans PROSERpine et RUBÉOLE.
ce qui donne une grande marge de réactivité. Le combustible est constitué de grappes d'oxyde d'uranium légèrement enrichi (1,51% et 3,49%). Les sources et les compteurs sont à l'extérieur du cœur (fig. 3).

4. Mesures du rapport $\beta_{\text{eff}} / \tau$

Ce rapport est obtenu par extrapolation des mesures de $\alpha$ effectuées sur le réacteur dont les écarts à la criticalité sont connus:

$$\alpha = \frac{\beta_{\text{eff}}}{\tau} \left[ 1 - \frac{\Delta k (1 - \beta_{\text{eff}})}{\beta_{\text{eff}}} \right] - \lambda.$$ 

A la criticalité, $\alpha = \alpha_c$ et $\beta_{\text{eff}} / \tau = \alpha_c + \lambda$.

L'étalonnage en réactivité peut se faire par approche sur-critique classique: on mesure le temps de doublement du réacteur pour différents chargements, et on en déduit, par l'intermédiaire de la formule de Nordheim, la valeur en réactivité différentielle du paramètre variable qui peut être la hauteur du cœur, le nombre d'éléments de combustible chargés ou la concentration en bore du modérateur.

En fait, la variation de $\alpha$ n'est linéaire que dans la mesure où $\beta_{\text{eff}}$ et $\tau$ sont des constantes; en réalité, le temps de vie est fonction lui aussi du laplacien géométrique par les termes de fuites ou bien de la teneur en bore par le terme d'absorption. De même le facteur d'efficacité des neutrons retardés dépend des fuites. Lorsque les mesures ne portent pas sur un milieu éloigné de la criticalité (entre $-50 \cdot 10^{-5}$ et $-1 000 \cdot 10^{-5}$), ces corrections sont cependant négligeables et la variation de $\alpha$ reste linéaire. Si l'on s'éloigne franchement de la criticalité, on peut alors soit corriger par le calcul les valeurs expérimentales, soit admettre une variation parabolique de $\alpha$ en fonction de $\Delta k$ et traiter en conséquence les points expérimentaux par la méthode des moindres carrés. Nous avons ainsi poussé très loin les mesures sur RUBÉOLE ($\Delta k = 17 000 \cdot 10^{-5}$), et les résultats obtenus par les deux méthodes concordent très bien. Par contre les corrections dues aux fuites que nous apportons par le calcul au rapport $\beta_{\text{eff}} / \tau$ sont très nettement sous-estimées, mais il est possible qu'une variation d'économie du réflecteur axial empoisonné, dont nous ne tenons pas compte, explique la différence.

Le tableau I compare les résultats expérimentaux pour les réacteurs ALIZÈ, AQUILON, MARIUS et PROSERPINE, aux valeurs calculées, le temps de vie étant obtenu par la formule:

$$\tau = \frac{\sum \left[ \frac{1}{V_1} \int_{\text{milieu}} \Phi^* \Phi_1 \, dV + \frac{1}{V_2} \int_{\text{milieu}} \Phi^* \Phi_2 \, dV \right]}{\sum \int_{\text{cœur}} (\nu \Sigma_f)_1 \Phi^* \Phi_1 \, dV + \int_{\text{cœur}} (\nu \Sigma_f)_2 \Phi^* \Phi_2 \, dV}.$$ 

Les intégrales sont obtenues à l'aide du code RIFIFI [7].

Les conditions expérimentales étaient les suivantes:

| ALIZÈ  | Cœur de 8X8 boîtes; chaque boîte est chargée de 12 plaques d'uranium enrichi $^{235}U$ et de 28 plaques d'aluminium. Variation de réactivité |
VALEURS EXPÉRIMENTALES ET CALCULÉES DES PARAMÈTRES $\beta_{\text{eff}}/\tau$, $\beta_{\text{eff}}$ ET $\tau$ SUR ALIZÉ, AQUILON, MARIUS ET PROSERPINE

<table>
<thead>
<tr>
<th></th>
<th>ALIZÉ</th>
<th>AQUILON</th>
<th>MARIUS*</th>
<th>PROSERPINE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{\text{eff}}/\tau$ (s^{-1})</td>
<td>Expérience</td>
<td>102,1 ± 2</td>
<td>6 ± 0,2</td>
<td>5,515</td>
</tr>
<tr>
<td></td>
<td>Calcul</td>
<td>107</td>
<td>6,1</td>
<td>5,456</td>
</tr>
<tr>
<td>$\beta_{\text{eff}}$ (10^{-5})</td>
<td>Expérience</td>
<td>-</td>
<td>-</td>
<td>715</td>
</tr>
<tr>
<td></td>
<td>Calcul</td>
<td>709</td>
<td>714</td>
<td>724</td>
</tr>
<tr>
<td>$\tau$ (µs)</td>
<td>Expérience</td>
<td>-</td>
<td>-</td>
<td>1296</td>
</tr>
<tr>
<td></td>
<td>Calcul</td>
<td>66,6</td>
<td>1170</td>
<td>1327</td>
</tr>
</tbody>
</table>

* Aucune marge d'erreur n'est donnée pour MARIUS, car ces résultats sont provisoires.

par changement de la concentration en bore: 5 points de mesure pour $\Delta k$ allant de $-30 \cdot 10^{-5}$ à $-280 \cdot 10^{-5}$.

AQUILON Cœur constitué par 88 grappes de 7 barres d'oxyde d'uranium naturel (diamètre: 22 mm) placées dans des tubes d'aluminium (diamètre intérieur: 106 mm), les grappes étant réparties suivant un réseau carré. Variation de réactivité par changement du niveau d'eau lourde: 6 points de mesure, $\Delta k$ allant de $-600 \cdot 10^{-5}$ à $-2 500 \cdot 10^{-5}$.

MARIUS Cœur constitué d'une zone centrale de 144 canaux au pas carré de 224 mm, entouré d'une zone périphérique de 94 canaux au pas carré de 192 mm. Les canaux sont au diamètre 70 mm et sont chargés d'uranium naturel de diamètre 31 mm. Variation de réactivité par déchargement des canaux périphériques: 7 points de mesure, $\Delta k$ allant de $94 \cdot 10^{-5}$ à $-920 \cdot 10^{-5}$.

PROSERPINE Solution de sulfate d'uranium (enrichissement 90%) acide à 0,1 N, la concentration en uranium étant de 45 g/l. Variation de réactivité par changement du niveau de la solution: 14 points de mesure, $\Delta k$ allant de $-100 \cdot 10^{-5}$ à $-1350 \cdot 10^{-5}$. Les valeurs de $\beta_{\text{eff}}$ sont obtenues en extrapolant les droites $\alpha = f(\Delta k)$ au prompt critique.

Une autre expérience a été faite dans ALIZÉ en faisant varier la réactivité par enlèvement de quelques plaques des boîtiers périphériques. Cette expérience a été répétée sur 3 cœurs de géométries différentes, mais
de mêmes caractéristiques (espace des plaques, rapport Al/U): un cœur carré de 3X3 boîtiers à 28 plaques d'\(235\)U, un cœur rectangulaire de 2X8 boîtiers à 28 plaques, et un ensemble couplé constitué de deux cœurs de 2X8 boîtiers à 28 plaques séparés par une lame d'eau de 19 cm.

Les rapports \(\beta_{\text{eff}}/\tau\) obtenus sont respectivement de \((158,3 \pm 3,5)s^{-1}, (167 + 5)s^{-1}\) et \((157 \pm 3)s^{-1}\), la valeur calculée pour le cœur carré étant 162 s^{-1}. Ces valeurs assez voisines semblent montrer que le rapport \(\beta_{\text{eff}}/\tau\) n'est pas très affecté par la géométrie du cœur, mais nous ne citons cette expérience que comme exemple de l'utilité de la méthode pour déterminer un des paramètres cinétiqes d'un réacteur lorsque l'on fait varier sa géométrie. D'autres comparaisons sont possibles sur RUBÉOLE.

Plusieurs expériences ont en effet été réalisées sur trois types différents de combustible:
- Type 12A: grappes de 12 barres UO\(_2\) à 1,51% gainées d'aluminium.
- Type 4B: grappes de 4 barres UO\(_2\) à 3,49% gainées d'aluminium.
- Type 6C: grappes de 6 barres UO\(_2\) à 3,49% gainées d'acier inox.

Le rapport \(\beta_{\text{eff}}/\tau\) a été mesuré pour les trois cœurs complètement chargés: les 202 canaux au pas de 11,18 cm avaient reçu leur combustible, la pile n'était donc pas réfléchie radialement.

Par ailleurs, les valeurs de ce rapport ont été calculées, et sur chacun de ces cœurs une nouvelle détermination expérimentale en a été faite en utilisant la technique de modulation. Les différentes valeurs sont portées dans le tableau II et l'on peut constater leur très bon groupement.

Ce tableau présente également la comparaison, moins bonne, entre valeurs expérimentales et calculées pour des cœurs des deux derniers types de combustible avec réflecteur radial; les réflecteurs sont simplement obtenus en vidant des canaux de leur combustible. Les épaisseurs de réflecteur dans les différents cas sont les suivantes:

- 4B réflecteur externe: 40,79 cm,
- 6C réflecteur externe: 44,17 cm,
- 6C réflecteur interne: 32,17 cm,
- 6C type Argonaut: réflecteur interne 25,63 cm, réflecteur externe 27,21 cm.

Les droites \(\alpha = f(\Delta k)\) obtenues dans les différents cas du combustible 6C sont présentées par la figure 4.

Il est possible, si l'expérience critique s'y prête comme c'est le cas pour RUBÉOLE, de déterminer l'économie de réflecteur par des mesures en neutrons pulsés.

Il suffit d'effectuer des mesures de \(\alpha\) en fonction de l'épaisseur du réflecteur radial, tout en conservant la même hauteur du cœur, celle qui correspond à la hauteur critique lorsque tous les canaux sont chargés (réflecteur radial nul). Chaque valeur de \(\alpha\) est alors corrigée par le calcul pour tenir compte de la variation du rapport \(\beta_{\text{eff}}/\tau\) au fur et à mesure que le réflecteur augmente. La valeur corrigée, \(\alpha_{202} = \alpha(\beta_{\text{eff}}/\tau)_{202}(\tau/\beta_{\text{eff}})_{202}\), est ainsi ramenée à celle que l'on mesurerait sur la pile non réfléchie (202 canaux) ayant le même écart à la criticalité \(\Delta k\).

La courbe \(\alpha_{202} = f(\Delta k)\) ayant été déterminée auparavant jusque assez loin de l'état critique (\(\Delta k\) entre \(-50 \cdot 10^{-5}\) et \(-15 000 \cdot 10^{-5}\)), on en déduit
### TABLEAU II
COMPARAISON DES VALEURS OBTENUES PAR LE CALCUL, PAR NEUTRONS PULSÉS ET PAR MODULATION SUR RUBÉOLE

<table>
<thead>
<tr>
<th></th>
<th>12A</th>
<th>4B</th>
<th>4B réflecteur externe</th>
<th>6C</th>
<th>6C réflecteur externe</th>
<th>6C réflecteur interne</th>
<th>6C type Argonaut</th>
</tr>
</thead>
</table>
| **$\beta_{\text{eff}}/\tau$**
| neutrons pulsés| $15.42 \pm 0.17$  | $11.39 \pm 0.12$ | $9.8 \pm 0.11$       | $15.98 \pm 0.10$  | $12.52 \pm 0.15$     | $11.99 \pm 0.15$     | $10.75 \pm 0.15$ |
| modulation     | $15.0 \pm 0.45$   | $11.74 \pm 0.35$ | $-16.05 \pm 0.48$     | $-12.52 \pm 0.15$ | $-11.99 \pm 0.15$    | $-10.75 \pm 0.15$    | $-10.75 \pm 0.15$ |
| calcul         | $15.26$           | $11.62$          | $-16.05 \pm 0.48$     | $-12.52 \pm 0.15$ | $-11.99 \pm 0.15$    | $-10.75 \pm 0.15$    | $-10.75 \pm 0.15$ |
| **$\beta_{\text{eff}}$**
| neutrons pulsés| $664 \pm 15$      | $661 \pm 20$     | $698 \pm 35$          | $645 \pm 20$      | $704 \pm 30$         | $708 \pm 30$        | $705 \pm 30$    |
| calcul         | $682.5$           | $677.2$          | $-676$                 | $-676$            | $-676$                | $-676$                | $-676$          |
| **$\tau$**
| neutrons pulsés| $431 \pm 10$      | $580 \pm 18$     | $712 \pm 36$          | $404 \pm 10$      | $562 \pm 15$         | $590 \pm 15$        | $656 \pm 20$    |
| calcul         | $447.2$           | $582.6$          | $714$                  | $414.7$           | $542.7$               | $565.2$              | $616.2$         |
\[ \Delta k \text{ de la pile réfléchie pour chacune des valeurs } \alpha_R \text{ corrigées. Cet écart correspond uniquement à une variation du laplacien géométrique } \Delta B^2 \text{ qui est intégralement imputable au laplacien radial } B_1^2, \text{ puisque dans ces expériences la hauteur du cœur reste inchangée. Le laplacien radial du cœur nu, mesuré par carte de flux, est égal à } 7,00 \text{m}^{-2}; \text{ le laplacien radial des cœurs réfléchis est donc } \]

\[ B_1^2 = 7,00 + \Delta B^2 = (2,405/R_{\text{ext}})^2. \]

On déduit immédiatement l'économie de réflecteur des valeurs du rayon extrapolé \( R_{\text{ext}} \) et du rayon géométrique moyen du cœur. La figure 5 compare les variations expérimentales et calculées (code RIFIFI [7]) de l'économie de réflecteur en fonction de l'épaisseur du réflecteur: l'accord est excellent pour les fortes épaisseurs, mais il laisse à désirer pour les faibles épaisseurs: les valeurs expérimentales sont certainement les meilleures dans cette zone où la théorie de la diffusion s'accorde mal d'un réflecteur à grandes cavités dont l'épaisseur est de l'ordre de quelques libres parcours de transport. Un point obtenu par carte de flux sur un cœur critique de
dimensions neutroniques très voisines confirme les résultats sur la pile la plus réfléchie.

Il est intéressant de noter que la technique des sources pulskées de neutrons s'applique ici correctement sur certains réacteurs à temps de vie très longs qui contiennent une grande quantité d'uranium 238, en particulier sur MARIUS chargé à environ 15 t d'uranium naturel et dont le temps de vie est de 1296 μs. Ces conditions imposent évidemment certaines précautions: les bouffées doivent être très espacées pour diminuer la composante continue importante. La figure 5 donne un exemple de l'importance de la partie de celle-ci, due aux neutrons retardés: l'expérience effectuée en envoyant une bouffée/s est pratiquement inexploitable. Le bruit de fond des fissions spontanées reste cependant très acceptable. Les résultats publiés ici sont obtenus en déterminant la constante de décroissance α par moindres carrés à partir des points expérimentaux: une fonction ϕ = Be^{-at} + F est ajustée en supprimant progressivement les premiers canaux jusqu'à obtenir une valeur de α constante.

Cette méthode entraîne cependant une erreur systématique dans le cas, comme celui de la figure 6, où la durée de l'analyse est très longue, car la contribution des premiers groupes de neutrons retardés ne peut plus être considérée comme constante. Nous avons donc mis au point un programme de dépouillement qui tient compte de la variation, pendant la durée de l'analyse, de la décroissance des deux premiers groupes de neutrons retardés, la contribution des groupes suivants pouvant réellement être considérée
Les nouveaux dépouillements à l'aide de ce programme sont en cours, et les premiers résultats confirment que l'effet n'est pas négligeable, et les valeurs indiquées pour MARIUS dans le Tableau I seront certainement déplacées de quelques pour cent, les nouvelles valeurs pour RUBÉOLE (tableau II) devant certainement rester à l'intérieur des marges d'erreur indiquées, car l'écart entre les résultats du nouveau et de l'ancien dépouillement ne semble être significatif que lorsque le temps de division du flux par un facteur 2 est supérieur à 60 ms.

5. Étalonnages d'absorbants neutroniques

Une mesure importante, qui intervient dans le fonctionnement d'un réacteur, est celle de l'antiréactivité introduite par un élément de contrôle; le résultat doit en être précis s'il s'agit d'ajuster un mode de calcul de barres de sécurité, on peut se contenter d'une approximation moins bonne pour la vérification de la réserve de réactivité d'un réacteur de puissance.

La technique des sources pulsées se montre très séduisante pour ces mesures et a déjà été intensivement utilisée [8, 9]. La méthode la plus rapide revient à mesurer, par une expérience préliminaire, la constante de décroissance du réacteur à l'état critique $\alpha_c = \beta_{\text{eff}}/\tau$, puis de déterminer...
la constante $\alpha$ après introduction dans le cœur de l'élément absorbant; la réactivité négative de cet élément est alors:

$$\Delta k = \frac{\beta_{\text{eff}}}{1 - \beta_{\text{eff}}} \left( 1 - \frac{\alpha}{\alpha_c} \right).$$

Cette manière de faire est excessivement rapide, une bonne valeur de $\alpha$ peut en effet être obtenue en une demi-heure ou une heure avec une précision de 1%. Ceci suppose cependant que la présence de l'absorbant dans le cœur ne modifie pas la valeur de $\alpha_c$, ce qui est le cas lorsque l'absorption apportée par l'élément de contrôle est faible devant celle du cœur. La valeur de $\alpha_c$ peut cependant être notablement modifiée si cette absorption est importante, et il n'est pas toujours facile de calculer la correction à apporter à $\alpha_c$ mesuré ou même de prévoir, si la géométrie du réacteur n'est pas simple, le signe de cette correction. La présence d'une barre de sécurité, dans un cœur de faibles dimensions entouré d'un réflecteur de grande importance neutronique, entraîne une diminution du temps de vie par suite de l'augmentation de l'absorption, mais également une déformation notable du flux dont résulte une augmentation de l'importance du réflecteur. Si le temps de vie des neutrons dans celui-ci est plus grand que dans le cœur même, le temps de vie global augmente.

Il est donc nécessaire d'interpréter avec beaucoup de prudence les mesures de réactivités négatives élevées, et autant que possible, de recaler les résultats à l'aide d'une détermination effectuée par une autre méthode. Nous donnons ici, à titre d'exemple, deux séries d'expériences menées sur RUBÉOLE et ALIZE qui montrent l'intérêt d'un recoupement expérimental.

**Rubéole**

L'étalonnage $\alpha = f(\Delta k)$ préalable a été effectué sur les deux cœurs 4B et 6C radialement non réfléchis:

Les résultats en sont présentés à la figure 7. Les valeurs de $\Delta k$ sont toujours obtenues à partir de la variation du laplacien géométrique (diminution de la hauteur du cœur) et de la mesure par approche sur-critique de $\frac{d\rho}{dH}$.

Un certain nombre de barres de cadmium constituées par des tubes de cadmium, qui traversent dans tous les cas complètement le cœur de haut en bas, ont été étalonnées par sources pulsées: la constante de décroissance est, à chaque fois, mesurée sur le cœur qui contient la ou les barres, la hauteur du cœur étant toujours la hauteur qui correspond à l'état critique lorsque la barre est enlevée. La comparaison directe de la valeur $\alpha$ mesurée à la courbe d'étalonnage correspondante donne une valeur de $\Delta k$ pour l'absorbant considéré, dont la valeur de la réactivité négative $\rho = \frac{\Delta k}{1 + \Delta k}$ est déduite.

Dans chaque cas, cette réactivité est mesurée par une autre méthode: le cœur qui contient l'absorbant à étalonner est amené à la criticalité par augmentation de sa hauteur; on mesure ainsi la variation du laplacien critique imposée par cet absorbant, c'est-à-dire sa réactivité négative en connaissant la réactivité différentielle $\frac{d\rho}{dH}$ mesurée par approche sur-critique.
que l'on intègre sur la différence de hauteur. Nous avons là une comparaison directe avec la méthode des sources pulsées.

Le tableau III montre cette comparaison entre les deux méthodes; alors que dans le réseau 4B les résultats sont parfaitement concordants, il n'en est pas de même pour le réseau 6C où les valeurs en sources pulsées présentent un écart systématique, proportionnel à $p$, avec les valeurs obtenues par variation du laplacien.

Les deux effets suivants se compensent dans le premier cas, mais pas dans le second: les courbes d'étalonnage, par changement de hauteur $\alpha = f(\Delta k)$, tiennent compte d'une diminution du temps de vie $\tau$ provoquée par l'augmentation du laplacien géométrique; les mesures avec absorbants en pile sont effectuées au laplacien géométrique qui correspond au laplacien critique du cœur sans absorbant; la variation du temps de vie $\tau$ due aux fuites est faible, car la pile n'est pas réfléchie, mais ce temps de vie est diminué par suite de l'augmentation de l'absorption totale. Pour le réseau 4B, l'effet de fuite, au cours de l'étalonnage $\alpha = f(\Delta k)$, compensant l'effet d'absorption lors que la barre est en pile, alors que l'effet d'absorption est supérieur dans le réseau 6C. Nous avons alors supposé que la variation de $\beta_{\text{eff}}/\tau$ pouvait être considérée comme linéaire en fonction de la réactivité $p$ et l'avons ajustée sur la mesure n° 8. La dernière colonne du tableau III indique les résultats en sources pulsées normalisés à cette mesure.
### ÉTALONNAGE DE BARRES D'ABSORBANTS SUR RUBÈOLE

<table>
<thead>
<tr>
<th>Cœur</th>
<th>N° de mesure</th>
<th>$\alpha$ ($s^{-1}$)</th>
<th>$-\rho (\times 10^5)$ par $\Delta B^2$</th>
<th>$-\rho (\times 10^5)$ sources pulsées</th>
<th>$-\rho (\times 10^5)$ sources pulsées normalisé au cas n° 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>4B</td>
<td>1</td>
<td>24,32</td>
<td>771</td>
<td>759 ± 11</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>40,05</td>
<td>1742</td>
<td>1685 ± 25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>70,97</td>
<td>3507</td>
<td>3500 ± 53</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>140,25</td>
<td>7644</td>
<td>7582 ± 115</td>
<td></td>
</tr>
<tr>
<td>6C</td>
<td>5</td>
<td>81,66</td>
<td>2652</td>
<td>2700 ± 40</td>
<td>2644 ± 40</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>169,29</td>
<td>6136</td>
<td>6350 ± 95</td>
<td>6060 ± 95</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>234,4</td>
<td>8615</td>
<td>9108 ± 135</td>
<td>8520 ± 135</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>260,7</td>
<td>9544</td>
<td>10235 ± 150</td>
<td>9544 ± 150</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>317,2</td>
<td>11445</td>
<td>12680 ± 190</td>
<td>11600 ± 190</td>
</tr>
</tbody>
</table>

**Alizé**

Divers absorbants ont également été étalonnés dans ALIZÉ par sources pulsées et par variation du titre critique en bore de l'eau. Ces réactivités négatives sont liées à des effets neutroniques très différents par la ré-partition des plaques absorbantes, soit l'une au centre, soit deux plaques éloignées l'une de l'autre; par leur sensibilité spectrale, les unes sont en cadmium, d'autres en alliage argent-indium-cadmium, ou en oxyde de terres rares. Dans chaque cas, le titre critique $T_c$ est mesuré par approche sur-critique, et la constante de décroissance $\alpha$ est mesurée au titre fixe 312 mg/l.

Le rapport $\beta_{eff}/T$ du cœur sans absorbant, dont le titre critique est 376,5 mg/l, est égal à 125 $s^{-1}$. La réactivité mise en jeu va jusqu'à $-6000 \cdot 10^{-5}$, et dans la gamme étudiée on constate (fig. 8) que $\alpha$ varie linéairement avec le titre critique. Ce résultat est d'une grande importance pratique, car il suffit de déterminer expérimentalement la droite $\alpha = f(T_c)$ à l'aide de deux ou trois mesures de $\alpha$ et de $T_c$ pour mesurer ensuite directement, à l'aide de $\alpha$, l'équivalence d'éléments absorbants en bore, par conséquent en réactivité.

Cette méthode est préférable à celle qui consiste à déduire la réactivité de la formule: $\Delta k = (\beta_{eff}/(1-\beta_{eff})) (1-\alpha/\alpha_c)$ pour les raisons indiquées précédemment; nous avons, en effet, également trouvé un écart systématique linéaire dû à la variation de $\alpha_c$, mais cette fois l'importance du réflecteur l'emporte sur l'augmentation de l'absorption. La comparaison de deux valeurs de $\alpha_c$ mesurées, l'une sur le cœur sans absorbant, au titre critique 375,5 mg/l et l'autre sur le cœur contenant deux plaques de cadmium d'effi-
ALIZÉ: variation linéaire du paramètre $\alpha$ en fonction du titre critique $r_c$ pour divers absorbants.

capacité $5500 \cdot 10^{-5}$ au titre critique $209.1 \text{ mg/l}$, met en évidence l'effet du réflecteur:
- sans plaque en cadmium: $\alpha_c = (125 \pm 54) \text{ s}^{-1}$
- avec 2 plaques en cadmium: $\alpha_c = (114 \pm 33) \text{ s}^{-1}$.

Un programme expérimental réalisé sur MARIUS a entraîné les mêmes conclusions: il est indispensable d'étañlîner les mesures en sources pulées par une mesure indépendante, variation de titre ou de laplacien critique.

6. Expériences pulées en pile mixte (RUBÉOLE).

Une méthode de mesure de la laplacien matière d'un réseau couramment utilisée en France, est la méthode dite de substitution [10, 11]. Elle consiste à mesurer la variation du laplacien critique provoquée par la substitution, dans un réseau de référence bien connu, d'éléments de combustible de ce réseau de référence par des éléments de combustible de réseau à étudier. La zone substituée est centrale, et son rayon est progressive-
ment augmenté jusqu'à environ un tiers du rayon du cœur de référence. On déduit le laplacien matière du réseau étudié de celui du réseau de référence par la théorie des perturbations.

Nous avons pensé que la méthode des sources pulsiées pouvait être utile à la mesure des différences de laplacien dans le cas où le réseau étudié est très peu réactif et où les dimensions de l'expérience ne sont pas suffisantes pour atteindre la criticalité. Nous avons donc supposé que nous ne pouvions pas augmenter la hauteur du cœur de RUBÉOLE au delà de celle qui correspond à la criticalité pour le réseau 4B, et nous avons substitué successivement dans 2, 8, 16, 26 et 44 canaux, en partant de l'axe du cœur, le combustible 4B par le combustible 12A moins réactif.

Une expérience pulsée est effectuée après chacune de ces substitutions, la hauteur du cœur restant inchangée; le résultat brut est une mesure de \( \alpha_s \)

\[
\alpha_s = \left( \frac{\beta_{\text{eff}}}{\tau} \right) + \frac{1}{\tau} \left( \frac{M^2}{\kappa} \right) \Delta B^2.
\]

Les valeurs \((\beta_{\text{eff}}/\tau)_s\) et \((1/\tau)(M^2/\kappa)_s\) sont celles du milieu mixte et varient donc avec \( \alpha_s \). La variation du laplacien \( \Delta B^2 \) se réfère à la pile critique de référence 4B, sur laquelle, pour une valeur \( \Delta B^2 \) donnée, la constante de décroissance serait \( \alpha_{4B} \neq \alpha_s \)

\[
\alpha_{4B} = \left( \frac{\beta_{\text{eff}}}{\tau} \right)_{4B} + \frac{1}{\tau_{4B}} \left( \frac{M^2}{\kappa} \right)_{4B} \Delta B^2.
\]

La courbe \( \alpha_{4B} = f(\Delta B^2) \) a été déjà obtenue expérimentalement au cours de la mesure du rapport \( \beta_{\text{eff}}/\tau \) pour le réseau 4B:

\[
\alpha_{4B} = 11,3714 + 49,589\Delta B^2.
\]

Les valeurs expérimentales \( \alpha_s \) doivent donc être corrigées pour les ramener aux valeurs \( \alpha_{4B} \) que l'on obtiendrait sur le cœur 4B; des valeurs corrigées et de la droite \( \alpha_{4B} = f(\Delta B^2) \) on en déduit la variation du laplacien. Deux corrections sont à apporter sur les valeurs \( \alpha_s \): la première porte sur le temps de vie, la seconde sur \( M^2/\kappa \). Un calcul est effectué sur chacune des substitution en supposant que les caractéristiques du réseau étudié 12A sont approximativement connues. Le temps de vie \( \tau_{4B} \) et le rapport \( (M^2/\kappa)_{4B} \) sont déterminés à partir des valeurs \( \tau_{4B} \), \( \tau_{12A} \), \( (M^2/\kappa)_{4B} \) et \( (M^2/\kappa)_{12A} \) en les pondérant par les poids statistiques des deux régions calculés à l'aide du code RIFIFI.

Le calcul du temps de vie \( \tau_s \) a été vérifié par la mesure de \( \tau_s \) sur le cœur dans lequel les 44 canaux centraux sont substitués, et l'accord est satisfaisant:

\[
\begin{align*}
(\beta_{\text{eff}}/\tau) & \left\{ \begin{array}{l}
\text{calculé} = 13,38 \text{ s}^{-1} \\
\text{mesuré} = 13,37 \pm 0,15 \text{ s}^{-1}
\end{array} \right.
\end{align*}
\]

\[
\begin{align*}
\tau_s & \left\{ \begin{array}{l}
\text{calculé} = 508,2 \mu s \\
\text{mesuré} = 490 \pm 25 \mu s.
\end{array} \right.
\end{align*}
\]
TABLEAU IV

RUBÈOLE: VARIATION DU LAPLACIEN DU RÉSEAU 4B PAR SUBSTITUTION DU RÉSEAU 12A

<table>
<thead>
<tr>
<th>Nombre de canaux substituts</th>
<th>αs (s⁻¹)</th>
<th>α corrigé (s⁻¹)</th>
<th>ΔB² pulsé* (m²)</th>
<th>ΔB² critique (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11,371</td>
<td>11,371</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>17,949</td>
<td>17,774</td>
<td>0,130</td>
<td>0,118</td>
</tr>
<tr>
<td>8</td>
<td>34,452</td>
<td>33,707</td>
<td>0,450</td>
<td>0,438</td>
</tr>
<tr>
<td>16</td>
<td>53,934</td>
<td>52,070</td>
<td>0,821</td>
<td>0,824</td>
</tr>
<tr>
<td>26</td>
<td>75,398</td>
<td>71,781</td>
<td>1,218</td>
<td>1,253</td>
</tr>
<tr>
<td>44</td>
<td>112,08</td>
<td>104,399</td>
<td>1,876</td>
<td>1,930</td>
</tr>
<tr>
<td>202</td>
<td>204,8</td>
<td>181,02</td>
<td>3,421</td>
<td>3,436</td>
</tr>
</tbody>
</table>

* L'incertitude sur ΔB² est de l'ordre de 2%.

Le tableau IV compare les variations de laplacien mesurées par cette méthode et par détermination de hauteur critique (la hauteur du cœur est augmentée jusqu'à la criticalité obtenue par approche sur-critique). Ce tableau indique également la mesure en « substitution totale », c'est-à-dire le cas extrême où le cœur est complètement chargé en combustible 12A. L'ensemble des résultats est satisfaisant.

Cet exemple, joint à la mesure de l'économie de réflecteur, montre que la méthode des sources pulsées peut être étendue largement dans le domaine de la physique des réacteurs.

B. PARAMÈTRES DE DIFFUSION ET ANISOTROPIE DU GRAPHITE

La technique des sources pulsées de neutrons appliquée aux modérateurs purs est riches d'enseignement, puisqu'elle permet d'atteindre le coefficient de diffusion, le coefficient de refroidissement, le temps de thermalisation des neutrons et l'anisotropie de la diffusion, paramètres qui rentrent en compte dans le calcul des réacteurs. Malheureusement, les résultats publiés par les principales équipes de recherche ne sont pas concordants, en grande partie parce que des améliorations ont été apportées au cours du temps au matériel de mesure et les précautions expérimentales se sont améliorées.

Nous avons effectué une série de mesures sur le graphite dont les résultats sont en bon accord avec les dernières valeurs de STARR et PRICE [12] en mesurant la constante de décroissance dans 23 blocs de graphite de dimensions différentes dont les laplaciens vont de 7 m² à 155 m². En se limitant à un développement à trois termes, cette constante est fonction du laplacien géométrique et dépend des paramètres d'absorption, de diffusion et de thermalisation.
\[ \alpha = \bar{V} \Sigma_a + D_0 B^2 - CB^4 \]

où \( \bar{V} \) : vitesse moyenne des neutrons,
\( \Sigma_a \) : section efficace d'absorption macroscopique,
\( D_0 \) : coefficient de diffusion,
\( C \) : coefficient de refroidissement.

Le matériel de détection était le même que sur les réacteurs, mais les sources, qui ne permettent pas un débit et une fréquence de répétition des bouffées suffisants, sont ici remplacées par un accélérateur électrostatique SAMES de 150 kV, les neutrons étant toujours produits par la réaction (d, t). Chaque empilement de graphite a été réalisé avec le souci de conserver une densité constante et d'éviter tout effet d'anisotropie. Les compteurs sont placés en sorte d'éliminer les premiers harmoniques. Un compte rendu plus détaillé de la réalisation de ces mesures pourra être trouvé en [13].

Nous avons ajusté le retard à l'analyse sur les petits empilements après une mesure d'indice de spectre effectuée sur le bloc de dimensions 60X 60X 60 m; deux sélecteurs, en temps-reçoivent chacun les impulsions d'un détecteur différent: l'un est un compteur à BF\(_3\) enrichi à 90% en \( ^{10}\)B et l'autre un compteur appauvri à 10% en \( ^{10}\)B. Ils sont placés côté-à-côte au centre d'une face latérale de l'empilement, leur axe perpendiculaire à cette face. Chacun d'eux est placé dans une chemise de cadmium; les neutrons, fuyant l'empilement, traversent les compteurs dans des directions voisines de celle de leur axe et sont absorbés différemment dans chacun d'eux, le premier étant noir à tous les neutrons dans le domaine thermique et le second ayant une efficacité variable avec l'énergie des neutrons dans ce même domaine. Le rapport de leurs taux de comptage varie donc en fonction du temps, tant que le spectre évolue après l'envoi d'une bouffée de neutrons.

La figure 9 montre que la mise à l'équilibre du spectre demande de l'ordre de 2 ms, et que cet équilibre subsiste ensuite. Toutes les mesures ont donc été effectuées avec un retard d'au moins 2 ms. Cette expérience permet de constater que l'approche à l'équilibre du spectre se fait suivant un mode exponentiel avec un temps de relaxation de 560 \( \mu s \). Cette méthode a été utilisée par STARR et de VILLIERS [14] qui trouvent 525 \( \mu s \) et déterminent, après étalonnage du rapport des 2 compteurs à l'équilibre, le coefficient de refroidissement \( C = (38 \pm 5) \cdot 10^5 \text{ cm}^4 \text{ s}^{-1} \).

Les paramètres ajustés sur les valeurs de \( \alpha \) sont les suivants

\[ \bar{V} \Sigma_a = (75 \pm 3) \text{ s}^{-1} \]
\[ D_0 = (216,6 \pm 3) \cdot 10^3 \text{ cm}^2 \text{ s}^{-1} \]
\[ C = (37,9 \pm 4) \cdot 10^5 \text{ cm}^4 \text{ s}^{-1} \]

pour notre graphite de densité 1,619 et à la température de 20 °C. On voit sur la figure 10 que les écarts des neuf premiers points à la parabole sont inférieurs à 5% et qu'il n'apparaît aucun écart systématique du côté des grands laplaciani qui permettrait d'envisager l'existence significative d'un terme en \( B^6 \). La valeur du libre parcours de transport, ramenée à un graphite de densité 1,60 se déduit de \( D_0 \)

\[ \lambda_t = (2,55 \pm 0,04) \text{ cm} \]
Figure 9

Graphite, empilement 60 x 60 x 60 cm³. Mise à l'équilibre du spectre des neutrons.

Figure 10

Graphite; écarts des points expérimentaux à la parabole.

Une seconde série de mesures a permis d'étudier l'anisotropie du graphite due à l'orientation préférentielle des cristaux lors du filage. Les empilements sont parallélépipédiques et sont constitués par des briques dont les axes de filage sont tous orientés dans le même sens; suivant la plus
petite dimension des empilements d'abord, puis suivant une direction perpendiculaire. Les mesures de la constante de décroissance sur un empilement de géométrie donnée mettent en évidence un écart systématique suivant l'orientation des briques. Chaque géométrie d'empilement fournit une relation linéaire entre les coefficients de diffusion dans le sens parallèle au filage, $D_n$, et dans le sens perpendiculaire $D_\perp$:

$$\beta_\perp = \alpha_\perp - \nabla \Sigma_a + C (B_{n}^{2} + B_{\perp}^{2})^2 = B_{n}^{2} D_n + B_{\perp}^{2} D_\perp$$

où $\nabla \Sigma_a$ et $C$ ont les valeurs précédemment déterminées. L'étude de trois blocs de dimensions différentes donne six relations liant les deux inconnues $D_n$ et $D_\perp$; les caractéristiques géométriques et neutroniques des six mesures sont rassemblées dans le tableau V.

**TABLEAU V**

GRAPHITE - MESURES D'ANISOTROPIE

<table>
<thead>
<tr>
<th>Dimensions du bloc (cm$^3$)</th>
<th>$B^2 \times 10^4$ (cm$^{-2}$)</th>
<th>$B_n^2$</th>
<th>$B_\perp^2$</th>
<th>$\alpha$ (s$^{-1}$)</th>
<th>$\beta$ (s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>240 x 240 x 100</td>
<td>12,4975</td>
<td>9,1743</td>
<td>3,3232</td>
<td>344,37</td>
<td>275,18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1,6616</td>
<td>10,8359</td>
<td>341,03</td>
<td>271,84</td>
</tr>
<tr>
<td>200 x 200 x 100</td>
<td>13,9305</td>
<td>9,1743</td>
<td>4,7563</td>
<td>371,71</td>
<td>304,05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2,3781</td>
<td>11,5524</td>
<td>370,06</td>
<td>302,40</td>
</tr>
<tr>
<td>160 x 160 x 80</td>
<td>21,4454</td>
<td>14,0812</td>
<td>7,3642</td>
<td>521,93</td>
<td>464,35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3,6821</td>
<td>17,7633</td>
<td>518,27</td>
<td>460,69</td>
</tr>
</tbody>
</table>

Une résolution par moindres carrés fournit la valeur des deux coefficients de diffusion:

$$D_n = 218,89 \cdot 10^3 \text{ cm}^2 \text{ s}^{-1}$$
$$D_\perp = 217,24 \cdot 10^3 \text{ cm}^2 \text{ s}^{-1}$$

d'où l'anisotropie:

$$\frac{D_n}{D_\perp} = 1,017 \pm 0,008.$$
Remarquons que la quantité \( \frac{1}{3} D_u + \frac{2}{3} D_\perp = 216.4 \cdot 10^3 \text{ cm}^2 \text{ s}^{-1} \) est en bon accord avec la valeur de \( D_0 = 216.6 \).

Les principaux résultats expérimentaux publiés sur le graphite sont présentés dans le tableau VI. Les dernières valeurs publiées du coefficient de diffusion ne montrent pas de désaccord notable, et ces valeurs sont nettement supérieures aux résultats trouvés antérieurement.

**TABLEAU VI**

**PRINCIPAUX RÉSULTATS EXPÉRIMENTAUX**

\( \rho = 1.6 \text{ g/cm}^3 \quad T = 20^\circ \text{C} \)

<table>
<thead>
<tr>
<th>Année</th>
<th>Auteurs</th>
<th>( \nu \xi_3 ) ( (s^{-1}) )</th>
<th>( D_u ) ( (10^5 \text{ cm}^2 \text{ s}^{-1}) )</th>
<th>( C ) ( (10^5 \text{ cm}^4 \text{ s}^{-1}) )</th>
<th>Variation de laplacien ( (10^6 \text{ cm}^4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1955</td>
<td>ANTONOV et al. [15]</td>
<td>82.0 ± 3</td>
<td>2.07 ± 0.03</td>
<td>12.5 ± 2</td>
<td>10 à 400</td>
</tr>
<tr>
<td>1956</td>
<td>BECKURTS [16]</td>
<td>127 ± 1</td>
<td>2.13 ± 0.02</td>
<td>16.3 ± 2.5</td>
<td>7 à 55</td>
</tr>
<tr>
<td>1958</td>
<td>DROULERS, LACOUR, RAJEVSKI*</td>
<td></td>
<td>2.10 ± 0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1959</td>
<td>STARR et PRICE [18]</td>
<td>71.2 ± 0.8</td>
<td>2.06 ± 0.02</td>
<td>12.4 ± 2.2</td>
<td>16 à 275</td>
</tr>
<tr>
<td>1961</td>
<td>LALANDE [19]</td>
<td>79.5 ± 3</td>
<td>2.07 ± 0.04</td>
<td>25.8 ± 2</td>
<td>7 à 390</td>
</tr>
<tr>
<td>1962</td>
<td>STARR et PRICE**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>type GBF</td>
<td>74.6 ± 0.6</td>
<td>2.14 ± 0.01</td>
<td>34 ± 3</td>
<td>18 à 180</td>
</tr>
<tr>
<td></td>
<td>type AA [12]</td>
<td>68.1 ± 0.8</td>
<td>2.19 ± 0.03</td>
<td>41.0 ± 4</td>
<td>20 à 137</td>
</tr>
<tr>
<td>1962</td>
<td>STARR et de VILLIERS [14]</td>
<td></td>
<td></td>
<td></td>
<td>38 ± 5</td>
</tr>
<tr>
<td>1962</td>
<td>KLOSE, KUCHLE, REICHARDT**</td>
<td>88.3 ± 1.2</td>
<td>2.13 ± 0.02</td>
<td>26 ± 5</td>
<td>7 à 240</td>
</tr>
<tr>
<td>1963</td>
<td>SAGOT et TELLIER</td>
<td>74.1 ± 3</td>
<td>2.19 ± 0.03</td>
<td>37.9 ± 4</td>
<td>7 à 155</td>
</tr>
</tbody>
</table>

* complété par une expérience exponentielle
** mesure par méthode de la longueur de diffusion complexe

Le résultat obtenu par la méthode de la longueur de diffusion complexe est un peu faible; il est possible qu'il existe encore, sur l'empilement utilisé de \( 180 \times 160 \times 160 \text{ cm} \), un effet de spectre. Sur un tel empilement, la correction de refroidissement est de l'ordre de 1.7% du terme diffusion, c'est-à-dire que l'écart de la vitesse moyenne à celle qui existerait en milieu infini est d'environ 1.7%. Le résultat publié par Lalande en 1961 est nettement plus faible que le nôtre; mais le nombre d'empilements étudiés,
particulièrement ceux de grandes dimensions, est plus réduit, et on peut mettre en doute la précision de la base de temps utilisée pour les premières mesures.

Le désaccord sur la valeur du coefficient de refroidissement est très important, et cette valeur augmente notablement de 1955 à 1962. Il est très probable que les retards à l'analyse étaient choisis trop courts, entraînant ainsi une mesure de \( \alpha \) entachée d'une erreur systématique. Il peut être considéré maintenant que ce retard à l'analyse doit être égal ou supérieur à 2 ms pour tous les empilements de faibles dimensions. Starr et Price, qui ont respecté cette procédure, obtiennent deux valeurs en parfait accord avec la nôtre et avec celle de Starr et de Villiers. Par contre Klose, Kuchle et Reichardt arrivent à une valeur beaucoup plus basse. Ces auteurs mentionnent bien que le retard souhaitable est de 2 ms, mais ils indiquent des retards utilisés plus faibles, allant, au-delà de 150 m², jusqu'à 1 ms. Dans ces conditions, les valeurs de \( \alpha \) sont trop élevées, ce qui entraîne une faible valeur de \( C \). Par ailleurs, la parabole ajustée sur les points expérimentaux, publiée à la page 943 de [20] n'est certainement pas celle qui correspond au meilleur jeu de paramètres: elle laisse pratiquement en dessous d'elle tous les points correspondants aux forts laplaciens, nombre d'entre eux étant éloignés de 6%. Il est bien évident, dans ces conditions, que de nouveaux ajustements du même type effectués en écartant successivement les mesures extrêmes de \( \alpha \) donneront des jeux de valeurs à chaque fois différents.

Compte tenu de l'incertitude annoncée par ces auteurs sur les hautes valeurs de \( \alpha \) et de l'erreur systématique introduite par des retards à l'analyse trop courts, il paraît difficile d'accorder une grande valeur au coefficient \( F \) du terme en \( B^6 \) qui a été publié, d'autant plus que les auteurs reconnaissent que le lissage des points est alors moins bon aux faibles valeurs du laplacien. Ils ne publient malheureusement pas cette courbe en \( B^6 \) ni les valeurs de \( \alpha \) mesurées.

Nous sommes par contre en bon accord avec ces auteurs sur les mesures de l'anisotropie: ils obtiennent

\[
\frac{D_6}{D_4} = 1,010 \pm 0,005, \\
\]

notre résultat étant 1,017 \( \pm \) 0,008.

Starr et Price ont par ailleurs mesuré l'anisotropie par une double expérience exponentielle et obtiennent une valeur \( D_n / D_4 = 1,012 \) qui recoupe parfaitement les expériences en sources pulsées.

Un nouveau programme de mesure débutera fin 1963 sur un nouveau graphite. Une partie importante en sera la mesure de l'anisotropie d'un réseau de canaux en fonction du diamètre du canal. Nous aurons donc l'occasion de reprendre la discussion en fonction des résultats qui auront été publiés lorsque ces mesures seront terminées.

CONCLUSION

Ce rapport n'a pas la prétention d'épuiser les possibilités de la technique des sources pulsées. Un article de GARELIS et RUSSEL récemment paru
qui poursuit le travail de SJOSTRAND [22], exploite plus complètement les données brutes d'une mesure d'absorbant en déterminant, en même temps que $\alpha$, la variation du rapport $\beta_{\text{eff}}/\tau$. Nous avons nous même essayé cette méthode sur ALIZÉ, mais nos résultats ne sont pas très précis, et notre méthode doit être améliorée. De même nous n'avons effectué aucune recherche spectrale, qui nécessite des sources de neutrons beaucoup plus intense que celles dont nous disposons.

Nous pensons qu'un laboratoire qui effectue des recherches en physique des réacteurs ne peut se passer de l'utilisation de cette technique étant donné le grand nombre d'informations qu'elle peut fournir; la plupart serviront à vérifier des formulaires de calcul, d'autres peuvent donner rapidement des résultats pratiques lors du démarrage d'un réacteur de puissance (temps de vie, réserve de réactivité), et enfin certaines sont des paramètres fondamentaux dont la détermination expérimentale sera toujours nécessaire, tels les caractéristiques de modérateurs.

RÉFÉRENCES

M. KÜCHLE: I should like to make one comment on this question of graphite measurements. We made the point at the Brookhaven Conference on neutron thermalization - and this was, I think, the general feeling there - that better results can be obtained by making a direct comparison of measured points than by means of deduced parameters. The latter may be influenced by the way the evaluation is carried out. If this technique were used, perhaps there would no longer be any discrepancy between our results and yours.

There is also one question I should like to ask on the subject of $\beta_{eff}$. What you measure is the difference in buckling between delayed and prompt critical. I should have thought that you would have to make use of some theoretical quantities to convert this into $\beta_{eff}$. Is this in fact the case?

M. SAGOT: I would agree with your comments. Unfortunately, I am not in possession of your experimental points so that I have not been able to compare them with our data.

As for $\beta_{eff}$, I did mention that the accuracy of our measurements was poor. The corrections that have to be made are within the relevant margins of error. However, this does not affect the fact that it is still possible to follow the evolution of $\beta_{eff}$ when one changes the characteristics of the core.

W. DRECHSEL: I have two questions in connection with the pulsed neutron sources you use in your experiments. Firstly, are they commercially available or do you make them yourselves? Secondly, how many neutrons do you obtain per burst of the neutron source?

M. SAGOT: The sources we have been using have been designed and built by the CEA. As far as I know, it is planned - I think by the Société anonyme d'études et de réalisations nucléaires - to make them available commercially some time this year. Our sources produce $4 \times 10^6$ neutrons per burst. The production of the commercial ones will, I think, be higher.

W.C. REDMAN: You mentioned the possibility of using a number of sources simultaneously as a means of suppressing harmonic effects. Have you actually used this technique? If so, I should be interested to know how important it is to have sources of equal strength and to what extent such equality has been achieved.

M. SAGOT: Yes, we have made use of a number of synchronized sources in our experiments but unfortunately I don't know enough about the electronics side to be able to give you any details. Source production can be equated to within 20% approximately. That is enough to considerably attenuate the harmonics of the source. Complete elimination can then be achieved by appropriate positioning of the detectors.
MEASUREMENTS OF $\delta \rho/\delta h$ IN A WATER-MODERATED REACTOR BY AN OSCILLATING MODERATOR-LEVEL TECHNIQUE

D. BABALA*

KJELLER RESEARCH ESTABLISHMENT, NORWAY

Abstract — Résumé — Аннотация — Resumen

MEASUREMENT OF $\delta \rho/\delta h$ IN A WATER-MODERATED REACTOR BY AN OSCILLATING MODERATOR-LEVEL TECHNIQUE. Measurements of the moderator-level coefficient of reactivity in the NORA reactor have been carried out by means of a pneumatic oscillator that varies the moderator height periodically with a trapezoidal wave-form. A detailed theory for this type of experiment has been developed which is of value both for interpreting the experimental results and for estimating the accuracy of the method.

UTILISATION D'UNE MÉTHODE D'Oscillation Du NIVEAU Du RALENTISSEUR POUR MESURER $\delta \rho/\delta h$ DANS UN RÉACTEUR RALENTI À L'EAU. On a effectué des mesures du coefficient de niveau du ralentisseur, dans le réacteur NORA, en utilisant un oscillateur pneumatique qui fait varier le niveau du ralentisseur suivant une courbe périodique de forme trapezoïdale. Ce type d'expérience a fait l'objet d'une théorie détaillée qui peut servir à la fois pour interpréter les résultats expérimentaux et évaluer l'exactitude de la méthode.

ИЗМЕРЕНИЕ ОТНОШЕНИЯ $\delta \rho/\delta h$ В РЕАКТОРЕ С ВОДНЫМ ЗАМЕДЛИТЕЛЕМ МЕТОДОМ КОЛЕБЛЕЮЩЕГОСЯ УРОВНЯ ЗАМЕДЛИТЕЛЯ. В реакторе NORA измерения коэффициента реактивности по уровню замедлителя проводились с помощью пневматического осциллятора, который периодически изменяет высоту уровня замедлителя по кривой в форме трапециoidalной волны. Для эксперимента этого типа разработана подробная теория, которая представляет ценность как для объяснения экспериментальных результатов, так и для оценки точности метода.

MEDICIÓN DEL COEFICIENTE $\delta \rho/\delta h$ EN UN REACTOR MODERADO POR AGUA, MEDIANTE UNA TÉCNICA DE OSCILACIÓN DEL NIVEL DEL MODERADOR. El autor midió el coeficiente de reactividad del nivel del moderador en el reactor NORA, mediante un oscilador neumático que modifica periódicamente el nivel del moderador, según una ley de variación en forma de onda trapezoidal. Propone una teoría detallada que sirve tanto para interpretar los resultados experimentales como para evaluar la precisión del método.

1. INTRODUCTION

In the NORA zero-power reactor, $\delta \rho/\delta h$ was measured for several cores by an oscillation technique. The method proved to be more accurate and less time consuming than the method of measuring the power doubling time after a step change in the moderator level.

The experimental equipment is mentioned only briefly in section 2, because of its more or less standard character. The theory of this type of experiment and a method of interpretation of the results of measurements are presented in detail in section 3. In section 4, the results of measurements are listed and compared with the theoretical predictions.

* IAEA Fellow, Nuclear Research Institute, Prague, Czechoslovak Socialist Republic.
2. EXPERIMENTAL EQUIPMENT

A schematic diagram of a pneumatic oscillator is shown in Fig. 1. The oscillator consists of two pressure vessels, which are connected through electronically operated valves to a tube dipped into the moderator. A manometer containing water darkened with a few drops of China ink is also connected to the tube. The pressure in the conduit is changed periodically by opening and closing the valves A and B. At the beginning of a period the valve A opens. The water is sucked up until the dark water in the manometer reaches a photocell, which closes the valve. In the middle of a period the valve B opens. The water falls down until a signal from the second photocell of the manometer closes the valve.

It is possible to adjust the rise-and-fall-times of the water to be approximately the same by means of setting the openings of the valves. The oscillation of the moderator level is, therefore, approximately trapezoidal and odd-harmonic.

The tube was dipped into the $\text{D}_2\text{O}$ reflector and the lower equilibrium position of the water level in the tube was about 50 cm above the level in the reactor tank to minimize the reflecting effect of the oscillating water column in the tube.

The neutron flux was detected by an ionization chamber placed in the graphite reflector surrounding $\text{D}_2\text{O}$. The signal was amplified, multiplied by sine and cosine, by means of a sine potentiometer, and integrated over five periods. The measured first harmonic of the reactor response was normalized to the average power $n_0$ [1].
3. THEORY OF THE EXPERIMENT

If a linear dependence of the reactivity on the water height is assumed, one has

$$\rho_1 = \frac{\partial \rho}{\partial h}, \quad h_1 = -\frac{\partial \rho}{\partial h} S^1 s_1,$$

where \( h \) and \( s \) are the water heights in the reactor tank and the tube respectively, and \( S^1 \) and \( S \) stand for the surfaces of the water in the reactor tank and the tube respectively; the subscripts denote the coefficients in the Fourier series of the type

$$f(t) = \sum_{k=-\infty}^{\infty} f_k e^{ik\omega t}, \quad f_k = f_k^*$$

(the cross means complex conjugate) throughout the whole paper*. Harmonic analysis of the zero-power-reactor kinetic equations gives

$$n_1 = \left( \frac{1}{T(\omega)} - \rho_0 \right)^{-1} \left[ \rho_1 n_0 + \sum_{k=1}^{\infty} \left( \rho_k^* n_{k+1} + \rho_{k+1} n_k^* \right) \right],$$

$$\rho_0 = -\frac{2}{n_0} \sum_{k=1}^{\infty} \text{Re}(\rho_k^* n_k),$$

where \( n \) is the reactor power and \( T(\omega) \) is the zero-power-reactor transfer function.

Assuming that the even harmonics of the reactor output and \( \rho_0 \) are negligible, one has

$$n_1 / n_0 = T(\omega) \rho_1.$$  \hspace{1cm} (4)

From (1) and (4) \( \partial \rho / \partial h \) can be calculated provided \( s_1 \) is known. The rise- and fall-times and the equilibrium positions of the water can be measured in the NORA oscillator. As an approximation, one can assume that \( |s_1| \) has the form

$$|s_1| \approx (s_b - s_a) \left[ \frac{1}{2} \right],$$

where

$$\left[ \frac{1}{2} \right] = \frac{1}{\pi} \left[ \frac{\cos x' - 1}{x'} \frac{\sin x'}{x'} \right],$$

where

* Except the subscripts \( a \) and \( b \) which are used beginning with Eq 5.
is the first harmonic of a trapezoidal wave, \( x' = \omega t_a = \omega t_b \), \( s_b \), \( s_a \), \( t_b \), \( t_a \) being the higher and lower equilibrium positions of the water in the tube and the rise- and fall-times, respectively.

Equations (1), (4) and (5) give

\[
\frac{\partial p}{\partial h} = \frac{S'}{S(s_a - s_b)} \left| \frac{h_1}{n_0} \right| \frac{1}{|T|},
\]

However, the water oscillations are not exactly trapezoidal (see Fig. 2) and it is difficult to record \( s_1 \) directly. Since the difference between \( |s_j| \) and the magnitude of first harmonic of a square wave is not large (5% for \( x' \approx 1.12 \), 10% for \( x' \approx 1.58 \)), one can expect that \( |s_1| \) will not be very much different from (5), but it might be of some interest to evaluate this correction. The rest of the section is concerned with the calculation of \( s_1 \).

First the relation between \( s_1 \) and the first harmonic \( p_1 \) of the air pressure in the conduit is derived.

3.1. Relation between \( s_1 \) and \( p_1 \)

Consider first the Navier-Stokes hydrodynamic equations for a laminar flow [2].

\[
\frac{\partial}{\partial t} \vec{u}(x, t) + \vec{u}(x, t) \nabla \vec{u}(x, t) = -\nabla \Omega(x) + \frac{1}{\mu} p'(x, t) + \nu \nabla^2 \vec{u}(x, t),
\]

where \( \vec{u}, \Omega, p', \mu, \nu \) are the water velocity, the gravitational potential (in this case \( \Omega = gz \)), the static pressure, the water density and the kinematic viscosity of the water, respectively.

To express the velocity in terms of the pressure difference \( (p-P) \) – see Fig.1 – integrate these equations along the path WXYZ. Assume that the surface of the moderator is large compared to the cross-section of the tube, so that the oscillation amplitude of the water level in the tank is negligible compared to the amplitude in the tube. Further, we can choose the part WXY
of the integration path so far from the tube that along this line the water is at rest.

The water motion is axially symmetric with respect to the tube axis, therefore it is convenient to introduce cylindrical co-ordinates with the $z$-axis coinciding with the axis of the tube.

Integrating the $z$-component of Eq.(8) along this path gives (the points $W$, $X$ and $Y$ may be assumed infinitely far away)

$$
- \int_{-d}^{d} \frac{\partial}{\partial t} u_z(r, z, t) \, dz + \int_{-d}^{d} \frac{\partial}{\partial t} v(r, t) \, dz + \int_{-d}^{d} u(r, z, t) \nabla u_z(r, z, t) \, dz
$$

$$
- \frac{\partial}{\partial t} s(r, t) = - g s(r, t) \left( p - P \right) + \nu \int_{-d}^{d} \nabla^2 u_z(r, z, t) \, dz + \nu \int_{-d}^{d} \nabla^2 v(r, t) \, dz. 
$$

The upper boundary $s$ generally depends on $r$. The water velocity in the tube was denoted by $v(r, t)$. The first integrals on the left-hand side and right-hand side represent the effect of the region in the tank where the water column dissipates. In this region the dependence of the water velocity on $r$ is much weaker than in the tube and we can replace these integrals by averages over the tube cross-section according to the general formula

$$
y(t) = \frac{2}{R^2} \int_{0}^{R} y(r, t) \, rdr,
$$

where $R$ is the inner radius of the tube.

Equation (9) can then be written as follows

$$
\left[ s(r, t) + d \right] \frac{\partial}{\partial t} v(r, t) + I(t) + \int_{-d}^{d} u(r, z, t) \nabla u_z(r, z, t) \, dz
$$

$$
- g s(r, t) - \frac{1}{\mu} (p - P) + \nu \left[ s(r, t) + d \right] \nabla^2 v(r, t) + \nu J(t),
$$

where

$$
I(t) = \int_{-d}^{d} \frac{\partial}{\partial t} u_z(z, t) \, dz, J(t) = \int_{-d}^{d} \nabla^2 u_z(z, t) \, dz.
$$

Harmonic analysis gives for the first harmonic (all the time dependent quantities in (11) expressed according to (2), the oscillations assumed odd-harmonic)
\[ \omega \nu [s_0'(r)+d] \nabla^2 s_1(r) + [g - \omega^2(s_0(r)+d)] s_1(r) + \frac{P_1}{\mu} + i_1 + \nu J_1 = 0. \]  

(13)

The quantity \( s_0(r) \) can be replaced by its average \( \bar{s}_0 \) over the tube cross-section, because the dependence of \( s_1(r) \) on \( r \) is much stronger. (The deviations of \( s(r,t) \) from the mean \( \bar{s}(t) \) have opposite signs for water motions in opposite directions during the oscillation, so that in the time integral

\[ s_0(r) = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} s(r,t) dt \]

the deviations approximately cancel.)

Denote

\[ G = \frac{P_1}{\mu} + i_1 + - \nu J_1, \quad \alpha^2 = \frac{g - \omega^2(\bar{s}_0 + d)}{\nu \omega (\bar{s}_0 + d)} \]

(14)

and introduce a new function

\[ \theta(r) = s_1(r) + \frac{G}{g - \omega^2(\bar{s}_0 + d)} \]

(15)

The equation (13) now becomes

\[ \nabla^2 \theta(r) - \alpha^2 \theta(r) = 0, \]

(16)

which is a Bessel equation.

Taking into account the boundary condition that the water velocity is zero on the tube wall and the requirement for \( \theta(r) \) to be finite on the axis \( r = 0 \), one has the solution of (16) [3, 4]:

\[ \theta(r) = \frac{G}{g - \omega^2(\bar{s}_0 + d)} \text{ber}(\alpha r) + i \text{bei}(\alpha r) \]

\[ \text{ber}(\alpha R) + i \text{bei}(\alpha R) \]

(17)

Define an "effective additional length" of the tube caused by the inertia of the oscillating water in the tank, as follows

\[ a(t) = -a_0 \omega^2 \bar{s}_1 \]

(18)

so that from (12)

\[ I_1 = -a_0 \omega^2 \bar{s}_1 \]

(19)

Similarly, define an "effective additional length" of the tube from friction in the tank
MEASUREMENTS OF $\partial \rho / \partial h$

$$b(t) = \frac{1}{\nabla^2 v(t)} \int_{-d}^{d} \nabla^2 u_z(z, t) dz. \quad (20)$$

From (12) it follows that

$$J_1 = b_0 \nabla^2 v_1. \quad (21)$$

One has

$$\nabla^2 v_1 = \frac{2}{R^2} \int_0^R \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_1(r)}{\partial r} \right) rd\tau = \frac{2k}{R} \left( \frac{\partial s_1(r)}{\partial r} \right)_{\tau = R}. \quad (22)$$

Assuming $\alpha R >> 1$, so that the higher powers of $1/\alpha R$ can be neglected and the Bessel functions can be replaced by their asymptotic representations, one obtains from (14), (15), (17), (19), (21) and (22) after some calculations, the relation between $s_1$ and $p_1$:

$$\overline{s_1} = -\frac{p_1}{\mu g} A, \quad (23)$$

where

$$A = \left[ 1 - \frac{\omega^2}{g} (\overline{s_0} + d + a_0) \right]^{-1} \left[ 1 - \frac{\sqrt{2}}{\alpha R} \left( 1 + \frac{b_0}{\overline{s_0} + d} \right) (1-i) \right]. \quad (24)$$

The first and second square brackets represent the effect of inertia and viscosity, respectively. They are both usually very small and thus even a very rough estimate of the magnitude of the effective lengths $a_0$ and $b_0$ is sufficient. In most cases $a_0$ and $b_0$ can simply be neglected. The time average $\overline{s_0}$ can be determined by the manometer (see below).

In case of turbulent flow, Eq. (8) contains an additional term (Reynolds equations) on the right-hand side [2]

$$\sum_{j=1}^{3} \nabla^i (w^j w^j)$$

(written in component form), where $w^i$ are the fluctuating components of the velocity. The first harmonic of this expression vanishes for odd-harmonic oscillations, so that the result is the same as for a laminar flow, namely (23).

A relation similar to (23) can be obtained for the manometer. Assume that the radius of curvature of the manometer tube in the lower part is sufficiently large compared to the tube radius $R'$, so that the manometer can be considered as a straight tube (see Fig. 1.) This is equivalent to the assumption that the effect of the tube curvature on the space distribution of the water velocity is negligible. In this case the velocity depends only on $r$
and only the z-component of the velocity is non-vanishing. The Navier-Stokes equation is

$$\frac{\partial}{\partial t} v(r, t) = -g \left[ 2\epsilon (z + \frac{1}{2}) - 1 \right] - \frac{1}{\mu'} \frac{\partial p'(z, t)}{\partial z} + \nu' \nabla^2 v(r, t),$$

(25)

where $\mu'$, $\nu'$, $\epsilon(z+1/2)$ and 1 are the density, the viscosity of the water in the manometer, the Heaviside's unit function and the length of the water column, respectively.

The equation (25) integrated along the dashed path in the manometer in Fig.1 and solved for the first harmonics, gives for $\alpha'R' \gg 1$,

$$\bar{s}_1^m = -\frac{P_1'}{2\mu'g} B,$$

(26)

where

$$B = \left[ 1 - \frac{\omega^2}{g} \frac{1}{2} \right]^{-1} \left[ 1 - \frac{\omega}{\alpha'R'}(1-1) \right]$$

(27)

and

$$(\alpha')^2 = \frac{2g - \omega^2}{\nu'\omega_1}.$$  

(28)

Taking $\bar{s}_0 + d = 100$ cm and using the constants of the NORA oscillator (CGS units) $\omega = 0.172$, $g = 980.7$, $R = 3.5$, $R' = 0.75$, $1 = 275$, $\nu(H_2O, 20^\circ C) \approx 10^{-2}$ $\nu(D_2O, 20^\circ C) \approx 1.12 \times 10^{-2}[5]$, one has $A \approx 0.997 + 6.02 \times 10^{-3}i$, $B \approx 0.975 + 2.95 \times 10^{-2}i$. The quantities $a_0$ and $b_0$ were neglected, because an error of 0.1% in $|A|$ can be caused by an error of 33 cm in $a_0$, or 88 cm in $b_0$, or 40 cm $\bar{s}_0$ in this case. In calculations the value of $\bar{s}_0$ given by the "equilibrium" relation

$$\bar{s}_0 \approx \frac{2\mu'}{\mu} \bar{s}_0^m \approx \frac{\mu'}{\mu} \times (\text{manometer amplitude})$$

(29)

can be used.

Under the assumption $p_1 \approx p_1''$, equations (23) and (26) give for the NORA oscillator

$$|\bar{s}_1| \approx 2\frac{\mu'}{\mu} 1.045 |\bar{s}_1^m|,$$

so that the first harmonics of the water oscillations in the tube and the manometer differ by 4.5% in this case.

If there were an equipment for recording $p_1$, the problem would be solved. Since there is no such equipment in NORA, an approximate method for determining $\bar{s}_1$ is proposed below.
3.2. Equation of state

Assume that in the conduit of the oscillator the usual gas-state equation holds,

\[ p(t)V(t) = \text{const} \times m(t) \]  \hspace{1cm} (30)

where \( V \) and \( m \) are the volume and the mass of the air contained in the conduit, respectively. The mass \( m \) changes periodically, having a higher and a lower constant value when the valves are closed. Denote the corresponding equilibrium quantities by subscripts \( a \) and \( b \) respectively, all other subscripts meaning the coefficients in Fourier series.

One has (the bars omitted from now on)

\[ s_{o,a,b} \approx \frac{2\mu}{\mu} s_{o,a,b}^m, \quad p_{o,a,b} = 2\mu'g(a-s_{o,a,b}^m), \]  \hspace{1cm} (31)

where \( a = P/2\mu'g \). Express the volume \( V \) as follows:

\[ V(t) = W - Ss(t) - cs^m(t), \]  \hspace{1cm} (32)

where \( c \) is the cross-section of the manometer and \( W \) is a constant.

If \( |c(B/A-1)/(b+c)| \ll 1 \) (where \( b = 2\mu'S/\mu \)) is assumed, it follows from (23), (26), (31) and (32) that

\[ V_1 \approx -(b+c) \frac{\mu}{2\mu'} s_1. \]  \hspace{1cm} (33)

Define the "pulse shape" \( \Delta \) of the mass oscillations by the equation

\[ (p_b V_b - p_a V_a) \Delta(t) = p(t)V(t) - p_a V_a. \]  \hspace{1cm} (34)

For odd-harmonic oscillations one gets

\[ (p_b V_b - p_a V_a) \Delta_1 = p_1 V_0 + p_0 V_1. \]  \hspace{1cm} (35)

Assuming \( |A-1| \ll 1 \), one obtains from (23), (26), (31), (32), (33) and (35)

\[ p_b V_b - p_a V_a = 2\mu'g(s_{a}^m - s_{b}^m) \left[ W + (b+c)(a-2s_{b}^m - s_{a}^m) \right], \]

\[ p_0 V_1 + p_1 V_0 = -\mu g s_1 \left[ W + (b+c)(a-2s_{b}^m) \right]. \]  \hspace{1cm} (36)

One has approximately \( s_{b}^m = (s_{b}^m + s_{a}^m)/2 \), so that from (35) and (36) follows

\[ \Delta_1(s_{b}^m - s_{a}^m) = \frac{\mu}{2\mu'} s_1. \]  \hspace{1cm} (37)
Substituting Eqs. (37) and (4) in (1), we get
\[
\frac{\partial \rho}{\partial h} = \frac{S^I}{b(s_a^m - s_b^m)} \frac{n_1}{n_0} \frac{1}{T\Delta_1}.
\] (38)

The problem was now reduced to determination of \( \Delta_1 \).

If the air streaming through the valves did not depend on the differences between the pressures in the conduit and the pressure vessels, \( \Delta(t) \) would be trapezoidal and Eq. (7) would be valid. However, the pressure oscillations in the conduit are quite large and \( \Delta(t) \) may be deformed in a way shown in Fig. 3. One can write \( \Delta \) as follows:
\[
\Delta(t) = \Box(t) + \epsilon(t),
\] (39)

where \( \epsilon(t) \) is equal to \( \epsilon_a \) and \( \epsilon_b \) during the rise- and fall-times respectively, and is zero elsewhere (see Fig. 3). Assume that \( \epsilon(t) \) is small, so that \( \epsilon_a \) and \( \epsilon_b \) can be approximated by quadratic functions
\[
\epsilon_a(x) = \alpha(x-x'), \quad \epsilon_b(x) = \beta(x-\pi)(x-x'-\pi),
\] (40)

\( \alpha \) and \( \beta \) being constants. The first harmonic of \( \epsilon \) is given by
\[
\text{Re}\epsilon_1 = \frac{\alpha - \beta}{2\pi} \left[ x' (1 + \cos x') - 2 \sin x' \right],
\] (41)

\[
\text{Im}\epsilon_1 = -\frac{\alpha - \beta}{2\pi} \left[ x' \sin x' + 2 \cos x' - 1 \right].
\]

Now, \( \partial \rho/\partial h \) is real and one has from (38)
\[
\text{Im} \{ T(\Box_1 + \epsilon_1)/n_1 \} = \text{Im}(T\Box_1/n_1) + \text{Im}(T/n_1) \text{Re}\epsilon_1 + \text{Re}(T/n_1) \text{Im}\epsilon_1 = 0. \] (42)
From (39) and (42) it follows that

$$\Delta_1 = \frac{\text{Im}(T) / n_1}{\text{Im}(T / n_1) + \text{Re}(T / n_1)} (1 + iE),$$

(43)

where $E = \text{Im} \epsilon_1 / \text{Re} \epsilon_1$. This, substituted in (38), gives the final result.

4. EXPERIMENTAL RESULTS

The oscillation method proved to be very accurate. The accuracy depends on the number of measurements (preferably with different oscillation amplitudes).

In NORA, in every case, about 10-15 measurements were taken (over 5 periods each) within 1 - 1.5 h. The data were processed by a code both according to (7) and (38). The statistical standard error was always approximately 0.3%. The maximum difference between the results obtained by the two formulae (7) and (38) was about 1.5%. In most cases the differences were smaller than the experimental errors, which proves that the "trapezoidal approximation" (7) was good. This, of course, depends on a particular type of oscillator.

<p>| TABLE I |
| MEASUREMENTS FOR SEVERAL CORES AND COMPARISON WITH THEORETICAL PREDICTIONS |
|---------|-----------------|-----------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th>Core</th>
<th>Oscillation measurement (pcm/cm)</th>
<th>Period measurement (pcm/cm)</th>
<th>Linde's code (pcm/cm)</th>
<th>Eq.(44) (pcm/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S2</td>
<td>282 ±6</td>
<td>331.8</td>
<td>262.1</td>
<td></td>
</tr>
<tr>
<td>S3</td>
<td>320 ±1.6*</td>
<td>255 ±5</td>
<td>334.5</td>
<td>274.1</td>
</tr>
<tr>
<td>S4</td>
<td>351.5 ±2.1</td>
<td>Failed ++</td>
<td>375.2</td>
<td>356.8</td>
</tr>
<tr>
<td>S6</td>
<td>278.0 ±1.2</td>
<td>184 ±18</td>
<td>266.5</td>
<td>243.3</td>
</tr>
</tbody>
</table>

* Statistical standard error.

++ The function $\rho(h)$ was quadratic, not even intersecting the axis $\rho = 0$.

As an illustration, results of measurements for several cores are given in Table I and compared with the theoretical predictions of both the Linde's code [6] and the formula

$$\frac{\partial \rho}{\partial h} = \frac{2\pi B^2}{H_{\text{extr.}}^3} \left( \frac{\tau + \frac{L^2}{1 + B^2 L^2}}{L} \right),$$

(44)

where $H_{\text{extr.}}$, $\tau$, $L^2$ and $B^2$ are the extrapolated height of the core, the neutron age, the diffusion length and the buckling, respectively. Measurements of
$\partial \rho / \partial h$ were also carried out by observing the asymptotic periods for several step changes in the water height. There is a large discrepancy between the results of these measurements and those obtained by the oscillation technique. This difference has not been satisfactorily explained, though there is some indication that the effect of the water meniscus (the fuel elements are not completely covered by water) is different for the two kinds of measurements. The oscillation method can be considered as more reliable.

There is not very good agreement between theory and experiment, but in general the numbers computed by the Linde's code are closer to the measured values than those given by (44).

REFERENCES


DISCUSSION

C.E. COHN: In a system like this where the water is introduced or removed very rapidly from the tank, one would expect waves or disturbances to be produced on the surface of the water. Did this in fact happen and, if so, do you think it perturbed your experiments significantly?

D. BABALA: I do not think this effect was very important and it was in fact neglected. The period of the water oscillations was about 35 s, while the rise- and fall-times of the water were about 6 - 8 s. For the amplitude of the oscillations we used (about 0.5 mm) I think that the motion of the water was sufficiently slow. I don't think there were any significant perturbances as a result of this effect.

R. PERSSON: Is the meniscus effect more pronounced in the case of the oscillating technique or in that of the conventional method? Do you think the meniscus would be produced following oscillations having an amplitude of less than 0.5 mm?

D. BABALA: We have not actually investigated the influence of the meniscus effect. I merely mentioned it as a possible explanation for the large difference between the values obtained by the two methods. Our impression is that the oscillating technique is more reliable.

N. RAJŚIĆ: In your two-group calculation of $\partial \rho / \partial h$, you did not take the influence of the radial reflector into account. This can, of course, cause a considerable difference in the calculated value. Do you think that this may perhaps be the reason for the discrepancies between your calculated and measured values of $\partial \rho / \partial h$?

D. BABALA: Yes, I think it is. However, the effect of the reflector was taken into account in the computations done by the LINDE code.
G. L. LUNIN: This is an extremely interesting method but I doubt whether it could be used for closely-spaced lattices because of the considerable errors liable to be produced by the shape of the meniscus in the water as a result of the upward and downward level fluctuations. We came up against this problem of the effect of meniscus shape on the accuracy of results in the course of fuel-assembly experiments carried out at the Novo-Voronezh station.

D. BABALA: Yes, I think you are right. The meniscus effect could certainly be very large in closely-spaced lattices. With doubling-time measurements we tried raising the water level continually and measuring with the fuel rods dry above the water level. However, the measurements were not in agreement with the theoretical data and we had some doubts about them. Once we even obtained a quadratic function of the reactivity on the water level which, when extrapolated, did not even intersect at the axis ρ = 0. Perhaps the meniscus effect was very large in these measurements too, but, as I said, we didn't go into details. It is planned to do so in the future.

I. McGILL: Do you think that the differences between your oscillating and doubling-time results are perhaps due in part to the uncertainty of the photoneutron data at present available for a D₂O system?

D. BABALA: The photoneutron effectiveness was calculated separately for each core, two different situations being envisaged. In one case, it was assumed that the gamma sources were distributed homogeneously in the fuel rod; in the second, that the gamma sources were concentrated in the middle of the rod. The values were fairly close to one another so that the photoneutron effectiveness was known relatively accurately. I don't think this could be the reason for the large differences that were found.

I. McGILL: How do you measure the water height and what accuracy can you achieve?

D. BABALA: The water height was measured by a dipstick with an accuracy of 0.1 mm. The method is described in the paper of Bryhn-Ingebrigtsen et al.* In the case of the oscillation measurements the accuracy was, of course, much higher. The oscillation amplitude was measured by means of the attached manometer.

I. McGILL: Another thing that interests me is whether you found any oscillations back on the water from the main tank. We find that there is an oscillation between the main tank and the tube even with the control tubes which hold the instruments. I should think there would be quite a swing back in the water tube when the pressure is levelled out so that — once you've done your rise and you stop — the water would go on oscillating between the tank and tube.

D. BABALA: The pressure didn't change abruptly because the rise- and fall-times of the water oscillations were adjusted by opening the valve. When the valve was opened, the speed increased because the pressure change was more abrupt. However, when the air streamed through a small opening in the valve, the pressure did not change so very much. Under those conditions the change was not very abrupt, so that the movement of the water was relatively slow.

THE IMPULSE RESPONSE OF AN EXPONENTIAL ASSEMBLY

R.E. UHRIG
UNIVERSITY OF FLORIDA, GAINESVILLE, FLA.
UNITED STATES OF AMERICA

Abstract — Résumé — Аннотация — Resumen

THE IMPULSE RESPONSE OF AN EXPONENTIAL ASSEMBLY. A spatial-dependent transfer function of an exponential assembly with rectangular geometry and the neutron source located at the origin in the middle of one side has been derived using diffusion theory and the usual approximations. If the end correction factor is neglected, the result

\[ G(x, y, z, s) = \frac{\Delta \delta(x, y, z, s)}{\Delta S(o, o, o, s)} \frac{2}{abD} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\rho_{mn}(s)} \cos \frac{ma}{a} \cos \frac{nb}{b} e^{-\rho_{mn}(s)Z}. \]  

(1)

where

\[ \rho_{mn}(s) = \left[ \frac{V}{VD} + \delta_{1mn}^{s} - \delta_{2mn}^{s} \right]^\frac{1}{2} = \left[ \frac{V}{VD} - s \right]^\frac{1}{2}. \]

(2)

The inverse Laplace transform of Eq. (1) gives the spatial dependent impulse response function (Green's function) to be

\[ g(x, y, z, t) = \frac{2}{ab} \sqrt{\frac{V}{\pi D}} \frac{1}{\sqrt{t}} e^{-z^2/4VDt} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \cos \frac{ma}{a} \cos \frac{nb}{b} e^{-VDt^2/4}. \]

(3)

If only the terms of Eq. (3) outside the double summation are considered (i.e. multiplicative and leakage effects are neglected) the result is

\[ g(x, y, z, t) = \frac{2}{ab} \sqrt{\frac{V}{\pi D}} \frac{1}{\sqrt{t}} e^{-z^2/4VDt} \frac{4V}{ab} P(z). \]

(4)

where \( P(z) \) is a Gaussian distribution term of the form

\[ P(z) = \frac{1}{\sigma \sqrt{2\pi}} e^{-z^2/2\sigma^2}. \]

(5)

with a time dependent standard deviation

\[ \sigma = \sqrt{2VDt}. \]

(6)

Hence, a pulse of thermal neutrons introduced at the origin spreads out with time in a symmetrical manner about the \( z=0 \) plane. However, the variation of amplitude with time at any position along the \( z \) axis
shows that a peak value of neutron density does move out from the origin with a decreasing amplitude. Although
the multiplication and transverse leakage influence the characteristics of the disturbance, it does propagate
away from the source in a manner similar to the propagation of neutron waves.

The propagation of a thermal-neutron pulse has been demonstrated experimentally in 1962 at the University
of Florida using a pulsed-neutron source and a "Thermalizer box". However, the method used for the experi-
ments reported here was the cross correlation between the pseudo-random binary (off-on) variation of source
strength and the resulting variation of neutron density in the exponential assembly. Data are given for experi-
ments carried out on both light- and heavy-water moderated assemblies using natural uranium. The results
are discussed in terms of the theoretical relations derived and the physical phenomena taking place. The
validity of the derived relationships and the need for considering higher harmonics for various arrangements of
fuel and moderator are discussed briefly.

RéPONSE D'UN ASSEMBLAGE EXPONENTIEL AUX IMPULSIONS. En utilisant la théorie de la diffusion et
les approximations habituelles, l'auteur a établi une fonction de transfert dépendant de l'espace d'un assemblage
exponentiel à géométrie rectangulaire, l'origine étant au milieu de l'un des côtés où la source de neutrons
est placée. Si l'on néglige le facteur de correction final, on obtient l'expression suivante:

\[
G(x, y, z, t) = \frac{\Delta G(x, y, z, t)}{\Delta S} = \frac{2}{abD} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\rho_{mn}^{(s)}} \cos \frac{m \pi x}{a} \cos \frac{n \pi y}{b} e^{-\rho_{mn}^{(t)} t} Z,
\]

dans laquelle

\[
\rho_{mn}^{(t)} = \left[ \frac{S}{VD} + B_{mn}^2 - B_{mn}^2 \right]^{\frac{1}{2}} = \left[ \frac{S}{VD} - \frac{m^2}{\rho_{mn}} \right]^{\frac{1}{2}}.
\]

L'inverse de la transformée de Laplace de l'équation (1) donne comme fonction de réponse aux impulsions
dépendant de l'espace (fonction de Green)

\[
g(x, y, z, t) = \frac{2}{ab} \sqrt{\frac{V}{\pi D}} \frac{1}{\sqrt{t}} e^{-z^2/4VDt} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\rho_{mn}^{(t)}} \cos \frac{m \pi x}{a} \cos \frac{n \pi y}{b} e^{-VDt}.
\]

Si l'on considère uniquement les termes de l'équation (3) sans tenir compte de la double sommation
c'est-à-dire si l'on néglige les effets de multiplication et des effets de fuite), on obtient le résultat suivant:

\[
\frac{2}{ab} \sqrt{\frac{V}{\pi D}} \frac{1}{\sqrt{t}} e^{-z^2/4VDt} = \frac{4V}{ab} P(z),
\]

dans lequel \( P(z) \) est un terme de distribution normale de la forme

\[
P(z) = \frac{1}{\sqrt{2\pi} \sigma} e^{-z^2/2\gamma^2}
\]

avec un écart type qui est fonction du temps

\[
\sigma = \sqrt{\frac{2VDt}{\pi}}.
\]

Par conséquent, une impulsion de neutrons thermiques fournie à l'origine se déploie avec le temps de
façon symétrique selon le plan \( z = 0 \). Toutefois, la variation d'amplitude en fonction du temps en un point
quelconque le long de l'axe de \( z \) montre qu'une valeur maximum du nombre volumique de neutrons se déplace
bien à partir de l'origine avec une amplitude décroissante. Bien que la multiplication et la fuite transversale
aient un effet sur les caractéristiques de la perturbation, cette perturbation se propage bien à partir de la source
de la même façon que les ondes neutroniques.

On a fait une expérience sur la propagation d'une impulsion de neutrons thermiques en 1962 à l'Université
de Floride en utilisant une source de neutrons pulés et une « boîte de thermalisation ». Toutefois, l'auteur a
utilisé pour les expériences ci-dessus mentionnées la méthode de corrélation entre la variation binaire pseudo-
aléatoire (ouvert-fermé) de l'intensité de la source et la variation du nombre volumique de neutrons qui en résulte dans l'assemblage exponentiel. L'auteur fournit des données sur les expériences faites avec des assemblages à uranium naturel et à eau ordinaire et des assemblages à uranium naturel et à eau lourde. Il examine les résultats, compte tenu des relations théoriques qu'il a établies et des phénomènes physiques qui se produisent. Il examine brièvement la validité de ces relations et la nécessité de tenir compte d'harmoniques d'ordre supérieur pour les diverses dispositions du combustible et du ralentisseur.

IMPULEUX CHARAKTERISTIKA EKSPONENCIALLIAKOY SBBORKI. Выводится пространственно зависимая функция переноса эксконенциальной сборки с прямоугольной геометрией и нейтронным источнико, расположенным в исходном пункте в середине одной стороны. При этом используются теория диффузии и обычных приближений. Если не учитывать конечный коэффициент поправки, то получается следующий результат:

\[ G(x, y, z, s) = \frac{\Delta f(x, y, z, s)}{\Delta S(0, 0, 0, s)} = \frac{2}{abD} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\rho_{mn}(s)} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-\rho_{mn}(s)z}, \]

где

\[ \rho_{mn}(s) = \left[ \frac{S}{VD} + B_{km}^2 - B_{mn}^2 \right]^\frac{1}{2} \left[ \frac{S}{VD} - B_{mn} \right]^\frac{1}{2}. \]

Обратное лапласово преобразование уравнения (A−1) дает пространственно зависимую функцию импульсной характеристики (функция Грина) следующего вида:

\[ g(x, y, z, t) = \frac{2}{ab} \sqrt{\frac{V}{\pi D}} \frac{1}{\sqrt{1}} e^{-2V^2(z)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-VD^2 \gamma_{mn}^2} \]

Если рассматриваются только члены уравнения (A−3) вне двойного сложения (т.е. не учитываются факторы размножения и утечки), то получается следующий результат:

\[ \frac{2}{ab} \sqrt{\frac{V}{\pi D}} \frac{1}{\sqrt{1}} e^{-2V^2(z)} = \frac{4V}{ab} P(z), \]

где \( P(z) \) является гауссовым членом распределения формы:

\[ P(z) = -\frac{1}{\sigma^2/2 \pi} e^{-z^2/\sigma^2}, \]

при зависимом от времени стандартном отклонении

\[ \sigma = \sqrt{2VDt}. \]

Таким образом, импульс тепловых нейтронов, введенный в исходной пунк, распространяется со временем симметрически по плоскости \( z = 0 \). Однако изменение амплитуды со временем в любом месте вдоль \( z = 0 \) показывает, что максимальная величина плотности нейтронов исходит из исходного пункта с уменьшением амплитуды. Хотя размножение и обратная утечка влияют на характеристики возмущения, это распространяется от источника подобно нейтронным волнам.

Распространение импульса тепловых нейтронов было продемонстрировано экспериментально в 1962 году во Флоридском университете при использовании импульсного источника
нейтронов и "ящика термализатора". На этот раз был использован метод перекрестной корреляции псевдосистемы двойного (выключения-включения) изменения сили источника и изменения в результате этого плотности нейтронов в экспоненциальной сборке. Приводятся данные для экспериментов, проведенных на сборках как на легкой, так и на тяжелой воде, на которых испытывается природный уран. Обсуждаются результаты с точки зрения полученной теоретической связи и физических явлений. Кратко обсуждаются действительность установленных связей и необходимость рассмотрения более высоких гармоников для различных расположений горючего и замедлителя.

RESPUESTA DE UN CONJUNTO EXPONENCIAL A LOS IMPULSOS. El autor recurre a la teoría de la difusión y a las aproximaciones corrientes a fin de derivar una función de transferencia dependiente del espacio para un conjunto exponencial con geometría rectangular y cuya fuente neutónica está situada en el centro de uno de los lados. Si se desprecia el factor de corrección final, el resultado es:

\[ G(x,y,z,t) = \frac{D}{abD} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\rho_{mn}(t)} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-\rho_{mn}(t)} Z, \]

\[(1)\]

donde

\[ \rho_{mn}(t) = \left[ \frac{S}{VD} + \frac{b^2}{a^2} \right] \]

\[(2)\]

La transformación laplaciana inversa de la ecuación (1) da para la función de respuesta al impulso, dependiente del espacio (función de Green),

\[ g(x,y,z,t) = \frac{2}{abV} \sqrt{\frac{V}{\pi D}} \frac{1}{\sqrt{t}} e^{-z^2/4VDt} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-VDt_{mn}}. \]

\[(3)\]

Si en la ecuación (3) se desprecia la doble suma (es decir, los efectos multiplicantivos y de escape) el resultado será

\[ g(x,y,z,t) = \frac{2}{abV} \sqrt{\frac{V}{\pi D}} \frac{1}{\sqrt{t}} e^{-z^2/4VDt} = \frac{4V}{ab} P(z), \]

\[(4)\]

donde \( P(z) \) constituye un término de distribución gaussiana de la forma

\[ P(z) = \frac{1}{\sigma \sqrt{2\pi}} e^{-z^2/2\sigma^2}, \]

\[(5)\]

con una desviación estándar, dependiente del tiempo,

\[ \sigma = \sqrt{2VDt}. \]

\[(6)\]

Por tanto, un impulso de neutrones térmicos introducido en el origen se distribuye con el tiempo simétricamente por el plano \( z = 0 \). No obstante, la variación de la amplitud con el tiempo en cualquier posición a lo largo del eje \( z \) muestra que, partiendo del origen, un máximo de la densidad neutónica se desplaza con amplitud decreciente. Aunque la multiplicación y las pérdidas transversales ejercen influencia sobre las características de la perturbación, ésta se propaga alejándose de la fuente de modo similar a la propagación de ondas neutónicas.

En 1962, se demostró experimentalmente en la Universidad de Florida la propagación de un impulso de neutrones térmicos empleando una fuente pulsante y una "caja termalizadora". Sin embargo, el método empleado para los experimentos descritos en la presente memoria consistió en establecer una correlación entre la variación binaria (paro-marcha) pseudo-aleatoria de la intensidad de la fuente y las variaciones de la densidad neutónica a que da lugar en el conjunto exponencial. La memoria expone datos relativos a experimentos
IMPULSE RESPONSE

1. INTRODUCTION

Historically, the impulse response of an exponential assembly has been obtained by injecting bursts of neutrons from a pulsed accelerator and studying the die-away of the neutron density as a function of time. This technique requires that the decay of the neutron flux be studied during the interval after the energy and spatial modes have died out and before the neutron flux has reached the background level. Problems of high background often present limitations on the usefulness of this technique. The pulsed neutron technique has been adequately described in the literature [1-21] and a considerable amount of work is presently under way.

The purpose of this paper is to describe an alternative method of measuring the impulse response of an exponential or subcritical assembly in which the background is not important. It involves cross-correlation between the input and output of the assembly where the input is a random or pseudo-random variation of neutron source strength and the output is neutron density or flux. This method is based upon a technique illustrated by LEE [22] and first applied to nuclear systems by BALCOMB et al. [23] and RAJAGOPAL [24]. A detailed description of how this method can be applied to subcritical or exponential assemblies has been given by STERN et al. [25], STERN and VALAT [26] and UHRIG [27], and a brief review of these presentations is given here.

For systems which can be characterized by a transfer function $G(s)$ and an impulse response $g(t)$, the cross-correlation between the input and the output for a time lag $\tau$ is given by [22]

$$\phi_{io}(\tau) = \int_{-\infty}^{\infty} \phi_{ii}(\lambda - \tau) g(\lambda) \, d\lambda, \quad (1)$$

where $\phi_{ii}(\lambda - \tau)$ is the auto-correlation of the input and $\lambda$ is the dummy time variable of integration. If the input $i(t)$ closely approximates "white noise," the auto-correlation function of the input is a $\delta$-function located at $\lambda = \tau$, that is

$$\phi_{ii}(\lambda - \tau) = C \delta(\lambda - \tau), \quad (2)$$

where $C$ is a constant. Hence, Eq. (1) becomes

$$\phi_{io}(\tau) = \int_{-\infty}^{\infty} C \delta(\lambda - \tau) g(\lambda) \, d\lambda. \quad (3)$$

The relationship between $\delta(\lambda - \tau)$ and $g(\lambda)$ is shown graphically in Fig. 1. Since
The impulse response function of a system and the auto-correlation function of a "white noise" input

![Fig. 1](chart)

The impulse response function of a system and the auto-correlation function of a "white noise" input the delta function is zero everywhere except at \( \lambda = \tau \), and the integral of a delta function is unity, we have

\[
\int_{-\infty}^{\infty} g(\lambda) \delta(\lambda - \tau) d\lambda = g(\tau),
\]

and Eq. (3) becomes

\[
\phi_{10}(\tau) = C g(\tau),
\]

i.e. the cross-correlation function \( \phi_{10}(\tau) \) is proportional to the impulse response function \( g(\tau) \).

The source transfer function for a nuclear reactor using a "one-point" or "lumped parameter" model is given by UHRIG [27] to be

\[
G(s) = \Delta N(s) \frac{A(s)}{\Delta S(s)} = \frac{1}{s + \frac{1 - k}{\ell} + k \left[ \beta - \sum_{i=1}^{n} \frac{\beta_i \lambda_i}{s + \lambda_i} \right]},
\]

where \( k \) is the effective reproduction constant, \( \ell \) is the effective neutron lifetime, \( \beta_i \) and \( \lambda_i \) are the delayed fraction and decay constant of the \( i \)-th group of delayed neutrons respectively. For one group of delayed neutrons, Eq. 6 simplifies to

\[
G(s) = \frac{\Delta N(s)}{\Delta S(s)} = \frac{s + \lambda}{s + \frac{1 - k}{\ell} \left( 1 - \beta \right)} \frac{(s + \lambda)}{s + \frac{\lambda(1 - k)}{1 - k(1 - \beta)}}.
\]

Plots of amplitude and phase of this transfer function are shown in Figs. 2 and 3, and it is readily apparent that the effect of the delayed neutrons as
Amplitude of the source transfer function of a "lumped parameter" model of a nuclear reactor

\[ f = 10^{-4} \, \text{s} \]
\[ \beta = 0.0064 \]
\[ \lambda = 0.08 \, \text{s}^{-1} \]

\[ G_R(t) = \left[ \frac{f}{1-k} \right] \left[ \frac{1}{\lambda t} + 1 \right] \left[ \frac{1}{1-k(1-\beta)} + 1 \right] \]

represented by the two break frequencies on the left portions of the curves are insignificant if \( k \) is less than 0.98. Hence, Eq.(7) can be further simplified to

\[ G(s) = \frac{\Delta N(s)}{\Delta S(s)} = \frac{1}{s + \frac{1-k(1-\beta)}{\beta}}. \]  

The impulse response of a subcritical or exponential assembly is obtained by taking the inverse Laplace transform of this transfer function to give

\[ g(t) = e^{-\alpha t}, \]

where \( \alpha \) is the negative of the "Rossi alpha",

\[ \alpha = \frac{1-k(1-\beta)}{\beta}. \]
Phase angle of the source transfer function of a "lumped parameter" model of a nuclear reactor

\[ s \]

\[ G_R(s) = \left( \frac{s}{1-k} \right)^{\frac{1}{\lambda+1}} \left( \frac{s}{\lambda(1-k)+1} \right)^{\frac{1}{\epsilon}} \]

Substitution of Eq. (9) into Eq. (5) gives the cross-correlation between the input and output to be proportional to the impulse response function, i.e.

\[ \phi_\theta(\tau) = C e^{-\alpha \tau} = C \exp \left\{ -\left( \frac{1-k(1-\beta)}{\epsilon} \right) \right\} \]

A comparison of the result obtained in the previous paragraph with the pulsed neutron technique will show that the cross-correlation function \( \phi_\theta(\tau) \) corresponds directly with the exponential decay of the fundamental mode in a pulsed system provided that the time variable is replaced with the correlation lag time \( \tau \). Hence, the cross-correlation technique can be used to replace the pulsed neutron experiment if the "lumped parameter" model of the system is valid. There are, furthermore, certain advantages associated with the cross-correlation technique which are quite significant. First, the cross-correlation process automatically removes the effect of any extraneous background variation of the neutron population which is uncorrelated with the input (i.e. their cross-correlation function is zero). Secondly, by use of a discrete binary "pseudo-random" source of neutrons, it is possible to operate with duty cycles of approximately 50%, whereas, with pulsing, the duty cycle is often 0.1% to 1%. With random variations at a high rate in the neutron source intensity, the delayed neutrons constitute a constant background which is not correlated with the neutron source.

When a pseudo-random binary "on-off" neutron source is used, there are many similarities between the phenomena taking place in this system and that taking place in neutron wave propagation [28, 29, 30, 31], particularly
when the pseudo-random variation is periodic in nature as used by UHRIG [27] and STERN and VALAT in Refs. [25] and [26]. With a periodic "pseudo-random" signal, the random neutron source might be more properly considered as a "multi-frequency" neutron source, and the technique under consideration here could be treated as a more general case of neutron wave propagation in which many frequencies are generated at once and superimposed. Furthermore, the fact that neutron wave phenomena have been shown to exist experimentally in exponential assemblies [28, 31] has implications regarding the validity of the model described in the foregoing discussion. In an attempt to take these phenomena into account, a more sophisticated model will be developed.

2. SPATIAL-DEPENDENT TRANSFER FUNCTION

Let us consider the case of an exponential or subcritical assembly with a neutron source located in one face. If one considers a region of the assembly where most of the neutrons have been produced by fission, the non-equilibrium neutron-diffusion equation as given by GLASSTONE and EDLUND [32]

\[ D \nabla^2 \phi(r, t) - \Sigma_a \phi(r, t) + k_{pe} \Sigma_a \phi(r, t) = \frac{1}{v} \frac{\partial \phi(r, t)}{\partial t} \]  

may be applied. The time \( t \) and space \( r \) variable are assumed to be separable. Division by \( D \) and use of the relationship

\[ \frac{\Sigma_a}{D} = \frac{1}{L^2} = \frac{1}{\nu vD} \]  

(13)
gives

\[ \nabla^2 \phi(r, t) + \beta^2_{M} \phi(r, t) = \frac{1}{vD} \frac{\partial \phi(r, t)}{\partial t}, \]  

(14)

where the material buckling \( \beta^2_{M} \) is defined as

\[ \beta^2_{M} = \frac{1}{L^2} (k_{pe} - 1) = \frac{1}{L^2} [k_m(1 - \beta) - 1]. \]  

(15)

Eq. (14) can be Laplace-transformed to give

\[ \nabla^2 \phi(r, s) + [\beta^2_{M} - \frac{s}{vD}] \phi(r, s) = 0. \]  

(16)

or

\[ \nabla^2 \phi(r, s) + \beta^2_{M}(s) \phi(r, s) = 0, \]  

(17)

where the complex material buckling \( \beta^2_{M}(s) \) is given by

\[ \beta^2_{M}(s) = \frac{\beta^2_{M} - \frac{s}{vD}}{vD}. \]  

(18)
An exponential assembly with rectangular geometry will be chosen for convenience in obtaining a solution to Eq. (17), and solutions for other geometries will be readily apparent. The boundary conditions chosen for a rectangular parallelepipedal assembly with dimensions (including extrapolated distances) $a, b, c$, having a neutron source at the centre of one face at the origin of the co-ordinate system, are

(i) The flux is everywhere finite and non-negative;
(ii) The flux is zero at the extrapolated boundaries, that is

\[ \phi(\pm a, y, z, s) = 0, \]  
\[ \phi(x, \pm b, z, s) = 0, \]
\[ \phi(x, y, c, s) = 0. \]

(iii) The neutron source is an isotropic point source located at the origin and the number of neutrons flowing out of the $z = 0$ plane per square centimetre-second in each mode must equal one-half the number produced in that mode by the source, since neutrons flow in both positive and negative directions from the $z = 0$ plane. Hence, the net current density in the $z$-direction from the $z = 0$ plane is

\[ -D \frac{\partial \phi(x, y, z, s)}{\partial z} \bigg|_{z = 0} = \frac{1}{2} S(x, y, z, s) \delta(x-x_0, y-y_0). \]  

When boundary conditions (i) and (ii) are imposed on Eq. (17), the solution as found by a separation of variable technique is

\[ \phi(x, y, z, s) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \cos \frac{m \pi x}{a} \cos \frac{n \pi y}{b} e^{-\rho_{mn}(s)z}, \]

\[ 1 - e^{-2\rho_{mn}(s)(c-z)}, \]  

where $A_{mn}$ are the coefficients of the Fourier cosine series and $\rho_{mn}(s)$ is the complex inverse relaxation length of the $mn$-th mode which is

\[ \rho_{mn}^2(s) = \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 - \beta_M^2 = B_{\bot mn}^2 - \beta_M^2, \]

\[ = B_{\bot mn}^2 - B_M^2 + \frac{s}{vD} = \gamma_{mn}^2 + \frac{s}{vD}. \]

The perpendicular component of the $mn$-th mode of the geometric buckling $B_{\perp mn}$ and the static inverse relaxation length $\gamma_{mn}$ are given by

\[ B_{\perp mn}^2 = \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2, \]

and
It should be noted that the portions of Eq. (23) in rectangular brackets, which is commonly known as the end correction factor, is approximately equal to unity except near the upper end of the assembly. This end correction factor is usually omitted even though it implies that the assembly is infinitely long in the z-direction and that the geometric buckling $B_{mn}^2$ is equal to its perpendicular component $B_{mn}^2$.

The source transfer function of the assembly can be obtained by considering a neutron source whose rate of emitting neutrons to the assembly varies sinusoidally. Since a negative source is a physical impossibility, a steady component equal to or greater than the amplitude of the sinusoidal variation is always present. This source condition is expressed by

$$S(x, y, z, s) = [S_0 + \Delta S(s)] \delta(x-x_0, y-y_0),$$

where $S_0$ is the amplitude of the steady component and $\Delta S(s)$ is the amplitude of the sinusoidal component. Since a subcritical system with a varying source can be considered as a linear system, it is possible to separate the sinusoidal and steady components and deal only with the sinusoidal component which can be represented by

$$S(x, y, z, s) = \Delta S(s) \delta(x, y)$$

when the source is located at the origin, i.e. $x_0 = 0, y_0 = 0$. This source can be expanded in a set of orthogonal functions

$$\Delta S(s) \delta(x, y) = \sum_{m=1}^{p} \sum_{n=1}^{q} \Delta S_{mn}(s) \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b}.$$ 

Multiplication by

$$\cos \frac{p\pi y}{a} \cos \frac{q\pi y}{b} \, dx \, dy$$

and integration between the extrapolated boundaries for x and y causes all terms in which $p \neq m$ and $q \neq n$ to vanish, leaving

$$\Delta S(s) \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \delta(x, y) \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \, dx \, dy$$

$$= \Delta S_{mn}(s) \int_{-a/2}^{a/2} \cos \frac{m\pi x}{a} \, dx \int_{-b/2}^{b/2} \cos \frac{n\pi y}{b} \, dy.$$ 

The left-hand integral is equal to unity because of the delta function, and integration of the right-hand side gives
Use of the source condition in Eq. (22) for each mode of Eq. (23) gives

\[ -D A_{mn} \cos \frac{m\pi x}{a} \frac{\cos \frac{m\pi y}{b}}{b} \left[ -\rho_{mn}(s) \right] e^{-\rho_{mn}(s)z} \bigg|_{z=0} = \frac{1}{2} \frac{4\Delta S(s)}{ab} \cos \frac{m\pi x}{a} \frac{\cos \frac{m\pi y}{b}}{b}, \]  

from which

\[ A_{mn} = \frac{2\Delta S(s)}{abD \rho_{mn}(s)}. \]  

This expression can be substituted in Eq. (23) to give

\[ \Delta \phi(x, y, z, s) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{2\Delta S(0, 0, 0, s)}{abD \rho_{mn}(s)} \cos \frac{m\pi x}{a} \cos \frac{m\pi y}{b} e^{-\rho_{mn}(s)z}. \]  

The source term \(\Delta S(0, 0, 0, s)\) can be taken outside the double summation, and Eq. (34) can be arranged in the form of a transfer function

\[ G(x, y, z, s) = \frac{\Delta \phi(x, y, z, s)}{\Delta S(0, 0, 0, s)} \]

\[ = \frac{2}{abD} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\rho_{mn}(s)} \cos \frac{m\pi x}{a} \cos \frac{m\pi y}{b} e^{-\rho_{mn}(s)z}, \]

where

\[ \rho_{mn}(s) = \left[ \frac{s}{vD} + B_{mn}^2 - B_M^2 \right]^{1/2} = \left[ \frac{s}{vD} + \gamma_{mn}^2 \right]^{1/2}. \]

It is informative to compare the spatial-dependent transfer function of Eq. (35) with the transfer function of the lumped parameter or "one-point" reactor model which is defined as

\[ G(s) = \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \int_{-c/2}^{c/2} G(x, y, z, s) \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} \cos \gamma_{mn} \cos \gamma_{mn} dx dy dz. \]  

The cosine terms are weighting functions which, because of the orthogonality relationship, eliminate all modes except the fundamental. This has been done because "point" reactor kinetics are "one-mode" reactor kinetics. Hence
IMPULSE RESPONSE

\[ G(s) = \frac{1}{2D} \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \int_{0}^{\infty} \frac{2}{abD} \frac{1}{\rho(s)} \cos^2 \left( \frac{\pi x}{a} \right) \cos^2 \left( \frac{\pi y}{b} \right) e^{-\rho(s)z} \, dx \, dy \, dz \]

\[ = \frac{1}{2D} \frac{1}{\rho^2(s)} . \]  

(38)

Since the effect of omitting the end correction factor in Eq.(12) is equivalent to assuming that the assembly is infinitely long in z-direction, the upper limit on the z-variable in Eq.(38) was changed to infinity. This also means that the geometric buckling \( B_g^2 \) is equivalent to its perpendicular component \( B_2^2 \). Hence, a combination of Eqs.(13), (15) and (24) gives

\[ \rho^2(s) = \frac{s}{vD} + B_g^2 - B_m^2 = \frac{1}{vD} \left[ s + \frac{B_g^2 L^2 + 1 - k_{pe}}{\ell_{\infty}} \right] . \]  

(39)

Substituting in Eq.(38), one gets

\[ G(s) = \frac{1}{2} \left[ \frac{1}{s + \frac{B_g^2 L^2 + 1 - k_{pe}}{\ell_{\infty}}} \right] . \]  

(40)

Since the effective neutron lifetime \( \ell \) and effective reproduction constant \( k \) are given by

\[ \ell = \frac{\ell_{\infty}}{1 + B_g^2 L^2} \]  

(41)

and

\[ k = \frac{k_{pe}}{1 - \beta} = \frac{k_{pe}}{(1 - \beta)(1 + B_g^2 L^2)} , \]  

(42)

Eq.(40) becomes

\[ G(s) = \frac{1}{2} \left[ \frac{1}{s + \frac{1 - k(1 - \beta)}{\ell}} \right] , \]  

(43)

which is identical to Eq.(8) except for the factor \( \sqrt{v} \). This difference arises because of the assumption that only half of the neutrons go into the assembly (Eq.22) and that Eq(35) involves neutron flux while Eq.(8) involves neutron density.

3. SPATIAL-DEPENDENT IMPULSE RESPONSE FUNCTION

The impulse response of a system can be obtained by taking the inverse Laplace transform of the transfer function. In the case of the exponential assembly, in which spatial effects are important, the impulse response can be obtained by taking the inverse Laplace transform of Eq.(35), i.e.
\[ g(x, y, z, t) = \mathcal{F}_s^{-1} \left[ G(x, y, z, s) \right] \]

\[
= \mathcal{F}_s^{-1} \left[ \frac{2}{abD} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\rho_{mn}(s)} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-\rho_{mn}(s)z} \right], \quad (44)
\]

where \( \rho_{mn}(s) \) is defined by Eq. (36). Use of the inversion integral gives

\[
g(x, y, z, t) = \frac{1}{2\pi j} \int_{\mu-j\infty}^{\mu+j\infty} e^{st} \left[ \frac{2}{abD} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{\rho_{mn}(s)} \right] \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-\rho_{mn}(s)z} ds \quad (45)
\]

which is most conveniently solved by a change of variable to put it into a form for which a solution is available. Let

\[
p = s + vD\gamma_{mn}^2 \quad (46)
\]

and substitute Eqs. (36) and (46) in Eq. (45)

\[
g(x, y, z, t) = \frac{1}{2\pi j} \int_{\mu+j\infty}^{\mu-j\infty} e^{st} \left[ \frac{2}{abD} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( e^{-vD\gamma_{mn}^2 t} \right) \right] \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \sqrt{\frac{vD}{p}} e^{\sqrt{vD/p}z} dp \quad (47)
\]

Use of Eq. (84) of Appendix 3 of CHURCHILL [33], gives the spatial-dependent impulse response function to be

\[
g(x, y, z, t) = \frac{2}{\sqrt{vD\pi}} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-vD\gamma_{mn}^2 t} \quad (49)
\]
It is instructive to expand the exponential term in the summation by use of Eqs. (13), (15) and (26) (in which the geometric buckling is considered equal to its perpendicular component) into

\[ e^{-\nu D \gamma_{mn}^2} = e^{-\nu D [B_{gm}^2 - B_m^2]} \]

\[ = \exp \left\{ -\nu D B_{gm}^2 + \frac{\Sigma_a}{D} (k_{pm} - 1) \right\} \tag{50} \]

\[ = \exp \left\{ -\nu D B_{gm}^2 - \nu \Sigma_a + \nu \Sigma_m (1 - \beta) \right\} t. \]

The macroscopic absorption cross-section can then be separated into two components, \( \Sigma_{au} \) representing the absorption in the fuel and \( \Sigma_{am} \) representing the absorption in the moderator, that is

\[ \Sigma_a = \Sigma_{au} + \Sigma_{am} \tag{51} \]

and \( \kappa_\infty \) for a one-thermal group of neutrons model is

\[ \kappa_\infty = \eta l = \eta \frac{\Sigma_{au}}{\Sigma_a}, \tag{52} \]

where \( \eta \) is the number of neutrons released per thermal neutron absorbed by the fuel. Substituting Eqs. (51) and (52) in (50), one gets

\[ e^{-\nu D \gamma_{mn}^2} = \exp \left\{ -\nu D B_{gm}^2 - \nu \Sigma_{au} - \nu \Sigma_{am} + \nu \eta \Sigma_{au} (1 - \beta) \right\} t. \tag{53} \]

Equation (53) can be split into four exponential terms having the following meaning:

(a) \( e^{-B_m^2 \rho_D t} \) is the probability that a neutron escapes leakage by diffusion from a system of buckling \( B_{gm}^2 \) during the diffusion time of length \( t \);

(b) \( e^{-\nu \Sigma_{au} t} \) is the probability that a thermal neutron escapes absorption in the uranium in the diffusion time of length \( t \);

(c) \( e^{-\nu \Sigma_{am} t} \) is the probability that a thermal neutron escapes absorption in the moderator in the diffusion time of length \( t \);

(d) \( e^{\nu \eta \Sigma_{au} (1 - \beta) t} \) is the probability that a neutron will be generated by fission during the interval \( t \).

When one compares Eq. (53) with a similar expression derived from pulsed neutron techniques, e.g. VALENTE [20, 21], there are two significant differences: the absence of the product of the fast leakage and resonance escape term \( pe^{-\beta \tau} \) and the absence of the "cooling" term \( CB_{gm}^2 \). Both omissions are expected since the theory used in deriving Eq. (53) is based entirely upon one-group thermal neutron theory.
An interesting comparison can be obtained by considering the portion of Eq.(49) outside of the double summation (i.e. multiplicative and leakage effects are ignored) to be a Gaussian distribution term of the form
\[ P(z) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\left(\frac{z^2}{2\sigma^2}\right)}, \]  
where the standard deviation is
\[ \sigma = \sqrt{2vDt}. \]  
Hence,
\[ \frac{2}{ab} \sqrt{\frac{v}{\pi Dt}} e^{-\frac{z^2}{4vDt}} = \frac{4vP(z)}{ab}. \]  
Since Eq.(55) indicates that the standard deviation and dispersion increases with time, a pulse of thermal neutrons injected at the origin at time \( t = 0 \) spreads out as time increases. Hence, it appears that a burst of thermal neutrons introduced at the origin simply dissipates in a symmetrical fashion about the \( z = 0 \) plane. However, when one considers the variation of amplitude as a function of time at points throughout the assembly, it is seen that a peak does move in the \( z \)-direction with a decreasing amplitude as time increases. Furthermore, the influence of the transverse leakage upon the shape of the pulse as it is propagated cannot be ignored.

The fact that a thermal neutron pulse will propagate has been demonstrated experimentally by PEREZ et al. [34] at the University of Florida in 1962. A pulsed neutron generator using the D-T reaction was employed in conjunction with a "neutron thermalizer box" to provide pulses of thermal neutrons to a graphite-moderating assembly. Measurements of neutron density at several positions along the \( z \)-axis with a \( BF_3 \) detector and a multi-channel analyser clearly showed the propagation of the pulse.

The shape of the impulse response function at a particular position \( x_i, y_i, z_i \), can be obtained from Eq.(49). Let us consider contributions of each of the terms
\[ e^{-\frac{z^2}{4vDt}}, \frac{1}{\sqrt{t}} \text{ and } e^{-vDy_i^2t/4}, \]  
to the amplitude of the impulse response at a position \( (0, 0, 10 \text{ cm}) \). Each of these quantities and their product in a graphite-moderated assembly is shown in Fig.4. For convenience, the characteristics of a rectangular graphite-moderated natural uranium assembly with which the author has worked [35] were chosen for the calculations. The shape of the curve for the product resembles the response of a subcritical assembly following the injection of a burst of neutrons.

The inclusion of terms other than \( e^{-vDy_i^2t/4} \) in the summation causes the impulse response curve to decrease more rapidly than if only the first mode is considered. This fact is quite significant in view of the experimental results obtained by STERN and VALAT [26] in which the majority of the curves, when fitted with a single exponential, gave a slope which was
Contributions of three terms of Eq. (40) to the impulse response of a graphite-moderated, natural uranium subcritical assembly

\[ v = 2.2 \times 10^5 \text{ cm/s} \]
\[ D = 0.903 \text{ cm} \]
\[ y_u = 9.884 \times 10^{-4} \text{ cm}^2 \]
\[ z = 10 \text{ cm} \]

greater than that predicted using calibrations obtained by pulsed neutron measurements.

The phenomena of pulsed propagation can be demonstrated graphically by plotting the impulse response at a point on the \( z \)-axis \( g(0, 0, z, t) \) as a function of time. For purposes of the plot, only the first term of the infinite summation of Eq. (49), i.e.

\[
g_{11}(0, 0, z, t) = \frac{2}{ab} \sqrt{\frac{v}{\pi D}} \frac{1}{\sqrt{t}} \exp \left[ -\frac{z^2}{4vDt} - vDy_1^2t \right] \]  

is shown in Fig. 5. It is apparent that the neutron density does spread out in a Gaussian-like manner about the \( z = 0 \) plane with the spatial dispersion increasing as a function of time. The variation of neutron density as a function of time at specific positions \((0, 0, z)\) on the \( z \)-axis of a graphite-moderated natural uranium assembly \([35]\) is shown in Fig. 6 for \( z = 4, 8, 12, 16, 20, 25, 30, 40; \) and 50 cm. It is apparent that the peak of the pulse does propagate along the \( z \)-axis with time. However, the pulse spreads out severely with time.
Figure 5

Spatial distribution of neutrons in a graphite-moderated, natural uranium assembly after a pulse of thermal neutrons has been injected at the origin.

\[ a = b = 157.5 \text{ cm} \quad c = 200.7 \text{ cm} \]
\[ D = 0.903 \text{ cm/s} \quad v = 2.2 \times 10^5 \text{ cm/s} \]
\[ \gamma_{11}^2 = 9.884 \times 10^{-4} \text{ cm}^{-2} \]

\[ g(0, 0, z, t) = \frac{2}{ab} \sqrt{\frac{v}{\pi D}} \cdot \frac{1}{\sqrt{t}} \exp \left[ - \left( \frac{z^2}{4vDt} \right) - vD \gamma_{11}^2 t \right] \]

and distance, and eventually the neutron density decays asymptotically to \( Ct \cdot e^{-vD \gamma_{11}^2 t} \), where \( C \) is a constant of proportionality.

The phenomena of pulse propagation can also be demonstrated mathematically by considering Eq. (57) and setting its time derivative equal to zero to determine the relationship between time and the location of the maximum value of the response.
The impulse response function at several positions along the z-axis of a graphite-moderated, natural uranium assembly

\[ a = b = 157.5 \text{ cm} \quad c = 200.7 \text{ cm} \]
\[ D = 0.903 \text{ cm} \quad v = 2.2 \times 10^5 \text{ cm/s} \]
\[ \gamma_{11} = 9.884 \times 10^4 \text{ cm}^{-2} \]

\[ g(0,0,z,t) = \frac{2m}{ab} \sqrt{\frac{v}{\pi D}} \exp \left[ -\frac{(z'^2)}{4vDt} - vD\gamma_{11}t \right] \]

\[ \frac{\partial g(0,0,z,t)}{\partial t} = \frac{2m}{ab} \sqrt{\frac{v}{\pi D}} \left\{ -\frac{1}{2} t^{3/2} \exp \left[ -\frac{v^2}{4vDt} - vD\gamma_{11}^2 t \right] \right\} \]

\[ + t^{-1/2} \exp \left[ -\frac{v^2}{4vDt} - vD\gamma_{11}^2 t \right] \left[ \frac{z^2}{4vDt^2} - vD\gamma_{11}^2 \right] \]

\[ = g(0,0,z,t) \left\{ -\frac{1}{2t} + \frac{z^2}{4vDt^2} - vD\gamma_{11}^2 \right\} = 0. \]  

(58)

Since \( g(0,0,z,t) \) cannot be zero, then

\[ -\frac{1}{2t} + \frac{z^2}{4vDt^2} - vD\gamma_{11}^2 = -\frac{2vDt + z^2 + 4vD^2t^2\gamma_{11}^2}{4vDt^2} = 0. \]

(59)
The quadratic equation
\[ 4v^2D^2\gamma_{11}^2 t^2 + 2vDt - z^2 = 0 \] (60)
can be solved for its roots
\[ t = \frac{-1 \pm \sqrt{1 + 4\gamma_{11}^2 z^2}}{4vD\gamma_{11}^2}. \] (61)

If the assumption
\[ 4\gamma_{11}^2 z^2 << 1 \] (62)
can be made, then the radical can be expanded so that
\[ \sqrt{1 + 4\gamma_{11}^2 z^2} \approx 1 + 2\gamma_{11}^2 z^2. \] (63)
The roots of Eq. (60) are then found to be
\[ t_1 = \frac{z^2}{2vD} \] (64)
and
\[ t_2 = -\frac{1 + \gamma_{11}^2 z^2}{2vD\gamma_{11}^2}. \] (65)
The negative root is meaningless; hence, the substitution of the first root into Eq. (57) gives
\[ g(0, 0, z, t) = \frac{2v}{ab\sqrt{\pi}} \frac{1}{z} \exp \left[ -1/2(1 + \gamma_{11}^2 z^2) \right]. \] (66)

Using the assumption of Eq. (62) one gets
\[ g(0, 0, z, t) = \frac{2}{ab\sqrt{\pi D}} e^{-1/2 \frac{1}{z}}, \] (67)
which indicates that the peak value falls off inversely with \( z \). Since Eq. (67) does not include any term involving \( \gamma_{11} \), the effect of multiplication and leakage upon the propagation of peaks through the assembly is negligible. Unfortunately, the assumption of Eq. (62) is valid only for small distances along the \( z \)-axis. In the case of the graphite-moderated, natural uranium assembly cited previously [35], it is valid only for \( z < 5 \) cm.

At this point a discussion of the difference between the injection of a burst of fast neutrons and a burst of slow neutrons in an exponential assembly is in order. When a burst of fast neutrons is injected into an assembly, they travel out over the assembly and become thermal neutrons at a considerable distance from their point of origin. As the assemblies become smaller, the distribution of these thermal neutrons from the fast burst be-
comes more nearly uniform throughout the assembly. The process of neutron leakage from the assembly then allows the neutron population to die out with the fundamental mode. In the case of a thermal neutron burst injected at one of the boundaries in the assembly, the neutrons begin to diffuse with an intensity along the $z$-axis which varies in a Gaussian-like manner with a dispersion $\sigma^2$ which increases with time. As has been shown previously, this results in a peak propagating along the $z$-axis with an amplitude varying inversely with distance from the origin for a short distance.

The process of waiting until the fundamental mode has been assumed in the three principal directions before taking measurements is a very important procedure in the pulsed fast neutron technique. However, if a source of thermal neutrons were used, it is apparent from the discussions in this paper that the neutron population would probably die away to a low value before a fundamental mode could be obtained.

It is apparent that the only way to use a thermal neutron source for a "pulsed" experiment of the conventional type would be to leave the neutron source on long enough that an equilibrium distribution exists, then suddenly remove the source and watch the die-away of the neutron population. Such a procedure is equivalent to the well-known "source jerk" technique. Obviously, such a process could not be repeated rapidly and hence, would lead to long experimental times. With the use of the fast source of neutrons, the resulting distribution of thermal neutrons is sufficiently spread out over the assembly that it is possible to obtain the fundamental mode at least a decade or two above background level.

Since the impulse response of an exponential assembly is determined by the cross-correlation of the neutron flux at a point in the assembly (the output) and the source intensity at the origin (the input) when the source variation is a "random" binary variation with a 50% duty cycle, it is apparent that there is not sufficient time for an equilibrium or fundamental mode distribution to be established in the assembly due to neutron leakage. However, the concept of disturbances propagating away from the source, as discussed earlier in this paper, is consistent with the nature of the physical phenomena taking place in the assembly under these conditions. Thermal neutrons injected at the origin as pulses are very narrow Gaussian-like distributions which spread on account of leakage and diffusion of neutrons away from the source as a function of time as shown in Fig. 5. Since an exponential assembly with a source input is a linear system, the superposition of source pulses gives an output which is the superposition of the responses to the individual pulses. This results in a series of peaks in neutron density of different heights being propagated along the $z$-axis with the amplitudes of the individual peaks decreasing as they propagate.

The importance of using a thermal neutron source is apparent when one considers that a fast source produces a spatial distribution of thermal neutrons in the assembly from slowing down. Whether this represents a serious deviation from the theory developed in this paper depends upon how far the neutrons travel before thermalization in comparison with the size of the assembly and whether the slowing-down time is a small or large fraction of the total effective neutron lifetime.
4. EXPERIMENTAL EQUIPMENT AND PROCEDURE

Experimental measurements of the impulse response function of exponential assemblies were carried out on both light-water and heavy-water-moderated assemblies which used natural uranium as fuel. Both assemblies are described in detail elsewhere \[36, 37\] and hence, only the salient features of the assemblies will be presented here. Both assemblies are cylindrical in geometry and use the Mark I, Savannah River-type fuel slugs contained in aluminium tubes, mounted parallel to the longitudinal axis of the assembly in grid plates. These plates were drilled to provide a triangular lattice with a 4-cm pitch. However, in the heavy-water experiment, a sufficient number of positions were left vacant so that the pitch was actually 12 cm. Both assemblies were wrapped with sheet cadmium to reduce the backscatter of neutrons from the surroundings.

In the case of the light-water assembly, a Philips Norelco neutron generator, model No. NS-100/8 was used with the neutron tube placed near the bottom of the assembly, as shown in Fig. 7. This neutron generator uses the D-T reaction to produce 14.3 MeV neutrons as described by BUHLER \[38\]. The detectors were enclosed in aluminium tubes and mounted horizontally so that they could be moved along the z-axis. Both He\(^3\) detectors and BF\(_3\) detectors were used.

In the case of the heavy-water experiment, the Texas Nuclear neutron generator, model No. 9501 was used, with the target mounted in the geometric centre of the tank as shown in Fig. 8. In this particular case, a deuterium target was used to produce ~2 MeV neutrons.

![Fig. 7](image_url)

*Light-water-moderated, natural uranium assembly showing location of neutron generator and detectors*
The neutron generators are turned on and off by the output from an 8-stage shift register which was wired with appropriate feedbacks to provide a maximum length linear shift register sequence [39]. This unit is driven with an internal clock which could be operated at eleven rates between 0.5 and 19 600 c/s. With eight stages, the shift register shifts 255 times before the output pattern is repeated. The unit is similar to the one described by STERN et al. [25].

The instrumentation system used in these experiments is shown in Fig. 9 and has been described in detail by KYLSTRA [40]. Most of the pieces of equipment are commercially available units which have been chosen with the specific experiment in mind. For instance, the linear amplifiers are the double-delay-line differential type and are capable of amplifying and transmitting 250 000 pulses per second. The fast responding rate meter circuits constructed by KYLSTRA [40] had five time constants which could be selected in accordance with the number of pulses per second being received. When set on the proper range for 100 000 pulses per second coming into the rate meter, the frequency response is down only 3 db at 6000 c/s. The bandpass filters were used to remove DC components and the higher frequencies to prevent aliasing errors. The FM tape recorder was used with the heavy-water experiments because the heavy-water assembly was located in a building distant from the data acquisition system. It does, however, provide the advantage that data can be recorded on the FM recorder at high speeds and played back at a slower speed to provide more points per unit time, on the data tracks. Slowing down the tape recorder upon playback will also alleviate limitations imposed by the upper frequency response limits of the bandpass filters.
Fig. 9

Block diagram of instrumentation system used in random pulsing experiment

Detector Nos. 1 and 2: Texas Nuclear Model 9501 He$^3$ detectors
PA: Hamner Model 361 preamplifiers
HVPS: Atomic Instrument Corp. Model 318 high-voltage power supply
LA: Hamner Model N318 linear amplifier with discriminator
RM: Rate meter circuit constructed by KYLSTRA [44]
Tape recorder: Minneapolis Honeywell Model 8100 FM tape recorder
SR: 8-stage shift register constructed at University of Florida [40]
BPF: Krohn-Hite Model 330A bandpass filter
Data acquisition system: Systems Engineering Laboratory Model SEL-600 (includes high-level multiplexer, analogue-to-digital converter, format control unit and Ampex Model FR-300 digital tape recorder).

The digital data acquisition system includes an 8-channel high-level multiplexer (capable of being expanded up to 48 channels), an analogue-to-digital (A/D) converter, a format control unit and an Ampex model FR-300 digital tape deck, which operates at 150 in/s. The A/D converter provides 11-bit (plus sign) binary representation of the data. The multiplexer is capable of operating at two rates: 15 000 samples per second and 28 571 samples per second. For the slower sampling rate, the format control unit arranges data on the digital tape in a form which is directly compatible with the University of Florida IBM-709 digital computer. For the faster sampling rate, it is necessary to use an IBM-1401 computer to convert the data from high to low density. Hence, tapes produced on this data acquisition system can be played back on the IBM-709 tape decks, thereby placing data directly into the computer.

Digital codes prepared by SELFridge [41] have been used to place the data into the computer and to carry out both auto-correlation of a single variable and cross-correlation between two variables. The technique of performing this correlation has been described by LEE [22] and ZIMMERMANN [42]. This code involved a least-squares fit of all the data points to a second-order curve which is then subtracted from the data to remove any trend or non-stationarity. Digital correlation techniques are then used to determine
the auto-correlation and cross-correlation functions. Provision has been written into the code to take only every n-th value of data recorded on the tapes to make the amount of computing time consistent with the precision of the results desired. Typically, the number of points involved in each correlation was between 5000 and 7000 points per channel. The output correlation function is normalized so that the peak positive value is unity.

5. TYPICAL RESULTS AND DISCUSSION

The results of a typical test run where the output of a neutron detector located 88.2 cm from the neutron generator which is cross-correlated with the output signal of the shift register are shown in Fig. 10. If the neutron generator follows the shift register signal, Fig. 10 represents the cross-correlation between the input source and the neutron density. Furthermore, if the auto-correlation of the input is a delta function as postulated previously, then Fig. 10 represents the impulse response of a light-water, natural uranium assembly at a position on the z-axis 88.2 cm from the source to an impulse applied to the assembly at the origin. The validity of these two assumptions is discussed later.

Fig. 10

Normalized cross-correlation between the shift register (source) and detector located 88.2 cm from the source in a light-water-moderated, natural uranium assembly.
The normalized cross-correlation function in Fig. 10 reaches its peak value of unity approximately 1.5 ms after the impulse was applied, then decreases rapidly to a value of approximately -0.4, and finally increases toward the zero value. For negative time lags, the cross-correlation function vacillated about zero. The negative values of the cross-correlation function were not expected from the theory derived in this paper. Fig. 11 shows two similar cross-correlations for the same run where the detectors are located at 44.5 and 103.8 cm from the source. It is apparent that the peak value for the 44.5-cm position occurs ~1 ms earlier than for the 103.8-cm position, and that the width of the peak for the 103.8-cm position is wider than for the 44.5-cm position. The difference between the two curves in the 3- to 10-ms range is probably the result of the normalization procedure used in calculating the cross-correlation functions. It should be realized that the magnitude of the cross-correlation function decreases with distance and that if the normalization procedure had not been used, the peak value for the 103.8-cm position would be much smaller than for the 44.5-cm position.
The presence of negative values on the cross-correlation function in Figs. 10 and 11 beyond 2 ms is attributed to the Philips Norelco neutron generator failing to follow the signal produced by the shift register. This phenomena was clearly visible on the oscilloscope. Figure 12 shows the

![Figure 12](image_url)

**Fig. 12**

Normalized auto-correlation functions of the outputs of the shift register and a detector in a light-water-moderated, natural uranium assembly

- ◊ Detector located 29.2 cm from source
- ○ Shift register output (source)

auto-correlation function of the shift register signal to be a good representation of a delta function with negligible side bands. However, the output of a detector placed only 29.2 cm from the source had an auto-correlation function which deviated significantly from a delta function. These same two auto-correlations are shown on a semi-logarithmic plot in Fig. 13, and the auto-correlation function of the detector output, 29.2 cm from the source, is found to a good approximation to be an exponential-cosine. Figure 14 shows auto-correlation functions of the outputs of two detectors located 29.2 and 73.0 cm from the source during a run with a partial fuel loading and indicates that these auto-correlation functions also resemble exponential cosines. Furthermore, the decay constants of the exponentials decrease as the distance from the source to the detector increases. This is expected because of the "filtering" effect of the intervening material between the source and the detector.

In discussing exponential-cosine auto-correlation functions and the nature of phenomena which produce them, BENDAT [43] cites two pheno-
Fig. 13

Normalized auto-correlation functions of the outputs of the shift register and a detector in a light-water-moderated, natural uranium assembly

- Detector located 29.2 cm from source
- Shift register output (source)

...mena which may be appropriate in this particular situation. One arrangement which will produce an exponential cosine is a system with an input which is a summation of sinusoids with random phases and amplitudes. Since the discrete, binary source used in this experiment is periodic in nature and can be represented by a Fourier series, such a phenomena is certainly possible in this system. Furthermore, when one considers the fact that the neutron source actually emits fast neutrons instead of thermal neutrons, the likelihood of having random phases and amplitudes in the "equivalent thermal neutron source" seems quite probable. Another phenomena cited by BENDAT [43] which produces an exponential-cosine auto-correlation function is the "shot effect" in which the input to a linear system consists of a large number of impulses occurring at random times and with random amplitudes. When the number of input impulses per second is sufficiently large, the superposition of the responses results in an output which has an exponential-cosine auto-correlation function. If the input is considered to be the thermal neutrons which result from the thermalization of the individual fast neutrons.
produced by the neutron generator, this phenomenon also offers a reasonable explanation for the exponential-cosine type of auto-correlation function. In one sense, these two phenomena may actually be considered as the same phenomenon viewed from a different viewpoint.

Figure 15 shows the cross-correlation between the shift register and the output of the detector located 32.5 cm above the source in a heavy-water assembly (without uranium). The peak value of the function is reached in about 1.5 ms after which it decreases to zero and vacillates about zero value. The absence of a large negative value for the cross-correlation function found in light-water systems is attributed to the fact that the Texas Nuclear neutron generator followed the output signal of the shift register. A plot of the first six milliseconds is shown in Fig. 16, and the data are fitted with a single exponential as indicated in Eqs. (9) and (10). For $k = 0$, the reciprocal of the decay constant is equal to the effective neutron lifetime. The data presented in Fig. 16 yields an effective neutron lifetime of 1.73 ms for
Fig. 15
Normalized cross-correlation of the outputs of the shift register and a detector located 32.5 cm above the source in a heavy-water assembly.

Fig. 16
Normalized cross-correlation of the outputs of the shift register and a detector located 32.5 cm above the source in a heavy-water assembly.
the heavy-water assembly. This value is in good agreement with the lifetime obtained from pulsed neutron experiments on the same assembly [44].

Figure 17 shows the cross-correlation between the shift register and the output of the detector located at 32.5 cm above the source in the heavy-water, natural uranium assembly. In general, the data are similar in nature to that shown for the heavy water alone, in the sense that the cross-correlation function decays to zero and then vacillates about the axis. When the first six milliseconds of data are plotted on a semi-logarithmic graph in Fig. 18, the results can be fitted to a single exponential with a "cosine-like" deviation of the data which is much more pronounced than that which occurred in the experiment with heavy water alone. It is postulated that this deviation may be caused by the thermal neutrons introduced in the system by the thermalization of fission neutrons produced throughout the system.

Fitting experimental data to Eq. (49) using several (or many) terms of the infinite summation is possible because the exponential term in the summation can be split up as in Eq. (50) so that Eq. (49) becomes

$$g(x, y, z, t) = \frac{2}{ab} \sqrt{\frac{\nu}{\pi D}} \frac{1}{\sqrt{t}} e^{-\frac{1}{4} \frac{\nu D t}{v D g_{mn} t}}$$

$$\times \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \cos \frac{m \pi x}{a} \cos \frac{n \pi y}{b} e^{-\nu D g_{mn} t}$$

$$= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a(e^{-\nu D g_{mn} t})$$

Fig. 17

Normalized cross-correlation functions between the outputs of the shift register and a detector located 32.5 cm above the source in a heavy-water-moderated, natural uranium assembly.
The response of a system at a position $x_i, y_i, z_i$ is

$$g(x_i, y_i, z_i, t) = h(x_i, y_i, z_i, t)e^{vDB/4Dm^2 t},$$

(69)

where $h(x_i, y_i, z_i, t)$ is by definition

$$h(x_i, y_i, z_i, t) = \frac{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \cos \frac{m\pi x_i}{a} \cos \frac{n\pi y_i}{b} e^{-vDB^2/4Dm^2 t}}{ab \sqrt{\frac{\pi D}{\nu} \int t \ e^{z_i^2/4Dt}}}.\ 

(70)

Since $h(x_i, y_i, z_i, t)$ is a function of time $t$, it can be calculated for a given position using as many terms of the summation as desired. After $g(x_i, y_i, z_i, t)$ has been measured experimentally, a plot of $[g(x_i, y_i, z_i, t)/h(x_i, y_i, z_i, t)]$ versus time $t$ on semi-logarithmic paper will yield the quantity $[B_0^4/\nu D]$. The use of the summation in Eq. (49) or (70) is based on the assumption that all modes propagate from the source at the same velocity. However, Perez and Uhrig [30] have shown that the velocity of propagation of low-frequency neutron waves for each mode is different. Therefore, the higher
spatial modes may either increase or decrease the amplitude of the impulse depending upon the location of the detector in the assembly. The deviation from the exponential curve shown in Figs. 16 and 18 in the 4- to 6-ms range may be caused by the effect of higher spatial modes. It is believed that the inclusion of the higher modes is quite important, particularly as the multiplication constant decreases. The inconsistent results reported by STERN and VALAT [26] in random pulsing of subcritical reactors support this postulation.

The necessity for developing a more sophisticated model than that presented in this paper is quite apparent, particularly when fast neutron sources are used. KYLSTRA [40] has developed the spatial-dependent transfer function of a nuclear system using Fermi-age theory to account for the slowing down of both source and fission neutrons. The extension of this work to a spatial-dependent impulse response function (or Green's function) is presently under way at the University of Florida. Even this model may be inadequate to explain the observed phenomena in light-water systems or any system in which a source of D-T (14.3 MeV) neutrons is used.

The experimental results on heavy-water assemblies reported in this paper appear to be in better agreement with theory than those obtained from the light-water experiments. Whether this is because of the use of the D-D reaction to produce source neutrons, the fact that the system used heavy water instead of light water, or the fact that the neutron generator followed the output of the shift register is not clear. In all probability, the better results are from a combination of these factors.

This work is continuing at the University of Florida and more elaborate experiments using more sophisticated equipment and data-processing techniques are being planned for the near future. A 4-MeV Van de Graaff accelerator is being installed later this year to provide a high-intensity source of low-energy neutrons. Additional subcritical facilities which should overcome some of the disadvantages inherent in the present assemblies are being planned for use with the new accelerator. More efficient and economical methods of processing data are being developed at the same time that improved precision of the measurements is being sought. Finally, considerable effort is being devoted to the theoretical aspects of this random pulsing and its relation to our neutron wave propagation work [28, 29, 30].

ACKNOWLEDGEMENTS

The author is indebted to many of his colleagues for help in this project. He is particularly indebted to Drs. R.B. PEREZ [30, 34], M.N. MOORE [45], and C.D. KYLSTRA [40] for the stimulating discussions which led to some of the conclusions included in this paper. The computer programme for processing data was developed over a period of months by Dr. R. SELFRIDGE [41] and many other members of the Department of Nuclear Engineering aided in the experiments and the data processing.

Financial support for this project has been provided by both the University of Florida from its Nuclear Sciences budget and by the United States Atomic Energy Commission under Contract (AT-(40-l)-3067 and by the loan of the natural uranium and heavy water used in these experiments.
REFERENCES

DISCUSSION

I. McGill: In the section of your paper entitled "Experimental equipment and procedure", you mention the use of filters to eliminate the higher frequencies to prevent aliasing errors. How serious would the effect be if you did not eliminate them?

R.E. Uhrig: In cutting out the high frequencies going into the digital data acquisition system, our practice is to eliminate all those which are greater than twice the upper frequency which we want to obtain in our data processing. This eliminates some of the aliasing errors. I just do not know how serious the problem might be if we did not take this precaution. Perhaps it is a problem which needs closer study. In any event, we have not tried working without the filter.

E. Behrens: Theoretically, you obtain a Poisson distribution for your flux behaviour. But this, of course, is only possible if you assume the absence of a boundary condition. It means you have a continuous spectrum of eigenvalues. I should like to know whether this assumption is justified in the special case of a heavy-water tank, as shown in Fig. 8 of your paper.

R.E. Uhrig: The omission of the so-called incorrection factor is equivalent to saying that the exponential assembly is infinite in length and therefore that the geometric buckling is equal to its perpendicular component. This assumption would seem to be valid as long as measurements are not made close to the upper end of the assembly. I think it is important to point...
out that there is a fundamental difference between the so-called fast pulse experiments and the thermal pulse experiments that we have been talking about. In the former, the neutrons move out over the assembly and then die away, due to the leakage in all directions. This is not the case in the type of experiment described in the paper. Our random pulsing is so rapid that what we have is essentially a driven assembly. The situation is very similar to that in the work we are doing on neutron wave propagation. In fact, since the source is periodic in nature, we assume that we are dealing with a neutron wave phenomenon in which we have many frequencies generated at once, instead of just a single frequency, as is the case in our neutron wave work.

D. WALTON: I wonder why you use your particular system of data acquisition, which at one stage involves the conversion of digital to analogue and analogue back to digital. In my opinion, this is not necessary and it has the drawback of introducing two notoriously inaccurate mechanisms. Moreover, at one stage you use a tape recorder which is also notoriously non-linear. Another consideration is that the system you use for triggering the neutron source does not give consistent results. In view of all these things, I should like to know whether you had any particular reason for using this system.

R.E. UHRIG: Our original experiments were carried out using an ionization chamber, but the frequency response of the chamber and micromicroammeter was such that we went to pulse chambers. Since the pseudo-random input was periodic in nature, we could have used one of our multi-channel analysers. However, these were tied up with other experiments, so we decided to use a general-purpose digital data acquisition system — more specifically, our IBM-709 computer — to process the data. Since this system required analogue inputs, we used specially designed fast-responding pulse-rate measuring circuits to provide the analogue signals. This system has proved fully satisfactory. As far as the triggering pulses are concerned, they are generated by a shift register which is driven by a synchronous clock. The neutron generator is turned off and on in a random binary fashion in accordance with the output of the shift register. The latter is entirely transistorized and works quite satisfactorily. Our problem with the input system is connected with the electronics of the neutron source, i.e. the fact that this source does not follow the shift register. It simply was not designed for the 50% duty cycle which we have.
USE OF THE NEUTRON DIE-AWAY TECHNIQUE TO TEST CONTROL ROD EFFECTIVENESS THEORIES

R.B. PEREZ*, G. DE SAUSSURE** AND E.G. SILVER**
UNIVERSITY OF FLORIDA, GAINESVILLE, FLA., UNITED STATES OF AMERICA

Abstract — Résumé — Аннотация — Resumen

USE OF THE NEUTRON DIE-AWAY TECHNIQUE TO TEST CONTROL ROD EFFECTIVENESS THEORIES. The calculation of control rod effectiveness is complicated by its dependence on both the neutron energy distribution and the geometry of the assembly. When one compares the theory with experimental results obtained from either reactors or subcritical systems, difficulties arise in the comparison because of the intrinsic complexity of such systems.

The neutron die-away technique affords the possibility of having an all-thermal neutron model, in which the neutron energy distribution can be separated from spatial effects. Hence, the geometrical factor of the control rod effectiveness can be studied without regard to the details of the neutron spectrum, and the results compared with a clean, simple experimental set-up. The method is based on the fact that in a neutron die-away experiment of the type described here, the buckling of the assembly is related to the decay constant of the fundamental mode by

\[ \beta^2 = (\lambda - \lambda_a)/D \]

\[ \lambda_a = \text{inverse lifetime of the neutrons in moderator (s}^{-1}) \]

\[ D = \text{diffusion constant (cm}^2/\text{s}). \]

The moderating assemblies used for these experiments were rectangular prisms of beryllium, built in several sizes from small (2.54 cm high, 7.3 cm square) blocks. Three types of cadmium control rods were used: thin 0.476 cm diameter rods; a cruciform-section rod; and hollow "thick" rods 7.3 cm x 7.3 cm cross-section.

The theoretical schemes tested were:

1. Nordheim-Scalettar
2. Hurwitz-Roe

The effect of a cruciform absorber was computed by using the Hurwitz-Roe conformal transformation technique and a value of 0.0188 cm^-2 was found for the buckling which compares with the experimental results of 0.0187 ± 0.0006 cm^-2. For the thick rods, both Nordheim-Scalettar and the diffusion code overestimated the experimental results by about 10%. However, the interaction between thick rods was correctly predicted by both methods. For thin rods, the Nordheim-Scalettar technique was extremely accurate. The disagreement found for the thick rods is to be expected when diffusion theory is used to describe the effect of absorbers with cross-sectional dimensions comparable to the neutron mean free path in the moderator.

Measurements with rods of intermediate sizes are being carried out to determine the point at which diffusion theory becomes inadequate.

EMPLOI DE LA MÉTHODE D'ABSORPTION DES NEUTRONS POUR VÉRIFIER LES THÉORIES SUR L'EFFICACITÉ DES BARRES DE COMMANDE. Le calcul de l'efficacité des barres de commande se trouve compliqué par le fait qu'elle dépend à la fois de la répartition énergétique des neutrons et de la géométrie de l'ensemble. La comparaison entre la théorie et les résultats expérimentaux obtenus sur des réacteurs ou des systèmes sous-critiques soulève des difficultés, en raison de la complexité intrinsèque de ces systèmes.

* On leave of absence from Instituto Nacional de Industria, Madrid, Spain.
** Affiliated with Oak Ridge National Laboratory, Oak Ridge, Tennessee.
La méthode d'absorption des neutrons permet d'utiliser un modèle à neutrons exclusivement thermiques, dans lequel on peut dissocier la répartition énergétique des neutrons des effets spatiaux. Il s'ensuit que l'on peut étudier le facteur géométrique de l'efficacité de la barre de commande sans tenir compte des détails du spectre des neutrons, et comparer les résultats à un dispositif expérimental simple non empoisonné. La méthode est fondée sur le fait que, dans une expérience d'absorption des neutrons du genre de celle que décrivent les auteurs, le laplacien de l'ensemble est fonction de la constante de décroissance du mode fondamental.

\[ B^2 = \frac{(\lambda - \lambda_a)}{D} \]

\[ \lambda_a = \text{inverse de la période des neutrons dans le modérateur (s}^{-1}\text{)} \]

\[ D = \text{constante de diffusion (cm}^2\text{/s}) \]

Le milieu ralentisseur utilisé pour ces expériences était constitué par des prismes rectangulaires de béryllium, de dimensions diverses, dont les plus petits étaient des blocs de 2,54 cm de haut et 7,3 de côté. Trois types de barres de cadmium ont été utilisés: des barres minces de 0,476 cm de diamètre, une barre de section cruciforme, et des barres creuses et «épaisses» ayant une section carrée de 7,3 de côté.

Les théories qui ont été vérifiées sont celles de:
1. Nordheim-Scalettar,
2. Hurwitz-Roe,
3. et celle du code de diffusion numérique.

Les auteurs ont calculé l'effet des barres cruciformes à l'aide de la méthode de transformation conforme de Hurwitz-Roe et ont trouvé pour le laplacien la valeur de 0,0188 cm\(^{-2}\) qui est voisine du résultat de l'expérience, 0,0187 ± 0,0006 cm\(^{-2}\). Pour les barres épaisses, les prévisions de la théorie de Nordheim-Scalettar et de la théorie du code de diffusion dépassent les résultats expérimentaux d'environ 10%. Les deux méthodes ont toutefois établi des prévisions correctes en ce qui concerne l'interaction entre les barres épaisses. Pour les barres minces, la méthode Nordheim-Scalettar est extrêmement précise. On peut s'attendre à trouver l'écart constaté pour les barres épaisses, lorsqu'on utilise la théorie de diffusion pour calculer l'effet des barres ayant une section de dimensions comparables à celle du libre parcours moyen des neutrons dans le ralentisseur.

On procède actuellement à des mesures avec des barres de dimensions intermédiaires pour déterminer le point à partir duquel la théorie de la diffusion n'est plus valable.

**ИСПОЛЬЗОВАНИЕ МЕТОДА СПАДА ПОТОКА НЕЙТРОНОВ ДЛЯ ПРОВЕРКИ ТЕОРИЙ ЭФФЕКТИВНОСТИ РЕГУЛИРУЮЩИХ СТЕРЖНЕЙ.** Расчеты эффективности регулирующих стержней усложняются их зависимостью как от распределения энергий нейтронов, так и от геометрии сборки. При сравнении теоретических данных с результатами экспериментов, проведенных на реакторах или критических системах, возникают трудности, связанные с внутренней сложностью таких систем.

Метод спада потока нейтронов позволяет получить модель всех тепловых нейтронов, в которой распределение энергии нейтронов может быть отделено от пространственных эффектов. Поэтому геометрический фактор эффективности регулирующего стержня может быть изучен без учета деталей нейтронного спектра, а результаты могут быть сравнены с результатами чистой простой экспериментальной установки. Метод основан на том факте, что в опыте с замыканием нейтронов, как он здесь описан, лапласиан сборки связан с постоянной распада основного вида по формуле:

\[ B^2 = \frac{\lambda - \lambda_a}{D}, \]

где \( \lambda_a \) — обратный полуpériode нейтронов в замедлителе (сек\(^{-1}\));
\( D \) — постоянная диффузии (см\(^2\)·сек\(^{-1}\)).

В замедляющих сборках, используемых для этих экспериментов, имелись прямоугольные и квадратные призмы различных размеров, составленные из небольших (7,3 х 7,3 х 5,24 см) блоков. Использовались три вида кадмиевых регулирующих стержней: тонкие диаметром 0,476 см, крестообразные и полые "толстые" стержни сечением 7,3 х 7,3 см.

Были проверены следующие теоретические схемы:
1. Нордхейма-Скалеттара;
2. Хурвитца-Рое;
3. Цифровой ход диффузии.
Эффект крестообразного поглотителя был рассчитан с помощью метода конформного преобразования Хурвитца—Рое. Для лапласиана было получено значение $0,0188 \text{ см}^{-2}$, что сравнимо с экспериментальными результатами, составившими $0,0187 \pm 0,0006 \text{ см}^{-2}$. Значение, полученные для толстых стержней с помощью схемы Нордхейма—Скалеттара и кода диффузии были выше экспериментальных результатов почти на $10\%$. Однако взаимодействие между толстыми стержнями было правильно предсказано с помощью обоих методов. Для тонких стержней чрезвычайно точные результаты были получены по методу Нордхейма—Скалеттара. Расхождения результатов для толстых стержней следует ожидать в тех случаях, когда для описания эффектов поглотителей, имеющих размеры сечений, сравнимые со средним свободным пробегом нейтрона в замедлителе, используется теория диффузии.

Чтобы определить точку, в которой теория диффузии становится непригодной, выполняются измерения со стержнями промежуточных размеров.

1. INTRODUCTION

The calculation of control rod effectiveness is complicated by its dependence on both the neutron energy distribution and the geometry of the assembly [1]. When one compares the theory with experimental results [2]...
obtained from either reactors or subcritical systems, difficulties arise in the comparison because of the intrinsic complexity of such systems.

The neutron die-away technique affords the possibility of having an all-thermal neutron model, in which the neutron energy distribution can be separated from spatial effects. Hence, the spatial factor of the control rod effectiveness can be studied without regard to the details of the neutron spectrum, and the results compared with a clean, simple experimental set-up.

2. THEORY OF THE EXPERIMENT

When the neutron population in the assembly has reached the asymptotic distribution, the time derivative operator in the non-stationary one-group balance equation [3] can be replaced by the time eigenvalue, \( \lambda \). After rearranging, the neutron equation can be recast in the form of the Helmholtz equation with the eigenvalue (buckling) given by [4]

\[
B^2 = \frac{\lambda - \lambda_s}{D_0},
\]

(1)

where

- \( \lambda_s \) = inverse lifetime of the neutrons in moderator (s\(^{-1}\))
- \( D_0 \) = diffusion constant (cm\(^2\)/s).

By measuring the time-eigenvalue \( \lambda \) before and after the insertion of neutron absorbers, the change in buckling can be determined by repeated use of Eq. (1).

A small correction was applied to Eq. (1) to take into account the cooling of the neutron spectral distribution caused by the preferential leakage of the faster neutron components [5].

In all configurations studied in this paper, the absorbers were fully inserted into the moderator; therefore, only the perturbation in the transverse buckling must be accounted for.

3. EXPERIMENTAL PROCEDURE

The moderating assemblies used for these experiments were rectangular prisms of beryllium built in several sizes from small, 1-in-high, 2 7/8-in-square blocks. Each block has a 3/16-in-diam. hole through its centre, so that the completed prisms contained a series of 3/16-in-diam. channels on 2 7/8-in centres. Three types of cadmium control rods were used: thin 3/16-in-diam. rods, which could be inserted in the 3/16-in channels; a cruciform-section rod, with four arms or blades each 0.030-in thick and 1 7/16-in wide, which could be inserted between a stack of blocks without appreciably distorting the assembly; and hollow "thick" rods, formed of 0.050-in cadmium sheet with a square, 2 7/8-by 2 7/8-in cross-section. Insertion of the thick rod required removal of a stack of beryllium blocks.
In each experiment the decay constant of thermal neutrons was measured in the clean beryllium assembly; then the cadmium rods were inserted and a new decay constant measured. Details of the pulsed neutron technique employed have been previously reported [6]. The changes in buckling corresponding to the measured changes in the decay constant were obtained from the results of previous work [6].

4. THEORY

Several theoretical schemes were tested and evaluated against the experimental results.

A. Nordheim-Scalettar method

The well known Nordheim-Scalettar method [7] which describes the control rod as a singularity in the neutron field was used in the framework of the formalism developed by SINYUTIN and SEMENOV [8] whose notation we closely follow in the present paper.

The regular and irregular parts of the flux are given respectively by

\[ \phi_{\text{reg}}(r, \theta) = \sum_{n=-\infty}^{\infty} A_n J_n(\mu r) e^{in\theta} \]  
and

\[ \phi_{\text{irreg}}(\mathbf{r}, \mathbf{r}_k, \nu_k) = \sum_{k} \sum_{m=-\infty}^{\infty} B_m^{(k)} Y_m(\mu |\mathbf{r} - \mathbf{r}_k|) e^{im\nu_k}, \]

where
\[ \mathbf{r} \] = observation point position vector
\[ \mathbf{r}_k \] = k\textsuperscript{th} control rod position vector
\[ \theta \] = angle of the position vector measured from an arbitrary direction
\[ \nu_k \] = angle of the \[ \mathbf{r} - \mathbf{r}_k \] (same reference direction)
\[ \mu^2 \] = radial buckling (cm\textsuperscript{-2})

By application of the zero-flux conditions on the absorber and at the extrapolated surface of the system and by use of the addition theorem for Bessel functions, one obtains the following set of coupled algebraic equations to determine the coefficients \( A_n, B_m^{(k)} \) and the eigenvalue, \( \mu \):

\[ \sum_{n=-\infty}^{\infty} A_n J_n(\mu r_1) J_n(\mu r_2) + B_m^{(k)} Y_m(\mu r_1) \]

\* The total buckling was obtained by adding the axial buckling to \( \mu^2 \).
\[ + \sum_{k \neq m}^{\infty} \sum_{n=-\infty}^{n} (-1)^{m+n} R^{(k)}_{m-n} (\mu r_{kl}) J_{\nu} e^{i(m-n)\theta_{kl}} \] (4)

on the \( i \)th control rod, and

\[ A_{n}J_{0}(\mu R) + \sum_{k}^{\infty} \sum_{m=-\infty}^{n} B_{m}^{(k)} Y_{0}(\mu R) J_{n-m}(\mu r_{kl}) e^{i(n-m)\theta_{kl}} = 0 \] (5)

on the extrapolated surface, where

\[ k = 1, 2, \ldots, N \) (number of control rods)
\[ m, n, \nu = 0, \pm 1, \pm 2, \ldots, \pm \infty \]

and

\[ R \] extrapolated radius of the assembly
\[ \rho \] effective radius of the control rod
\[ r_{kl} \] distance between the \( k \)th and \( l \)th rod
\[ \theta_{kl} \] angle of \( r_{kl} \) with respect to the reference direction
\[ \gamma_{k} \] angle of \( r_{k} \) with respect to the reference direction

For the actual computations, the series (2) and (3) were truncated at \( n, m, \nu = 0 \) for thin rods and at \( m, n, \nu = 0, \pm 1 \), for the thicker absorbers.

**Two thin rods** (configuration (2) in Fig. 1)

The method will be illustrated for the case of two thin rods. Because of symmetry, \( B_{m}^{(1)} = B_{m}^{(2)} \); then from Eq. (4) and Eq. (5) we obtain, respectively

\[ A_{0}J_{0}(\mu R)J_{0}(\mu R) + B_{0}^{(1)} Y_{0}(\mu R) + Y_{0}(\mu r_{12}) J_{0}(\mu R) = 0 \] (6)

and

\[ A_{0}J_{0}(\mu R) + 2B_{0}^{(1)} Y_{0}(\mu R)J_{0}(\mu R) = 0 \] (7)

where we used the zero-flux boundary conditions on one of the rods and at the external surface. The compatibility condition yields the following secular equation for the eigenvalue \( \mu \):

\[
\begin{vmatrix}
J_{0}(\mu R)J_{0}(\mu R) & Y_{0}(\mu R) + Y_{0}(\mu r_{12}) J_{0}(\mu R) \\
J_{0}(\mu R) & 2Y_{0}(\mu R)J_{0}(\mu R)
\end{vmatrix} = 0
\] (8)

By developing the determinant (8) one could obtain the eigenvalue from the transcendental equation

\[
\frac{J_{0}(\mu R)}{Y_{0}(\mu R)} = \frac{2J_{0}(\mu R)J_{0}(\mu r_{12})}{Y_{0}(\mu R) + Y_{0}(\mu r_{12}) J_{0}(\mu R)}; \quad r_{12} = 2r.
\] (9)
Fig. 1

Configurations used in decay-constant measurements with cadmium rods in beryllium assemblies

(1) - (4) - Configurations for thin-rod measurements

(5) - Configuration for cruciform-rod measurements

(6) - (9) - Configurations for hollow-square-rod measurements

(10) - (12) - Configurations for two-rod measurements

(All dimensions in inches)

However, for thin absorbers (compared with the dimensions of the assembly) the difference between the perturbed and unperturbed eigenvalue is small, i.e.

$$\delta \mu = \mu_1 - \mu_0 \ll 1 \left( \mu_0 = \frac{2.405}{R} \right)$$

and a very accurate, explicit expression for $\delta \mu$ can be derived as shown in WEINBERG and WIGNER [9], (see Appendix A) by neglecting terms in $(\delta \mu)^2$. The result is

$$\delta \mu = 2 \left\{ 1.543 R^{-1} [J_0(\mu_0 r)]^2 \left[ \ln R \rho - 0.762 - 0.0181 [J_0(\mu_0 r)]^2 \frac{\pi}{2} Y_0(\mu_0 r_{12}) \right] \right\}.$$  (11)

It is interesting to compare the result, Eq. (11), with the one resulting from the mere superposition of the effect of two absorbers [9] located at a distance $r$ from the centre of the assembly.
Therefore, except for the small difference between the terms 0.771 and 0.762 - 0.0181 \((J(\mu_0 r))^2\), Eq. (11) predicts the linear addition of the effect of both control rods when

\[ Y_0(\mu_0 r_{12}) = Y_0(4.81 r/R) = 0 \]  

or for \(2r_c = 0.37R\). If the absorbers are separated by less than the critical distance a "shadowing" effect takes place. "Anti-shadowing" effect will appear in the opposite case [9].

**Ring of four (4) thin rods (configuration (3), Fig. 1)**

Because of symmetry

\[ B_0^{(3)} = B_0^{(2)} = B_0^{(3)} = B_0^{(4)} = B_0 \]

and following steps similar to the previous case, one obtains

\[
\delta \mu = 4 \left[ 1.543 [J_0(\mu_0 r)]^2 \right. \left. R^{-1} \left\{ \ln \frac{R}{\rho} - 0.762 - 0.036 [J_0(\mu_0 r)]^2 \right. \right] \\
\left. - \frac{\pi}{2} \left[ 2Y_0(\sqrt{2\mu_0 r}) + Y_0(2\mu_0 r) \right] \right]^{-1},
\]

where \(r\) = distance from a member of the ring to the centre of the assembly.

Notice that the effect of the four absorbers does not add linearly because of the interaction factor in the curly brackets. Again, the "shadowing" or "anti-shadowing" phenomena depends on the sign of the interaction term.

**Double ring of four absorbers each (configuration (4), Fig. 1)**

Invoking symmetry, we have

\[ B_0^{(1)} = B_0^{(2)} = B_0^{(3)} = B_0^{(4)} = B_{01} \]  

(first ring)

and

\[ B_0^{(5)} = B_0^{(6)} = B_0^{(7)} = B_0^{(8)} = B_{02} \]  

(second ring).

Therefore, there are three coefficients, \(A_0, B_{01}, \text{ and } B_{02}\) to be determined. The necessary equations are obtained by making the flux zero on any of the rods in the first and second ring, and also at the extrapolated external boundary. The compatibility condition yields the secular determinant

\[
\begin{vmatrix}
J_0(\mu R) & 4Y_0(\mu R)J_1(\mu r_0) & 4Y_0(\mu R)J_0(\mu r_1) \\
J_0(\mu r_0)J_0(\mu \rho) & (Y_0(\mu \rho + I_1(\mu)) & I_2(\mu) \\
J_0(\mu r_1)J_0(\mu \rho) & I_3(\mu) & Y_0(\mu \rho) + I_2(\mu)
\end{vmatrix} = 0,
\]
where

\( r_0 = \) distance from a member of the first ring to the centre of the assembly

\( r_1 = \) the same as above for the second ring

and

\[
I_1(\mu) = 2Y_0(\sqrt{2\mu r_0}) + Y_0(2\mu r_0)
\]

(16)

\[
I_2(\mu) = 2Y_0(\sqrt{2\mu r_1}) + Y_0(2\mu r_1)
\]

(17)

\[
I_3(\mu) = 2Y_0(\mu \sqrt{r_0^2 + r_1^2}) + Y_0(\mu (r_1 - r_0)) + Y_0(\mu (r_1 + r_0)).
\]

(18)

Introducing the approximations described in Appendix A, we obtain

\[
\delta\mu = \frac{1}{F} \left[ 4 \{ 1.543 \left[ J_0(\mu_0 r_0) \right]^2 R^{-1} H_1^{-1} \} + 4 \{ 1.543 \left[ J_0(\mu_0 r_1) \right]^2 R^{-1} H_2^{-1} \} - (2)(4) \{ 2.47 \left[ J_0(\mu_0 r_0) \right] \left[ J_0(\mu_0 r_1) \right] R^{-1} (x) I_3(\mu_0)(H_1 H_2)^{-1} \} \right],
\]

(19)

where \( F \) is a correction term close to unity.

\[
F = 1 - \left( \frac{\pi/2}{H_1 H_2} \right)^2 \left\{ \left[ J_0(\mu_0 r_0) \right]^2 + \left[ J_0(\mu_0 r_1) \right]^2 \right\}
\]

\[
+ 2 \left[ J_0(\mu_0 r_0) \right] \left[ J_0(\mu_0 r_1) \right] \frac{H_1 H_2}{H_1 H_2}
\]

(20)

and

\[
H_1 = \ln \frac{R}{\rho} - 0.762 - \frac{\pi}{2} I_1(\mu_0)
\]

(21)

\[
H_2 = \ln \frac{R}{\rho} - 0.762 - \frac{\pi}{2} I_2(\mu_0).
\]

(22)

Inspection of the above equations shows that except for a small correction factor \( F \), the overall effect is a result of the linear superposition of the effects of the two rings and one interaction factor between both rings of absorbers. The influence of the "non-linear" term is given by the sign of the \( I_3(\mu_0) \) factor which in turn depends on the separation between rings (see Eq. (18)).
Thick absorbers

For the computation of single thick absorbers the series were truncated in the second term. The resulting secular determinant was

\[
\begin{vmatrix}
J_0(\mu R) & 0 & Y_0(\mu R)J_0(\mu r) & 2Y_0(\mu R)J_1(\mu r) \\
0 & J_1(\mu R) & Y_1(\mu R)J_1(\mu r) & Y_1(\mu R)g(\mu r) \\
J_0(\mu r)J_0(\mu R) & 2J_1(\mu r)J_0(\mu R) & Y_0(\mu R) & 0 \\
J_1(\mu r)J_0(\mu R) & J_1(\mu R)g(\mu r) & 0 & Y_1(\mu R)
\end{vmatrix}
= 0. \tag{23}
\]

where \( g(\mu r) = J_0(\mu R) + J_2(\mu R) \).

The computation of the lowest root of the secular equations was programmed for the ORACLE computer.

B. Transformation to cylindrical symmetry

The Nordheim-Scalettar method applies to systems with cylindrical symmetry only. The equivalent cylinders were obtained by assuming the same height as the beryllium blocks. The effective radius was computed by equating the transverse bucklings in cartesian and cylindrical geometries.

To convert the square hollow cadmium boxes to cylindrical rods two different techniques were used. First, the equivalent radius of the rod was obtained by using the technique HURWITZ and ROE [10], based on the conformal transformation of a polygon into a cylinder. This model is based on the assumption that there is an infinite moderating material surrounding the absorber. A second semi-empirical scheme, called the weighted equivalent radius model, was used to take into account the relative dimensions of moderator and absorber. The method is based on equating the product of the cross-sectional dimensions and the statistical weight (computed at the centre of the absorber) for both the prism and the cylinder, i.e.

\[
\left( \frac{\rho_0 - \delta}{R} \right)^2 W_{\text{cylinder}} = \left( 1 - \frac{2\delta^2}{ab} \right) W_{\text{prism}}, \tag{24}
\]

where

\[
W_{\text{cylinder}} = \left[ \pi R \left( J_1(\nu_0) \right)^2 \right] \left[ J_0 \left( \nu_0 \frac{R}{R} \right) \right] (\text{cm}^2) \tag{25}
\]

\[
W_{\text{prism}} = 4(ab)^{-1} \cos^2 \left( \frac{\pi x}{a} \right) \cos^2 \left( \frac{\pi y}{b} \right) \tag{26}
\]

\( \rho_0 = \) equivalent radius of the absorber (cm)
\( \delta = \) extrapolation length (cm)
\( l = \) side of the square absorber (cm)
\( a, b = \) extrapolated transverse dimensions of the moderator (cm)
\( R = \) extrapolated radius of the equivalent cylinder (cm)
\( \nu_0 = 2.405 \)
From Eqs. (24), (25) and (26) we obtain

$$\rho_0 - \delta = \rho_{\text{eff}} = 0.5392 \left[ \frac{\cos \frac{\pi X}{a} \cos \frac{\pi Y}{b}}{J_0(v_0 r/R)} \right] (1 - 2\delta).$$  \hspace{1cm} (27)

Table I gives the values of the equivalent absorber radius computed by the two methods.

C. Numerical code computations

The one-group of neutrons diffusion equation was solved using the IBM-704 PDQ code [11]. Since two energy groups are the minimum required for PDQ to iterate, a spurious fast group was included with a very large removal cross-section, a very small diffusion coefficient and a vanishing absorption cross-section. This combination of parameters results in immediate localized scattering into the thermal group of all neutrons in the fast group. The decay constant $\lambda$ is related to the "multiplication" $k$ by the relation $\lambda = \nu \Sigma_t / k$. By setting $\nu \Sigma_t = 1/\nu$, one obtains $\lambda = 1/k$. For the cases of unpoisoned beryllium, using the values of $\nu \Sigma_a$ and $D$, obtained from the neutron pulsed experiments [6], the calculated and experimental values of $1/k$ agree very well.

This code was used to perform x-y geometry calculations, using at the cadmium surface, the boundary condition

$$\frac{D \nabla \phi}{\phi} = \frac{\lambda_T}{3d},$$

where $\lambda_T$ is the beryllium transport mean-free path and "d" the extrapolated length taken as $0.77 \lambda_T$.

To investigate the possibility that better results might be obtained from the PDQ code by a change in the boundary condition at the surface of the cadmium box, a series of computations was performed in which the extrapolation length "d" was varied within wide limits. Results are shown in Table II and indicate that the changes in the buckling by the PDQ code are not sensitively dependent upon the extrapolation length.

6. RESULTS AND COMPARISON WITH THE THEORETICAL CALCULATIONS

A. Thin cadmium rods

Four measurements were made using the 3/16-in-diam. cadmium rods in a beryllium assembly 20 1/8 X 20 1/8 X 20-in high. The buckling of this assembly, without rods, was $1.05 \times 10^{-2}$/cm$^2$, its decay constant was $1.59 \pm 0.02$ k.$\nu$. Rod configurations are shown in Fig.1 and the experimental values of the decay constant, the corresponding bucklings, and the bucklings computed by the Nordheim-Scalettar method are given in Table III. The extrapolation length into the rod was obtained from the work of DAVISON...
TABLE I
EQUIVALENT ABSORBER RADIUS

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Hurwitz-Roe [10] (cm)</th>
<th>Semi-empirical (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3)</td>
<td>3.099</td>
<td>2.62</td>
</tr>
<tr>
<td>(4)</td>
<td>3.099</td>
<td>2.61</td>
</tr>
<tr>
<td>(5)</td>
<td>3.099</td>
<td>2.57</td>
</tr>
<tr>
<td>(6)</td>
<td>3.099</td>
<td>2.22</td>
</tr>
</tbody>
</table>

TABLE II
BUCKLING COMPUTED BY THE PDQ CODE FOR THE CASE OF A THIN-WALLED CADMIUM ROD INSERTED IN A BERYLLIUM ASSEMBLY, FOR VARIOUS ASSUMED EXTRAPOLATION LENGTHS (Configuration 6)*

<table>
<thead>
<tr>
<th>$d/\lambda_r$</th>
<th>$g^2$ buckling (cm$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.68</td>
<td>0.03508</td>
</tr>
<tr>
<td>0.71</td>
<td>0.03491</td>
</tr>
<tr>
<td>0.74</td>
<td>0.03474</td>
</tr>
<tr>
<td>0.77</td>
<td>0.03456</td>
</tr>
<tr>
<td>0.83</td>
<td>0.03439</td>
</tr>
</tbody>
</table>

* The configuration is shown in Fig. 1. The assembly was 16 in high, and the rod was 16 in long.

and KUSHNERIUK [12]. Comparison of results show an excellent agreement between experiment and calculation in all cases considered.

B. Single, large hollow rods

Six measurements were made using one of the 2 7/8- × 2 7/8-in hollow cadmium rods described above. The first two measurements were made with a 16-in-long rod in a 16-in-high, 14 3/8-in beryllium assembly. The unpoisoned assembly had a decay constant of 2.56±0.02 $k_e$, giving a buckling of $1.86 \times 10^{-2}$/cm$^2$. Of the two measurements made the first was performed
TABLE III
EXPERIMENTAL DECAY CONSTANTS AND BUCKLINGS AND COMPARISON WITH THEORY FOR THIN RODS

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Decay constant $(k_c)$</th>
<th>$B^2$, buckling (cm$^{-1}$) (experimental)</th>
<th>$B^2$, buckling (cm$^{-1}$) (computed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1.70 \pm 0.02$</td>
<td>$0.0114 \pm 0.0002$</td>
<td>$0.0111$</td>
</tr>
<tr>
<td>2</td>
<td>$1.74 \pm 0.02$</td>
<td>$0.0119 \pm 0.0002$</td>
<td>$0.0118$</td>
</tr>
<tr>
<td>3</td>
<td>$1.91 \pm 0.02$</td>
<td>$0.0132 \pm 0.0002$</td>
<td>$0.0132$</td>
</tr>
<tr>
<td>4</td>
<td>$2.06 \pm 0.02$</td>
<td>$0.0144 \pm 0.0002$</td>
<td>$0.0144$</td>
</tr>
</tbody>
</table>

with the cadmium box on the vertical axis of the assembly, the second, with the axis of the box displaced 2 7/8-in from the assembly axis.

The following four measurements with the single, large hollow rods were made with a 20-in-long rod in a 20-in-high, 20 1/8- X 20 1/8-in beryllium assembly. The assembly without control rod has a decay constant of $1.58 \pm 0.02$ $k_c$ and a buckling of $1.05 \times 10^{-2}$/cm$^2$. The four measurements were made with the axis of the cadmium box first coincident with the axis of the assembly, then displaced successively, 2 7/8, 5 3/4, and 8 5/8 in from the centre line.

The computational schemes used were the Nordheim-Scalettar method using either the Hurwitz-Roe theory or the weighted equivalent radius recipe, and the PDQ, x-y geometry calculation. Table IV, which summarizes the data from the experiments with single, large hollow cadmium rods, shows that the results obtained with the weighted equivalent radius are better, although none of the theories used gives good agreement with the experiment.

C. Two large hollow rods

Three measurements were made using a pair of 2 7/8- X 2 7/8-in cadmium boxes, with their lengths matched to the height of the particular beryllium assembly. The results which are given in Table V show disagreement between experiment and calculation. This, of course, has to be expected from the lack of agreement in the single-box results noted above. The IBM PDQ numerical code was used for the calculations.

D. Cruciform-section rods

A single experiment was performed with the cruciform-section rod described above. The 20-in-long rod was placed vertically at the centre of a 20-in-high, 17 1/4-in assembly. Without the rod this assembly has a decay constant of $1.87 \pm 0.02$ $k_c$ and a buckling of $1.29 \times 10^{-2}$/cm$^2$. With the rod inserted the measured decay constant was $2.58 \pm 0.07$, corresponding to a buckling of $(1.87 \pm 0.06) \times 10^{-2}$/cm$^2$. 
<table>
<thead>
<tr>
<th>Configuration (a)</th>
<th>Size of assembly (in)</th>
<th>Bucklings (cm$^{-2}$)</th>
<th>Measured</th>
<th>Computed NS-HR(b)</th>
<th>Computed NS-WER(c)</th>
<th>Computed PDQ(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>16 X 14 3/8 x 14 3/8</td>
<td>0.0335 ± 3 x 10$^{-4}$</td>
<td>0.034</td>
<td>0.0324</td>
<td>0.0345</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>16 X 14 3/8 x 14 3/8</td>
<td>0.0321 ± 2 x 10$^{-4}$</td>
<td>0.0285</td>
<td>0.027</td>
<td>0.0263</td>
<td></td>
</tr>
<tr>
<td>8; b=0</td>
<td>20 x 20 1/8 x 20 1/8</td>
<td>0.0160 ± 1 x 10$^{-4}$</td>
<td>0.0170</td>
<td>0.0163</td>
<td>0.0172</td>
<td></td>
</tr>
<tr>
<td>9; b=2 7/8</td>
<td>20 x 20 1/8 x 20 1/8</td>
<td>0.0132 ± 2 x 10$^{-4}$</td>
<td>0.0145</td>
<td>0.0140</td>
<td>0.0146</td>
<td></td>
</tr>
<tr>
<td>9; b=5 3/4</td>
<td></td>
<td></td>
<td>0.0123</td>
<td>0.0123</td>
<td>0.0125</td>
<td></td>
</tr>
<tr>
<td>9; b=8 5/8</td>
<td></td>
<td></td>
<td>0.0107 ± 1 x 10$^{-4}$</td>
<td>0.0110</td>
<td>0.0109</td>
<td>------</td>
</tr>
</tbody>
</table>

(a) Configurations are shown in Fig. 1; b is the distance from the axis of the assembly to the axis of the rod. Rod length was equal to the assembly height.

(b) NS-HR indicates computation by Nordheim-Scalettar method, using the Hurwitz-Roe recipe for equivalent cylinder.

(c) NS-WER indicates computation by Nordheim-Scalettar method, using weighted equivalent radius method for equivalent cylinder.

(d) PDQ calculation; x-y geometry.
TABLE V
COMPARISON OF THEORETICAL AND EXPERIMENTAL EIGENVALUES (BUCKLINGS) FOR BERYLLIUM BLOCKS WITH TWO LARGE HOLLOW CADMIUM RODS

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Size of assembly (in)</th>
<th>Measured</th>
<th>Computed PDQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>16(\times) 14 3/8(\times)14 3/8</td>
<td>0.041 ± 0.015</td>
<td>0.037</td>
</tr>
<tr>
<td>11</td>
<td>16(\times) 14 3/8(\times)14 3/8</td>
<td>0.042 ± 0.005</td>
<td>0.040</td>
</tr>
<tr>
<td>12</td>
<td>20(\times)20 1/8(\times)20 1/8</td>
<td>0.017 ± 0.002</td>
<td>0.0185</td>
</tr>
</tbody>
</table>

The theoretical result was obtained by transforming the cruciform rod into a cylindrical rod making use of Hurwitz-Roe conformal transformation technique, and then using the Nordheim-Scalettar theory. The result was 0.0188, in excellent agreement with the experiment.

7. CONCLUSIONS

The following conclusions may be made from these experiments: (1) The Nordheim-Scalettar method is adequate for calculations of changes in bucklings associated with one or more absorbing rods of small diameter compared with the transport mean-free path in the moderator. (2) The Hurwitz-Roe recipe to convert a cruciform-section rod to an equivalent cylinder gives results that agree with experiment. (3) Both the Nordheim-Scalettar method and the PDQ code fail to predict correctly the buckling of the configurations containing box-like cadmium rods, the cross-sectional dimensions of which are comparable with the transport mean-free path in the moderator.

Both the Nordheim-Scalettar method and the PDQ code are based on diffusion theory and use the concept of extrapolation length to compute the buckling. This model may not be adequate for assemblies containing a large volume of black absorber, although this is somewhat surprising since the model permits very accurate computation of bucklings for cubical assemblies of even only a few mean-free paths. Measurements with rods of intermediate sizes might make it possible to determine the point at which diffusion theory becomes inadequate.

The neutron die-away technique is shown to be a powerful tool to obtain eigenvalues of the Helmholtz equation which is yet to be fully exploited. Immediate applications of this technique could be made to evaluate the effects of air gaps or coolant channels in the core of nuclear reactors by properly simulating the situation in "heterogeneous" moderators.

Finally, the experimental results shown here could be analysed using more involved theories which would probably include transport corrections for the larger absorbers.
APPENDIX A

SIMPLIFIED EXPRESSIONS FOR THIN ABSORBERS

Because of the smallness of the radius of the rod and the change in the eigenvalue $\delta \mu$, the following approximations can be made:

(a) \[ \lim_{\rho \to 0} J_0(\rho \mu) = 1 \]

(b) \[ \lim_{\rho \to 0} J_0(\rho \mu) = -\frac{2}{\pi} \ln \left( \frac{2}{\gamma \mu \rho} \right) = -\frac{2}{\pi} \ln \left( \frac{2R}{(2.405)\gamma \rho} \right) \]

where
\[ \mu = \mu_0 \]
\[ \gamma = \text{Euler constant.} \]

By developing $J_0(\rho \mu)$ and $Y_0(\rho \mu)$ in Taylor series we obtain

(c) \[ J_0(\rho \mu) = -0.519 \delta \mu R \left[ 1 - 0.208 (\delta \mu) R \right] \]

from
\[ J_0(0) = 0 \]
\[ J_0'(x) = -J_1(x) \]
\[ J_0''(x) = J_1(x) - \frac{1}{x} J_0(x) \]
\[ J_1(\rho \mu) = J_1(2.405) = 0.519 \]

(d) \[ Y_0(\rho \mu) = 0.510 - 0.103 (\delta \mu) R \]

from
\[ Y_0'(x) = -Y_1(x) \]
\[ Y_1(0) = 0.103 \]
\[ Y_0(0) = 0.510. \]

REFERENCES

PULSED AND STATIC NEUTRON MEASUREMENTS IN LIGHT-WATER AND HEAVY-WATER EXPONENTIALS

T.F. PARKINSON, R.B. PEREZ, D.N. FRY, R.H. HARTLEY, S.P.D. SMITH AND N.J. DIAZ
UNIVERSITY OF FLORIDA, GAINESVILLE, FLA., UNITED STATES OF AMERICA

Abstract — Résumé — Аннотация — Resumen

PULSED AND STATIC NEUTRON MEASUREMENTS IN LIGHT-WATER AND HEAVY-WATER EXPONENTIALS. The exponential experiment has long been used to determine the material buckling of a multiplying medium. More recently, the pulsed-neutron technique has been applied to measure the time eigenvalue \( \lambda \) from which \( k_{\infty} \) may be deduced. The latter method has the potential advantage of requiring smaller lattice samples than the static exponential method. However, to establish the validity of the pulsed method, intercomparisons between the pulsed and the static techniques are needed. We have performed such measurements with both \( H_2O \) and \( D_2O \) moderators in bare and reflected systems; a tank 1 m in diameter was used for the experiments.

To correlate the measured decay constants with pertinent parameters of the system, a modified one-group theoretical model (hereafter called model A) was used. Thus \( \lambda \) is given by a simple relationship involving \( \lambda_0, k_{\infty} \tau, L^2 \) and \( B^2 \) where \( B^2 \) satisfies the two-region critical equation. The fast non-leakage probability was represented by Fermi Age theory. The computational scheme involved first estimating the value of the decay constant \( \lambda \), and then iterating until a value of \( B^2 \) was found which was consistent with the critical equation and the boundary conditions.

Two other models were also tested:
Model B — a two-region, one-group system is assumed in which \( B^2 \) in the Fermi kernel is taken as the geometrical buckling of the equivalent bare system with reflector savings.
Model C — a two-group bare system with reflector savings is assumed. Comparison of the calculated and experimental decay constants indicates that only Model A gives satisfactory agreement. The following conclusions can be drawn:
(1) The presence of the reflector has a large influence on the physics of the pulsed system. The expedient of assuming a reflector savings is not generally valid.
(2) The value of the decay constant \( \lambda \) is very sensitive to the assumed value of the fast non-leakage probability.

The experiments indicate that \( k_{\infty} \) can be determined for small heterogeneous assemblies using the pulsed-neutron technique. Work is currently in progress on hollow uranium rods which permit simulation of organic coolants.

MESURES PAR LES MÉTHODES PULSÉE ET STATIQUE DANS LES ENSEMBLES EXPONENTIELS A EAU LÉGÈRE ET A EAU LOURDE. Les expériences exponentielles ont été longtemps utilisées pour déterminer le laplacien matière d'un milieu multiplicateur. A une date plus récente, on a fait appel à la méthode des neutrons pulsés pour mesurer la valeur propre de temps \( \lambda \) dont on peut déduire \( k_{\infty} \). L'avantage de cette dernière méthode, par rapport à la méthode exponentielle statique, est de permettre l'emploi de cellules plus petites. Cependant, pour déterminer sa validité, il faut comparer ses résultats à ceux des méthodes statiques. Les auteurs ont fait les mesures correspondantes en employant comme talentisseur \( H_2O \) et \( D_2O \) dans des systèmes avec et sans réflecteur; les expériences ont eu lieu dans une cuve de 1 m de diamètre.

Pour établir une corrélation entre les constantes de décroissance mesurées et les paramètres pertinents du système, les auteurs ont fait appel à un modèle théorique à un groupe modifié (appelé ci-après modèle A). Ils ont ainsi obtenu par une formule simple où entrent \( \lambda_0, k_{\infty}, \tau, L^2 \) et \( B^2 \), et dans laquelle \( B^2 \) satisfait à l'équation critique à deux régions. La probabilité d'absence de fuite de neutrons rapides était exprimée par la théorie de l'âge de Fermi. La méthode de calcul a consisté à évaluer d'abord la valeur de la constante de décroissance \( \lambda \) et ensuite à opérer par itération jusqu'à ce que l'on ait trouvé pour \( B^2 \) une valeur compatible avec l'équation critique et les conditions aux limites.
Модели B и C: Наиболее важное влияние отражательной добавки оказывают на физику импульсной системы. Методы импульсных нейтронов и экспоненциальных методы дают удовлетворительное совпадение. Можно сделать следующие выводы:

1. Присутствие отражателя сильно влияет на физику импульсной системы. Методы до- 
бовки не всегда ценен.

2. Значение постоянной распада \( \lambda \) очень чувственно к предполагаемому значению 
вероятности извлечения тепловых нейтронов.

Эксперименты показали, что для небольших гетерогенных сборок \( k \) можно опреде- 
лить методом импульсных нейтронов. В настоящее время проводятся работы по пустоте- 
лым урановым стержням, которые позволяют имитировать органические теплоносители.

Измерение импульсных и статических нейтронов в экспоненциальных установках с обычной и тяжелой водой. Долгое время для определения материального лапласиана разнообразной среды использовался экспоненциальный эксперимент. Совсем недавно для измерения собственного значения времени \( \lambda \) использовался экспоненциальный эксперимент методом импульсных нейтронов. Однако для установления правомерности импульсного метода необходимо произвести сравнение между импульсным и статическим методами. Мы выполнили такие измерения с замедлителями из \( {\mathrm{H}}_2{\mathrm{O}} \) и \( \text{D}_2\text{O} \) в системе с отражателем и без такового. Для экспериментов использовался бак диаметром 1 м.

Были опробованы также две другие модели: Модель B — двухзонная одногрупповая система, в которой \( B_2 \) в функции влияния Ферми применяется как геометрический лапласиан эквивалентной системы без отражателя с таковым. Модель C — двухзонная одногрупповая система без отражателя с отражателем. В двумерной системе без отражателя с отражателем. Методы импульсных нейтронов и экспоненциального метода дают удовлетворительное совпадение. Можно сделать следующие выводы:

1. Присутствие отражателя сильно влияет на физику импульсной системы. Метод пред- 
дкусной отражательной добавки не всегда ценен.

2. Значение распада \( \lambda \) очень чувственно к предполагаемому значению 
вероятности извлечения тепловых нейтронов.

Эксперименты показали, что для небольших гетерогенных сборок \( k \) можно опреде- 
лить методом импульсных нейтронов. В настоящее время проводятся работы по пустоте- 
лым урановым стержням, которые позволяют имитировать органические теплоносители.

МЕДИЦИНЕ ПО МЕТОДОМ РАСЧЕТА КРУГЛОГО ВОРОТНИКА В КОНДУКТУАХ ЭКСПОНИЕНЦИАЛЬНЫХ СОДЕРЖАНИЕ АГАУ МЕДИЦИНЕ.

T.F. PARKINSON et al.
PULSED AND STATIC NEUTRON MEASUREMENTS

También se ensayaron dos otros modelos, a saber:

Modelo B: Se imagina un sistema de dos zonas y un solo grupo en el que el factor de Fermi se toma como laplaciano geométrico del sistema desnudo equivalente, teniendo en cuenta la economía del reflector.

Modelo C: Se imagina un sistema desnudo de dos grupos en el que se tiene en cuenta la economía del reflector. Al comparar las constantes de desintegración calculadas y obtenidas experimentalmente, se comprueba que sólo asegura una concordancia satisfactoria el modelo A. Esto permite llegar a las siguientes conclusiones:

1. La presencia del reflector ejerce una influencia considerable sobre la física del sistema pulsado. La hipótesis según la cual se admite una economía debida al reflector no es válida en todos los casos.
2. El valor de la constante de decrecimiento $\lambda$ depende en gran medida del valor atribuido a la probabilidad de ausencia de escape de los neutrones rápidos.

Los experimentos indican que la técnica de los neutrones pulsados permite determinar $k^*$ en conjuntos heterogéneos de pequeñas dimensiones. Los autores están llevando a cabo trabajos con barras huecas de uranio que permiten simular el empleo de refrigerantes orgánicos.

1. INTRODUCTION

The exponential experiment has long been used to determine the material buckling of a multiplying medium. More recently, the pulsed neutron technique has been applied to measure the time eigenvalue, $\lambda$, from which $k^*$ may be deduced. The latter method has the potential advantage of requiring smaller lattice samples than the static exponential method. However, to establish the validity of the pulsed method, inter-comparisons between the pulsed and the static techniques are needed.

Earlier measurements of the dynamic characteristics of subcritical systems using the pulsed neutron technique include the work of CAMPBELL and STELSON [1], BORST [2], CARPENTIER et al. [3], WESTFALL and WALTNER [4], and MEISTER [5]. The purpose of the present experiments has been to make a critical comparison of the pulsed and static methods to establish the usefulness of the former for the determination of lattice parameters. We have performed measurements with both $\text{H}_2\text{O}$ and $\text{D}_2\text{O}$ moderators in bare and reflected systems.

2. DESCRIPTION OF APPARATUS

A. $\text{H}_2\text{O}$-moderated subcritical

The $\text{H}_2\text{O}$-moderated subcritical facility is shown in Fig. 1; since it is similar to many installations that have been described in the literature [2,3], the details of construction are omitted. Not shown in Fig. 1 is a re-entrant tube that permits insertion of a pulsed neutron source at the centre of the tank.

B. $\text{D}_2\text{O}$ subcritical facility

The $\text{D}_2\text{O}$-moderated subcritical facility is fuelled with either solid or hollow natural uranium fuel elements. The tank (Fig. 2) is constructed of
type 6061 aluminium, 0.25 in thick, has an outer diameter of 38.3 ± 0.5 in and is 72 in high. The side of the tank is completely wrapped in cadmium, 0.030 in thick, to provide a "black" boundary for thermal neutrons. A re-entrant tube, 2.5 in inside diameter and 21 in long is installed in the side of the vessel to receive a pulsed neutron source; the re-entrant tube may be removed if necessary.
The top of the tank is covered with a Lucite top 0.75 in thick. A rack sits above the Lucite top to provide a platform for the flux-traversing mechanism. An aluminium traverse tube, 1 in inside diameter and 70 in long extends down into the water and provides a method of protecting neutron detectors from immersion in the water. A small check valve in the Lucite top is used to introduce a blanket of dry nitrogen over the heavy water.

The top is also fitted with two rectangular ports approximately 6 in x 8 in with 0.25-in Lucite covers which allow access to all the fuel elements by simply removing the covers and rotating the Lucite top.

For static neutron measurements, a 3-c Pu-Be source is located on the vertical centre line near the top of the lattice.

C. Fuel and lattice configurations

The arrangement of the fuel elements in the exponential tank is shown in Fig. 3. For the pulsed experiments in H₂O-moderated lattices, the fuel elements were 25.8 in high. For all other experiments, both pulsed and static, the height of the fuel elements was 50.6 in. Thus, for the shorter-core configuration, the re-entrant tube was above the core so that all lattice positions were filled. For the full-height core, it was necessary to omit a few fuel elements due to the presence of the re-entrant tube.
Pitch No. of rods
(cm)

- 7.9  55
- 11.9  51
+ 15.9  29

D. Instrumentation

The pulsed neutron generator used in these experiments is a 150-kV Cockcroft-Walton accelerator (Texas Nuclear Model 9500) with pre-acceleration pulsing. Either the d(d,n)He³ or the T(d,n)He⁴ reaction which produces neutrons of energy 8 MeV or 14.1 MeV, respectively, may be utilized; the former reaction was used for the D₂O experiments while the latter was used for the H₂O experiments. By deflecting the ion beam before acceleration, pulse widths from 1μs to 10⁴μs can be obtained. The pulse rise- and decay-times are approximately 0.5μs. For most of the work described here, a pulse width of 4.0 ms and a repetition rate of 39 pulses/s were used. With D-D neutrons, an intensity of about 10⁸ n/s is obtainable while with D-T neutrons, the intensity is approximately 10¹⁰ n/s.

The detector which was used for the pulsed neutron data was a "Texlium" He³ counter with an active length of 12 in. An Ortec Au-Si surface barrier solid detector with a Li foil was used for some of the static measurements; In foils were also used for some of the traverses. The Au-Si detector has
active dimensions of $7 \times 7$ mm and detects approximately 20 out of 1000 thermal $\text{n/cm}^2$. The counting equipment consists of an Ortec Model 101 low-noise preamplifier, an Ortec Model 201 low-noise amplifier, and an RCL Model 10D decade scaler.

The decay of the neutron pulse was recorded using a 256-channel time analyser, Technical Measurement Corporation Model CN-110. The system is made up of four modules — a basic unit or digital computer, a logic plug-in programme, a data-handling unit and a digital recorder.

3. THEORETICAL ANALYSIS

A. Static measurements

The fundamental solution of the differential equation for the steady-state flux distribution in a finite cylindrical system is the well-known relation

$$\phi = A J_0\left(\frac{j_1 r}{R}\right) \sinh \kappa (\tilde{h} - z),$$

(1)

where $j_1$, $\tilde{R}$ and $\kappa$ are related by

$$\kappa^2 = \left(\frac{j_1}{\tilde{R}}\right)^2 + L^{-2}$$

(2)

and $L^2 = D/\Sigma_a$

$j_1$ = the first root of $J_0(r) = 2.405$,
$\tilde{h}$ = the extrapolated height of the system, and
$\tilde{R}$ = the extrapolated radius of the system.

In a multiplying medium, Eq. (2) takes the form

$$\kappa^2 = \left(\frac{j_1}{\tilde{R}}\right)^2 - B_m^2,$$

(3)

where $B_m^2$ is the material buckling, and, for a three-group model is given by

$$B_m^2 = Z - FZ^2 + 2F^2Z^3,$$

(4)

where

$$Z = \frac{(k_{\infty} - 1)/M^2}{Z = \frac{(L^2 \tau_1 + \tau_1 \tau_2 + \frac{1}{2} \tau_2^2)/M^2}{F}$$
\[ M^2 = L^2 + \tau \]
\[ \tau_1 = \alpha_1 \tau = \int_{E_1}^{E_0} \frac{E}{E_t} \frac{dE}{E} = 0.378 \tau \]
\[ \tau_2 = (1 - \alpha_1) \tau = \int_{E_2}^{E_1} \frac{E}{E_t} \frac{dE}{E} = 0.622 \tau \]
\[ k_\infty = \eta _{pf}. \]

The nomenclature used in the above equations and in equations to follow is similar to that in Ref. [6].

The detailed calculations for the \( H_2O \)-moderated lattice are given in Ref. [7].

The formulae used to obtain the lattice parameters for the \( D_2O \)-moderated lattices are summarized below; with minor modifications, the assumptions are identical to those described in detail in Ref. [6]. The methods described here have been incorporated into a Fortran code (BUCKSHOT) [8] which gives the lattice parameters both at ambient temperature and at elevated temperatures up to 240°C (see Table I).

**Thermal utilization and diffusion area, \( f \) and \( L^2 \)**

Values for the thermal utilization, \( f \), and the diffusion area, \( L^2 \), are obtained via a mono-energetic, P-3 computer code, CEPTR [9]. The thermal flux averages in each region obtained from CEPTR are used to calculate \( f \), \( L^2 \) and \( \Sigma^\text{eff} \), where

\[ L^2 = \frac{D^\text{cell}}{\Sigma^\text{eff}}. \]

**Diffusion coefficients, \( D \)**

The diffusion coefficients for the Al cladding and for the uranium are obtained from the relationship

\[ D = \frac{1}{3}(\Sigma_t - \Sigma_s \mu_0), \quad \text{(5)} \]

where

\[ \Sigma_t = \Sigma_a + \Sigma_s \]
\[ \mu_0 = \frac{2}{3} \frac{A}{M} \]
\[ A = \text{atomic weight.} \]
<table>
<thead>
<tr>
<th>Pitch (cm)</th>
<th>Moderator</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_u$</td>
<td>$V_{Al}$</td>
</tr>
<tr>
<td>3.97</td>
<td>H$_2$O</td>
<td>0.3695</td>
</tr>
<tr>
<td>7.9</td>
<td>D$_2$O</td>
<td>0.0931</td>
</tr>
<tr>
<td>11.9</td>
<td>D$_2$O</td>
<td>0.0413</td>
</tr>
<tr>
<td>15.9</td>
<td>D$_2$O</td>
<td>0.0233</td>
</tr>
</tbody>
</table>
A detailed account of the calculation of $D$, $\Sigma_2$, $\Sigma_1^*$ and $\Sigma_2^*$, the $\text{D}_2\text{O}$ transport scattering cross-sections used in CEPTR, is given in Ref. [8].

**Thermal neutron reproduction factor, $\eta$**

The thermal neutron reproduction factor, $\eta$, is assumed to be 1.327 for all lattices. Spectral hardening within the fuel element is not taken into account.

**Fast fission factor, $\epsilon$**

The convention used in calculating $\epsilon$ is that all fissions in $\text{U}^{238}$ are included in $\epsilon$, but the associated $(n,\gamma)$ reactions are included in $p$. Thus,

$$\epsilon = 1 + \frac{\nu_f - 1}{\nu_{\text{th}}} \delta,$$

where $\delta$ is the ratio of the number of fissions in $\text{U}^{238}$ to the number in $\text{U}^{235}$. The values for $\nu_f$ and $\nu_{\text{th}}$ in natural uranium are taken as 2.51 and 2.46, respectively. $\delta$ is obtained by fitting an equation to a family of curves which gives $\delta$ as a function of the radius of the fuel assembly and of the relative amounts of $\text{D}_2\text{O}$, U, and structural materials [6].

**Neutron age, $\tau$**

The neutron age for fission neutrons in pure $\text{D}_2\text{O}$ is 120 cm at a temperature of 20°C. To obtain the neutron age, $\tau$, of a lattice, this figure must be corrected for the presence of: (1) light water, (2) cladding material, and (3) uranium. A decrease in $\tau$ of 4 cm$^2$ is produced by adding 1% light water. It is assumed that the cladding material does not moderate neutrons at all and that the inelastic scattering cross-section of uranium makes it half as good a moderator as $\text{D}_2\text{O}$. Then, since $\tau$ is inversely proportional to the moderator density, the slowing-down area or neutron age in the lattice is given by

$$\tau = (120.0 - 400 F_{\text{H}_2\text{O}}) \left(1 - \frac{V_{\text{Al}}}{V_{\text{cell}}} - \frac{V_u}{V_{\text{cell}}}\right)^{-2},$$

where

- $F_{\text{H}_2\text{O}}$ = the molecular fraction of $\text{H}_2\text{O}$ in the moderator
- $V_{\text{cell}}$ = the total volume of the cell
\[ V_{\text{Al}} = \text{the total volume of aluminium in the cell} \]
\[ V_u = \text{the total volume of uranium in the cell} \]

**Resonance escape probability, \( p \)**

The recipes used for calculating the resonance escape probability follow closely those reported in Ref. [6]. The resonance escape probability, \( p \), is defined as

\[
\Delta p = \exp(-\Delta p) = 1 - \Delta p + \frac{1}{2} (\Delta p)^2 - \frac{1}{6} (\Delta p)^3, \tag{8}
\]

where

\[
\Delta p = \frac{A \phi_c N_1 V_u RI}{\xi \Sigma_s V_{D_2O}}
\]

and

\[ A = \text{normalization parameter to adjust the calculated and measured bucklings} \]
\[ \phi_c = \text{resonance flux ratio} \]
\[ V_u = \text{total volume of uranium-238 in cell} \]
\[ N_1 = \frac{\rho^{238} a}{\text{at. wt.}} = \text{number of fuel atoms per cm}^3 \text{ with} \]
\[ \rho^{238} = \text{density of U}^{238} \]
\[ a = 0.60234, \text{ when} \]
\[ RI = \text{resonance integral (in barns)} \]
\[ V_{D_2O} = \text{total volume of D}_2\text{O in cell} \]
\[ \xi \Sigma_s = \text{slowing-down power of the moderator}. \]

The correction due to the resonance flux dip in the moderator is expressed in Ref. [6] as a sum of the contributions from the resonance flux from neighbouring assemblies. In the present calculation, this sum is replaced by the ratio, \( \phi_c \), of the flux in the centre of the cell to the average flux in the cell, as given by the expression
\[ \phi_c = 1 + 0.0089 \left[ \left( \frac{\tau_L}{\tau_M} \right)^{1/2} s - 6.2 \right]^2, \]  

(9)

where

\[ \left( \frac{\tau_L}{\tau_M} \right)^{1/2} = \left( 1 - \frac{V_{Al}}{V_{cell}} - \frac{V_u}{V_{cell}} \right)^{-1} \]

and \( s \) is the triangular lattice pitch in inches.

The value of \( \xi \Sigma_s \), the slowing-down power of the moderator, is calculated using appropriate molecular fractions of \( \text{D}_2\text{O} \) and \( \text{H}_2\text{O} \).

The resonance integral for uranium metal is that of Hellstrand in Ref. [10]

\[ \text{RI} = 2.95 + 25.8 \left( S_{\text{eff}} / M \right), \]

where

\[ M = \text{mass of uranium metal in the fuel element} \]
\[ = p^{238} V_u \]

and \( S_{\text{eff}} \) is the effective surface defined as

\[ S_{\text{eff}} = S_1 + \left( 1 / G_0 \right) \left( \text{EC} S_2 \right). \]

\( S_1 \) is the outer surface of the rod and \( S_2 \) is the interior surface. The quantity \( \text{EC} \), the effectiveness of the inner surface for resonance energy absorptions and for the solid rods used in the present experiments, is one. Al cladding is considered to have the same transport cross-section as the \( \text{D}_2\text{O} \).

The quantity, \( G_0 \), corrects for the flux dip of resonance energy neutrons within the fuel assembly. It is the ratio of flux at the outer surface to the average flux in the fuel. To obtain \( G_0 \), a one-group, three-region, diffusion theory calculation was made.

**Macroscopic absorption cross-sections, \( \Sigma \),**

Macroscopic absorption cross-sections are averages over Maxwellian energy distributions at the actual moderator temperature. Since the purity of the \( \text{D}_2\text{O} \) will affect the absorption cross-section, \( \Sigma_{\text{D}_2\text{O}} \) is calculated using the appropriate molecular fraction of \( \text{H}_2\text{O} \).
B. Pulsed measurements

A rigorous calculation of the decay constant, $\lambda$, involves complex factors depending on the energy and spatial neutron distributions. One hopes, however, that relatively simple theoretical models will give sufficiently accurate answers. The situation is complicated because of the radial reflector which cannot always be accounted for by the expedient of introducing a reflector savings. In fact, the reflector savings is itself a function of the decay constant and the transient neutron population in the reflector should be accounted for in the calculations.

The simplest reasonably accurate model appears to be a two-group, two-region scheme. However, hand computations become too lengthy, although the method is well suited for numerical diffusion codes. The compromise we have adopted is to use a two-region, modified one-group model with Maxwellian-weighted cross-sections.

The non-equilibrium equations are

$$\frac{1}{v} \frac{\partial \phi_c (r,t)}{\partial t} = (D_c \nabla^2 - \Sigma_c) \phi_c (r,t) + k\Sigma_c P(B^2) \phi_c (r,t)$$  \hspace{1cm} (10)

and

$$\frac{1}{v} \frac{\partial \phi_R (r,t)}{\partial t} = (D_R \nabla^2 - \Sigma_R) \phi_R (r,t), \hspace{1cm} (11)$$

where the sub-indices "c" and "R" stand for core and reflector, respectively, and $P(B^2)$ is the fast non-leakage factor.

When the asymptotic distribution has been reached, the flux can be separated in space and time and one obtains the quasi-static equations

$$(\nabla^2 + B^2) \phi_c (r) = 0 \hspace{1cm} (12)$$

and

$$(\nabla^2 - \kappa^2) \phi_R (r) = 0, \hspace{1cm} (13)$$

where

$$B^2 = \frac{\alpha_c}{D_{0c}} [k_w P(B^2) - 1] + \frac{\lambda}{D_{0c}} \hspace{1cm} (14)$$

and

$$\kappa^2 = \frac{\alpha_R}{D_{0R}} - \frac{\lambda}{D_{0R}} \hspace{1cm} (15)$$
One notes that the separation constant, \( B^2 \), in Eq. (11), is the so-called material buckling when \( \lambda = 0 \), i.e., for a prompt critical system.

The set of Helmholtz equations, (12) and (13), together with the pertinent boundary conditions, yields the following secular equation for the eigenvalue, \( \lambda \):

\[
\frac{J_0(-c r_0)}{D_c J_1(c r_0)} = \frac{J_0(l_R r_1) Y_0(l_R r_0) - Y_0(l_R r_1) J_0(l_R r_0)}{D_R [J_0(l_R r_1) Y_1(l_R r_0) - Y_0(l_R r_1) J_1(l_R r_0)]},
\]

where

\[
l_c^2 = B^2 - \left( \frac{\pi}{H} \right)^2.
\]

\[
l_r^2 = \kappa^2 - \frac{\lambda}{D_R}.
\]

\( r_0 \) is the effective radius of the core and the oscillatory Bessel functions are used in the reflector because \( l_r^2 \) is negative.

At this point, the nature of the fast non-leakage factor is to be defined. The non-escape probability for the fast neutrons will depend on both the size of the system and the decay constant, \( \lambda \). To introduce these features into the non-leakage factor, a Fermi slowing-down kernel is used in the form

\[
P(B^2) = e^{-B^2 r}.
\]

Therefore, Eq. (17) becomes

\[
\lambda = \alpha_c + D_{0c} B^2 - \alpha_c \kappa \omega e^{-B^2 r},
\]

or by expanding the exponential in Eq. (20)

\[
\lambda = \alpha_c + D_{0c} B^2 - \alpha_c \kappa \omega (1 - B^2 \tau + \frac{1}{2} \tau^2 B^4).
\]
Eq. (21) is not to be taken as an explicit relation for the decay constant since \( B^2 \) itself is a function of \( \lambda \) through the secular Eq. (16) and Eqs. (17) and (18). Therefore, an iteration procedure is required. One starts by guessing a value of \( B^2 \) in Eq. (21); a value for \( \lambda \) is then obtained. Both values are introduced into the secular equation until a consistent pair of \( \lambda \) and \( B^2 \) is found satisfying Eq. (16) and Eq. (21) simultaneously.

This model has given good agreement with the experimental results. Tables II and III show the comparison between experiment and theory for a few configurations.

4. RESULTS

A. Static measurements

**Radial buckling measurements**

The conventional method for determining the radial buckling is to fit the measured radial flux distribution to a zero-order Bessel function. Alternatively, if the axial buckling is measured for a lattice whose material buckling is well known, then the radial buckling may be obtained from the relationship \( B_r^2 = B_m^2 + \kappa_r^2 \). We have measured the radial buckling using both these techniques for the 11.9 cm spacing; the values obtained for the two measurements were in good agreement. For the other lattice loadings, the radial buckling will depend on the amount of moderator surrounding the core. The values given in Table III are based on the known material bucklings for each lattice pitch.

**Axial buckling measurements**

Most of the reported axial bucklings were obtained with In foils. Inter-comparisons between In foil data and the Au-Si counter data showed good agreement. In-Cd ratios were measured to determine the region where the neutron spectrum was constant.

B. Pulsed measurements

With the exponential tank filled with \( \text{D}_2\text{O} \) only, a signal-to-background ratio of approximately 2000 was obtained. This ratio decreased to about 400 when a lattice was loaded in the tank. The increased background is likely a result of the photoneutron production from fission-product gamma radiation.

The influence of the higher spatial modes was studied by measuring the neutron population at several points in the system as a function of time. The data were normalized to the integral counts of a standard detector placed in a fixed point inside the subcritical. Axial and radial space-time plots of the flux were obtained for the \( \text{H}_2\text{O} \) subcritical and radial plots for the \( \text{D}_2\text{O} \) subcritical (Figs. 4 to 8).
TABLE II

PULSED AND STATIC MEASUREMENTS IN NATURAL URANIUM, H₂O-MODERATED LATTICES

<table>
<thead>
<tr>
<th>Pitch (cm)</th>
<th>$V_m/V_u$</th>
<th>No. of rods</th>
<th>Core radius (cm)</th>
<th>$\lambda$ (s⁻¹)</th>
<th>$\sigma^2$ (m⁻²)</th>
<th>$B_{t}^2$ (m⁻²)</th>
<th>$B_{m}^2$ (m⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.97</td>
<td>1.14</td>
<td>217</td>
<td>31.3</td>
<td>5017</td>
<td>5486 ± 459</td>
<td>44 ± 1</td>
<td>-</td>
</tr>
<tr>
<td>3.97</td>
<td>1.14</td>
<td>169</td>
<td>27.7</td>
<td>5020</td>
<td>5122</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3.97</td>
<td>1.14</td>
<td>127</td>
<td>24.1</td>
<td>5025</td>
<td>5166</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3.97</td>
<td>1.14</td>
<td>91</td>
<td>20.5</td>
<td>5030</td>
<td>5421 ± 201</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3.97</td>
<td>1.14</td>
<td>61</td>
<td>16.9</td>
<td>5040</td>
<td>5358</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3.97</td>
<td>1.14</td>
<td>37</td>
<td>13.3</td>
<td>5050</td>
<td>5413</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3.97</td>
<td>$\infty$</td>
<td>0</td>
<td>0.0</td>
<td>4930</td>
<td>4870</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
TABLE III

PULSED AND STATIC MEASUREMENTS IN NATURAL URANIUM, D\textsubscript{2}O-MODERATED LATTICES

<table>
<thead>
<tr>
<th>Pitch (cm)</th>
<th>$V_m/V_u$</th>
<th>No. of rods</th>
<th>Core radius (cm)</th>
<th>$\lambda$ (s$^{-1}$) \hspace{1cm}</th>
<th>$\kappa^2$ (m$^{-2}$)</th>
<th>$B_f^2$ (m$^{-2}$)</th>
<th>$B_m^2$ (m$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.9</td>
<td>9.2</td>
<td>55</td>
<td>32.5</td>
<td>1231 ± 24 \hspace{1cm}</td>
<td>14.3 ± 0.7</td>
<td>18.7</td>
<td>4.4 ± 0.9</td>
</tr>
<tr>
<td>11.9</td>
<td>21.7</td>
<td>51</td>
<td>46.5</td>
<td>820 ± 16 \hspace{1cm}</td>
<td>12.2 ± 0.5</td>
<td>20.5</td>
<td>8.3 ± 0.6</td>
</tr>
<tr>
<td>11.9</td>
<td>21.7</td>
<td>33</td>
<td>38.1</td>
<td>847 ± 17 \hspace{1cm}</td>
<td>12.5 ± 0.5</td>
<td>20.8</td>
<td>8.3 ± 0.6</td>
</tr>
<tr>
<td>11.9</td>
<td>21.7</td>
<td>16</td>
<td>27.3</td>
<td>745 ± 15 \hspace{1cm}</td>
<td>14.2 ± 0.5</td>
<td>22.5</td>
<td>8.3 ± 0.6</td>
</tr>
<tr>
<td>11.9</td>
<td>21.7</td>
<td>6</td>
<td>16.6</td>
<td>- \hspace{1cm}</td>
<td>17.6 ± 0.6</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>15.9</td>
<td>41.4</td>
<td>29</td>
<td>46.3</td>
<td>758 ± 22 \hspace{1cm}</td>
<td>14.7 ± 0.7</td>
<td>22.0</td>
<td>7.3 ± 0.8</td>
</tr>
<tr>
<td>15.9</td>
<td>41.4</td>
<td>17</td>
<td>36.2</td>
<td>- \hspace{1cm}</td>
<td>15.1 ± 0.7</td>
<td>22.4</td>
<td>7.3 ± 0.8</td>
</tr>
<tr>
<td>15.9</td>
<td>41.4</td>
<td>6</td>
<td>22.0</td>
<td>- \hspace{1cm}</td>
<td>17.8 ± 0.7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>∞</td>
<td>0</td>
<td>0</td>
<td>639 ± 19 \hspace{1cm}</td>
<td>25.8 ± 0.5</td>
<td>24.4</td>
<td>1.39 ± 0.6</td>
</tr>
</tbody>
</table>
From a comparison of our radial plots in the D₂O subcritical with the axial and radial plots of MEISTER [5], it appears that the axial modes die out slower than the radial modes. The results are shown in Table IV.

The longer waiting times required to obtain the asymptotic distribution as described in Ref. [5] are very likely influenced by the use of 14-MeV neutrons as compared with the D-D neutrons used in our experiment. Because of poor counting statistics, we started the analysis of the decay data at 0.3 ms and 4 ms for the H₂O and D₂O subcriticals, respectively. In each case the measurements were made at several points in the system. Agreement was obtained within an experimental error of 2% in the decay constant for the D₂O measurements. This error was estimated from the 1% error obtained from the least-squares data fitting plus a 1% error attributed to contamination from higher modes. For the H₂O subcritical, the results are much more sensitive to the contamination by the higher spatial modes. The measurements in H₂O at several positions varied by 8%. Experimental results for several lattice pitches and fuel loadings are plotted on Fig. 8 and are given in Tables II and III for both H₂O and D₂O subcritical systems.

The data, obtained as counts per channel, are fitted to a single exponential plus a background. A numerical IBM-709 code computes an estimate of the parameters by the non-iterative CORNELL [11] method and uses the estimates as input for a non-linear general least-squares programme.
Radial flux distribution for wide-spaced fuel as a function of time after pulse (in a H$_2$O-moderated lattice).

Fig. 5

Radial flux distribution as a function of time after the end of the pulse (D$_2$O subcritical zero loading).

Fig. 6
Fig. 7.
Theoretical fit of the radial flux distribution 4.5 ms after the end of the pulse

Fit to $J_4(2.405 \frac{r}{503})$

 Experimental

Fig. 8
Radial flux distribution as a function of time after the end of the pulse ($D_2O$ subcritical, 33 uranium rods)
### Table IV

**Waiting Time, $t_{\text{asymp.}}$ (ms), to Attain the Asymptotic Distribution**

<table>
<thead>
<tr>
<th>Moderator</th>
<th>$t_{\text{asymp.}}$ (ms)</th>
<th>Axial plot</th>
<th>Radial plot</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_2O$</td>
<td>$&gt;2$</td>
<td></td>
<td>0.9</td>
</tr>
<tr>
<td>$D_2O$</td>
<td>16</td>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

**Fig. 9**

Effect of core radius on the decay constant in $H_2O$- and $D_2O$-moderated lattices

- $H_2O$, $V_m/V_u = 1.14$
- $D_2O$, $V_m/V_u = 1.92$ (Ref. [3])
- $H_2O$, $V_m/V_u = 22.6$
5. CONCLUSIONS

For the H$_2$O-moderated system, the decay constant is found to be very insensitive to the core radius when the core is surrounded by an infinite H$_2$O reflector. This result is a consequence of the large absorption cross-section of H$_2$O. For the D$_2$O-moderated system, on the other hand, the decay constant goes through a broad maximum and then decreases, becoming zero when the critical radius is reached. The measured decay constants are in satisfactory agreement with calculated values in the region of this broad maximum (see Fig. 9). The calculated values are based on a two-region, modified one-group model. Thus, it appears feasible to determine lattice parameters using the pulsed neutron technique in a 1-m tank with core radii as small as 35 cm, although this minimum value will depend on the mean free path within the core.

To firmly establish this minimum value, additional measurements of the neutron spectrum in the reflector and core are planned.

ACKNOWLEDGEMENTS

This work was made possible by the loan of uranium, D$_2$O and Pu-Be by the Division of Education and Training, United States Atomic Energy Commission. W. F. Fagen was responsible for the design of the H$_2$O subcritical facility. Others who gave valuable assistance with the experiments and calculations include F. A. Primo, G. W. Fogel, R. Booth, R. Cockrell and R. B. Razminas.

REFERENCES


MEDIDA DE LA LAPLACIANA EN UNA EXPERIENCIA EXPONENCIAL

E. RODRÍGUEZ MAYQUEZ
JUNTA DE ENERGÍA NUCLEAR, MADRID, ESPAÑA

Abstract — Résumé — Аннотация — Resumen

BUCKLING MEASUREMENT IN AN EXPONENTIAL EXPERIMENT. The paper describes the method of data processing now being used in the laboratories of the Junta de Energía Nuclear (Nuclear Energy Board), Madrid, for determining the buckling in an exponential experiment. The method is based on a least-squares operation on the flux values measured at different points, which makes it possible to eliminate the contributions due to harmonics as well as other disturbances.

MESURE DU LAPLACIEN DANS UNE EXPÉRIENCE EXPONENTIELLE. L’auteur décrit la méthode de traitement des données actuellement employée au Laboratoire de la Commission de l’énergie atomique à Madrid pour déterminer le laplacien, dans une expérience exponentielle. Il s’agit essentiellement d’un ajustement par la méthode des moindres carrés des flux mesurés en différents points, ce qui permet d’éliminer l’influence des harmoniques et autres perturbations.

ИЗМЕРЕНИЕ ЛАПЛАСИАНА В ЭКСПОНЕНЦИАЛЬНОМ ОПЫТЕ. Авторы описывают метод обработки данных, которыми в настоящее время пользуются в лаборатории Комиссии по атомной энергии в Мадриде для измерения лапласиана в экспоненциальном опыте. Речь идет, главным образом, об уточнении методом наименьших квадратов потоков, измеряемых в различных точках, что позволяет устранить воздействие гармонических и других возмущений.

MEDIDA DE LA LAPLACIANA EN UNA EXPERIENCIA EXPONENCIAL. Se describe el método de tratamiento de datos, actualmente en uso en los laboratorios de la Junta de Energía Nuclear de Madrid, para la determinación de la laplaciana en una experiencia exponencial. Básicamente se trata de un ajuste por mínimos cuadrados de los flujos medidos en diferentes puntos que permite eliminar las contribuciones de armónicos, y otras perturbaciones.

INTRODUCCIÓN

El presente trabajo sólo tiene por objeto describir los criterios que han servido de guía en el tratamiento de datos experimentales para la determinación de la laplaciana en una experiencia exponencial. A la hora de redactar este trabajo no se tiene aún confirmación experimental completa de la bondad y eficacia de estos criterios, pero es de esperar se tenga para la redacción definitiva.

La experiencia exponencial, actualmente en curso en los laboratorios de la Junta de Energía Nuclear en Madrid, consiste en un retículo de elementos combustibles de carburo de uranio natural y un orgánico como simulador del refrigerante y agua pesada como moderador. La simetría externa es cilíndrica con las fuentes de neutrones (Sb-Be) en su base. En estas condiciones, y no teniendo en cuenta por ahora la existencia de armónicos y efectos de reflector, la distribución de flujos es

\[ \Phi(r, z) = A J_0(\alpha r) \sin(\gamma(H - z)) \]
siendo el valor de la laplaciana material que se busca

\[ B^2 = \alpha^2 - \gamma^2 \]

donde

\[ \alpha = \frac{2.405}{R_{\text{ext}}} \]

Dada la dificultad de calcular el radio extrapolado \( R_{\text{ext}} \) es conveniente determinar tanto \( \gamma \) como \( \alpha \) de las distribuciones experimentales axiales y radiales.

Para esta determinación experimental pueden existir dos criterios generales. Uno de ellos tiende a medir muchos puntos con la obtención de varios valores para \( \alpha \) y \( \gamma \). Una vez eliminados aquellos valores que muestran todavía alguna perturbación sistemática por las condiciones especiales en que se hizo la medida (contaminación de armónicos, efectos de reflector, etc.) se calcula el valor medio para \( \alpha \) y \( \gamma \).

El otro criterio tiende a medir menos puntos con la obtención de pocos valores para \( \alpha \) y \( \gamma \), en los que la ausencia de perturbación se halla asegurada por un tratamiento más cuidadoso de los datos. Ello resulta más tedioso, pues el tratamiento de datos no puede hacerse tan rutinario como en el caso del primer criterio y además exige personal experimentado para hacer todas las fases del tratamiento. En cambio es más difícil que se escapen errores sistemáticos.

La experiencia exponencial de la JEN consta de 37 celdas, de las cuales sólo 6 son distintas por su posición geométrica relativa y de ellas tres pueden estar afectadas por efectos de reflector. Las medidas de cada \( \alpha \) particular exigen ya un examen cuidadoso de las condiciones de medida. Ello decidió a utilizar definitivamente el segundo criterio.

No es objeto de este trabajo hacer la crítica de las correcciones a las que habrá que someter el valor de la laplaciana obtenida en una experiencia exponencial de dimensiones relativamente pequeñas, debido a diferencias entre espectros de neutrones.

El tratamiento de datos sigue los siguientes pasos:

1° Determinación experimental de la distribución radial y axial de flujos de neutrones.

2° Estimación por medida de la fuente de neutrones de los armónicos más probables y corrección de esta contribución en las distribuciones experimentales.

3° Correcciones por reflector.

4° Comprobación de la bondad de las correcciones efectuadas y ligeras modificaciones de dichas correcciones.

DETERMINACIÓN DE LA DISTRIBUCIÓN DE FLUJOS DE NEUTRONES

La determinación experimental de la distribución radial y axial del flujo de neutrones se ha hecho utilizando la técnica de sondas de activación,
pues las pequeñas perturbaciones que introducen son fáciles de corregir y medir.

La actividad de cada sonda calibrada es medida por varios contadores y varias veces en cada contador obteniéndose así una buena estadística y un control del perfecto estado de funcionamiento del equipo de contadores. Las actividades son referidas a las de una sonda tomada como patrón sometida siempre al mismo flujo. Los datos de las medidas son tratados por el programa CA02 en la calculadora digital UNIVAC-UCT de la Junta de Energía Nuclear. Junto con los resultados de dicho tratamiento (con medias ponderadas, exclusión de datos anómalos, etc.) se da también amplia información sobre la marcha de los cálculos, lo que permite una mejor estimación de la bondad de los resultados. Para cada punto se hacen varias irradiaciones de sondas permitiendo comprobar que los errores asignados a cada medida están de acuerdo con la fluctuación obtenida entre las diferentes irradiaciones. Las distribuciones son corregidas de efectos de temperatura y cambios de pureza isotópica del agua pesada.

ESTUDIO DE LA FUENTE DE NEUTRONES

La fuente de neutrones está constituida por cinco fuentes de antimonio-berilio cuyos neutrones son moderados en un pedestal de grafito colocado como base de la experiencia exponencial. Las dimensiones de este pedestal, distribución de las cinco fuentes de neutrones e intensidad relativa de las mismas, fue estudiada para conseguir la mínima contaminación de armónicos en la zona de medidas de la experiencia.

No obstante se comprueba la ausencia real de armónicos o su pequeña contribución haciendo medidas solamente con agua pesada sin elementos combustibles, y en la zona inferior del recipiente, para poder determinar con suficiente exactitud los armónicos existentes en la zona de separación entre grafito y agua pesada. La magnitud de las correcciones a efectuar en las zonas de trabajo es aceptable, dando confianza a la bondad de las correcciones a efectuar.

Se comprueba también la ausencia práctica de reflexiones por los objetos circundantes, tal como se esperaba por el blindaje exterior con carburo de boro de las paredes del recipiente.

AJUSTE DE LA DISTRIBUCIÓN RADIAL

La distribución experimental radial del flujo de neutrones, ya corregido de las perturbaciones ha de ajustarse a la función

$$\Phi(r) = A J_0(\alpha r).$$

Dicho ajuste se ha hecho por mínimos cuadrados ponderados con la calculadora digital de la JEN, utilizando el programa AMCJ. Dicho programa efectúa las siguientes operaciones:
1° Ajusta por mínimos cuadrados los mejores valores para $A$ y $a$ para sólo tres puntos de medida, los tres de radio mínimo. Asimismo calcula la $\chi^2$ de dichos puntos para este ajuste.

2° Repite los ajustes para cuatro, cinco, etc., hasta completar la totalidad de puntos experimentales obteniéndose también las $\chi^2$ correspondientes.

3° Normaliza todos los puntos experimentales para cada uno de estos ajustes permitiendo dibujar cómodamente gráficas de todos los puntos para cada ajuste.

Se estudian tres pasos de reticulados. En uno de ellos no existen problemas de reflexión pero en los otros el efecto de reflector del agua pesada lateral se manifestará por una variación tendenciosa de los valores de $a$ al empezar a intervenir los puntos exteriores y por una desviación manifiesta de los puntos exteriores cuando en el ajuste todavía no intervienen éstos. Es posible determinar, pues, hasta qué radio se manifiesta la existencia del reflector.

No obstante, y con el fin de mejorar la estadística, se intenta corregir este efecto en los puntos perturbados, pues la forma de corrección se puede estimar aunque sea difícil la determinación de algunos parámetros que intervienen en ella. Un tanteo de dichos parámetros con el criterio de la constancia de los valores que se encuentren para $a$ y la no desviación sistemática de los puntos más exteriores permite ampliar la zona que contribuye al cálculo correcto de $a$. Los valores de relaciones de $Cd$ ayudan a confirmar la extensión de esta zona.

En cuanto a los armónicos, su contribución, además de pequeña, es fácilmente calculable una vez conocido el término fuente. De todas maneras, y especialmente en el caso del reticulado sin efecto de reflector, la sistematización por zonas de las desviaciones de los puntos experimentales sobre la curva de mejor ajuste y el criterio de la $\chi^2$ da información suficiente sobre la mejor corrección de contribución de armónicos. Medidas de $a$ en tres planos diferentes confirman la bondad de los resultados.

**AJUSTE DE LA DISTRIBUCIÓN AXIAL**

La distribución axial de neutrones, determinada experimentalmente y corregidas de las perturbaciones existentes, ha de cumplir la expresión

$$ \Phi(z) = BShy(H - z) $$

siendo $H$ la altura extrapolada que no se conoce con exactitud suficiente. Hay, pues, que hacer un ajuste por mínimos cuadrados de los tres parámetros $B, H$ y $y$. Dicho ajuste también se ha hecho con la calculadora digital con el programa EXPO que efectúa operaciones equivalentes al utilizado para el ajuste de la distribución radial. La única diferencia consiste en partir del ajuste para los cinco puntos más cercanos al borde superior.

Siguiendo la misma técnica que en el caso anterior es posible asegurar la bondad de las correcciones y hasta qué altura no se manifiestan perturbaciones mal corregidas. Se hacen medidas en tres ejes diferentes con la misma idea de confirmación de la bondad de los resultados obtenidos.
CONCLUSIÓN

Esta técnica de tratamiento de datos ha sido probada con éxito con los datos obtenidos en una pequeña experiencia exponencial de uranio natural y agua ligera. Es de esperar que a la hora de redactar definitivamente este trabajo, para la publicación de las Actas del Simposio*, exista ya la comprobación con las experiencias en carburó de uranio y agua pesada, actualmente en curso en la División de Física de la Junta de Energía Nuclear.

DISCUSSION

O.H. CRITCHLEY: The description of the measurement technique by which the number of pick-up heads is reduced to lessen perturbations is of great interest. Clearly, the presence in an experiment of a measuring probe is synonymous with perturbation. Consequently, I would like to know what quantitative criterion was used to define the number of measuring points used in relation to the hoped-for accuracy of the experimental data and the inferences drawn therefrom.

E. RODRIGUEZ-MAYQUEZ: Our method was to estimate the number of points necessary for obtaining sufficient accuracy in the final results. However, in irradiating the activation probes, only part of the total number are put in, so as to reduce perturbation and, in particular, to avoid mutual interference. This considerably simplifies the calculation of perturbation, which is obtained with more than sufficient accuracy.

O.H. CRITCHLEY: Of course, as a caveat, it must be stated that the number of measuring points is reduced, so the ignorance about the conditions in the experiment must increase, although the individual point measurement may be made more exact. Is it, therefore, correct to assume that this was made good by increasing the number of traverses?

E. RODRIGUEZ-MAYQUEZ: As I indicated in reply to the previous question, we first estimate the minimum number of points. For this we already have an idea of the accuracy that can be expected from the flux measurements at each of the points. To be able to estimate the number of points and positions sufficient for us to get an acceptable final error, it is enough, using a least-squares method, to estimate various hypothetical cases involving different contaminations of harmonics and reflections. By comparing the results for different radii or axes with different contaminations, and having the certainty that these contaminations can be detected by the number of points taken, we can verify that the corrections for the perturbations are adequate.

O.H. CRITCHLEY: Have you attempted to deal in a generalized quantitative manner with this philosophy of measurement?

E. RODRIGUEZ-MAYQUEZ: I regret to say that I have made no quantitative generalization of the ideas expressed in my paper. I have merely endeavoured to reach a solution for the special case of our measurements. However, if this method of exponential experiments is continued, we shall no doubt find it necessary to make a fuller and more generalized study.

* Hasta el momento de imprimir estas Actas no se recibió la memoria definitiva del autor.
THE EVALUATION OF BUCKLING AND DIFFUSION COEFFICIENTS FROM TWO-REGION EXPERIMENTS

R. PERSSON
AKTIEBOLAGET ATOMENERGI, STUDSVIK, NYKOPING, SWEDEN

Abstract — Résumé — Аннотация — Resumen

THE EVALUATION OF BUCKLING AND DIFFUSION COEFFICIENTS FROM TWO-REGION EXPERIMENTS.
The progressive substitution technique which is described in the literature has the aim of giving the buckling difference between a small test region and the reference lattice surrounding the test region. With voided fuel assemblies one is forced to consider the difference \( \delta D \) between diffusion coefficients in the various regions. Using measurements with single fuel assemblies one is able to determine \( \delta D \) experimentally both in the radial and in the axial directions. The method of evaluation discussed in this paper can be described as a modified two-group perturbation theory taking perturbed fluxes into account.

ÉVALUATION DU LAPLACIEN ET DES COEFFICIENTS DE DIFFUSION A PARTIR D'EXPÉRIENCES SUR DEUX RÉGIONS. La méthode de substitution progressive décrite par plusieurs auteurs a pour but d'obtenir la différence des laplaciens entre une petite région témoin et le réseau de référence qui l'entoure. Avec des ensembles de combustible à cavités, on est forcé de considérer la différence \( \delta D \) entre les coefficients de diffusion dans plusieurs régions. Avec des mesures sur des ensembles à un seul combustible, on peut déterminer \( \delta D \) expérimenteralement dans la direction radiale et dans la direction axiale. La méthode d'évaluation exposée dans le mémoire peut être considérée comme une théorie de perturbation à deux groupes modifiée pour tenir compte des flux perturbés.

ОЦЕНКА ЛАПЛАСИАНА И КОЭФФИЦИЕНТОВ ДИФФУЗИИ В ОПЫТАХ С ДВУМЯ ОБЛАСТЯМИ. Метод постепенного замещения, который описывается в литературе, имеет своей целью определить различия в величине лапласиана в небольшой испытываемой области и эталонной решетке, окружающей испытуемую область. При вакуумированных топливных сборках возникает необходимость рассмотрения различий \( \delta D \) между коэффициентами диффузии в различных областях. С помощью измерений на сборках из отдельных топливных блоков можно экспериментально определять \( \delta D \) как в радикальном, так и в осевом направлениях. Обсуждаемый метод оценки может быть определен как измененная двухгрупповая теория возмущения, учитывающая возмущенные потоки.

EVALUACIÓN DE LAPLACIANOS Y DE COEFICIENTES DE DIFUSIÓN EN EXPERIMENTOS DE DOS ZONAS. La finalidad de la técnica de sustitución progresiva descrita por diversos autores es dar la diferencia de los laplacianos entre una reducida zona testigo y el retículo de referencia que la rodea. Al utilizar conjuntos combustibles con cavidades, es preciso considerar la diferencia \( \delta D \) entre los coeficientes de difusión de las diversas zonas. Esta diferencia se puede determinar experimentalmente, tanto en dirección radial como axial, por medio de mediciones en un solo conjunto combustible. El método de evaluación expuesto en la memoria podría considerarse como una teoría de la perturbación de dos grupos, modificada para tener en cuenta los flujos perturbados.

1. INTRODUCTION

The primary purpose of the progressive substitution method is to find the material buckling of a uniform test lattice from measurements of buckling differences, caused by stepwise replacement of the central part of a reference lattice by the test lattice. If the diffusion coefficient is different in the test and reference regions this has to be taken into account, making
the analysis more complicated. If the diffusion coefficient is dependent on the direction, i.e. if there is anisotropy in the neutron diffusion, the critical buckling is no longer a material constant and we have to specify the geometry related to the critical buckling and give the degree of anisotropy to be able to transfer the results to other geometric conditions. The difference between the diffusion coefficients of two regions can be measured by complementary experiments using a single test fuel assembly and varying the perturbation axially and radially. The axial measurements can be applied only in a critical facility, which of course also gives the highest accuracy in the radial case. However, reversible substitution experiments with complete loadings in an exponential assembly can also be used to determine differences between diffusion coefficients, though with poor accuracy for the axial component. The theory applied is discussed below and some experimental results are also given.

2. THEORY

2.1. General

Usually perturbation theory is only applied if the perturbations are small. The range of validity of the perturbation formulae can, however, be increased considerably if one properly takes into account the perturbed fluxes when calculating the weight functions. Though the perturbed fluxes are not known beforehand, they are found from the analysis of the experiment and can therefore be applied iteratively.

2.2. One-group theory

Let us assume a cylindrical reactor containing different core regions (subscript $i$) each with a one-group flux $\phi_i$ and a diffusion coefficient $D_{kl}$ and a buckling $B_{kl}$ in the direction $k$ ($k = r$ and $z$). Regarding the whole reactor as one unit we introduce an unperturbed fictitious flux $\phi_0$ related to the extrapolated geometry of the core which has the critical buckling $B_0 = \Gamma \sum_i B_{k0}$. By multiplying the diffusion equation of every region with $\phi_0$ and integrating over the whole system we can derive the following relationship [1]

$$\sum_i \sum_k \left[ D_{kl} \left( B_{kl}^0 - B_{k0}^0 \right) \right] \phi_0 \phi_i \, dV$$

$$- \left( D_{kl} - D_{k0} \right) \int \left( \nabla_{kl} \phi_0 \right) \phi_i \, dS_{kl} = 0. \tag{1}$$

The meaning of $\nabla_{kl} \phi_0$ is the $k$-component of the outward gradient of $\phi_0$ on the surface element $dS_{kl}$ of region $i$. The diffusion coefficient $D_{kl}$ introduced is related to the reference region (subscript 1), but it could really be any constant, since

$$\sum_i \int \left( \nabla_{kl} \phi_0 \right) \phi_i \, dS_{kl} = 0. \tag{2}$$
BUCKLING AND DIFFUSION COEFFICIENTS

When $D_{kl} = D_{lk}$ and if we define a statistical weight $W_i$ as

$$W_i = \int \phi_0 \phi_i \mathrm{d}V / \sum_j \int \phi_0 \phi_j \mathrm{d}V, \quad (3)^*$$

we find that Eq. (1) is simplified to

$$\sum_i \sum_k B_{ki}^2 W_i = \sum_k B_{k0}^2 \quad (4)$$

or

$$\sum_i B_i^2 W_i = B_0^2. \quad (4a)$$

Introducing

$$\left( \nabla_{kl} \phi_0 \right) \phi_i \mathrm{d}S_{kl} / \sum_i \int \phi_0 \phi_i \mathrm{d}V = B_{k0}^2 \Omega_{kl}, \quad (5)^{**}$$

we may write Eq. (1) in its general form as

$$\sum_i \sum_k \left[ B_{ki} \left( B_{ki}^2 - B_{k0}^2 \right) W_i - \left( B_{ki} - D_{kl} \right) B_{k0}^2 \Omega_{kl} \right] = 0. \quad (6)$$

(i = region subscript)

So far no approximations have been made. As long as a one-group theory is valid Eq. (6) combined with Eqs. (3) and (5) is exact.

2.3. Two-group theory

Applying a two-group diffusion theory [1] we get a formula similar to Eq. (1).

$$\sum_i \sum_k \left[ \left( B_{ki}^2 - B_{k0}^2 \right) \int \phi_0 \psi_1 \mathrm{d}V - \left( D_{ki}^2 - D_{kl}^2 \right) \int \nabla_{ki} \phi_0 \psi_1 \mathrm{d}S_{kl} \right. \right.$$

$$\left. - \int \left[ D_{ki}^2 \phi_0 \left( \nabla_{ki} \theta_1 \right) - D_{kl}^2 \left( \nabla_{kl} \phi_0 \theta_1 \right) \right] \mathrm{d}S_{kl} \right] = 0, \quad (7)$$

* $W_i$ corresponds to $W_{g1} W_{r1}$ in Ref. [2].

** $\Omega_{rl}$ corresponds to $(U_{g1}-W_{g1}) W_{r1}$ and $U_{z1}$ to $(U_{z1}-W_{z1}) W_{r1}$ in Ref. [2]. It means that the surface integral $\int (\nabla_{ki} \phi_0) \phi_1 \mathrm{d}S_{kl}$ in Eq. (6) can be written as the difference between two volume integrals $\int (\nabla_{ki} \phi_0) \phi_1 \mathrm{d}V - B_{k0} \int \phi_0 \phi_1 \mathrm{d}V$. 
where

\[ D_{ki}^f = D_{ki}^f + S_1 D_{ki}^{th} \]  
\[ D_{ki}^u = D_{ki}^u + T_1 D_{ki}^{th} \]  
\[ \phi_f = \psi_i + \theta_i \]  
\[ \phi_i^{th} = S_1 \psi_i + T_1 \theta_i \]

The superscripts \( f \) and \( th \) of \( D \) and \( \phi \) refer to the fast and thermal neutron groups respectively. \( S \) and \( T \) (\( T < 0 \) in practical cases) are coupling coefficients \((S_1 \) and \( S_2 \) respectively in Ref. [3]). The flux \( \psi \) is related to the ordinary material buckling \((\mu^2 \) in Ref. [3]) whereas \( \theta \) is a transient term related to the negative root of the critical equation \((- \nu^2 \) in Ref. [3]).

2.4. Modifications

When comparing Eq. (7) with Eq. (1) we find that \( \phi_i \) is continuous but \( \psi_i \) discontinuous between regions. If we use the same definitions of the functions \( W_i \) and \( \Omega_{ki} \) as given in Eqs. (3) and (5) the continuity condition for \( \psi \) has to be satisfied which is ensured by requiring that

\[ C_1 \psi_i (r) = C_i \psi_i (r) \]  

at the interface between the two regions. The diffusion coefficients \( D_{ki}^f \) and \( (D_{ki}^u - D_{ki}^{th}) \) are then to be multiplied by \( C_1/C_i \). This ratio is, in good approximation, found to be

\[ \frac{C_1}{C_i} = \frac{S_1 - T}{S_1 - T}, \]

where

\[ T = \frac{T_1 Y(r) + T_i}{Y(r) + 1} \]  

with

\[ Y(r) = \frac{\nu_1 I_1(\nu_1 r) / I_0(\nu_1 r)}{\nu_1 K_1(\nu_1 r) / K_0(\nu_1 r)} \]  

The \( \nu \)-value is connected with the negative root of the two-group critical equation.

Equation (11) means that the ratio \( C_1/C_i \) is not a material constant but somewhat dependent on the size of the test region. In terms of two-group
constants the equivalent one-group quantities $D_{ki}$ and $(D_{ki} - D_{kl})$ in Eq. (6) divided by $D_{kl}$ have to be written as

$$\frac{D_{kl}}{D_{ki}} = \frac{D_{kl}^{th} + S_i D_{kl}^{th} (1+G_i)}{D_{kl}^{th} + S_i D_{kl}^{th} (1+G_i)}$$ (12)

and

$$\frac{D_{kl} - D_{kl}}{D_{kl}} = \frac{D_{kl}^{th} + S_i (D_{kl}^{th} - D_{kl}^{th})}{D_{kl}^{th} + S_i D_{kl}^{th} (1+G_i)} (13)$$

where

$$G_i = \frac{(S_2 - S_1)(D_{kl}^{th} + \bar{T})}{(S_2 - D_{kl}^{th} + \bar{T})}$$ (14)

G corresponds to $-\Gamma/(1+\Gamma)$ in Ref. [2].

Bearing in mind that $\bar{T} < 0$ (in practical cases) we see that $G_i$ << 1. Further we notice that Eq. (12) cannot be derived directly from Eq. (13) which is important to remember when evaluating experimental results.

The last surface integrals of Eq. (7) show the effect of the transient terms. From a closer examination [1] we conclude that these surface integrals could be replaced by two one-group terms similar to those discussed above and related to a transition region of a certain thickness. We find

(a) That the transition region has to have such a position that its statistical weight $W(r - \delta r_1; r + \delta r_2)$ is symmetric around the radius $r$ corresponding to the two-zone interface, i.e. $W(r - \delta r_1; r) = W(r; r + \delta r_2)$; and

(b) That the thickness $(\delta r_1 + \delta r_2)$ has to be chosen properly.

Condition (a) is equivalent to the assumption that the buckling of the transition region in the limiting case of identical spectra approaches the mean of the test and reference bucklings, which seems to be plausible.

The question is how to choose the transition region. It seems impossible to get it from the conventional cell definition, where the fuel is placed in the centre of the cell. However, if we apply a new unconventional definition of cells with the fuel assemblies in the corners of the cells (square, rhombic or triangular), we get more kinds of cells than there are kinds of fuel assemblies. Cells with more than one kind of fuel are defined as transition cells forming a transition region between the test lattice and the reference lattice.

In addition we may point out that the test and reference lattices can have different pitches (e.g. by a factor of $\sqrt{2}$ or 2 in a square lattice) and we are still able to get a well-defined transition region from the new cell concept. By means of the conventional cell definition, it is not possible in such cases to define a two-zone interface.

In summary, a comparison between the one-group Eq. (6) and two-group Eq. (7) shows the possibility of modifying the two-group expression into a form which is equivalent to the one-group formula. Thus Eq. (7) can be rewritten as
\[
\sum \sum (1 + G_i) D_{kl}^H \left[ \frac{D_{kl}^H}{D_{kl}^L} (B_{kl}^F - B_{kl}^M) W_i - \frac{D_{kl}^L}{D_{kl}^H} B_{kl}^S \Omega_{kl} \right] = 0, \tag{15}
\]

with

\[
W_i \quad \text{according to Eq. (3)}
\]

\[
\Omega_{kl} \quad \text{according to Eq. (5)}
\]

\[
D_{kl}^L \quad \text{according to Eq. (7a)}
\]

\[
G_i \quad \text{according to Eq. (14)}.
\]

\(D_{kl}^H\) may be called the effective diffusion coefficient of the reference region weighted over the neutron spectrum. The ratio \(D_{kl}^H/D_{kl}^L\) is a factor giving the effect of changes in \(D_f\) and \(D_m\). \(G_i\) is a correction term due to spectral mismatch. When \(S_i = S_1\) we have \(G_i = 0\).

Equation (15) should be adequate if the transition region is chosen properly.

2.5. Flux corrections

When calculating \(W_i\) and \(\Omega_{kl}\) in the first approximation we put \(\phi_i = \phi_0\). By doing this we are able to take the real form of the regions into account, which is of special importance for the transition region. With the approximate values of \(B_{kl}^F\) obtained by this first approach we can now find correction factors to \(W_i\) and \(\Omega_{kl}\) by comparing values of \(W_i\) and \(\Omega_{kl}\) calculated for circular cylinders, on one hand with \(\phi_i = \phi_0\), on the other with \(\phi_i\) determined by \(B_{kl}^F\). We notice that \(\phi_i\) is the one-group flux and continuous between regions. Any discontinuity in the current \(\nabla \phi\) is taken care of by the transition region.

When the test region is placed in an eccentric position there is an additional effect caused by perturbed fluxes which has not been considered above. If the eccentricity (\(r = r_0\)) is small, we can derive [4] a simplified correction factor \(E\), by which the measured buckling difference (\(B_{20} - B_{21}\)) has to be multiplied.

\[
E = 1 - 0.65 (B_{20}^2 - B_{21}^2) r_0^2 \tag{16}
\]

For instance if \(B_{20}^2 - B_{21}^2 = 2 \times 10^{-4} \text{ cm}^{-2}\) and \(r_0 = 20 \text{ cm}\), we get \(E = 1 - 0.052 = 0.948\).

2.6. Definition of \(D\)

The diffusion coefficients considered above are cell averages. Applying the conventional cell definition we may write the cell-average diffusion coefficient \(D\) as

\[
D = \frac{D_f V_f \phi_f + D_m V_m \phi_m}{V_f \phi_f + V_m \phi_m}, \tag{17}
\]

where the subscripts \(f\) and \(m\) refer to the fuel and the moderator respectively. However, the volume \(V_f\) includes, besides the fuel assembly, an annular region of the adjacent moderator. The thickness of this region should be of the order of one mean-free path [5]. The value of \(D_f\) is then also an
average quantity. From Eq.(17) we can easily see how $\delta D / \delta D_f$ varies with the cell size. Assuming $\phi_m / \phi_f$ to be independent of coolant we find the ratio $\delta D_\text{z} / \delta D_\text{r}$ to be equal to $\delta D_\text{z} / \delta D_f$ and consequently independent of the lattice pitch. Restricting the assumption to the case $\phi_m / \phi_f = 1$ we find that $\delta D$ is inversely proportional to the cell area. These relationships have been used as a guide when plotting the experimental data.

3. EXPERIMENTS

3.1. Experimental procedure

The experiments reported here are restricted to studies of void effects, but the measurements were performed both in a critical facility (RO) and in an exponential assembly (ZEBRA) [2]. To facilitate the change of coolant from air to D$_2$O the shroud tubes were equipped with pneumatic valves at the bottom.

The experimental procedure in RO, which has no graphite reflector, was the following:

(a) Determination of $\delta D_\text{r}$ from critical height measurements with a single test fuel assembly in various radial positions of a reference lattice. In each position two consecutive measurements were performed, one with air and the other with D$_2$O as coolant in order to suppress temperature effects. It is essential for good results to know the difference between the two critical heights accurately.

(b) Determination of $\delta D_\text{z}$ from the variation of critical height with depth of void column in a single fuel assembly. This fuel assembly was either alone in the reference lattice or surrounded by test fuel assemblies of the same kind in the adjacent lattice positions. The lattice pitch of the test region was most often different from the reference pitch. At each void depth the critical height measurement was always followed by a check with D$_2$O as coolant to minimize systematic errors in height differences.

(c) Determination of critical bucklings from progressive substitutions. The test regions were of rectangular form and contained 1, (2), 4, 6, 9, 12 etc. test fuel assemblies. The size of the test regions was increased gradually by adding one row of fuel assemblies to one side of the preceding rectangular (or square) region. This means that we used smaller and, in consequence, more numerous test regions than is the practice at other laboratories [6].

For the experiments in ZEBRA we used complete loadings and performed two series with progressive substitutions of the coolant, namely going from D$_2$O to air and vice versa. However, the measurements had to be restricted to small coolant-to-fuel ratios (<1), because it was impossible to get reliable results with large void channels. The transient effect from the bottom part was too dominating in the latter cases.

The experiments were evaluated by means of Eq.(6) or Eq.(15). If $B_\text{H} = B_\text{F}$, i.e. if the buckling values are related to the same extrapolated radius, Eq.(6) can be rearranged in the following form
\[ (B_{z1}^2 - B_{z0}^2) \left[ \sum_i D_{zl} W_i + \sum_i (D_{zl} - D_{z1}) \Omega_{zl} \right] \]

\[ = \sum_i (B_{zl}^2 - B_{z1}^2) D_{zl} W_i - \sum_i \sum_k B_{kl}^2 (D_{kl} - D_{k1}) \Omega_{k1}. \]  

\[ \text{Eq. (18)} \]

\[ B_{z1}^2 \] is the axial buckling of the reference region filling the whole tank. 
\[ B_{z0}^2 - B_{z1}^2 \] is the buckling difference measured when a perturbation is introduced into the reference core. With a single fuel assembly there are only two regions (\( i = 1 \) and \( 2 \)). Since \( \Sigma W_i = 1 \), Eq. (18) is then equivalent to

\[ \left( B_{z1}^2 - B_{z0}^2 \right) \left[ 1 + \frac{D_{z2} - D_{z1}}{D_{z1}} (W_2 + \Omega_{z2}) \right] = \left( B_{z2}^2 - B_{z1}^2 \right) \frac{D_{z2}}{D_{z1}} W_2 \]

\[ - B_{z1}^2 \frac{D_{z2} - D_{z1}}{D_{z1}} \Omega_{z2} - B_{z1}^2 \frac{D_{z2} - D_{z1}}{D_{z1}} \Omega_{z2}. \]  

\[ \text{Eq. (19)} \]

When the perturbation is varied only axially (radially) the function \( \Omega_{z2}(\Omega_{z2}) \) is constant and can be made quite small by suitable experimental arrangements.

A single test fuel assembly surrounded by reference fuel corresponds to transition cells according to the new cell concept (see section 2.4). Since the change of diffusion coefficients takes place within a small and rather well-defined volume, we can use the conventional cell definition when analysing diffusion coefficient experiments with single fuel assemblies. The buckling values obtained in this connection are only used in the flux corrections, which generally are small. However, when an inner fuel assembly of a test region of 9 or 12 was used in the diffusion coefficient experiments, the flux corrections were by no means negligible, being of the order of 10 to 30\% in the actual cases.

3.2. Critical measurements

Measurements on metal rods, diameter 3.05 cm, canned in Al (ID 3.15 cm; OD, 3.45 cm) and placed in Al shrouds (ID, 6.3 cm; OD, 6.5 cm) have been reported elsewhere [7]. The substitutions were limited to the coolant and, since the whole tank was filled with the same kind of fuel, complete substitutions in both directions were possible. The results are collected in Table I.

The void effect in 27-rod clusters of UO\(_2\), diameter 1.35 cm with three different internal spacings, has been investigated in various lattices. The changes of the one-group diffusion coefficients evaluated are found in Table II, which also gives more data about the clusters. Figure 1 shows the ratio of \( \delta D_z/\delta D_r \) for one type of cluster versus lattice pitch. In Fig. 2 the \( \delta D_z/D \) ratios are plotted versus the inverse of the cell area. The lines drawn are just linear fits to the experimental points (compare with section 2.6).
TABLE I
SUBSTITUTION MEASUREMENTS WITH URANIUM*

<table>
<thead>
<tr>
<th>Measured quantity</th>
<th>Numerical results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_z$(air) $D_2(D_2O)$</td>
<td>1.120 ± 0.018</td>
</tr>
<tr>
<td>$D_r$(air) $D_1(D_2O)$</td>
<td>1.083 ± 0.004</td>
</tr>
<tr>
<td>$B^4(D_2O) - B^4$(air)</td>
<td>0.250 ± 0.028 m$^{-2}$</td>
</tr>
<tr>
<td>Substitution of a single element</td>
<td></td>
</tr>
<tr>
<td>Substitution up to 16 elements</td>
<td>0.282 ± 0.015 m$^{-2}$</td>
</tr>
<tr>
<td>Complete loadings</td>
<td>0.274 ± 0.005 m$^{-2}$</td>
</tr>
</tbody>
</table>

* U, diam. 3.05 cm, canned in Al(ID, 3.15 cm; OD, 3.45 cm) and placed in Al shrouds (ID, 6.3 cm; OD, 6.5 cm). Square lattice pitch: 19.0 cm. Radial buckling: 4.37 m$^{-2}$. (See Ref. [7] for more information.)

The ratio $\delta D_z/\delta D_r$ due to void in a 27-rod cluster (c-c: 2.4 cm) versus lattice pitch (Compare Table II)

Clusters of 12 rods of UO$_2$, diameter 1.35 cm, have also been studied and the results are given in Table III. For comparison with exponential measurements (see below) the $\delta D_r/D$ value is plotted in Fig. 3.
# TABLE II

## DETERMINATION OF CHANGES OF DIFFUSION COEFFICIENTS

<table>
<thead>
<tr>
<th>Centre-to-centre spacings (cm)</th>
<th>$V_{\text{coolant}}$</th>
<th>$V_{\text{fuel}}$</th>
<th>Al shroud ID (cm)</th>
<th>OD (cm)</th>
<th>Coolant annulus (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8</td>
<td>0.97</td>
<td>-</td>
<td>11.27</td>
<td>11.5</td>
<td>0.8</td>
</tr>
<tr>
<td>2.4</td>
<td>2.50</td>
<td>-</td>
<td>14.4</td>
<td>14.7</td>
<td>0.8</td>
</tr>
<tr>
<td>3.0</td>
<td>4.47</td>
<td>-</td>
<td>17.0</td>
<td>17.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lattice arrangement</th>
<th>Square lattice pitch in test region (cm)</th>
<th>[\frac{D_z(\text{air}) - D_z(D_2O)}{D_z\text{ref}}] (D_z\text{ test})</th>
<th>[\frac{D_T(\text{air}) - D_T(D_2O)}{D_T\text{ref}}] (D_T\text{ test})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single test fuel assembly in reference lattice</td>
<td>c-c: 1.8</td>
<td>0.289 ± 0.009</td>
<td>0.225 ± 0.041</td>
</tr>
<tr>
<td></td>
<td>c-c: 2.4</td>
<td>0.756 ± 0.012</td>
<td>0.656 ± 0.012</td>
</tr>
<tr>
<td></td>
<td>c-c: 3.0</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Test region of 9 or 12 fuel assemblies</td>
<td>c-c: 1.8</td>
<td>0.160 ± 0.015</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>c-c: 2.4</td>
<td>—</td>
<td>0.412 ± 0.020</td>
</tr>
<tr>
<td></td>
<td>c-c: 3.0</td>
<td>—</td>
<td>0.360 ± 0.025</td>
</tr>
<tr>
<td></td>
<td>c-c: 3.0</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>15.0</td>
<td>0.160 ± 0.015</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>17.0</td>
<td>0.412 ± 0.020</td>
<td>0.755 ± 0.040</td>
</tr>
<tr>
<td></td>
<td>19.0</td>
<td>0.360 ± 0.025</td>
<td>0.675 ± 0.040</td>
</tr>
</tbody>
</table>

*Test fuel: 27-rod clusters of $\text{UO}_2$, diam. 1.35 cm, canned in Al(ID, 1.37 cm; OD, 1.52 cm or 1.57 cm). Reference fuel: $\text{U}$, diam. 3.05 cm, canned in Al(ID, 3.15 cm; OD, 3.45 cm).

The perturbed one-group fluxes of the large test regions have been taken into account but no two-group corrections have been applied. See also Figs. 1 and 2, where the ratio $\delta D_2/\delta D_T$ corresponds to $[D_z(\text{air}) - D_z(D_2O)]/D_T(\text{air}) - D_T(D_2O)$. The perturbed ratio $\delta D_2/D$ to $[D_z(\text{air}) - D_z(D_2O)]_{\text{test}}/D_z\text{ ref}$. 

\[D_z(\text{air}) - D_z(D_2O)\]
The relative change of $D_z$ due to void in lattices of 27-rod clusters versus the inverse of the cell area
(Compare Table II)

Comparison between values of $\delta D_z/D$ due to void as determined from exponential and critical experiments with 12-rod clusters
(Compare Table III)
### Table III

**DETERMINATION OF CHANGES OF DIFFUSION COEFFICIENTS**

<table>
<thead>
<tr>
<th>Kind of measurement</th>
<th>Square lattice pitch (cm)</th>
<th>No. of fuel assemblies in test</th>
<th>Change of axial buckling ($m^{-2}$)</th>
<th>$\frac{\delta D}{D}$</th>
<th>$\frac{\delta D^*}{D}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical substitutions</td>
<td>19.0</td>
<td>1</td>
<td>$&lt;0.0250 \pm 0.0005$</td>
<td>0.220</td>
<td>0.148</td>
</tr>
<tr>
<td>Exponential substitutions</td>
<td>17.0</td>
<td>4 (air)</td>
<td>1.53 ± 0.05</td>
<td>0.25</td>
<td>0.190</td>
</tr>
<tr>
<td></td>
<td></td>
<td>12 (air)</td>
<td>2.22 ± 0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>24 (air)</td>
<td>1.33 ± 0.006***</td>
<td>0.25</td>
<td>0.190</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 ($D_2O$)</td>
<td>-1.40 ± 0.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>12 ($D_2O$)</td>
<td>-2.39 ± 0.06</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Test fuel: 12-rod clusters of $UO_2$, diam. 1.35 cm, canned in $Al$ (ID, 1.37 cm; OD, 1.52 cm) Centre-to-centre distance: 1.8 cm.

Shroud tube: $Al$, OD 9.8 cm, OD 9.0 cm.

Reference fuel: $U$, diam. 3.05 cm, canned in $Al$ (ID, 3.15 cm; OD, 3.45 cm), only in the critical experiments.

** Compare Fig. 3

*** The difference between the total bucklings of the two cases is $1.33 - 0.81 = 0.52 m^{-2}$, taking the radial buckling into account.

---

### 3.3. Exponential measurements

Because of streaming effects this investigation had to be limited to a fuel assembly with a small coolant-to-fuel ratio. Twelve-rod clusters of the same type as mentioned in section 3.2 above were studied. The total number of clusters in the tank (diam., 100 cm) was 24 and the coolant was substituted in two steps with 4 and 12 fuel assemblies respectively. The results are given in Table III and Fig. 3.

The radial buckling of the system was assumed to be 21.00 m$^{-2}$ with $D_2O$ as coolant. The introduction of air as coolant gave a change of $D_r$ which is closely related to the effective extrapolation length 1. We find that $\frac{\delta D_r}{D_r} = 61/1$. The extrapolated radius in the $D_2O$ case was 52.5 cm and the mean radius of the core 47.0 cm. The effective extrapolation length including a thin $D_2O$ reflector was then $l = 5.5$ cm. The change of the radial buckling is obtained as

$$\delta B_r^2 = -21.00 \times 2 \times \left(1 - \frac{3}{2} x + \ldots\right),$$

where $x = (5.5/52.5)(\delta D_r/D_r)$. By means of Eq. (6) we find another relationship between $\delta B_r^2$ and $\delta D_r/D_r$. By correlation we get $\delta D_r/D_r = 0.190 \pm 0.009$ and $\delta B_r^2 = -0.81 \pm 0.10 m^{-2}$, assuming $l = 5.5 \pm 0.5$ cm. The value of $\delta D_r/D$ is related to the sum of two approximately equal quantities (1.53 and 1.40 or...
2.22 and 2.39 in Table III) while $\delta D_z/D$ results from the difference between the same quantities. The error limits of $\delta D_z/D$ are consequently much larger.

4. CONCLUSIONS

The formulae given relate in an explicit way the changes of the measured quantities, i.e. axial bucklings, to the values of bucklings and diffusion coefficients of the various regions. Though the weight functions depend on the perturbed quantities, the dependence is not strong and adjustments can be applied iteratively. According to Eq.(3), i.e. the definition of $W_i$, one has to use the product of the unperturbed flux $\phi_0$ and the actual one-group flux $\phi_i$. Streaming effects related to the function $\Omega_{kl}$ of Eq.(5) are found to be independent of the gradient of the actual flux. When calculating $\Omega_{kl}$ one has to use the perturbed flux multiplied by the gradient of the unperturbed flux, i.e. $\phi_i(V\phi_0)$.

The one-group flux $\phi_i$ is determined by the buckling of each region. However, the continuity conditions between the test and reference regions are relaxed and only $\phi_i$ is assumed to be continuous, while any discontinuity of $D_l \nabla \phi_l$ is taken care of by introducing a transition region.

The non-separability of flux in axial void measurements may in the case of large void-to-fuel ratios give rise to some systematic error. The problem has not yet been studied thoroughly.

The new definition of cells (see section 2.4) takes the coupling effect between different kind of fuel assemblies into account and thus it brings the theory of homogeneous media closer to the heterogeneous method.

When applying the formulae to experiments one is faced with another problem. The two-group correction term $G$ is determined by factors related to two different regions. According to the new cell definition we get a transition region between the test and reference regions. One is then forced to define some average properties of cells consisting of two kinds of fuel. This problem can be serious in diffusion coefficient experiments with a single test fuel assembly, though the uncertainty of $1 + G$ is probably less than 5% since $G$ is smaller than 0.1 in practical cases. In a large test region $G$ should be well known but in diffusion coefficient studies necessary flux corrections may be considerable, giving rise to still larger uncertainties. The exponential measurements on coolant substitutions were found capable of giving $\delta D_t/D$ values with rather good accuracy and only a small two-group correction. However, streaming effects limited this method to small coolant-to-fuel ratios.

The ratio of $\delta D_z/\delta D_t$ is practically independent of lattice pitch and of spectral mismatch and it can therefore be used most directly in a comparison with theoretical expressions. However, since a computer programme fitted to cluster geometries is still in a stage of preparation at this laboratory [8] no comparison with theoretical values has been made.

The results with 27-rod clusters and large coolant-to-fuel ratios show a spread which cannot be explained by imperfectness of the method of analysis. The main reason is certainly non-uniformity and irregularities of the fuel assemblies. Spacers were only placed at the top and bottom parts
of the clusters. Similar experiences have been made at the Savannah River Laboratory [9].

The buckling analysis of the progressive substitutions is thought to be self-explanatory and well established, so it has not been treated in detail here. A graphical form of the analysis is rather completely explained in Refs. [2] and [7].

ACKNOWLEDGEMENT

The author is much indebted to all his colleagues at the company who have taken part in stimulating discussions and criticism, and to all those who have devoted their time to the tedious experiments and numerical work.

REFERENCES

[1] PERSSON, R., Perturbation analysis applied to substitution measurements of buckling, to be published.

DISCUSSION

R. MEIER: You have made two different types of measurement of the change in the radial diffusion coefficient due to the voiding of a lattice. With the first method, a voided element was moved from the centre to the boundary of the reactor in a critical facility. With the second, a successive substitution technique was used in a subcritical facility. Measurements of void effects performed in the NORA reactor have shown that there are interactions between neighbouring voided channels. Under certain circumstances, therefore, your two methods should not give identical results. I am thinking particularly of the case of low moderator-to-fuel-volume ratio lattices.

R. PERSSON: I disagree. In the case of the interaction effect shown by the measurements performed in the NORA reactor, it was the total change of reactivity which was affected. By separating the $\phi^2$-dependent and the $(\nabla\phi)^2$-dependent effects, we would probably find that the change of the diffusion coefficient caused by void is not affected by neighbouring channels, if these are at least two or three transport mean free paths away. However, the multiplying properties (related to the $\phi^2$-dependent function) of a cell are more strongly coupled to neighbouring cells. I think that our two methods of measuring changes of radial diffusion coefficients should there-
fore give identical results. The values shown in Fig. 3 of the paper give
strong support to this opinion.

R. MEIER: Measurements are more precise when you void the whole
system successively than when you merely move one element.

R. PERSSON: We have made such measurements with reference fuel,
as you are certainly aware. However, since the large fuel assemblies with
which the paper deals cannot be permitted to become critical, I cannot
comment on the point you raise.

G.B. ZORZOLI: What is the nature of the uncertainty in your $\Delta B^2$
results?

R. PERSSON: The errors in the quoted $\Delta B^2$ values (which were ob-
tained by the substitution technique (see Table I)) are statistical errors caused
by uncertainty as to the water heights. The errors in the $\Delta B^2$ values meas-
ured directly are mainly from uncertainties regarding the differences be-
tween extrapolated radii.

| TABLE I |
| EXPERIMENTS WITH METAL RODS, 30.5 cm IN DIAMETER |

<table>
<thead>
<tr>
<th>Shroud diameter (cm)</th>
<th>Coolant</th>
<th>Pitch (cm)</th>
<th>$\Delta B^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref.</td>
<td>--</td>
<td>15</td>
<td>$\Delta B^2$</td>
</tr>
<tr>
<td>Test</td>
<td>--</td>
<td>15/2</td>
<td>$\Delta B^2$</td>
</tr>
<tr>
<td>Ref.</td>
<td>--</td>
<td>21</td>
<td>$\Delta B^2$</td>
</tr>
<tr>
<td>Test</td>
<td>--</td>
<td>21/2</td>
<td>$\Delta B^2$</td>
</tr>
<tr>
<td>Ref.</td>
<td>Al 6.3/6.5</td>
<td>$D_p$O</td>
<td>$\Delta B^2$</td>
</tr>
<tr>
<td>Test</td>
<td>Al 6.3/6.5</td>
<td>air</td>
<td>$\Delta B^2$</td>
</tr>
<tr>
<td>Ref.</td>
<td>Al 6.3/6.5</td>
<td>air</td>
<td>$\Delta B^2$</td>
</tr>
<tr>
<td>Test</td>
<td>Al 6.3/6.5</td>
<td>$D_p$O</td>
<td>$\Delta B^2$</td>
</tr>
</tbody>
</table>

G.B. ZORZOLI: In section 2.6 of your paper, the cell diffusion coef-
ficient is defined in a rather simplified formula. Would a more sophisti-
cated formula, that of Benoist for example, enable you to make a more satis-
factory interpretation of the experimental results?

R. PERSSON: Equation (17) of the paper shows the diffusion coefficient
averaged over a cell. You will note that the volume $V_f$ also includes an
annular region of the adjacent moderator, in accordance with the recommend-
dations in the paper by Benoist listed as Ref. [5] at the end of the paper.
The purpose of Eq. (17) is to show how D varies with cell size for different
simplifying assumptions. The relationships obtained are merely intended
to serve as guides in plotting the experimental data. It would be of great
help to take the flux distribution into account, but I do not think that a more
sophisticated formula is necessary. Mention should also be made of similar experiments at the Savannah River Laboratory reported in DP-832 of June 1963*, where various methods of analysis are compared.

M. SAGOT: Are the examples of controls given for your four-element substitution method valid for fuel placed in air channels?

R. PERSSON: The method has been checked with metal rods (diam. 3.05 cm) placed in aluminium tubes (inside diam. 8.8 cm, outside diam. 9.0 cm). The agreement with the full-scale result was very good, as may be seen in Ref.[7]. Measurements both in the exponential and in the critical assembly on 12-rod clusters including the four-element substitution gave consistent results, as shown in Fig. 3 of the paper. I therefore conclude that the substituted region of four elements is sufficiently large.

M. SAGOT: What precision can be expected of this method when the reference lattice and the tested lattice have air channels of different dimensions, i.e. when the two lattices have different anisotropies (e.g. in the range of 6-15%).

R. PERSSON: You mention only the degree of anisotropy, but the quantities $\delta D_z/D$ and $\delta D_r/D$ are more important than the ratio $D_z/D_r$. Different diffusion coefficients between different parts are taken care of by the method of analysis, independent of the degree of anisotropy. The precision that one may expect depends, of course, on the magnitude of the various diffusion coefficients, but also on the sign and the size of the buckling difference. As a rough rule, one may say that $\delta (D_z/D) = \pm 0.01$ corresponds to $\delta (\Delta B) = \pm 0.01 \times B^2 \text{m}^{-2}$ and $\delta (D_r/D) = \pm 0.01$ gives $\delta (\Delta B) = \pm 0.01 \times \Delta B \text{m}^{-2}$. In this connection, I would like to refer again to the measurements made at the Savannah River Laboratory (see Rep. DP-832), where 19-rod and 31-rod clusters have been studied in 4-in and 5-in housing tubes. You may also find information there which has a bearing on your first question.

MEASUREMENT OF THE ANISOTROPY OF DIFFUSION CONSTANT IN MEDIA WITH EMPTY CHANNELS

M. ČOPIČ, T. KALIN, G. PREGL AND F. ŽERDIN
NUCLEAR INSTITUTE JOZEF STEFAN, LJUBLJANA, YUGOSLAVIA

Abstract — Résumé — Аннотация — Resumen

MEASUREMENT OF THE ANISOTROPY OF DIFFUSION CONSTANT IN MEDIA WITH EMPTY CHANNELS. Using the pulsed-neutron source technique, the diffusion constant was measured in systems with empty channels. Plexiglas was used as the neutron diffusing material. From separate sets of measurements on rectangular blocks the diffusion constants parallel and perpendicular to channels were determined. The average value of the diffusion constant was also obtained experimentally from measurements on cubes. The difference between both diffusion constants, \( D_p - D_L \), agrees with theoretical predictions inside the limits of experimental errors, yet the average diffusion constant lies systematically below the predictions of Behrens' theory.

MESURE DE L'ANISOTROPIE DE LA CONSTANTE DE DIFFUSION DANS DES MILIEUX A CANAUX VIDÉS. En se servant de la méthode de la source des neutrons pulsés, les auteurs ont mesuré la constante de diffusion dans des systèmes à canaux vides. Comme matériau diffusant les neutrons, ils ont utilisé du plexiglas. A partir de séries de mesures différentes faites sur des blocs rectangulaires, ils ont déterminé les constantes de diffusion parallèle et perpendiculaire aux canaux. Ils ont également obtenu expérimentalement la valeur moyenne de la constante de diffusion à l'aide de mesures faites sur des cubes. La différence entre les deux constantes de diffusion, \( D_p - D_L \), concorde avec les prévisions théoriques dans les limites des erreurs d'expérience; cependant, la valeur moyenne de la constante de diffusion reste systématiquement inférieure aux prévisions établies par la théorie de Behrens.

ИЗМЕРЕНИЕ АНИЗОТРОПИИ ПОСТОЯННОЙ ДИФФУЗИИ В СРЕДЕ С ПУСТЫМИ КАНАЛАМИ. С помощью источника импульсных нейтронов в системах с пустыми каналами измерялась постоянная диффузии. Органическое стекло использовалось в качестве материала для рассеивания нейтронов. В результате проведения раздельных серий испытаний на прямоугольных блоках определялись постоянные диффузии в направлениях параллельно и перпендикулярно каналам. Среднее значение постоянной диффузии было получено также опытом путем в результате измерений на кубических блоках. Различие между этими двумя постоянными диффузии \( D_p - D_L \) согласуется с теоретическими предположениями в пределах экспериментальных ошибок, тем не менее средняя постоянная диффузии систематически лежит ниже предположений теории Беренса.

MEDICIÓN DE LA ANISOTROPIA DE LA CONSTANTE DE DIFUSIÓN EN VARIOS SISTEMAS CON CANALES VACÍOS. Utilizando la técnica de la fuente neutónica pulsada, los autores midieron la constante de difusión en varios sistemas con canales vacíos. Como material difusor de neutrones emplearon polimetacrilato de metilo. Basándose en varias series de mediciones efectuadas en bloques rectangulares, los autores determinaron las constantes de difusión en sentido paralelo y perpendicular a los canales. Obtuvieron experimentalmente el valor medio de la constante de difusión a partir de mediciones efectuadas en cubos. La diferencia entre ambas constantes de difusión, \( D_p \) y \( D_L \), concuerda con los valores predichos teóricamente dentro de los límites de error experimental; sin embargo, la constante media de difusión está sistemáticamente inferior a los valores calculados según la teoría de Behrens.

INTRODUCTION

The applicability of the diffusion theory depends upon the consistency of essentially two constants, the absorption cross-section and the diffusion constant. They describe the rate of absorption and the leakage rate of neutrons in the system. Due to its simplicity we apply the diffusion theory
even in systems where one should be rather suspicious about its correctness, say in systems penetrated by empty channels. Yet, if we are able to find the proper values for the diffusion constants and for the average absorption cross-section, so that the neutron balance is correctly described by them, the validity of the diffusion approximation can be extended also to such systems. A great deal of effort was spent to calculate the diffusion constants parallel and perpendicular to channels. The theory of BEHRENS [1] is based on the assumption that the diffusion area associated with a given direction in a system with voids is proportional to the mean square path in that direction per scatter and to the average number of scattering events during a neutron lifetime. For the case of the cylindrical channels widely separated, so that for a »λ where a is the lattice pitch and λ the mean free path, Behrens finds that

\[ \frac{L_x}{L_m} = 1 + 2\phi(1 + \frac{R}{\lambda}) \] (1)

and

\[ \frac{L_y}{L_m} = 1 + 2\phi(1 + \frac{R}{2\lambda}), \] (2)

with \( \phi \) equal to the ratio of the volume of channels to the volume of material and \( R \) the radius of the channel. Assuming that the average absorption cross-section is just \((1 + \phi)\) times smaller than the absorption in solid material, one obtains for the diffusion constants

\[ \frac{D_x}{D_m} = 1 + \left( \frac{V_c}{V} \right)(1 + 2\frac{R}{\lambda}) \] (3)

and

\[ \frac{D_y}{D_m} = 1 + \left( \frac{V_c}{V} \right)(1 + \frac{R}{\lambda}), \] (4)

where \( V_c \) is the volume of channel and \( V \) the volume of the unit cell. Using the diffusion theory, GRANT [2] proposes a semi-empirical formula for \( D_x \)

\[ \frac{D_x}{D_m} = 1 + \phi \left[ 1 + \left( \frac{R - \frac{1}{2}a V_c}{V} \right) \right] \frac{1}{(R + \lambda)}. \] (5)

LALETIN [3] calculated the leakage out of a cell of the height \( H \) with the central cylindrical channel and finds for a sinusoidal flux variation along the channel

\[ \frac{D_n}{D_m} = 1 + \left( \frac{V_c}{V} \right)[1 + 2\frac{R}{\lambda} - C(\frac{R}{\lambda})(\frac{R}{H})], \] (6)

where higher terms in \( \frac{R}{\lambda} \) and \( \frac{R}{H} \) have been neglected. The constant \( C \) has the value 7.225 according to his results and \( 3\pi^2/4 \) according to PERSHAGEN [4]. BENOIST [5] calculates the diffusion constants from the transport theory and obtains as the first approximation the same results as Behrens. Yet, he gives also three corrections to the results of Behrens. The first one results from the consecutive passages of a neutron through the same channel. The second one is from interactions among different channels.
The third one is applied to systems of finite dimensions with positive $B^2$ and is of the same form as that obtained by Laletin.

There are few experimental results reported in the literature. In almost all cases they are obtained by static measurements of the neutron flux relaxation parallel and perpendicular to the channels in standard exponential stacks. In general, one can conclude that Eq. (1) describes quite well the parallel diffusion area in contrast to the perpendicular one which is systematically below the theoretical prediction of Behrens, Eq. (2). Similar conclusions were made by Schaefer and Parkyn [6] from the results of Monte Carlo calculations.

Because there were no pulsed neutron source experiments on such systems reported in the literature and to obtain the experimental results on simple systems which would be directly comparable to the theoretical results the present study was undertaken. The pulsed neutron source technique has, besides, the advantage that the flux variation is sinusoidal and thus nearer to the real situation in reactor systems. The complete report of the experiments will be published elsewhere [7].

Due to still relatively large errors in our experiments some doubt appeared regarding the physical meaning and the cause of the observed differences between experiments and theory. Furthermore, the reliability of the pulsed neutron experiments on small systems is in question as a result of difficulties in extrapolating the results to zero buckling. Therefore, Monte Carlo calculations were performed, trying to simulate as near as possible the pulsed neutron source measurements. The Monte Carlo calculations are still not complete, yet enough information is available to warrant the first comparison with experimental results, and perhaps with theory. To make the comparison possible, the experiments are first briefly described and then the available results of our Monte Carlo calculations are reviewed.

EXPERIMENTS

Using the pulsed neutron source technique one has to measure the time constant of the decay of neutron density in the lowest mode. For a system with a regular array of empty cylindrical channels in the $z$-direction we assumed the following dependence of the time constant upon the geometrical bucklings in $x$-, $y$- and $z$-directions,

$$\alpha = \nu \Sigma_a + D_x B^2_x + B^2_y + D_n B^2_z - C_x (B^4_x + B^4_y) - C_n B^4_z - \ldots$$

(7)

Because the extrapolation distances are influenced by the presence of channels they have to be determined by an iterative procedure.

The basic idea is that one can determine $D_\lambda$ from the initial slope if $\alpha$ is plotted versus $B^2_x$ and $B^2_y$ for constant values of $B^2_z$ and that $D_n$ is determined from measurements on the rectangular blocks with different heights $c$ keeping constant the sides $a$ and $b$ (Fig. 1).

To make the change of bucklings in different directions easy enough, we built our systems from unit blocks. The unit block is a cube with the side $a_0$ and with a cylindrical hole drilled through its axis. Plexiglas was
Decay constants for rectangular blocks of Plexiglas with empty channels as functions of the reduced geometric bucklings, where $B_{\text{im}}^2$ is equal to the corresponding values of $(B_1^2 + B_2^2)$ for systems 33Z and 44Z, or to $B_2^2$ for systems XY3 and XY4, respectively.

Selected as the diffusing medium for it is easy to manipulate and because its diffusion parameters are similar to the parameters of ordinary water.

Altogether, 125 unit blocks of the size $a_0 = 2.90$ cm and 125 unit blocks of the size $a_0 = 3.60$ cm have been used with the radius of the cylindrical hole equal to $R = 0.85$ cm in both cases. To identify the measured blocks the notation XYZ is used, where X, Y and Z are the numbers of unit blocks along the x, y and z axes, respectively.

Four sets of experiments were performed. First, the solid blocks were measured to determine the diffusion parameters of Plexiglas. For this purpose the holes in unit blocks with $a_0 = 2.90$ cm were filled with Plexiglas rods (Table I). In the second run we measured XXX systems, in the third run, $D_1$ and $D_2$ on XY3, XY4, 33Z and 44Z systems, using the same small-size unit blocks. The fourth set of measurements was performed on cubes built from larger unit blocks with $a_0 = 3.60$ cm.

A 31-MeV betatron was used as a pulsed source of collimated gamma rays (Fig. 2). Neutrons with an average energy around 1 MeV were produced by ($\gamma$, n) reaction in a copper target. Approximately $2 \times 10^5$ neutrons were produced in each burst, with the repetition rate of 50 bursts per second. The length of each neutron burst was about 4 $\mu$s.

The relatively large fast neutron background from the betatron was eliminated by 30 to 60 cm of borated paraffin shield around the enclosure in which the neutron source and the diffusing system were placed. The walls of the paraffin shield and of the diffusing system were lined with 1-mm Cd sheets.

Neutrons emerging from the diffusing system were normally detected by two BF$_3$ counters. Pulses from both counters were amplified and led to
# ANISOTROPY OF DIFFUSION CONSTANT

## TABLE I

### TIME CONSTANTS OF PLEXIGLAS BLOCKS

<table>
<thead>
<tr>
<th>System</th>
<th>Buckling</th>
<th>Time constant</th>
<th>Estimated error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(B_3^2 + B_2^2)$</td>
<td>$B_2^2$</td>
<td>$a \times 10^{-3}$</td>
</tr>
<tr>
<td>XYZ</td>
<td>(cm$^{-2}$)</td>
<td>(cm$^{-2}$)</td>
<td>(s$^{-1}$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$a_x = 2.90$ cm</td>
<td>$R = 0.85$ cm</td>
<td>$a_x = 3.60$ cm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>309</td>
</tr>
<tr>
<td>222</td>
<td>0.433</td>
<td>0.187</td>
<td>36.60</td>
</tr>
<tr>
<td>223</td>
<td>0.433</td>
<td>0.096</td>
<td>31.50</td>
</tr>
<tr>
<td>233</td>
<td>0.392</td>
<td>0.096</td>
<td>26.30</td>
</tr>
<tr>
<td>322</td>
<td>0.212</td>
<td>0.187</td>
<td>26.80</td>
</tr>
<tr>
<td>442</td>
<td>0.125</td>
<td>0.187</td>
<td>23.30</td>
</tr>
<tr>
<td>244</td>
<td>0.279</td>
<td>0.058</td>
<td>21.40</td>
</tr>
<tr>
<td>333</td>
<td>0.212</td>
<td>0.086</td>
<td>21.03</td>
</tr>
<tr>
<td>394</td>
<td>0.212</td>
<td>0.058</td>
<td>18.19</td>
</tr>
<tr>
<td>453</td>
<td>0.168</td>
<td>0.096</td>
<td>19.00</td>
</tr>
<tr>
<td>553</td>
<td>0.147</td>
<td>0.096</td>
<td>18.00</td>
</tr>
<tr>
<td>355</td>
<td>0.212</td>
<td>0.039</td>
<td>16.80</td>
</tr>
<tr>
<td>336</td>
<td>0.212</td>
<td>0.028</td>
<td>16.25</td>
</tr>
<tr>
<td>633</td>
<td>0.135</td>
<td>0.096</td>
<td>16.90</td>
</tr>
<tr>
<td>443</td>
<td>0.125</td>
<td>0.096</td>
<td>16.88</td>
</tr>
<tr>
<td>553</td>
<td>0.083</td>
<td>0.096</td>
<td>14.60</td>
</tr>
<tr>
<td>444</td>
<td>0.125</td>
<td>0.058</td>
<td>14.05</td>
</tr>
<tr>
<td>445</td>
<td>0.125</td>
<td>0.039</td>
<td>12.74</td>
</tr>
<tr>
<td>544</td>
<td>0.104</td>
<td>0.058</td>
<td>13.20</td>
</tr>
<tr>
<td>446</td>
<td>0.125</td>
<td>0.028</td>
<td>12.00</td>
</tr>
<tr>
<td>554</td>
<td>0.083</td>
<td>0.058</td>
<td>12.05</td>
</tr>
<tr>
<td>555</td>
<td>0.083</td>
<td>0.039</td>
<td>10.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>36.60</td>
</tr>
<tr>
<td>222</td>
<td>0.304</td>
<td>0.152</td>
<td>24.20</td>
</tr>
<tr>
<td>333</td>
<td>0.144</td>
<td>0.072</td>
<td>13.70</td>
</tr>
<tr>
<td>444</td>
<td>0.084</td>
<td>0.042</td>
<td>9.60</td>
</tr>
<tr>
<td>555</td>
<td>0.056</td>
<td>0.028</td>
<td>7.40</td>
</tr>
</tbody>
</table>

### Solid blocks

<table>
<thead>
<tr>
<th>System</th>
<th>Buckling</th>
<th>Time constant</th>
<th>Estimated error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(B_3^2 + B_2^2)$</td>
<td>$B_2^2$</td>
<td>$a \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>(cm$^{-2}$)</td>
<td>(cm$^{-2}$)</td>
<td>(s$^{-1}$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>222</td>
<td>0.478</td>
<td>0.239</td>
<td>28.80</td>
</tr>
<tr>
<td>223</td>
<td>0.478</td>
<td>0.113</td>
<td>24.30</td>
</tr>
<tr>
<td>332</td>
<td>0.227</td>
<td>0.239</td>
<td>20.20</td>
</tr>
<tr>
<td>333</td>
<td>0.227</td>
<td>0.113</td>
<td>16.25</td>
</tr>
<tr>
<td>334</td>
<td>0.227</td>
<td>0.066</td>
<td>14.50</td>
</tr>
<tr>
<td>443</td>
<td>0.132</td>
<td>0.113</td>
<td>12.60</td>
</tr>
<tr>
<td>444</td>
<td>0.132</td>
<td>0.066</td>
<td>11.20</td>
</tr>
<tr>
<td>445</td>
<td>0.132</td>
<td>0.043</td>
<td>10.50</td>
</tr>
<tr>
<td>446</td>
<td>0.132</td>
<td>0.030</td>
<td>9.80</td>
</tr>
</tbody>
</table>
two input channels of the multichannel analyser*. The channel width of 10 μs was used. For the time analysis of the neutron decay in the smallest system, 222, only one detector was used and the channel width adjusted to 4 μs.

Because of the fixed repetition rate of the betatron we had to use the orbit expansion pulse of the betatron to start the analysing cycle of the multichannel analyser.

The two sets of data from the time analyser, i.e. from odd and from even channels, were separately fitted to a simple exponential function using the least-square method. In this way the decay constants and their standard deviations were obtained. To eliminate the initial transition period, possible contribution of the higher modes, and the possible spurious background, the least-square fit was repeated on each set of data up to ten times, omitting successively initial points one at a time. In this way we introduced delays up to 400 μs after the end of the neutron burst.

The results for solid blocks of Plexiglas are presented in Table II together with the results of SEEMANN [8] for comparison. His results were recalculated from the reported values of the decay constants to obtain the information on the standard deviations. The differences between both sets

* We used TMC 256-channel analyser with 212 and 213 plug-in units. The standard 212 plug-in unit has been modified so that it can accept pulses from two counters, storing analysed counts from the first input in odd channels and the counts from the second input in even channels.
ANISOTROPY OF DIFFUSION CONSTANT

TABLE II

NEUTRON DIFFUSION PARAMETERS FOR POLYMETHYL METHACRYLATE (Plexiglas)

<table>
<thead>
<tr>
<th></th>
<th>$v E_a$ (eV)</th>
<th>D (cm$^2$ s$^{-1}$)</th>
<th>C (cm$^4$ s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present work</td>
<td>4015 ± 350</td>
<td>36500 ± 2000</td>
<td>2880 ± 2200</td>
</tr>
<tr>
<td>Seemann</td>
<td>4300 ± 100</td>
<td>34000 ± 1100</td>
<td>-640 ± 2200</td>
</tr>
</tbody>
</table>

of the diffusion parameters are mainly caused by the essentially linear fit on the data of Seemann. From his data one obtains a slightly positive contribution of the diffusion cooling effect.

The transport mean free path obtained from our results is equal to 0.44 cm and thus, the extrapolation distance should be around 0.32 cm if the usual expression for the extrapolation distance is used. Yet, according to NELKIN [9] and KLADNIK [10] the extrapolation distance for a group of thermal neutrons in hydrogenous material, like Plexiglas, which is very similar to water, should be a bit higher, around 0.34 cm. Thus instead of 0.71 $\lambda_t$, we used throughout 0.76 $\lambda_t$ as the extrapolation distance.

The same value of the extrapolation distance was used as the starting point in iterative procedure to determine the neutron diffusion parameters of systems with empty channels. All data from Table I for unit blocks $a_0 = 2.90$ cm with holes were fitted to Eq. (7) and the parameters determined by the least-square method. The least-square fit was repeated with the corrected values of the geometrical bucklings, using new values for the extrapolation distances determined from $D_x$ and $D_y$ by

$$d_{ex} = 0.76 \lambda_t = 0.76 \times 3D/v,$$

respectively for x and y and for z directions. The results are presented in Table III. The geometrical bucklings are given in Table I.

The experimental data for the cubes built from larger unit blocks were fitted to

$$a = v L_x + D B^2 - C B^4,$$

where $B^2$ is the sum of all three bucklings. Thus only the average diffusion constant

$$\bar{D} = \frac{1}{3} (2D_x + D_y)$$

is determined for systems built from blocks with $a_0 = 3.60$ cm. The results are given also in Table III.
### TABLE III

**NEUTRON DIFFUSION PARAMETERS OF SYSTEMS WITH EMPTY CHANNELS.**

**EXPERIMENTS ON PLEXIGLAS**

<table>
<thead>
<tr>
<th>System</th>
<th>$v \Sigma_0$ ($\text{s}^{-1}$)</th>
<th>$D_\perp$ (cm$^2$ s$^{-1}$)</th>
<th>$D_n$ (cm$^2$ s$^{-1}$)</th>
<th>$\Sigma$ (cm$^2$ s$^{-1}$)</th>
<th>$C_\perp$ or $A_\perp$ (cm$^4$ s$^{-1}$)</th>
<th>$C_n$ or $A_n$ (cm$^4$ s$^{-1}$)</th>
<th>$\bar{C}$ (cm$^4$ s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>XYZ*</td>
<td>3360 ± 300</td>
<td>51 400 ± 5400</td>
<td>79 400 ± 5 400</td>
<td>9 000 ± 6 400</td>
<td>50 600 ± 27 000</td>
<td></td>
<td>50 600 ± 27 000</td>
</tr>
<tr>
<td>XYZ**</td>
<td>3120 ± 400</td>
<td>55 100 ± 4 700</td>
<td>88 200 ± 10 000</td>
<td>15 000 ± 9 800</td>
<td>41 000 ± 22 000</td>
<td></td>
<td>41 000 ± 22 000</td>
</tr>
<tr>
<td>XXX</td>
<td>3950 ± 370</td>
<td></td>
<td></td>
<td>47 800 ± 3 400</td>
<td></td>
<td></td>
<td>54 000 ± 6 900</td>
</tr>
</tbody>
</table>

$a_0 = 2.90 \text{ cm}$

$R = 0.85 \text{ cm}$

* Fitted to Eq. (7).

** Fitted to Eq. (17).
MONTE CARLO CALCULATIONS

The mono-kinetic problem of neutron transport in small systems with empty channels was solved for spherically symmetric scattering law in the laboratory system by Monte Carlo technique. The initial density distribution of neutrons was sinusoidal in all three directions. The history of each neutron was followed from its birth at randomly selected position \( (x_0, y_0, z_0) \) where \( x_0, y_0 \) and \( z_0 \) are drawn from a sinusoidal probability distribution, rejecting those pairs of \( x_0 \) and \( y_0 \) which would place the birth of a neutron in the empty channel. The probability for the absorption was taken equal to 0.0087 and the ratio \( \lambda/a = 0.15 \) with \( R/a = 0.293 \).

The history of each neutron is followed through the system and each time it crosses a channel its predetermined path length, drawn from the exponential probability distribution, is prolonged for the length of the passage through the channel. The neutron is lost either by absorption or by leakage. The neutron is lost by leakage when it crosses any of the boundary planes of the system. This is important for neutrons lost through the channels.

In the course of calculations the following results are accumulated: the number of passages through channels, the average length and the average square length of the passages through channels, the number of neutrons escaped from the system, the number of neutrons escaped through the empty channels, the average squares of distances between births and disappearances of neutrons along \( x- \), \( y- \) and \( z- \) directions, the average neutron lifetime and the average number of collisions per lifetime. Furthermore, for a few cases the collision densities along three directions were recorded.

The Monte Carlo programme is run on the medium-size computer ZUSE Z-23 which is relatively slow and therefore not many histories could be run in one experiment, especially not for large systems. The preliminary results of Monte Carlo calculations are given in Table IV. The final report is in preparation [11].

Unfortunately, there is no theory developed that would describe in detail such systems as investigated here. We have little theoretical support in analysis of the data obtained and one cannot escape the impression of arbitrariness in fitting the data to different analytical forms, like Eq.(7). It is the trial and error procedure to find the simplest form that describes all the data within the limits of "experimental" errors.

What we would like to get are the limiting values of some functions for \( B_x^2, B_y^2 \) and \( B_z^2 \) equal to zero. Thus, it is necessary to find how these functions behave at small values of \( B_x^2, B_y^2 \) and \( B_z^2 \). As can be seen, e.g. from Eq.(6), proposed by Laletin, \( D_n \) approaches its limiting value at \( B_x^2 = 0 \) proportionally to \( -B_x \). The easiest way to check this assumption is to plot \( (1/2)\sigma <z^2> \) versus \( B_x \). If the same is made for \( (1/2)\sigma <x^2> \) plotted versus \( B_x \) one can conclude that they approach their limiting values proportionally to \( -B_x \) or \( -B_x \) respectively, as can be seen from Fig.3.

The geometrical bucklings \( B_x^2, B_y^2 \) and \( B_z^2 \) were calculated for each system separately using the extrapolation distances determined from the mean-square distances \( <x^2> \) and \( <z^2> \)

\[
2d_{exl} = 0.71 \times 3 <x^2> d / \nu \quad (11)
\]
TABLE IV
RESULTS OF MONTE CARLO CALCULATIONS *

<table>
<thead>
<tr>
<th>System</th>
<th>$&lt;x^2&gt;$</th>
<th>$&lt;z^2&gt;$</th>
<th>$\alpha$</th>
<th>$B_x^2$</th>
<th>$B_z^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$30 \times 30 \times 30$</td>
<td>26.17</td>
<td>48.065</td>
<td>3.949 ± 0.300</td>
<td>0.0013</td>
<td>0.0013</td>
</tr>
<tr>
<td>$20 \times 20 \times 20$</td>
<td>24.41</td>
<td>29.09</td>
<td>4.488 ± 0.250</td>
<td>0.0028</td>
<td>0.0028</td>
</tr>
<tr>
<td>$6 \times 6 \times 20$</td>
<td>14.33</td>
<td>19.18</td>
<td>8.027 ± 0.400</td>
<td>0.0294</td>
<td>0.0294</td>
</tr>
<tr>
<td>$14 \times 14 \times 14$</td>
<td>22.70</td>
<td>25.465</td>
<td>4.920 ± 0.220</td>
<td>0.0057</td>
<td>0.0057</td>
</tr>
<tr>
<td>$10 \times 10 \times 10$</td>
<td>18.165</td>
<td>25.331</td>
<td>6.119 ± 0.500</td>
<td>0.0110</td>
<td>0.0110</td>
</tr>
<tr>
<td>$10 \times 10 \times 6$</td>
<td>12.962</td>
<td>17.245</td>
<td>8.195 ± 0.200</td>
<td>0.0110</td>
<td>0.0286</td>
</tr>
<tr>
<td>$6 \times 6 \times 10$</td>
<td>12.614</td>
<td>17.456</td>
<td>8.121 ± 0.400</td>
<td>0.0296</td>
<td>0.0296</td>
</tr>
<tr>
<td>$6 \times 6 \times 6$</td>
<td>9.555</td>
<td>13.521</td>
<td>9.667 ± 0.500</td>
<td>0.0296</td>
<td>0.0286</td>
</tr>
<tr>
<td>$6 \times 6 \times 4$</td>
<td>8.313</td>
<td>10.601</td>
<td>11.960 ± 0.500</td>
<td>0.0296</td>
<td>0.0615</td>
</tr>
<tr>
<td>$4 \times 4 \times 6$</td>
<td>7.759</td>
<td>10.825</td>
<td>13.358 ± 0.400</td>
<td>0.0638</td>
<td>0.0286</td>
</tr>
<tr>
<td>$4 \times 4 \times 4$</td>
<td>6.554</td>
<td>7.900</td>
<td>14.795 ± 0.600</td>
<td>0.0638</td>
<td>0.0615</td>
</tr>
<tr>
<td>$6 \times 6 \times 3$</td>
<td>7.307</td>
<td>8.674</td>
<td>14.212 ± 0.600</td>
<td>0.0296</td>
<td>0.1051</td>
</tr>
<tr>
<td>$3 \times 3 \times 6$</td>
<td>5.235</td>
<td>7.986</td>
<td>18.490 ± 0.500</td>
<td>0.1089</td>
<td>0.0286</td>
</tr>
<tr>
<td>$3 \times 3 \times 3$</td>
<td>4.093</td>
<td>4.860</td>
<td>22.894 ± 0.700</td>
<td>0.1089</td>
<td>0.1051</td>
</tr>
<tr>
<td>$6 \times 6 \times 2$</td>
<td>5.229</td>
<td>4.525</td>
<td>20.167 ± 0.700</td>
<td>0.0296</td>
<td>0.2199</td>
</tr>
<tr>
<td>$2 \times 2 \times 6$</td>
<td>2.951</td>
<td>5.180</td>
<td>28.536 ± 0.600</td>
<td>0.2259</td>
<td>0.0286</td>
</tr>
<tr>
<td>$2 \times 2 \times 2$</td>
<td>2.360</td>
<td>2.579</td>
<td>40.016 ± 0.800</td>
<td>0.2259</td>
<td>0.2199</td>
</tr>
</tbody>
</table>

Extrapolated to zero buckling:
$\infty \times \infty \times \infty$

|          | 30.18  | 43.42*** | 3.621 ± 0.109*** | 0.0000 | 0.0000 |

* Basic data: Lattice pitch a = 2.90 cm
Channel radius R = 0.85 cm
Material mean free path $\lambda = 0.435$ cm
Absorption probability $w_a = 0.0087$
Average velocity $\langle v \rangle = 2.48 \times 10^5$ cm s$^{-1}$

** Extrapolation of $\frac{1}{2} <z^2> \alpha = D_x^\alpha - A_x B_m$ to $B_x = 0$.

*** Assumed to be $\bar{v}_a = v \frac{\Sigma_{in}}{1 + \phi}$.

Thus, the extrapolation distances are functions of the size of the system. Even if the scatter of the points is very large, the linear dependence can be proved, because if $(1/2)\alpha <x^2>$ or $(1/2)\alpha <z^2>$ are plotted versus
The values of $\alpha <x^2>$ are independent of $B_z$, and vice versa, $\alpha <z^2>$ independent of $B_x$.

Thus, we can assume that the following two equations hold:

\[(1/2)\alpha <z^2> = D^*_z - A_z B_z \tag{13}\]
\[(1/2)\alpha <x^2> = D^*_x - A_x B_x \tag{14}\]

We designated the diffusion constants in Eqs. (13) and (14) with an asterisk because they are not necessarily the diffusion constants that properly describe the leakage out of the system. And, indeed, if one calculates $\nu T (1 + \phi)^{-1} (1/2)\alpha <x^2> (B^2_x + B^2_y) + (1/2)\alpha <z^2> B^2_z$, one finds all values lie systematically below the corresponding values of $\alpha$. Yet, one can hope that the terms $-A_z B_z$ and $-A_x B_x$ properly describe the variation of the diffusion constants appearing in $\alpha$. Thus we added the terms $A_z (B^2_x + B^2_y)$ and $A_x B^2_z$ to corresponding values of $\alpha$ and plotted new values versus $B^2$ where

\[B^2 = B^2_x + B^2_y + B^2_z\tag{15}\]

and

\[\alpha^* = \alpha + A_z (B^2_x + B^2_y) + A_x B^2_z.\tag{16}\]

As is seen from Fig. 4, all points for cubes lie on one straight line, the slope of which is equal to $D$. In addition, we get two other lines, for columns and for slabs, the slopes of them giving $D_n$ and $D_1$. Therefore, we can con-
Corrected values of \( \alpha \) (see Eq. (16) for \( \alpha^0 \)) are plotted versus \( B^2 \).

The slopes give the diffusion constants.

The columns give \( D_n \), cubes \( D \) and slabs \( D_a \).

conclude that the proper form, by which the dependence of \( \alpha \) on \( B_x \), \( B_y \) and \( B_z \) is described, is

\[
\alpha = v \Sigma_\alpha + D_L (B_x^2 + B_y^2) + D_n B_z^2 - A_L (B_x^2 + B_y^2) - A_n B_z^2 .
\] (17)

No terms proportional to \( B_x \), \( B_y \) and \( B_z \) are necessary, at least not in the limits of our "experimental" errors. This is understandable because, due to mono-kinetic treatment of the problem, no diffusion cooling effects are to be expected.

The results of the least-square fit of our Monte Carlo data for \( \alpha \) are given in Table V, together with graphically determined values of \( L_\alpha^2 \), \( L_n^2 \), \( A_L \) and \( A_n \) from Fig. 3.

CONCLUSIONS

It is too early to make definite conclusions because the results are obtained for one pair of geometrical parameters only, i.e. for \( a/\lambda = 6.67 \) and \( R/\lambda = 1.935 \). Nevertheless, the analysis shows that from the pulsed neutron experiments reliable results can be obtained if data are fitted to Eq. (17) instead of Eq. (7), as is done with our results reported in Table V. The question how the diffusion cooling terms affect the results has not yet been resolved.

With regard to different theoretical expressions and postulates the situation is not clear because of still large errors in the results obtained either from the experiment or from the Monte Carlo calculations. The experiment gives diffusion constants and the coefficients \( A_n \) and \( A_L \). From the Monte Carlo calculations we obtained also \( L_\alpha^2 \) and \( L_n^2 \), determined from the mean-square distances. It appears, as is shown in Table V, that the diffusion
ANISOTROPY OF DIFFUSION CONSTANT

TABLE V

COMPARISON WITH THE THEORIES

<table>
<thead>
<tr>
<th></th>
<th>Experiment</th>
<th>Monte Carlo</th>
<th>Behrens Eqs. (1), (2) (3) and (4)</th>
<th>Grant Eq. (5)</th>
<th>Laletin Eq. (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_n/D_m$</td>
<td>2.42 ± 0.30</td>
<td>2.47 ± 0.33</td>
<td>2.326</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$D_A/D_m$</td>
<td>1.51 ± 0.15</td>
<td>1.72 ± 0.20</td>
<td>1.798</td>
<td>1.502</td>
<td>-</td>
</tr>
<tr>
<td>$\beta/D_m$</td>
<td>1.81 ± 0.16</td>
<td>1.97 ± 0.26</td>
<td>1.974</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$(D_n - D_A)/D_m$</td>
<td>0.91 ± 0.30</td>
<td>0.75 ± 0.45</td>
<td>0.528</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$D_n/D_A$</td>
<td>1.60 ± 0.22</td>
<td>1.43 ± 0.31</td>
<td>1.294</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$L_n/L_m$</td>
<td>-</td>
<td>2.97</td>
<td>3.186</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$L_A/L_m$</td>
<td>-</td>
<td>2.10</td>
<td>2.463</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{L}/L_m$</td>
<td>-</td>
<td>2.39</td>
<td>2.704</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$(L_n^2 - L_A^2)/L_m^2$</td>
<td>-</td>
<td>0.87</td>
<td>0.723</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$L_n/L_A$</td>
<td>-</td>
<td>1.41</td>
<td>1.294</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$A_n/D_m$</td>
<td>1.12 ± 0.60 cm</td>
<td>1.77 cm</td>
<td>-</td>
<td>1.03 cm</td>
<td>-</td>
</tr>
<tr>
<td>$A_A/D_m$</td>
<td>0.41 ± 0.23 cm</td>
<td>0.73 cm</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$A/DM$</td>
<td>0.64 ± 0.29 cm</td>
<td>1.08 cm</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

coefficients, calculated from the mean-square distances would be too low.
To get the correct values for the diffusion coefficients, one has to calculate
them from the total leakage from the system. In this respect, Laletin's
result [3] can be regarded as basically correct, except that his coefficient
$C$ is too low if applied to the average diffusion coefficient parallel to channels
for an array of channels with a sinusoidal flux distribution also in the plane
perpendicular to channels. There is no theory with respect to $D_A$ for small
systems, so the meaning of $A_A$ is not clear. Namely, it is evident from our
Monte Carlo calculations that for very large systems the distribution of
neutrons approaches a constant across the unit cell ($v_{E_A}$ is almost equal to
$v_{E_A} T (1 + \phi)^{-1}$) in contrast to small systems where the depression of the neutron
flux in the vicinity of channels becomes appreciable ($v_{E_A} T (1 + \phi)^{-1}$).
This fact complicates calculations appreciably.

ACKNOWLEDGEMENTS

Our thanks are due to Mr. J. Grad and Miss Mira Petrišić for the pro-
gramming and operation of the programmes on ZUSE Z-23 computer. This
work is performed under the Yugoslav Federal Atomic Energy Commission's Contract No. 04-3-21.

REFERENCES


DISCUSSION

H.R. LUTZ: What was the running time for the ZUSE Z-23 computer when you did your Monte Carlo calculations?

G. PREGL: The time was 400 h.

H.R. LUTZ: I do not think you can say that your Monte Carlo calculations are closer to the Behrens calculations than the experiment, because both show a rather large error, which extends over a much greater range than the observed discrepancies.

G. PREGL: The results I have given are not the latest ones. We now have more accurate results.

P. BENOIST: With respect to the comment by Mr. Lutz, I think, that as far as the axial coefficient is concerned, the Monte Carlo calculation should yield exactly the same result as the Behrens theory, because the latter is accurate for the empty channels in the axial direction. In the case of the radial coefficient, the Monte Carlo calculation should give a value lower than that from the Behrens theory, which disregards the angular correlation terms. These comments, of course, apply only to the independent terms of the bucklings.

G. PREGL: What you say is true, and these results are due to the fact that the inclusion of cross-dams lowers the perpendicular, and in our Monte Carlo calculations all the cross-dams are included. At present I have no explanation for the great discrepancy between the Monte Carlo calculations and the Behrens predictions.

G.B. ZORZOLI: How many events did you follow in your Monte Carlo calculations? The explanation for the unexpected discrepancy in your results might lie with this figure.

G. PREGL: The number of events followed, ranged from 100 to 1000, depending on the size of the system under consideration.

P. BENOIST: I think that this paper is especially interesting as a systematic experimental study of diffusion coefficients in small systems with channels. Very few experimental results or measurements are avail-
able for investigation of this subject and there is no satisfactory theory to account for streaming effects in small-dimension systems, which is what we are concerned with in the case of exponential experiments. As a result of the experiments described by Mr. Pregl, the streaming terms dependent on the buckling in the diffusion coefficients can be shown, and thus theoretical studies can be directed towards effects connected with the finite dimensions of the system. Knowledge of such effects would appear to be necessary for the correct interpretation of critical or exponential experiments involving channels, since they generally relate to systems of comparatively small dimensions, where there is often no reason, at least a priori, to consider these terms to be negligible.
DETERMINATION OF NEUTRON TEMPERATURE USING BORON FILTERS

T. RZESZOT AND E. WARDA
INSTITUTE OF NUCLEAR RESEARCH, WARSAW, POLAND

Abstract — Résumé — Аннотация — Resumen

DETERMINATION OF NEUTRON TEMPERATURE USING BORON FILTERS. A method of neutron temperature determination using boron filters has been elaborated. The accuracy of the relative measurements has been estimated to be ± 4°K. By means of this method the distribution of neutron temperatures in the WWR-S reactor core and in a graphite block has been obtained.

DÉTERMINATION DE LA TEMPÉRATURE DES NEUTRONS A L'AIDE DE FILTRES EN BORE. Les auteurs ont mis au point une méthode de détermination de la température des neutrons à l'aide de filtres en bore. Ils ont estimé que les mesures relatives étaient exactes à ± 4°K près. Les auteurs ont également obtenu, à l'aide de cette méthode, la répartition énergétique du flux de neutrons dans le cœur du réacteur WWR-S et dans un bloc de graphite.

ОПРЕДЕЛЕНИЕ ТЕМПЕРАТУРЫ НЕЙТРОНОВ С ПОМОЩЬЮ БОРНОГО ФИЛЬТРА. Разработан метод определения температуры нейтронов с помощью борного фильтра. Подсчитано, что точность относительных измерений составляет ± 4° К. С помощью этого метода в общих чертах определяется распределение температур нейтронов в активной зоне реактора WWR-S и в графитовом блоке.

DETERMINACIÓN DE TEMPERATURAS NEUTRONICAS MEDIANTE FILTROS DE BORO. Los autores han desarrollado un método para medir la temperatura neutrónica por medio de filtros de boro. Calculan que la exactitud de los valores relativos hallados es de ± 4°K. Por medio de este método han obtenido la distribución de temperaturas neutónicas en el cuerpo del reactor WWR-S y en un bloque de grafito.

THEORY

In our laboratory the boron filter method for neutron temperature measurements has been worked out. The method needs standard and simple electronic equipment and can be useful for neutron spectrum measurements in multiplying and moderating media. The determination of the temperature of the neutron beam extracted from the medium under investigation is performed in the following way.

The neutron beam is successively filtered by a set of six calibrated boron glass filters of different thicknesses and by one cadmium filter, and is detected by means of a proportional BF₃ counter. For each boron filter the transmission α is defined as

\[ α(h) = \frac{n(h) - n_{Cд}(h)}{n(0) - n_{Cд}(0)} \]  

where

- \( α(h) \) = neutron transmission for the boron filter of thickness \( h \)
- \( n(0) \) = counting rate for the unfiltered beam

\[ \text{321} \]
\[ n(h) = \text{counting rate for the beam passing through the boron filter of thickness } h \]

\[ n_{Cd}(h) = \text{counting rate for the beam passing through boron and cadmium filters} \]

\[ n_{Cd}(0) = \text{counting rate for the beam passing through cadmium filter only.} \]

Let us denote the unknown neutron density spectrum in the beam by \( N(E) \). The number of neutrons, \( N'(E) \), in the energy interval \( dE \) passing through the filter of thickness \( h \) takes the following form:

\[
N'(E)dE = N(E)\exp\left[-(\Sigma_a(E) + \Sigma_s)h\right]dE, \quad (2)
\]

where

\[
\Sigma_a = \Sigma_a^0 E_0^{1/2} E^{-1/2} \text{ is the macroscopic absorption cross-section}
\]

\[
(\Sigma_a^0 = \text{macroscopic absorption cross-section of the filter for the energy } E_0 = 0.0253 \text{ eV})
\]

\[
\Sigma_s = \text{macroscopic scattering cross-section of the filter, which is assumed to be independent of energy.}
\]

Assuming the efficiency of the \( BF_3 \) counter to be \( 1/\nu \), and integrating expression (2) over the whole energy range, we obtain the expression for \( n(h) \)

\[
n(h) = \int_0^\infty N(E) \exp\left[-(\Sigma_a(E) + \Sigma_s)h\right]dE. \quad (3)
\]

Similarly

\[
n_{Cd}(h) = \int_0^\infty N(E)\exp\left[-(\Sigma_a(E) + \Sigma_s)h\right] \exp[-\Sigma_{Cd}^0(E)l]dE, \quad (4)
\]

where \( \exp[-\Sigma_{Cd}^0(E)l] \) is the transmission function for a cadmium filter of thickness \( l \). Now we can determine \( \alpha(h) \) in the following form:

\[
\alpha(h) = \frac{\int_0^\infty N(E)\chi(E)\exp\left[-(\Sigma_a(E) + \Sigma_s)h\right]dE}{\int_0^\infty N(E)\chi(E)dE}, \quad (5)
\]

where \( \chi(E) = 1 - \exp[-\Sigma_{Cd}^0(E)l] \).

We assume the neutron density spectrum to be of a form

\[
N_1(E) = 2\pi^{-1/2} [\mu(z)]^{-1} N_x E^{1/2} \exp[-E \cdot E^{-1/2}] E^{-3/2} dE \quad \text{for } E < E_c
\]

\[
N_2(E) = 2^{-1}E_c^{1/2} N_x E^{3/2} dE, \quad \text{for } E > E_c
\]

where

\[
\mu(z) = \text{erf } z^{1/2} - 2\pi^{-1/2} z^{1/2} \exp(-z) \]

\[ z = E_c \cdot E^{-1} \]
\[ N_T = \text{the fraction of thermal neutrons}, \]
\[ N_E = \text{the fraction of epithermal neutrons}. \]

From the conditions \( N_T + N_E = 1 \) and \( N(T) = N(E) \), we have

\[ N_T(z) = \mu(z)[\mu(z) + 4\pi^{-1/2} z^{3/2} \exp(-z)]^{-1}. \] (8)

Evaluating an effective cadmium cut-off energy to be 0.381 eV for our cadmium filter of 0.6-mm thickness, the following theoretical expression for transmission is obtained:

\[ \alpha^{th}(x, s, z) = \left\{ c_1(z)2\pi^{-1/2} \int_0^z t^{1/2} \exp[-(t + xt^{-1/2})] \, dt + c_2(z)[\exp(-xs^{-1}B)] \right\}^{-1}, \] (9)

where
\[ x = \Sigma_s h, \]
\[ t = E \cdot E_T^{-1} \]
\[ s = E_0^{1/2} \cdot E_T^{-1/2} \]
\[ c_1(z) = N_T(z)[\mu(z)]^{-1} \]
\[ c_2(z) = z^{1/2}N_E(z) \]
\[ B = E_0^{1/2}(0.381)^{-1/2} \]

For the set of our boron glass filters, the function \( \alpha^{th}(x, s, z) \) has been tabulated by a digital computer centre, that is it has been tabulated for the parameter \( s \) in the range 0.80 - 0.96 stepwise every 0.01, and for the parameter \( z \) in the range 5.0 - 7.0 stepwise every 0.1.

If we denote the theoretical and experimental value of the transmission for the \( i \)-th filter by \( \alpha^{th}_i \) and \( \alpha^{ex}_i \), respectively, multiplied by \( \exp(\Sigma_i h_i) \), we can choose the parameters \( s \) and \( z \) such that the value of the expression

\[ \delta^2 = \Sigma_i [\alpha^{th}_i(s, z) - \alpha^{ex}_i]^2 \] (10)

is the minimum. Knowing the parameter \( s \), we can easily evaluate the temperature of the neutron beam.

**METHOD OF MEASUREMENT**

One fuel element (of 43) has been removed from the WWR-S reactor core, and a 6-m-long aluminium tube of 60-mm inner diameter, passing through the whole core, has been situated vertically in its place (No. 4/22
or 31, see Fig. 1). A cadmium tube constituting the first part of the collimator inserted in the aluminium tube permits the neutron beam to be extracted from the scatterer located in the aluminium tube. The position of the scatterer can be changed in the reactor core along the y-axis within the aluminium tube. The scheme of the experimental arrangement is shown in Figs. 2 and 3. The temperature of the neutron beam extracted from the scatterer is determined by means of the method described above.

The filters and the BF$_3$ counter are placed in the second part of the collimator which is mounted above the aluminium tube after the neutron beam emerging from the reactor has been adjusted. The boron filters are made
of glass plates containing ~3.5% pure boron. These filters have been calibrated on the neutron crystal spectrometer. For each of the filters, the constant $A$ from the relation

$$I = I_0 \exp(-A \lambda)$$

has been determined. $I_0$ and $I$ are intensities of the neutron beam before and after passing through the filter, respectively; $\lambda$ is the neutron wavelength for the monochromatic neutron beam. This constant calculated for a 1-mm thickness of the filter is $A^* = A/h = 0.1876$.

For a single neutron temperature measurement the set of six boron filters of thickness 1.01, 1.84, 3.86, 4.87, 5.70 and 7.71 mm is used. $\Sigma$, evaluated for our filters appears to be $\Sigma_i = 0.255$ cm$^{-1}$. A proportional $\text{BF}_3$ counter, NUM 30 type, equipped with typical amplifying and counting circuits is used for neutron detection. It has been put in a position perpendicular to the neutron beam since in that position in the energy region under consideration it has an almost $1/v$ efficiency characteristic according to experiments carried out by means of the crystal spectrometer.

RESULTS

- The distribution of the neutron temperature in the reactor core has been measured using different scatterers shifted along the $y$-axis of the alumi-
nium tube: a lead sample, 32 mm thick *, a graphite sample, 40 mm thick, a water sample, 30 mm thick, and by changing the water level in the aluminium tube by adding a known volume of water. The results of measurements are shown in Figs. 4-13. The axis of ordinates represents the temperature of the neutron beam in °K or the inverse cadmium ratio \( \frac{n_{Cd}}{n_{q}} \), whereas the axis of abscissas shows the position of the scatterer in cm measured from the centre of the reactor core. It can be seen that for scatterers of higher atomic mass the distribution of neutron temperature in the reactor core is more symmetrical than for water samples.

\* The lead sample used was not of nuclear-grade purity.
To determine the thermalization constant and the diffusion cooling constant for graphite the distribution of the neutron temperature in the graphite stack has been obtained by means of the filter method. The graphite stack of several different thicknesses, 11, 22, 33, 55 and 66 cm, was irradiated on one side by the neutron beam of known temperature from a horizontal channel of the WWR-S reactor. The results obtained do not agree with the calculation done by WEISS [5], and will be repeated with a graphite of better nuclear parameters. Corrections for the re-entrant hole and flux gradient [1, 2] were in our case negligible.
Fig. 8
Neutron temperature distribution in the reactor core (channel 4/22) measured by shifting of 3-cm water layer

○ Measurements performed on 28, 29 August 1962
× Measurements performed on 23 October 1962

Fig. 9
Inverse cadmium ratio distribution in the reactor core (channel 4/22) measured by shifting of 3-cm water layer

○ Measurements performed on 28, 29 August 1962
× Measurements performed on 23 October 1962
Fig. 10
Neutron temperature distribution in the reactor core (channel 4/22) measured by shifting the graphite sample

Fig. 11
Inverse cadmium ratio distribution in the reactor core (channel 4/22) measured by shifting the graphite sample
Fig. 12
Neutron temperature distribution in the reactor core (channel 4/22) measured by shifting of the lead sample

Fig. 13
Inverse cadmium ratio distribution in the reactor core (channel 4/22) measured by shifting of the lead sample
REMARKS

The measurements performed hitherto have not aimed at the determination of the absolute temperature distribution in the core*. For this reason, corrections concerning the neutron temperature due to the transport theory, have not for the moment been introduced. All necessary corrections as well as the interpretation of our results are now in elaboration. Owing to the considerable sensitivity of the filter method, variations of the neutron beam temperature can be followed up.

Since the Maxwellian distribution is a good approximation of the neutron spectra in poisoned water [3] and in uranium-water lattices [4], the filter method can be a simple and useful method for neutron temperature distribution measurements in water-moderated assemblies, as well as in other moderators and multiplying media.

ACKNOWLEDGEMENTS

The authors would like to thank S. Białowas and L. Szadkowski for their technical aid and assistance in the measurements.

REFERENCES


* The determination by the filter method of the temperature of the neutron beam from the horizontal channel of the reactor (350 ± 14°K) [8] has shown good agreement with the measurements that have been performed with the neutron crystal spectrometer (350 ± 7°K) [9].
SOME EXPERIMENTAL CHARACTERISTICS OF THE GRAPHITE, WATER-MODERATED CRITICAL ASSEMBLY FOR THE SECOND POLISH RESEARCH REACTOR


INSTITUTE OF NUCLEAR RESEARCH, WARSAW, POLAND

Abstract — Résumé — Аннотация — Resumen

SOME EXPERIMENTAL CHARACTERISTICS OF THE GRAPHITE, WATER-MODERATED CRITICAL ASSEMBLY FOR THE SECOND POLISH RESEARCH REACTOR. A critical assembly for the second Polish Research Reactor, consisting of tubular 20% $^{235}$U enriched fuel elements in the water-graphite lattice surrounded by the graphite reflector, has been constructed and put into operation in the Institute of Nuclear Research at Swierk near Warsaw. First experimental characteristics of the assembly are presented.

QUELQUES CARACTÉRISTIQUES EXPÉRIMENTALES DE L'ENSEMBLE CRITIQUE RALENTI A L'EAU ET AU GRAPHITE, DESTINÉ A L'ÉTUDE DU 2e RÉACTEUR DE RECHERCHE DE POLOGNE. Un ensemble critique, qui est destiné à l'étude du 2e réacteur de recherche de Pologne et se compose d'éléments combustibles tubulaires d'uranium enrichi à 20% dans un réseau d'eau et de graphite entouré d'un réflecteur en graphite, a été construit et mis en service à l'Institut de recherches nucléaires de Swierk près de Varsovie. Les auteurs présentent les premières caractéristiques expérimentales de l'ensemble.

НЕКОТОРЫЕ ЭКСПЕРИМЕНТАЛЬНЫЕ ХАРАКТЕРИСТИКИ КРИТИЧЕСКОЙ СБОРКИ С ГРАФИТО-ВОДЯНЫМ ЗАМЕДЛИТЕЛЕМ ДЛЯ ВТОРОГО ПОЛЬСКОГО ИССЛЕДОВАТЕЛЬСКОГО РЕАКТОРА. Сконструирована критическая сборка для второго польского исследовательского реактора, состоящая из трубчатых топливных элементов с 20%-ным обогащением ураном-235 в водно-графитовой решетке, окруженной графитовым рефлектором. Эта сборка пущена в эксплуатацию в Институте ядерных исследований в Сверке около Варшавы. Даются первые экспериментальные характеристики сборки.

CARACTERÍSTICAS EXPERIMENTALES DEL CONJUNTO CRÍTICO MODERADO POR GRAFITO Y AGUA PARA EL SEGUNDO REACTOR POLACO DE INVESTIGACIÓN. En el Instituto de Investigaciones Nucleares de Swierk cerca de Varsovia se ha construido un conjunto crítico destinado al segundo reactor polaco de investigación, que consiste en elementos combustibles tubulares de uranio enriquecido al 20% en $^{235}$U en un reticulado de agua y grafito rodeado por un reflector de grafito. La memoria presenta las primeras características experimentales de ese conjunto crítico.

1. INTRODUCTION

The critical assembly ANNA [1] was built in the Institute of Nuclear Research at Święerk in 1961–1963 and went critical for the first time on 12 June 1963. The assembly is a mock-up facility for the high flux research reactor of the Soviet RPT-type, to be built in Poland.

The purpose of constructing the critical assembly ANNA is (1) to investigate the neutron parameters in various core configurations important
for reactor design, (2) to test various experimental loadings, e.g. power loops, before placing them in the high flux core, and (3) to develop advanced theoretical and experimental reactor physics techniques.

This paper presents the results of measurements performed with the first core configuration in June and July 1963.

2. SHORT DESCRIPTION OF THE FACILITY

The assembly is located in Hall A of the Reactor Engineering Building. The vertical cross-section of the assembly is shown in Fig. 1 and the horizontal arrangement of the core in Fig. 2.

The core with the reflector forms a graphite pile of an octagonal horizontal cross-section and the following over-all dimensions; height, 245 cm; diameter, 270 cm. The 45 graphite blocks in the central part of the assembly are provided with vertical holes of 7.5-cm diam. Fuel channels or graphite plugs can be inserted into these holes according to the core loading requirements.

The first core configuration is a square matrix of 14 cm X 14 cm pitch.

The fuel channel consists of an aluminium tube passing through the whole graphite block and connected to the water filling system. A fuel element is placed inside the channel. The fuel element, of 100-cm active length, is constructed of three concentric fuel tubes and a central supporting aluminium tube (see Fig. 3).

The "meat" of each sandwich-type fuel tube contains 61.5 wt. % UO₂ of 20% enrichment and 38.5 wt. % Al. Canning as well as all the spacing and supporting parts inside the fuel channel are made of aluminium. Each fuel element contains 250 g of U²³⁵.

Two of the fuel elements can be moved remotely along the fuel channel from below the core, and are used as control and safety rods. In addition, there are four boron steel safety rods surrounding the central fuel channel.

As the fuel channels are usually filled with light water, the core represents a heterogeneous two-moderator system. The bottom and top reflectors are also a heterogeneous mixture of graphite, water and aluminium. The radial reflector is made of nuclear-grade graphite. The material parameters of the assembly are given in Tables I and II.

The graphite pile is placed on a bottom steel plate mounted on steel supports. The water header and dump tank are located underneath. The whole assembly structure is surrounded by a radial biological shield made of heavy concrete 60 cm thick. The top shield consists of a 10-cm thick paraffin-boron layer and 15-cm thick steel plate.

The control system [2] consists of: two pulse channels, one provided with a fission chamber, and the other with a BF₃ proportional counter; two linear DC channels provided with current ionization chambers; and two logarithmic DC channels provided with current ionization chambers. The detectors are placed in vertical channels in the radial reflector.

The health physics system [3] consists of six gamma-radiation logarithmic monitors with high-pressure ionization chambers and two fast- and slow-neutron monitors, both located in various parts of the reactor hall.
Fig. 1

Vertical cross-section of the facility

1. Support plate
2. Framework
3. Iron pole
4. Upper framework
5. Foundation
6. Boron-paraffin layer
7. Cast iron plate
8. Fuel channel
9. Safety rod
10. Water dump tank
11. Header
12. Valve
13. Upper platform
14. Mechanism of the safety rod
15. Safety rod wire
Both mobile fuel elements and four safety rods provide for the automatic shutdown of the assembly in case of emergency. The assembly is operated from the control room adjacent to the hall.

The experimental channels for the measurement of neutron density traverses are situated vertically in graphite between the fuel element channels and in the reflector along the x- and y-axes. Thin aluminium supporting tubes for wire and foil measurements may be introduced into the central part of the fuel element.
Fig. 3

Cross-section of the fuel element

1. Fuel "meat"
2. Fuel tube spacers
3. Fuel element channel
4. Central aluminium supporting rod
5. Aluminium canning
TABLE I
CORE COMPOSITION (CORE A)

<table>
<thead>
<tr>
<th>Material</th>
<th>Volume fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Core</td>
</tr>
<tr>
<td>Graphite ($\rho = 1.70 , \text{g/cm}^3$)</td>
<td>0.7746</td>
</tr>
<tr>
<td>Water</td>
<td>0.1156</td>
</tr>
<tr>
<td>Aluminium</td>
<td>0.1010</td>
</tr>
<tr>
<td>Uranium dioxide</td>
<td>0.0088</td>
</tr>
</tbody>
</table>

3. THEORETICAL CHARACTERISTICS

The assembly appears to be an under-moderated system so that the neutron spectrum is rather hard. The slowing-down from water amounts to $\sim 70\%$ of the total slowing-down power. The system is strongly heterogeneous and the parameters obtained through the homogenization procedure are, in fact, space-dependent. It makes the theoretical analysis rather complicated.

As the first step of the theoretical analysis two-group calculations have been performed with the data given in Table III. The thermal utilization factor has been calculated in the diffusion approximation with an assumption that the slowing-down densities in the water-filled fuel channel and in the surrounding graphite are proportional to the corresponding slowing-down powers. This assumption has been verified by a two-group calculation of the cell [4]. The results of these two approaches differ from each other by less than $1\%$.

The resonance integral has been calculated numerically for the actual geometry of the fuel channel [5]. The method used is an extension of MÜLLER's [6] approach to multi-layer cell systems. Substituting an equivalent slab system for the cylindrical fuel structure, the Dancoff corrections have been estimated.

From the analysis of various available experimental and theoretical data concerning the fast effect in uranium-water mixtures the quantity $\epsilon - 1$ has been found to be negligible in our system.

The anisotropy of the thermal diffusion parameters has been taken into account. The neutron age has been assumed isotropic.

The calculated microscopic and migrational parameters listed in Table III have been used as input data for the two-group equations which in the core have the form:
The assumption of the diffusion equation in such a form permits reducing the three-dimensional problem to a two-dimensional one. The value of $B_2^2$ has been taken from experiment (see section 6), being determined there with considerable accuracy. Nearly the same value of $B_2^2$ has been obtained from simple one-dimensional calculations.

The results of the $B_2^2$ and $B_2^0$ determinations are given in Table VII.
4. APPROACH TO CRITICALITY

The first approach to criticality was performed in the period from 7-12 June 1963. In view of large uncertainties in the critical calculations, severe safety precautions were undertaken [7].

The start-up Ra-α - Be neutron source of $3 \times 10^6$ n/s strength has been introduced inside the central channel between both mobile fuel elements, and placed in the bottom reflector (see Fig. 2).

In addition to the neutron detection channels of the control system four pulse counting units with BF$_3$ proportional counters have been used for the determination of the inverse count rate $N_0/N$ versus fuel loading. The BF$_3$ counters have been positioned around the core in the upper part of the radial reflector channels (channels PN-4, W-2, P-7, Z-5 in Fig. 2) to avoid direct source effects, i.e. to detect mainly neutrons originating in the core.

Twenty-five fuel channels filled with water in a 5 X 5 square matrix (channels (B to F) X (2 to 6)) have been prepared for loading, and both mobile fuel elements with their electromechanical instrumentation have been installed in the core. The following sequence of fuel loading operations has been chosen:

1. Lowering of the mobile fuel elements outside the graphite pile,
2. Positioning of the new fuel charge in the core,
3. Gradual introduction of the mobile fuel element up to the predetermined height with a speed not exceeding 2.5 mm/s,
4. Measurement of the neutron count rate, and
(5) Analysis of the inverse count rate curve and evaluation of the new critical mass (Fig. 4, Curve 1).

The criterion for the amount of fuel charge to be loaded in the next step was that this charge should be less than half of the difference between the evaluated critical mass and the actual fuel mass in the core.

To evaluate the effectiveness of the safety rods two additional operations have been carried out after each loading: (1) dropping of one safety rod and (2) dropping of the remaining three safety rods. The negative reactivity introduced into the subcritical core has been determined by plotting separate inverse count rate curves (Fig. 4, Curves 2 and 3).

The critical mass corresponding to this core (core A) and radial reflector configuration amounts to 15.6 fuel elements (3900 g U$^{235}$), i.e. the 16th fuel element has been inserted only 60 cm into the core. The critical masses with one and four safety rods in the core have been estimated to be 16.38 fuel elements (4095 g U$^{235}$) and 17.77 fuel elements (4442 g U$^{235}$), respectively.
To symmetrize the core configuration composed of graphite only the reactivity worths of the fuel elements and of the radial reflector material have been estimated in relation to the central mobile fuel element of 3.75% reactivity. The average reactivity worth of the peripheral fuel element (distance from the centre $r = 29$ cm) amounts to 3%, that of water contained in the reflector channel next to the core boundary ($r \approx 40$ cm) is $-0.45\%$, and that of the graphite plug replacing the water and aluminium thimble is $0.33\%$. The aluminium thimble contributes $-0.04\%$ only. Thus the replacement of water-filled channels in the radial reflector by graphite plugs leads to considerable decrease of the critical mass.

Taking into account these reactivity values the minimum critical mass feasible in this lattice has been evaluated to be 11.5 fuel elements. A rectangular matrix of $3 \times 4$ fuel elements has been adopted as the most symmetrical and the surplus reactivity has been removed by lowering both of the central mobile fuel elements by 35 cm. This corresponds to a decrease of the core height by 5.5 cm ($H_{\text{eff}} = 94.5$ cm). This core configuration, called core B, has been chosen for further experimental and theoretical analysis presented in the following sections.

5. Reactivity Effects

The determination of reactivity worths in the ANNA assembly has been hitherto carried out for the mobile fuel elements and for the safety rods by means of (a) analysis of the inverse count rate curves, and (b) rod-drop method.

A Analysis of the inverse count rate curves (core A)

In the near-critical state ($1 - k_{\text{eff}} \ll 1$) the subcritical multiplication of neutrons is described by the formula

$$k_{\text{eff}} = 1 - N_0 / N.$$  \hfill (3)

The reactivity worth of the safety rod (mobile fuel element) may thus, for the actual subcritical loading, be estimated in the following way

$$\Delta k_{\text{SR}}^{(\text{MFE})} = \left( \frac{N_0}{N} \right)^{\text{SR}}^{(\text{MFE})} - \frac{N_0}{N},$$  \hfill (4)

where the index SR refers to the safety rod and the index MFE to the fuel element.

These $\Delta k$ values depend on the respective $k_{\text{eff}}$, therefore the true values of $\Delta k$ have to be extrapolated to $k_{\text{eff}} = 1$ for the clean core. The results for one mobile fuel element, one safety rod, and four safety rods are given in Table IV together with the theoretical estimations [8].

The last row in Table IV gives the ratio of $\frac{1}{4} \Delta k$ for four safety rods to $\Delta k$ for one safety rod and may be considered as a measure of the shadowing effect. The theoretical values have been calculated by the two-group Scalettar-Nordheim method, the control rods being regarded as grey bodies.
TABLE IV

REACTIVITY WORTHS DETERMINED BY THE ANALYSIS OF THE INVERSE COUNT RATE CURVES
(CORE A)

<table>
<thead>
<tr>
<th>Rod (element)</th>
<th>Quantity</th>
<th>Experiment</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobile fuel element</td>
<td>$\Delta k_{MFE}$</td>
<td>3.75%</td>
<td></td>
</tr>
<tr>
<td>Safety rod No. 1</td>
<td>$\Delta k_{SR}$</td>
<td>4.55%</td>
<td>4.48%</td>
</tr>
<tr>
<td>Four safety rods</td>
<td>$\Delta k_{4SR}$</td>
<td>16.80%</td>
<td>16.88%</td>
</tr>
<tr>
<td>$\frac{1}{4} \frac{\Delta k_{4SR}}{\Delta k_{SR}}$</td>
<td>0.928</td>
<td>0.935</td>
<td></td>
</tr>
</tbody>
</table>

B. Rod-drop method

The determination of reactivity worths of the mobile fuel elements and safety rods has been carried out by means of measurements of the neutron density time-dependence after a sudden drop of the safety rod into the core or a sudden removal of the mobile fuel element from the core (core B).

The neutron density time-dependence is described by the relation

$$ n(t) = n_0 \left( \frac{\beta_l}{\beta}, \lambda_i, \frac{1}{\beta_{eff} k}, \frac{-\rho}{\beta_{eff}}, t \right), \tag{5} $$

where the $f$-function represents a sum of exponential decays in which the first term depends practically on $1/\beta_{eff} k$.

The measured $n(t)$ function has been analysed by three techniques [9]:

(1) Hogan's integral technique where

$$ \frac{-\rho}{\beta_{eff}} = \frac{n_0}{\int n(t) dt} \sum \frac{\beta_i}{\lambda_i \beta} \tag{6} $$

(2) Schultz's technique where

$$ \frac{-\rho}{\beta_{eff}} = C \left( \frac{n_0 T}{\int n(t) dt} - 1 \right), \tag{7} $$

$C$ denoting a constant and $T$ the integrating term, usually 15 s.
(3) Technique of comparison of the function \( \frac{n}{n_0}(\rho/\beta_{\text{eff}}) \) with the theoretical decay curves.

Four pulse counting units (the same as in the critical experiment) have been used for measuring the neutron density changes. One of the channels has been connected to two scalers operating alternately in 3.3-s time intervals. The remaining channels have been provided with scalers summing up the pulses in the intervals of 90 to 200 s. The count rates measured before the rod (element) drop have given the \( n_0 \) value. Owing to the presence of the neutron source in the assembly the integration in the integral technique has been carried out up to \( t = 200 \) s because then the integral \( \int_0^t n(t') \, dt' \) reaches 99% of the infinite integral value, and this value has been corrected for the neutron source effect.

The results are given in Table V.

### TABLE V

<table>
<thead>
<tr>
<th>Safety rod (mobile fuel element)</th>
<th>Techniques</th>
<th>Probable ( \Delta k (\rho) ) value (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mobile fuel element (from 76-cm height)</td>
<td>Integral 3.5</td>
<td>Schultz 3.3</td>
</tr>
<tr>
<td>Safety rod No. 2</td>
<td>Integral 4.8</td>
<td>Schultz 5.3</td>
</tr>
<tr>
<td>Safety rod No. 3</td>
<td>Integral 4.7</td>
<td>Schultz 4.6</td>
</tr>
<tr>
<td>Safety rods Nos. 2, 3, 4</td>
<td>Integral 11.8</td>
<td>Schultz 11.0</td>
</tr>
</tbody>
</table>

As the fourth safety rod is symmetric to the third one the shadowing effect amounts to

\[
\frac{\Delta k_{SR_{2,3,4}}}{\Delta k_{SR_2} + 2\Delta k_{SR_3}} = \frac{11.4}{14.6} = 0.78.
\]

The accuracy of the reactivity-worth determination is estimated to be 10%.
6. NEUTRON DENSITY AND FLUX DISTRIBUTIONS

A. Measurements in the unloaded assembly [10]

As the criticality parameters of this assembly strongly depend on the neutron migration properties of the core and reflector composition an attempt has been made to determine the magnitude of the diffusion length in the core without fuel elements.

For this purpose neutron density distributions induced by the Ra-α-Be source have been measured in vertical channels with miniature BF$_3$ counters along the z-axis in the x-direction (Fig. 5), and with In activation foils along the z-axis in the y-direction (Fig. 6).

As the system has a complicated geometry the analytical solution of the diffusion equations is impossible. Diffusion parameters have therefore been determined from the neutron density balance in small regions.
Neutron density traverses in unloaded core along z-axis in the y-direction measured with bare (1, 2, 3) and cadmium-covered (1', 2', 3') in foils (Ra-α-Be source at z= 270 cm)

1, 1' = Channel (B-Q) 4
2, 2' = Channel (A-B) 4
3, 3' = First reflector channel (y= 59.5 cm)

The results are summarized in Table VI. They show a strong anisotropy effect in the vertical and horizontal directions amounting to

$$\frac{L_v}{L_h} = 1.135$$

As shown in Fig. 7 there is a significant decrease of neutron density in the water channels as compared with the neighbouring graphite channels. This effect may account for the difference between the experimental and theoretical L-values.

B. Measurements in the critical state [11]

The purpose of the measurements of neutron density distributions carried out with activation detectors (Cu and Au foils and Cu wires) and a
TABLE VI
DIFFUSION LENGTHS FOR THE UNLOADED CORE

<table>
<thead>
<tr>
<th>Region</th>
<th>$L_u$ (cm)</th>
<th>$L_v$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core: graphite-water</td>
<td>16.2</td>
<td>13.0</td>
</tr>
<tr>
<td>Radial reflector: graphite</td>
<td>43.4</td>
<td>43.4</td>
</tr>
</tbody>
</table>

A miniature semiconductor probe has been the determination of the reactor buckling, the spectral index $r$, and the comparison with theoretical calculations.

The neutron density traverses in a number of vertical experimental channels in the $xz$-plane are shown in Fig. 8 whereas those in the $yz$-plane are shown in Fig. 9.

The axial buckling $B_z$ has been determined from these curves by fitting them to a $\cos B_z z$ function within the region of constant Cd ratio. This procedure failed for the traverses in the horizontal directions $x$ and $y$ as only few experimental points were available, and, moreover, the Cd ratio varied significantly along these traverses.

The thermal and fast neutron flux distributions have been calculated from the respective experimental neutron density traverses, assuming the neutron spectrum to be of Westcott's form

$$\Phi(E) = n_0 \sqrt{\frac{4T_n}{\pi T_0}} \left\{ (1 - rb) e^{\frac{E}{E_{Tn}} - r \frac{\Delta}{E}} \right\}.$$ (8)

The spectral index $r \sqrt{T_n/T_0}$ has been determined from the Au-Cd ratios and its variation along the $x$- and $y$-traverses is shown in Fig. 10.

The thermal and fast fluxes are related to the respective neutron densities by the formulae:

$$\Phi_{th} = n_0 (1 - rb) \sqrt{\frac{4T_n}{\pi T_0}}$$ (9)

and

$$\Phi_f = n_0 \frac{r}{b} \sqrt{\frac{4T_n}{\pi T_0}} \ln \frac{E_2}{E_1}.$$ (10)

where $E_2 = 2$ MeV, $E_1 = 4$ kT.
Neutron density traverses in the unloaded core for various distances from the source, $\text{Ra-}^{\alpha}-{\text{Be}}$ ($z = 180, 160, 140 \text{ cm}$)

- Measuring channel in graphite
- Measuring channel in water

(a) Along the x-axis
(b) Along the y-axis
The respective reaction rates for $1/\nu$ detectors with $\Phi_{th}$ and $\Phi_{r}$ for $x$- and $y$-traverses are shown in Fig. 11. To determine the $B_x$ and $B_y$, an attempt has been made to fit these flux distributions to those from the two-group theory for a slab core of finite dimensions surrounded by an infinite reflector. Discrepancies between experimental and calculated values of $B_x$, $B_y$, $r$ and $L^2$ are observed. It seems that the theoretical values of $r$ and $L^2$ are too low. The buckling values determined from experiment and theory are given in Table VII.

C. Intracell measurements

As the flux distribution, particularly within this type of cell, is important for the determination of the criticality parameters such as $f$, $\rho_{28}$ and $\rho_{25}$,
first measurements have been made to determine the variation of a parameter characterizing the disadvantage factor along the fuel element. For this purpose neutron density distributions in the centre of the fuel element, \( n_c \), in relation to those at the cell boundary, \( n_1 \) and \( n_2 \), have been obtained by means of Cu wire and foils activation.

This disadvantage parameter defined as \( 2n_c/(n_1+n_2) \) amounts to 1.055 for the wire probe and 1.047 for the Cu foils in the middle of the fuel element height, implying that there is a strong peaking of the neutron density inside the water-filled central fuel tube. The variation of this parameter along the fuel channel (shown in Fig. 12) is quite considerable, namely it changes from 1.055 in the middle of the fuel element height to \( \sim 0.93 \) at the fuel element ends.
D. Absolute determination of the effective neutron flux

The effective neutron flux has been determined by three independent techniques:

(1) $4\pi$-proportional flow counter [12] (for Au foils),
(2) $\beta-\gamma$-scintillation coincidence unit [13] (for Au foils) and
(3) Liquid scintillation unit [14, 15] (for In and Mn foils).

All measurements have been performed at an effective flux level in the middle of the assembly amounting to

$$\Phi_{\text{eff}} = n\Phi_0 = 6 \times 10^6 \text{ n/cm}^2\text{s}.$$ 

The results obtained by the above-mentioned techniques agree within ± 5%.
Fig. 11

Effective neutron flux, thermal flux and fast flux traverses in horizontal medium plane

\[ \varphi = \frac{4T_0}{\pi T_0} \]

\[ x = \phi_{th} \]

\[ \Delta = \phi_f \]

(a) Along the x-axis

(b) Along the y-axis

**TABLE VII**

EXPERIMENTAL AND THEORETICAL VALUES OF $B^2$ AND THE EFFECTIVE REFLECTOR SAVING $\delta$ (CORE B)

<table>
<thead>
<tr>
<th></th>
<th>$B_x^2$ (m$^{-2}$)</th>
<th>$B_y^2$ (m$^{-2}$)</th>
<th>$B_z^2$ (m$^{-2}$)</th>
<th>$\delta_x$ (cm)</th>
<th>$\delta_y$ (cm)</th>
<th>$\delta_z$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>4.32</td>
<td>28.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theory</td>
<td>15.35</td>
<td>11.85</td>
<td>4.44</td>
<td>19.5</td>
<td>17.75</td>
<td></td>
</tr>
</tbody>
</table>
ACKNOWLEDGEMENTS

The authors wish to acknowledge the assistance of Miss E. Fiszer, Mr. A. Janikowski, Mrs. T. Kulikowska, and Mr. B. Szczechla in calculations, measurements and critical assembly operation, and of the technicians Messrs. S. Bialowas, K. Dembiński, J. Jabłoński, M. Jokiel, L. Kowalski, K. Kwiatek, J. Maziarz, L. Szatkowski, M. Zwierzyński, and Mrs. D. Łysik, Mrs. S. Piekoszewska, and Mrs. E. Weiss.

The authors are indebted to Miss K. Kowalska for her valuable remarks and general assistance in the course of preparation of this paper.

REFERENCES


[9] SOUŻYK, J. i SUWALSKI, W., Kinetyczne pomiary reaktywności, to be published.


[13] MALEWSKI, S., Absolute determination of disintegration rates by means of $\beta\gamma$ scintillation coincidence unit, to be published.


ИССЛЕДОВАНИЕ ФИЗИЧЕСКИХ СВОЙСТВ АКТИВНОЙ ЗОНЫ ВВЭР НА КРИТИЧЕСКИХ СБОРКАХ

Г.Я. АНДРИАНОВ, В.А. ВОЗНЕСЕНСКИЙ, А.Н. КАМЫШАН Л.В. КОМИССАРОВ, В.А. КУЗЬМИЧЕВА, Г.Л. ЛУНИН В.Н. СЕМЕНОВ и В.И. ХАЛИЗЕВ

СССР

Abstract — Résumé — Аннотация — Resumen

STUDY OF THE PHYSICAL PROPERTIES OF THE VVER CORE IN CRITICAL ASSEMBLIES. Data calculated for lattices made up from elements enriched to different degrees were checked experimentally. In the course of the experiments, we measured the values of the physical parameter \( \kappa_3 \), the neutron migration area \( M^2 \) and the multiplication constant \( k_o \) for three types of "clean assembly", consisting of units of natural uranium oxide, units of uranium oxide enriched to 1.5\% (with \( \text{U}^{235} \)) and units enriched to 2\%.

In addition, \( \kappa_4 \) and \( k_m \) were determined for various mixed assemblies. Through these experiments we were able to work out a law for averaging the neutron-physical parameters in these assemblies. The experimental method used permitted determination of the form of the critical equation describing the lattices in question.

The effectiveness of control elements was also studied for assemblies of units with 1.5\% enriched uranium oxide. Interaction between control elements was closely studied on a critical assembly made up of such units.

ÉTUDE DES PROPRIÉTÉS PHYSIQUES DU CŒUR VVER DANS DES ASSEMBLAGES CRITIQUES. On a fait la vérification expérimentale des données obtenues par le calcul pour des réseaux constitués d'éléments plus ou moins enrichis. A la suite de ces expériences, on a mesuré les grandeurs du laplacien matière \( \kappa_3 \), de l'aire de migration des neutrons \( M^2 \) et du facteur de multiplication \( k_o \) pour trois types d'«assemblages non empoisonnés», formés de cartouches d'oxyde d'uranium naturel, de cartouches d'oxyde d'uranium enrichi jusqu'à 1,5\% (en \( \text{U}^{235} \)) et de cartouches de combustible enrichi à 2\%.

En outre, on a déterminé les grandeurs \( \kappa_4 \) et \( k_m \) pour un certain nombre d'assemblages mixtes. Grâce à ces expériences, on a pu dégager les principes permettant de déterminer les valeurs moyennées des paramètres physiques des neutrons dans des assemblages de ce genre. La méthode utilisée au cours des expériences a permis d'établir la forme de l'équation critique qui caractérise les réseaux étudiés.

Les recherches sur l'efficacité du mécanisme de commande ont été effectuées sur des assemblages de cartouches contenant de l'oxyde d'uranium enrichi à 1,5\%. On a étudié en détail les interactions entre les barres de commande à l'aide d'un assemblage critique formé par des cartouches des types mentionnés ci-dessus.

ИССЛЕДОВАНИЕ ФИЗИЧЕСКИХ СВОЙСТВ АКТИВНОЙ ЗОНЫ ВВЭР НА КРИТИЧЕСКОЙ СБОРКЕ. Проводилась экспериментальная проверка расчетных данных, относящихся к решеткам, собранным из элементов того или иного обогащения. В результате экспериментов были измерены величины материального параметра \( \kappa_3 \), площади миграции нейтронов \( M^2 \) и коэффициента размножения \( k_o \) для трех типов "чистых сборок", состоящих из кассет с естественной окисью урана, кассет с окисью урана, обогащенной до 1,5\% (по изотопу U-235), и из кассет с 2-процентным обогащением.

Кроме того, были определены величины \( \kappa_4 \) и \( k_m \) для ряда смешанных сборок. Эти эксперименты позволили выявить закон усреднения нейтронно-физических параметров в таких сборках. Используемая в экспериментах методика позволила определить форму критического уравнения, которое описывает исследуемые решетки.

Исследования эффективности органов регулирования проводились на сборках кассет, содержащих окись урана 1,5-процентного обогащения. Интерференция действия органов регулирования подробно изучена на критической сборке, состоящей из указанных кассет.

ESTUDIO DE LAS PROPIEDADES FÍSICAS DEL CUERPO VVER EN LOS CONJUNTOS CRÍTICOS. Se comprobaron experimentalmente los datos teóricos relativos a reticulados constituidos por elementos combustibles
Г. Я. АНДРИАНОВ и др.

де distintos grados de enriquecimiento. Los experimentos permitieron medir las magnitudes del laplaciano \( k_j \), del área de migración de los neutrones \( M^2 \) y del coeficiente de multiplicación \( k_m \) para tipos de «conjuntos limpios», formados por elementos combustibles de óxido de uranio natural, elementos combustibles de óxido de uranio enriquecido hasta el 1,5% (en el isótopo \( ^{235}U \)), y elementos combustibles enriquecidos al 2%.

Además se determinaron las magnitudes del \( k_j \) y \( k_m \) para toda una serie de conjuntos mixtos. Estos experimentos permitieron establecer la ley de los valores medios de los parámetros neutrónicos en dichos conjuntos. El método empleado en los experimentos facilita la determinación de la forma de la ecuación crítica que describe los reticulados estudiados.

Se examinó la eficacia de los órganos del control en conjuntos de elementos combustibles que contengan óxido de uranio enriquecido al 1,5%. En un conjunto crítico integrado por los elementos indicados, se investigó detalladamente la influencia recíproca ejercida por los órganos de control.

**ВВЕДЕНИЕ**

В реакторах типа ВВЭР в качестве тепловыделяющих элементов используются стержни из спеченной двуокиси урана различного обогащения с покрытием из сплава циркония. Тепловыделяющие элементы скомплексованы в кассеты, оболочкой которой также служит сплав циркония. К началу проектирования не имелось достаточно точных данных о нейтронно-физических параметрах решеток с подобными тепловыделяющими элементами. В связи с этим выявились необходимость экспериментальной проверки расчетных данных, относящихся к решеткам, собранным из элементов того или иного обогащения (так называемых "чистых сборок"). В реакторах предусматривается комплектация активных зон из кассет с различным обогащением урана. Кроме того, в целях достижения глубокого выгорания во время кампании будут производиться частичные перегрузки реактора. Таким образом, в реакторе всегда будет присутствовать горючее с разной степенью выгорания. В связи с этим потребовалось изучить физические параметры смешанных сборок с крупномасштабной гетерогенностью, а также проверить законы усреднения нейтронно-физических параметров в таких сборках.

В качестве органов регулирования в реакторах ВВЭР используются компенсирующие кассеты (КК). Верхняя часть КК представляет собой толстый шестигранный стержень, содержащий воду внутри поглощающего чехла из бористой стали. Стержни имеют топливные надставки, которые вводятся в активную зону реактора одновременно с выведением поглотителя. Стержни такой конфигурации будут захватывать наряду с тепловыми нейтронами значительную долю надтепловых и быстрых нейтронов, подпадающих внутрь стержня и замедляющихся там. Расчет эффективности таких толстых поглотителей с замедлителем внутри довольно сложен. Большие деформации нейтронного распределения в реакторах с толстыми стержнями должны приводить к сильному взаимному влиянию стержней друг на друга; эффективность отдельного стержня сильно зависит от конкретного расположения всех остальных, присутствующих в активной зоне. Поэтому необходимо экспериментальное определение эффективности поглотителей при самых различных расположениях стержней в активной зоне. Вышеуказанные соображения определили направление экспериментальных работ, описываемых в настоящем докладе.

В результате экспериментов были измерены величины материального параметра \( k_j \), площади миграции нейтронов \( M^2 \) и коэффициента размножения
ИССЛЕДОВАНИЕ ФИЗИЧЕСКИХ СВОЙСТВ АКТИВНОЙ ЗОНЫ

Қ для трех типов "чистых сборок", состоящих из кассет с естественной окисью урана, кассет с окисью урана, обогащенной до 1,5% (по изотопу U-235), и из кассет с 2-процентным обогащением.

Кроме этого, были определены величины Қ для ряда смешанных сборок. Эти эксперименты позволили выявить закон усреднения нейтронно-физических параметров в таких сборках. Используемая в экспериментах методика позволила определить форму критического уравнения, которое описывает исследуемые решетки.

Исследования эффективности органов регулирования проводились на сборках кассет, содержащих окись урана 1,5-процентного обогащения. Интерференция действия органов регулирования подробно изучена на критической сборке, состоящей из указанных кассет.

2. МЕТОДИКА ЭКСПЕРИМЕНТОВ

Для определения нейтронно-физических параметров сборок была выбрана так называемая методика "надкритической достройки реактора", основанная на анализе критического уравнения.

Сущность этой методики заключается в следующем.

Обозначим меру надкритичности реактора величиной $k_{2ф}$, тогда:

$$k_{2ф} = \frac{\Gamma(k^2M^2)k_0}{k_2}.$$  \hspace{1cm} (1)

Здесь функция $\Gamma(k^2M^2)$ есть вероятность для нейтрона избежать утечки в процессе замедления и диффузии, представляющая собой Фурье-образ функции влияния точечного источника нейтронов деления; $k_0$ - коэффициент размножения нейтронов в бесконечной среде; $k^2 = k_0^2 + \delta k^2$, причем $k_0$ - геометрический параметр в критическом состоянии, а величина $\delta k^2$ характеризует отклонение размера реактора от критического.

Реактивность реактора связана с величиной $k_{2ф}$ следующим соотношением:

$$\rho = \frac{\delta k_{2ф}}{k_{2ф}},$$  \hspace{1cm} (2)

где $\delta k_{2ф} = k_{2ф} - 1$. Из (1) и (2), после несложных преобразований, получим:

$$\rho = \frac{\delta k_{2ф}}{k_{2ф}} = M^2 \frac{\partial \ln \Gamma(k^2M^2)}{\partial (k^2M^2)} \frac{\delta k^2}{k^2}.$$  \hspace{1cm} (3)

Для цилиндрического реактора, достроенного по высоте в случае разделения переменных, имеем:
\[ \delta \kappa^2 = - \frac{2 \pi^2 \delta h}{(h_k + \delta_z)^3}, \quad (4) \]

где \( h_k \) — критическая высота реактора;
\( \delta z \) — суммарная осевая экстраполированная добавка отражателя;
\( \delta h \) — превышение высоты реактора над критической.

Из (3) и (4) получим:

\[ \frac{\delta \rho}{\delta h} = - M^2 \frac{\partial \ln \tilde{f}(k^2 \rho^2)}{\partial (k^2 \rho^2)} \bigg|_{k^2 = k^2_0} \cdot \frac{2 \pi^2}{(h_k + \delta_z)^3} . \quad (5) \]

Обозначим величину

\[ - M^2 \frac{\partial \ln \tilde{f}(k^2 \rho^2)}{\partial (k^2 \rho^2)} \bigg|_{k^2 = k^2_0} = \psi(k^2_0 \rho^2), \quad (6) \]

tогда

\[ (h_k + \delta_z) = \left[ \psi(k^2_0 \rho^2) \right]^{\frac{1}{3}} \left( \frac{\delta \rho}{\delta h} \cdot \frac{1}{2 \pi^2} \right)^{-\frac{1}{3}} . \quad (7) \]

Величину \( \psi(k^2_0 \rho^2)^{\frac{1}{3}} \) можно определить из наклона экспериментальной пря-мой, представляющей зависимость критической высоты реактора \( h_k \) от величини \( \left( \frac{\delta \rho}{\delta h} \cdot \frac{1}{2 \pi^2} \right)^{\frac{1}{3}} \). Определяя функцию \( \psi(k^2_0 \rho^2) \) при различных значениях геометрического параметра \( k^2_0 \) (но при одинаковых значениях \( \rho^2 \)), можно определить величину площади миграции, экстраполируя полученную в экспериментах функцию \( \psi(k^2_0 \rho^2) \) к значению при \( k^2_0 = 0 \).

Действительно в этом случае:

\[ - \frac{\partial \ln \tilde{f}(k^2_0 \rho^2)}{\partial (k^2_0 \rho^2)} \bigg|_{k^2_0 \to 0} = 1, a, \]

\[ \psi(k^2_0 \rho^2) \bigg|_{k^2_0 \to 0} = M^2 . \]

Чтобы из этих опытов определить величину коэффициента размножения нейтронов \( k_m \), необходимо знать функцию \( \tilde{f}(k^2 \rho^2) \), т.е. форму критического уравнения. Действительно,

\[ k_m = [ \tilde{f}(k^2 \rho^2) ]^{-1} . \]
ИССЛЕДОВАНИЕ ФИЗИЧЕСКИХ СВОЙСТВ АКТИВНОЙ ЗОНЫ

Кроме этого, получение формы критического уравнения в эксперименте имеет и самостоятельный интерес, поскольку знание функции влияния необходимо при расчетах запаса реактивности реактора, эффективности регулирующих стержней и т.д. Поэтому большое внимание в настоящей работе уделялось экспериментальному определению формы критического уравнения.

Наиболее общей записью критического уравнения является выражение его в так называемой моментной форме

\[ k = \sum_{n=0}^{-1} (-1)^n \frac{k_0^{2n} R^{2n}}{(2n+1)!} = 1. \] (8)

Здесь величина суммы представляет собой функцию \( f(K^2 M^2) \), т.е. вероятность для нейтрона избежать утечки в процессе замедления и диффузии; \( R_{2n} \) — пространственные моменты функции влияния, причем \( M^2 = \frac{1}{6} R^2 \).

Вычисление моментов функции влияния, а также экспериментальное их определение представляет значительные трудности. Поэтому на практике принимается какая-либо определенная модель замедления, характеризующая данную мультиплицирующую среду, и на основе этой модели выводится определенный вид критического уравнения. Можно указать две крайние модели замедления. Одна из них предполагает, что нейтроны теряют энергию непрерывно. При этом описание утечки нейтронов в процессе замедления выражается функцией

\[ e^{\phi M^2}. \]

а критическое уравнение записывается в виде:

\[ k = e^{-\phi M^2} = 1. \]

Такая модель замедления называется возрастной. Другой крайностью является, так называемая, модель однократного столкновения. При этом считается, что нейтрон при первом же соударении теряет всю свою энергию. Критическое уравнение в этом случае будет выражаться следующим образом:

\[ \frac{k}{(3\phi^2 M^2)^2} \arctg(3\phi^2 M^2) = 1. \] (9)

Промежуточной моделью является одногрупповая модель, которая дает следующее критическое уравнение:

\[ \frac{k_0}{1 + \phi^2 M^2} = 1. \] (10)
При экспериментальном исследовании формы критического уравнения можно идти двумя путями. Первый из них заключается в измерении моментов распределения нейтронов при анализе уравнения в моментной форме. Однако использование этого уравнения приводит к необходимости определения в эксперименте большого количества параметров, причем, если величину второго момента можно определить с достаточно высокой точностью, то величины последующих моментов из-за экспериментальных ошибок определяются со все более возрастающей относительной ошибкой.

Можно идти и по другому пути, пытаясь ввести в критическое уравнение какой либо один экспериментальный параметр (кроме \( k_\infty, k_4 \) и \( M^2 \)), отражающий механизм замедления, но такой, чтобы в новую форму критического уравнения входили все вышеописанные как частные случаи. Именно этот путь был избран в настоящей работе.

Поиски универсальной формы критического уравнения привели к следующей связи величин \( k_\infty, M^2 \) и \( k_4 \) в критическом уравнении

\[
\frac{k_\infty}{1 + \frac{k_4^2 M^2}{n}} = 1 .
\]

Здесь \( n \) — экспериментальный коэффициент, отражающий характер замедления в интересующей среде.

Написанное выше уравнение было предложено Комиссаровым Л. В. и названо эмпирическим критическим уравнением. При различных значениях \( n \) такая форма критического уравнения описывает все случаи, приведенные выше. Действительно, при \( n \to \infty \) получаем возрастное критическое уравнение, а при \( n = 1 \) одногрупповое. При \( n = 0,38 \) разложение в ряд эмпирического уравнения дает следующее:

\[
\frac{k_\infty}{1 + \frac{k_4^2 M^2}{0,38}} = k_\infty(1 - k_4^2 M^2 + 1,81 k_4^4 M^4 - 3,79 k_4^6 M^6 + 8,42 k_4^8 M^8) .
\]

Разложение в ряд уравнения, соответствующего модели однократного столкновения, дает:

\[
\frac{k_\infty}{(3k_4^2 M^2)^{\frac{1}{2}}} = \text{arctg}(3k_4^2 M^2)^{\frac{1}{2}} = k_\infty(1 - k_4^2 M^2 + 1,80 k_4^4 M^4 - 3,86 k_4^6 M^6 + 9,00 k_4^8 M^8 . . . ) .
\]

Как видно, первые пять членов обоих рядов весьма близки друг к другу. Таким образом, при \( n = 0,38 \) эмпирическое уравнение описывает картину,
исследование физических свойств активной зоны  

близкую к модели однократного столкновения. При обработке экспериментов, описываемых в данной работе, была использована эмпирическая форма критического уравнения. В этом случае

\[
\psi (\kappa_0^2 M^2) = \frac{M^2}{1 + \kappa_0^2 M^2 n},
\]

(12)

\[
\psi^{-1} (\kappa_0^2 M^2) = \frac{1}{M^2} + \frac{\kappa_0^2}{n}.
\]

(13)

Таким образом, экспериментальные результаты по измерению \( \psi^{-1} (\kappa_0^2 M^2) \) можно представить в виде линейной зависимости от геометрического параметра \( \kappa_0^2 \), причем начальная ордината прямой определяет площадь миграции, а котангенс угла наклона — значение величины \( n \), отражающей характер замедления в изучаемой среде.

3. ЭКСПЕРИМЕНТАЛЬНАЯ УСТАНОВКА

Для измерения физических параметров активной зоны реакторов типа ВВЭР был специально построен экспериментальный стенд. Стенд включает в себя следующие основные узлы: открытый бак критических сборок, пульт контроля управления и защиты критической сборки, две емкости на 40 м³ каждая. Одна из них предназначена для хранения дистиллированной воды, а другая для хранения и приготовления раствора борной кислоты.

Стенд оборудован двумя насосами, системой трубопроводов и задвижек, лебедками для перемещения органов управления и защиты и люками аварийного сброса воды из бака критических сборок. Все баки, арматура и трубопроводы выполнены из нержавеющей стали.

Бак критических сборок представляет собой сварную цилиндрическую конструкцию, покрытую снаружи слоем теплоизоляции. На дне бака находится теплообменник, позволяющий подогревать активную зону паром до температуры 90—95°С. На теплообменник установлена решетка, имеющая 349 втулок, в которые входят хвостовики кассет. Для аварийного сброса воды в баке предусмотрены 3 люка, расположенные на различной высоте от дна бака. Они могут быть открыты дистанционно с пульта управления. Бак критических сборок оборудован двумя измерительными системами, предназначенными для контроля и измерения уровня воды в баке. Один из уровнемеров поплавкового типа предназначался для грубого (точность ~1 мм) измерения уровня, другой, электронный прецизионный уровень-номер, служил для точного определения изменения уровня воды (точность ~0,01 мм).

Для регулирования мощности и аварийного прекращения цепной реакции использовались восемь поглощающих стержней. Эти стержни представляли собой кадмевые полосы, покрытые тонким слоем нержавеющей стали. Стержни размещались между кассетами критической сборки так, что при взводе всех стержней активная зона становилась однородной.
Пульт управления и контроля отделен от бака критических сборок бетонной защитой со смотровым окном, залитым водой. На пульте управления расположены регистрирующие приборы контрольно-измерительной аппаратуры, кнопки управления насосами, ключи и лебедки управления стержнями аварийной защиты и ручного регулирования, а также световая и звуковая сигнализация.

Тепловыделяющие элементы активной зоны реакторов типа ВВЭР представляют собой циркониевые трубы внешним диаметром 10,2 мм с толщиной стенки 0,6 мм, заполненные таблетками из спеченной двуокиси урана. Диаметр таблетки составляет 8,7 мм, плотность двуокиси урана ≈ 9,9 – 10,1 г/см³. Тепловыделяющие элементы группируются в рабочих кассетах по 91 элементу в каждой. Некоторые кассеты содержат по 90 элементов. В этих кассетах центральный элемент заменен полой циркониевой трубкой. Тепловыделяющие элементы в кассете образуют треугольную решетку с шагом 14,3 мм. Толщина стенки кассеты 2,1 мм. Длина активной части кассет составляет 2500 мм. Кассеты в активной зоне образуют треугольную решетку с шагом 147 мм. Схема кассеты и общий вид ее приведены в работах [1, 2]. Соотношение между объемами UO₂, воды и циркониевого сплава в сборках равно 1:1,78:0,45.

Первоначальные опыты по определению эффективности органов регулирования проводились на поглотителях-макетах. Эти макеты имеют шестигранную форму с тем же внешним размером, что и топливная кассета. Кадмиевая стенка толщиной 0,5 мм плотно прилегает с внутренней стороны к алюминиевому шестигранному чехлу с толщиной стенки 1 мм. Внутри шестигранного чехла находится вода. Такая конструкция по эффективности близка к штатному поглотителю и в то же время достаточно удобна для теоретического описания.

4. ОПИСАНИЕ ЭКСПЕРИМЕНТОВ

1) Измерение величины \( \delta \rho / \delta h = f(h_k) \)

Для определения \( M^2, k_{\infty} \), а также величины \( \rho \) были проделаны эксперименты с "чистыми сборками", состоящими из кассет 1,5 и 2-процентного обогащения и с одной из смешанных сборок, состоящей из кассет с 2-процентным и естественным ураном с соотношением чисел тех и других кассет 1:1.

Эксперимент по определению \( \delta \rho / \delta h = f(h_k) \) производился следующим образом. В бак критических сборок устанавливалось максимально имеющееся количество кассет какого-либо обогащения. Достигалась критическое состояние системы медленным повышением уровня воды в баке. Затем измерялась функция реактивности в зависимости от уровня воды в баке.

Реактивность \( \rho \) определялась по установившемуся периоду реактора на основании формулы "обратных часов". При этом расчетным путем учитывался вклад в цепную реакцию мгновенных и запаздывающих нейтронов. Выходы запаздывающих нейтронов и постоянные распада ядер-эмиттеров запаздывающих нейтронов для U²³⁵ и U²³⁶ взяты из работы [3]. Функция \( \rho(h_k) \) экстраполировалась к значению \( \rho(h_k) = 0 \). Таким образом опреде-
ИССЛЕДОВАНИЕ ФИЗИЧЕСКИХ СВОЙСТВ АКТИВНОЙ ЗОНЫ

Сборки из кассет 1,5-процентного обогащения.

Исследовался критический уровень, соответствующий данной загрузке. При этом вводилась расчетная поправка, учитывающая конечное изменение уровня воды над соответствующим значением \( h_k \). Для иллюстрации на рис. 1 показана функция \( \rho(h) \) для одной из сборок. Далее из системы извлекалось некоторое количество кассет и опыты повторялись. Таким образом, величина \( \frac{\delta \rho}{\delta h} \) была измерена для 6—8 уровней, соответствующих различному количеству загруженных кассет каждого обогащения.

Полученные на опыте результаты измерения величин \( \frac{\delta \rho}{\delta h} \) обрабатывались на быстродействующей электронной счетной машине. Методом наименьших квадратичных отклонений определялся наклон функции \( \frac{\delta \rho}{\delta h} \) в зависимости от высоты \( h_k \). Из наклона функции \( \frac{\delta \rho}{\delta h} \) определялась величина \( \psi(k^2 M^2) \); экстраполяция функции \( \frac{\delta \rho}{\delta h} \) к значению \( \frac{\delta \rho}{\delta h} = 0 \) дает величину суммарной аксиальной добавки отражателя \( \delta_2 \). Функции \( \frac{\delta \rho}{\delta h} \) для сборок с различным обогащением кассет приведены на рис. 2, 3 и 4.

Кроме вышеописанных экспериментов было проведено измерение величины \( \frac{\delta \rho}{\delta h} = f(h_k) \) для сборок, состоящих из кассет 2- и 1,5-процентного обогащения, когда в качестве замедлителя использовался слабый раствор борной кислоты.

Опыты были проведены при двух концентрациях \( \text{H}_3\text{BO}_3 \) (1,2 г/л и 1,55 г/л). Методика проведения этих опытов была такой же, как и в экспериментах с "неотравленной" водой. Измеренные при этом функции \( \frac{\delta \rho}{\delta h} \) приведены на рис. 5 и 6.
Сборки из кассет 2-процентного обогащения. \((\delta \rho / \delta h)^{1/4} = A h + B\).

- сборки из 2-процентных кассет с водяным отражателем:
  \(B = 1,64 \pm 0,11; A = 0,1098 \pm 0,0019; \delta_z = 15,0 \pm 1,3\) см.

- сборки из 2-процентных кассет с отражателем из естественных кассет:
  \(A = 0,1083 \pm 0,0009; B = 1,76 \pm 0,09; \delta_z = 16,3 \pm 0,9\) см.

Для сборок из кассет с окисью урана 2-процентного обогащения при малом их количестве (12–13), соответствующем большим критическим уровням (\(h_k = 130\) см и \(h_k = 110\) см), наблюдается отклонение поведения функции \((\delta \rho / \delta h)^{1/4} = f(h_k)\) от прямой.

Это, повидимому, объясняется тем, что систему, состоящую из малого количества кассет, нельзя рассматривать как эквивалентный реактор без отражателя с учетом экстраполированной добавки радиального отражателя. В этом случае размер системы (радиус ~25 см) сравним с длиной экстраполированной добавки по радиусу (\(\delta_r = 7\) см) и прилежащая к границе системы область, в которую распространяются возмущения, вносимые в функцию влияния отражателем, становится сравнимой с размером активной зоны. Для проверки этого предположения был проделан контрольный опыт. В этом опыте кассеты из окиси урана 2-процентного обогащения были окруженны специфическим отражателем, состоявшим из кассет с естественным ураном, в котором были равномерно расставлены полосы из кадмия. Количество кадмия в отражателе было подобрано таким образом, чтобы экстраполированная добавка отражателя приблизительно совпадала с величиной \(\delta_z\) для воды.

Контрольный опыт показал, что в этом случае даже при малом количестве кассет не наблюдается функции \((\delta \rho / \delta h)^{1/4}\) от теоретической прямой. Полученная при этом зависимость \((\delta \rho / \delta h)^{1/4} = f(h_k)\) приведена на рис.2.
ИССЛЕДОВАНИЕ ФИЗИЧЕСКИХ СВОЙСТВ АКТИВНОЙ ЗОНЫ

Рис. 3
Сборки из кассет 1,5-процентного обогащения. \( (\partial p/\partial h)^{-1} = Ah + B \).
А = 0,1062 ± 0,0008; В = 1,86 ± 0,09; \( h_{0} = 17,5 ± 1,0 \) см.

Рис. 4
Смешанные сборки. \( N_{25}:N_{ект} = 1:1 \). \( (\partial p/\partial h)^{-1} = Ah + B \).
А = 0,1045 ± 0,0001; В = 2,06 ± 0,14; \( h_{0} = 19,7 ± 1,5 \) см.
Сборки из кассет 2-процентного обогащения с отравленной (бором) водой. \((\delta p/\delta h)^{1/2} = Ah + B\).

- отравление I.
  \[A = 0,1067 \pm 0,0009; B = 1,85 \pm 0,09; \delta_x = 17,3 \pm 1,0 см.\]
- отравление II.
  \[A = 0,1067 \pm 0,0009; B = 1,73 \pm 0,08; \delta_x = 16,2 \pm 0,9 см.\]

Измерение величины \(\delta p/\delta h\) производилось еще для трех смешанных сборок, состоящих из равномерной смеси:

а) естественных и 1,5-процентных кассет с соотношением чисел кассет 1:2;
б) 2- и 1,5-процентных кассет с соотношением 1:1;
в) 2- и 1,5-процентных кассет с соотношением 1:2.

Эти измерения величины \(\delta p/\delta h\) были сделаны с малой точностью и поэтому результаты их не учитывались при определении формы критического уравнения и определения площади миграции, а были использованы для определения величины \(k_0^{2}\) и \(k_{cm}\) смешанных решеток.

2) Определение величины \(k_0^{2}\)

При проведении вышеописанных экспериментов при некоторых критических уровнях измерялись распределения плотности тепловых и быстрых нейтронов по радиусу и высоте критических сборок.

Измерение производилось с помощью малогабаритных импульсных ионизационных камер деления, где в качестве индикатора тепловых нейтронов использовались тонкие слои урана-235, а в качестве детектора быстрых нейтронов — слои урана, обедненного в 100 раз по изотопу \(U^{235}\). Использовались многослойные цилиндрические камеры с внешним диаметром 8 мм, наполненные аргоном до 20 ата. В рабочем объеме камеры помещают-
ИССЛЕДОВАНИЕ ФИЗИЧЕСКИХ СВОЙСТВ АКТИВНОЙ ЗОНЫ

Сборки из кассет 1,5-процентного обогащения с отравленной (бором) водой. \( (\frac{\partial \rho}{\partial h})^\frac{1}{2} = Ah + B. \)

- отравление I.
  \( A = 0,1040 \pm 0,001; B = 2,11 \pm 0,13; \delta z = 20,3 \pm 1,4 \text{ см.} \)
- отравление II.
  \( A = 0,1029 \pm 0,0008; B = 2,27 \pm 0,10; \delta z = 22,1 \pm 1,2 \text{ см.} \)

С собой слои урана-235 или урана-238, причем в камерах, применяемых для измерения поля по высоте активной зоны, эти слои имели длину около 1 см, а в камерах для измерения поля по радиусу—около 10 см.

При измерении распределения полей нейтронов по высоте критических сборок, камера помещалась в центральную трубку одной из кассет и последовательно перемещалась по высоте.

При измерении распределений по радиусу камера делений последовательно перемещалась из кассеты в кассету.

Для ряда сборок на основании этих измерений определялись эффективные добавки отражателей по радиусу \( (\delta_r) \) и по высоте \( (\delta_h) \). Результаты экспериментов обрабатывались на быстро действующей электронной счетной машине. При этом в расчет принимались лишь те экспериментальные точки, которые лежат в области равновесного спектра нейтронов.

Для измерения величины \( k^2_0 \) для решетки, состоящей из естественных кассет, была собрана трехзонная сборка с кольцевым запалом из кассет 2- и 1,5-процентного обогащения. Центральная зона этой сборки состояла из 37 кассет с естественным ураном. С помощью вышеописанных камер деления были измерены распределения быстрых и тепловых нейтронов по высоте и радиусу центральной зоны (рис.7). Методом наименьших квадратов была подобрана функция \( I_0(k, r) \), описывающая распределение плотности нейтронов по радиусу центральной зоны сборки. Высотная часть геометрического параметра определялась из обработки экспериментальной кривой распределения плотности нейтронов по высоте сборки.
Величина $\kappa_0^2$ для чистых сборок определялась еще одним способом. В процессе проведения экспериментов по определению величины $\delta\rho/\delta h$ для каждой из изучавшихся сборок был получен ряд критических состояний при различном количестве загруженных кассет. Методом наименьших квадратов были подобраны функции

$$\kappa_0^2 = \frac{\pi^2}{(h_k + \delta_e)^2} + \frac{\mu^2}{(R_k + \delta_r)^2},$$

наилучшим образом описывающие эти критические ансамбли. При этом задавалась величина $\delta_e$, определенная из измерения нейтронного поля по высоте, а величины $\kappa_0^2$ и $\delta_r$ определялись в результате обработки уравнения (14). При определении величины $\kappa_0^2$ этим способом в расчет принимались лишь те сборки, которые имели правильную геометрическую форму. Результаты экспериментов по определению величины $\kappa_0^2$ приведены в табл.1. Указанные в таблице величины определены для всех сборок (исключая сборку из кассет с естественным ураном) методом обработки уравнения (14) с использованием значений $\delta_e$, указанных в четвертой графе таблицы.

По данным экспериментов вычислялась функция

$$\psi^{-1}(\kappa_0^2) = \frac{1}{M^2} + \frac{\kappa_0^2}{n}.$$

Экстраполяция этой прямой к значению при $\kappa_0^2 = 0$ дает величину площади
ИССЛЕДОВАНИЕ ФИЗИЧЕСКИХ СВОЙСТВ АКТИВНОЙ ЗОНЫ  

Рис. 8

\[ \phi^{-1} = \frac{1}{M^2} + \frac{k^2}{\bar{n}}; \quad M^2 = 49,7 \pm 0,9; \quad \bar{n} = 1,04 \pm 0,08. \]

миграции, а наклон — величину \(1/\bar{n}\) (рис. 8). Обработка прямой способом наименьших квадратов дает следующее:

\[ M^2 = 49,7 \pm 0,9 \text{ см}^2; \quad \bar{n} = 1,04 \pm 0,08. \]

На основании полученных данных были вычислены величины коэффициента размножения \(k\) для различного типа сборок. Эти величины приведены в табл. 2. На рис. 9 величины \(k_n\) и \(k_p\) представлены как функции обогащения урана изотопом \(U^{235}\). Для сборок, состоящих из различного сорта кассет, обогащение усреднено с учетом количества кассет того и другого сорта.

Результаты, приведенные в табл. 2 были получены на основании данных из экспериментов по "надkritической достройке". В этих экспериментах компенсация надkritичности системы после очередного замера производилась путем разгрузки системы по радиусу, т.е. непосредственным изменением величины радиального геометрического параметра. Однако в ряде случаев такое изменение радиальных размеров активной зоны оказывается недоступным или нежелательным. В связи с этим встает вопрос о возможности эквивалентного изменения величины геометрического параметра каким-либо иным способом. Для этой цели наиболее естественным представляется введение в активную зону поглощающих стержней или создание в ней водяных полостей, изменяющих утечку нейтронов в радиальном направлении.

В связи с этим были проведены соответствующие опыты на сборках из кассет 1,5-процентного обогащения.

После измерения дифференциальной реактивности при определенном уровне воды в систему вводилось то или иное число поглотителей (либо
Таблица 1

РЕЗУЛЬТАТЫ ЭКСПЕРИМЕНТОВ ПО ОПРЕДЕЛЕНИЮ ВЕЛИЧИНЫ $k_0^2$

<table>
<thead>
<tr>
<th>Тип сборки</th>
<th>$k_0^2$, см$^{-4}$</th>
<th>Экстраполированная добавка по высоте, см</th>
<th>Экстраполированная добавка по радиусу, см</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Из экстраполяции $(b/b_0)^1 = 0$</td>
<td>Из измерений поля по высоте</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Естественные кассеты</td>
<td>25,9 ±0,3</td>
<td>19,7 ±0,8</td>
<td>-</td>
</tr>
<tr>
<td>Кассеты 1,5%</td>
<td>36,1 ±0,3</td>
<td>17,5 ±1,0</td>
<td>16,9 ±0,5</td>
</tr>
<tr>
<td>Кассеты 2%</td>
<td>56,7 ±0,5</td>
<td>16,8 ±1,2</td>
<td>13,9 ±0,3</td>
</tr>
<tr>
<td>Кассеты 2% с отражателем из естественных кассет</td>
<td>57,0±0,7</td>
<td>19,2±0,9</td>
<td>-</td>
</tr>
<tr>
<td>Смесь кассет: 1,5% и 2%; 1:1</td>
<td>47,1±0,7</td>
<td>-</td>
<td>15,5±0,5</td>
</tr>
<tr>
<td>Смесь кассет: 1,5% и 2%; 2:1</td>
<td>43,4±0,5</td>
<td>-</td>
<td>15,9±0,5</td>
</tr>
<tr>
<td>Смесь кассет 1,5% и естественных кассет; 2:1</td>
<td>16,7±0,1</td>
<td>-</td>
<td>19,7±0,5</td>
</tr>
<tr>
<td>Смесь кассет 2% и естественных кассет; 1:1</td>
<td>18,2±0,1</td>
<td>19,7±1,5</td>
<td>19,5±0,5</td>
</tr>
<tr>
<td>Кассеты 2% (отравление I)</td>
<td>42,3±0,5</td>
<td>17,3±1,0</td>
<td>16,9±0,5</td>
</tr>
<tr>
<td>Кассеты 1,5% (отравление I)</td>
<td>23,3±0,3</td>
<td>20,2±1,4</td>
<td>19,2±0,5</td>
</tr>
<tr>
<td>Кассеты 2% (отравление II)</td>
<td>38,0±0,5</td>
<td>16,2±0,9</td>
<td>-</td>
</tr>
<tr>
<td>Тип сборки</td>
<td>$a \cdot 10^4$, см$^{-2}$</td>
<td>Экстраполированная добавка по высоте, см</td>
<td>Экстраполированная добавка по радиусу, см</td>
</tr>
<tr>
<td>---------------------</td>
<td>-------------------------</td>
<td>------------------------------------------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Из экстраполяции $(\lambda_0 / \lambda h) = 0$</td>
<td>Из измерений поля по высоте</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Кассеты 1,5%</td>
<td>19,3 ± 0,3</td>
<td>22,1 ± 1,2</td>
<td>-</td>
</tr>
</tbody>
</table>

* Использовано значение $\lambda_0 = 13,9 ± 0,3$ см.
### Таблица 2

ВЕЛИЧИНА КОЭФФИЦИЕНТА РАЗМНОЖЕНИЯ \( k^* \) ДЛЯ РАЗЛИЧНОГО ТИПА СБОРОК

<table>
<thead>
<tr>
<th>Тип сборки</th>
<th>Естественные кассеты</th>
<th>Кассеты 1,5%</th>
<th>Кассеты 2%</th>
<th>Кассеты 2% с отражателем из естественных кассет</th>
<th>Смесь кассет 1,5% и 2%; 1:1</th>
<th>Смесь кассет 1,5% и 2%; 2:1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k^* )</td>
<td>0,867 ± 0,005</td>
<td>1,179 ± 0,007</td>
<td>1,282 ± 0,006</td>
<td>1,283 ± 0,007</td>
<td>1,234 ± 0,006</td>
<td>1,216 ± 0,005</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Тип сборки</th>
<th>Смесь кассет 1,5% и естественных кассет; 2:1</th>
<th>Смесь кассет 2% и естественных кассет; 1:1</th>
<th>Кассеты 2% (отравление I)</th>
<th>Кассеты 1,5% (отравление I)</th>
<th>Кассеты 2% (отравление II)</th>
<th>Кассеты 1,5% (отравление II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k^* )</td>
<td>1,083 ± 0,002</td>
<td>1,090 ± 0,002</td>
<td>1,210 ± 0,005</td>
<td>1,116 ± 0,003</td>
<td>1,193 ± 0,004</td>
<td>1,096 ± 0,002</td>
</tr>
</tbody>
</table>
часть кассет заменялась водяными полостями), затем производилось очередное измерение дифференциальной реактивности для другого уровня воды и т.д. Результаты эксперимента аппроксимировались линейной зависимостью \( (\delta\rho/\delta h)^{-1} = Ah + B \).

При обработке опытных данных учитывалось эффективное изменение величины радиального геометрического параметра, вызванное введением поглотителей. Результаты обработки этих опытов приведены на рис. 10. Наклоны прямых, характеризующих зависимость дифференциальной реактивности от высоты, для всех указанных способов изменения величины радиального геометрического параметра практически совпадают. Величина \( \psi(k_g^2 M^2) \), полученная таким способом совпадает с величиной \( \psi(k_g^2 M^2) \), полученной в эксперименте по надкритической достройке с разгрузкой по радиусу для сборки, состоящей из 1,5-процентных кассет. Таким образом, введение в систему поглощающих стержней или замена части топливных кассет водяными полостями эквивалентны разгрузке системы по радиусу.

В результате обработки экспериментов, проведенных на сборках, состоящих из кассет 1,5-процентного обогащения и кадмиевых шестиугольных
374. Г.Я. АНДРИАНОВ и др.

Сборки из кассет 1,5-процентного обогащения.
Δ, □ — сборки из кассет 1,5-процентного обогащения и кадмиевых поглотителей.
\((\Theta_\alpha/\Theta_\beta)^4 = A\beta + B\). 
\(A = 0,1059 \pm 0,0009; B = 1,93 \pm 0,09; \delta_\alpha = 18,2 \pm 1,0 \text{ см.}\)
о — сборки из кассет 1,5-процентного обогащения.

поглотителей, были получены следующие значения \(k_m\) и \(M^2\) (использовано одногрупповое критическое уравнение):

\[
\begin{align*}
  k_m &= 1,182 \pm 0,005; \\
  M^2 &= 50,5 \pm 1,0 \text{ см}^2.
\end{align*}
\]

В пределах ошибок значения этих параметров совпадают с соответствующими величинами, приведенными в табл. 2 для системы из 1,5-процентных кассет. Такое согласие указывает на то, что даже при наличии в уранводных системах рассматриваемого типа поглощающих стержней достаточно больших размеров в среде между поглотителями справедливо одногрупповое диффузионное приближение.

3) Сравнение эксперимента с расчетом

В процессе проектирования реактора ВВЭР-1 проводились расчеты величин \(k_m\), \(k_0^2\) и \(M^2\) для решеток, содержащих уран с различным обогащением. В табл. 3 приведено сравнение расчетных данных с результатами экспериментов.
ИССЛЕДОВАНИЕ ФИЗИЧЕСКИХ СВОЙСТВ АКТИВНОЙ ЗОНЫ

Расчет величин \( k_{\infty} \) для "чистых" сборок проводился по методике, изложенной в работах [4 и 5]. Эта методика учитывает особенности энергетического спектра нейтронов в уран-водных размножающих системах.

Расчет величин \( k_{\infty} \) для "смешанных" решеток проводился для систем, собранных из кассет двух разновидностей, различающихся обогащением урана. При расчете предполагалось, что размещение кассет одного сорта среди кассет другого практически равномерное, так что активная зона имеет закономерно повторяющуюся структуру. Вычисление \( k_{\infty} \) для таких "смешанных" решеток выполнялось по следующей формуле:

\[
k_{\infty} = \frac{k_{\infty 1} N_1 + k_{\infty 2} N_2}{N_1 + N_2} + \Delta k_{\infty 0} = k_{\infty 0} + \Delta k_{\infty 0},
\]

где \( k_{\infty 1} \) и \( k_{\infty 2} \) - коэффициенты размножения для кассет 1-го и 2-го сорта соответственно;

\( N_1, N_2 \) - числа кассет 1-го и 2-го сорта в системе;

\( \Delta k_{\infty 0} \) - поправка к \( k_{\infty 0} \), учитывающая перетечку тепловых нейтронов между кассетами разных сортов.

Эта поправка может быть легко рассчитана, если известны функции, описывающие микроход плотности нейтронного поля по кассетам активной зоны.

Из табл. 3 следует, что с учетом возможных ошибок эксперимента наблюдается удовлетворительное согласие расчетных и экспериментальных результатов.

Таблица 3

<table>
<thead>
<tr>
<th>Решетки</th>
<th>Эксперимент</th>
<th>Расчет</th>
</tr>
</thead>
<tbody>
<tr>
<td>Кассеты 2%</td>
<td>49,7 ± 0,9</td>
<td>47,4</td>
</tr>
<tr>
<td>Кассеты 1,5%</td>
<td>49,7 ± 0,9</td>
<td>47,9</td>
</tr>
<tr>
<td>Смесь кассет 2% и 1,5%; 1:1</td>
<td>51,4 ± 0,9</td>
<td>49,3</td>
</tr>
<tr>
<td>Кассеты с естественным ураном</td>
<td>49,7 ± 0,9</td>
<td>49,7</td>
</tr>
<tr>
<td>Смесь кассет 1,5% и 2%; 2:1</td>
<td>49,7 ± 0,9</td>
<td>47,7</td>
</tr>
<tr>
<td>Смесь кассет 2% и естественных кассет; 1:1</td>
<td>49,7 ± 0,9</td>
<td>47,7</td>
</tr>
<tr>
<td>Смесь кассет 1,5% и естественных кассет; 2:1</td>
<td>49,7 ± 0,9</td>
<td>47,7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>10^4, см^{-2}</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Эксперимент</td>
<td>57 ± 0,7</td>
<td>1,283 ± 0,007</td>
</tr>
<tr>
<td>Расчет</td>
<td>58</td>
<td>1,281</td>
</tr>
<tr>
<td>Эксперимент</td>
<td>36,1 ± 0,5</td>
<td>1,179 ± 0,004</td>
</tr>
<tr>
<td>Расчет</td>
<td>34,5</td>
<td>1,172</td>
</tr>
<tr>
<td>Эксперимент</td>
<td>47,1 ± 0,5</td>
<td>1,234 ± 0,005</td>
</tr>
<tr>
<td>Расчет</td>
<td>-</td>
<td>1,230</td>
</tr>
<tr>
<td>Эксперимент</td>
<td>40,4 ± 0,5</td>
<td>1,216 ± 0,005</td>
</tr>
<tr>
<td>Расчет</td>
<td>-</td>
<td>1,215</td>
</tr>
<tr>
<td>Эксперимент</td>
<td>18,2 ± 0,1</td>
<td>1,090 ± 0,007</td>
</tr>
<tr>
<td>Расчет</td>
<td>-</td>
<td>1,100</td>
</tr>
<tr>
<td>Эксперимент</td>
<td>16,7 ± 0,1</td>
<td>1,083 ± 0,005</td>
</tr>
<tr>
<td>Расчет</td>
<td>-</td>
<td>1,088</td>
</tr>
</tbody>
</table>
4) Измерение эффективности поглотителей

На сборках, состоящих из 85 кассет 1,5-процентного обогащения, подробно была изучена эффективность поглотителей компенсирующих кассет (КК), выполненных из кадмия. Поглотители размещаются в сборке вместо извлекаемых топливных кассет. Картограммы размещения поглотителей приведены на рис. 11. Путем подъема уровня воды в баке достигается критическое состояние сборки. Эффективность регулирующих стержней получается путем интегрирования зависимости \( \frac{\delta \rho}{\delta h} = f(h) \) в пределах от \( H_1 \) до \( H_2 \), причем \( H_1 \) и \( H_2 \) - критические высоты сборки без поглотителей и с поглотителями

\[
\Delta \rho = \int_{H_1}^{H_2} \frac{\delta \rho}{\delta h} \, dh .
\] (15)
Для однородных сборок выражение (15) можно заменить следующим:

\[ \Delta \rho = \frac{\Delta k^2 M^2}{\kappa_\infty}, \]

где

\[ \Delta k^2 = \frac{\pi^2}{(H_1 + \delta^2)^2} - \frac{\pi^2}{(H_2 + \delta^2)^2}. \]

Величина отношения \( M^2/\kappa_\infty \) находится из наклона прямых \( (\delta \rho/\delta h)^{-1} = f(h_k) \), а суммарная осевая добавка отражателя берется из обработки экспериментов по измерению распределения нейтронов по высоте сборки.

Рис. 12

Эффективность поглотителей "КК" (эксперимент и расчет). Поглотители расположены по кольцу на радиусе \( r_0 \) от оси сборки. \( R/M = 11,3; r_1/R = 0,094 \)

\( r_0 \) — радиус расположения поглотителя;
\( r_1 \) — эквивалентный радиус шестигранного кольцевого поглотителя;
\( R \) — радиус сборки с учетом добавки радиального отражателя;
\( M \) — длина миграции;
\( N \) — число поглотителей, которые расположены на радиусе \( r_0 \) от оси сборки;
\( N + 1 \) — число стержней с учетом стержня, расположенного на оси сборки.

Результаты экспериментов по измерению эффективности поглотителей при различном их размещении в сборке представлены на рис.12,13. Здесь приведены эффективности отдельно взятых стержней, а также групп стержней, симметрично расположенных вокруг оси сборки. Поглощающая способность стержней сильно зависит от взаимного расположения и числа поглотителей. Величина коэффициента интерференции, определяемого как отношение эффективности группы стержней к сумме эффективностей отдельно взятых поглотителей, полученной в отсутствии остальных, изменяется в функции от \( r_0/R \) от 0,9 до 1,7 и проходит через максимум. По-
Эффективность поглотителей "КК" (эксперимент и расчет). Поглотители расположены по кольцу на радиусе \( r_0 \) от оси сборки. \( R/M = 11,3; \ r_1 / R = 0,094 \)

\( r_0 \) — радиус расположения поглотителя;
\( r_1 \) — эквивалентный радиус шестигранного кольцевого поглотителя;
\( R \) — радиус сборки с учетом добавки радиального отражателя;
\( M \) — длина миграции;
\( n \) — число стержней с учетом стержня, расположенного на оси сборки.

Положение максимума зависит от присутствия центрального \((N+1)\)-го поглотителя.

Экспериментальная точка, соответствующая семи поглотителям (один в центре и шесть на расстоянии \( r_0 / R = 0,49 \)), получена из измерений осевого распределения плотности нейтронов в опыте с частично извлеченным центральным поглотителем. В этом случае при полностью погруженном центральном поглотителе сборка была подкритична.

На рис.12 и 13 сплошной линией показаны результаты расчетов, выполненных по методике, изложенной в работе [6]. Следует отметить хорошее согласие эксперимента с расчетом.

ВЫВОДЫ

На основании результатов, полученных в экспериментах с критическими сборками из кассет активной зоны реакторов ВВЭР, можно сделать следующие выводы:

1. Результаты экспериментов с однородными мультиплицирующими решетками хорошо описываются одногрупповым критическим уравнением вплоть до значений материального параметра \( k_0^2 = 60 \cdot 10^{-4} \text{см}^{-2} \). Величина \( n \), отличие которой от единицы характеризует степень отклонения условий критичности от одногруппового, оказались равной \( 1,04 \pm 0,08 \).
2. Результаты экспериментов по определению $k_\infty$ и $k_0$ для "чистых" сборок, состоящих из кассет 1,5- и 2-процентного обогащения, а также для смешанных сборок указывают на удовлетворительное согласие с расчетом.

3. Измерение эффективности поглотителей показало, что введение стержней можно рассматривать как изменение геометрического параметра реактора. Экспериментальные значения эффективности хорошо согласуются с расчетными результатами, полученными по методике, изложенной в работе [6]. Это согласие позволяет более широко использовать расчеты определения эффективности стержней в различных состояниях реактора ВВЭР.

В заключение авторы выражают благодарность руководителям работ Скворцову С.А. и Лазукову Н.А. за постоянный интерес, проявляемый к работе и за ряд ценных указаний, полученных во время подготовки доклада.

**ЛИТЕРАТУРА**


IX

ORGANIC-MODERATED OR COOLED ASSEMBLIES
1. INTRODUCTION

In the autumn of 1957 a feasibility study of the D\(_2\)O-moderated organic-cooled power reactor concept, called DOR, was initiated at the Research Establishment Risø of the Danish Atomic Energy Commission. After initial survey calculations it became evident that to make reliable predictions as to the reactor physics properties of this reactor type it would be necessary to have available results of measurements of this kind of reactor lattice, with which the theory used could be compared. The results of a number of investigations on D\(_2\)O moderated and cooled lattices could be found in the literature, but because of the fact that the DOR concept uses a coolant, a mixture of terphenyls, which is a much better moderator than the D\(_2\)O moderator, it was considered insufficient to compare the theoretical methods used with pure D\(_2\)O lattices only. The use of a very efficient moderator as coolant will have influence on the resonance capture due to strong slowing-down in the coolant, that is in the fuel channel, and for the same reason also on the thermal neutron flux distribution in the lattice cell. In addition, as the coolant in the reactor is at a high temperature, (of the order of 300°C) while the moderator is only at around 70°C, some thermal neutrons coming from the cold moderator will be scattered by hydrogen nuclei in the hot coolant, whereby the energy of the neutrons will be increased. This means a hardening of the neutron spectrum.

For these reasons and because no experimental data on DOR lattices could at that time be obtained elsewhere, it was decided to carry out experimental investigations in this field at Risø as part of the DOR project.
2. SELECTION OF TYPE OF EXPERIMENT

2.1. Possible types of experiments

There exists a number of types of experiment which can give information on the properties of reactor lattices, e.g. exponential experiments, critical experiments, "just-subcritical" experiments and measurements on small assemblies using either a pulsed or a constant neutron source. Of these the "just subcritical" experiment was discarded at once, because the cost and construction time for such an experiment would amount to almost the same as for a critical facility, while the amount of information that could be obtained is considerably less. The use of small assemblies with either pulsed or constant neutron sources was also discarded immediately because of the very heterogeneous lattice to be considered, the need for rather advanced techniques, at least in the case of a pulsed neutron source, and the belief that sufficient information would not be obtained.

With respect to the remaining two possibilities, the exponential and the critical experiment, a number of pro's and con's was taken into account in making the final decision. These are discussed below.

2.2. Advantages of exponential versus critical experiments

A very important advantage of an exponential experiment is its cost which is considerably lower than that of a critical experiment. As indicated in section 4.2, experience has shown that the price of a D₂O exponential facility including D₂O and fuel is of the order of US $0.5 million, while it has been estimated that the cost of a D₂O critical facility would be at least three times as much due to the larger amounts of materials required and the larger and more complicated construction work involved. It should also be noticed that the operation cost of an exponential experiment is considerably less than that of a critical experiment, among other reasons because of the lower staff requirement.

Another important advantage is that the time required for the construction of an exponential facility is significantly shorter than the construction time for a critical experiment. This is partly a result of the difference in size of the two facilities and partly due to the fact that while for a critical facility it is usually necessary to construct a special building with the necessary shielding arrangements, this is not the case for an exponential experiment.

A great advantage in the use of an exponential experiment is that the safety restrictions are much less severe than for a critical facility. By designing the experiment in such a way that it under no circumstances will go critical with the fuel available, the safety precautions against criticality can be limited to literally none. This makes, for example, changes in the lattice configurations much easier and speeds up the operation of experiment.

2.3. Disadvantages of exponential versus critical experiments

There are however also a number of disadvantages connected with exponential experiments as compared with critical ones. One of the most
important is the poorer accuracy which can be obtained in buckling measurements. In a critical facility this type of measurement is usually based on well-defined water level measurements, while for an exponential experiment it is based on more inaccurate flux distribution measurements, in particular as the calculation of the buckling involves taking the difference between two almost equal terms. For low buckling lattices, however, such as DOR lattices with natural uranium, only very large (and very expensive) critical facilities can be used for criticality measurements with uniform lattices, and therefore the substitution method, which is less accurate than a criticality measurement with full lattice, will in practice always be used.

Another important disadvantage of the exponential experiment is that the flux level in such an experiment is usually considerably lower than the one that can be obtained in a critical facility. This means that a number of measurements, such as the determination of resonance escape probability and neutron temperature, are either impossible or very difficult to carry out in an exponential facility. Because of the lower flux level the exponential experiment is also more sensitive to a number of disturbing effects, e.g. spontaneous fission. In the case of the DOR lattice investigations this point was, however, not considered essential, since a very powerful neutron source, the Danish reactor DR 1, was available for an exponential experiment.

Because an exponential facility cannot be brought to criticality, certain kinds of experiment cannot be carried out in this type of facility. For example, this is true for kinetic investigations and control rod reactivity worth measurements. The value of kinetic investigations with a critical facility for design work on a power reactor is, however, rather limited because of the much lower power and usually lower temperatures and smaller size.

Finally the fact that an exponential experiment is smaller in size makes it more sensitive to geometrical irregularities, such as incorrect pitch values. Also anisotropic effects, such as streaming in voided channels, are more correctly represented in the larger critical facility.

2.4. Conclusion

Based on the foregoing points it was decided to construct an exponential experiment. The main reasons for this choice were that the costs involved would be lower, that the construction time would be shorter whereby results would be available faster, and that an exponential experiment was found to be able to give the information desired, i.e. experimental values of reactor physics constants of DOR lattices.

The exponential experiment which was subsequently constructed was named EXPO.

3. DESCRIPTION OF EXPO

3.1. Mechanical structure

In Fig. 1 a vertical cross-section of the EXPO facility is shown. As may be seen, EXPO is placed on top of the DR 1, which serves as a neutron
source for the experiment. DR 1 is a small homogeneous reactor with a maximum power of 2.3 kW. The core which is contained in a spherical 30.5-cm diameter stainless steel vessel, is composed of 7600 g of 20% enriched uranyl sulphate dissolved in H₂O. The core is imbedded in the middle of a 132-cm high, 152-cm diameter graphite cylinder, which serves as a reflector.

On top of the reactor is placed a steel cover which, in case of an accident, will prevent water from penetrating into the reactor.

An aluminium shutter tank is positioned on top of the cover plate. This tank, which has a height of approximately 60 cm, can be filled with H₂O to provide the necessary shielding when the reactor is used for other purposes than as a neutron source for EXPO.

Above the shutter tank is a cylindrical steel tank containing a 70-cm high graphite pedestal. The bottom flange of the cylinder rests on a recess in the top concrete shield of the reactor. The inside bottom of the cylinder is a 10-mm aluminium plate resting on a protruding inner rim at the bottom of the cylinder. The graphite pedestal also rests on this rim, on top of the aluminum plate. The inside walls of the cylinder are lined with boron plastic to reduce the activation of the steel. As the steel tank is cylindrical, while the opening in the top shield is quadratic, the space between the top shield and the steel cylinder is filled with concrete blocks to improve the shielding.
The experimental tank rests on the top flange of the steel cylinder. It has a diameter of 152 cm and a height of 180 cm. The tank is made of a 6-mm aluminium plate and is clad on the outside with a layer of boron plastic.

The top of the tank is covered by a steel box, in which are movable beams carrying the fuel elements. The movement of the beams is performed by means of two parallel screw spindles placed inside the top box, but operable from the outside and the fuel elements can be moved along the beams by means of sealed manipulator rods. This design is a modified version of the fuel manipulation system of the Swedish ZEBRA facility [1]. The roof of the top box is made of Plexiglass which allows visual inspection of the arrangement in the tank from the manoeuvring platform surrounding the top box. A number of aluminium lids in the Plexiglass cover permits the introduction and removal of fuel elements as can be seen in Fig. 2.

3.2. D$_2$O system

When not in use the heavy water is kept in a 3500-1 stainless steel storage tank that may be completely shut off from the rest of the system. The heavy water is pumped into the exponential tank by a pump which may also be used to circulate the D$_2$O between the exponential tank and a heating and cooling system.
The heavy water is purified by circulating it through a small loop containing an ion-exchange filter. The purity of the D$_2$O is measured by conductivity cells positioned at the entrance and the exit of the purification loop.

3.3. Protective gas system

To protect the heavy water from contamination with light water, a protective atmosphere of nitrogen is maintained in the top box at an overpressure of 100 mm water head.

The nitrogen, which is supplied from a pressure bottle of dry nitrogen, flows first through a reduction valve which reduces the pressure to 1.5 atm, then through a silica gel filter and finally through a second reduction valve before it enters the experimental tank.

The protective atmosphere is also maintained in the drying-out system. This system makes it possible to heat and circulate the gas in the system and to pass it through a water or liquid air-cooled condenser to remove moisture. The system is used to remove H$_2$O when the experimental tank has been opened to the outer atmosphere for a prolonged time, but can also be used to remove D$_2$O.

A diagram of the piping system is shown in Fig. 3.

3.4. Instrumentation

The experiment is provided with a central, vertically moving BF$_3$ counter which automatically measures the flux in a preset number of positions. To correct for changes in reactor power during measurements a fixed BF$_3$ reference counter is placed in the graphite pedestal. The maximum neutron flux at the bottom of the experimental tank is $4 \times 10^7$ n/cm$^2$ s at a reactor power of 2 kW.

The movable BF$_3$ counter may be exchanged with a vertical central foil holder which is used for both flux plotting, Cd-ratio measurements, neutron temperature measurements and other foil measurements.

The remaining instrumentation and technical installations have mainly to do with the safe containment and handling of the D$_2$O to avoid losses and contamination with H$_2$O.

The D$_2$O level in the exponential tank is measured by a level indicator consisting of a tubular condenser of 100-mm length, which can be mounted at different levels in the experimental tank. The change in capacity by the water entering the condenser is indicated on a meter registering between 0 and 100% filling of the condenser. An alarm may be activated when the D$_2$O level passes beyond a preset interval.

The alarm will also be activated in case the pressure of the protecting nitrogen atmosphere in the experimental tank passes beyond a preset range. All flanges and other joints in the D$_2$O system are provided with leak detectors, consisting of foil-electrodes wrapped around the flanges and isolated from these by filter paper. In case of leaks, alarm is given. Finally, the alarm is activated if too high temperatures should appear in the D$_2$O system.

The alarm consists of both an audible and a visual signal and is given both in the reactor hall and the reactor control room, where the instrumentation panel of the facility is situated.
3.5. Fuel elements

The fuel elements consist of 19-rod clusters in aluminium "pressure" tubes which may or may not be surrounded by aluminium "calandria" tubes. The rods consist of 1/2-in diameter fuel rods, canned in 1-mm thick aluminium tubes. Both natural uranium metal and natural UO₂ rods are available. The "pressure" tube may be filled with different fluids. When simulating DOR conditions Diphyl (identical to Dowtherm A) is used.

Figure 4 gives a view of a partly dismantled fuel element.

4. CONSTRUCTION TIME SCHEDULE AND COSTS

4.1. Time schedule

Preliminary design investigations on EXPO were carried out during the summer and autumn of 1958. The actual mechanical design was started early in 1959 and went on until the spring of 1960. The procurement and manufacture of the components began during the summer of 1959 and the assembling work started in October 1959. In August 1960 the mechanical construction of the facility was finished and during the autumn test operations were carried out using light water. In November 1960 the D²O was brought into the EXPO facility. In January 1961 fuel was for the first time loaded into the experimental tank.
4.2. Cost breakdown

The breakdown of the total cost of EXPO is shown in Table I. 
As 1 Danish kr. is equal to approximately US $ 0.15, the total cost 
figure corresponds to US $ 0.45 million.

The design effort comprised 24 man-months of graduate engineers, 
32 man-months of non-graduate engineers and 6 man-months of technical 
assistants.

It should be noticed that no overhead has been included in the cost of 
the design work and for the part of the mechanical system and the electronic 
and control system manufactured at Risø.

5. EXPERIMENTAL PROGRAMME

5.1. Fuel configurations

As mentioned above, the main purpose with the construction of EXPO 
was to obtain empirical data suited for checking the reactor physics calcu-
lations on the DOR reactor project.

It was felt that to check the theoretical methods used it would be desir-
able to be able to carry out measurements on a number of fuel-coolant com-
binations. For this reason two sets of fuel rods, the one containing natural 
uranium metal, the other natural UO$_2$, were procured.

In the present programme the following combinations have been, or are 
being, examined:

UMA series: natural uranium metal, 19-rod cluster surrounded by 
air and contained in a pressure tube.

UM series: natural uranium metal, 19-rod cluster surrounded by 
Diphyl (Dowtherm A) and contained in a pressure tube.

UMC series: same as UM but with additional air gap and calandria tube 
surrounding pressure tube.

UOA series: same as UMA but with natural UO$_2$ instead of metal.

UO series: same as UM but with natural UO$_2$ instead of metal.

UOC series: same as UMC but with natural UO$_2$ instead of metal.
**TABLE I**

BREAKDOWN OF THE TOTAL COST OF EXPO

<table>
<thead>
<tr>
<th>Materials:</th>
<th>Millions of Danish kr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy water (3.4 t)</td>
<td>1.46</td>
</tr>
<tr>
<td>Uranium metal (3 t)</td>
<td>0.80</td>
</tr>
<tr>
<td>UO&lt;sub&gt;2&lt;/sub&gt; (1.5 t)</td>
<td>0.38</td>
</tr>
<tr>
<td>Graphite (2.5 t)</td>
<td>0.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Construction:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical system</td>
<td>0.26</td>
</tr>
<tr>
<td>Electronic and control system</td>
<td>0.07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Design work</td>
<td>0.14</td>
</tr>
<tr>
<td><strong>Total cost</strong></td>
<td><strong>3.14</strong></td>
</tr>
</tbody>
</table>

---

**Fig. 5**

Cross-section of an EXPO fuel element with calandria tube
(All dimensions in mm)

Figure 5 shows a cross-section through a UMC or UOC fuel element from which all geometrical dimensions may be obtained. If necessary, further combinations can be made with the existing components, e.g. by using a different coolant such as H<sub>2</sub>O or D<sub>2</sub>O.
5.2. Material buckling

The measured quantities consist first of all of the material buckling, \( B_{\text{th}} \), which is determined in the usual way through measurements of the exponentially decaying axial flux and of the radial flux distribution. There is, however, one complication. Working with rather large pitch values from 16 cm and upward in a 150-cm diameter tank, it is impossible to make a direct determination of the radial buckling because of the small number of lattice points left unperturbed by the reflector which usually will exist between the lattice and the tank wall.

To correct for this effect the axial flux distribution was in a few cases measured as a function of the reflector thickness (by changing the number of fuel elements while the lattice pitch was kept constant). The results of these measurements were then used for adjustment of a two-group calculation, which subsequently was applied to correct for the effect of the reflector.

5.3. Fine structure flux distribution

Information on the properties of the single lattice cells is obtained by making fine structure flux measurements in some representative cases. Manganese foils are used to measure the mean fluxes in the fuel rods and in the coolant as well as the flux distribution in the moderator.

From these measurements the thermal utilization factor \( f \) and the diffusion length \( L \) may be inferred using appropriate cross-section values.

5.4. Spectral quantities

In the theoretical investigations the neutron temperature, \( T_n \), and the Westcott \( r \)-factor were used to characterize the neutron spectrum. Therefore it was desirable to determine \( T_n \) and \( r \) experimentally.

These quantities are measured in EXPO by the activity ratio between manganese and lutetium foils. The ratio of activity between Mn\(^{55}\) and the fast decaying (3.7 h) isotope \( \text{Lu}^{176} \) yields the quantity \( r \sqrt{T_n/T} \), where \( T \) is the physical temperature, and from the ratio of activity between Mn\(^{55}\) and the slowly decaying (6.8 d) isotope \( \text{Lu}^{177} \) the neutron temperature may be obtained when \( r \sqrt{T_n/T} \) is known. With the above-mentioned method the latter quantity is best determined for large \( r \)-values. For small \( r \)-values the Cd ratio for gold is preferable. The lutetium is used in the form of aluminium-lutetium foils weighing 54 mg/cm\(^2\) and containing 37% natural Lu. The foils are made by pressing, sintering and rolling a mixture of \( \text{Lu}_2\text{O}_3 \) and aluminium powder.

5.5. Conversion ratio

The initial conversion ratio has been determined by measuring the ratio of neptunium formation to fission. The ratio was found by carrying out \( \gamma \)-spectroscopical analysis on uranium metal foils irradiated in the fuel rods. The analysis using a 256 channel multichannel analyser is rather involved, and a new method based on chemical separation of Np from urani-
um and fission products similar to the one given in Ref. [2] is being
developed.

5.6. Other measurements

As a matter of routine the diffusion length in the heavy water is measured
at regular intervals. The Fermi-age in heavy water has also been measured,
using a small sample of enriched uranium as the fission neutron source.
Since the neutrons coming up through the graphite pedestal are very
well thermalized, rather strong transients in the flux distribution occur-
early near the tank bottom when fuel is introduced into the tank. These transients
have been examined by Cd-ratio measurements. The Cd ratio has also been
measured in different positions in the tank to determine the variation of
the neutron spectrum in the tank.

5.7. Temperature coefficient measurements

A few temperature coefficient measurements have been carried out by
measuring the change in buckling between 20 and 60°C. This temperature
range may later on be extended from 10 to 80°C.

5.8. Flux levels

The maximum obtainable flux in EXPO is \(4 \times 10^7\) \(\text{n/cm}^2\ \text{s}\) near the
bottom of the tank. \(\text{BF}_3\) counting and most of the ordinary foil activations
may easily be carried through with fluxes close to this level.
When lutetium and uranium foils are irradiated inside the fuel the
maximum intensity is just on the edge of being too low. To obtain sufficient
activity irradiation, periods of 5 h for lutetium foils and 8 h for uranium
foils have been necessary.
The chemical separation will probably make it possible to cut down on
the latter irradiation time, which is mainly determined by the background
radiation from natural uranium.

5.9. Future development

Preliminary investigations have been made regarding the construction
of a single fuel element in which the coolant could be heated to a temperature
comparable to the coolant temperature of the DOR reactor. In such an
element the influence of the hot coolant on the fine structure flux, the
effective neutron temperature and the conversion ratio could be examined
by essentially the same methods as have been used hitherto.

6. OPERATIONAL EXPERIENCE

6.1. Electronic equipment

One of the most usual difficulties in operating an experimental facility
like EXPO is the electronic equipment. In the case of EXPO the main diffi-
culty has been in connection with the BF3 counters, the reason being that the geometrical size of the counter is too small compared with the range of the α-particles produced inside the counter when counting neutrons. Consequently the pulse spectrum broadens, leaving only a very narrow minimum between the noise in the lower end and the signal in the upper end to set the discriminator level.

The obvious remedy would be to increase either the size of the counters or the gas pressure in the counters.

Having only commercially available counters to choose among, these two possibilities have been ruled out, and the effort has been concentrated on reducing the noise level by careful design of the rather long cables leading to the measuring panel in the control room. A strange source of noise was the Kalundborg Broadcasting Station about 40 km from Risø, whose 0.245 Mc/s signal was superposed on the pulse spectrum. Another troublesome source of noise was the control rod mechanism of the DR 1 reactor.

After having reduced the noise, good results have been obtained by setting the discriminator level very carefully using a single-channel analyser.

6.2. D2O losses

Next to obtaining useful experimental results the most important function in operating a heavy-water facility is to safeguard the heavy water against losses and contamination with H2O.

The main quantity of D2O lost from the facility is carried away by the protective gas leaking out of the system. The rate of leakage when the facility is left by itself is 3.3 l/h or 27 900 l/yr. When one of the aluminium lids in the Plexiglass cover, as in Fig. 2, is open the rate of escape will be determined by the reduction valve of the N2 system, fixing the rate of escape at 120 l/h. Since the start-up of EXPO the total consumption of N2 has been 260 000 l, of which the major part has escaped during periods with open lids, when manipulation with the fuel has been performed. Assuming that the escaping N2 is saturated with D2O vapour the total loss of D2O from gas leakage should be 260 000 × 0.024 = 6600 g. Other losses will occur mainly as droplets on objects such as foil holders and fuel elements taken up directly from the heavy water. Special rags are used to dry these objects. The heavy water contained in these rags is later extracted in a special plant treating also heavy water rags from other departments. The quantity of D2O disappearing from EXPO in this manner is small compared with the above-mentioned loss.

6.3. Degradation of D2O

The isotopic composition of the heavy water in EXPO has been kept under constant observation by infra-red absorption measurements on samples taken every month. As this type of measurement is essentially a relative measurement, a calibration is made by comparing with a standard.

In Fig. 6 the points represent the results from the monthly sampling and the straight line is a least-squares fit corresponding to a degradation of 0.035 wt. % per year. With a heavy-water inventory of 3.4 t this means that 1 kg of light water penetrates into the facility per year.
Contamination of the D₂O may be caused by the following sources:
1. H₂O introduced into the system with the protective gas.
2. H₂O introduced into the system deposited on the surface of fuel elements.
3. H₂O diffusing through openings during fuel manipulation.
4. H₂O diffusing through Plexiglass cover.

The first source of contamination may easily be estimated since the total quantity that has passed through the system up to now, 276 000 l with a H₂O content not exceeding 0.02 mg/l, corresponds to a total of 5.6 g.

The second source is difficult to estimate as it depends very much on the surface condition of the fuel elements. If these are covered by a heavy layer of oxide they certainly contain an amount of crystalline water, which will rapidly change its isotopic composition by exchange with its surroundings. In connection with EXPO the philosophy has been to keep the oxide layer as thin as possible by cleaning the surface very thoroughly. This cleaning which includes a treatment with NaOH and HNO₃ is repeated each time a new fuel configuration is introduced. As the heavy oxidation primarily takes place in contact with the heavy water, the contamination will mainly be caused by elements going in and out of the facility several times before being recleaned. To keep this kind of contamination low the measuring programme should be planned so as to minimize the number of elements returning to the facility uncleaned. Although the magnitude of this kind of contamination is not known, it can be said with confidence that it cannot be responsible for any substantial part of the observed degradation, in which case there should be abrupt changes in the H₂O concentration, detectable in Fig. 6. However, according to Fig. 6 the degradation takes place at a steady rate, independent of the number of manipulations with fuel elements.

![Fig. 6](image-url)

Isotopic purity of the D₂O in EXPO as a function of time
A calculation of the third source has been performed for EXPO. With one aluminium lid removed, leaving an opening of 200 cm² and an upward directed speed of 0.17 cm/s, corresponding to the 2 l/min delivered by the N₂ system, the leakage of H₂O from saturated air above the opening would be \(4.3 \times 10^{-4}\) g/min. If two covers are removed the in-leakage would be \(4.3 \times 10^{-3}\) g/min. As the covers - usually one at a time - are open on the order of \(\frac{1}{2}\) h/d, this source of contamination is very far from being significant, compared with the normal rate of 2.7 g/d.

From the foregoing it would seem likely that the main source of H₂O in-leakage comes from diffusion through the Plexiglass covers. Some rather coarse experiments using a Plexiglass-covered steel box and two simple hygrometers have been made to verify this. Unfortunately these experiments have given results too small, by a factor of 10, to explain the actual rate of in-leakage. The disagreement has so far not been explained.

6.4. D₂O purity and corrosion

To observe possible sources of corrosion, the conductivity and the concentration of a great number of ions are measured in the monthly samples. The conductivity is also measured every time D₂O is pumped into the experimental tank.

The most prominent of the ions kept under observation are Al, Ca, Cr, Cu, Fe, Ce, NO₃, and ND₄. The ions Cu and Cl are considered to be particularly dangerous from the point of view of corrosion in an aluminium system. Only Ca, Cr, and Cu have occurred in concentrations significantly above the level considered permissible. By circulation through the ion-exchange filter it has, however, always been possible to reduce the concentration to a safe level. The conductivity is always kept between \(1 \times 10^{-6}\) and \(0.3 \times 10^{-6}\ \Omega^{-1}\ cm^{-1}\), the last and lowest value just after circulation has been performed. From this value the conductivity will grow rapidly at first and then level off to a very slow increase at \(0.5 \times 10^{-6}\ \Omega^{-1}\ cm^{-1}\). Introduction of a large number of fuel elements usually causes the conductivity to make an upward jump.

The heavy water in EXPO has been heated on two occasions, once to 40°C and once to 60°C. In both cases a considerable amount of condensate accumulated in the top box. The isotopic purity was found to be significantly lower in the condensate than in the main body of heavy water, which is quite natural since a major part of the 2.7 g(H₂O)/d penetrating into EXPO must be concentrated mainly in the condensate. Also the content of Cu and Cl was rather high. Nevertheless it was considered safe to return the condensate to the main body. After circulation of the D₂O through the ion-exchange filter the conditions were brought back to normal.

6.5. Other corrosion problems

There have been two cases of corrosion inside the fuel elements. A minor one occurred at the lower end of the fuel rods when some of the fuel rods by mistake were left overnight, filled with carbon tetrachloride to destroy the last traces of Diphyl. To make things worse, the tetrachloride had been contaminated with water. A second, more serious case of corrosion took place inside a number of fuel rods filled with uranium metal. Owing
to too small an inner diameter in a few of the canning tubes, the uranium slugs could not get into these tubes. As it would take too long a time to have new canning tubes fabricated, the slugs were forced in using alcohol as a lubricant. Apparently the alcohol did not have sufficient time to evaporate before the tubes were closed, resulting in corrosion inside the tubes and causing the canning wall to bulge.

6.6. Handling of organic coolant

The handling of the organic coolant has presented no major problem. Apart from its rather unpleasant smell Diphyl has two properties which may cause difficulties. The first is that rubber, such as the sealing ring next to the fuel bundle in Fig. 4, will swell considerably in contact with Diphyl, both in liquid and vapour form. In the case of the seal in Fig. 4 the circumference of the rubber O-ring increased nearly 50% in one year.

The second property which may cause trouble if ignored is the large thermal expansion. When the fuel in EXPO was taken up to 60°C the liquid in the fuel elements had to be lowered 10 cm to avoid excessive pressure.

7. CONCLUDING REMARKS

As the EXPO facility has now been operating for about one and a half years, it may be reasonable to ask whether EXPO has been able to serve its intended purpose. Fortunately this question can be answered affirmatively.

As described in section 5, a number of experimental investigations has been carried out in the EXPO facility and the results of these have been of great value for development of theoretical reactor physics calculation methods for the DOR project. A number of the results has already been reported in Chapter 3 of Ref. [3].

An example of how the experimental results have influenced the theoretical work is shown in Fig. 7, where theoretical curves and experimental points for the material buckling as a function of the uranium-to-moderator volume ratio are given for the UM lattice. Curve 1 is an early curve, calculated by rather coarse methods before the first EXPO measurements had been carried out. When the first results became available it was obvious that better theoretical methods had to be worked out. It was believed that the discrepancy was mainly from incorrect calculation of the resonance escape probability, \( p \), and the thermal utilization factor, \( f \), and more sophisticated methods for calculating these two quantities were hence developed. Curve 2, which is identical to the theoretical curve in Fig. 3.6 of Ref. [3], was determined by the new method and very good agreement with measurements was obtained. Unfortunately it was later found that in the calculation of curve 2 incorrect cross-section values had been used and that the improved theory corresponded instead to curve 3. Further improvements on the calculation of \( p \) resulted in curve 4 which is in excellent agreement with the experimental results. It should be noticed that in the development of better theoretical methods it has always been attempted to develop methods
Theoretical buckling curves and measured buckling values for the UM lattice as a function of the fuel-to-moderator volume ratio

1. Theoretical
2. Theoretical
3. Theoretical
4. Theoretical

Experimental

which better describe the physical reality of the phenomena considered rather than introduce "effective" constants.

Based on the experience with EXPO it is felt that an exponential experiment with a strong neutron source will be able to give practically as good answers to most of the reactor physics problems connected with power reactor design as can a critical facility. Considering also that the amount of experimental reactor physics data from exponential and critical experiments and from power reactors is steadily increasing additional information can often be derived from these data. Hence it is felt that in many cases sufficient information for the design of a reactor can be obtained from an exponential experiment together with the general available knowledge of reactor physics.

Some people will argue that soon there will be no need at all for experimental reactor physics investigations, except in very special cases, because reactor theory will have reached a stage where it is as accurate as experiments. This may be true for the bigger countries with large experience in reactor design and which have large computers available, but it is not so likely in smaller, less advanced countries. Further it could be mentioned that a condition for the development of good calculation methods is that all important effects involved are known, that the time and money involved in preparing and running big computer codes are not negligible, and that direct experimental evidence always will tend to be more trust-
worthy than results of complicated computer calculations which can never be better than the men who prepare them.

So there is every reason to expect that exponential facilities will remain important tools in the field of reactor physics for still quite some time to come.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the valuable assistance rendered by members of the reactor physics section, in particular Mr. C. F. Højerup, in the preparation of this report.

REFERENCES


DISCUSSION

P. BENOIST: I should like to mention that we recently developed expressions, at Saclay, for calculating the probability of primary collisions in a cluster of rods of any configuration, located in a material environment or in a vacuum. It would be an exaggeration to say that these expressions are very simple, but at least they are not over-complicated. A computer programme is being prepared for these expressions; for the same accuracy the calculation time will be much less than in the case of the Monte Carlo method. These probabilities can be used for calculating the different factors involved in cell problems, such as f, p, ε and diffusion coefficients. They can also be used for calculating the effects of interaction between different rods in light-water lattices and in fast or intermediate reactors. The expressions will be published in Report CEA-2278.

You, too, mentioned a method of calculating these probabilities, and I should like to ask you for some details of it.

H. NELTRUP: It is an approximative method, using isotropic double P_0 approximation for the collision probability in annular zones. The escape probability is calculated by numerical integration of the K_{13} function. The collision density in each zone is then given by a number of linear equations.

H.R. LUTZ: You mentioned that you have to use a chemical separation technique for your U_{238} activation measurements and that you cannot directly measure the Np_{239} γ-activity, because of the high U_{nat} background. With your flux of 4×10^7 n/cm^2s, however, I think this technique should be feasible. Could you comment on this?
H. NELTRUP: That was the flux at the bottom of the fuel; inside these very heavy fuel clusters the flux is actually much lower. We found that the natural uranium background is substantial in this process.

H.R. LUTZ: What activation times did you use?

H. NELTRUP: Roughly eight hours.

H.R. LUTZ: You can gain quite a factor by activating longer, and neptunium has a half-life of 56 h.

H. NELTRUP: Yes, but it is rather difficult to get longer irradiation times, as we do not work shifts.

J. DE VILLIERS: I am interested in knowing how sure you are that the diffusion of light water into the system is through the plexiglass cover of your assembly. I understand that the people at Würenlingen also have a plexiglass cover, on which they pasted an aluminium foil for this purpose. Is it certain that the light water enters in this way?

H. NELTRUP: Various careful experiments have failed to reveal any other source.

R. MEIER: In our Swiss subcritical facility, MINOR, a plexiglass cover of 0.5 cm thickness is used for the protection of the D₂O. The degradation observed over the last three years of operation amounted to 0.17% per year. However, this is due primarily to the extensive use of the facility, with frequent fuel changing and imperfect drying out of the tank. In particular, we find a stronger degradation after a time of high activity at the facility. For one and a half years the plexiglass was covered by adhesive Al-foils (Al-Scotch), but no improvement was noticed. The effect observed in EXPO, which is four times smaller than that found in MINOR, might well be due to diffusion through the plexiglass.

H. NELTRUP: Our degradation rate is not very serious, but if degradation were caused by fuel handling we would expect some rather abrupt jumps in the curve that I showed you, and there are none. We have also tried to keep our fuel as clean as possible, and we cleaned off the oxides when changing the type of fuel.

R. MEIER: Perhaps I might mention that for the upgrading of heavy water from 99.4 to 99.9 we pay two Swiss francs per litre — about half a dollar per litre. To treat about 4 t, as was done recently, it cost us about Sfr. 10 000 and this operation has to be performed once every three years.
"ROSTO" ORGANIC-MODERATED CRITICAL FACILITY. As a part of the Italian Project for an Organic Moderated Nuclear Power Plant, the National Commission for Nuclear Energy (CNEN) has installed a critical facility, known as ROSPO, planned to perform criticality studies of the nuclear characteristics of cores to be tested in a 60 MW(t) prototype reactor (PRO).

The first core loading consists of MTR-type fuel elements, with flat, 90% enriched, stainless-steel clad plates; the main physical question to be answered concerns the possibility of obtaining, with this core, a proper reactivity excess, in order to compensate for temperature and poisoning effects, and to ensure a convenient lifetime to the power reactor. The variable parameter is the uranium/stainless steel ratio in the fuel element, and the experiments are performed by studying cores where some of the 16 active plates of each fuel element are substituted by dummy (stainless steel) plates. For three of these cores (0, 3 and 5 dummy plates per element respectively), detailed calculations have been carried out to obtain flux plots, reactivity effects and $k_{\text{eff}}$ versus number of elements. The calculation was based upon a modified two-group theory that accounts for fissions in the epithermal zone. The following codes were used: THESIS for thermal, and MUFT 4 for epithermal spectra; FLIP 1 (transport theory) and TUT-T5 (Monte Carlo) for fluxes in the unit cell; PDQ-02 coupled to WANDA 4 (diffusion theory in xy and slab geometries) for reactivities and macroscopic fluxes. The results are shown and compared with those obtained for the same cores by AGIP Nucleare and by the CNEN Bologna Computation Centre; in spite of the analogy of the methods used, large discrepancies were observed, which are thought to derive from differences in the libraries of cross-sections.

This seems to confirm the necessity of critical experiments as the most valid help in choosing and checking calculation methods, nuclear codes and libraries.

The measurement of $k_{\text{eff}}$ versus the number of fuel elements, performed for the three mentioned uranium/stainless steel ratios in ROSPO allows a detailed analysis of these problems.
упомянутых отношений уран —нержавеющая сталь в ROSPO, дает возможность провести под-
analyse détaillée.  
permètent mieux que tout autre moyen de choisir et de vérifier les méthodes de calcul, les codes nucléaires 
робный анализ этих проблем. 
Измерение К, зависящего от количества топливных элементов, выполненное для трех упомянутых отношений уран—нержавеющая сталь в ROSPO, дает возможность провести подробный анализ этих проблем.

"РОСПО"—КРИТИЧЕСКАЯ УСТАНОВКА С ОРГАНИЧЕСКИМ ЗАМЕДЛИТЕЛЕМ. Как часть итальянского проекта ядерной силовой установки с органическим замедлителем Национальный комитет по атомной энергии смонтировал критическую установку, известную под названием ROSPO, предназначенную для проведения критических исследований ядерных ха-
рakteristik активных зон, подлежащих испытанию в прототипном реакторе (PRO) мощностью 60 мгвт.

Первая загрузка активной зоны состоит из топливных элементов типа MTR с плоскими пластинами 90%-ного обогащения в оболочке из нержавеющей стали. Основной физический вопрос, на который необходимо ответить, связан с возможностью получения в этой активной зоне соответствующего избытка реактивности в целях компенсирования температурного эффекта и эффекта отравления и обеспечения подходящей продолжительности компании энергетического реактора. Изменяющимся параметром является отношение уран—нержавеющая сталь в топливном элементе, и в настоящее время проводятся опыты путем изучения активных зон, в которых некоторые из 16 активных пластин каждого топливного элемента замещаются макетами (нержавеющая сталь). Для трех из этих активных зон (3 и 5 пластин—макетов на каждый топливный элемент соответственно) проведены подробные расчеты с целью получе-
ния графиков потоков, изучения эффектов реактивности и К, в зависимости от количества элементов. Расчет был основан на измененной теории двух групп, которая учитывает деления в надтепловой зоне. Применялись следующие коды расчета: THESIS—для спектров тепловых нейтронов и MUFT 4—для спектров надтепловых нейтронов; FLIP-1 (теория переноса) и TUT-T5 (Монте Карло) для потоков в единичной ячейке; PDQ-02, связанная с WANDA-4 (теория диффузии в геометрических формах ху и форме пластин) для измерения реактивности и макроскопических потоков. Приводятся результаты, которые сравниваются с результатами, полученными для тех же активных зон институтом AGIP Nucleare и вычислительным центром в Болонья. Несмотря на сходство примененных методов, отмечались большие расхождения, которые, как полагают, возникают из-за различий в данных о поперечных сечениях.

Это обстоятельство, по-видимому, подтверждает необходимость критических опытов как наиболее эффективного средства при выборе и проверке методов расчета, ядерных кодов и собранных данных.

Измерение К, зависящего от количества топливных элементов, выполненное для трех упомянутых отношений уран—нержавеющая сталь в ROSPO, дает возможность провести подр-
обный анализ этих проблем.

LA INSTALACIÓN CRÍTICA "ROSPO" CON MODERADOR ORGÁNICO. Como parte del proyecto italiano de una central nucleoeléctrica con reactor de moderador orgánico, el Comitato Nazionale per l'Energia Nu-
cleare ha construido la instalación crítica ROSPO, destinada a la realización de estudios de las características nucleares, en el estado crítico, de cuerpos de reactor que luego se ensayarán en un reactor prototipo de 60 MW(t), el PRO.

La primera carga del cuerpo consta de elementos combustibles del tipo MTR (reactor de ensayo de materiales), con placas planas, enriquecidas al 90 por ciento y revestidas de acero inoxidable; la principal incógnita física se refiere a la posibilidad de obtener, con dicho cuerpo, un exceso de reactividad apropiado, a fin de compensar los efectos de temperatura y de envenenamiento y de asegurar que el reactor de potencia tenga una duración aceptable. El parámetro variable es la relación uranio/acero inoxidable en el elemento combustible; los experimentos se ejecutan estudiando cuerpos en los que algunas de las 16 placas activas de que consta cada elemento combustible se reemplazan con placas simuladas (de acero inoxidable). Para tres de estos cuerpos (de 3 y 5 placas simuladas por elemento, respectivamente) se efectuaron cálculos detallados a fin de obtener diagramas de flujo, de los efectos de reactividad y de k, en función del número de elementos. El cálculo se basó en una teoría de dos grupos modificada, que tiene en cuenta las fisiones en la zona epité-
mica. Se utilizaron las siguientes claves: THESIS para los espectros térmicos, y MUFT 4 para los epitérmicos; FLIP 1 (teoría del transporte) y TUT-T5 (Monte Carlo) para los flujos en la celda unitaria; PDQ-02 com-
binaida con WANDA 4 (teoría de la difusión en geometrías x y de placas) para las reactividades y los flujos macroscópicos. Los autores exponen los resultados y los comparan con los obtenidos para los mismos cuerpos por
INTRODUCTION

As a part of the Italian project for an organic-moderated nuclear power plant, the Comitato Nazionale per l'Energia Nucleare (C.N. E.N.) has installed a critical facility, known as R.O.S.P.O. (Reattore Organico Sperimentale Potenza 0) designed to perform criticality studies of the nuclear characteristics of cores to be tested in a 30-MW(t) prototype reactor P.R.O. (Programme Reattore Organico).

The first core loading consists of box-type fuel elements with flat, 90% enriched, stainless steel clad plates, moderated with Santowax. The main physical problem concerns the possibility of obtaining, with this core, a proper reactivity excess to compensate for temperature and poisoning effects and to ensure a convenient lifetime to the power reactor. The variable parameter is the uranium-stainless steel ratio in the fuel elements, and the experiments (devoted to determining the value of this parameter which allows a P.R.O. first core loading with the necessary reactivity excess) are performed by testing, in the R.O.S.P.O. facility, cores where some of the 16 active plates in each fuel element are substituted by dummy (stainless steel) plates. For three of these cores (0, 3 and 5 dummy plates per element, respectively) detailed calculations have been carried out to obtain flux plots, reactivity effects, and $K_{eff}$ versus the number of elements. The calculations, as described in the following section, have been based on a modified two-group theory that accounts for fissions in the epithermal zone.

The results were compared with those obtained for similar cores by AGIP Nucleare and by the C.N.E.N., Bologna Computation Centre; in spite of the analogy of methods used, some discrepancies were observed, which are thought to derive from differences in the libraries of cross-sections.

This seems to confirm the necessity for critical experiments, as the most valid assistance in choosing and checking calculation methods, nuclear codes and libraries. Such experiments, to be performed for three uranium-stainless steel ratios, will allow a detailed analysis of these problems.

Measurements of $K_{eff}$ versus the number of fuel elements, as well as reactivity effects and rod worth, were recently started at the R.O.S.P.O. Organic-Moderated Critical Facility. In section II the results for cores with five dummy plates per element are reported.

* The scattering and transport cross-sections for hydrogen bound in Santowax have been taken from KAPL-1643. The other cross-sections used in calculations have been taken from BNL-325.
I. CALCULATIONS

A. Core zoning

In Fig. 1, a layout of the R. O. S. P. O. core is shown. In order to approach the proper geometrical array, the reactor has been divided into twelve zones for calculation purposes: a central core, two radial reflector zones, and five upper and four lower axial reflector zones.

![Core layout and zoning diagram](image-url)
The core itself has been regarded as consisting of elementary cells, each containing a fuel element with the pertinent moderator and control rod follower. The two radial reflector zones represent the inner reflector (divided into elementary cells, in which control rod followers are present) and the outer homogeneous zone (in which clean Santowax is present). In the axial reflector the presence of the end fittings and the grid plates has been taken into account by dividing the top reflector into five zones, and the bottom reflector into four zones.

B. Nuclear constants

Nuclear constants in each zone have been calculated by means of a modified two-group theory that accounts for fissions in the epithermal zone. The following methods have been used in calculating nuclear constants.

First group (fast constants)

In calculating fast constants for the clean core, the elementary cell was homogenized, and the MUFT-4 code was employed. This code [1] furnished the fast constants averaged over the first group (lethargy threshold = 16.59), accounting for fast and epithermal capture, resonance absorption and fast and epithermal fissions. Owing to the fast flux flatness over the cell, inhomogeneities were not accounted for. The same method was followed for calculating fast constants in the reflectors.

Second group (thermal constants)

In calculating the thermal constants over the cell, one must account for inhomogeneity and disadvantage factors from the thermal flux microscopic distribution. The following procedure has been employed.

The first step consisted in homogenizing the elementary cell (Table I); thermal cross-sections were then calculated by the THESIS code [2]. This code, starting from the absorption and scattering characteristics of the cell (and considering the moderator as a mono-atomic gas with a proper mass number and energy dependent scattering cross-sections), yields the thermal spectrum and the requested thermal cross-sections averaged over the spectrum itself. The results were then used to calculate the differential scattering cross-sections (developed in Legendre polynomials) and macroscopic scattering and absorption cross-sections in fuel plates, cladding and intra-plates moderator. These constants were put into the FLIP-1 code* to determine the intracell thermal flux in the active region of the cell. The resulting disadvantage factors were applied to average the thermal cross-sections over this region.

Then the TUT-T5 code [4] was applied to determine, by the Monte Carlo method, the absorption rate of thermal neutrons in the different materials of the elementary cell. The cell was divided into five different compositions,

* The FLIP-1 code [3] calculates the flux distribution in an infinite periodical system of flat plates by transport theory. In the R.O.S.P.O. fuel elements fuel plate thickness and spacing are small compared with the fuel plate size, so the approximation to an infinite slab can be considered as satisfactory.
### TABLE 1

**HOMOGENIZATION OF THE ELEMENTARY CELL**

<table>
<thead>
<tr>
<th></th>
<th>Number densities $10^{-12}$ cm$^{-3}$ (190°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H</td>
</tr>
<tr>
<td><strong>Elementary cell</strong></td>
<td></td>
</tr>
<tr>
<td>Cell with rod follower</td>
<td>11/5</td>
</tr>
<tr>
<td>10/3</td>
<td>298.1</td>
</tr>
<tr>
<td>10/0</td>
<td>297.8</td>
</tr>
<tr>
<td>Cell with control rod*</td>
<td>11/5</td>
</tr>
<tr>
<td>10/3</td>
<td>290.1</td>
</tr>
<tr>
<td>10/0</td>
<td>286.1</td>
</tr>
<tr>
<td><strong>Radial reflectors</strong></td>
<td></td>
</tr>
<tr>
<td>Inner zone</td>
<td>349.1</td>
</tr>
<tr>
<td>Outer zone</td>
<td>356.6</td>
</tr>
<tr>
<td><strong>Top reflectors</strong></td>
<td></td>
</tr>
<tr>
<td>Zone 1</td>
<td>306.6</td>
</tr>
<tr>
<td>Zone 2</td>
<td>249.0</td>
</tr>
<tr>
<td>Zone 3</td>
<td>242.8</td>
</tr>
<tr>
<td>Zone 4</td>
<td>197.6</td>
</tr>
<tr>
<td>Zone 5</td>
<td>278.2</td>
</tr>
<tr>
<td><strong>Bottom reflectors</strong></td>
<td></td>
</tr>
<tr>
<td>Zone 1</td>
<td>312.3</td>
</tr>
<tr>
<td>Zone 2</td>
<td>274.8</td>
</tr>
<tr>
<td>Zone 3</td>
<td>322.7</td>
</tr>
<tr>
<td>Zone 4</td>
<td>74.44</td>
</tr>
</tbody>
</table>

*Volumes of elementary cell are calculated here by excluding the black region of the control rod. Boron thickness in the control rod is 170 mg/cm$^2$.\)
Fuel element compositions calculated by the TUT-T5 code as shown in Fig. 2: using as input the macroscopic thermal cross-sections already obtained from the THESIS code for each composition and the above-mentioned thermal constants of the active region, the absorption rate of thermal neutrons in the different compositions (hence the thermal flux distribution) was calculated by the TUT-T5 code.

The neutron flux distribution was assigned as a weight to the number densities of materials in the cell; the THESIS code, into which the so-corrected number densities were put, was then applied to yield the thermal constants averaged over the cell.
The nine axial reflector zones and the outer zone of axial reflectors were homogenized (Table I), and the thermal constants were calculated by the THESIS code without any further elaboration.

Dealing with the inner zone of the radial reflector, the thermal constants yielded by the THESIS code for stainless steel (control rod followers) and Santowax, were put into the PDQ-02 code \([5]\) to get the flux distribution in the reflector cell. Then, the neutron flux distribution was assigned as a weight to the number densities which were newly put into the THESIS code, to yield the thermal constants in the inner radial reflector zone. Actually this procedure is the same as that followed in calculating thermal constants of elementary cells, except for using the PDQ-02 code instead of the TUT-T5.

**C. Reactivity calculations**

In Fig. 3, a schematic flow diagram of the procedure involved in calculating nuclear constants and \(K_{\text{eff}}\) is shown. The effective multiplication factor of the core has been calculated by the PDQ-02 and WANDA-4 \([7]\) codes through the procedure described below.

The WANDA-4 code was first applied in the axial direction to calculate \(K_{\text{eff}}\) of cores which were axially zoned, as described in section I, assuming several estimated values of the radial buckling \(B_R\). The yielded values of \(K_{\text{eff}}\) were applied to obtain the corresponding values of total \(B^2\), by solving the equation
The values of $B_2^2$ were then obtained (by subtracting $B_2^k$ from $B_2^2$) and plotted versus $B_2^k$.

The PDQ-02 code was then applied to calculate $K_{eff}$ of a core with an assigned number of fuel elements, giving several estimated values for the axial buckling $B_2^2$. The values of $B_2^2$ versus the assigned values of $B_2^2$ obtained from the yielded $K_{eff}$, solving Eq. (1), were then plotted versus $B_2^k$ on the same graph as above. The intersection between the plots gave the values of $B_2^2$ and $B_2^k$ for the core with the assigned number of fuel elements, and $K_{eff}$ was then obtained.

This method of evaluating the geometrical buckling of a reflected core requires a small number of calculations, provided that the estimated values of $B_2^2$ and $B_2^k$ are close to the effective ones.

D. Reactivity effects

Control rod worth

The R.O.S.P.O. reactor is controlled by cruciform-shaped boron control rods, clad with stainless steel. The boron thickness is 170 mg/cm². Control rods are arranged on a square lattice, with 173.2 mm pitch. To evaluate single control rod worth, $K_{eff}$ of the clean core has been compared with $K_{eff}$ of the poisoned core (in which the clean cells were substituted by poisoned cells, constituting the pertinent fourth of a control rod), which had been calculated by the PDQ-02 code assigning the value of $B_2^k$ obtained in section C. The differential worth of banked control rods has been obtained by the WANDA-4 code, applied in the axial direction to yield $K_{eff}$ of the bank-poisoned core. Two-group nuclear constants for the poisoned cells had been calculated by the following procedure:

1. The extrapolated length in the black region was evaluated by the PRO-29 code [8], which yields extrapolation length and effective cross-sections in thin regions, based on Wackpress and Gelbard’s theory.
2. The nuclear constants over the clean region of the cell (excluding the black region) were calculated by the same procedure as in the clean elementary cell.
3. Using the values obtained in (1) and (2), the PDQ-02 code was applied to get the fast and thermal macroscopic constants over the homogenized poisoned cell.

Void coefficient

Nuclear constants were calculated for an elementary cell in which the density of the moderator contained inside the fuel box was reduced to a fraction of 0.9 and 0.8. The procedure was the same as that followed in calculating nuclear constants for the elementary cell. Nuclear constants of partially voided cells were later used for calculating the void coefficient of the reactor.
This coefficient was obtained by comparing the multiplication factor of the clean core with $K_{\text{eff}}$ of a partially voided core, calculated by the method described in section C (except for applying the WANDA-4 code also in the radial direction instead of the PDQ-02).

**Temperature effect**

To obtain the slow temperature coefficient, nuclear constants of the elementary cell were calculated at different temperature values ranging from 190 to 350°C. The procedure was the same as that followed in calculating nuclear constants at 190°C. The effects of temperature on material densities, geometrical size and cross-sections were taken into account. Inputs of the THESIS code were modified for number density changes due to differences in densities, and for cross-section value changes.

Inputs of the TUT-T5 and FLIP-1 codes were also modified, accounting for variations of the geometrical array of cell components. Nuclear constants of the elementary cell at different temperature values were used to calculate $K_{\text{eff}}$ versus temperature of the clean core (hence the temperature coefficient of the reactor), according to the same procedure as that described in section C, except for using the WANDA-4 in the radial direction instead of the PDQ-02.

No attempt was made to calculate the fast temperature coefficient from the Doppler effect in $^{238}\text{U}$ resonances. This coefficient can be considered as negligible because of the very high enrichment (90% in $^{235}\text{U}$) of fuel elements.

**E. Calculation results**

In Tables II and III the nuclear constants of core cells and reflectors for all cases of interest are reported. In Fig. 4 the calculated values of $K_{\text{eff}}$ versus the number of fuel elements for clean cores at 190°C are shown. Peripheral fuel element worth, as deduced by calculating the reactivity increments due to loading a fuel element progressively, is plotted in Fig. 5.

Figure 6 shows the reactivity worth of the moderator level for the 11/5 core loaded with 52 elements.

The worth of banked control rods in the 11/5 core loaded with 52 fuel elements is shown in Fig. 7.

Values of $K_{\text{eff}}$ for the 16/0 core at different temperatures and void fractions are shown in Table IV. The deduced values of the temperature and void coefficients are reported in Table V. Worths of single control rods in the 11/5 core loaded with 52 elements are shown in Table VI.

**II. EXPERIMENTS**

**A. Experimental programme**

The chief aim of the first cycle of experiments in R. O. S. P. O. is to determine the nuclear characteristics of the fuel elements to be loaded in the P. R. O. reactor in order to ensure a proper reactivity excess to the full
### Table II

<table>
<thead>
<tr>
<th>Constants</th>
<th>R. O. S. P. O. 16/0</th>
<th>R. O. S. P. O. 13/3</th>
<th>R. O. S. P. O. 11/5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>190°C</td>
<td>232°C</td>
<td>288°C</td>
</tr>
<tr>
<td><strong>Thermal</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>D (cm)</strong></td>
<td>0.353852</td>
<td>0.368557</td>
<td>0.391914</td>
</tr>
<tr>
<td><strong>Σa (cm⁻¹)</strong></td>
<td>0.093187</td>
<td>0.090806</td>
<td>0.087420</td>
</tr>
<tr>
<td><em><em>νΣa</em> (cm⁻¹)</em>*</td>
<td>0.138065</td>
<td>0.134993</td>
<td>0.130450</td>
</tr>
<tr>
<td><strong>Fast</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>D (cm)</strong></td>
<td>1.344861</td>
<td>1.377138</td>
<td>1.433703</td>
</tr>
<tr>
<td><strong>Σa (cm⁻¹)</strong></td>
<td>0.004573</td>
<td>0.004549</td>
<td>0.004503</td>
</tr>
<tr>
<td><em><em>νΣa</em> (cm⁻¹)</em>*</td>
<td>0.005267</td>
<td>0.005247</td>
<td>0.005204</td>
</tr>
<tr>
<td><strong>Στ (cm⁻¹)</strong></td>
<td>0.021157</td>
<td>0.020462</td>
<td>0.019336</td>
</tr>
<tr>
<td><strong>r (cm²)</strong></td>
<td>52.2671</td>
<td>55.0595</td>
<td>60.1409</td>
</tr>
<tr>
<td><strong>K∞</strong></td>
<td>1.42280</td>
<td>1.42601</td>
<td>1.42865</td>
</tr>
</tbody>
</table>

* ν = 2.47
Fig. 4

$K_{eff}$ versus number of fuel elements (190°C)

- Calculated values
- Experimental points

Fig. 5

Fuel element worth in 11/5 core (190°C)
### TABLE III

NUCLEAR CONSTANTS OF CORE CELLS AND REFLECTORS

<table>
<thead>
<tr>
<th>Zone</th>
<th>Thermal constants (190°C)</th>
<th>Fast constants (190°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D$ (cm)</td>
<td>$\Sigma_a$ (cm$^{-1}$)</td>
</tr>
<tr>
<td>Top reflectors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zone 1</td>
<td>0.32988</td>
<td>0.029847</td>
</tr>
<tr>
<td>Zone 2</td>
<td>0.35072</td>
<td>0.051591</td>
</tr>
<tr>
<td>Zone 3</td>
<td>0.35431</td>
<td>0.05279</td>
</tr>
<tr>
<td>Zone 4</td>
<td>0.36670</td>
<td>0.066842</td>
</tr>
<tr>
<td>Zone 5</td>
<td>0.34363</td>
<td>0.040132</td>
</tr>
<tr>
<td>Bottom reflectors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zone 1</td>
<td>0.32750</td>
<td>0.027478</td>
</tr>
<tr>
<td>Zone 2</td>
<td>0.34566</td>
<td>0.04029</td>
</tr>
<tr>
<td>Zone 3</td>
<td>0.32930</td>
<td>0.02229</td>
</tr>
<tr>
<td>Zone 4</td>
<td>0.36392</td>
<td>0.085778</td>
</tr>
<tr>
<td>Radial reflectors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inner zone</td>
<td>0.31191</td>
<td>0.010274</td>
</tr>
<tr>
<td>Outer zone</td>
<td>0.30760</td>
<td>0.0084165</td>
</tr>
</tbody>
</table>

**Fig. 6.**

Reactivity effect of moderator level in 11/5 core (52 fuel elements, 190°C)
loaded core. As indicated by the results of the P. R. O. project studies on a 96 fuel element core, a $K_{ex} = 0.17$ is necessary to compensate for temperature, poisons, baffles and burn-out effects, and to allow a further reactivity excess to be available for experiments.

The R.O.S.P.O. experimental plan is described below.

**Measurements on fuel elements with a ranging ratio of uranium-to-steel weight**

This set of experiments is performed to find the uranium-stainless steel concentration in the R.O.S.P.O. fuel elements, such that a core, in the above-mentioned final conditions, has a $K_{ex} = 0.17$. The proper concentration is approached by testing three cores with different uranium-stainless steel ratio: the homogeneous composition is simulated by assembling the fuel elements of each core with a definite number of stainless steel dummy plates, alternating with the active ones.

Defining $\omega$ (concentration of an active plate, in which a "meat" of UO$_2$-stainless steel is clad in stainless steel) as:

$$\omega = \frac{\text{UO}_2 \text{ weight}}{\text{UO}_2 \text{ weight} + \text{stainless steel weight (in the "meat")}}$$
### TABLE IV

VALUES OF $K_{\text{eff}}$ AT DIFFERENT TEMPERATURES AND VOID FRACTIONS

<table>
<thead>
<tr>
<th>Core loading (No. of fuel elements)</th>
<th>$K_{\text{eff}}$</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11/5 190°C</td>
<td>13/3 190°C</td>
<td>16/0 190°C</td>
<td>16/0 232°C</td>
<td>16/0 288°C</td>
<td>16/0 350°C</td>
<td>16/0 190°C Void fraction = 0.1</td>
</tr>
<tr>
<td>12</td>
<td>0.8892</td>
<td>0.9412</td>
<td>0.9293</td>
<td>0.9129</td>
<td>0.8930</td>
<td>0.9102</td>
<td>0.8864</td>
</tr>
<tr>
<td>16</td>
<td>0.9426</td>
<td>0.9959</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>0.9319</td>
<td>0.9911</td>
<td>1.0470</td>
<td>1.0370</td>
<td>1.0215</td>
<td>1.0032</td>
<td>1.0174</td>
</tr>
<tr>
<td>28</td>
<td>0.9753</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.9850</td>
<td>1.0488</td>
<td>1.1046</td>
<td>1.0976</td>
<td>1.0832</td>
<td>1.0664</td>
<td>1.0781</td>
</tr>
<tr>
<td>40</td>
<td>1.0025</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>1.0155</td>
<td>1.0793</td>
<td>1.1371</td>
<td>1.1298</td>
<td>1.1164</td>
<td>1.1003</td>
<td>1.1107</td>
</tr>
<tr>
<td>52</td>
<td>1.0584</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table V

VALUES OF THE TEMPERATURE AND VOID COEFFICIENTS

<table>
<thead>
<tr>
<th>Core loading (No. of fuel elements)</th>
<th>16/0 Core</th>
<th>Void fraction = 0.1</th>
<th>Void fraction = 0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Temperature coefficient $\frac{d\rho}{dT}$ (pcm/°C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>190°C</td>
<td>223°C</td>
<td>288°C</td>
<td>350°C</td>
</tr>
<tr>
<td>12</td>
<td>-21</td>
<td>-27</td>
<td>-37</td>
</tr>
<tr>
<td>21</td>
<td>-15</td>
<td>-20</td>
<td>-27</td>
</tr>
<tr>
<td>30</td>
<td>-12</td>
<td>-16</td>
<td>-22</td>
</tr>
<tr>
<td>37</td>
<td>-10</td>
<td>-14</td>
<td>-20</td>
</tr>
</tbody>
</table>

### Table VI

WORTH OF SINGLE CONTROL RODS

<table>
<thead>
<tr>
<th>Rod No.</th>
<th>Distance from centre of core (cm)</th>
<th>$\Delta K_{\text{eff}}$ (pcm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>31.2</td>
<td>855</td>
</tr>
<tr>
<td>.3</td>
<td>26</td>
<td>1540</td>
</tr>
<tr>
<td>7</td>
<td>19.4</td>
<td>2450</td>
</tr>
<tr>
<td>8</td>
<td>8.6</td>
<td>4150</td>
</tr>
</tbody>
</table>

The fictitious concentration ($\omega_f$), corresponding to a definite fuel element configuration, is

$$\omega_f = \frac{\omega n_a}{n_a + n_d \frac{P_d}{P_a}}$$

where:
- $n_a$ = number of active plates per element
- $n_d$ = number of dummy plates per element
- $P_a$ = stainless steel weight in the "meat" of an active plate
P_d = stainless steel weight in the zone of a dummy plate corresponding to the 'meat' of the active plates.

The concentrations (and related configurations) to be tested are shown in Table VII.

### Table VII

<table>
<thead>
<tr>
<th>ω (%)</th>
<th>n_d/n_d</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.17</td>
<td>16/0</td>
</tr>
<tr>
<td>18.10</td>
<td>13/3</td>
</tr>
<tr>
<td>15.36</td>
<td>11/5</td>
</tr>
</tbody>
</table>

For each of these cores, critical mass, peripheral fuel element worth, reactivity variations depending on moderator level and temperature, and control rod worth are measured, at the temperature corresponding to the condition of "cold core" for the P. R. O. reactor (190°C). Measurements of the reactivity dependence on the moderator level Δρ/ΔH (in addition to peripheral fuel element worth, Δρ/ΔN) are performed to evaluate the reflector saving, avoiding the direct measurement of buckling by flux distribution, which shows some difficulties especially connected with the characteristics of the plant and the core geometry. The values of critical mass, Δρ/ΔN and Δρ/ΔH, are utilized to determine the proper fictitious concentration for the full loaded core to give the necessary K_{ex}, while the measurements of the temperature coefficient and the control rod worth allow a partial check of the fixed K_{ex} for the P. R. O. core and the negative reactivity inserted by the control system.

**Corrections for inhomogeneities and geometry**

By the second set of experiments the value of ω_f, determined in the first approach, is corrected, accounting for the inhomogeneous configuration of the fuel elements tested in R. O. S. P. O. and for some geometrical differences between P. R. O. and R. O. S. P. O. fuel elements.

Calculations have been carried out by AGIP Nucleare using the FLIP-1 code [9] to evaluate the effect of a heterogeneous distribution of the plates in the R. O. S. P. O. element. The results have shown that the expected value of ΔK resulting from the inhomogeneity is small (some %), provided that the plate's distribution is properly chosen. The best distribution of the dummy stainless steel plates in the 11/5 and 13/3 cases is shown, with the calculated ΔK, in Table VIII. The indicated distributions have been applied in assembling fuel elements for the first set of experiments.

The experimental determination of the reactivity effects from the inhomogeneous configuration of the R. O. S. P. O. elements will be obtained...
G. BITELLI et al.

TABLE VIII

BEST DISTRIBUTION OF THE DUMMY STAINLESS STEEL PLATES

<table>
<thead>
<tr>
<th>Core</th>
<th>Order of stainless steel plates</th>
<th>$K_{\text{eff}}(\text{homog.}) - K_{\text{eff}}(\text{inhomog.})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11/5</td>
<td>2, 5, 8, 12, 15</td>
<td>$\approx 500$ pcm</td>
</tr>
<tr>
<td>13/3</td>
<td>4, 8, 13</td>
<td>$\approx 60$ pcm</td>
</tr>
</tbody>
</table>

by a series of measurements of $K_{\text{eff}}$ in cores having the same uranium-stainless steel ratio but a different degree of inhomogeneity in the distribution of the dummy plates. From these measurements, the $K_{\text{eff}}$ corresponding to a core with homogeneously assembled elements will be obtained by extrapolation.

The correction for the different stainless steel contents of the R. O. S. P. O. elements, in respect of the P. R. O. ones (due to some geometrical differences), will be obtained by measuring the reactivity dependence on the stainless steel content in R. O. S. P. O. fuel elements.

Full core test

Once the proper uranium-stainless steel ratio that ensures the necessary $K_{\text{ex}}$ for a 96 fuel element core has been obtained, one must verify this value of $K_{\text{ex}}$, comparing it with the balancing negative reactivity effects. To do this, a full core will be loaded in the R. O. S. P. O. reactor; the moderator temperature will be brought to the "hot reactor" point (350°C); poisons and burn-out will be properly simulated while baffle and control rod follower worth will be directly measured. In addition, the worth of the final control rods designed for the P. R. O. prototype will be measured in the R. O. S. P. O. full core to determine the negative reactivity insertible by the control system. Flux distribution measurements will also be done in the final core to get a direct value of buckling and reflector saving.

B. Measurements and results

Operation of the R. O. S. P. O. reactor was started in June 1963. The reactor went critical on 10 June with a core loading of 39½ fuel elements of 11/5 type arranged on an approximately rectangular geometry. The core loading was then rearranged to approach a cylindrical geometry, and the measure of $K_{\text{eff}}$ versus the number of fuel elements was prosecuted up to a core loading of 52 fuel elements.

The loading steps and core configurations are shown in Fig. 1. The measured values of $K_{\text{eff}}$ are compared with calculation results in Fig. 4,
and reported in Table IX, together with peripheral fuel element worth and uranium and stainless steel contents in the core.

In the core loaded with 51 and 52 fuel elements, measurements with a partial moderator level have been performed to determine the critical height and the reactivity dependence on core height. The results are reported in Table X.

A preliminary interpretation of the experimental results shows that the calculation procedure described in section I leads to an underestimation of the values of \(K_e\) and \(M^2\). The experimental critical mass is greater than the theoretical value, as a result of a greater leakage term than that calculated in the experimental core, but \(K_{ex}\) of a fully loaded core is expected to be greater than the calculated value. It is expected that the final uranium-stainless steel concentration for the P. R. O. reactor will be very close to that represented by R. O. S. P. O. fuel elements of 11/5 type.

In the 52 fuel element core the reactivity dependence on core temperature in the range of 190–230°C has been measured, leading to a value of 20 pcm/°C. Measurements of differential worth have been performed by period measurements for control rods which are taken at various distances from the core central axis. The results are shown in Fig. 8.
TABLE X

RESULTS OF MEASUREMENTS
WITH A PARTIAL MODERATOR LEVEL

<table>
<thead>
<tr>
<th>No. of fuel elements</th>
<th>Moderator level (in the active part of fuel element) (cm)</th>
<th>Reactivity (pcm)</th>
<th>Critical height (cm)</th>
<th>dp/dH (pcm/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>51</td>
<td>66.7</td>
<td>470</td>
<td></td>
<td>296 ± 5</td>
</tr>
<tr>
<td></td>
<td>65.7</td>
<td>165</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>65.5</td>
<td>109</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>65.3</td>
<td>54</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>65.1 ± 0.3</td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>65.7</td>
<td>606</td>
<td></td>
<td>300 ± 5</td>
</tr>
<tr>
<td></td>
<td>65.5</td>
<td>540</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>65.3</td>
<td>476</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>64.8</td>
<td>332</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>64.4</td>
<td>207</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>64.1</td>
<td>121</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>63.7</td>
<td>0</td>
<td>63.7 ± 0.3</td>
<td></td>
</tr>
</tbody>
</table>
R. O. S. P. O. CRITICAL FACILITY

Fig. 8
Worth of control rods versus height of extraction in 11/5 core
(52 fuel elements, 190°C)

REFERENCES


[8] AGIP NUCLEARE, "Calcolo di distanze estrapolate e di sezioni d'urto effettive per regioni sottili mediante il programma PRO-29". Rapporto Tecnico 3. 2. 7. -187.

CRITICAL ANALYSIS OF THE PROGRESSIVE SUBSTITUTION METHOD FOR MATERIAL BUCKLING MEASUREMENTS

G. CASINI AND J. MÉGIER
EURATOM JOINT NUCLEAR RESEARCH CENTRE, ISPRA, ITALY

Abstract — Résumé — Аннотация — Resumen

CRITICAL ANALYSIS OF THE PROGRESSIVE SUBSTITUTION METHOD FOR MATERIAL BUCKLING MEASUREMENTS. A critical analysis of the methods of interpretation for progressive substitution measurements is in progress at the Centre of Ispra (EURATOM). These studies are intended for the interpretation of material buckling measurements to be performed in the ECO (Orgel Critical Experience) reactor starting in 1964.

The report deals with the method of interpretation for progressive-substitution measurements by a two-group theory applied to a reflected reactor, the regions of which are supposed to be cylindrical and homogenized. Some of the points analyzed are:

(1) Limits of validity of the method with increasing differences between the neutronic properties of the test and reference lattices.
(2) Effect of the cylindrical homogenization as a function of the number of substituted rods.
(3) Effect of the approximations in the reflector calculation.

ANALYSE CRITIQUE DE LA MÉTHODE DE SUBSTITUTION PROGRESSIVE UTILISÉE POUR MESURER LE LAPLACIEN MATIÈRE. Le Centre d’Ispra (EURATOM) procède actuellement à une analyse critique des méthodes d'interprétation des mesures faites par substitution progressive. Ces études ont pour objet d'interpréter les mesures du laplacien matière, qui seront effectuées dans le réacteur ECO (Expérience Critique Orgel) à partir de 1964.

Le mémoire a trait à la méthode d'interprétation des mesures obtenues par substitution progressive: on applique une théorie à deux groupes à un réacteur avec réflecteur, dont les régions sont supposées être cylindriques et homogénéisées. Parmi les points analysés figurent:
1. Limites de validité de la méthode lorsque augmentent les écarts entre les propriétés neutroniques du réseau testé et du réseau de référence.
2. Effets de l'homogénéisation cylindrique en fonction du nombre de barres substituées.
3. Effets des approximations sur les calculs concernant le réflecteur.

КРИТИЧЕСКИЙ АНАЛИЗ МЕТОДА ПОСТЕПЕННОГО ЗАМЕЩЕНИЯ, ПРИМЕНЯЕМОГО ДЛЯ ИЗМЕРЕНИЯ МАТЕРИАЛЬНОГО ЛАПЛАСИАНА. В научно-исследовательском центре в Испре (Евратом) осуществляется критический анализ методов интерпретации результатов измерений, проводимых методом постепенного замещения. Эти исследования направлены на получение интерпретации результатов измерений материального лапласiana, которые должны быть выполнены, начиная с 1964 года, на реакторе ECO (Оргельский критический опытный реактор).

Дается описание метода интерпретации результатов измерений методом постепенного замещения на основании двухгрупповой теории, применяемой к реактору с отражателем, зоны которого, как предполагается, являются цилиндрическими и гомогенизованными. Некоторые из анализируемых пунктов:
Пределы пригодности метода с возрастающими различиями между нейтронными свойства-ми испытательной и эталонной решеток.
Значение цилиндрической гомогенизации как функции ряда замещаемых стержней.
Значение приближений в расчете отражателя.

ANALISIS CRÍTICO DEL MÉTODO DE SUSTITUCIÓN PROGRESIVA PARA MEDICIONES EN LAPLACIANOS MATERIALES. En el Centro de Ispra (EURATOM) se está realizando un análisis crítico de los métodos de interpretación de las mediciones por sustitución progresiva. Su finalidad es evaluar los resultados de la medición de los laplacianos materiales del reactor ECO (Experimento Crítico Orgel), que debe comenzar en 1964.
La memoria trata de un método de interpretación basado en una teoría de dos grupos aplicada a un reactor con reflector cuyas zonas se suponen cilíndricas y homogéneas. Entre otros puntos, analiza los siguientes:

1. Los límites de validez del método cuando aumentan las diferencias entre las propiedades neutrónicas del reticulado que se ensaya y el de referencia.

2. El efecto de la homogeneización cilíndrica en función del número de barras sustituidas.

3. El efecto de las aproximaciones en el cálculo del reflector.

1. INTRODUCTION

A complete series of buckling measurements will be performed in 1964 on ECO (Expérience Critique Orgel), a natural uranium-fuelled, heavy-water-moderated critical assembly now in construction at the EURATOM Joint Nuclear Research Centre at Ispra [1]. First the neutronic characteristics of the pile loaded with the "reference" fuel elements (clusters of 19 rods of uranium metal contained in a double tube filled with organic liquid) will be determined by flux mapping. Then the buckling of variously fuelled lattices will be measured by the progressive substitution technique.

The aim of this report is to illustrate the method established at EURATOM, to interpret the substitution experiments and to discuss the approximations involved in view of defining the range of applicability of the method itself.

2. OUTLINE OF THE METHOD IN USE AT EURATOM

The substitution approach to determine the buckling of a lattice has been used for several years in various laboratories [2, 3, 4]. Methods have been established to interpret the experimental results; particular attention has been reserved to the "progressive" substitution measurements on heavy-water-moderated critical assemblies performed by French and Swedish national centres.

It is foreseen that the same technique will be used in the ECO experimental programme. Therefore a method of interpretation, developed on the basis of the French work [5], has been established and programmed for IBM-7090 (BERTHE code) at the EURATOM Joint Nuclear Research Centre at Ispra.

A two-group, two-homogenized-core region diffusion model has been assumed. Practically no approximation in the analytical development of the theory is made. The effectiveness of the radial graphite reflector and of the aluminium tank between the core and the reflector are directly incorporated in the programme instead of using a reflector coefficient as in the French method [5].

A series of check calculations with a normal two-group diffusion code (WANDA and RIFIFI codes [6, 7]) have been found to be satisfactory. In the appendix the main lines of the developed method are indicated.

In particular the formalism which takes into account the aluminium tank has been found to be necessary. To study this point two BERTHE calculations have been performed for a typical series of ECO substitution experiments. The buckling of the reference lattice was 4.346 m⁻². The difference
between the test and reference lattice was -0.905 m\(^2\). The series of substitutions is given in Fig. 1. The corresponding heavy-water level variations (ΔH\(_1\)) as calculated by RIFIFI are listed in Table I.

In the first BERTHE calculation a value of q\(_t\) (see appendix) of 0.012 (corresponding to an aluminium thickness of 1 cm) was assumed. In the second one the tank effect was not included (q\(_t\) = 0). A difference of 0.20 m\(^2\) in ΔB\(^2\) was found; this shows the importance of being able to evaluate the influence of the thin aluminium layer between core and reflector for such type of calculations.

3. Input Data Influence

Table II shows the influences on the buckling difference between test and reference region (ΔB\(^2\)) caused by small variations in the input nuclear data of the test region and in the measured heavy-water levels during the replacements (ΔH\(_1\)). The influence of the diffusion coefficients seems to be fairly important. However the uncertainty of these parameters for the test lattice as compared with that for the reference region is certainly less than 5%. Therefore an error inferior to 0.02 m\(^2\) can be expected for this effect.
### TABLE I

**HEAVY-WATER LEVEL VARIATIONS FOR SUBSTITUTION EXPERIMENTS**

<table>
<thead>
<tr>
<th>No. of substituted rods</th>
<th>Radius of substituted zone (cm)</th>
<th>$\Delta H_1$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>26.49</td>
<td>2.25</td>
</tr>
<tr>
<td>9</td>
<td>35.54</td>
<td>4.11</td>
</tr>
<tr>
<td>13</td>
<td>42.72</td>
<td>5.93</td>
</tr>
<tr>
<td>21</td>
<td>54.29</td>
<td>9.49</td>
</tr>
<tr>
<td>25</td>
<td>59.24</td>
<td>11.21</td>
</tr>
</tbody>
</table>

### TABLE II

**INFLUENCE OF INPUT DATA ON BUCKLING DIFFERENCE BETWEEN TEST AND REFERENCE REGION**

<table>
<thead>
<tr>
<th>Parameter symbol*</th>
<th>Numerical value</th>
<th>Parameter variation</th>
<th>Variation on $\Delta B^2$ (m$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{1r}$</td>
<td>1.257</td>
<td>+5%</td>
<td>-0.0234</td>
</tr>
<tr>
<td>$D_{1t}$</td>
<td>0.819</td>
<td>+5%</td>
<td>-0.0263</td>
</tr>
<tr>
<td>$L_{1}^2$</td>
<td>85.88</td>
<td>+5%</td>
<td>-0.0002</td>
</tr>
<tr>
<td>$T_{1}$</td>
<td>106.36</td>
<td>+5%</td>
<td>-0.0001</td>
</tr>
<tr>
<td>$p_{1}$</td>
<td>0.8717</td>
<td>+2%</td>
<td>0.0</td>
</tr>
<tr>
<td>$H_1$</td>
<td>-</td>
<td>$+0.1$ cm</td>
<td>+0.0150</td>
</tr>
</tbody>
</table>

* * D: Diffusion coefficient
  L$^2$: Diffusion area
  T: Slowing-down area
  p: Resonance escape probability

Subscript 1: Test region.
Subscript r: "Fast group".
Subscript t: "Thermal group".

As far as the error in level is concerned, the accuracy in the ECO level measurement being 0.2 mm, the influence on $\Delta B^2$ could be reduced to less than 0.005 m$^{-2}$. 
4. APPROXIMATIONS INVOLVED IN THE METHOD

As we said, the scheme used to determine from experimental values of the critical level the buckling of the test lattice involves a certain number of approximations. In the following chapters we will try to evaluate the errors which can be expected in the ECO operation cases mentioned in the introduction.

The main effects to be treated are: (a) the cylindrical-shape approximation in the test region and (b) the heterogeneity effects.

(a) Cylindrical-shape approximation in the test region

In BERTHE the test region is considered to be cylindrical. The radius of the region is calculated as:

\[ R_i = \frac{d(N_i/\pi)^{1/2}}{2} \]

where:

- \( N_i \) = number of substituted rods and
- \( d \) = pitch of the lattice.

The choice of this radius is somewhat arbitrary because in the boundary region between the two zones it is not possible to define an equivalent cell for each rod. To examine the cause of this uncertainty we have compared the normal BERTHE results with those obtained by increasing the radii of the different substituted regions by a fixed amount. Table III gives the results for \( \Delta R_i = +2 \text{ cm} \).

**TABLE III**

**COMPARISON OF THE NORMAL BERTHE RESULTS WITH THOSE OBTAINED BY INCREASING RADII**

<table>
<thead>
<tr>
<th>No. of substituted rods</th>
<th>( \Delta B^2 ) (m²)</th>
<th>( \delta(\Delta B^2) ) (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-0.507</td>
<td>-0.0022</td>
</tr>
<tr>
<td>9</td>
<td>-0.905</td>
<td>-0.0054</td>
</tr>
<tr>
<td>13</td>
<td>-1.284</td>
<td>-0.0120</td>
</tr>
<tr>
<td>21</td>
<td>-1.528</td>
<td>-0.003</td>
</tr>
<tr>
<td>25</td>
<td>-1.702</td>
<td>-0.005</td>
</tr>
</tbody>
</table>

The calculations have been performed for the ECO pile geometry; a pitch of 22.6 cm and five substitutions (5, 9, 13, 21, 25 elements replaced) have been considered. As can be seen, the effect remains in all cases relatively small.

The influence of the form of the boundary line has also been studied by comparing for the ECO substituted configurations the results of a one-dimensional and a two-dimensional calculation.
In the first calculation (WANDA programme) the pile has been schematized as in BERTHE: three cylindrical regions (test region, reference region and radial reflector) have been considered. The radius of the test region varied according to the number of fuel elements substituted.

For each pile configuration a critical extrapolated height was obtained; this value (together with the nuclear data used in WANDA) was introduced as input data into a two-dimensional calculation (EQUIPOISE programme [8]). In this second case the boundary limit between the two-core regions was determined according to the cell hypothesis as indicated, for the different substitutions, in Fig. 1.

The EQUIPOISE results (based on the critical heights obtained by WANDA) are shown in Table IV; they are expressed as $k_{\text{eff}}$ variations (in $\text{pcm} = 10^{-5}$) from the critical conditions. These figures give a direct indication of the error involved in the cylindrical-shape approximation for the boundary line. Taking into account that in ECO 1 pcm of reactivity corresponds to about 0.2 mm variation in the critical level and comparing with the data of Table II (1 mm in $\Delta H_i$ means a variation of 0.0156 in $\Delta B^2$), it can be concluded that this effect is relatively small, even though it is slightly more pronounced in the configurations with 13 and 25 rods which are farthest away from the circular shape.

A general conclusion can be derived from these results: apart from the heterogeneity effects of the lattices (which will be studied in the following sections), the form (and thus the number of rods) of the region does not seem to play an important part in the substitution technique.

In this perspective the WANDA–EQUIPOISE comparison has been extended to the case of 17 replaced rods (Fig. 2) where the central zone configuration is less symmetrical. The results are given in Table V.

The error introduced by the cylindrical-shape approximation remains fairly small. This means that such a substitution could be introduced in the replacement operations to increase the number of experimental data or to eliminate, if the heterogeneity effects are not too important, the 25-rod replacement.
MATERIAL BUCKLING MEASUREMENTS

![Diagram of 17-rod substitution](image)

**Fig. 2**

17-rod substitution

**TABLE V**

EXTENSION OF THE WANDA-EQUIPOISE COMPARISON

<table>
<thead>
<tr>
<th>Pitch, (cm)</th>
<th>$\Delta B^2$ (pcm)</th>
<th>$\delta k/k$ (pcm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.8</td>
<td>-2.36</td>
<td>+10.0</td>
</tr>
<tr>
<td>21.0</td>
<td>-0.905</td>
<td>-7.0</td>
</tr>
</tbody>
</table>

(b) Heterogeneity effects

As previously said, in the BERTHE method the test and reference regions are homogenized; this means that the heterogeneity effects of the lattices are supposed to be independent from the dimensions of the regions. However, for under-moderated lattices this hypothesis is not justified. This fact prompted us to study the limits of validity of such an assumption in the ECO substitution measurements.

The analysis is not concerned with fuel elements containing void where the anisotropy of the neutron streaming is important. The main effects to be considered then are: (i) resonance absorption and (ii) thermal absorption. Indeed for the pitches which will be considered in the ECO reactor (>18.0 cm) only a quite small fraction of $^{238}U$ fissions in a fuel element are produced by neutrons coming from other rods. Thus the heterogeneity fast effects can be neglected.

The resonance absorption effect can be illustrated in the following manner: let us consider an infinite heterogeneous lattice. The resonance absorption in a rod can be considered as the sum of the absorption of neutrons born in the rod itself plus the absorption of neutrons born in the surrounding rods. All these absorptions do not depend on the intensity of the fission source because the lattice is infinitely extended. However if we have two connected lattices, as in the case of substitution experiments, some rods of one lattice are surrounded also by rods of the other lattice. If the
uranium section per channel is quite different for the two lattices the fast source will also be different and the absorption for the boundary rods will be different from that of an infinite lattice.

Analytically, if \( p_e \) indicates the resonance escape probability for rod 0 in an infinite moderator sea and \( p_0 \) the same in the lattice, the resonance absorption can be written

\[
a = \frac{1 - p_0}{1 - p_e} = \sum_{j=0}^{\infty} \frac{S_j}{S_0} e^{-r_j/4\tau_R},
\]

where:
- \( \tau_R \) = slowing-down area to the resonance energy
- \( r_j \) = distance from rod 0 to the rod j of the lattice
- \( S_j \) = intensity of the fast source of rod j.

For the sake of simplicity in formula (1) a line source has been assumed to calculate, on the basis of the Fermi-age model, the slowing-down distribution of the neutrons. Furthermore the resonance absorption has been assumed to occur only at a certain energy level (single-energy approximation).

In the case of an infinite lattice Eq. (1) becomes

\[
a = \sum_{j=0}^{\infty} e^{-r_j/4\tau_R}.
\]

The correction introduced by the presence of the other type of rods can be written in the following form:

\[
\frac{\Delta a}{a} = \frac{\Delta (1 - p_0)}{1 - p_0} = 1 - \frac{\sum_{j=0}^{\infty} \left(S_j/S_0\right) e^{-r_j/4\tau_R}}{\sum_{j=0}^{\infty} e^{-r_j/4\tau_R}}.
\]

In practice the calculation has been done as follows: let us consider the pile configuration of Fig. 3(a). For each rod of the test and reference lattice \( \Delta a/a \) according to Eq. (3) has been evaluated. Then the core has been divided into five regions as indicated in Fig. 3(b). The resonance absorption of each of these regions has been calculated by correcting the p-values (for the infinite lattice) according to the \( \Delta a/a \) already determined. On the basis of these data a WANDA calculation has been performed; the critical height so obtained has been compared to the value corresponding to the normal two-region-core model used in BERTHE.

The results for an ECO case are shown in Table VI. The data are the following:
- Pitch: 18.8 cm
- \( \tau_R \) : 55.0 cm²
- Fission-source ratio between test and reference lattices: 0.67
- \( B_{\text{Ref}} \) : 4.076 m²
- \( \Delta B^2 \) : -1.394 m²
MATERIAL BUCKLING MEASUREMENTS

TABLE VI
RESULTS FOR AN ECO CASE

<table>
<thead>
<tr>
<th>No. of substitutions</th>
<th>( \Delta H ) (cm)</th>
<th>( \Delta H' ) (cm)</th>
<th>( \Delta H - \Delta H' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2.02</td>
<td>2.13</td>
<td>-0.11</td>
</tr>
<tr>
<td>9</td>
<td>4.66</td>
<td>4.67</td>
<td>-0.01</td>
</tr>
<tr>
<td>13</td>
<td>6.06</td>
<td>6.10</td>
<td>-0.04</td>
</tr>
<tr>
<td>21</td>
<td>10.35</td>
<td>10.53</td>
<td>-0.18</td>
</tr>
<tr>
<td>25</td>
<td>13.33</td>
<td>13.41</td>
<td>-0.08</td>
</tr>
</tbody>
</table>

\( \Delta H \) are the variations of the critical level according to the different substitutions, as calculated by WANDA for the two-region-core configuration. \( \Delta H' \) are the corresponding results from the many-region calculations (5 in the case of the substitution 9). On the basis of the two series of \( \Delta H \) so obtained, two BERTHE calculations have been performed: the results showed a difference of only 0.004 m\(^2\) in \( \Delta B^2 \).
In the calculation the sources have been assumed to be linear. In fact in the practical case here considered the fuel elements have a diameter of about 7.0 cm. If the distance between two rods $r_j$ in the $\Delta a/a$ calculation is considered to be the distance between the boundaries of two fuel channels, the resonance absorption effect increases fairly sensibly. A second calculation based on this hypothesis has been done and the results are given in Table VII. The corresponding $\Delta B^2$ variation is $0.024 \, m^{-2}$. Therefore it seems that, at least for the ECO pile configurations, this effect does not become too important, also for the most under-moderated lattices.

Two main thermal absorption effects can be envisaged: (a) a spatial effect and (b) a spectrum effect.

Point (a) may be considered analogous to the resonance absorption effect and treated similarly. Specifically the neutron flux distribution across the cell is supposed to be the same as for an infinite lattice; near the boundary between the two-core zones this condition is not verified. The corresponding effect on the criticality can be easily evaluated by comparing the results of a homogenized two-region calculation (as in BERTHE) with a heterogeneous calculation (of the Feinberg-Galanin type) where the difficulties connected with the superposition of the microscopic and macroscopic flux distribution are automatically by-passed.

This effect has already been studied in Saclay and it has been found to be fairly small for some of the heavy-water lattices tested in Aquilon II [9]. Similar calculations to check the validity of this hypothesis for ECO experiments are now in progress at EURATOM.

Point (b) is connected with the fact that in the boundary region the thermal spectrum will be intermediate between those which would correspond to the two infinite lattices of the test and reference region.

This effect could be sensible if the uranium sections per cell of the test and reference regions (i.e. the moderating ratios, because the pitch is the same in all the pile) are sensibly different. To check this point we proceeded in the following manner. It was assumed that in the boundary region (constituted by two rings of test and reference rods respectively) the thermal

<table>
<thead>
<tr>
<th>No. of substitutions</th>
<th>$\Delta H$ (cm)</th>
<th>$\Delta H'$ (cm)</th>
<th>$\Delta H - \Delta H'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2.02</td>
<td>2.23</td>
<td>-0.21</td>
</tr>
<tr>
<td>9</td>
<td>4.66</td>
<td>4.71</td>
<td>-0.05</td>
</tr>
<tr>
<td>13</td>
<td>6.06</td>
<td>6.06</td>
<td>0.0</td>
</tr>
<tr>
<td>21</td>
<td>10.35</td>
<td>10.69</td>
<td>-0.34</td>
</tr>
<tr>
<td>25</td>
<td>13.33</td>
<td>13.46</td>
<td>-0.13</td>
</tr>
</tbody>
</table>
spectrum averaged cross-sections of the fuel were the half-sums of the values corresponding to the two infinite lattices.

On the basis of this data a four-region-core WANDA calculation has been compared with a normal two-region calculation. The results are indicated in Table VIII.

### Table VIII

<table>
<thead>
<tr>
<th>No. of substituted rods</th>
<th>$\Delta H$ (cm)</th>
<th>$\Delta H'$ (cm)</th>
<th>$\Delta H - \Delta H'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>2.65</td>
<td>2.62</td>
<td>0.03</td>
</tr>
<tr>
<td>25</td>
<td>7.32</td>
<td>7.19</td>
<td>0.13</td>
</tr>
</tbody>
</table>

To calculate the spectrum-averaged cross-sections the French method [10] has been used. The two lattices were constituted of single rods of 2.5 and 1.75 cm radius directly immersed in heavy water. The Horowitz and Tretiakoff parameters for the two lattices were:

- Moderator temperature: $T_m = 293^\circ K$,
- Epithermal factor: $r = 0.0046$ and $0.010$ and
- Heterogeneity factor: $z = 1.5$ and $2.0$.

As can be seen, the moderating conditions of the two lattices are very different. The $\Delta B^2$ variation caused by the spectrum effects is $0.029 \text{ m}^2$; by taking into account that this is a rather extreme case, we can conclude that for natural uranium lattices the effect on spectrum of lattice heterogeneities is generally rather small. This probably would not be true for fuels containing plutonium where the condition of a similar epithermal factor in test and reference region is more stringent [11].

5. CONCLUSIONS

The preceding theoretical analysis gives some confidence in the use of the BERTHE method to interpret the progressive substitution measurements in the ECO reactor. In particular the hypothesis of cylindrical shape for the test region seems to be fairly justified. In these conditions the extension of the substitution configurations to less symmetrical cases could be envisaged. Some caution must however be exercised as far as the moderating conditions for reference and test lattice are concerned. If such conditions are sensibly different from one another the heterogeneity effects cannot be disregarded. In particular, the 5-rod substitution could be difficult to interpret.
ACKNOWLEDGEMENTS

The authors wish to express their appreciation to Professor A. Kind for his continuous support and useful criticism. They are grateful to Messrs. R. Naudet and M. Chabrilliac of the Commissariat à l'énergie atomique (Saclay) for most helpful discussions and for supplying the results of comparison calculations.

Finally they would like to thank Mr. C. Daolio who programmed the BERTHE code and Mr. S. Stramaccia who has performed the work of preparing the data for computer calculations.

APPENDIX

I. SURVEY OF THE REFLECTOR CALCULATION WITHOUT THIN LAYER

1.1 In the case of a two-region bare reactor the critical equation for the core can be written as follows [12]:

\[ U + V S = 1, \]

with

\[ U = \frac{J_0}{\mu_2 R_e} \left( \frac{Y_2 - \mu_2 A Y_1}{J_0 - \mu_2 A J_1} \right), \]

\[ V = \frac{K_0 - (t_1/\lambda_2)\mu_2 A K_1}{K_0 - (t_1/\lambda_2)\mu_2 A K_0} \left[ \frac{J_0}{\mu_2 R_e} \right] \left( \frac{Y_2 - \mu_2 A Y_1}{J_0 - \mu_2 A J_1} \right), \]

\[ S = \frac{(T_1 - T_2)(S_1 - S_2)}{(T_1 - S_1)(S_1 - T_2)}, \]

where: \( J_0, J_1, Y_1, K_0, K_1 \): Bessel functions
- \( \mu_2^2 \): positive radial buckling of the reference lattice
- \( -\nu_2^2 \): negative radial buckling of the reference lattice
- \( \mu_1^2 \): positive radial buckling of the test lattice
- \( -\nu_1^2 \): negative radial buckling of the test lattice

\[ \Lambda = \frac{1}{\nu_1^2} \left( \frac{J_0}{\mu_1 R_e} \right), \quad \Lambda' = \frac{1}{\nu_1^2} \left( \frac{J_0}{\mu_1 R_e} \right), \]

\( t_1, t_2, \lambda_1, \lambda_2 \): terms including the ratios between the diffusion coefficients of the test and reference lattices
\( R_e \): central test zone radius
\( R_e \): core extrapolated radius.

\( S_1, T_1 \): Coupling coefficients between fast and thermal fluxes in the homogenized test zone.
\( S_2, T_2 \): Coupling coefficients between fast and thermal fluxes in the homogenized reference zone.

The fast and thermal fluxes in the reference zone are linear combinations of the functions \( J_0(t_1) + \epsilon Y_0(t_1) \) and \( I_0(t_2) + \epsilon' Y_0(t_2) \) where \( \epsilon \) and \( \epsilon' \) are constant numbers. The critical condition leads to the value

\[ -\frac{I_0'}{Y_0} \]
1.2. In the case of a reactor with lateral reflector the expressions U and V are unchanged except that the value given to $\epsilon$ by the critical condition is no longer $-1/(Y_0/Y_{0s}R_e)$. We just outline here the calculations, first established by JONSSON in a private communication [12]. The boundary conditions at the interface between core and reflector can be written, if we suppose the same fast and thermal diffusion coefficient in the reference zone and the reflector. For the fluxes

$$\begin{pmatrix} \phi_{2R} \\ \phi_{3R} \end{pmatrix}_{R_1} = \begin{pmatrix} 1 & m_2 \\ S_2 & m_2 T_2 \end{pmatrix} \begin{pmatrix} X_2 \\ Y_2 \end{pmatrix}_{R_1} = M_{2} \begin{pmatrix} X_2 \\ Y_2 \end{pmatrix}_{R_1}$$

$$\begin{pmatrix} \phi_{3R} \\ \phi_{3R} \end{pmatrix}_{R_1} = \begin{pmatrix} 1 & 0 \\ S_3 & m_2 T_3 \end{pmatrix} \begin{pmatrix} X_3 \\ Y_3 \end{pmatrix}_{R_1} = M_{3} \begin{pmatrix} X_3 \\ Y_3 \end{pmatrix}_{R_1}$$

(1)

so that

$$\begin{pmatrix} X_2 \\ Y_2 \end{pmatrix}_{R_1} = M_{2}^{-1} M_{3} \begin{pmatrix} X_3 \\ Y_3 \end{pmatrix}_{R_1}$$

For the currents

$$\begin{pmatrix} X_2^* \\ Y_2^* \end{pmatrix}_{R_1} = M_{2}^{-1} M_{3} \begin{pmatrix} X_3^* \\ Y_3^* \end{pmatrix}_{R_1}$$

(2)

where the subscripts mean: $r$: fast group; $t$: thermal group; 2: reference zone; 3 reflector (1 would be the test zone); $R_1$ indicates that the functions are calculated at the interface between core and reflector, the radius of which is $R_1$

and where:

$$X_2 = X_2(\mu_{2t}) + \epsilon Y_0 \nu_{2t}$$

$$Y_2 = (\nu_{2t} + \nu_{3t}) Y_0$$

$$X_3 = X_3(\mu_{3t}) - (1/\nu_0) \nu_{3t} R_2 \cdot K_0 X_{3t}$$

$$Y_3 = Z_0 (\nu_{3t})$$

$R_2$: outer reflector radius, including the extrapolation distance.

$S_2$, $T_2$, $S_3$: coupling coefficients between fast and thermal fluxes.

Equations (1) and (2) can be expressed with the single condition:

$$\begin{pmatrix} X_2 + \lambda X_2^* \\ Y_2 + \lambda Y_2^* \end{pmatrix}_{R_1} = M_{2}^{-1} M_{3} \begin{pmatrix} X_3 + \lambda X_3^* \\ Y_3 + \lambda Y_3^* \end{pmatrix}_{R_1}$$

(where $\lambda$ is an arbitrary parameter)
which leads to the two equations

\[(X_2 + \lambda X^*_2)_{R_1} + m_2(Y_2 + \lambda Y^*_2)_{R_1} = 0\]  \(\text{(3)}\)

and

\[(X_2 + \eta X^*_2)_{R_1} + m_2 \frac{T_2}{S_2} (Y_2 + \epsilon Y^*_2)_{R_1} = 0\]  \(\text{(4)}\)

or:

\[(I_0 K) + \epsilon(Y_0 K) + m_2(\lambda K) + m_2\epsilon'(\lambda' K) = 0\]  \(\text{(3')}\)

and

\[(I_0 \eta) + \epsilon(Y_0 \eta) + m_2 \frac{T_2}{S_2} (I_0 \eta) + m_2 \frac{T_2}{S_2} \epsilon'(\lambda' K) = 0\]  \(\text{(4')}\)

where \(K\) is such that \(X_2 + \lambda X^*_2 = 0; \lambda = K = - (Z_0 / Z_0^*)_\mu R_1\)

and:

\(K'\) is such that \(Y_2 + \lambda Y^*_2 = 0; \lambda = K' = - (Z_0 / Z_0^*)_\mu R_1\)

and:

\[\eta = K' + (K - K') \frac{S_2}{S_2}, \quad \epsilon = K' + (K - K') \frac{S_2}{T_2},\]

\[(I_0 K) = I_0 (\mu_2 R_1) - \mu_2 K J_1 (\mu_2 R_1), \quad (Y_0 K) = Y_0 (\mu_2 R_1) - \mu_2 K J_1 (\mu_2 R_1).\]  \(\text{(5)}\)

etc.

Eliminating \(m_2\) between (3') and (4') and noting that \(\epsilon'(K_0 K)_{R_1} \propto (I_0 K)_{R_1}\) and that \(\epsilon'(K_0 \eta) \propto (I_0 \eta)_{R_1}\) one finds the new value of \(\epsilon:\)

\[\epsilon = - \left( \frac{J_0 - \mu_2 \lambda J_1}{Y_0 - \mu_2 \lambda J_1} \right) \mu_2 R_1\]  \(\text{(6)}\)

where \(\Lambda_1\) means the expression \(\Lambda_1 = \frac{1}{\mu_2} \left( \frac{J_0 + \epsilon Y_0}{Y_0 + \epsilon Y_0} \right) \mu_2 R_1\).

Equations (3) and (4) give for this expression the value

\[\Lambda_1 = \frac{\eta(I_0 K) - (T_2/S_2) K(I_0 \eta)}{(I_0 K) - (T_2/S_2)(I_0 \eta)}.\]  \(\text{(7)}\)

Substituting the value of \(\epsilon\) in (6) for \(- (I_0 / Y_0)_{R_2} R_1\) in 1.1, one obtains the expression for \(U\) and \(V\) in the case of a reactor with lateral reflector.

II. DIFFUSION-COEFFICIENTS CORRECTION AND THIN LAYER FORMALISM

These extensions will lead to a more general formulation of \(\Lambda_1\) and, once more, to a new value of \(\epsilon\) to be substituted in the former expressions of \(U\) and \(V\).
2.1. The boundary conditions, in the case where there is a thin weak absorber layer between reference zone and reflector, and when one takes into account the difference between the diffusion coefficients of these two zones, can be expressed in the following way:

The neutronic currents at \((R_{1-})\) and \((R_{1+})\), \((R_{2-})\) and \((R_{2+})\) being respectively the inner and outer radii of the thin layer located at the radius \(R_t\) \((R_t = R_i - (d/2); R_t = R_i + (d/2); d: \) thin layer thickness), can be written to a very good approximation

\[
\begin{align*}
  j^+_{(R_{1+})} &= j^+_{(R_{1-})} e^{-q} \approx j^+_{(R_{1-})} (1 - q) \\
  j^-_{(R_{1-})} &= j^-_{(R_{1+})} e^{+q} \approx j^-_{(R_{1+})} (1 - q),
\end{align*}
\]

where \(q = \Sigma_a d / (\Sigma_a absorption cross-section of the thin layer material; j^+: outward current; and j^-: inward current).

Let us consider now, in diffusion theory, the expressions of the currents at the boundaries \((R_{1-})\) and \((R_{1+})\) of the two-region reference lattice (region 2) and reflector (region 3).

\[
\begin{align*}
  j^+_{(R_{1+})} &= \frac{\phi_2}{4} - \frac{D_2}{2} \frac{d\phi_2}{dr} \\
  j^-_{(R_{1-})} &= \frac{\phi_3}{4} - \frac{D_3}{2} \frac{d\phi_3}{dr}
\end{align*}
\]

\(\phi_2\): flux in reflector at \((R_{1+})\) \\
\(\phi_3\): flux in reference region at \((R_{1-})\) \\
\(D_3\): reflector diffusion coefficient \\
\(D_2\): reference region diffusion coefficient

and similar expressions for \(j^-_{(R_{1+})}\) and \(j^-_{(R_{1-})}\).

Considering these current expressions and the absorption law, (8) and (9), and neglecting the thickness of the layer, i.e. supposing that the changes in current and flux produced by the layer take place discontinuously at the boundary \(R_t\), one finds the following boundary conditions at \(R_t\):

\[
\begin{align*}
  \phi_3 &= \phi_3 + 2q D_3 \phi^*_2 \\
  \phi^*_3 &= \frac{q}{2D_3} \phi_3 + \frac{D_2}{D_3} \phi^*_2
\end{align*}
\]

2.2. In our case, with two groups of neutrons and matrix formalism the boundary conditions can be written

\[
\begin{pmatrix}
  \phi_3 \\
  \phi_2
\end{pmatrix}_{R_1} = \begin{pmatrix}
  \phi_3 \\
  \phi_2
\end{pmatrix}_{R_1} + \begin{pmatrix}
  2q, D_2, 0 \\
  0, 2q, D_2
\end{pmatrix} \begin{pmatrix}
  \phi_2 \\
  \phi_2
\end{pmatrix}_{R_1}
\]

(10)'\]

\[
\begin{pmatrix}
  \phi_3' \\
  \phi_2'
\end{pmatrix}_{R_1} = \begin{pmatrix}
  q, 0 \\
  0, q
\end{pmatrix} \begin{pmatrix}
  \phi_3 \\
  \phi_2
\end{pmatrix}_{R_1} + \begin{pmatrix}
  D_3, 0, 0 \\
  0, D_3, 0
\end{pmatrix} \begin{pmatrix}
  \phi_2' \\
  \phi_2'
\end{pmatrix}_{R_1}
\]

(11)'}
where \( q_r = L_a d \) (fast group)
\( q_t = L_a d \) (thermal group)

Remembering (cf. 1.2) that
\[
\begin{align*}
\phi_{2r} &= M_1 \begin{pmatrix} X_2 \\ Y_2 \\ \lambda \end{pmatrix} \\
\phi_{3r} &= M_1 \begin{pmatrix} X_3 \\ Y_3 \\ \lambda \end{pmatrix}
\end{align*}
\]
and the same relationships for the flux derivatives, we can again condense (10\#) and (11\#) into a single equation

where:

\[
\begin{align*}
M_2^{-1} M_2 \begin{pmatrix} X_2 + \lambda X_2' \\ Y_2 + \lambda Y_2' \end{pmatrix} &= R_1 \\
&= \begin{pmatrix} M_1 \frac{q_r}{2D_3} \\ 0 \end{pmatrix} X_2 + \begin{pmatrix} M_1 \frac{q_t}{2D_3} \end{pmatrix} Y_2 \\
&+ \begin{pmatrix} M_1 + 2qD_2 \frac{D_2 r}{D_3 r} \\ M_1 + 2qD_2 \frac{D_2 r}{D_3 r} \end{pmatrix} \begin{pmatrix} X_2' \\ Y_2' \end{pmatrix} \\
&+ \begin{pmatrix} M_1 + 2qD_2 \frac{D_2 r}{D_3 r} \end{pmatrix} \begin{pmatrix} X_2' \\ Y_2' \end{pmatrix} \\
&= \begin{pmatrix} M_1 \frac{q_r}{2D_3} \\ 0 \end{pmatrix} X_2 + \begin{pmatrix} M_1 \frac{q_t}{2D_3} \end{pmatrix} Y_2 \\
&+ \begin{pmatrix} M_1 + 2qD_2 \frac{D_2 r}{D_3 r} \\ M_1 + 2qD_2 \frac{D_2 r}{D_3 r} \end{pmatrix} \begin{pmatrix} X_2' \\ Y_2' \end{pmatrix} \\
&+ \begin{pmatrix} M_1 + 2qD_2 \frac{D_2 r}{D_3 r} \end{pmatrix} \begin{pmatrix} X_2' \\ Y_2' \end{pmatrix}
\end{align*}
\]

Setting \( q_r = q_t = 0 \) leads to the simple diffusion coefficient correction, while setting \( q_r = q_t = 0 \) and \( D_2 = D_3 \) leads to the formalism used in 1.2.

One finds again two expressions similar to (3) and (4):

\[
X_{r R_1} \begin{pmatrix} 1 + K \frac{q_r}{2D_3} \\ K \frac{D_2 r}{D_3 r} + 2qD_2 r \\ m_2 \end{pmatrix} + m_2 \begin{pmatrix} Y_{r R_1} \begin{pmatrix} 1 + K \frac{q_t}{2D_3} \\ K \frac{D_2 r}{D_3 r} + 2qD_2 r \\ m_2 \end{pmatrix} \end{pmatrix} = 0
\]

\[
X_{r R_1} \begin{pmatrix} 1 + K \frac{q_r}{2D_3} \\ K \frac{D_2 r}{D_3 r} + (K - K') \frac{S_2}{2D_3 r} + 2qD_2 r \\ m_2 \end{pmatrix} + m_2 \begin{pmatrix} Y_{r R_1} \begin{pmatrix} 1 + K \frac{q_t}{2D_3} \\ K \frac{D_2 r}{D_3 r} + (K - K') \frac{S_2}{2D_3 r} + 2qD_2 r \\ m_2 \end{pmatrix} \end{pmatrix} = 0
\]

\[I \quad II\]
\[
+ m_s \sum \left\{ Y_{0t} \left[ 1 + K \frac{q_s}{2 D_{s0}} + (K - K') \frac{S_2}{2 D_{s0}} \right] + Y_{2t} \left[ K' \frac{D_{2t}}{D_{s0}} + (K - K') \frac{S_2}{2 D_{s0}} + 2 q_s D_{2t} \right] \right\} = 0 \quad (13)
\]

(12) and (13) can be written like (3') and (4'):

\[
[I_0K] + \varepsilon [Y_0K] + m_s [I_0K] + m_s \varepsilon [K_0K] = 0 \quad (12')
\]

\[
[I_0\eta] + \varepsilon [Y_0\eta] + m_s \frac{T_0}{S_2} [I_0\xi] + m_s \frac{T_0}{S_2} \varepsilon [K_0\xi] = 0, \quad (13')
\]

where:

\[
[I_0K] = I_{t\left(\mu_2 R_1\right)} \left( 1 + K \frac{q_s}{2 D_{s0}} - \mu_2 \left( \frac{D_{2t}}{D_{s0}} + 2 q_s D_{2t} \right) \right) I_1(\mu_2 R_1),
\]

\[
[I_0\eta] = I_{t\left(\mu_2 R_1\right)} \left( 1 + K \frac{q_s}{2 D_{s0}} + \nu_2 \left( \frac{D_{2t}}{D_{s0}} + 2 q_s D_{2t} \right) \right) I_1(\mu_2 R_1),
\]

and similar expressions for \([Y_0K]\) and \([K_0K]\)

and where

\[
[I_0\eta] = I_{t\left(\mu_2 R_1\right)} \left[ 1 - \mu_2 [II] \right] I_1(\mu_2 R_1),
\]

\[
[I_0\xi] = I_{t\left(\nu_2 R_1\right)} \left[ III + \nu_2 [IV] I_1(\nu_2 R_1) \right]
\]

and similar expressions for \([Y_0\eta]\) and \([K_0\xi]\)

Again, eliminating \(m_2\) between (12') and (13') we find the same formalism for the expression of \(\varepsilon\):

\[
\varepsilon = \frac{\left[ I_0K - \nu_2 \Delta_1 [I_0K] \right]}{Y_0 \mu_2 \nu_2 Y_1 I_1(\mu_2 R_1)}
\]

as given by (6) but \(\Delta_1\) is now given by:

\[
\Delta_1 = \frac{\varepsilon [I_0K] - \frac{T_0}{S_2} (KD) [I_0\xi]}{[1 + (\eta \eta)] [I_0K] - \frac{T_0}{S_2} [1 + (KQ)] [I_0\xi]}.
\]
where

\[ H = \left[ \Pi \right] - K' - K - K' - K + 2qD + D^2r + \left( K + K' \right) - \left( \frac{D^2r}{2D_3t} \right) + 2qD^2t \]

\[ (KD) = K\frac{D^2r}{D_3r} + 2qD^2t \]

\[ 1 + (\eta q) = [I] = 1 + K\frac{q_r}{2D_3t} + (K - K') \frac{\pi^2}{D_2} \frac{q_r}{2D_3t} \]

\[ 1 + (Kq) = 1 + K\frac{q_r}{2D_3t} \]

It is therefore easy to see that the successive extensions of the method to include the eventualities of lateral reflector, non-equal diffusion coefficients in reflector and reference region, and a thin weak absorber layer between core and reflector affect only the term \( \epsilon \) in the expressions \( U \) and \( V \) which can be written under the general form

\[ U = -\epsilon \left( \frac{Y_0 - \mu_0 Y_1}{I_0 - \mu_0 A Y_1} \right) \mu_2 R_0 \]

\[ V = \left( \frac{K_0 - (t_1/t_2) \mu_2 A K_1}{K_0 - (t_1/t_2) \mu_2 A K_1} \right) \mu_2 R_0 \left[ \left( \frac{I_0 - \mu_0 A Y_1}{I_0 - \mu_0 A Y_1} \right) + \epsilon \left( \frac{Y_0 - \mu_0 A Y_1}{I_0 - \mu_0 A Y_1} \right) \mu_2 R_0 \right] \]

REFERENCES

DISCUSSION

R. PERSSON: The errors caused by the cylindrical-shape approximation and by heterogeneity are no doubt related to the perturbation of the reactor during the measurement. If that is so, the error of the buckling in the test region is larger.

G. CASINI: To pass from the $\Delta k/k$ values to $\Delta B^2$ variation, one must first establish the $\Delta H$ variation and then calculate the $\Delta B^2$ variation on the basis of the data of Table II of the paper (1 mm corresponds to about 0.015 m$^2$ in $\Delta B^3$).

U. FARINELLI (Chairman): I might perhaps comment that we did similar calculations in connection with substitution experiments in the ROSPO reactor and got very similar results.
AN APPLICATION OF HETEROGENEOUS REACTOR THEORY TO SUBSTITUTION EXPERIMENTS

G. BLAESER
EURATOM JOINT NUCLEAR RESEARCH CENTRE, ISPRA, ITALY

Abstract — Résumé — Аннотация — Resumen

AN APPLICATION OF HETEROGENEOUS REACTOR THEORY TO SUBSTITUTION EXPERIMENTS. An interpretation of substitution experiments by heterogeneous theory avoids many difficulties which are typical of the homogenized treatment (as, for example, the determination of the coupling coefficients of the two zones). As a consequence it seems feasible to reduce sensibly the number of substitutions necessary for many experiments. A suitable method for the determination of the fuel element parameters which enter into the heterogeneous theory will be discussed. There exists a direct relationship between these parameters and some quantities that can be measured in oscillation experiments.

APPLICATION DE LA THÉORIE DES RÉACTEURS HÉTÉROGÈNES AUX EXPÉRIENCES DE SUBSTITUTION. L'interprétation des expériences de substitution à l'aide de la théorie des réacteurs hétérogènes évite de nombreuses difficultés caractéristiques du traitement homogénisé (comme c'est le cas, par exemple, pour déterminer les constantes de couplage pour les deux zones). Il semble, par conséquent, possible de diminuer sensiblement le nombre de substitutions nécessaire pour un grand nombre de ces expériences. Il est procédé à l'examen d'une méthode permettant de déterminer les paramètres des éléments combustibles qui entrent dans la théorie des réacteurs hétérogènes. Il existe un rapport direct entre ces paramètres et certaines quantités que l'on peut mesurer au cours d'expériences d'oscillation.

ПРИМЕНЕНИЕ ТЕОРИИ ГЕТЕРОГЕННОГО РЕАКТОРА К ОПЫТАМ ПО ЗАМЕЩЕНИЮ. Интерпретация опытов по замещению на основании гетерогенной теории дает возможность избежать многих трудностей, которые характерны для объяснения этих опытов с точки зрения теории гомогенизации (как, например, определение коэффициентов связи двух зон). Как следствие представляется возможным заметно уменьшить число замещений, необходимых для большинства экспериментов. Обсуждается соответствующий метод определения параметров топливных элементов, которые входят в гетерогенную теорию. Существует прямая связь между этими параметрами и некоторыми количественными величинами, которые могут измеряться в опытах с осцилляцией.

APLICACIÓN DE LA TEORÍA DE LOS REACTORES HETEROGÉNEOS A LOS EXPERIMENTOS DE SUSTITUCIÓN. Cuando los experimentos de sustitución se interpretan con ayuda de una teoría heterogénea se evitan muchas de las dificultades características del tratamiento homogéneo (como, por ejemplo, determinación de los coeficientes de acoplamiento de dos zonas). Este enfoque permite en muchos casos reducir notablemente el número de sustituciones necesario. El autor describe un método adecuado para determinar los parámetros del elemento combustible que intervienen en la teoría heterogénea. Existe una relación directa entre estos parámetros y ciertas cantidades que se pueden medir mediante experimentos de oscilación.

I. REVIEW OF THE DIFFERENT FORMULATIONS OF HETEROGENEOUS REACTOR THEORY

Heterogeneous calculation methods have been widely discussed in the literature [1 - 7]. They can be subdivided into "kernel methods" and "finite difference methods". Examples of the first type are the theories discussed
in Refs. [1–5], whereas the methods of HASSIT [6] and STUMMEL [7]
belong to the second type.

In the kernel methods the fuel elements are represented by neutron
sources and sinks, of which the contribution to the neutron density at any
particular point is expressed by means of a suitable propagation kernel.
This procedure leads directly to a set of linear homogeneous algebraic
equations for the fluxes at the surfaces of the fuel elements.

In the finite difference methods one tries to solve the differential
equations of multi-group theory within the moderator region after having
them approximated by finite difference equations. At the surface of the fuel
elements the differential equations must satisfy certain boundary conditions
which can be incorporated in the system of finite difference equations.

As an example, let us consider a cylindrical reactor containing N fuel
elements embedded in some moderator. We first derive the equations of
the kernel method: Let \( P(|\vec{r}' - \vec{r}|) \) be the slowing-down kernel (which can
approximately be taken equal to the slowing-down kernel of an infinite me-
dium), \( i_k \) the number of neutrons absorbed and \( q_k \) the number of neutrons
produced per unit time and unit length of the \( k \)th fuel element. Then the
diffusion equation for the thermal flux \( \phi \) becomes

\[
DV^2 \phi(\vec{r}) - \Sigma_a \phi(\vec{r}) + \sum_{k=1}^{N} q_k P(|\vec{r}_k - \vec{r}|) - \sum_{k=1}^{N} i_k \delta(\vec{r}_k - \vec{r}) = 0,
\]

(1)

where \( D \) and \( \Sigma_a \) are the (thermal) diffusion coefficient and the absorption
cross-section of the moderator.

Equation (1) has to be supplemented by the boundary conditions on the
surface \( \Gamma \) of the reactor

\[
\phi(\vec{r}) = 0, \quad \text{if } \vec{r} \in \Gamma.
\]

(2)

Let \( G(\vec{r}, \vec{r}') \) be the Green's function of the operator \(-V^2 + (1/L^2)\) (where
\( L^2 = D/\Sigma_a \), i.e. the solution of

\[
[V^2 - (1/L^2)] G(\vec{r}, \vec{r}') = -\delta(\vec{r} - \vec{r}'),
\]

which vanishes for \( r \in \Gamma \). Then an equivalent formulation of Eqs. (1) and (2) is

\[
\phi(\vec{r}) = \sum_{k=1}^{N} \left[ (q_k/D)F(\vec{r}, \vec{r}_k) - (i_k/D)G(\vec{r}, \vec{r}_k) \right],
\]

(3)

where

\[
F(r, r_k) = \int G(\vec{r}, \vec{r}') P(|\vec{r}' - \vec{r}_k|) d^3 r.
\]

(4)
According to the definition of "blackness" $\beta$, the absorption in a lump of absorber material is given by $J_{in} \beta S$, where $S$ is the surface of the lump and $J_{in}$ the incident current. If the unperturbed flux near the fuel element is not too far from isotopic (which is the case in D$_2$O), $J_{in} = \phi/4$ and

$$i_k = \frac{\pi}{2} a_k \beta_k \phi_k = \gamma_k \phi_k \text{ with } \gamma_k = \frac{\pi}{2} a_k \beta_k,$$

(5)

where $a_k$ is the radius of the $k$th fuel element, $\beta_k$ its blackness and $\phi_k$ the flux at its surface. If $\eta_k$ is the number of secondary neutrons leaving the $k$th fuel element for one neutron absorbed in it, then

$$q_k = \eta_k \gamma_k \phi_k.$$

(6)

Thus finally one obtains the set of homogeneous algebraic equations

$$\phi_j = \sum_{k=1}^{N} \left( \gamma_k / D \right) \left[ \eta_k F(\mathbf{r}_j, \mathbf{r}_k) - G(\mathbf{r}_j, \mathbf{r}_k) \right] \phi_k.$$

(7)

For a critical reactor the determinant of the coefficient must vanish (critical condition).

The main restrictions of this method lie in the assumption that the presence of the fuel elements does not change the slowing-down process. Thus, any contribution by the fuel elements to the slowing down (by elastic scattering in the coolant or inelastic scattering in the fuel) cannot be taken into account as such but only by modifying the value of the Fermi-age $\tau$. Also, resonance absorption has to be included in the "effective rod-blackness" $\beta$, together with some correction for the difference in age between resonance neutrons and thermal neutrons. The finite extension of the fuel elements can be taken into account by some slight modifications of the general theory.

The same problem will now be formulated according to multi-group theory in the following way. One has to solve the multi-group diffusion equations in the moderator

$$D_j \nabla^2 \phi_i(\mathbf{r}) - \Sigma_{a,i} \phi_i(\mathbf{r}) + \sum_{j=1}^{g} \Sigma_{i \rightarrow j} \phi_j(\mathbf{r}) = 0,$$

(8)

where the fluxes $\phi_i$ satisfy the following boundary conditions:

$$\phi_i(\mathbf{r}) = 0 \text{ for all } i \text{ and } \mathbf{r} \in \Gamma,$$

(9)

where $\Gamma$ denotes the reactor surface, and

$$\frac{\partial \phi_i}{\partial n}(\mathbf{r}) = \sum_{j=1}^{g} \sigma^{(k)}_{i,j} \phi_j(\mathbf{r}) \text{ for } \mathbf{r} \in \Gamma_k,$$

(10)
where $\Gamma_k$ denotes the surface of the $k$th fuel element. Thus in this method each fuel element is described by a $g \times g$ "surface matrix" $a_{ij}$ connecting the surface fluxes and its normal derivatives. Since the labour connected with the calculation increases strongly with the number of groups $g$, one usually chooses $g = 2$ which for well-moderated thermal reactors is usually sufficient, at least for a suitable choice of the fast relaxation length which gives the best fit of the slowing-down distribution. In this case a fuel element is described by four parameters $a_{ij}$, whereas in the kernel method one has only two parameters ($\gamma$ and $\eta$) available for the characterization of a fuel element.

In particular, one can include in the four parameters a description of the epithermal absorption and of the slowing down taking place in the fuel element itself. For the actual calculation Eq. (8) and the boundary conditions of Eqs. (9) and (10) are converted into a set of homogeneous algebraic equations by replacing the derivatives by difference operators, and are then solved numerically, using a source iteration procedure.

The finite difference method is obviously more flexible and more general, but on the other hand it leads to longer computations. In fact, whereas the final algebraic system of equations in the kernel method is of rank $N$ for a reactor with $N$ elements, its rank in the case of the finite difference method exceeds even $2N$ since there must be more than one mesh point in the moderator between two mesh points representing fuel element positions. However, the coefficients have a much simpler form in the case of the finite difference method, so that the difference in computation time between these two methods is less than one would expect from a comparison of the ranks alone.

II. APPLICATION OF THE THEORY TO THE INTERPRETATION OF SUBSTITUTION EXPERIMENTS

Critical experiments, in which an increasing number of fuel elements of the reference lattice is successively replaced by elements of a new lattice, are called substitution experiments. Up to now the evaluation of such a type of experiment has been performed by using two-group theory and treating the two different fuel regions as concentric homogeneous domains. Since the homogenization procedure presupposes that a certain extension of the lattice has been realized, it is not possible to reduce the number of substituted elements below a certain minimum, otherwise the interpretation on the basis of a homogenized theory would not be possible. The theoretical difficulties are especially apparent if one tries to calculate the "coupling factor" $S$ of the standard method from basic data [8].

As a result of the substitution experiment the classical method yields the buckling of the new lattice, which is a parameter typical of the lattice as a whole, and which depends also on the lattice pitch.

An interpretation of a substitution experiment by a heterogeneous method must determine at least two parameters (using a kernel method) or even four parameters (if one uses the two-group finite difference method). These parameters are properties of the fuel element and should not depend on the lattice pitch. (This latter requirement gives a check on the approxi-
mations used in the method chosen.) It is clear that for a determination of the two parameters of the kernel method one must measure two independent quantities. One of these two quantities should be the critical height ($D_2O$ level in the case of the ECO (Expérience Critique Orgel) critical facility now in construction at the Centre commun de recherche nucléaire, Ispra [9]). The other quantity could either be the detailed thermal flux shape in the vicinity of the new fuel element (giving, in particular, the logarithmic derivative of the thermal flux at the surface of the fuel element), or even the critical height for a different lattice pitch. In fact, though the fuel element parameters should be independent of the lattice pitch $b$, they enter nevertheless with different weights in the total criticality balance of the reactor as a whole ($\eta_k \gamma_k$ having the coefficients $F(\tau_j, \tau_k)$, $\gamma_k$ having the coefficients $G(\tau_j, \tau_k)$, $j = 1, \ldots, N$). These relative weights depend sensibly on the lattice pitch, in a manner that can easily be calculated by the heterogeneous method. For a given reference lattice a substitution of $n_1$ elements by elements of the new lattice yields a critical height $h$, which is a function of $n_1, \gamma_0, \eta_0, \gamma_1, \eta_1, b$, where the index 0 denotes the reference values and the index 1 the new values. For fixed values of $n_1, \gamma_0, \eta_0$ and $b = b_1$ we have (assuming the differences between the parameters of the two lattices to be small)

$$\Delta h(b_1) = \frac{\partial h(b_1)}{\partial \eta} \Delta \eta + \frac{\partial h(b_1)}{\partial \gamma} \Delta \gamma.$$

(11a)

Similarly for the same arrangement, but a pitch $b = b_2$, we have

$$\Delta h(b_2) = \frac{\partial h(b_2)}{\partial \eta} \Delta \eta + \frac{\partial h(b_2)}{\partial \gamma} \Delta \gamma.$$

(11b)

The coefficients $\partial h/\partial \eta$ and $\partial h/\partial \gamma$ are calculated by the theory. Since their dependence on $b$ is different, the determinant

$$\left| \begin{array}{cc} \frac{\partial h(b_1)}{\partial \gamma} & \frac{\partial h(b_1)}{\partial \eta} \\ \frac{\partial h(b_2)}{\partial \gamma} & \frac{\partial h(b_2)}{\partial \eta} \end{array} \right|$$

does not vanish and therefore the system of Eqs.(11a) and (11b) can be solved for $\Delta \eta$ and $\Delta \gamma$. In the appendix the results of numerical calculations for a typical example are given; they illustrate the ease with which the theory can be applied.

III. CONNECTION WITH THE QUANTITIES DETERMINED IN OSCILLATION EXPERIMENTS

In an oscillation measurement one determines the absorption and production integrals of the sample. These integrals can be written in the following way [10]:

\begin{align*}
\Delta h(b_1) &= \frac{\partial h(b_1)}{\partial \eta} \Delta \eta + \frac{\partial h(b_1)}{\partial \gamma} \Delta \gamma, \\
\Delta h(b_2) &= \frac{\partial h(b_2)}{\partial \eta} \Delta \eta + \frac{\partial h(b_2)}{\partial \gamma} \Delta \gamma.
\end{align*}
\[ A(\bar{F}) = S \int \beta(E) J_{in}(E, \bar{F}) \, dE, \quad P(\bar{F}) = S \int \eta(E) \beta(E) J_{in}(E, \bar{F}) \, dE, \]

where \( S \) is the sample surface and \( J_{in}(E, \bar{F}) \) the unperturbed current incident on the sample per unit interval of energy. For a long cylindrical sample of radius \( a \) and a nearly isotropic unperturbed flux at \( \bar{F} \) we have, by the same arguments that lead to Eq. (5),

\[ A(\bar{F}) = \int \gamma(E) \phi(E, \bar{F}) \, dE, \quad P(\bar{F}) = \int \eta(E) \gamma(E) \phi(E, \bar{F}) \, dE, \]

where

\[ \gamma(E) = \frac{\pi}{2} a \beta(E), \]

and where \( \phi(E, \bar{F}) \) is the unperturbed flux per unit energy. If the absorption is mainly caused by the thermal neutrons, and the spectrum of the unperturbed flux can be represented by some normalized Maxwellian expression \( M(E) \) such that for thermal energies

\[ \phi(E, \bar{F}) = M(E) \phi_0(\bar{F}), \]

then we have

\[ A(\bar{F}) = \gamma \phi_0(\bar{F}), \quad P(\bar{F}) = \eta \gamma \phi_0(\bar{F}), \]

where

\[ \gamma = \int \gamma(E) M(E) \, dE \]

\[ \eta = \frac{\int \eta(E) \gamma(E) M(E) \, dE}{\int \gamma(E) M(E) \, dE}. \]

We thus see that the parameters of the kernel-type heterogeneous theory can be determined also by suitable oscillation measurements.

**APPENDIX**

In section II it was proposed to determine the parameters \( \gamma \) and \( \eta \) from measurements of the critical height at two different lattice pitches. However, it is not clear a priori whether these measurements could be analysed sufficiently enough for an independent determination of the two parameters, or in other words, whether the coefficients \( \partial h/\partial \eta \) and \( \partial h/\partial \gamma \) are sufficiently independent and sensitive to the value of the lattice pitch to allow the resolution of the linear equations

\[ \Delta h(b_1) = c_\eta(b_1) \Delta \eta + c_\alpha(b_1) \Delta \alpha \]

\[ \Delta h(b_2) = c_\eta(b_2) \Delta \eta + c_\alpha(b_2) \Delta \alpha \]
for \( \Delta \eta \) and \( \Delta \alpha = \Delta \gamma / 2\pi D \). In these equations \( b_1 \) and \( b_2 \) denote the two lattice pitches, and \( \Delta b(\cdot) \) the corresponding changes in the critical height resulting from the substitution of the elements whose parameters \( \eta \) and \( \alpha = \gamma / 2\pi D \) differ by the quantities \( \Delta \eta \) and \( \Delta \alpha \) respectively from those of the reference lattice. The coefficients \( c_\eta \) and \( c_\alpha \) are the derivatives \( \partial h/\partial \eta \) and \( \partial h/\partial \alpha \) respectively; taken at the indicated values of the lattice pitch.

To obtain an idea about the behaviour of the coefficients \( c_\eta, c_\alpha \), a model case has been studied, using the code TRIHET IV which has been developed by the French company, Groupement Atomique Alsacienne Atlantique. For this case the reference lattice consisted of 80 natural uranium rods (radius 1.6 cm) arranged in a quadratic array in a heavy-water moderator, contained in a tank of 3.0 m diameter. Instead of determining the critical height for each lattice pitch it was more convenient to fix the height at 1.5 m and to calculate the criticality factor. This change in the approach has no effect on our conclusions since it amounts to a multiplication of the determinant of the coefficients in Eq.(1) by the constant and non-vanishing factors \( dh(b_1)/dk \) and \( dh(b_2)/dk \).

### TABLE I

**k\text{eff} AS A FUNCTION OF THE LATTICE PITCH b**

(REFERENCE LATTICE)

<table>
<thead>
<tr>
<th>b (cm)</th>
<th>k\text{eff}</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.0608</td>
</tr>
<tr>
<td>15</td>
<td>1.0493</td>
</tr>
<tr>
<td>20</td>
<td>1.0012</td>
</tr>
<tr>
<td>25</td>
<td>0.9291</td>
</tr>
</tbody>
</table>

Since resonance absorption is not essential to the problem in question (although it must, of course, be provided for in the final form of the analysis) it was neglected, and thus the parameters \( \eta \) and \( \alpha \) of the reference lattice were chosen to be \( \alpha = 0.475 \) and \( \eta = 1.30 \). The values of \( k_{\text{eff}} \) obtained for this reference lattice are shown in Table I. For the simulation of the substitution experiment the parameters of the four central rods were changed by the amounts \( \Delta \eta \) and \( \Delta \alpha \).

Case 1: \( \Delta \eta = 0.244 \quad \Delta \alpha = -0.075 \)

Case 2: \( \Delta \eta = -0.143 \quad \Delta \alpha = 0.0 \)

The corresponding values of the change \( \delta \) in \( k_{\text{eff}} \) are given in Table II.

From these results one can now determine the quantities \( e_\eta, e_\alpha \), which enter the relation

\[
\delta(b) = e_\eta(b) \Delta \eta + e_\alpha(b) \Delta \alpha,
\]

and which are related to the quantities \( c_\eta, c_\alpha \) by

\[
e_\eta(b) = -c_\eta(b) \frac{dk}{dh(b)} \quad e_\alpha(b) = -c_\alpha(b) \frac{dk}{dh(b)}.
\]
TABLE II

CHANGE IN CRITICALITY RESULTING FROM THE SUBSTITUTION

<table>
<thead>
<tr>
<th>b (cm)</th>
<th>( \delta ) (units of 100 pcm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
</tr>
<tr>
<td>10</td>
<td>13.8</td>
</tr>
<tr>
<td>15</td>
<td>16.8</td>
</tr>
<tr>
<td>20</td>
<td>18.6</td>
</tr>
<tr>
<td>25</td>
<td>20.5</td>
</tr>
</tbody>
</table>

One finds

\[ e_\eta(b) = \delta_{\text{Case } 2} / -0.143 \]

\[ e_\alpha(b) = (\delta_{\text{Case } 1} - e_\eta(b) \times 0.244) / -0.075 \]

The results are given in Table III. One sees immediately that \( e_\eta \) and \( e_\alpha \) have a different functional dependence on the lattice pitch. Therefore, one can solve the equations

TABLE III

DETERMINATION OF \( e_\eta \) AND \( e_\alpha \)

<table>
<thead>
<tr>
<th>b (cm)</th>
<th>( e_\eta(b) )</th>
<th>( e_\alpha(b) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>59.44</td>
<td>9.378</td>
</tr>
<tr>
<td>15</td>
<td>70.63</td>
<td>5.783</td>
</tr>
<tr>
<td>20</td>
<td>77.62</td>
<td>4.524</td>
</tr>
<tr>
<td>25</td>
<td>87.41</td>
<td>10.667</td>
</tr>
</tbody>
</table>

\[ \delta(b_1) = e_\eta(b_1) \Delta \eta + e_\alpha(b_1) \Delta \alpha \]

\[ \delta(b_2) = e_\eta(b_2) \Delta \eta + e_\alpha(b_2) \Delta \alpha \]  \hspace{1cm} (1')

that result from substitutions at the two pitches \( b_1 \) and \( b_2 \). One thus obtains

\[ \Delta \eta = R_{11} \delta(b_1) + R_{12} \delta(b_2) \]

\[ \Delta \alpha = R_{21} \delta(b_2) + R_{22} \delta(b_2) \]  \hspace{1cm} (4)
where the elements of $R_{ij}$, the inverse matrix of

$$
\begin{pmatrix}
e_\eta(b_1) & e_\alpha(b_1) \\
e_\eta(b_2) & e_\alpha(b_2)
\end{pmatrix}
$$

are given by

$$
R_{11} = e_\alpha(b_1)/D \\
R_{12} = -e_\alpha(b_2)/D \\
R_{21} = -e_\eta(b_1)/D \\
R_{22} = e_\eta(b_2)/D,
$$

with

$$
D = e_\eta(b_1)e_\alpha(b_2) - e_\alpha(b_1)e_\eta(b_2).
$$

For example, choosing $b_1 = 10$ cm, $b_2 = 20$ cm, we get $D = -459.014$ and

$$
R_{11} = -0.00986 \\
R_{12} = 0.02043 \\
R_{21} = 0.1691 \\
R_{22} = -0.1295.
$$

From the magnitude of these coefficients one can conclude that a precision of the order of 10 pcm will determine the parameter $\eta$ of the substituted elements with a maximal error of about 2% and the coefficient $\alpha$ to within 5%. The relative importance of the two parameters for the criticality of a lattice consisting only of the substitution elements is such that the uncertainty in $\alpha$ has not more effect than the uncertainty in $\eta$, and thus the overall criticality of such a lattice will be known to within some hundred pcm.

The above considerations are based on the linearized Eq.(1) or (1') which are valid only when terms of higher order in $\Delta\eta$ and $\Delta\alpha$ can be neglected. For the practical analysis of substitution experiments, charts will be provided on which (for a given lattice pitch) the contours of constant critical height will be drawn in an $\eta\alpha$-plane. By the use of such charts, substitution experiments can be easily analysed, also in the cases where the linear approximation is not suitable.

**REFERENCES**

2. FEINBERG, S.M., Proc. UN Int. Conf. PUAE 5 (1956) 484.
3. MEETZ, K., Proc. 2nd. UN Int. Conf. PUAE 16 (1958) 611.
5. LIGOU, J., Personal communication.

**DISCUSSION**

U. FARINELLI (Chairman): Although this paper has been presented at the session concerned with organic-moderated or cooled assemblies, I do not think that its applications are limited to organic systems. The progress-
ive substitution method is becoming increasingly important, owing to the need to keep an eye on costs.

C. MANDRIN: Mr. Blaesser is proposing in his paper that account should be taken of resonance absorptions by adjusting the absorption coefficient (thermal blackness) $\beta$ and by modifying the thermal neutron age in the heterogeneous method of Feinberg and Galanin. Other authors - especially Horning - use more complicated methods, introducing a third group (the resonance neutron group). I wonder whether Mr. Blaesser could estimate what agreement might be expected between the simplified method he proposes and the experimental measurements.

G. BLAESSER: Thus far, we have used the simplified theory of resonance absorption only on a trial basis. We intend to use it as long as it works, i.e. as long as the corrected heterogeneous parameters are independent of the lattice pitch. (The success of the procedure depends on the errors in the calculation of the $P$-factor being cancelled out to some degree by the experimental measurement of the fuel blackness $\beta$, or of the equivalent factor $\gamma$. The only remaining question is whether the theory will enable us to predict the change in the resonance factor $P$ sufficiently well to achieve independence of the lattice pitch with the corrected values of $\eta$, $\beta$ or $\gamma$. If we could do this, we would not have to bother about finding ways of dividing the blackness of the rod into a thermal part and a resonance part. All we would need to use is the effective blackness, and this is what would enter into subsequent calculations of reactor criticality. If it turned out that the experiments could not be interpreted in this way, we should have to use the two-group finite difference formulation, which makes allowance for the effective resonance integrals of the fuel rods.

B. VITTOZ: How do you take account of the finite dimension of the moderator or of a reflector different from the moderator? I am referring here to the general theory, not to your work in particular.

G. BLAESSER: As to the general case, one can take other reflectors into account by modifying the effective boundary conditions (e.g. using a mixed boundary condition) on the core reflector boundary in the case of the finite difference methods, or by modifying the Green's function in the case of the kernel methods. We have not tried this so far because we can use the reactor without an external reflector (i.e. we can shut off the graphite reflector and consider only the reflection due to the heavy water).

H. R. LUTZ: Have you applied your formalism to existing experimental results?

G. BLAESSER: We have not had time to apply these methods to the substitution experiments done so far, but we do intend to apply them. The reason for this is that the heterogeneous method we have been using involves a rather limited number of elements which can be substituted. Now, however, working in collaboration with a French company, the Groupeement Atomique Alsacien Atlantique (GAAA), we have developed a very flexible heterogeneous programme which allows us to treat up to 400 fuel elements where we have a symmetry of 4, and up to 100 fuel elements where we have no symmetry. We are planning to use this code in doing some theoretical analyses of experimental data reported in the literature.

H. R. LUTZ: What do you think would be the minimum number of elements needed for getting meaningful results?
G. BLAESER: In our calculations, the precision with which the heterogeneous parameters can be determined depends on the number of elements used in the substitution. If the number of rods substituted is too small, the statistical weight, or the influence on the over-all reactivity of the substituted zone, is so small that the measurements do not have the required accuracy. We therefore use something like half a dozen elements, at least. However, considering the 24 elements used in the homogeneous approach, the number of elements replaced can easily be reduced by half and, given the high price of carbide fuel, this alone would result in a very appreciable saving of money - an amount, in fact, which would pay for the work of our group for several years.

R. PERSSON: Are you planning to use any special experimental techniques to measure differences more accurately?

G. BLAESER: Special techniques for measuring small differences in the lattice parameters have not yet been developed in the case of the heterogeneous method, but this subject will be investigated in the future.
PANEL DISCUSSION
F. MILES (Chairman): I think the most satisfactory procedure will be to ask the members of the Panel to summarize the results of the Symposium under the five headings into which we have divided the subject matter. I shall therefore ask Mr. Ward to speak on heavy-water reactors, Mr. Hellens on light-water lattices, Mr. Hitchcock on graphite lattices, Mr. Kuznetsov on fast reactors and Mr. Breton on the various techniques used in this work.

A. WARD: I came to this Symposium with fixed ideas about the state of the art in regard to heavy-water-moderated reactors. As is usual at the end of a symposium, my head is in a whirl, so I think all I can do in the few minutes allotted to me is to pass on some random thoughts.

First of all, I was impressed very much by the development of the art of acquiring information from exponential facilities. There have been many papers on this subject, and I think the accuracy of the measurements is very much to the credit of the people doing the experiments. I believe it was at the First Geneva Conference that Mr. Persson, in connection with buckling measurements for heavy-water lattices, remarked that the values of buckling derived for a given lattice were inversely proportional to the diameter of the tank in which measurements were made. We seem to have come a long way since then. The Savannah River Laboratory in particular has been responsible for a very wide range of buckling measurements for D2O-moderated lattices. I seem to recall Dr. Graves saying that he felt sufficient information of this type was now available. I therefore think our efforts should be turned to arriving at a much better understanding of neutron behaviour in a lattice, and several papers at this Symposium did actually give us more detailed information about what goes on in the lattice cell. I was very pleased to learn that Mr. Poole, who reported on his chopper measurements, has at last started to make measurements using D2O as the moderating substance. It has now taken third place, after graphite and H2O, in his programme.

Perhaps I could just run briefly through the various types of reactor coolant and comment on them. During one of our sessions we heard some papers about organic cooling—a method highly favoured for heavy-water reactors; in fact, the use of organic-cooled reactors and also of carbide fuel will probably be very important future developments. Fortunately, from the reactor physics point of view, the main directive to design engineers in respect of organic-cooled heavy-water reactors is fairly straightforward: we simply tell them to put the minimum amount of organic coolant into the reactor.

The work on void coefficients in coolant channels and on the reactivity effect of these voids is extremely interesting, and the precision of the
measurements is remarkable. As regards heavy-water-moderated reactors with heavy-water coolant (used as a boiling coolant), I suppose the main application of this work is in enhancing reactor safety and control. I myself am not really in a position to assess the problem posed by the use of voids in the coolant of such a reactor, but I do not think the reactivity investment in the coolant is too great, so that the problem is probably quite manageable and we can hope to have a satisfactory solution.

One other fact I would like to comment on is the lack of experimental results relating to reaction rates, neutron spectra, etc., in either irradiated or simulated fuel. This I think is one of the fields where we will need a great deal more work. Such information is not easy to obtain and perhaps we should pay some attention to Mr. Peterson's advice about making the best use of the results from operating reactors, as they become available.

R. HELLENS: For the sake of brevity, I shall comment on measurements in light-water lattices only from the point of view of their importance to nuclear design theory. In reviewing the available data from such systems last winter, we found that measurements had been reported for some 280 uranium-metal lattices, 85 uranium-dioxide lattices and about 20 zone or multi-region assemblies. Off-hand, it would seem that this mass of information should now provide an adequate basis for theory, but closer inspection of the data unfortunately shows rather marked deficiencies. Perhaps the best way of indicating future experimental needs is to summarize some of these present shortcomings.

Our knowledge of experimental behaviour in light-water lattices is limited by the lack of precision in earlier measurements. It is difficult to specify exactly what precision is desired, but for the sake of discussion let us say that the uncertainty in the various parameters of the lattice should not affect the reactivity by more than about 0.2% over a substantial range of lattice configurations: for example, if the thermal utilization is about 0.8, the error in the disadvantage factor should be less than about 1%. The choice of a criterion of this type thus makes the desired accuracy rather strongly dependent on lattice configuration, enrichment, water-to-uranium ratio, etc. Let us consider the situation with regard to some of these parameters in greater detail.

First, thermal utilization. During this Symposium, Dr. Hardy* presented the results of a rather extensive series of disadvantage factor measurements and calculations, and there was a fairly consistent tendency for the experimental results to lie about 3 to 6% below the theoretical. I gather that there is at the moment no obvious explanation of this discrepancy.

Secondly, the fast-fission ratio. Although there has been little discussion at this meeting about the fast effect, it should be recalled that theory (either Monte Carlo or analytical approximations) predicts a smaller variation of $\delta^{28}$ with water-uranium volume ratio than is actually observed. These discrepancies are worth perhaps 0.4% in reactivity over the normal volume-ratio range and are thus a subject of continuing interest, at least to the theorists.

* HARDY, J. et al.: Thermal neutron spectral and spatial distributions in light-water-moderated uranium lattices, these Proceedings II.
Thirdly, resonance capture. In his paper, Dr. Engelder* indicates that epi-to sub-cadmium capture ratios in U\(^{238}\) can now be measured to ±1%, although few such high-precision measurements actually exist for light-water lattices. I think Dr. Lutz would agree that this margin of error is what can be looked for also in the cadmium-cover technique, if this technique is used with very considerable care. In wet lattices such precision is adequate but, in the under-moderated assemblies, results with an uncertainty of this magnitude are not very definitive and they do not help the theory a great deal. It seems fair to say that measurements of resonance capture have been perhaps the weakest point in water-lattice studies for many years and now that techniques have been improved we can hope that a large number of good measurements will be made.

Fourthly, neutron leakage. This is the most difficult quantity to interpret in water-lattice studies since its description depends on the type of critical equation being used. Two methods of measurement are common: observation of the change in reactor period - or of poison concentration for criticality - with reactor size, and measurement of the indium activation about a plane source of fission neutrons. The latter has a simple theoretical interpretation but measurements exist only for water, for some thorium-water lattices and, if memory serves, for a few cadmium-loaded uranium-water lattices. To be useful, these measurements should include the fourth as well as the second spatial moment, since the influence of the former can frequently be about 3% in reactivity. Of course, the reactor period measurements of leakage at finite buckling measure a combination of the moments of the slowing-down distribution and very frequently include some fast-fission cycling as well. Dr. Lunin gave a very interesting paper** dealing systematically with this problem from the point of view of the general moment-expansion critical equation. Poisoned exponential experiments extending to zero buckling provide one of the most direct tests of neutron leakage and the theory seems to be in fairly good agreement with measurements.

Fifthly, material buckling. This is the most common measurement of all and I think it provides the best test of our ability to compute criticality. Measurements of buckling at Brookhaven National Laboratory during the last two years have shown a significant discrepancy of 1–4 m\(^2\) between the results obtained by the flux-shape and the variable-loading methods. It is possible to account theoretically for these differences. The flux-shape method tends to yield systematically high bucklings whereas the variable-loading method apparently under-estimates the correct value. The problem becomes unusually severe in lattices with large material buckling, and it may well be necessary to poison such systems in order to carry out valid exponential experiments.

A. HITCHCOCK: We heard comparatively little during this Symposium about the requirements placed on us physicists by design and operating engineers and about the theoretical methods by which one proceeds from experiment to the final reactor situation. However, if we are going to try and

---

* ENGELDER, T.C. et al., Role of critical, exponential, and small lattice experiments in the design of spectral shift control reactor, these Proceedings II.

** ЛУНИН Г.Л. и др., Исследование физических свойств активной зоны ВВЭР на критических сборках, these Proceedings III.
determine what kind of experiments are needed in the future, these requirements must be borne in mind. We must remember that a practical reactor is hot and poisoned and contains fuel subjected to many and continually changing irradiations and our experimental work therefore has to be directed towards dealing with the properties of the irregular lattices we meet in practice, in relation to conditions of power and temperature. Our aim must thus be the development of experimentally-tested methods of calculation which can cope with these problems and, as Dr. Ward has most pertinently remarked, this is best done by defining exactly the neutron balance in a complicated situation. It is fortunate that advances in the theory of thermalization and the advent of large computing machines have brought us to the point where, in simple situations with a single moderator, such as are found in the graphite problems, this goal is not entirely unattainable although certainly there is still a great deal to be done.

However, it is perhaps now possible to say, as a result of this meeting, what types of work seem likely to be profitable. Firstly, there are the basic spectrum studies in simple systems — described at this Symposium by Dr. Poole and by other workers elsewhere — which are designed to develop the theories of thermalization. Secondly, there is the general study of basic uniform lattices: I would agree here with Dr. Ward that we must be prepared to justify and improve our theoretical methods by very detailed studies of a comparatively small number of lattices rather than by the familiar long series of simple buckling measurements in cold, clean lattices where exponential piles are used.

Two tasks which still remain in connection with basic lattices are the study of systems containing plutonium and thorium and the study of heated lattices. I am convinced, myself, that we shall make far more progress by comparing not only the end result of our theories (i.e. buckling) but also the intermediate results (i.e. the various reaction rates), going into as much theoretical detail as is necessary to account properly for the different effects. I am thinking, in particular, of such an approach as detailed studies with large numbers of resonance detectors. In this connection, it has been gratifying to hear so many papers on the technique of measuring $\rho^{28}$ and related quantities. It may perhaps be not too much to hope that our ever-present fear of systematic error, which has bedevilled this kind of measurement and its interpretation for so long, may soon be allayed. I have been wondering whether it would be possible, soon, to develop similar techniques for dealing with the reaction rates in the plutonium isotopes, in the case of plutonium-bearing fuel. I was a little disappointed not to hear more during this Symposium on that perennial and vexed question of the choice between fission chambers and fissile foils.

Finally, I think it was most significant that, in the session specifically concerned with graphite, so many of the papers dealt with the design and use of heated systems. The rate of operation in a system of this kind is normally extremely slow and I hope it may be possible, at some future meeting organized by the Agency, to have an opportunity to discuss the various design features which have accelerated work in these large and ponderous hot facilities. For myself, I would suggest that we must make every effort to concentrate a great number of measurements into a single run and so get more information out of a single cycle of operations than
TRENDS IN EXPONENTIAL AND CRITICAL EXPERIMENTATION

has been customary in the past. The other great task which I think confronts us is the general problem of the irregular lattice and, apart from control-rod worth studies, this still seems to be an extremely open field. I was especially impressed by the contribution to this Symposium of the small team from Imperial College* which showed once again—if it was necessary to show it—that good work is not the prerogative of the large government-financed organization. The reports which we have heard on a series of extremely interesting irregular lattices in the Savannah River heavy-water facility will certainly merit further study when we get home. I note that these experiments were analysed on the basis of the theory associated with the names of Feinberg and Galanin and were apparently in tolerably good agreement with it. I find it gratifying that this theory is at last being used in practical cases.

Summing up, I would say, on the basis of what we have heard at this meeting, that in future—at least in the graphite systems—we shall be using our subcritical facilities, often heated, for detailed studies of the neutron balance and reaction-rate distributions within the cells of a uniform lattice, and our critical facilities for the study of power distribution and reactivity in irregular lattices and probably also for the measurement of coefficients of reactivity.

V.A. KUZNETSOV: We have today come to the end of a comprehensive exchange of opinions and ideas on an interesting range of questions connected with work on subcritical and critical assemblies of widely differing reactor types. About two years have passed since the Vienna Symposium on the Physics of Fast and Intermediate Reactors, and it was hardly to be expected that very much new material on fast-reactor physics could have become available since then. Yet, it is a pleasure to note that some very interesting material on the subject has been presented at this Symposium. The papers presented here show that certain definite trends have developed in fast-reactor research. These trends are very similar, regardless of the country or the reactor in question, and the reports we have heard on critical experiments prove that fast-reactor physicists are confronted with common problems and difficulties. Our exchange of opinions at this meeting has yielded welcome information on the status of the problem in various countries. We have heard very interesting papers by delegates from the United States and the United Kingdom on fast reactors, and we have heard about the MAZURCA project and the plans of our French colleagues to set up a fast-neutron critical mock-up. Work done on this subject in the Soviet Union has also been described.

I should like to discuss only a small group of questions which compel our immediate attention, because an analysis of all the papers presented at this Symposium is a difficult task—and one to which we can address ourselves only when we return to our respective countries.

What are the main difficulties facing fast-reactor physicists in the different countries? For one thing, there are the problems connected with temperature coefficients of reactivity in fast reactors and it is gratifying to learn from the papers presented that considerable attention is now being

* GRANT, P.J. and JAMES, C.G., Use of an exponential assembly to investigate boundary conditions in neutron diffusion, these Proceedings III.
paid to this subject. The problem of the reactivity coefficients is a very serious one for all types of reactor and it has been studied very little. It would appear that we shall have to devote greater attention to this subject in future, because advances in the relevant field of reactor theory are still inadequate. What the theoreticians need is more experimental information in this matter. In investigating temperature coefficients of reactivity, it seems to me that we should pay particular attention to the separation of these coefficients into their different parts, i.e. to determining the contribution of each component to the coefficient as a whole. I have in mind, for example, the reactivity Doppler coefficient: this may prove to be the simplest of these components - which would account for its having been the first one studied in various countries. Another promising avenue of research reported in some of the papers was the work done on temperature coefficients of reactivity in systems having different core components and uranium-plutonium ratios. We have also heard reports on interesting work concerned with determining the influence of screens and reflectors on these coefficients. I might say that the reason for my placing so much emphasis on temperature coefficients of reactivity is that I consider this to be one of the most complex problems facing fast-reactor physics at the present time.

I should like to make special mention of the paper presented by Dr. Cohn of the United States, in which he drew attention to what I would consider a number of promising methods for investigating the physical properties of reactors. These methods, and in particular the so-called statistical methods, are becoming increasingly important in several countries. A subject that merits serious attention and which may eventually yield valuable information on reactor physics is the time-space correlation between the noises of neutron detectors. There are interesting possibilities in the use of silicon semiconductor detectors to study the hard part of fast-reactor spectra, a subject which was also mentioned by Dr. Cohn. Unfortunately, no results of work along these lines were reported at the present Symposium but it is to be hoped that they will be described in the literature or at subsequent meetings.

A welcome development at this Symposium was the great amount of attention paid to pulsed-neutron methods of studying subcritical reactors, an approach which is significant for all types of reactor, not only fast ones. It is likely to be widely used in the future. Another subject which has applications for all types of reactor is research on two-region and multi-region facilities with insertion units, by means of which the parameters of larger assemblies can be studied. More specifically, I have in mind the work on the ZPR-III described by our United States colleagues. Two other subjects which are of great interest in connection with future research on critical and subcritical experiments are the thermalization of neutrons, with which Dr. Poole dealt in his paper, and more detailed investigation of the effectiveness of neutron absorbers.

D. BRETON: During this Symposium various experimental techniques have been considered, and I think they should be briefly reviewed. First of all, there is the exponential experiment, which has been mentioned several times and which finds very many applications in reactor physics studies. You may remember that one of the questions raised by the Agency
was whether exponential experiments are more useful than critical experiments and, while it is difficult to form ideas and conclusions immediately after a Symposium, it seems to me that various difficulties are associated with exponential experiments, which does not mean, of course, that they should be systematically rejected. Certain papers nevertheless did draw attention to differences in results.

I have noted with satisfaction that the progressive substitution method is gaining more and more supporters. It does seem to possess several advantages, in particular because of the limited number of fuel rods required and because of its relatively simple application.

In connection with buckling measurements, I was also interested to note the extension of the so-called PCTR techniques, developed at Hanford, and I particularly welcomed the indications that there were prospects for their use outside the thermal reactor field.

I was pleased to note that fine structure measurements are being further developed. I know by experience that it is an extremely lengthy and costly procedure to develop and perfect experimental techniques, and I think the efforts being made in various laboratories to measure what is generally known as $\rho^{28}$ are particularly important. Fine structure measurements can be carried out in different types of installation, and those which would seem to merit most attention are those made in assemblies supplied by large neutron sources, such as reactors.

Another technique which has been mentioned several times and which has been stressed by other experts is the pulsed-neutron technique. This approach seems very promising, and in conjunction with exponential experiments it can give additional information to that gained from exponential experiments alone. Apart from determining the diffusion coefficient, these $\alpha$-coefficient measurements enable us to undertake studies on moderation and to obtain the diffusion cooling term $C$, thereby opening up a field of experimentation for the new theories now appearing in connection with phonon spectra and moderation. These measurements by pulsed-neutron techniques have also found particularly interesting applications in the measurement of control-rod interaction; a number of papers gave interesting results obtained by this method and compared them with results obtained by the more conventional methods, showing that there is good agreement between them. These methods also permit determination of the term $\beta/\tau$, an extremely important factor in reactor studies.

I was surprised that mention was made only once or twice of the possibilities offered by fluctuation measurements and background noise studies in reactors. It appears to me that this technique, which is admittedly extremely difficult to apply, should be further developed and extended.

I think the time-of-flight measurements can also be included in the category of special techniques. We have all been very interested in Mr. Poole's description of the spectral measurements which he made using this technique. The method obviously lends itself to the performance of useful work on moderation and thermalization problems. Mention has been made of still other techniques for spectral measurements, these being based on the use of detectors or fission chambers. Actually, very few papers have dealt with measurements made with fission chambers and
this problem of choosing between the foil-type detector and the chamber as the means of detection should perhaps have been treated more fully.

The last point for inclusion under special techniques was of particular interest to me, namely the problem of oscillations. I think it is a problem which is extremely difficult to deal with, yet one which can give valuable information on reactor activities provided it is very accurately formulated.

Some of the papers presented have indicated possible ways of measuring capture and fission factors, and one speaker showed us how these factors can be used in conjunction with progressive substitution methods. I think these techniques should have extremely useful applications in evaluating the neutron properties of irradiated fuel elements.

Having reached the end of this Symposium, we may ask whether all the questions raised by the Agency in calling the meeting have been answered. In my opinion, a number of points have not been treated in great detail, in particular those relating to the cost of experiments, and this is perhaps regrettable, because in the choice of programmes and techniques financial questions are often of the utmost importance. It would have been very interesting to compare the costs, for example, of buckling measurements carried out using progressive substitution methods and exponential methods. I agree, however, that it is always very difficult to estimate the cost of experiments, and in my own paper I exercised some degree of precaution in citing figures. In connection with prices, another point which has not been mentioned is the cost of fuel for the critical experiments themselves. A critical experiment requires a stock of fuel, the price of which often exceeds the cost of the experiment itself, and this is a factor which in my opinion affects the choice between a critical experiment and an exponential experiment. However, I shall not try, because it would be beyond my ability at present, to reply to this question, or all the other questions raised by the Symposium and by the Agency itself.

In conclusion, I might recall that it had been mentioned in the invitation to the Symposium that there was a tendency in some quarters to feel that critical experiments had a limited future. I must confess that I came to Amsterdam thinking that it might be necessary to suggest that the Agency call a conference to devise useful tasks to which physicists previously concerned with critical experiments might turn their energies. However, in the light of what we have heard at this meeting, I think that the Agency should now consider organizing another symposium, this one to discuss programmes that could be undertaken in connection with critical experiments.

F. MILES (Chairman): Are there any further comments?

F. FEINER: It seems to me that there is one important question that has been almost completely neglected at this Symposium, namely the adequacy of the nuclear data with which our calculations have to be performed. As more sophisticated calculational techniques become available in the future, I think we shall have to consider whether there needs to be a shift in emphasis from the integral experiments we have been talking about here to what one might call differential experiments, i.e. cross-section measurements as a function of energy, which would allow calculations to be made with more precision. It seems to me that the uncertainties due to method
have become sufficiently small in many instances to make the accuracy of calculations hinge more on the nuclear data than on the method used.

A. WARD: I am sure we would all agree that the state of some of our nuclear data is unsatisfactory. I know personally that many of us are not satisfied with accepted world values for even the 2200 m/s cross-sections of plutonium-239 nor are we happy about the resonance integrals that are accepted as world values. I believe, however, that this was one of the subjects explicitly excluded from the agenda of the Symposium.

F. MILES (Chairman): This is a very big problem and we would have to devote at least another five days to it. I can inform you, however, that the Agency is considering the possibility of doing work along this line and of setting up an international group for the collection of nuclear data. Activities of this kind are, of course, already under way at other places, including Chalk River and Brookhaven, but I do not think that we can get into the subject here.

Since there are no further comments, I shall now bring this Symposium to a close, but first I would like to state one general impression. The question of what experiments or calculations are needed for any particular reactor development project will still have to be evaluated separately in each case, but I think we are now much closer to the answers. My own feeling is one of genuine amazement at the progress which has been made. Experimental and theoretical techniques are far more sophisticated than they were ten years ago and much closer agreement is now demanded between theory and experiment. For example, ten years ago there were often unexplained discrepancies of the order of 10% in reactivity whereas nowadays we look for agreement in the one per cent range.
SYMPOSIUM ON
EXPONENTIAL AND CRITICAL EXPERIMENTS
HELD AT AMSTERDAM, 2-6 SEPTEMBER 1963

CHAIRMEN OF SESSIONS

Session 1  A. WARD  Atomic Energy of Canada Ltd.,
            Chalk River, Ontario
Session 2  D. BRETON  Centre d'études nucléaires
de Saclay
Session 3  C.G. CAMPBELL  Atomic Energy Establishment,
            Winfrith, Dorchester, Dorset
Session 4  J. PELSER  Reactor Centrum Nederland,
            Petten
Session 5  W.C. REDMAN  Argonne National Laboratory,
            Argonne, Illinois
Session 6  R. PERSSON  AB Atomenergi, Studsvik,
            Nyköping
Session 7  V.A. KUZNETSOV  Physical Institute of Obninsk
Session 8  N. RAİŞIĆ  Boris Kidrič Institute for
            Nuclear Sciences, Belgrade
Session 9  U. FARINELLI  Centro di Studi Nucleari della
            Casaccia, Rome

SECRETARIAT OF THE SYMPOSIUM

Scientific Secretaries:  J.W. WEBSTER  Division of Reactors, IAEA
                        A. MERTON  Division of Reactors, IAEA
Editor:  A. HAMENDE  Division of Scientific and
         Technical Information, IAEA
Records Officer:  L. LIEBERMAN  Division of Languages, IAEA
Administrative
Secretary:  P. GHELABONI  Division of Scientific and
            Technical Information, IAEA
<table>
<thead>
<tr>
<th>Name</th>
<th>Institution</th>
<th>Nominating State or Organization</th>
</tr>
</thead>
<tbody>
<tr>
<td>F. Accinni</td>
<td>Centro Informazioni Studi Esperienze Segrate, Milan</td>
<td>Italy</td>
</tr>
<tr>
<td>J.G. Ackers</td>
<td>Reactor Centrum Nederland, Petten</td>
<td>Netherlands</td>
</tr>
<tr>
<td>E. Andersen</td>
<td>Institutt for Atomenergi, Kjeller</td>
<td>Norway</td>
</tr>
<tr>
<td>R. Andres</td>
<td>AEG Kemennergieversuchanlage, Grossweilheim</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>G. Apelqvist</td>
<td>The State Power Board, Fack</td>
<td>Sweden</td>
</tr>
<tr>
<td>J. Arkuszewski</td>
<td>Institute of Nuclear Research, Warsaw</td>
<td>Poland</td>
</tr>
<tr>
<td>A. Ascari</td>
<td>Centro Ricerche Nucleari SORIN, Saluggia (Vercelli)</td>
<td>Italy</td>
</tr>
<tr>
<td>D. Babala</td>
<td>Institutt for Atomenergi, Kjeller</td>
<td>Norway</td>
</tr>
<tr>
<td>A.R. Baker</td>
<td>Fast Reactor Physics Division, Atomic Energy Establishment, Winfrith, Dorchester, Dorset</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>J. Bárdor</td>
<td>Nuclear Research Institute of the Czechoslovak Academy of Sciences, Prague</td>
<td>Czechoslovak Socialist Republic</td>
</tr>
<tr>
<td>E. Behrens</td>
<td>Siemens-Schuckertwerke AG, Erlangen</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>P. Benoist</td>
<td>Centre d'études nucléaires de Saclay, Gif-sur-Yvette (S et O)</td>
<td>France</td>
</tr>
<tr>
<td>P. Berthet</td>
<td>Ecole Polytechnique de l'Université de Lausanne</td>
<td>Switzerland</td>
</tr>
<tr>
<td>G. Blaesser</td>
<td>EURATOM, Joint Research Centre, Ispra (Varese) Italy</td>
<td>European Atomic Energy Community</td>
</tr>
<tr>
<td>M. Bogaardt</td>
<td>Reactor Centrum Nederland, Petten</td>
<td>Netherlands</td>
</tr>
<tr>
<td>R.E. Bouwmeester</td>
<td>Graviner (Gosport) Ltd. , Atomic Energy Division, The Hague</td>
<td>Netherlands</td>
</tr>
<tr>
<td>A.R. Braun</td>
<td>N.V. Neratoom, The Hague</td>
<td>Netherlands</td>
</tr>
<tr>
<td>D. Breton</td>
<td>Centre d'études nucléaires de Saclay, Gif-sur-Yvette (S et O)</td>
<td>France</td>
</tr>
<tr>
<td>Name</td>
<td>Institution</td>
<td>Nominating State or Organization</td>
</tr>
<tr>
<td>-----------------------</td>
<td>------------------------------------------------------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>K. Bryhn-Ingebrigtsen</td>
<td>Institutt for Atomenergi, Kjeller</td>
<td>Norway</td>
</tr>
<tr>
<td>M. Bustraan</td>
<td>Reactor Centrum Nederland, Petten</td>
<td>Netherlands</td>
</tr>
<tr>
<td>C.G. Campbell</td>
<td>Atomic Energy Establishment, Winfrith, Dorchester, Dorset</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>A. Campise</td>
<td>Atomics International, Canoga Park, Calif.</td>
<td>United States of America</td>
</tr>
<tr>
<td>F. Casali</td>
<td>Comitato Nazionale per l'Energia Nucleare, Bologne</td>
<td>Italy</td>
</tr>
<tr>
<td>G. Casarelli</td>
<td>Centro Ricerche Nucleari SORIN, Saluggia (Vercelli)</td>
<td>Italy</td>
</tr>
<tr>
<td>G. Casini</td>
<td>EURATOM, Joint Research Centre, Ispra (Varese), Italy</td>
<td>European Atomic Energy Community</td>
</tr>
<tr>
<td>L.M. Caspers</td>
<td>Reactor Institute Delft</td>
<td>Netherlands</td>
</tr>
<tr>
<td>D. Clinch</td>
<td>Central Electricity Generating Board, Berkeley Nuclear Laboratories, Berkeley, Glos.</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>C. Clouet d'Orval</td>
<td>Centre d'études nucléaires de Saclay, Gif-sur-Yvette (S et O)</td>
<td>France</td>
</tr>
<tr>
<td>J. Coehoom</td>
<td>Reactor Centrum Nederland, Petten</td>
<td>Netherlands</td>
</tr>
<tr>
<td>C.E. Cohn</td>
<td>Argonne National Laboratory, Argonne, Ill.</td>
<td>United States of America</td>
</tr>
<tr>
<td>P.C.G.L. Colle</td>
<td>Centre d'étude de l'énergie nucléaire, Mol-Donk</td>
<td>Belgium</td>
</tr>
<tr>
<td>W. Cooper</td>
<td>Department of Nuclear Science and Technology, Royal Naval College, Greenwich, London</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>S. Corno</td>
<td>AGIP Nucleare, S. Donato Milanese, Milan</td>
<td>Italy</td>
</tr>
<tr>
<td>O.H. Critchley</td>
<td>Inspectorate of Nuclear Installations, Ministry of Power, Thames House South, Millbank, London</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>R. Cuniberti</td>
<td>Politecnico di Torino, Turin</td>
<td>Italy</td>
</tr>
<tr>
<td>C.J. van Daatselaar</td>
<td>Reactor Institute Delft</td>
<td>Netherlands</td>
</tr>
<tr>
<td>H. van Dam</td>
<td>Reactor Institute Delft</td>
<td>Netherlands</td>
</tr>
<tr>
<td>J. Debrue</td>
<td>Centre d'étude de l'énergie nucléaire, Brussels</td>
<td>Belgium</td>
</tr>
<tr>
<td>Name</td>
<td>Institution</td>
<td>Nominating State or Organization</td>
</tr>
<tr>
<td>------------</td>
<td>------------------------------------------------------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>H. Deckers</td>
<td>Centre d’étude de l’énergie nucléaire, Mol-Donk</td>
<td>Belgium</td>
</tr>
<tr>
<td>E. Deligat</td>
<td>Commissariat à l’énergie atomique, Paris</td>
<td>France</td>
</tr>
<tr>
<td>J. De Villiers</td>
<td>Atomic Energy Board, Pretoria</td>
<td>South Africa</td>
</tr>
<tr>
<td>D. Dingee</td>
<td>Battelle Memorial Institute, Columbus, Ohio</td>
<td>United States of America</td>
</tr>
<tr>
<td>L. Di Palo</td>
<td>Comitato Nazionale per l’Energia Nucleare, Rome</td>
<td>Italy</td>
</tr>
<tr>
<td>F. Domaniç</td>
<td>University of Ankara</td>
<td>Turkey</td>
</tr>
<tr>
<td>W. Drechsel</td>
<td>Brown Boveri/Krupp Reaktorbau GmbH, Jülich</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>F. Ebenoldt</td>
<td>Centre d’études nucléaires de Cadarache</td>
<td>France</td>
</tr>
<tr>
<td>B.T. Eendebak</td>
<td>Reactor Institute Delft</td>
<td>Netherlands</td>
</tr>
<tr>
<td>T.C. Engelder</td>
<td>Babcock &amp; Wilcox Co., Lynchburg, Va.</td>
<td>United States of America</td>
</tr>
<tr>
<td>W. Fader</td>
<td>Pratt &amp; Whitney Aircraft, CANEL, Middletown, Conn.</td>
<td>United States of America</td>
</tr>
<tr>
<td>U. Farinelli</td>
<td>Centro di Studi Nucleari della Casaccia, Rome</td>
<td>Italy</td>
</tr>
<tr>
<td>J. Feijen</td>
<td>Technological University, Eindhoven</td>
<td>Netherlands</td>
</tr>
<tr>
<td>F. Feiner</td>
<td>Knolls Atomic Power Laboratory, Schenectady, N.Y.</td>
<td>United States of America</td>
</tr>
<tr>
<td>P. Fischer</td>
<td>General Atomic Division, General Dynamics Corporation, San Diego, Calif.</td>
<td>United States of America</td>
</tr>
<tr>
<td>P. Fornaciari</td>
<td>AGIP Nucleare, S. Donato Milanese, Milan</td>
<td>Italy</td>
</tr>
<tr>
<td>E.A. Fossoul</td>
<td>Société Belge pour l’industrie nucléaire BELGONUCLEAIRE, Brussels</td>
<td>Belgium</td>
</tr>
<tr>
<td>H.R. Franzen</td>
<td>Institute of Atomic Energy, S. Paulo-Capital</td>
<td>Brazil</td>
</tr>
<tr>
<td>A. Friedman</td>
<td>United States Atomic Energy Commission, Paris, France</td>
<td>United States of America</td>
</tr>
<tr>
<td>Z. Gabrovšek</td>
<td>Josef Stefan Nuclear Institute, Ljubljana</td>
<td>Yugoslavia</td>
</tr>
<tr>
<td>W. D.J. Gestel</td>
<td>N.V. Neratom, The Hague</td>
<td>Netherlands</td>
</tr>
<tr>
<td>Y.E. Girard</td>
<td>Centre d’études nucléaires de Saclay, Gif-sur-Yvette (S et O)</td>
<td>France</td>
</tr>
<tr>
<td>Name</td>
<td>Institution</td>
<td>Nominating State or Organization</td>
</tr>
<tr>
<td>------------------</td>
<td>------------------------------------------------------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>J.A. Goedkoop</td>
<td>Reactor Centrum Nederland, Petten</td>
<td>Netherlands</td>
</tr>
<tr>
<td>P. Grant</td>
<td>Imperial College of Science and Technology, London</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>N. Grassam</td>
<td>University of Southampton, Highfield, Southampton, Hampshire</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>W.E. Graves</td>
<td>E.I. du Pont de Nemours &amp; Co., Savannah River Laboratory, Aiken, South Carolina</td>
<td>United States of America</td>
</tr>
<tr>
<td>R. Green</td>
<td>Atomic Energy of Canada Ltd., Chalk River, Ontario</td>
<td>Canada</td>
</tr>
<tr>
<td>H. Grimm</td>
<td>Österr. Studiengesellschaft für Atomenergie, Vienna</td>
<td>Austria</td>
</tr>
<tr>
<td>M. Guebén</td>
<td>Centre et Sud, S.A., Brussels</td>
<td>Belgium</td>
</tr>
<tr>
<td>J. Günther</td>
<td>Arbeitsministerium B.-W., Stuttgart</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>H. Harder</td>
<td>Deutsche Babcock &amp; Wilcox Dampfkessel-Werke AG, Oberhausen</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>J. Hardy, Jr.</td>
<td>Bettis Atomic Power Laboratory, Westinghouse Electric Corp., Pittsburgh, Pa.</td>
<td>United States of America</td>
</tr>
<tr>
<td>M.J. Harris</td>
<td>University of Manchester</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>R. Hecker</td>
<td>Jülich Nuclear Research Establishment, Jülich</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>R.J. Heijboer</td>
<td>Reactor Centrum Nederland, The Hague</td>
<td>Netherlands</td>
</tr>
<tr>
<td>A.W. van der Heijden</td>
<td>Reactor Institute Delit</td>
<td>Netherlands</td>
</tr>
<tr>
<td>R. Heineman</td>
<td>Hanford Laboratories, General Electric Co., Richland, Wash.</td>
<td>United States of America</td>
</tr>
<tr>
<td>W. Heintz</td>
<td>Physikalisch-Technische Bundesanstalt, Braunschweig</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>R. Hellens</td>
<td>Brookhaven National Laboratory, Upton, N.Y.</td>
<td>United States of America</td>
</tr>
<tr>
<td>E. Hellstrand</td>
<td>AB Atomenergi, Studsvik, Nyköping</td>
<td>Sweden</td>
</tr>
<tr>
<td>P.B. Hemmig</td>
<td>Reactor Division, United States Atomic Energy Commission, Washington, D.C.</td>
<td>United States of America</td>
</tr>
<tr>
<td>H.-H. Hennies</td>
<td>Interatom, Cologne</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>Name</td>
<td>Institution</td>
<td>Nominating State or Organization</td>
</tr>
<tr>
<td>----------------</td>
<td>------------------------------------------------------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>H. Henssen</td>
<td>Interatom, Cologne</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>M. Herry</td>
<td>Electricité de France, Clamart (Seine)</td>
<td>France</td>
</tr>
<tr>
<td>A. Hitchcock</td>
<td>UKAEA Reactor Group, Windscale Works, Seascale, Cumberland</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>C.F. Højrup</td>
<td>Danish Atomic Energy Commission, Risø, Roskilde</td>
<td>Denmark</td>
</tr>
<tr>
<td>M. Israel</td>
<td>Electricité de France, Clamart (Seine)</td>
<td>France</td>
</tr>
<tr>
<td>R. Janin</td>
<td>Electricité de France, Clamart (Seine)</td>
<td>France</td>
</tr>
<tr>
<td>K.J. de Jong</td>
<td>N.V. Neuratoom, The Hague</td>
<td>Netherlands</td>
</tr>
<tr>
<td>H.G. Kaper</td>
<td>Mathematisch Instituut der Rijksuniversiteit, Groningen</td>
<td>Netherlands</td>
</tr>
<tr>
<td>P.R. Kasten</td>
<td>Jülich Nuclear Research Establishment, Jülich</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>J.A.H. Kersten</td>
<td>N.V. KEMA, Arnhem</td>
<td>Netherlands</td>
</tr>
<tr>
<td>S.A. Khan</td>
<td>Pakistan Atomic Energy Commission, Karachi</td>
<td>Pakistan</td>
</tr>
<tr>
<td>J.M. Kim</td>
<td>Department of Nuclear Engineering, Queen Mary College, University of London</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>G.H. Kinchin</td>
<td>Atomic Energy Establishment, Winfrith, Dorchester, Dorset</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>H.R. Kleijn</td>
<td>Reactor Institute Delft</td>
<td>Netherlands</td>
</tr>
<tr>
<td>H. Kohler</td>
<td>Deutsche Babcock &amp; Wilcox Dampfessel-Werke AG, Oberhausen</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>K.A. Konoplev</td>
<td>Physical-Technical Institute, USSR Academy of Sciences, Leningrad</td>
<td>Union of Soviet Socialist Republics</td>
</tr>
<tr>
<td>H. Koskinen</td>
<td>Institute of Technical Physics, Institute of Technology, Otaniemi</td>
<td>Finland</td>
</tr>
<tr>
<td>M. Küchle</td>
<td>Karlsruhe Nuclear Research Centre, Karlsruhe</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>R. Küster</td>
<td>Brown Boveri/Krupp Reaktorbau GmbH, Manheim</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>Name</td>
<td>Institution</td>
<td>Nominating State or Organization</td>
</tr>
<tr>
<td>--------------------</td>
<td>------------------------------------------------------------------------------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td>V.A. Kuznetsov</td>
<td>Physical Institute of Obninsk</td>
<td>Union of Soviet Socialist Republics</td>
</tr>
<tr>
<td>G. Lautenbach</td>
<td>Reactor Centrum Nederland, Petten</td>
<td>Netherlands</td>
</tr>
<tr>
<td>G.G. Lesnoni</td>
<td>SIMEA, ENEL, Latina</td>
<td>Italy</td>
</tr>
<tr>
<td>D. Lockey</td>
<td>The Nuclear Power Group, Radbroke Hall, Knutsford, Cheshire</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>J.E. Lunde</td>
<td>OECD Halden Reactor Project, Halden, Norway</td>
<td>European Nuclear Energy Agency</td>
</tr>
<tr>
<td>G.L. Lunin</td>
<td>Institute of Atomic Energy, USSR Academy of Sciences, Moscow</td>
<td>Union of Soviet Socialist Republics</td>
</tr>
<tr>
<td>H.R. Lutz</td>
<td>Federal Institute for Reactor Research, Würenlingen</td>
<td>Switzerland</td>
</tr>
<tr>
<td>I. McGill</td>
<td>Atomic Energy Research Establishment, Harwell, Berks.</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>M.H. McTaggart</td>
<td>Atomic Weapon Research Establishment, Aldermaston, Berks.</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>L. McVeán</td>
<td>Argonne National Laboratory, Idaho Falls, Idaho</td>
<td>United States of America</td>
</tr>
<tr>
<td>C. Mandrin</td>
<td>Ecole Polytechnique de l'Université de Lausanne</td>
<td>Switzerland</td>
</tr>
<tr>
<td>W. Mansfield</td>
<td>Queen Mary College, University of London</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>H. Marković</td>
<td>Boris Kidrić Institute for Nuclear Sciences, Belgrade</td>
<td>Yugoslavia</td>
</tr>
<tr>
<td>A.P. Marks</td>
<td>Australian Atomic Energy Commission, Research Establishment, Sutherland,</td>
<td>Australia</td>
</tr>
<tr>
<td></td>
<td>New South Wales</td>
<td></td>
</tr>
<tr>
<td>R. Martinelli</td>
<td>Comitato Nazionale per l'Energia Nucleare, Rome</td>
<td>Italy</td>
</tr>
<tr>
<td>J. Massieux</td>
<td>Commissariat à l'énergie atomique, Paris</td>
<td>France</td>
</tr>
<tr>
<td>R. Meier</td>
<td>Federal Institute for Reactor Research, Würenlingen</td>
<td>Switzerland</td>
</tr>
<tr>
<td>E.A.C. MeiJlink</td>
<td>Scientific Council for Nuclear Affairs, The Hague</td>
<td>Netherlands</td>
</tr>
<tr>
<td>K. Miyazaki</td>
<td>Nuclear Energy Research Laboratory, Nippon Electric Co. Ltd., Kawasaki</td>
<td>Japan</td>
</tr>
<tr>
<td>Name</td>
<td>Institution</td>
<td>Nominating State or Organization</td>
</tr>
<tr>
<td>----------------------</td>
<td>-----------------------------------------------------------------------------------------------</td>
<td>-----------------------------------</td>
</tr>
<tr>
<td>Miss A.M. Moncassoli</td>
<td>FIAT, Sezione Energia Nucleare, Turin</td>
<td>Italy</td>
</tr>
<tr>
<td>J. Moore</td>
<td>UKAEA Reactor Group, Windscale Works, Seascale, Cumberland</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>H. Morewitz</td>
<td>Atomics International Division, North American Aviation, Inc., Canoga Park, Calif.</td>
<td>United States of America</td>
</tr>
<tr>
<td>I.A. Mossop</td>
<td>UKAEA Reactor Group, Windscale Works, Seascale, Cumberland</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>M. Muysken</td>
<td>Reactor Centrum Nederland, The Hague</td>
<td>Netherlands</td>
</tr>
<tr>
<td>T. Nagasuna</td>
<td>Mitsubishi Atomic Power Co., Inc., Kitabukuro-machi, Ohmiya-shi, Saitama</td>
<td>Japan</td>
</tr>
<tr>
<td>A. Nedelik</td>
<td>Österreichische Studiengesellschaft für Atomenergie, Vienna</td>
<td>Austria</td>
</tr>
<tr>
<td>H. Neltrup</td>
<td>Danish Atomic Energy Research Establishment, Riso, Roskilde</td>
<td>Denmark</td>
</tr>
<tr>
<td>W. W. Nijs</td>
<td>Reactor Centrum Nederland, The Hague</td>
<td>Netherlands</td>
</tr>
<tr>
<td>H. S. Olsen</td>
<td>OECD Halden Reactor Project, Halden, Norway</td>
<td>European Nuclear Energy Agency</td>
</tr>
<tr>
<td>P. F. Palmedo</td>
<td>Centre d'études nucléaires de Saclay, Gif-sur-Yvette (S et O)</td>
<td>France</td>
</tr>
<tr>
<td>N. Papadopoulos</td>
<td>Greek Atomic Energy Commission, Athens</td>
<td>Greece</td>
</tr>
<tr>
<td>J. Pelser</td>
<td>Reactor Centrum Nederland, Petten</td>
<td>Netherlands</td>
</tr>
<tr>
<td>D. Perricos</td>
<td>Institutt for Atomenergi, Kjeller</td>
<td>Norway</td>
</tr>
<tr>
<td>R. Persson</td>
<td>AB Atomenergi, Studsvik, Nyköping</td>
<td>Sweden</td>
</tr>
<tr>
<td>G. T. Petersen</td>
<td>General Electric Company, Vallecitos Atomic Laboratory, Pleasanton, Calif.</td>
<td>United States of America</td>
</tr>
<tr>
<td>C. Pleinevaux</td>
<td>EURATOM, Brussels, Belgium</td>
<td>European Atomic Energy Community</td>
</tr>
<tr>
<td>M. J. Poole</td>
<td>Atomic Energy Research Establishment, Harwell, Berks.</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>Name</td>
<td>Institution</td>
<td>Nominating State or Organization</td>
</tr>
<tr>
<td>-----------------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>D. Popović</td>
<td>Division of Safeguards, International Atomic Energy Agency, Vienna, Austria</td>
<td>International Atomic Energy Agency</td>
</tr>
<tr>
<td>G. Pregl</td>
<td>Josef Stefan Nuclear Institute, Ljubljana</td>
<td>Yugoslavia</td>
</tr>
<tr>
<td>G.A. Price</td>
<td>Brookhaven National Laboratory, Upton, N.Y.</td>
<td>United States of America</td>
</tr>
<tr>
<td>A.E. Profio</td>
<td>Massachusetts Institute of Technology, Cambridge, Mass.</td>
<td>United States of America</td>
</tr>
<tr>
<td>N. Ražič</td>
<td>Boris Kidrić Institute for Nuclear Sciences, Belgrade</td>
<td>Yugoslavia</td>
</tr>
<tr>
<td>A. Ramalho</td>
<td>Laboratorio de Física e Engenhria Nucleares, Sacavem</td>
<td>Portugal</td>
</tr>
<tr>
<td>W.C. Redman</td>
<td>Argonne National Laboratory, Argonne, Ill.</td>
<td>United States of America</td>
</tr>
<tr>
<td>J. Rembser</td>
<td>Arbeits- und Sozialministerium Nordrhein-Westfalen, Düsseldorf</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>G. Riesch</td>
<td>EURATOM, Brussels, Belgium</td>
<td>European Atomic Energy Community</td>
</tr>
<tr>
<td>H.A. Ritter</td>
<td>Arbeits- und Sozialministerium Nordrhein-Westfalen, Düsseldorf</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>P. Robert</td>
<td>Ecole Polytechnique de l'Université de Lausanne</td>
<td>Switzerland</td>
</tr>
<tr>
<td>E. Rodríguez-Mayquez</td>
<td>Junta Energía Nuclear, Ciudad Universitaria, Madrid</td>
<td>Spain</td>
</tr>
<tr>
<td>L. Ruby</td>
<td>Lawrence Radiation Laboratory, Dept. of Nuclear Engineering, University of California, Berkeley, Calif.</td>
<td>United States of America</td>
</tr>
<tr>
<td>E. Rutgers</td>
<td>Ecole Polytechnique de l'Université de Lausanne</td>
<td>Switzerland</td>
</tr>
<tr>
<td>T. Rzeszot</td>
<td>Institute of Nuclear Research, Warsaw</td>
<td>Poland</td>
</tr>
<tr>
<td>M. Sagot</td>
<td>Centre d'études nucléaires de Saclay, Gif-sur-Yvette (S et O)</td>
<td>France</td>
</tr>
<tr>
<td>Y. Sakurai</td>
<td>College of Engineering, Osaka University, Miyakojima, Osaka</td>
<td>Japan</td>
</tr>
<tr>
<td>V. Sangiust</td>
<td>Politecnico di Milano, Milan</td>
<td>Italy</td>
</tr>
<tr>
<td>Name</td>
<td>Institution</td>
<td>Nominating State or Organization</td>
</tr>
<tr>
<td>-----------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>A. Sauve</td>
<td>Commissariat à l'énergie atomique, Paris</td>
<td>France</td>
</tr>
<tr>
<td>J. Schatvet</td>
<td>Institutt for Atomenergi, Kjeller</td>
<td>Norway</td>
</tr>
<tr>
<td>E. Schmidt</td>
<td>Deutsche Babcock &amp; Wilcox Dampfeksel-Werke AG, Oberhausen</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>W. Schwarzer</td>
<td>Technischer Überwachungsverin Rheinland, Aachen</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>B. Semenov</td>
<td>Division of Reactors, International Atomic Energy Agency, Vienna, Austria</td>
<td>International Atomic Energy Agency</td>
</tr>
<tr>
<td>K. Slipičević</td>
<td>Elektrotehnički Facultet, Belgrade</td>
<td>Yugoslavia</td>
</tr>
<tr>
<td>H.B. Smets</td>
<td>OECD European Nuclear Energy Agency, Paris, France</td>
<td>European Nuclear Energy Agency</td>
</tr>
<tr>
<td>J. Smit</td>
<td>Institutt for Atomenergi, Kjeller, Norway</td>
<td>Netherlands</td>
</tr>
<tr>
<td>R. Solanilla</td>
<td>Comisión Nacional de Energía Atomica, Buenos Aires</td>
<td>Argentina</td>
</tr>
<tr>
<td>R. Stamm'ler</td>
<td>Reactor Centrum Nederland, The Hague</td>
<td>Netherlands</td>
</tr>
<tr>
<td>M.E. Stiévenart</td>
<td>Société Belge pour l'industrie nucléaire, Brussels</td>
<td>Belgium</td>
</tr>
<tr>
<td>T. Stoltz</td>
<td>Naval Reactor Branch, Ministry of Defence, The Hague</td>
<td>Netherlands</td>
</tr>
<tr>
<td>P. Storrer</td>
<td>EURATOM, Centre d'études nucléaires de Cadarache, France</td>
<td>European Atomic Energy Community</td>
</tr>
<tr>
<td>S. Takač</td>
<td>Boris Kidrić Institute for Nuclear Sciences, Belgrade</td>
<td>Yugoslavia</td>
</tr>
<tr>
<td>A. Tas</td>
<td>Reactor Centrum Nederland, Petten</td>
<td>Netherlands</td>
</tr>
<tr>
<td>G.C. Tavemier</td>
<td>Société Belge pour l'industrie nucléaire, Brussels</td>
<td>Belgium</td>
</tr>
<tr>
<td>J. Thomasssen</td>
<td>Institutt for Atomenergi, Kjeller</td>
<td>Norway</td>
</tr>
<tr>
<td>T.J. Thompson</td>
<td>Massachusetts Institute of Technology, Cambridge, Mass.</td>
<td>United States of America</td>
</tr>
<tr>
<td>J. Thurnheer</td>
<td>Ecole Polytechnique de l'Université de Lausanne</td>
<td>Switzerland</td>
</tr>
<tr>
<td>R. Uhrig</td>
<td>Dept. of Nuclear Engineering, University of Florida, Gainesville, Fla.</td>
<td>United States of America</td>
</tr>
<tr>
<td>Name</td>
<td>Institution</td>
<td>Nominating State or Organization</td>
</tr>
<tr>
<td>--------------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>J.C. Vidal</td>
<td>Commissariat à l'énergie atomique, Paris</td>
<td>France</td>
</tr>
<tr>
<td>R. Vidal</td>
<td>Centre d'études nucléaires de Fontenay-aux-Roses</td>
<td>France</td>
</tr>
<tr>
<td>B. Vittoz</td>
<td>Ecole Polytechnique de l'Université de Lausanne</td>
<td>Switzerland</td>
</tr>
<tr>
<td>M. Vořísek</td>
<td>Nuclear Research Institute of the Czechoslovak Academy of Sciences, Prague</td>
<td>Czechoslovak Socialist Republic</td>
</tr>
<tr>
<td>F. de Waegh</td>
<td>Société Belge pour l'industrie nucléaire, Brussels</td>
<td>Belgium</td>
</tr>
<tr>
<td>J. Walker</td>
<td>Physics Department, The University of Edgbaston, Birmingham</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>S.K. Wallace</td>
<td>Rolls-Royce &amp; Associates Ltd., Derby</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>D. Walton</td>
<td>Nuclear Engineering Department, Victoria University of Manchester</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>A. Ward</td>
<td>Atomic Energy of Canada Ltd., Chalk River, Ontario</td>
<td>Canada</td>
</tr>
<tr>
<td>R. van der Wart</td>
<td>Wetenschappelijke Raad voor de Kernenergie, The Hague</td>
<td>Netherlands</td>
</tr>
<tr>
<td>J.W. Weale</td>
<td>Atomic Weapon Research Establishment, Aldermaston, Berks.</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>J.J. Went</td>
<td>N.V. KEMA, Amhem</td>
<td>Netherlands</td>
</tr>
<tr>
<td>R. Widera</td>
<td>Deutsche Babcock &amp; Wilcox Dampfkesselwerke AG, Oberhausen</td>
<td>Federal Republic of Germany</td>
</tr>
<tr>
<td>C. Wik Dahl</td>
<td>AB Atomenergi, Studsvik, Nyköping</td>
<td>Sweden</td>
</tr>
<tr>
<td>J. Wolberg</td>
<td>Technion, Israel Institute of Technology, Department of Nuclear Science, Haifa</td>
<td>Israel</td>
</tr>
<tr>
<td>M. Yamada</td>
<td>Fuji Deuki Sei O., Kawasaki</td>
<td>Japan</td>
</tr>
<tr>
<td>P. Zaleski</td>
<td>Centre d'études nucléaires de Cadarache,</td>
<td>France</td>
</tr>
<tr>
<td>W.L. Zijp</td>
<td>Reactor Centrum Nederland, Petten</td>
<td>Netherlands</td>
</tr>
<tr>
<td>G.B. Zorzoli</td>
<td>Centro Informazioni Studi Esperienze, Milan</td>
<td>Italy</td>
</tr>
<tr>
<td>K. Zuehlke</td>
<td>Kernreaktor Bau- und Betriebges. m.b.H., Karlsruhe</td>
<td>Federal Republic of Germany</td>
</tr>
</tbody>
</table>
AUTHOR INDEX

Roman numerals are volume numbers.
Arabic numerals underlined refer to the first page of a paper by the author concerned.
Other Arabic numerals denote the page numbers of other references, principally discussion comments.
Literature references have not been indexed.

Accinni, F.: I 401; II 20, 159, 587
Acker, J.G.: II 233
Aline, P.G.: II 209
Andersen, E.: II 21
Andrianov, G.Ya.: III 355
Arkuszewski, J.: II 445; III 333
Armari, R.J.: I 227
Auerbach, T.: II 85
Babala, D.: III 191, 202, 203
Bach, D.R.: II 391
Baker, A.R.: I 155, 298
Baird, Q.L.: I 3
Bardes, R.G.: III 35
Barrett, L.G.: II 257
Bednarz, R.: III 333
Bee, T.: I 277
Behrens, E.: II 57; III 239
Bennett, E.F.: I 227
Benoist, P.: II 181; III 318-9, 399
Berthet, P.: I 355
Bigham, C.B.: II 457
Bistline, J.A.: II 391
Bittelli, G.: III 401
Blaessser, G.: II 507; III 158-9, 443, 452, 453
Boskewski, T.: II 509
Bouzyk, J.: III 333
Brenner, M.W.: I 227
Breton, D.: I 47, 64, 225; II 338; III 112, 462-4
Bremscher, M.M.: I 227
Brown, J.R.: III 35
Bruna, J.G.: II 313
Bryn-Ingebritsen, K.: II 291
Bunch, S.L.: II 391
Bustraan, M.: I 321; II 233
Cadilhac, M.: II 313
Campan, J.L.: I 277
Campbell, C.G.: I 29; II 41, 158, 454
Campise, A.V.: I 46, 123, 124, 276, 317, 354; II 207, 255, 289, 359; III 25, 159
Casini, G.: II 108, 477; III 423, 441
Caumette, P.: I 277
Cerbone, R.J.: II 391
Clark, R.H.: II 257
Coehoorn, J.: II 233, 255, 256, 572
Cogne, F.: I 47
Colle, P.C.G.L.: I 259, 277
Čopič, M.: III 305
Critchley, O.H.: III 69, 287
Dąbrowksi, C.: III 333
Davey, W.G.: I 85
Deckers, H.: I 277
Desprets, A.: II 277
Diaz, N.J.: III 259
Dingee, D.A.: II 207, 256
Domsławski, A.: III 333
Drake, M.K.: III 35
Drechsel, W.: III 189
Dubrovska, B.G.: I 299
Ebersoldt, F.: I 197; III 25, 125
Engelder, T.C.: II 257, 289, 290, 444; III 61-2
Erikson, V.O.: II 291
Fischer, G.J.: I 85
Fischer, P.U.: III 35, 60
Fry, D.N.: III 259
Gavin, P.: I 355
Gelbard, E.: II 339

478
Gereseva, L.A.: I 299
Gibson, I.H.: I 469
Girard, Y.E.: I 47; II 108, 455
Glazkov, Yu.Yu.: I 299
Golubev, V.I.: I 261
Goodfellow, H.: I 159
Govaerts, P.: I 197
Grant, P.J.: III 127, 137
Graves, W.E.: II 479
Green, R.E.: II 457, 477
Grifoni, S.: III 401
Grümm, H.: III 27, 33
Hamada, H.: III 63
Hardy, J.: II 339, 359, 445
Harris, M.J.: I 371, 401, 402, 443; II 18-19
Hartley, R.H.: III 259
Hayata, K.: II 43
Heer, W.: II 85
Heinemann, R.E.: I 65, 83-4; II 506-7
Hellens, R.L.: II 21, 41; III 458-9
Hellstrand, E.: I 194; II 455
Hennelly, E.J.: II 479
Henssen, H.: I 479-80
Heyboer, R.J.: II 233
Hishida, H.: III 63
Hitchcock, A.: II 84; II 56, 454, 507; III 459-61
Højjerup, C.F.: II 559, 606
Huber, R.J.: I 227
Jacquemart, R.: III 163
James, C.G.: III 127
Jósefowicz, E.T.: III 333
Kaczmarek, W.: III 333
Kalin, T.: III 305
Kamaev, A.V.: I 299
Kaminker, D.M.: II 197
Kamyshan, A.N.: III 355
Kaplan, I.: II 109, 147
Kaufmann, S.G.: I 227
Kersten, J.A.H.: I 337, 354
Khalizev, V.I.: III 355
Kim, J.M.: I 457
Kinchin, G.H.: I 29, 46; II 337-8; III 69
Kisil, I.M.: I 299
Kleijn, H.R.: III 113, 124-5
Klein, D.: II 339
Komissarov, V.A.: III 355
Kondo, T.: II 43
Konoplev, K.A.: II 197, 207
Koskinen, H.: I 445, 456
Kremser, J.: III 163
Kubowski, J.: III 333
Küchle, M.: II 57, 575, 585, 586; III 189
Kuzmicheva, V.A.: III 355
Kuznetsov, V.A.: I 125, 276, 317; III 461-2
Lanning, D.D.: II 109
Larvin, S.O.: II 161
Lautenbach, G.: II 337, 559
Ludinga, F.: II 233
Lunde, J.E.: II 527, 559
Lunin, G.L.: III 203, 355
Lutz, H.R.: II 84, 401-2; II 59, 85, 107-8; III 318, 399, 400, 452
McGill, I.: I 258; II 57, 256; III 203, 239
McTaggart, M.H.: I 159
McVeany, R.L.: I 85, 123, 124
McWhorter, R.J.: II 209
Malewski, S.: III 333
Mandrin, C.: I 355, 403, 443; III 452
Mansfield, W.K.: I 457, 467
Marković, H.: II 509
Marks, A.P.: III 25
Martinelli, R.: III 401
Mégier, J.: III 423
Meier, R.W.: I 479; II 59, 82-3, 85; III 302, 303, 400
Miles, F.: III 457, 464, 465
Moore, J.: III 3, 25, 60
Moret-Bailly, J.: II 313
Morewitz, H.: II 444
Mossop, I.A.: II 19, 20, 255-6, 476-7
Mukae, M.: II 43
Naganuma, T.: II 43, 56, 57
Nagashima, K.: III 63
Naudet, R.: I 47
Neltrup, H.: II 83; III 383, 399, 400
Nijs, W.W.: II 233
Nikolaev, M.N.: I 261
Nuytsen, M.: II 233
Øjford, K.: II 291
Ogrzewalski, Z.: III 333
Ogura, S.: II 43
<table>
<thead>
<tr>
<th>Author</th>
<th>Index Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Olgaard, P.L.</td>
<td>III 383</td>
</tr>
<tr>
<td>Olsen, H.S.</td>
<td>II 561, 573</td>
</tr>
<tr>
<td>Orlov, M.Yu.</td>
<td>I 261</td>
</tr>
<tr>
<td>Palmedo, P.F.</td>
<td>II 19, 181, 194, 454-5</td>
</tr>
<tr>
<td>Parkinson, T.F.</td>
<td>III 259</td>
</tr>
<tr>
<td>Paterson, W.J.</td>
<td>I 159</td>
</tr>
<tr>
<td>Perez, R.B.</td>
<td>III 241, 259</td>
</tr>
<tr>
<td>Perricos, D.C.</td>
<td>II 161, 179</td>
</tr>
<tr>
<td>Persson, R.</td>
<td>I 456; II 41, 83, 107, 194, 289, 559; III 124, 125, 137, 202, 289, 302-4, 441, 453</td>
</tr>
<tr>
<td>Petersen, G.T.</td>
<td>I 26; II 20, 56, 209, 230, 231</td>
</tr>
<tr>
<td>Petrov, Yu.V.</td>
<td>II 197</td>
</tr>
<tr>
<td>Pikulik, R.G.</td>
<td>II 197</td>
</tr>
<tr>
<td>Plumlee, K.E.</td>
<td>I 3</td>
</tr>
<tr>
<td>Poole, M.J.</td>
<td>II 338,444-5; III 87, 111, 112</td>
</tr>
<tr>
<td>Pound, D.C.</td>
<td>III 35</td>
</tr>
<tr>
<td>Pregl, G.</td>
<td>III 305, 318</td>
</tr>
<tr>
<td>Price, G.A.</td>
<td>II 3, 18, 19, 20, 158, 231</td>
</tr>
<tr>
<td>Profio, A.E.</td>
<td>I 258, 370, 467; II 41, 109, 526, 586</td>
</tr>
<tr>
<td>Purica, I.I.</td>
<td>III 71</td>
</tr>
<tr>
<td>Račățianu, E.</td>
<td>III 71</td>
</tr>
<tr>
<td>Raišić, N.</td>
<td>II 509, 526; III 202</td>
</tr>
<tr>
<td>Redman, W.C.</td>
<td>I 3, 26-7, 124, 227, 258, 276; II 158, 178-9, 337, 444, 445; III 32-3, 60, 111, 112, 189</td>
</tr>
<tr>
<td>Rembser, R.</td>
<td>I 336</td>
</tr>
<tr>
<td>Robert, P.</td>
<td>I 355</td>
</tr>
<tr>
<td>Robin, M.</td>
<td>III 141</td>
</tr>
<tr>
<td>Rodríguez-Mayquez, E.</td>
<td>III 283, 287</td>
</tr>
<tr>
<td>Ruby, L.</td>
<td>II 55-6</td>
</tr>
<tr>
<td>Rzeszot, T.</td>
<td>III 321, 333</td>
</tr>
<tr>
<td>Sagot, M.</td>
<td>I II 256; III 163, 189, 304</td>
</tr>
<tr>
<td>Sakurai, Y.</td>
<td>III 63, 69</td>
</tr>
<tr>
<td>Sampson, J.B.</td>
<td>III 35</td>
</tr>
<tr>
<td>Santandrea, E.</td>
<td>III 401</td>
</tr>
<tr>
<td>Saussure, de, G.</td>
<td>III 241</td>
</tr>
<tr>
<td>Schatvet, J.</td>
<td>II 291</td>
</tr>
<tr>
<td>Schmidt, E.</td>
<td>II 339</td>
</tr>
<tr>
<td>Schmitt, A.P.</td>
<td>I 135</td>
</tr>
<tr>
<td>Schofield, P.</td>
<td>III 87</td>
</tr>
<tr>
<td>Seferian, N.</td>
<td>III 71</td>
</tr>
<tr>
<td>Sekiya, T.</td>
<td>III 63</td>
</tr>
<tr>
<td>Semenov, V.N.</td>
<td>III 355</td>
</tr>
<tr>
<td>Silver, E.G.</td>
<td>III 241</td>
</tr>
<tr>
<td>Sinclair, R.N.</td>
<td>III 87</td>
</tr>
<tr>
<td>Skolnik, W.</td>
<td>II 391</td>
</tr>
<tr>
<td>Slovacek, R.E.</td>
<td>II 391</td>
</tr>
<tr>
<td>Smets, H.B.</td>
<td>II 57</td>
</tr>
<tr>
<td>Smith, R.D.</td>
<td>I 29</td>
</tr>
<tr>
<td>Smith, S. P.D.</td>
<td>III 259</td>
</tr>
<tr>
<td>Snidow, N.L.</td>
<td>II 257</td>
</tr>
<tr>
<td>Solanilla, R.</td>
<td>II 361</td>
</tr>
<tr>
<td>Stewart, H.B.</td>
<td>III 35</td>
</tr>
<tr>
<td>Stiévenart, M.</td>
<td>I 197</td>
</tr>
<tr>
<td>Storrer, F.</td>
<td>I 135, 155, 197, 225; III 160</td>
</tr>
<tr>
<td>Sugeno, M.</td>
<td>II 43</td>
</tr>
<tr>
<td>Suita, T.</td>
<td>III 63</td>
</tr>
<tr>
<td>Suwalski, W.</td>
<td>III 333</td>
</tr>
<tr>
<td>Szechter, A.</td>
<td>III 333</td>
</tr>
<tr>
<td>Takač, S.</td>
<td>II 509</td>
</tr>
<tr>
<td>Tas, A.</td>
<td>I 321, 336; II 233, 359</td>
</tr>
<tr>
<td>Tavernier, G.</td>
<td>I 135</td>
</tr>
<tr>
<td>Tellier, H.</td>
<td>III 163</td>
</tr>
<tr>
<td>Thomassen, J. A.</td>
<td>II 161</td>
</tr>
<tr>
<td>Thompson, T. J.</td>
<td>I 194, 479; II 107, 109, 147</td>
</tr>
<tr>
<td>Thurnheer, J.</td>
<td>I 355</td>
</tr>
<tr>
<td>Trédiakoff, O.</td>
<td>III 141</td>
</tr>
<tr>
<td>Uhrig, R. E.</td>
<td>II 57; III 69, 205, 239-40</td>
</tr>
<tr>
<td>Ukkestad, A.</td>
<td>II 291</td>
</tr>
<tr>
<td>Utzinger, E.</td>
<td>II 59</td>
</tr>
<tr>
<td>Uznadze, O.P.</td>
<td>I 261</td>
</tr>
<tr>
<td>Van Dam, H.</td>
<td>III 113</td>
</tr>
<tr>
<td>Van der Heijden, A.W.</td>
<td>III 113</td>
</tr>
<tr>
<td>Van Dievoet, J.</td>
<td>I 135</td>
</tr>
<tr>
<td>Vendryes, G.</td>
<td>I 135</td>
</tr>
<tr>
<td>Vidal, R.</td>
<td>III 141, 159, 160, 161</td>
</tr>
<tr>
<td>Villiers, de, J.</td>
<td>III 400</td>
</tr>
<tr>
<td>Vittoz, B.</td>
<td>I 355, 370; III 111, 452</td>
</tr>
<tr>
<td>Volpe, J. J.</td>
<td>II 339</td>
</tr>
<tr>
<td>Voznesensky, V. A.</td>
<td>III 355</td>
</tr>
<tr>
<td>Waegh, de, F.</td>
<td>II 289</td>
</tr>
<tr>
<td>Walker, J.</td>
<td>I 402, 469, 479, 480; II 231; III 111</td>
</tr>
<tr>
<td>Wallace, S.K.</td>
<td>II 586</td>
</tr>
<tr>
<td>Walton, D.G.</td>
<td>I 371; II 19, 159; III 240</td>
</tr>
<tr>
<td>Ward, A.G.</td>
<td>II 449, 454, 455, 507, 585; III 457-8, 465</td>
</tr>
</tbody>
</table>
AUTHOR INDEX

Warda, E.: III 321
Weinstein, S.: II 391
Went, J.J.: I 337, 354
Windsor, H.H.: II 161
Wolberg, J.R.: I 84, 258, 401;
II 18, 147, 158, 159, 311, 476
Wood, J.I.: II 291
Zaleski, C.P.: I 277, 298
Žerdin, F.: III 305
Zijp, W.L.: II 233
Zorzoli, G.B.: II 108, 587, 606;
III 303, 318
Zvonarev, A.V.: I 261

TRANSLITERATION INDEX

Andrianov, G. Ya.
Voznesensky, V. A.
Gerasova, L. A.
Glazkov, Yu. Yu.
Golubev, V. I.
Dubrovsky, B. G.
Zvonarev, A. V.
Kamaev, A. V.
Kaminsker, D. M.
Kamyshon, A. N.
Kisil, I. M.
Komissarov, L. V.
Konoplev, K. A.
Kuzmicheva, V. A.
Kuznetsov, V. A.
Lunin, G. L.
Nikolaev, M. N.
Orlov, M. Yu.
Petrov, Yu. V.
Pikulik, R. G.
Semenov, V. N.
Uznadze, O. P.
Khalizev, V. I.
Orders for Agency publications can be placed with your bookseller or any of our sales agents listed below:

**AUSTRALIA**  
T. & H. Hunter Publications,  
23 McKillop Street  
Melbourne, C.1

**AUSTRIA**  
Georg Fromme & Co.  
Spengergasse 39  
Vienna V

**BELGIUM**  
Office International de Librairie  
30, Avenue Marnix  
Brussels 5

**BRAZIL**  
Livraria Kosmos Editora  
Rua do Rosario, 135-137  
Rio de Janeiro  
Agencia Expoente Oscar M. Silva  
Rua Xavier de Toledo, 140-1º Andar  
(Calixa Postal No. 5, 614)  
Sao Paulo

**ITALY**  
Agenzia Editoriale Internazionale  
Organizzazioni Universali  
(A.E.I.O.U.)  
Via Meravigli 16  
Milan

**JAPAN**  
Maruzen Company Ltd.  
6, Tori Nichome  
Nihombashi  
P.O. Box 605  
Tokyo Central

**NETHERLANDS**  
N.V. Martinus Nijhoff  
Lange Voorhout 9  
The Hague

**NEW ZEALAND**  
Whitcombe & Tombs, Ltd.  
G.P.O. Box 1894  
Wellington, C.1

**PAKISTAN**  
Karachi Education Society  
Haroon Chambers  
South Napier Road  
P.O. Box No. 4886  
Karachi, 2

**POLAND**  
Osfodek Rozpowszechniana  
Wydawnistw Naukowych  
Polska Akademia Nauk  
Palac Kultury i Nauki  
Warsaw

**SPAIN**  
Libreria Bosch  
Ronda Universidad, 11  
Barcelona

**USSSR**  
See under USSR
IAEA publications can also be purchased retail at the United Nations Bookshop at United Nations Headquarters, New York, at the news-stand at the Agency's Headquarters, Vienna, and at most conferences, symposia and seminars organized by the Agency.

In order to facilitate the distribution of its publications, the Agency is prepared to accept payment in UNESCO coupons or in local currencies.

Orders and inquiries from countries where sales agents have not yet been appointed may be sent to:

Publications Sales Unit, International Atomic Energy Agency, Kärntner Ring 11, Vienna I, Austria