POLYNOMIAL CURVE FITTING FOR CONTROL ROD WORTH USING LEAST SQUARE NUMERICAL ANALYSIS

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THE CONTENT

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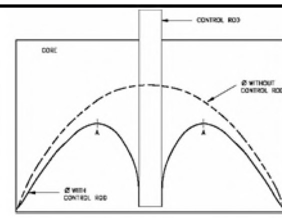
- Problem statement
- Curve fitting
- SCILAB 5.3.3 in action
- Results & Discussion

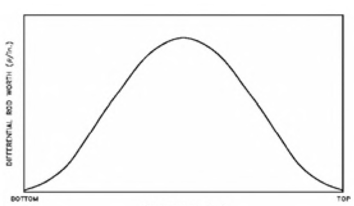
Flux Distribution & Control Rod Effectiveness

Neutron flux distribution depends on;

i. Reflector effect
ii. Fuel burn-up
iii. Temperature effect
iv. Control rod worth

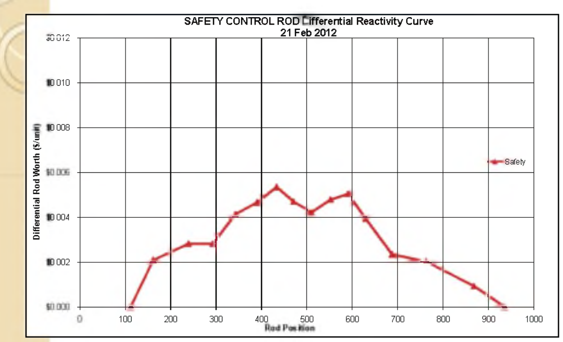


- Control rod worth is the change in reactivity caused by control rod motion.
- If additional rods are added to this simple reactor, the most effective location is where the flux is maximum, that is, at point A
- Numerous control rods are required for a reactor that has a large amount of excess reactivity (that amount of reactivity in excess of that needed to be critical)



- Differential control rod worth is the reactivity change per unit movement of a rod and is normally expressed as ρ/inch or Δk/k per inch.
- The integral rod worth at a given withdrawal is merely the summation of all the differential rod worth up to that point of withdrawal
- •It is also the area under the differential rod worth curve at any given withdrawal position

PROBLEM STATEMENT



CURVE FITTING

Describes techniques to fit curves (*curve fitting*) to discrete data to obtain intermediate estimates.

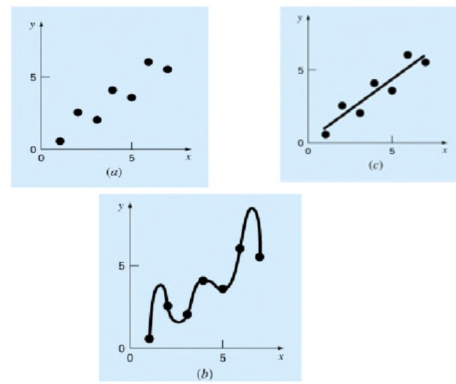
There are two general approaches for curve fitting:

- **Least Squares regression:**

  Data exhibit a significant degree of scatter. The strategy is to derive a single curve that represents the general trend of the data.

- **Interpolation:**

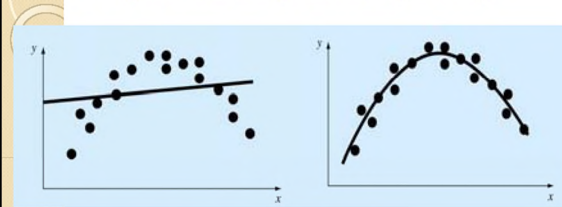
  Data is very precise. The strategy is to pass a curve or a series of curves through each of the points.



Polynomial Regression

- Some engineering data is poorly represented by a straight line.
- For these cases a curve is better suited to fit the data.
- The least squares method can readily be extended to fit the data to higher order polynomials.

Polynomial Regression (cont'd)



A parabola is preferable

Polynomial Regression (cont'd)

- **A 2nd order polynomial (quadratic)** is defined by:

$$y = a_o + a_1 x + a_2 x^2 + e$$

- The residuals between the model and the data:

$$e_i = y_i - a_o - a_1 x_i - a_2 x_i^2$$

- The sum of squares of the residual:

$$S_r = \sum e_i^2 = \sum \left(y_i - a_o - a_1 x_i - a_2 x_i^2\right)^2$$

Polynomial Regression (cont'd)

$$\frac{\partial S_r}{\partial a_o} = -2\sum (y_i - a_o - a_1 x_i - a_2 x_i^2) = 0$$

$$\frac{\partial S_r}{\partial a_1} = -2\sum (y_i - a_o - a_1 x_i - a_2 x_i^2)x_i = 0$$

$$\frac{\partial S_r}{\partial a_2} = -2\sum (y_i - a_o - a_1 x_i - a_2 x_i^2)x_i^2 = 0$$

$$\sum y_i = n \cdot a_o + a_1 \sum x_i + a_2 \sum x_i^2$$

$$\sum x_i y_i = a_o \sum x_i + a_1 \sum x_i^2 + a_2 \sum x_i^3$$

$$\sum x_i^2 y_i = a_o \sum x_i^2 + a_1 \sum x_i^3 + a_2 \sum x_i^4$$

3 linear equations with 3 unknowns (a_o, a_1, a_2), can be solved

Polynomial Regression (cont'd)

- A system of 3x3 equations needs to be solved to determine the coefficients of the polynomial.

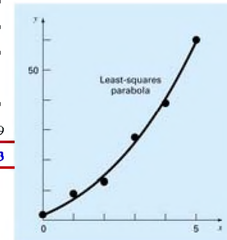
$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{Bmatrix}$$

- The standard error

$$s_{y/x} = \sqrt{\frac{S_r}{n-3}}$$

Polynomial Regression- Example (cont'd)

x_i	y_i	y_{model}	e_i^2	$(y_i-y')^2$
0	2.1	2.4786	0.14332	544.42889
1	7.7	6.6986	1.00286	314.45929
2	13.6	14.64	1.08158	140.01989
3	27.2	26.303	0.80491	3.12229
4	40.9	41.687	0.61951	239.22809
5	61.1	60.793	0.09439	1272.13489
15	152.6		3.74657	2513.39333



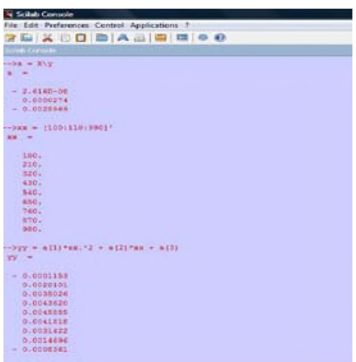
- The standard error of estimate:

$$s_{y/x} = \sqrt{\frac{3.74657}{6-3}} = 1.12$$

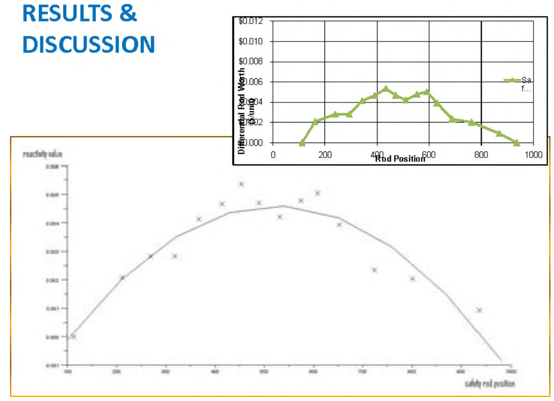
- The coefficient of determination:

$$r^2 = \frac{2513.39 - 3.74657}{2513.39} = 0.99851, \quad r = \sqrt{r^2} = 0.99925$$

SCILAB 5.3.3 in action



RESULTS & DISCUSSION



RESULTS & DISCUSSION

Residual e_i : 0.00034

Error : 0.5%

$f(x) = -0.003x^2 + 0.26x - 2.6E^{-5}$ for PUSPATI TRIGA Reactor's 2012 safety rod calibration.

THANK YOU