

# INFLUENCE OF PAULI PRINCIPLE AND POLARIZATION ON $^{16}\text{O} + ^{16}\text{O}$ INTERACTION POTENTIAL

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In the work have studied the dependence of the interaction potential on taking into account the Pauli principle as well as monopole and quadrupole polarization within approaches based on the energy-density formalism and two-center shell model wave functions for  $^{16}\text{O} + ^{16}\text{O}$  system. In the adiabatic approximation it is shown that the contribution of the Pauli principle and polarization in colliding nuclei radically changes the behavior of interaction potential.

## 1. Introduction

Atomic nuclei interaction is one of the most important problems in nuclear physics. During all time a large number of different approaches were created, including microscopic approaches. Resonating group method [1] (RGM) at present is one of the most successive approximations. In the framework of the RGM the Pauli principle is taken into account exactly and interaction potential of the nuclei is calculated not only in frozen density approximation but also with taking into account cluster polarization, in particular - monopole polarization [2 - 8] and quadrupole polarization in the algebraic version of the method [9 - 17]. Calculations were carried out at relatively low energies of the colliding nuclei and they indicated a significant effect of polarization on the nuclei interaction. But due to the extreme complexity of the above approaches such calculations have been performed only for the s-shell nuclei interaction.

If we want to consider the interaction of nuclei heavier than the s-shell nuclei we must choose more simple approach. Our aim is to calculate the real part of nucleus-nucleus interaction potential taking into account Pauli principle, monopole and quadrupole polarization of the interacting nuclei. It is obviously that the polarization of the interacting nuclei should manifest itself most clearly when they approach is sufficiently slow. Therefore, we use adiabatic approximation that is relative motion of the interacting nuclei is slow and reconstruction of the nuclei is full at any given distance between their centers of mass.

In the work to calculate  $\rho$  and  $\tau$  we use two-center shell model [18 - 23]. As in [18], we represent the total wave function of a system in the form of Slater determinant constructed on the three-dimensional harmonic oscillator functions with two oscillator wells whose centers are separated by a distance  $R$ . The coordinate axis  $z$  along which the nuclear motion occurs is directed along the straight line connecting the centers of masses of the nuclei. Our wave function contain two oscillator lengths ( $b$  – for  $x$  and  $y$  axis,  $c$  - for  $z$ ) as the parameters. Thus we have one preferred direction -  $z$  axis, and the potential interaction of the nuclei is presented as a function of  $R$ . In addition minimizing the energy for each  $R$  we define current values of the oscillator lengths  $b$  and  $c$ .

We use that approach to calculate the real part of the interaction potential of two  $^{16}\text{O}$  nuclei. In our calculations we take into account the Pauli principle as well as the polarization of the nuclei, i.e. deformation of nucleon orbitals in the interacting nuclei. In the next section we will present the formalism of the used model and in third section we discuss obtained results.

## 2. The formalism of the model

In the energy density approach potential energy of interaction between two nuclei is determined as the difference of whole system energy at a finite distance  $E(R)$  and the sum of binding energies of the separate nuclei  $E_1 + E_2 = E(\infty)$  at infinite distance:

$$V(R) = E(R) - E_1 - E_2. \quad (1)$$

Here

$$E_{1(2)} = \int h[\rho_{1(2)n}(\vec{r}), \rho_{1(2)p}(\vec{r}), \tau_{1(2)n}(\vec{r}), \tau_{1(2)p}(\vec{r})] d\vec{r} \quad (2)$$

$$E = \int h[\rho_n(\vec{r}), \rho_p(\vec{r}), \tau_n(\vec{r}), \tau_p(\vec{r})] d\vec{r}, \quad (3)$$

where  $\tau_{1(2)n}(\vec{r})$  and  $\tau_{1(2)p}(\vec{r})$  - kinetic energy densities for neutrons and protons of a single first (second) nucleus,  $\rho_{1(2)n}(\vec{r})$  and  $\rho_{1(2)p}(\vec{r})$  - neutrons and protons density distributions in the nuclei, expression for the energy density  $h$ , written for the Skyrme forces, which depends on the density we have taken from [24]. We use Skyrme forces due to the fact that their using in calculations in Hartree - Fock method leads to good results in describing binding energies of nuclei as well gives us good results in the framework of semiclassical approach with sufficiently heavy nuclei.

Next, to simplify the solution of the problem we neglect the spin-orbit and Coulomb interactions. Considering nuclei with  $N = Z$ , we assume that the neutron density is the same as the proton one and each of them is equal to half the total nucleon density. The same relates to the kinetic energy density.

To describe the state of  $^{16}\text{O} - ^{16}\text{O}$  system we use two-center shell model, so in each of two oscillator wells whose centers have coordinates  $-R/2$  and  $R/2$  on the axis  $z$  are 8 neutrons and 8 protons which completely filling the  $s$ - and  $p$ -state. Oscillatory length for  $x$  and  $y$  axes are the same but they differs from the  $z$  axis oscillator length. Our system is cylindrically symmetric and, respectively, in cylindrical coordinates  $\tau$  and  $\rho$  depends only on two variables  $r$  and  $z$ .

### 3. Results

The purpose of our calculations is to study the effect of antisymmetrization, as well as monopole and quadrupole polarization on the  $^{16}\text{O} - ^{16}\text{O}$  interaction potential with using different Skyrme forces dependent on the density. Since using different Skyrme forces parameterization does not lead to any qualitative changes we will show the results only for the SkP parametrization [23], a well established to describe the binding energies of nuclei in a wide range of mass numbers.

We investigate first the effect of Pauli principle on the nucleus-nucleus interaction potential. As a result of the Pauli principle accurate accounting potential well depth is reduced drastically, and at small distances between the nuclei we can see significant repulsion. Similarly, it seems necessary to take into account the fact that during the interaction process nuclei change their size and shape that is polarized. Most evidently we can see this at low energies of their relative motion. Note that results previously were cited in [25], similar results were obtained in [18].

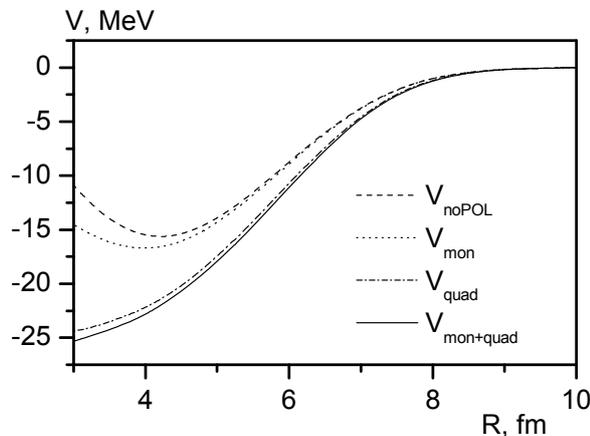


Fig. 1.  $^{16}\text{O} - ^{16}\text{O}$  interaction potential. There  $V_{noPOL}$  - potential without polarization;  $V_{mon}$  - potential with monopole polarization;  $V_{quad}$  - potential with quadrupole polarization;  $V_{mon+quad}$  potential with monopole and quadrupole polarizations. In all cases the Pauli principle is taken into account exactly.

increasing their size but remains spherically. Recall that in the spherically symmetric oscillator model mean square radii of nuclei is proportional to the oscillator radii. Fig. 2,  $b$  shows the behavior of oscillator radius associated with  $z$  axis along which the nuclei is moving at a fixed value of the oscillator radius  $b = b_\infty$  ( $x$ -axis and  $y$ ). Compared to the previous case the changes of the radius are more significant. They makes up to tens of percents of the initial value  $c_\infty = b_\infty$  that tell us that quadrupole mode of motion manifests itself as a much gentler than a monopole one.

The behavior of  $b$  and  $c$  for the case of both monopole and quadrupole polarization is shown in Fig. 2,  $c$ . While  $c$  is increasing similar to the previous case  $b$  firstly even slightly decreasing. That is, in general, where the nucleus overlaps slightly and our approximation can be considered quite reasonable, the nuclei are trying to maximize an elongated shape due to the attraction that arises between their nucleons.

The behavior of the total nucleon density for two  $^{16}\text{O}$  nuclei (its cross-section by  $zx$  ( $zy$ ) plane) is shown in Fig. 3. We begin our consideration from center mass distance 9 fm where their density is already beginning to overlap by their tails. You can see that the greatest impact on nucleon density make quadrupole polarization leading to a significant spreading of the density along  $z$  axis as the nuclei approaching.

Effect of polarization for two  $^{16}\text{O}$  nuclei interaction potential is shown in Fig. 1. The upper curve in this figure was obtained in "frozen density" approximation (here and below we refer to results obtained with exact accounting of the Pauli principle). Below we can see the curve with possibility of a monopole nuclear polarization that is nuclei can change their size without changing shape. Single monopole polarization don't give us significant impact on the capture well width and depth, it leads only for a few potential depth increasing and decreasing repulsion at small distances. Comparison of these curves indicates that quadrupole mode of motion makes the dominant contribution to these changes in the potential.

Fig. 2 gives an oscillator radii behavior depending on the distance between the nuclei. Figure 2a shows the oscillator radius  $b$  depending on the distance between the nuclei for monopole polarization only. The curve indicates that at that approach nuclei swell more,

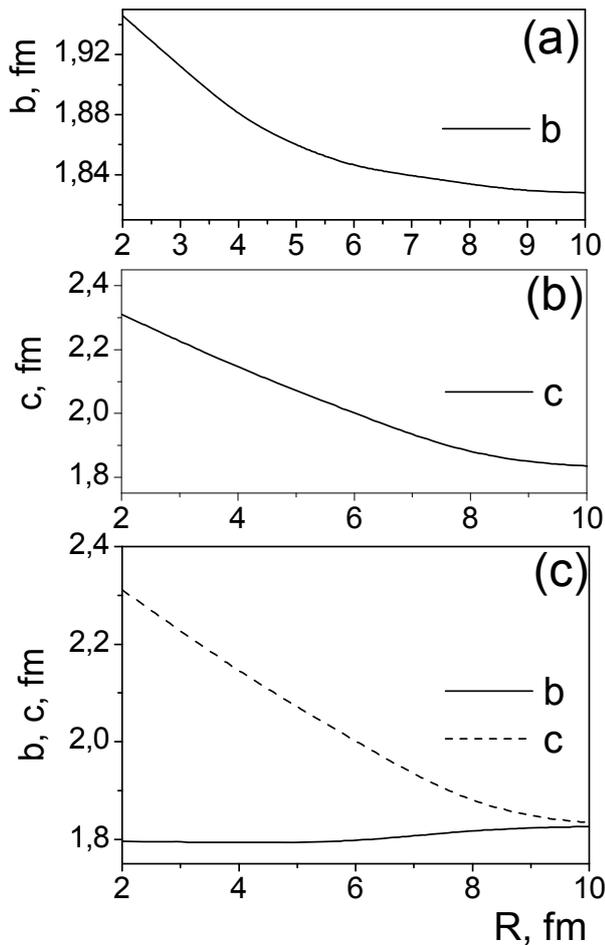


Fig. 2. Oscillator radii dependent from the distance between centers of mass of the nuclei in the case of monopole (a), quadrupole (b), the monopole and quadrupole polarizations (c).

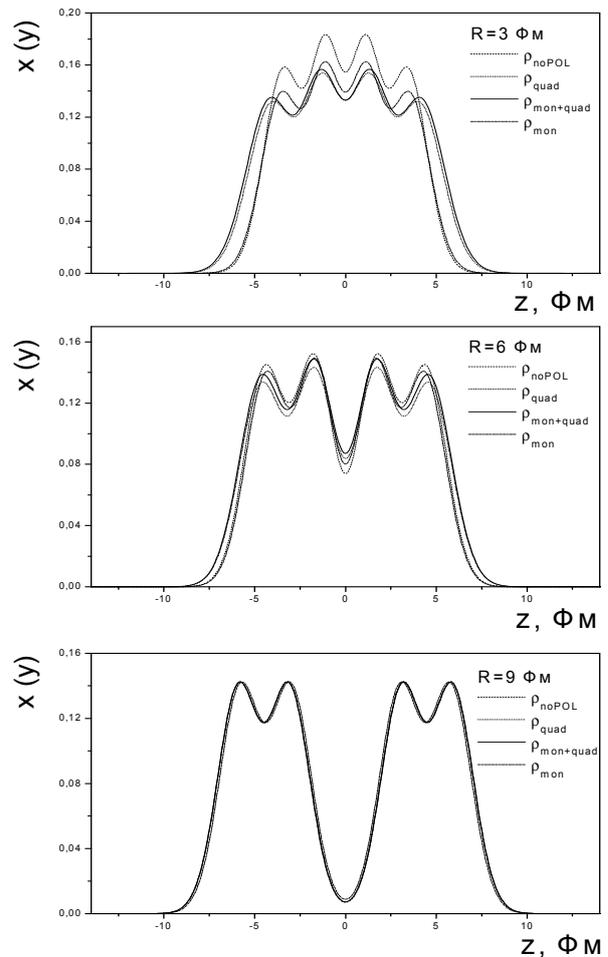


Fig. 3. Cross sections of nucleon densities by  $xz$  ( $zy$ ) plane without polarization ( $\rho_{noPOL}$ ), with monopole polarization only ( $\rho_{mon}$ ), with quadrupole polarization only ( $\rho_{quad}$ ), with both monopole and quadrupole polarizations ( $\rho_{mon+quad}$ ).

#### 4. Conclusions

Consideration of the nuclear-nuclear interaction potential for  $^{16}\text{O}+^{16}\text{O}$  system in the adiabatic approximation with an exact accounting of the Pauli principle and the polarization of the interacting nuclei indicates that the inclusion of both factors has a very significant impact on the results. At the same time, those factors act in different directions. Pauli principle leads to very large changes in the potential which reduces attraction between the nuclei while polarization, especially a quadrupole, increases the attraction markedly. At the same time the polarization itself, due to an earlier and stronger overlapping of nucleon densities, increases the effect of the Pauli principle, but generated by the polarization attraction overpowers the repulsion generated by the Pauli principle intensification.

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