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A COMPARISON BETWEEN MARKOVIAN MODELS AND BAYESIAN NETWORKS FOR TREATING SOME DEPENDENT EVENTS IN RELIABILITY EVALUATIONS

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ABSTRACT

Bayesian networks have become a very handy tool for solving problems in various application areas. This paper discusses the use of Bayesian networks to treat dependent events in reliability engineering typically modeled by Markovian models. Dependent events play an important role as, for example, when treating load-sharing systems, bridge systems, common-cause failures, and switching systems (those for which a standby component is activated after the main one fails by means of a switching mechanism). Repair plays an important role in all these cases (as, for example, the number of repairmen). All Bayesian network calculations are performed by means of the Netica™ software, of Norsys Software Corporation, and Fortran 90 to evaluate them over time. The discussion considers the development of time-dependent reliability figures of merit, which are easily obtained, through Markovian models, but not through Bayesian networks, because these latter need probability figures as input and not failure and repair rates. Bayesian networks produced results in very good agreement with those of Markov models and pivotal decomposition. Static and discrete time (DTBN) Bayesian networks were used in order to check their capabilities of modeling specific situations, like switching failures in cold-standby systems. The DTBN was more flexible to modeling systems where the time of occurrence of an event is important, for example, standby failure and repair. However, the static network model showed as good results as DTBN by a much more simplified approach.

1. INTRODUCTION

Treatment of dependent events in reliability engineering has been an important issue [1]. On one side, hundreds or even thousands of failure events have to be taken into account on large systems, and fault trees have been suitably used, although they are very limited to treat dependent events. On the other side, in many instances dependencies have to be modeled even for simpler systems and fault trees may strongly underestimate failure probabilities.

Markovian reliability approaches [1,2] have also been considered because of their ability to treat many dependencies and repair. Two shortcomings arise in this context: the typically used on-off approach (component or system works or fails) leads to an exponential number of coupled differential equations to be considered and aging requires the use of supplementary variables or stages for system modeling [3].

One alternative to face dependent events are Bayesian networks [4]. They are an application of Bayes' Theorem and are suitable for modeling dependent probabilities.

The purpose of this paper is to discuss the application of Bayesian networks to model some typical dependent events that arise in reliability engineering. The first case to be considered refers to load-sharing systems [1], where redundant components may have different failure rates because failures may increase the load on surviving components. For a two-component redundant system, this means that the failure of one of them implies that the surviving component will have a higher failure rate. Next, we consider cold-standby systems, in which the standby component in a redundancy is activated only upon failure of the main component by means of a switching device, whose failure is also modeled. The third case relates to bridge systems, in which redundant lines are connected to improve their reliability. Finally, common-cause failures are approached. Parametric models [5] have been used to quantify their contribution to system failure but when one has to consider higher redundant systems, the quantification process requires consideration of partial common-cause failures, which cannot be modeled by means of fault trees.

All cases to be considered in this work can be modeled by Markovian approaches and, as will be shown, also through Bayesian networks. So, a question arises: is it worth modeling these dependent event cases by Bayesian networks? The answer to this question is that, as Bayesian networks also model fault trees they are a natural candidate to model larger systems with variable number of dependent events. However, other features have to be considered, as time-dependent results, which are easily obtained through Markovian models.

This paper is organized as follows. Section 2 discusses the Markovian and block diagram models to be used for comparison purposes. Section 3 is dedicated to the discussion of the Bayesian network models developed for the same cases previously discussed in Section 2. Results and discussion for both sets of models are the purpose of Section 4. Conclusions and recommendations are the subject of Section 5.

2. MARKOVIAN AND BLOCK DIAGRAM MODELS FOR CASE STUDIES

We discuss in this section some case studies through Markovian and block diagram models considered in order to highlight the main features to be addressed [1-3,5]. The first case relates to load-sharing systems. Next, two-component cold-standby systems are treated, in which switching to a standby component is performed immediately after the main one fails. The third case to be discussed relates to a bridge system, which is useful in many safety systems, as, for instance, the residual heat removal system of Angra 1 nuclear power station. Finally, a discussion is performed on common-cause failures.

2.1. Load-sharing systems

Let us initially consider a two component parallel system. Fig. 1 illustrates the Markov transition diagram for this case, where A, B are system components and a bar over a letter indicates component failure, $\lambda_{A,B}$ are the failure rates with no failed components and $\lambda^*_{A,B}$ are the failure rates for one component when the other one is failed. Typically, $\lambda^*_{A,B} > \lambda_{A,B}$. System states are consecutively numbered and system success at time t means the system is in states 1, 2 or 3.

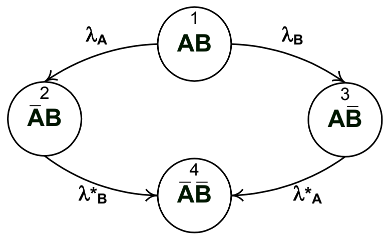

Figure 1: Markov transition diagram for the two-component load-sharing system without repair

The system equation to be solved for this case is as follows:

$$\frac{d\overline{p}(t)}{dt} = \overline{M}\overline{p}(t) \tag{1}$$

where $\overline{p}(t) = [p_1(t),..., p_4(t)]$ is the state probability vector, that is, its components are the probabilities of being in each state. An initial condition is due and the system is supposed to be in state 1 at $t = 0$. On the other hand, in Eq. (1):

$$M = \begin{bmatrix} -(\lambda_A + \lambda_B) & 0 & 0 & 0 \\ \lambda_A & -\lambda_B^* & 0 & 0 \\ \lambda_B & 0 & -\lambda_A^* & 0 \\ 0 & \lambda_B^* & \lambda_A^* & 0 \end{bmatrix} \tag{2}$$

is the transition rate matrix, which displays the transition rates for the possible transitions, displayed in Fig. 1. The solution to the system displayed in Eqs. (1) and (2) is given by:

$$R(t) = e^{-(\lambda_A+\lambda_B)t} + \frac{\lambda_A}{\lambda_A + \lambda_B - \lambda_A^*}\left[e^{-\lambda_B^* t} - e^{-(\lambda_A+\lambda_B)t}\right] + \frac{\lambda_B}{\lambda_A + \lambda_B - \lambda_A^*}\left[e^{-\lambda_A^* t} - e^{-(\lambda_A+\lambda_B)t}\right] \tag{3}$$

It should be noted that the solution displayed in Eq. (3) does not consider component repair and this is the reason why it is in closed form.

2.2. Cold-standby redundancy

In this case, the standby component is turned on whenever the main one fails by means of a switch whose reliability is not 100%. System success means that component 1 works for the mission time or, alternatively, when it fails, switching to component 2 is successful and this latter works for the remaining time to accomplish the mission (Fig. 2).

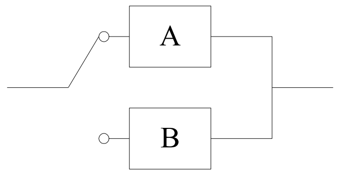

Figure 2: Two-component cold-standby system

Fig. 3 displays the Markovian model for the system shown in Fig. 2. Here R_S stands for the switching reliability, $\lambda_{A,B}$ are the component working failure rates and λ^*_B is the standby failure rate.

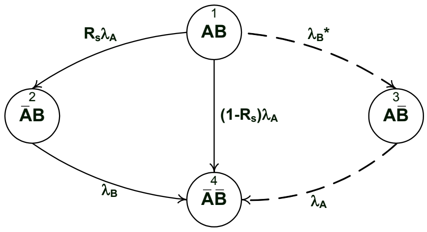

Figure 3: Markovian transition diagram for the cold-standby system

The Markov equations to be solved without considering standby failure are as follows:

$$\begin{aligned} \dot{p}_1(t) &= -\lambda_A p_1(t) \\ \dot{p}_2(t) &= R_s\, \lambda_A p_1(t) - \lambda_B p_2(t) \\ \dot{p}_4(t) &= (1-R_s)\lambda_A p_1(t) + \lambda_B p_2(t) \end{aligned} \tag{4}$$

and their solution is given by:

$$\begin{aligned} \mathrm{p}_1(t) &= e^{-\lambda_A t} \\ \mathrm{p}_2(t) &= \frac{\lambda_A R_S}{\lambda_A - \lambda_B}\left(e^{-\lambda_B t} - e^{-\lambda_A t}\right) \\ \mathrm{p}_4(t) &= 1 - \mathrm{p}_1(t) - \mathrm{p}_2(t) \end{aligned} \tag{5}$$

so that:

$$R(t) = p_1(t) + p_2(t) \tag{6}$$

2.3. Bridge systems

This is the only case for which no Markovian model will be used because one would have to handle a state transition diagram with 32 states. This would be useful only if repair were to be considered.

In this case, a new component is inserted to connect the two redundant lines of a system, as is the case with component *C* in Fig. 4.

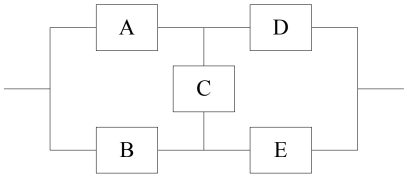

Figure 4: Bridge system

By means of a technique known as pivotal decomposition [1], it is possible to estimate the bridge system reliability. In this case, component *C* is considered a pivot and the total probability theorem is used to estimate system reliability as:

$$R = P(S \mid C)R_C + P(S \mid \overline{C})\overline{R}_C \tag{7}$$

where

$$P(S \mid C) = (R_A + R_B - R_A R_B)(R_C + R_D - R_C R_D) \tag{8}$$

and

$$P(S \mid \overline{C}) = R_A R_B + R_D R_E - R_A R_B R_D R_E \tag{9}$$

so that:

$$\begin{aligned} R(t) = &(R_A + R_B - R_A R_B)(R_D + R_E - R_D R_E)R_C \\ &+ (R_A R_B + R_D R_E - R_A R_B R_D R_E)(1 - R_C) \end{aligned} \tag{10}$$

2.3. Common-cause failures

For the case of common-cause failures, a two component active parallel system with identical components is considered and the corresponding state transition diagram is displayed in Fig. 5. Notice that the common-cause failure takes the system directly to the failed state.

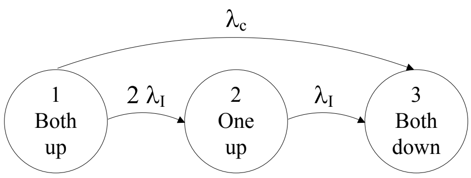

Figure 5: Two-component active parallel system with common-cause failure

For this case, the system equations to be solved are:

$$\frac{dp_1(t)}{dt} = -(2\lambda_I + \lambda_C)p_1(t)$$
$$\frac{dp_2(t)}{dt} = 2\lambda_I p_1(t) - \lambda_I p_2(t) \qquad (10)$$
$$\frac{dp_3(t)}{dt} = \lambda_C p_1(t) + \lambda_I p_2(t)$$

Subject to the typical $p_1(0) = 1$ initial condition. The solution is $R(t) = p_1(t) + p_2(t)$, that is:

$$R(t) = \frac{1}{\lambda_I + \lambda_C}\left[2\lambda_I e^{-\lambda_I t} - (\lambda_I - \lambda_C)e^{-2(\lambda_I + \lambda_C)t}\right] \qquad (11)$$

3. BAYESIAN NETWORK MODELS FOR CASE STUDIES

Bayesian networks are acyclic graphs directed by nodes and edges. Each node represents the probability that a given variable is in certain states. If a node A is dependent on a set of nodes B_i, the probability is calculated using the Bayes' Theorem, given the conditional probabilities $P(A|B_i)$ and the joint probability [6].

The developed models by means of Bayesian networks are discussed in this section. Two models developed are discussed: 1) a static model for load-sharing systems, bridge systems and common cause failure (in this model the input parameters are the component reliabilities), 2) a discrete-time Bayesian network [4] used in the cold-standby system case, where input parameters are the probability of failure for each discrete time interval. In both cases the reliability of each component is calculated from the knowledge of its probability distribution. The networks presented were performed by Netica™ software, of Norsys Software Corporation, and implemented in Fortran 90 to evaluate them over time. The purpose of modeling is to calculate the reliability of the different component configurations.

3.1. Load-sharing systems

As previously discussed, in this arrangement one component affects the functioning of the other and taking into account the definition presented in graphs [6], one can create a link between the state of a component with the state of the other. Thus, one has the Bayesian network configuration of Fig. 6 and Eqs. (12) and (13) for calculating system reliability.

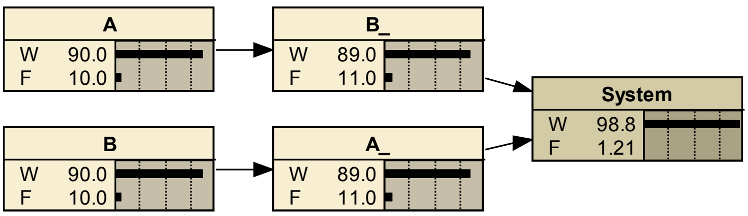

Figure 6: Static Bayesian network for the two-component load-sharing system without repair.

$$P(system\ work) = P(A\_) + P(B\_) + P(A\_)P(B\_) \tag{12}$$

where $P(A\_)$ is the reliability of component A given by

$$P(A\_) = P(A \mid B)P(B) + P(A \mid \overline{B})P(\overline{B}) \tag{13}$$

and $P(A \mid B)$ is the reliability of component A, given B is working and $P(A \mid \overline{B})$ is the reliability of component A, given B is failed.

Considering a static network means that system reliability will always be calculated by Eq. (12) at any instant of time. Reliabilities will be changed depending on the conditional probability distribution used. At this point, one can see that the model through a Bayesian network is not limited by the probability distribution considered.

3.2. Cold standby systems

This system takes into account that at the time of demanding the standby component there is a chance that this transition fails. This transition may be made through electrical relays and valves, for example. Considered as switching or demand failures, these components are also susceptible to failures. The proposed scheme in this case is called Discrete-time Bayesian Network (DTBN) [4] and it is represented in Figure 7 in a simplified manner. The states of the components are the probability of failure in a time interval $[t_i, t_f[$. Since one does not consider the probability of component *B* failure in standby, the conditional probability table (CPT) for *B* is Table 1.

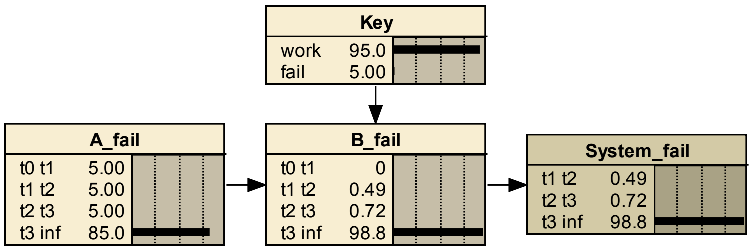

Figure 7: Discrete-time Bayesian network for cold standby systems.

Table 1 can be interpreted as follows: only component *B* may fail at time *j+1*, as component *A* has failed at time *j* and the switching has not failed. The switch failure means that component *B* is failed at time *j+1*. According to Ref. [4], P_{ij} is the failure probability of *B* in the interval *[i, i+Δ[* given that it has survived until time *j*, that is,

$$P_{ij} = \begin{cases} \dfrac{P(B=1)}{\mathrm{P(B > j)}} & \text{if i > j} \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

Table 1: Conditional Probability Table for $P(\overline{B})$

$P(\overline{A})$	Key	$P(\overline{B})$			
		$[t_0,t_1[$	$[t_1,t_2[$	$[t_2,t_3[$	$[t_3,t_\infty]$
$[t_0,t_1[$	Work	0	$P_{1,2}$	$P_{1,3}$	$P_{1,\infty}$
$[t_0,t_1[$	Fail	0	1	0	0
$[t_1,t_2[$	Work	0	0	$P_{2,3}$	$P_{2,\infty}$
$[t_1,t_2[$	Fail	0	0	1	0
$[t_2,t_3[$	Work	0	0	0	1
$[t_2,t_3[$	Fail	0	0	0	1
$[t_3,t_\infty[$	Work	0	0	0	1
$[t_3,t_\infty]$	Fail	0	0	0	1

On the other hand, the system will fail when component *B* fails. This means that the probability that the system fails in a time interval $[t_i, t_f[$ will equal the total failure probability of *B* in this time interval, which explains the way the switching was modeled.

3.3. Bridge systems

The bridged system (Fig. 4) can be modeled by a Bayesian network (Fig. 8), considering nodes *A*, *B*, *C*, *D* and *E* as the reliabilities of each of these components. The probability table of node "Bridge System" has all possible system configurations and among them are specified those for which the system will work. For example, if *A*, *C* and *E* work the system will operate, but if *A*, *C*, and *E* are failed the system will be failed. Applying Bayes' theorem for this configuration, one obtains the same Eq. (10) previously obtained by the pivotal method.

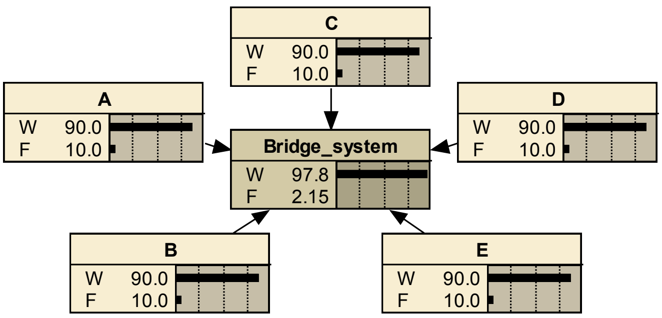

Figure 8: Bayesian network for bridge system

3.4. Common-cause failures

This model aims to perform the reliability evaluation for a system whose components are subject to failures that affect all components in addition to being subject to independent failures. Fig. 9 shows the static Bayesian network proposed.

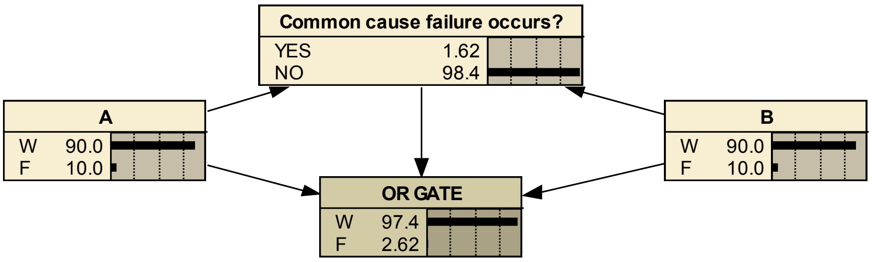

Figure 9: Static Bayesian network for the two-component active parallel system with common-cause failure

The conditional probability table of node "Common cause failure occurs?" allows the occurrence of a common-cause failure only when the two components are working, otherwise the probability is zero and the components undergo independent failures.

Table 2 is the “OR_gate” CPT and it shows the states where the system is failed or working. The total probability of system working at a time *t* is given by Eq. (15).

Table 2: Conditional Probability Table for common cause failure

A	B	Common cause failure occur?	Or gate
W	W	YES	F
W	W	NO	W
W	F	YES	F
W	F	NO	W
F	W	YES	F
F	W	NO	W
F	F	YES	F
F	F	NO	F

$$P(or\_gate = work) = P(A)P(B)P(common\_failure\_occur? = no) + P(A)P(\overline{B}) + P(\overline{A})P(B) \tag{15}$$

4. RESULTS AND DISCUSSION

The models using Markov chains (MC) or pivotal decomposition and Bayesian networks (BN) presented in Sections 2 and 3, respectively, were simulated for the four cases discussed: load-sharing systems, cold-standby systems, bridge systems and common-cause failures. The main purpose of the simulations was to compare the results for both approaches for relatively simple cases in order to explore the advantages of Bayesian networks to calculate reliability.

Table 3 shows the variables values used in simulations. It is assumed that all components are in their useful lifetime, which means that their failure times follow exponential distributions, to allow for comparison with the Markovian approach. In the case of the DTBN a time interval equal to 2 time units was used.

Table 3: Failure rates and probability values assumed

Components failure rates (*i = A, B, C, D or E*)	λ_i	0.001
Failure rate of *A* when *B* is failed	λ_A^*	0.0015
Failure rate of *B* when *A* is failed	λ_B^*	0.0015
Common failure rate	λ_C	0.0002
Switching reliability	R_s	0.95

Table 4 shows the results for t = 10, 100, 200 and 500. As can be observed, the results through the proposed Bayesian networks are very close to those obtained from the Markov chains, with a relative error less than 3%. As repair was not considered, it makes no sense to consider larger time intervals, where system reliabilities are less than 80%.

Table 4: Results for Bayesian Network and Markov Chain

Reliability for t = 10			
	Bayesian Network	Markov Chain	relative error
Load-sharing systems	0.999920	0.999880	0.00%
Cold-standby redundancy	0.999560	0.999460	0.01%
Bridge system	0.999800	0.999800	0.00%
Common-cause failure	0.997940	0.999230	0.13%
Reliability t = 100			
	Bayesian Network	Markov Chain	relative error
Load-sharing systems	0.9903	0.9869	0.3%
Cold-standby redundancy	0.9906	0.9908	0.0%
Bridge system	0.9806	0.9806	0.0%
Common-cause failure	0.9747	0.9836	0.9%
Reliability for t = 200			
	Bayesian Network	Markov Chain	relative error
Load-sharing systems	0.9614	0.9523	0.9%
Cold-standby redundancy	0.9875	0.9727	1.5%
Bridge system	0.9274	0.9274	0.0%
Common-cause failure	0.9409	0.9520	1.2%
Reliability for t = 500			
	Bayesian Network	Markov Chain	relative error
Load-sharing systems	0.8015	0.7865	1.9%
Cold-standby redundancy	0.8744	0.8946	2.3%
Bridge system	0.6695	0.6695	0.0%
Common-cause failure	0.8102	0.8101	0.0%

In general, the differences between BN and MC grow with time in the cases of load-sharing systems and of cold-standby redundancy. Meanwhile, the bridge system has the exact solution and the common-failure model has an intersection point at t = 500 when the RB reliability becomes greater than that calculated by MC.

The DTBN was more flexible to modeling systems where the time of occurrence of an event is important, for example, standby failure and repair. However, the static network model showed as good results as DTBN with much more simplified approach.

5. CONCLUSIONS

This paper discussed the reliability modeling of some typical dependent events for two component systems by means of Markovian models or pivotal decomposition and Bayesian networks, without considering repair. The intention was to check the validity of Bayesian networks to model these dependent events because Markovian models can become quite cumbersome due to the exponential growth of the number of coupled differential equations to solve.

Bayesian networks produced results in very good agreement with those of Markov models and pivotal decomposition. Static and discrete time Bayesian networks were used in order to check their capabilities of modeling specific situations, like switching failures in cold-standby systems. Static Bayesian networks results were much closer to those of Markovian models like the ones of discrete time Bayesian networks, even using a simpler model. One should bear in mind that this will not be the case for repair, for example, because its modeling will have to be made through discrete time Bayesian networks.

Bayesian networks can treat many kinds of dependencies and have the advantage of using expert opinion for estimating probabilities for which scarce data is available. On the other hand, Markovian models can produce time-dependent results, but are limited to equipment in its useful life, although they are also able to treat the dependencies referred to in the context of Bayesian networks. But Bayesian networks have the potential of modeling aging equipment much easier than Markovian model.

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