IMPROVED LUMPED MODELS FOR TRANSIENT COMBINED CONVECTIVE AND RADIATIVE COOLING OF A TWO-LAYER SPHERICAL FUEL ELEMENT

Alice Cunha da Silva\textsuperscript{1} and Jian Su\textsuperscript{2}

\textsuperscript{1}Departamento de Engenharia Nuclear, Escola Politécnica
Universidade Federal do Rio de Janeiro
21941-972 Cidade Universitária, RJ
alicecs@poli.ufrj.br

\textsuperscript{2}Programa de Engenharia Nuclear - COPPE
Universidade Federal do Rio de Janeiro
21941-972 Cidade Universitária, RJ
sujian@nuclear.ufrj.br

ABSTRACT

The High Temperature Gas cooled Reactor (HTGR) is a fourth generation thermal nuclear reactor, graphite-moderated and helium cooled. The HTGRs have important characteristics making essential the study of these reactors, as well as its fuel element. Examples of these are: high thermal efficiency, low operating costs and construction, passive safety attributes that allow simplification of the respective plants. The Pebble Bed Modular Reactor (PBMR) is a HTGR with spherical fuel elements that named the reactor. This fuel element is composed by a particulate region with spherical inclusions, the fuel UO\textsubscript{2} particles, dispersed in a graphite matrix and a convective heat transfer by Helium happens on the outer surface of the fuel element. In this work, the transient heat conduction in a spherical fuel element of a pebble-bed high temperature reactor was studied in a transient situation of combined convective and radiative cooling. Improved lumped parameter model was developed for the transient heat conduction in the two-layer composite sphere subjected to combined convective and radiative cooling. The improved lumped model was obtained through two-point Hermite approximations for integrals. Transient combined convective and radiative cooling of the two-layer spherical fuel element was analyzed to illustrate the applicability of the proposed lumped model, with respect to different values of the Biot number, the radiation-conduction parameter, the dimensionless thermal contact resistance, the dimensionless inner diameter and coating thickness, and the dimensionless thermal conductivity. It was shown by comparison with numerical solution of the original distributed parameter model that the improved lumped model, with $H_{2,1}/H_{1,1}/H_{0,0}$ approximation yielded significant improvement of average temperature prediction over the classical lumped model.

1. INTRODUCTION

The High Temperature Gas cooled Reactor (HTGR) is a fourth generation thermal nuclear reactor, graphite-moderated and helium cooled. The HTGRs have important characteristics making essential the study of these reactors, as well as the thermal analysis of spherical fuel elements. Examples of these are: high thermal efficiency, low operating costs and construction, passive security attributes that allow simplification of the respective plants [1]. The Pebble Bed Modular Reactor (PBMR) is a HTGR with spherical fuel elements that named the reactor [2]. The PBMR fuel element is composed of a particulate
Figure 1: Fuel Element Design for PBMR

region with spherical inclusions, the TRISO particles, dispersed in a graphite matrix and a convective heat transfer by Helium happens on the outer surface of the fuel element. The fuel element is composed of two macroscopic parts, the outer lay of graphite and the matrix with spherical inclusions of TRISO, these are composed of four layers with kernel of UO$_2$.

In this work we studied the behavior of the fuel temperature in a case of Loss of flow accident (LOFA) followed by automatic reactor shutdown. When it happens in a modular HTGR, the negative temperature reactivity coefficient prevents reactivity excursions of the reactor power generation and the reactor is shutdown automatically as a part of the safety features. However, the decay heat generation after the shutdown, mainly derived from the decay of the fission products, must be considered. Considering only the convective boundary condition, Pessoa [3] proposed a spherical resolution by seven convective heat transfer equations using integral transform technique and finite difference methods. The analysis of Transient conduction of multi-layer systems is more complicated when radiative cooling or heat at the boundaries are considered.


We propose the study of the transient heat conduction in two layers composite sphere
subjected to combined convective and radiative cooling in a situation of LOFA. The analysis of this work was carried out by using both the lumped parameter models and the distributed parameter model for comparison, which is solved by an implicit finite difference method. The improved lumped model was obtained through Hermite approximations for integrals $H_{2,1} / H_{1,1} / H_{0,0}$ [7].

2. THE MATHEMATICAL FORMULATION

Consider one-dimensional transient heat conduction in two-layers composite sphere. It is assumed that the thermophysical propeties of the composite are homogeneous, isotropic and independent of the temperature. At $t = 0$, the spherical body is exposed to an environment of loss of flow accident (LOFA). The mathematical formulation of the problem is given by

$$\rho_f c_f \frac{\partial T_f}{\partial t} = \frac{k_f}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T_f}{\partial r} \right) + q''', \quad (1)$$

for the matrix,

$$\rho_c c_c \frac{\partial T_c}{\partial t} = \frac{k_c}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T_f}{\partial r} \right), \quad (2)$$

for the graphite cladding, where $T_f$ and $T_c$ are temperatures in the matrix and the graphite cladding, $\rho_f, \rho_c$ their densities, $c_f, c_c$ their specific heats, $k_f, k_c$ their respective thermal conductivities, and $q'''$ the volumetric heat generation in the matrix. The heat generation in the cladding is neglected.

The equations are to be solved together with the following initial, boundary, and interfacial conditions:

$$T_f(r, 0) = T_{f0}(r), \quad (3)$$

$$-k_f \frac{\partial T_f}{\partial r} \bigg|_{r=0} = 0, \quad (4)$$

$$-k_f \frac{\partial T_f}{\partial r} \bigg|_{r=r_{fo}} = h_g(T_f(r_{fo}, t) - T_c(r_{ci}, t)), \quad (5)$$

$$T_c(r, 0) = T_{c0}(r), \quad (6)$$

$$-k_c \frac{\partial T_c}{\partial r} \bigg|_{r=r_{ci}} = h_g(T_f(r_{fo}, t) - T_c(r_{ci}, t)), \quad (7)$$

$$-k_c \frac{\partial T_c}{\partial r} \bigg|_{r=r_{co}} = h(T_f(r_{co}, t) - T_\infty) + \varepsilon \sigma (T^4 - T_s^4), \quad (8)$$

where $h$ is the heat transfer coefficient between the cladding and the coolant, $h_g$ is the heat transfer coefficient for the gap, $r_{fo}$ is the radius on the outer surface of the matrix, $r_{ci}$ is the radius on the inner surface of the cladding, $r_{co}$ is the radius on the outer surface of the cladding, $\varepsilon$ is the surface emissivity, and $\sigma$ is the Stefan-Boltzmann constant.

It should be noted that in general the environmental fluid temperature, in this case the temperature of the helium, differs from the radiation sink temperature. It is convenient to introduce the adiabatic surface temperature $T_a$, defined by

$$h(T_a - T_\infty) + \varepsilon \sigma (T^4 - T_s^4) = 0. \quad (9)$$

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The boundary condition Eq. (8) can be rewritten with use of the adiabatic surface temperature

$$-k_c \frac{\partial T_c}{\partial r} |_{r=r_{co}} = h(T_f(r_{co}, t) - T_a) + \varepsilon \sigma (T_f^4(r_{co}) - T_a^4).$$  \hfill (10)

Introducing the following dimensionless variables,

$$\theta_f = \frac{T_f}{T_{ref}}, \quad \theta_c = \frac{T_c}{T_{ref}},$$

$$R = \frac{r}{r_{co}}, \quad \tau = \frac{k_f t}{\rho_f c_f r_{co}^2},$$

$$K = \frac{k_f \rho_c c_c}{k_c}, \quad Bi = \frac{h r_{co}}{k_c},$$

$$G = \frac{r_{co}^2 g}{k_f T_{ref}}, \quad N_{rc} = \frac{\varepsilon \sigma r_{co} T_i^3}{k_c},$$

we get the following dimensionless equations

$$\frac{\partial \theta_f}{\partial \tau} = \frac{1}{R^2} \frac{\partial}{\partial R} \left( R^2 \frac{\partial \theta_f}{\partial R} \right) + G(\tau), \hfill (11)$$

$$\frac{\partial \theta_c}{\partial \tau} = K \frac{\partial}{\partial R} \left( R^2 \frac{\partial \theta_c}{\partial R} \right), \hfill (12)$$

with the following initial, boundary, and interfacial conditions:

$$\theta_f(R, 0) = \theta_{f0}(R), \hfill (13)$$

$$\frac{\partial \theta_f}{\partial R} \bigg|_{R=0} = 0, \hfill (14)$$

$$-\frac{\partial \theta_f}{\partial R} \bigg|_{R=R_{fo}} = Bi_{gf}(\theta_f(R_{fo}, \tau) - \theta_c(R_{ci}, \tau)), \hfill (15)$$

$$\theta_c(R, 0) = \theta_{c0}(R), \hfill (16)$$

$$-\frac{\partial \theta_c}{\partial R} \bigg|_{R=R_{ci}} = Bi_{gc}(\theta_f(R_{fo}, \tau) - \theta_c(R_{ci}, \tau)), \hfill (17)$$

$$-\frac{\partial \theta_c}{\partial R} \bigg|_{R=1} = Bi(\theta_c(1, \tau) - \theta_a) + N_{rc}(\theta_c^4(1, \tau) - \theta_a^4). \hfill (18)$$

The parameter $N_{rc}$ characterizes a ratio of the heat conduction resistance to the heat radiation resistance, this parameter plays an analogical role as the Biot number in case of convective heat transport between the body and the environment.
2.1. Lumped Models

The lumped parameter model proposes to decrease the computational costs of the numerical methods and computational codes available, in cases of multidimensional problems of diffusion in one or more regions by reducing the number of independent variables. This is achieved by integrating partial differential equations in relation to spatial variables that we seek to eliminate.

The corresponding spatially averaged dimensionless temperatures are defined by

\[
\theta_{f,av}(\tau) = \frac{\int_0^{R_{fo}} 4\pi R^2 \theta_f(R, \tau) dR}{(4/3)\pi R_{fo}^3} = \frac{3}{R_{fo}^3} \int_0^{R_{fo}} R^2 \theta_f(R, \tau) dR, \tag{19}
\]

\[
\theta_{c,av}(\tau) = \frac{\int_{R_{ci}}^1 4\pi R^2 \theta_c(R, \tau) dR}{(4/3)\pi (1 - R_{ci}^3)} = \frac{3}{(1 - R_{ci}^3)} \int_{R_{ci}}^1 R^2 \theta_c(R, \tau) dR. \tag{20}
\]

We operate Eq.(11) by \((3/R_{fo}^3) \int_0^{R_{fo}} R^2 dR\), using the definition of average temperature, Eq.(19) and the boundary conditions, we get,

\[
\frac{d\theta_{f,av}(\tau)}{d\tau} = -\frac{3}{R_{fo}} Bi_{gf}(\theta_f(R_{fo}, \tau) - \theta_c(R_{ci}, \tau)) + G(\tau). \tag{21}
\]

Similarly, we operate Eq.(12) by \((3/(1 - R_{ci}^3)) \int_{R_{ci}}^1 R^2 dR\), using the definition of average temperature, Eq.(20) and the boundary conditions, we get,

\[
\frac{d\theta_{c,av}(\tau)}{d\tau} = 3K(1 - R_{ci}^3)(R_{ci}^2 Bi_{gc}(\theta_f(R_{fo}, \tau) - \theta_c(R_{ci}, \tau)) - Bi(\theta_c(1, \tau) - \theta_a) - N_{rc}(\theta_c^4(1, \tau) - \theta_a^4)). \tag{22}
\]

2.1.1. Classical Lumped Model

In the classical lumped parameter model, the boundary temperatures are assumed to be the same as the average temperatures:

\[
\theta_f(R_{fo}, \tau) \approx \theta_{f,av}(\tau), \tag{23}
\]

\[
\theta_c(R_{ci}, \tau) \approx \theta_{c,av}(\tau), \tag{24}
\]

\[
\theta_c(1, \tau) \approx \theta_{c,av}(\tau). \tag{25}
\]

The approach is limited to low Biot numbers.

2.1.2. Improved Lumped Model

The idea of the improved analysis is to provide a better relation between the boundary potentials and the averaged potentials, which are to be developed from Hermite-type approximations of the integrals. These approximations allow the expression of average temperature integrals and the heat flow integrals by the values of the temperature and heat fluxes at the boundary.
The Hermite approximation for integral is given by:

$$\int_{a}^{b} y(x) dx = \sum_{\nu=0}^{\alpha} C_{\nu} y^{(\nu)}(a) + \sum_{\nu=0}^{\beta} D_{\nu} y^{(\nu)}(b)$$

where \( y(x) \) and the derivatives \( y^{(\nu)}(x) \) are defined for all \( x \in (a, b) \). Suppose that the numerical values of \( y^{(\nu)}(a) \), for \( \nu = 0, 1, \ldots, \alpha \), and \( y^{(\nu)}(b) \), for \( \nu = 0, 1, \ldots, \beta \), are well known. The general expression for the Hermite approximation \( H_{\alpha,\beta} \) is:

$$\int_{a}^{b} y(x) dx = \sum_{\nu=0}^{\alpha} C_{\nu}(\alpha, \beta) h^{\nu+1} y^{(\nu)}(a) + \sum_{\nu=0}^{\beta} C_{\nu}(\beta, \alpha)(-1)^{\nu} h^{\nu+1} y^{(\nu)}(b) + O(h^{\alpha+\beta+3}),$$

where \( h = b - a \), and

$$C_{\nu}(\alpha, \beta) = \frac{(\alpha + 1)!(\alpha + \beta + 1 - \nu)!}{(\nu + 1)!(\alpha - \nu)!(\alpha + \beta + 2)!}.$$  

In this case we utilized the \( H_{2,1} \) approximation for the average temperature of the matrix, \( H_{1,1} \) approximation for the average temperature of outer graphite layer and \( H_{0,0} \) approximation for the average heat fluxes in both region.

$$H_{0,0} \rightarrow \int_{a}^{b} y(r) dr \simeq \frac{h}{2} (y(a) + y(b))$$

$$H_{1,1} \rightarrow \int_{a}^{b} y(r) dr \simeq \frac{h}{2} (y(a) + y(b)) + \frac{h^2}{12} (y'(a) - y'(b))$$

$$H_{2,1} \rightarrow \int_{a}^{b} y(r) dr \simeq \frac{h}{5} (3y(a) + y(b)) + \frac{h^2}{20} (3y'(a) - y'(b)) + \frac{h^3}{60} y''(a)$$

2.2. Initial Temperatures

For the initial temperatures we consider that the reactor was in a steady state regime before the LOFA happens. So

$$\frac{\partial \theta_f}{\partial \tau} = 0, \quad \text{and} \quad \frac{\partial \theta_c}{\partial \tau} = 0,$$

and the power is constant.

To solve this problem we utilize the transient system with no power decay and changed the heat transfer coefficient of the transient regime to the heat transfer coefficient of a steady state regime.

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2.3. Power Decay

Todreas and Kazimi [13] presented the following empirical formula for the ratio between the decay power of a reactor $P$, after the shutdown, and the normal power $P_0$, before the shutdown:

\[
\frac{P}{P_0} = 0.1[(t+10)^{-0.2} - (t+t_s+10)^{-0.2} + 0.87(t+t_s+2\times10^7)^{-0.2} - 0.87(t+2\times10^7)^{-0.2}] \tag{33}
\]

This formula depends on the time since the reactor have starts the criticality to the time that the shutdown starts. In this work we use a year, the behavior of the power decay for this amount of time is shown in Fig. 2.

3. RESULTS AND DISCUSSION

Both Classical and Improved models are solved numerically. The solutions are then compared with an implicit finite difference solution of the dimensionless differential equation. Fig. 3 shows the averaged fuel and cladding temperatures obtained by both Classical and Improved lumped formulation (case 1). Fig. 4 and 5 show the averaged fuel and cladding temperatures for $N_{rc} = 0.087$ and $Bi = 1.0$ (case 2), $N_{rc} = 0.087$ and $Bi = 10.0$ (case 3), $N_{rc} = 1$ and $Bi = 0.02$ (case 4), and $N_{rc} = 1$ and $Bi = 1$ (case 5).

In case 1, we observed that in all methods the average temperature of the graphite layer raises slightly before starting to decrease along with the matrix average temperature. This may be because as soon as the LOFA happens, the sudden decrease of the cooling capacity and the starting of the power decay does not happen in the same proportion. Then, the matrix delivers heat to the graphite layer, but this does not have enough heat transfer capacity to delivers the heat in the same proportion. So the temperature starts to increase. But as the power is getting lower eventually both began to decrease together. In case 2 and 3, the heat transfer capacity is increased so we do not observe the increase of the graphite layer temperature.
Figure 3: Average Fuel and Cladding temperatures during LOFA by Lumped models.

(a) Classical Lumped Parameter

(b) Improved Lumped Parameter

Figure 4: Comparison between the Classical Model, Improved Model and Finite Difference Method for the Matrix.

(a) $N_{rc} = 0.087$ and $Bi = 1.0$

(b) $N_{rc} = 0.087$ and $Bi = 10.0$
Figure 5: Comparison between the Classical Model, Improved Model and Finite Difference Method for the Graphite Layer.

(a) $N_{rc} = 0.087$ and $Bi = 1.0$

(b) $N_{rc} = 0.087$ and $Bi = 10.0$

In case 4 and 5 we increased the radiation conduction parameter $N_{rc}$, and we observe a great difference between the lumped models. Comparing with the finite difference method, the improved lumped model presented consistence, but the classical lumped model presented great error.

Figure 6: Comparison between the Classical Model, Improved Model and Finite Difference Method for the Matrix.

(a) $N_{rc} = 1$ and $Bi = 0.02$

(b) $N_{rc} = 1$ and $Bi = 1$

4. CONCLUSIONS

We presented an improved lumped model for transient heat conduction in a two-region heterogeneous spherical fuel element subjected to combined convective and radiative cooling. Based on comparison with a finite difference solution of the original distributed parameter model, we concluded that the improved lumped model based on Hermite approximations
for the averaged temperatures and the heat fluxes, $H_{2,1}$, $H_{1,1}$, and $H_{0,0}$ achieved a significant improvement over the classical lumped parameter model as it provided more accurate results for increased Biot and $N_{rc}$ numbers, we also concluded that for increased $N_{rc}$ numbers the classical lumped model is not recommended for it presented great error margins. For more accuracy in the lumped model, a study on the small increase of the graphite layer temperature should be carried out to guarantee that there is no incipient melting of this layer and higher order Hermite approximation should be used.

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