REFRACTIVE NEUTRON LENS

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Compound concave refractive lenses are used for focusing neutron beam. Investigations of spectral and focusing properties of a refractive neutron lens are presented. Resolution of the imaging system on the base of refractive neutron lenses depends on material properties and parameters of neutron source. Model of refractive neutron lens are proposed. Results of calculation diffraction resolution and focal depth of refractive neutron lens are discussed.

Introduction

The microscopy on the base of a cold neutron beam (velocities around 2200 m/s and wavelength in nanoscale range) are useful for observing distribution of atomic nuclear. Refractive optics are used for a cold neutrons with wavelength 0,2-10 nm[1-4]. In small angle approximation the beam deflection angle after refraction doesn’t exceed $(2\delta)^{1/2}$, δ is the decrement of refraction index ($n=1-\delta+i\ (\lambda/4\pi)\ \mu$). Thus the focal length of the individual neutron lens is equal to 100-200 m and it is inappropriate for practical using. The focal distance of stack of N refractive lenses is N times smaller than the focal distance of individual lens due to the geometrical optics. So the stack of the refractive lenses can be suitable for experiments. The main idea to use refractive neutron lenses for SANS[1,2] and beam diverging reduction was observed for small angle scattering experiments[2,3]. Typical refractive neutron lens consists of 30 individual lenses, made from low absorbing material MgF_2 , with radius 25mm. Used in experiments refractive neutron lenses have focal distance 8,7m and transmission 50% for 1.32nm neutron beam, and measured gain was 9,7 [3], As it was proposed in 1999 refractive neutron lenses can be used for new forms of neutron microscopes [4]. Here we analyze optical properties of material for refractive neutron lens and discuss resolution and depth of field of refractive neutron lens.

Model of the refractive neutron lens

So wave length of cold neutron beam is much less than general sizes of lenses, the principles of geometrical optics for the description of distribution of radiation in system of lenses are appropriated. We assume, that the incident neutron beams are parallel of an optical axis in lens. As optical way of neutron beam through the system of *N* lenses is increased in *N* times in comparison with the way in one lens, it is possible to propose the lens model, which assume that system of *N* lenses acts as one thin lens with a complex refraction index n* for non magnetic material expressed as:

$$n^* = 1-\delta N + i\ (\lambda/4\pi)\ \mu N = 1 + (n-1) N, \qquad (1)$$

where μ is the linear absorbing coefficient of the material for neutrons, n is the refraction index of lens material. The δ is the decrement of refraction index ($n=1-\delta+i\ (\lambda/4\pi)\ \mu$) describes refractive properties of the material:

$$\delta = \lambda^2 \rho\ b_c\ / (2\pi), \qquad (2)$$

where ρ is the atomic number density, and b_c is the coherent scattering length. Values of (ρ b_c) the scattering-length density is small and, for example, for Si equals $2*10^{-6}$ A^{-2}. Refractive index of the most materials is less than 1 on the value 10^{-5}-10^{-6} (fig.1), so the focusing lens must be biconcave. Ideal shape for individual lens is parabolic.

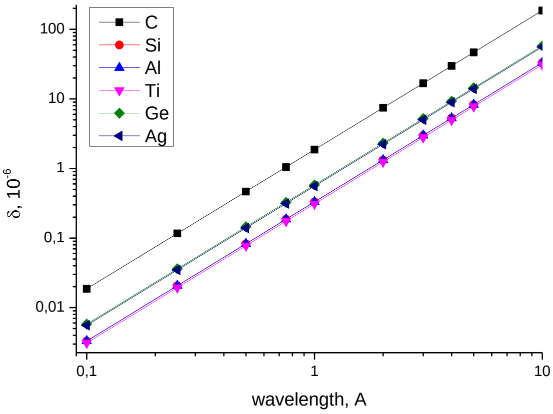

Fig. 1. The values of δ for materials

The ideal focal length of compound refractive lens is inversely to the number of individual refractive lens, at thin lens approximation when length of the compound refractive lens is smaller than the distance between the compound refractive lens and the focal spot. The length of the compound lens is proportional to the number of individual refractive lenses. If the length of the compound refractive lens is larger than ideal focal length the thin lens approximation doesn’t work and the focal length of a compound X-ray lens will be negative. The focal length depends on the number of individual lens of “thick” compound lens. Optical properties of the “thick” lens depends on the lens length. Limit of the focal length of a "thick" lens defines as the maximum number of individual lenses for “thick” lens. The maximum number N_{max} of individual lenses for “thick” neutron lens may be calculated as:

$$F \approx L = 2RN_{max}, \qquad (3)$$

where *L* is the length of the ideal refractive neutron lens, $F=R/(2\delta N_{max})$ is the focal distance. According to equation (3) N_{max} is equal to (fig.2.):

$$N_{max} = 0{,}5\ /\ (\delta)^{1/2}\ . \qquad (4)$$

According to equation (4) the focal length of the compound refractive lens has the limit value. Using individual lenses of decreasing radius according to the size of the compressed neutron beam it is possible to reduce the focal length F of the compound refractive lens.

The depth of field *dF*, the minimum size of the lens focal spot R_{diff} and resolution are defined by diaphragm size R_D placed at the entrance of the lens.

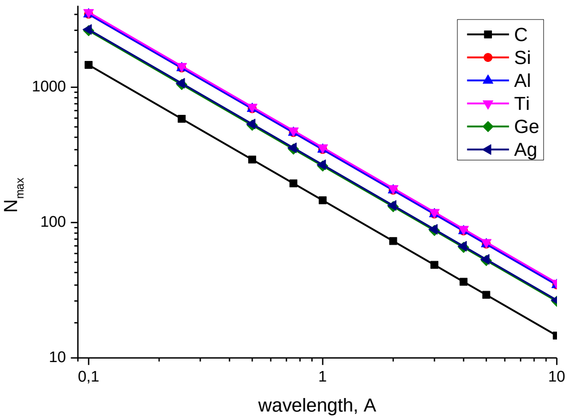

Fig. 2. The values of N_{max} for materials

Resolution of the refractive lens is the smallest separation of two objects that can be resolved by a lens:

$$R_{diff} = 1{,}22 \cdot \lambda \frac{F}{2R_D} \ge \lambda \cdot \qquad (5)$$

Depth of field *dF* of the refraction lens in thin lens approximation is:

$$dF = 0.5 \cdot \lambda \frac{F^2}{R_D^{\,2}} \cdot \qquad (6)$$

The equation (5) gives that (F/R_D) should be close to 1. In practice the value of (F/R_D) is more than 10^3. Thus, it is reasonable to increase value of diaphragm radius (aperture) or decrease focal length. One of the promising way is to produce short focal lens.

Another way for increasing the value of (F/R_D) is to produce neutron lens with large radius of the diaphragm. Large in radius lens consists of large number of individual lenses. Large number of individual lenses (for decreasing focal length) decrease transmission of the intensity through the refractive neutron lens. The kinoform lenses are designed in optics for decreasing absorption and for increasing the aperture of the refractive lens.

For the parabolic short focal refractive lenses the radius of the diaphragm or aperture of the refractive lens is limited by absorption as:

$$R_D = \sqrt{\frac{R}{2\mu N}} \le R \cdot \qquad (7)$$

Using this result in equations (5) and (6) we get the values of R_{diff} and *dF* as the function of the wave length, focal length and material parameters (μ/δ):

$$R_{diff} = 1{,}22 \cdot \lambda \cdot \sqrt{F\frac{\mu}{\delta}} = 1{,}22\sqrt{\frac{\pi}{2}}\sqrt{\frac{F\mu}{\rho b_c}} = 1.5\sqrt{\frac{F}{F_0}}, \qquad (8)$$

$$dF = 0.5 \cdot \lambda \cdot F\frac{\mu}{\delta} = \frac{\pi}{4}\frac{F\mu}{\lambda\rho b_c} = 0.8\frac{F}{dF_0}, \qquad (9)$$

The values of resolution factor $F_0=\rho b_c/\mu$ and the values of depth of field factor $dF_0=\lambda F_0=\lambda\rho b_c/\mu$ and the values of R_{diff} and *dF* in units of focal distance are shown in fig.3,4.

The results of calculation shows the value resolution depends only on resolution factor and can be around of microns. The value of depth of field increase with increasing of wavelength, and for Ag the dependence is small.

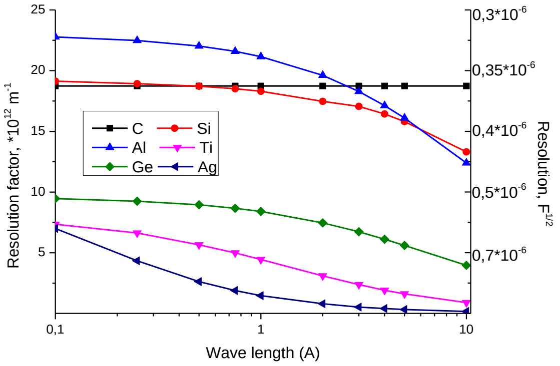

Fig. 3. The values of resolution factor $F_0=\rho b_c/\mu$ and the values of resolution R_{diff} in units of focal distance for materials

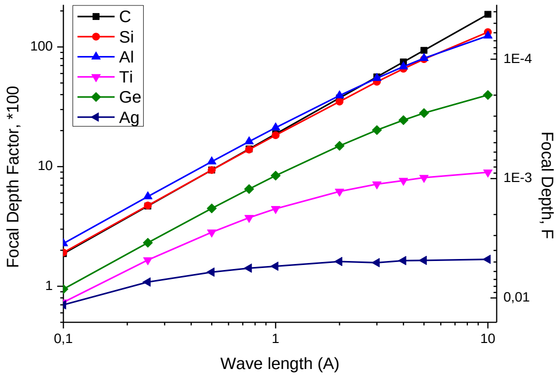

Fig. 4. The values of depth of field factor $dF_0=\lambda\rho b_c/\mu$ and the values of depth of field *dF* in units of focal distance for materials

The equations (8) and (9) doesn't take into account scattering neutrons in material, the real value is smaller then calculated results. The values of resolution factor and the values of focal depth factor should be as small as possible for the neutron microscope.

Conclusions

Model of the refractive neutron lens is proposed. System of *N* lenses acts as one thin lens with a complex refraction index n*. The maximum number N_{max} of individual lenses for "thick" neutron lens is calculated. Refractive neutron lens properties (resolution, focal depth) as function of resolution factor $F_0=\rho b_c/\mu$ and depth of field factor $dF_0=\lambda F_0=\lambda\rho b_c/\mu$ are calculated. It is shown that micro resolution of the refractive neutron optics is far from the wavelength in size and its open possibilities for progress in refractive neutron optics.

References

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