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A FRIENDLY MAPLE MODULE FOR ONE AND TWO GROUP REACTOR MODEL

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ABSTRACT

The well known two energy groups core reactor design model is revisited. A simple and friendly Maple module was built to cover the steps calculations of a plate reactor in five situations: 1. one group bare reactor, 2. two groups bare reactor, 3. one group reflected reactor, 4. 1-1/2 groups reflected reactor and 5. two groups reflected reactor. The results show the convergent path of critical size, as it should be.

1. INTRODUCTION

Neutrons from fission have very high speeds and must be slowed mainly by moderator like, for example, water or graphite, to maintain the chain reaction. This process is hard to be described accurately. In order to solve the neutron diffusion equation, the energy range is divided in several groups with particular features. This is the standard procedure.

There are several complications. For example, not all neutrons are created with the same energy and they do not lose the same amount of energy. So, the moderation is not continuous and they can jump to distant energy group.

One group calculations cannot represent that complex process and is very limited and restricted to academic works, although it can be employed as a starting point for a real project. The two groups approach has the advantage of being able to describe the neutron moderation, albeit in a limited way [1-5].

Looking for that way, it is interesting to follow the evolution of a critical reactor size, as an example, with several calculation methods.

The Maple module here presented enables rapid solution and graphical visualization of a certain restricted class of bare and reflected homogeneous reactor problems, within reasonable limits of accuracy.

This paper has the following structure: Section II refers to one and two groups basic equations definition; Section III solves that equations for bare and reflected one dimensional reactor core; Section IV analyzes the different critical size and Section V close this paper with some final remarks and conclusion.

2. DEVELOPMENT

We split this section in two subsections. The first is devoted to one energy group details. The second refers to the two energy groups in general form.

2.1. One Group

The one group approach is the simplest way for reactor calculation. Usually, its result is used as a first guess for the project. For a multi region reactor the j-region equation can be written as:

$$-D^j \nabla^2 \varphi^j + \Sigma_a^j \varphi^j = \frac{\nu \Sigma_f^j}{k_{eff}} \varphi^j \quad . \tag{1}$$

The terms are:
D^j is the diffusion coefficient;
Σ_a^j is the absorption macroscopic cross section;
$\nu\Sigma_f^j$ is the mean neutron emitted by fission times the fission macroscopic cross section;
k_{eff} is the effective multiplication factor and
φ^j is the neutron flux solution.

For a bare reactor core we have only one equation and two constants. One constant will be selected with the appropriate symmetry conditions and the other by power normalization.

For a reflected core we have four constants, two for the core and two for the reflector. Interface conditions for the flux and current and boundary conditions will set three constants. The last constant will be set by power normalization.

2.2. Two Group

A more elaborate model can be achieved by the two group model. One has two equations for each j-region. For a reflected reactor we have four equations, two for the core region:

$$-D_1^c \nabla^2 \varphi_1^c + \Sigma_{R1}^c \varphi_1^c = \frac{\chi_1}{k_{eff}} \left\{ \nu\Sigma_{f1}^c \varphi_1^c + \nu\Sigma_{f2}^c \varphi_2^c \right\} \tag{2}$$

$$-D_2^c \nabla^2 \varphi_2^c + \Sigma_{R2}^c \varphi_2^c = \frac{\chi_2}{k_{eff}} \left\{ \nu\Sigma_{f1}^c \varphi_1^c + \nu\Sigma_{f2}^c \varphi_2^c \right\} + \Sigma_{2\leftarrow 1}^c \varphi_1^c \tag{3}$$

and two for the reflector:

$$-D_1^r \nabla^2 \varphi_1^r + \Sigma_{R1}^r \varphi_1^r = 0 \tag{4}$$

$$-D_2^r \nabla^2 \varphi_2^r + \Sigma_{R2}^r \varphi_2^r = \Sigma_{2\leftarrow 1}^r \varphi_1^r \tag{5}$$

Equations (2)-(5) describe a homogeneous reactor. This can be arranged as operator notation:

$$\widetilde{\boldsymbol{\theta}}^{\boldsymbol{j}} \vec{\boldsymbol{\varphi}}^{j} = \vec{\mathbf{0}}$$

with $\widetilde{\boldsymbol{\theta}}^{\boldsymbol{j}} = \left\{\mathbb{I}\nabla^2 + \mathbb{A}^{\mathrm{j}}\right\}$, where $\mathbb{I}$ is the 2×2 identity matrix, $\mathbb{A}^{\mathrm{j}}$ is the 2×2 matrix with with the cross section parameters as elements, defined as:

$$a_{11}^j = \frac{\frac{\chi_1}{k_{eff}} \nu\Sigma_{f1}^j - \Sigma_{R1}^j}{D_1^j} \qquad a_{12}^j = \frac{\frac{\chi_1}{k_{eff}} \nu\Sigma_{f2}^j}{D_1^j}$$

$$a_{21}^j = \frac{\frac{\chi_2}{k_{eff}} \nu\Sigma_{f1}^j + \Sigma_{2\leftarrow 1}^j}{D_2^j} \qquad a_{22}^j = \frac{\frac{\chi_2}{k_{eff}} \nu\Sigma_{f2}^j - \Sigma_{R2}^j}{D_2^j}$$

The j index runs from c to the core and r to the reflector. The $\vec{\boldsymbol{\varphi}}^{j}$ is the j-region, core or reflector, two components column vector (for example: fast and thermal flux for a conventional thermal reactor).

The j-region and g energy group macroscopic cross-section constants are:

D_{g}^j- diffusion coefficient;
Σ_{Rg}^j - removal cross section;
$\nu\Sigma_{fg}^j$ - mean fission emitted neutrons times fission cross section;
χ_g - fission spectrum;
$\Sigma_{2\leftarrow 1}^j$ - transfer $1 \rightarrow 2$;
k_{eff} - effective multiplication factor.

A glance over the equations shows that there are four solutions for each region. Therefore, we have four constants for the core and four constants for the reflector, a total of eight constants to be determined. As in the past case, the last constant can be set by power normalization.

3. SOLUTION FOR EACH MODEL AND THE MAPLE INTERFACE

In this section one discusses the results of each model and, at the end one makes a discussion about its performance. First of all, one presents the tables 1 and 2 with all representative constants by region.

Table 1: Thermal Reactor Core Group Constants [2]

	One Group	Two Group	
		1	2
$\nu\Sigma_f$	0.02664	0.00847	0.18514
Σ_R	0.02327	0.02619	0.121
Σ_a	0.02327	0.01207	0.121
D	1.16928	1.2627	0.3543
χ	1.0	1.0	0.0

Table 2: Refletor Group Constants [2]

	One Group	Two Group	
		1	2
Σ_R	0.01388	0.0494	0.0197
Σ_a	0.01388	0.0004	0.0197
D	0.44629	1.13	0.16

3.1. Bare Reactor - One and Two Group Model

One group bare reactor is the simplest calculation. The equation (2.1.1) gives the criticality equation:

$$k_{eff} = \frac{\nu\Sigma_f}{\Sigma_a + DB^2} \tag{3.1.1}$$

where B^2 is called geometric buckling. This simple equation is suitable for a Maple representation. Figure 1 shows the opening page.

NUCLEAR REACTOR CORE

ONE AND TWO ENERGY GROUPS ANALYSIS

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The well known one and two energy groups core reactor design model is revisited. This simple and friendly MAPLE module covers the calculation steps in five situations:

1. One group bare reactor,
2. Two groups bare reactor,
3. One group reflected reactor,
4. One and half groups reflected reactor and
5. Two groups reflected reactor.

Figure 1: Maple Reactor Calculation file opening page.

The Figure 2 shows the Maple calculation resources for one group bare reactor. In pressing the buttons the user can change the curve color. Changing the value of Meter the user can vary the reactor leakage or absorption and the plot is automatically updated.

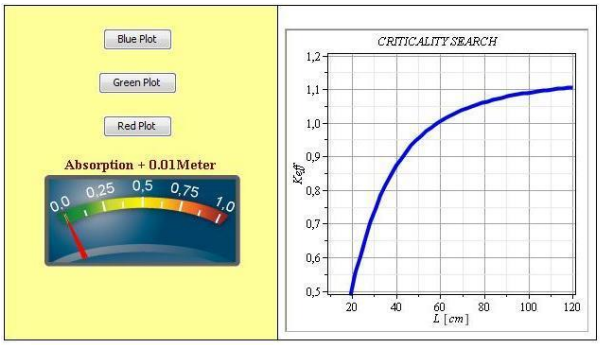


Figure 2: One group bare reactor plot. The Meter changes absorption.

One clearly sees that the first calculation estimates a critical size of 60 cm. We will see that this value will change as the calculations are more sophisticated, such as when adding a reflector.

We now proceed with our two group analysis, assuming the validity of a one-group description of the moderation process theory and diffusion theory for the thermal neutron.

Figure 3 shows the two group calculation for a bare reactor. In an unreflected reactor core, the thermal to fast flux ratio is a constant.

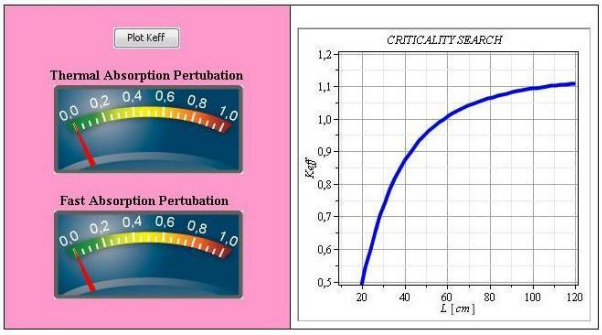


Figure 3: Two group bare reactor plot. The Meter changes Thermal and Fast absorption.

3.2. Reflected Reactor - One and Two Group Model

The addition of a reflector establishes a boundary between the simplified calculation of bare reactor and reflected reactor complex calculation, even in one energy group. For that task, the third modulus does that. Figure 4 shows the effect of a reflector when its dimension is changed. The saturation occurs no matter how additional reflector is added. For this particular case, 20 cm can be considered an infinite reflector.

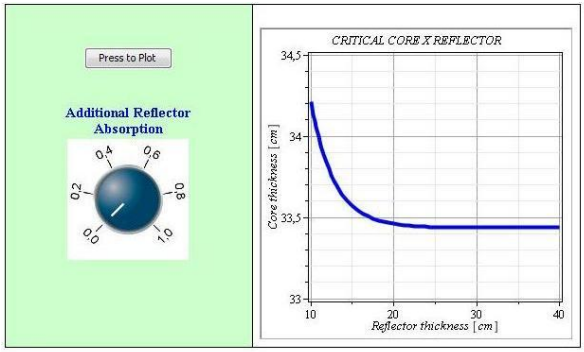


Figure 4: One group plot. The Meter changes the reflector absorption.

3.3. Reflected Reactor – 1-1/2 Group Model

For the two group reflected reactor model the calculations start to become exhaustive and tedious. However, some simplification, without much loss of accuracy, can be done if we note that, for this reactor, the thermal diffusion coefficient is much smaller than the fast diffusion coefficient. This model is called 1-1/2 Group. It allows quick results. The thermal leakage of the core and reflector are neglected. The price you pay is the loss of flux continuity on regions interface. Figure 5 shows the fourth calculation modulus getting a more accurate critical reactor core size.

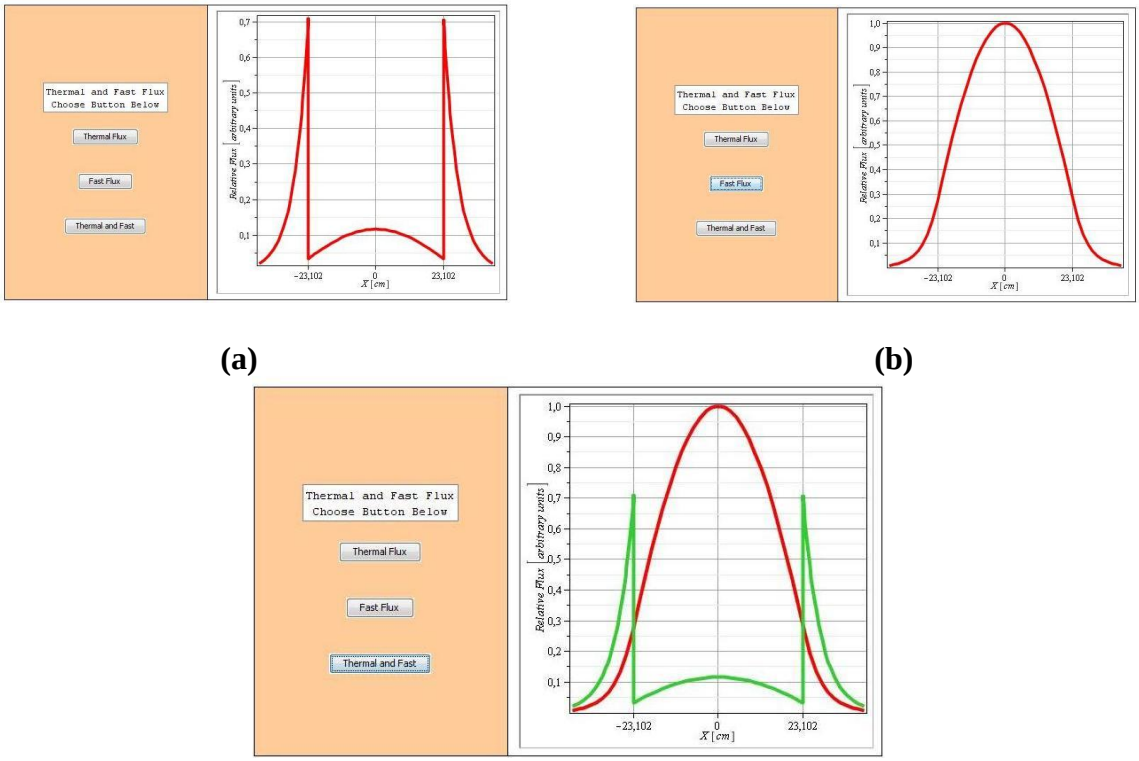
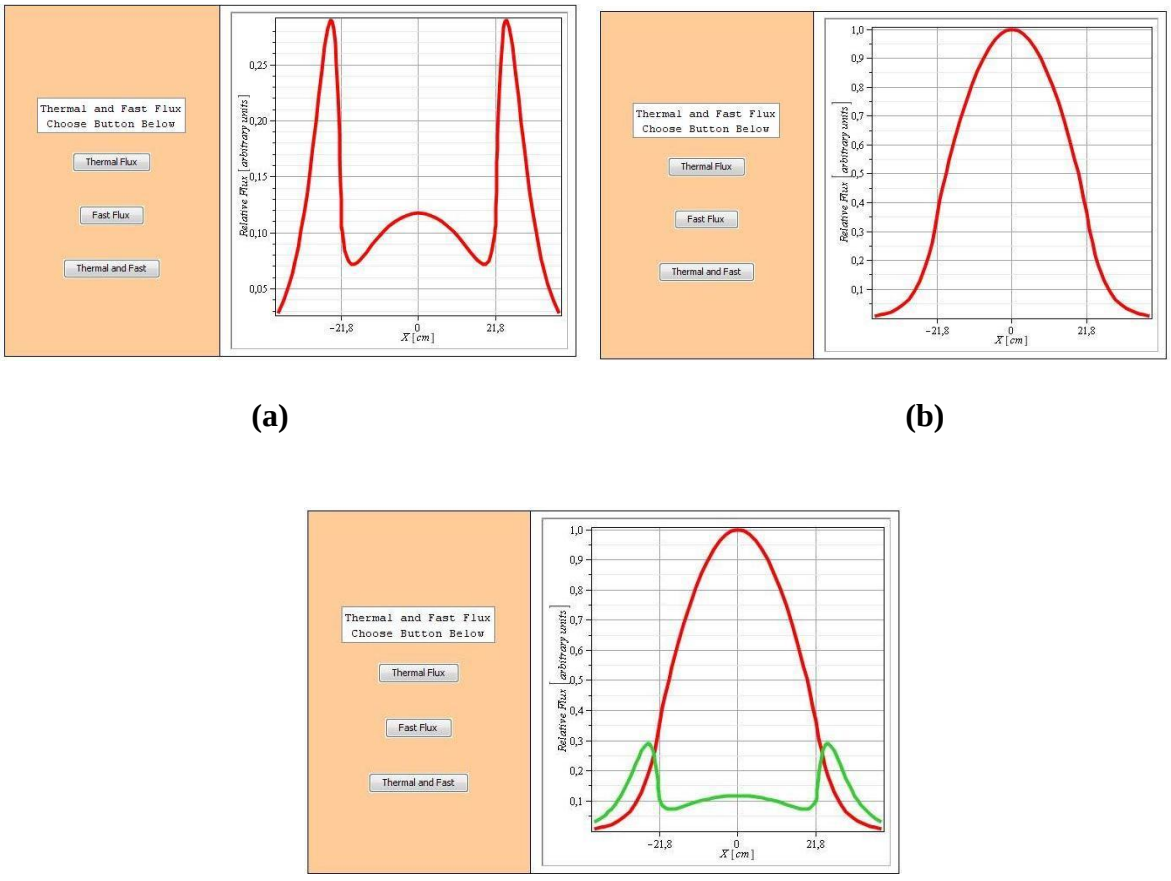


Figure 5: 1-1/2 Group Model (a) Thermal flux, (b) Fast flux and (c) Thermal and Fast.

3.4. Reflected Reactor – Two Group Model

Finally, we have the complete calculation of two group reflected reactor. It involves an order four determinant and it becomes almost impossible on hand calculation. The fifth modulus was developed to get the final most accurate result. Figures 6(a) and 6(b) show each individual flux and Figure 6-(c) shows the details of the relative size of thermal and fast fluxes.



(a) **(b)**

(c)

Figure 6: The more realistic Two Group Model (a) Thermal flux, (b) Fast flux and (c) Thermal and Fast.

3. CONCLUSIONS

In Reactor Physics, calculations can become extremely tedious and time consuming. This graphical interface, built on the Maple platform, enables quick and safe viewing of several integrated results. For an user taking the first steps in reactor physics, this worksheet can be a valuable resource. Providing the correct cross section data of each medium, all parameters of interest will be provided clearly and concisely. Furthermore, using the buttons, some variables may be readily varied. The changes can be seen immediately on the plot window on

the side. More calculations are being added to account for more important details. Finally, the authors wish it to be shared with multiple users.

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