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A SPECTRAL NODAL METHOD FOR EIGENVALUE S_N TRANSPORT PROBLEMS IN TWO-DIMENSIONAL RECTANGULAR GEOMETRY FOR ENERGY MULTIGROUP NUCLEAR REACTOR GLOBAL CALCULATIONS

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ABSTRACT

A spectral nodal method is developed for multigroup x,y-geometry discrete ordinates (S_N) eigenvalue problems for nuclear reactor global calculations. This method uses the conventional multigroup S_N discretized spatial balance nodal equations with two non-standard auxiliary equations: the spectral diamond (SD) auxiliary equations for the discretization nodes inside the fuel regions, and the spectral Green's function (SGF) auxiliary equations for the non-multiplying regions, such as the baffle and the reflector. This spectral nodal method is derived from the analytical general solution of the S_N transverse integrated nodal equations with constant approximations for the transverse leakage terms within each discretization node. The SD and SGF auxiliary equations have parameters, which are determined to preserve the homogeneous and the particular components of these local general solutions. Therefore, we refer to the offered method as the hybrid SD-SGF-Constant Nodal (SD-SGF-CN) method. The S_N discretized spatial balance equations, together with the SD and the SGF auxiliary equations form the SD-SGF-CN equations. We solve the SD-SGF-CN equations by using the one-node block inversion inner iterations (NBI), wherein the most recent estimates for the incoming group node-edge average or prescribed boundary conditions are used to evaluate the outgoing group node-edge average fluxes in the directions of the S_N transport sweeps, for each estimate of the dominant eigenvalue in the conventional Power outer iterations. We show in numerical calculations that the SD-SGF-CN method is very accurate for coarse-mesh multigroup S_N eigenvalue problems, even though the transverse leakage terms are approximated rather simply.

1. INTRODUCTION

We present in this paper a spectral nodal method for eigenvalue transport problems in two-dimensional rectangular geometry for energy multigroup nuclear reactor global calculations. As the mathematical model, we use the steady-state, multigroup, x,y-geometry transport equation with isotropic scattering in the discrete ordinates (S_N) formulation [1]. The main advantage of the nodal methods is the reduction of the number of discretized points. This implies potentially significant reduction of time and computer memory storage compared with the classical numerical methods applied to fixed source problems or multiplication eigenvalue problems. Therefore we describe a hybrid spectral nodal method for energy multigroup for coarse-mesh nuclear reactor S_N calculations. To solve the transverse-integrated S_N nodal equations, we generalize the spectral diamond (SD)

method that we developed for numerically solving slab-geometry S_N eigenvalue problems with no spatial truncation error [2]. In the present work, we approximate the transverse leakage through the edges of each spatial node by constants, so we call our method the SD-constant nodal method that we use in the fuel regions of the nuclear reactor core [3]. In the non-multiplying regions, e.g., reflector and baffle, we use the spectral Green's function-constant nodal method; hence the hybrid characteristic of the present SD-SGF-CN method. In order to converge the numerical solution for each S_N"fixed source" problem (inner iterations) in each outer iteration (power method), we use the partial one-node block inversion (NBI) iterative scheme.

In the next section, we present the mathematical model we use in this work. In Section 3 we describe the multigroup SD-SGF-CN hybrid method. In Section 4 we describe the partial NBI iterative scheme. In Section 5 we present some numerical results and concluding, in Section 6 a brief discussion is given.

2. MATHEMATICAL MODEL

Let us consider the multigroup S_N equations in a rectangular domain $D = \{(x,y) \in \mathbb{R}^2 | 0 \leq x \leq L_x; 0 \leq y \leq L_y\}$ with isotropic scattering

$$\left(\mu_m \frac{\partial}{\partial x} + \eta_m \frac{\partial}{\partial y}\right) \Psi_{m,g}(x,y) + \sigma_{Tg}(x,y)\, \Psi_{m,g}(x,y) = \frac{1}{4} \sum_{g'=1}^{G} \sigma_{Sg'g}(x,y) \sum_{n=1}^{M} \Psi_{n,g'}(x,y)\, \omega_n + \frac{\chi_g}{4k_{eff}} \sum_{g'=1}^{G} \nu\sigma_{fg'}(x,y) \sum_{n=1}^{M} \Psi_{n,g'}(x,y)\, \omega_n \quad , \tag{1}$$

where:

- $(x,y) \in D$, $g = 1:G$, $m = 1:M$ with $M = N(N+2)/2$,
- N = order of the angular quadrature set,
- $\Psi_{m,g}$ = group angular flux of neutrons migrating in direction μ_m,
- σ_{Tg} = group total macroscopic cross section,
- $\sigma_{Sg'g}$ = scattering macroscopic cross section from group g' to group g,
- $\sigma_{fg'}$ = fission macroscopic cross section induced by neutrons in energy group g',
- k_{eff} = effective multiplication factor,
- ν average number of neutrons emitted in each fission event,
- (μ_m, η_m) discrete ordinates,

- ω_m quadrature weights.

At this point, let us consider an arbitrary spatial grid $\Gamma_{xI} \times \Gamma_{yJ}$ on D wherein each spatial cell is termed node Γ_{ij} of width h_{xi} $(i = 1 : I)$ and height h_{yj} $(j = 1 : J)$, as shown in Fig. 1. Each node Γ_{ij} has constant cross sections that we denote as σ^{ij}_{Tg}, $\sigma^{ij}_{Sg'g}$ and $\sigma^{ij}_{fg'}$.

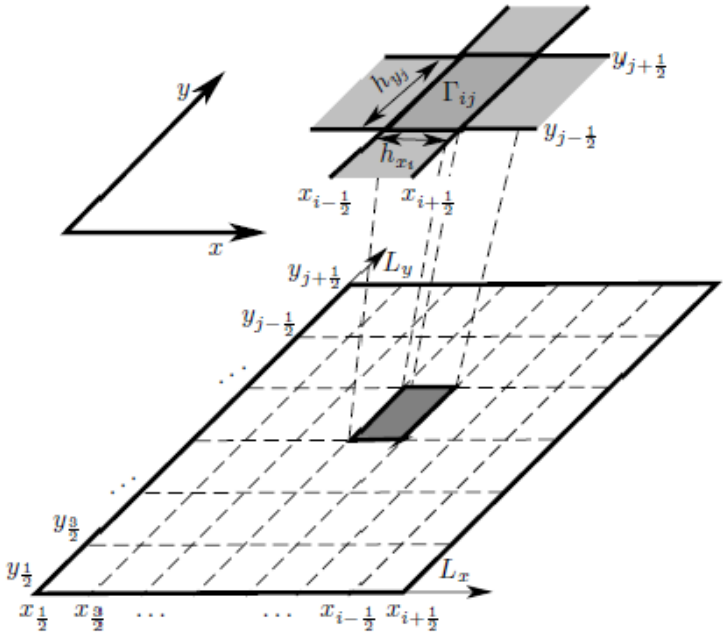

Figure 1: Spatial grid.

To obtain the multigroup one-dimensional transverse-integrated S_N nodal equations, we follow the standard procedure [4]. By applying the transverse integration operator

$$\frac{1}{h_{y_j}} \int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} \bullet dy \tag{2}$$

on Eq.(1), we obtain the multigroup one-dimensional transverse-integrated S_N nodal equation for the x coordinate direction

$$\mu_m \frac{d}{dx} \widetilde{\Psi}^j_{m,g}(x) + \frac{\eta_m}{h_{y_j}} \left(\Psi^{j+\frac{1}{2}}_{m,g}(x) - \Psi^{j-\frac{1}{2}}_{m,g}(x) \right) + \sigma^{ij}_{Tg} \widetilde{\Psi}^j_{m,g}(x) =$$
$$\sum_{g'=1}^{G} \frac{\sigma^{ij}_{Sg'g}}{4} \sum_{n=1}^{M} \widetilde{\Psi}^j_{n,g'}(x)\, \omega_n + \sum_{g'=1}^{G} \frac{\chi_g \nu \sigma^{ij}_{fg'}}{4k_{eff}} \sum_{n=1}^{M} \widetilde{\Psi}^j_{n,g'}(x)\, \omega_n \quad , \tag{3}$$

$x \in \Gamma_{ij}, \quad g = 1 : G, \quad m = 1 : M, \quad i = 1 : I, \quad j = 1 : J$, where we have defined the node-edge average angular flux in the y direction

$$\frac{1}{h_{y_j}} \int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} \Psi_{m,g}(x,y)\, dy = \widetilde{\Psi}^j_{m,g}(x)\,, \quad g = 1 : G, \quad m = 1 : M, \quad j = 1 : J\,. \tag{4}$$

Now we define the group node-edge average angular flux in the x direction

$$\frac{1}{h_{x_i}} \int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \Psi_{m,g}(x,y)\, dx = \widehat{\Psi}^i_{m,g}(y)\ , \quad g = 1 : G, \quad m = 1 : M, \quad j = 1 : J\,. \tag{5}$$

By following similar procedure, we obtain the multigroup one-dimensional transverse-integrated S_N nodal equations for the y coordinate direction.

Equation (3) together with the corresponding equations for the y-direction form two linear system of GM ordinary differential equations in $2GM$ unknown quantities.

The transverse leakage term in Eq.(3) is given by

$$\frac{\eta_m}{h_{y_j}}\left(\Psi_{m,g}^{j+\frac{1}{2}}(x)-\Psi_{m,g}^{j-\frac{1}{2}}(x)\right)\ . \tag{6}$$

and the leakage term in the x direction is given by

$$\frac{\mu_m}{h_{x_i}}\left(\Psi_{m,g}^{i+\frac{1}{2}}(y)-\Psi_{m,g}^{i-\frac{1}{2}}(y)\right)\ . \tag{7}$$

At this point we remark that the multigroup transverse-integrated S_N nodal equations are exact. Thus far, we have made no approximations. Furthermore, for each spatial node Γ_{ij} we use the following definitions:

$$\widetilde{\Psi}_{m,g}^{j}\left(x_{i+\frac{1}{2}}\right)=\widetilde{\Psi}_{m,g}^{i+\frac{1}{2},j} \tag{8}$$

and

$$\widehat{\Psi}_{m,g}^{i}\left(y_{j+\frac{1}{2}}\right)=\widehat{\Psi}_{m,g}^{i,j+\frac{1}{2}}\ ,\quad g=1:G\ ,\quad m=1:M\ . \tag{9}$$

Therefore, to obtain a unique solution to the system for a spatial grid $\Gamma_{xI}\times\Gamma_{yJ}$ composed of $I\times J$ discretization nodes, besides the node boundary conditions, which include continuity conditions and the boundary conditions to the S_N problem, we also need to approximate the transverse leakage terms. In this paper, we approximate the transverse leakage terms by constants. That is

$$\frac{\eta_m}{h_{y_j}}\left(\Psi_{m,g}^{j-\frac{1}{2}}(x)-\Psi_{m,g}^{j+\frac{1}{2}}(x)\right)\approx\frac{\eta_m}{h_{y_j}}\left(\widehat{\Psi}_{m,\mathtt{g}}^{i,j-\frac{1}{2}}-\widehat{\Psi}_{m,g}^{i,j+\frac{1}{2}}\right)=\widehat{L}_{m,g}^{ij} \tag{10}$$

and

$$\frac{\mu_m}{h_{x_i}}\left(\Psi_{m,g}^{i-\frac{1}{2}}(y)-\Psi_{m,g}^{i+\frac{1}{2}}(y)\right)\approx\frac{\mu_m}{h_{x_i}}\left(\widetilde{\Psi}_{m,g}^{i-\frac{1}{2},j}-\widetilde{\Psi}_{m,g}^{i+\frac{1}{2},j}\right)=\widetilde{L}_{m,g}^{ij}\quad . \tag{11}$$

By substituting Eq. (10) into Eq. (3), we obtain

$$\begin{aligned}\mu_m\frac{d}{dx}\widetilde{\Psi}_{m,g}^{j}(x)+\sigma_{Tg}^{ij}\widetilde{\Psi}_{m,g}^{j}(x)=\sum_{g'=1}^{G}\frac{\sigma_{Sg'g}^{ij}}{4}\sum_{n=1}^{M}\widetilde{\Psi}_{n,g'}^{j}(x)\,\omega_n+\\ \sum_{g'=1}^{G}\frac{\chi_g\nu\sigma_{fg'}^{ij}}{4k_{eff}}\sum_{n=1}^{M}\widetilde{\Psi}_{n,g'}^{j}(x)\,\omega_n+\widehat{L}_{m,g}^{ij}\quad ,\end{aligned} \tag{12}$$

$x\in\Gamma_{ij},\quad g=1:G,\quad m=1:M,\quad i=1:I,\quad j=1:J.$

The general solution of Eq.(12) in Γ_{ij} with flat approximations for the transverse leakage terms are given by

$$\widetilde{\Psi}_{m,g}(x) = \widetilde{\Psi}^{H}_{m,g}(x) + \widetilde{\Psi}^{P}_{m,g}\,. \tag{13}$$

Here the superscript P indicates the particular solution that depends upon the approximations we make in the transverse leakage terms. The superscript H indicates the homogeneous component of the solution, which satisfies the homogeneous equation associated with Eq.(12), obtained by setting the transverse leakage equal to zero. By substituting $\widetilde{\Psi}^{P}_{m,g}$ into Eq. (12), we seek for spatially constant particular solution to the resulting equation. Therefore, we obtain

$$\frac{1}{4}\sum_{g'=1}^{G}\sum_{n=1}^{M}\left[4\sigma^{ij}_{Tg}\delta_{g'g}\delta_{n,m} - \overline{\sigma}^{ij}_{g'g}\left(k_{eff}\right)\omega_n\right]\widetilde{\Psi}^{P}_{n,g'} = \widehat{L}^{ij}_{m,g} \quad , \tag{14}$$

$$g = 1:G, \quad m = 1:M, \quad i = 1:I, \quad j = 1:J \quad .$$

Equation (14) forms a system of MG algebraic linear equations in MG unknowns $\widetilde{\Psi}^{P}_{m,g}$, where we have defined

$$\overline{\sigma}^{ij}_{g'g}\left(k_{eff}\right) = \sigma^{ij}_{Sg'g} + \frac{\chi^{ij}_{g}\nu\sigma^{ij}_{fg'}}{k_{eff}} \quad . \tag{15}$$

Here $\delta_{g'g}(\delta_{nm})$ is the Kronecker delta.

A similar expression is obtained for the particular solution of the transverse-integrated equations for the y coordinate direction.

To obtain the homogeneous components of the general solution, we perform a spectral analysis of the homogeneous equations associated with Eq.(12). Thus, we consider the ansatz

$$\widetilde{\Psi}^{j}_{m,g,\vartheta^x}(x) = a^{x}_{m,g}\left(\vartheta^x\right)e^{-x/\vartheta^x} \quad , \quad g = 1:G \quad , \quad m = 1:M, \quad j = 1:J\,. \tag{16}$$

We substitute ansatz (16) into Eq.(12). The result can be written as the eigenvalue problem

$$\frac{1}{4\mu_m}\sum_{g'=1}^{G}\sum_{n=1}^{M}\left[4\sigma^{ij}_{Tg}\delta_{g'g}\delta_{mn} - \overline{\sigma}^{ij}_{g'g}\left(k_{eff}\right)\omega_n\right]a^{x}_{n,g'}\left(\vartheta^x\right) = \frac{1}{\vartheta^x}a^{x}_{m,g}\left(\vartheta^x\right)\,, \tag{17}$$

$g = 1:G, \quad m = 1:M, \quad i = 1:I, \quad j = 1:J$. Such an eigenvalue problem is also obtained for the y-direction. This spectral analysis is described in details in [3].

3. THE MULTIGROUP *SD-SGF-CN* HYBRID METHOD

Let us integrate the x, y-geometry S_N equations with isotropic scattering, given in Eq. (1) over, an arbitrary node Γ_{ij}. The result is the multigroup S_N balance equation

$$\frac{\mu_m}{h_{x_i}}\left(\widetilde{\Psi}^{i+\frac{1}{2},j}_{m,g} - \widetilde{\Psi}^{i-\frac{1}{2},j}_{m,g}\right) + \frac{\eta_m}{h_{y_j}}\left(\widehat{\Psi}^{i,j+\frac{1}{2}}_{m,g} - \widehat{\Psi}^{i,j-\frac{1}{2}}_{m,g}\right) + \sigma^{ij}_{Tg}\overline{\Psi}^{ij}_{m,g} =$$

$$\sum_{g'=1}^{G}\frac{\sigma^{ij}_{Sg'g}}{4}\sum_{n=1}^{M}\overline{\Psi}^{ij}_{n,g'}\omega_n + \sum_{g'=1}^{G}\frac{\chi^{ij}_{g}\nu\sigma^{ij}_{fg'}}{4k_{eff}}\sum_{n=1}^{M}\overline{\Psi}^{ij}_{n,g'}\omega_n\,, \tag{18}$$

$g = 1 : G, \quad m = 1 : M \quad i = 1 : I$ e $j = 1 : J$, where the group node-average angular flux is defined as

$$\overline{\Psi}^{ij}_{m,g} = \frac{1}{h_{x_i} h_{y_j}} \int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}} \Psi_{m,g}(x,y) dx dy \, . \tag{19}$$

The multigroup balance equation (18) combined with appropriate continuity conditions on the node edges and boundary conditions imposed at the outer boundaries of the domain form an underdetermined system; therefore we need auxiliary equations in order to obtain the same number of equations as unknowns. In the non-multiplying regions of the domain, e.g., reflector and baffle, we use the multigroup SGF auxiliary equations [3]

$$\overline{\Psi}^{ij}_{m,g} = \sum_{g'=1}^{G} \sum_{\mu_n > 0} {}^{x}\theta^{ij}_{m,g' \to n,g} \widetilde{\Psi}^{i-\frac{1}{2},j}_{n,g'} + \sum_{g'=1}^{G} \sum_{\mu_n < 0} {}^{x}\theta^{ij}_{m,g' \to n,g} \widetilde{\Psi}^{i+\frac{1}{2},j}_{n,g'} + \widehat{G}^{ij}_{m,g} \tag{20}$$

and

$$\overline{\Psi}^{ij}_{m,g} = \sum_{g'=1}^{G} \sum_{\eta_n > 0} {}^{y}\theta^{ij}_{m,g' \to n,g} \widehat{\Psi}^{i,j-\frac{1}{2}}_{n,g'} + \sum_{g'=1}^{G} \sum_{\eta_n < 0} {}^{y}\theta^{ij}_{m,g' \to n,g} \widehat{\Psi}^{i,j+\frac{1}{2}}_{n,g'} + \widetilde{G}^{ij}_{m,g} \, , \tag{21}$$

$m = 1 : M, \quad i = 1 : I, \quad j = 1 : J, \quad g = 1 : G$. In the fuel regions, we use the SD auxiliary equations

$$\overline{\Psi}^{ij}_{m,g} = \sum_{g'=1}^{G} \sum_{n=1}^{M} {}^{x}\gamma^{ij}_{m,g' \to n,g} \left(\widetilde{\Psi}^{i-\frac{1}{2},j}_{n,g'} + \widetilde{\Psi}^{i+\frac{1}{2},j}_{n,g'} \right) + \widehat{H}^{ij}_{m,g} \tag{22}$$

and

$$\overline{\Psi}^{ij}_{m,g} = \sum_{g'=1}^{G} \sum_{n=1}^{M} {}^{y}\gamma^{ij}_{m,g' \to n,g} \left(\widehat{\Psi}^{i,j-\frac{1}{2}}_{n,g'} + \widehat{\Psi}^{i,j+\frac{1}{2}}_{n,g'} \right) + \widetilde{H}^{ij}_{m,g} \, , \tag{23}$$

$m = 1 : M, \quad i = 1 : I, \quad j = 1 : J, \quad g = 1 : G$.

In these auxiliary equations, the parameters θ, γ, G and H are determined by requiring that the general solution of Eq. (12), given by Eq. (13), exactly satisfies Eq. (20) or Eq. (22) for the x-direction and Eq. (21) or Eq. (23) for the y-direction, in accordance with the type of medium, i.e., non-multiplying or fuel regions respectively. We remark that, for each estimate of the dominant eigenvalue k_{eff} in the outer iterations, we need to solve an eigenvalue problem given in Eq. (17) for each fuel zone and, then, re-evaluate the parameters γ in the auxiliary equations (22) and (23), since the parameters θ are needed only for the non-multiplying zones of the domain.

The balance equations (18), combined with the auxiliary equations (20), (21), (22) and (23) and appropriate continuity and boundary conditions form the SD-SGF-CN equations. The multigroup SD auxiliary equation is convenient for multiplying media as it indicates that an interior neutron source (due to fission) exists inside node Γ_{ij}.

4. ITERATIVE ALGORITHM

Power method [5] is traditionally used to converge the dominant solution of S_N eigenvalue problems. For each estimate of the dominant eigenvalue k_{eff} in the outer iterations, we

solve a "fixed-source" S_N problem using inner iterations. The "one-node block inversion" (NBI) scheme iterates on the node-edge average angular fluxes by performing "node-block inversions". That is, the partial NBI scheme uses the most recent estimates available for the incoming node-edge average angular fluxes in each energy group to evaluate the exiting fluxes, that constitute the incoming fluxes for the adjacent nodes in the directions of the transport sweeps [4]. To illustrate this concept, let us consider Fig. 2 representing

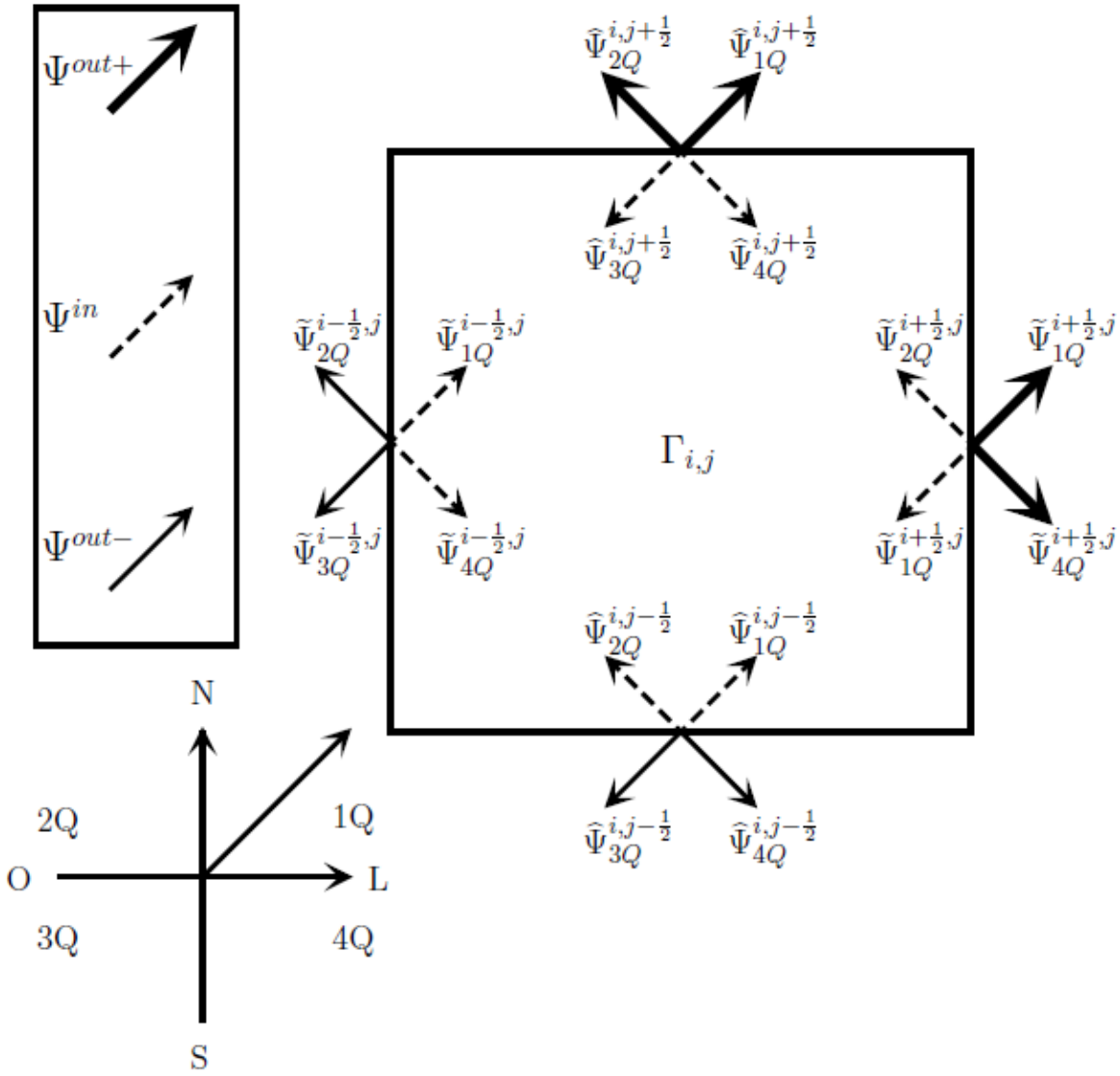

Figure 2: SW to NE transport sweep.

an arbitrary spatial node Γ_{ij} with the group node-edge average angular fluxes. Each arrow in Fig. 2 represents $N(N+2)/8$ directions and G groups in each quadrant [3]. For the sweep indicated (from SW to NE), the outward arrows (thicker arrows) in the north and east of Γ_{ij} represent the outgoing node-edge average angular fluxes we need to calculate, because they form the incoming node-edge average angular fluxes for the adjacent nodes in this sweep. The inward arrows (dashed arrows) represent the incoming node-edge average angular fluxes, that are known, or at least we use the most recent estimates available for them. Finally, the outward arrows (thinner arrows) in the south and west of Γ_{ij} represent

the node-edge average angular fluxes, that can be calculated, but are not needed for this sweep. They represent the outgoing quantities that are needed for the NE to SW sweep. An analogous convention is followed for the NW to SE and SE to NW transport sweeps. As we see, we need to iterate only on the group node-edge average angular fluxes. The equations of the multigroup *SD-SGF-CN* method are solved iteratively, covering the two-dimensional domain in the four sweeping directions, using the partial *NBI* scheme until a prescribed stopping criterion is satisfied. Otherwise, a new estimate for k_{eff}, is re-evaluated to calculate the sweeping matrices to be used in the inner iterations. We emphasize at this point that this algorithm goes on until the stopping criterion for the scalar flux and the stopping criterion for k_{eff} are satisfied.

5. NUMERICAL RESULTS

In this section, we perform a numerical experiment to illustrate the accuracy of the present hybrid *SD-SGF-CN* method for coarse-mesh nuclear reactor global calculations. We present a four energy-group ($G = 4$) model problem, and Fig. 3 shows $\frac{1}{4}$ of a two-dimensional (x, y) nuclear reactor core cut perpendicular to its z-axis, whose material zones have the multigroup energy data given in Ref. [3].

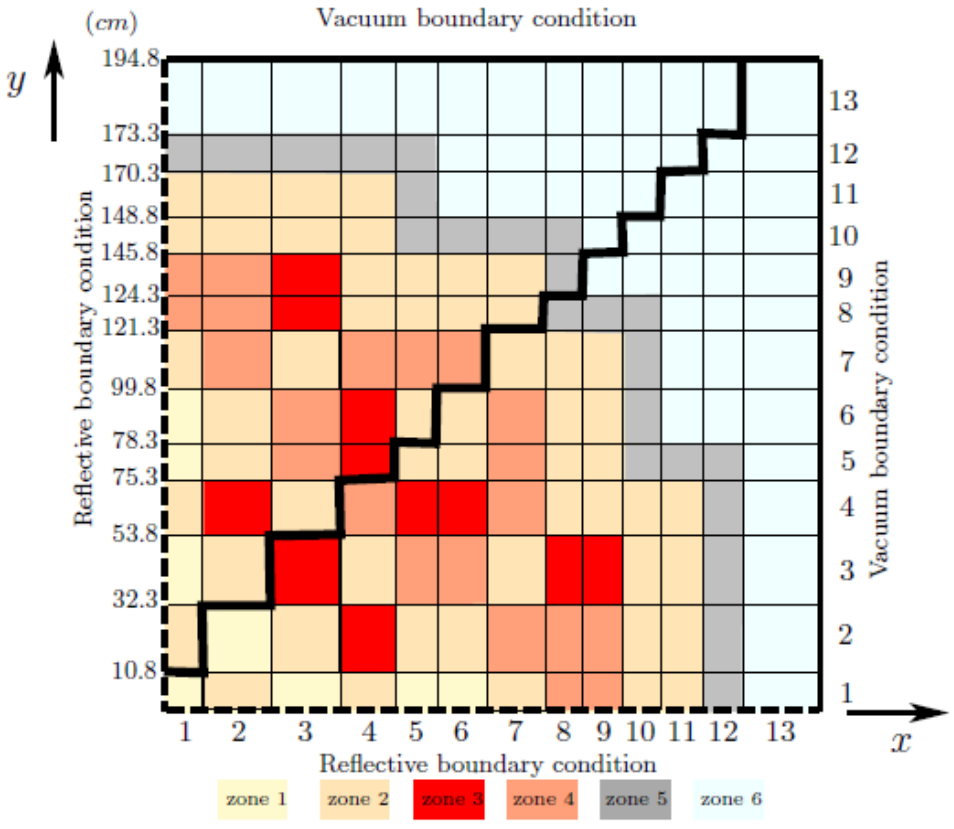

Figure 3: Model problem.

Table 1 shows the dominant eigenvalue (effective multiplication factor k_{eff}) generated by the present multigroup *SD-SGF-CN* method, with the S_4 level symmetric angular

quadrature set [1]. The reference result has been generated by the traditional Diamond Difference (DD) method [1] on a fine grid, composed of 40 spatial cells per region in each spatial direction (x and y).

Table 1: Effective multiplication factor (k_{eff})

Spatial grid [a] Γ_n	Iterations	k_{eff}	[b] Relative deviations (%)
Γ_2	227	1.125600	0.013
Γ_3	364	1.125380	0.007
Γ_4	518	1.125400	0.005
Γ_5	678	1.125501	0.004
Γ_6	835	1.125433	0.002
[c] DD	5465	1.125456	–

[a] $2^n/4$ spatial nodes per region in each spatial direction.

[b] Relative deviation with respect to the DD fine-mesh solution.

[c] Diamond Difference method.

According to Table 2, the maximum deviation generated by the *SD-SGF-CN* method was 6.87% in region (10, 4). In performing nuclear reactor global calculations, let us suppose that the user considers that relative deviations in the effective multiplication factor, greater than 0.5%, with respect to the reference result, are not acceptable. In this case, according to Table 1, the *SD-SGF-CN* method satisfies this condition on a spatial grid composed of 1×1 node per region.

Table 2: Maximum relative deviations (%) in the power density distribution

Spatial grid [a] Γ_n	[b] Relative deviations (%)	[c] Region
Γ_2	6.87	(10, 4)
Γ_3	3.56	(10, 4)
Γ_4	2.65	(10, 4)
Γ_5	1.26	(8, 7)
Γ_6	1.24	(8, 7)

[a] $2^n/4$ spatial nodes per region in each spatial direction.

[b] Relative deviation with respect to the DD fine-mesh solution.

[c] Region numbered in Figure 3, cf. ordered pair (x, y).

6. CONCLUSIONS

Based on the numerical results presented in the previous section, we list a number of general conclusions:

- The multigroup *SD-SGF-CN* hybrid method, as described in this paper, generated very accurate solution, even considering flat leakage approximations.
- Considering the numerical experiment presented in the previous section, the relative deviations for k_{eff}, with respect to the reference, are all smaller than 0.5%.
- We observed that the eigenvalue problems given in Eq. (17) can show some eigenvalues and eigenvectors components which are not real numbers. However, we can ensure that the parameters θ and γ that preserve the relationships between the node-average angular fluxes and the node-edge average angular fluxes, are all real [3].

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