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INTEGRAL TRANSFORM SOLUTION OF BENDING PROBLEM OF CLAMPED ORTHOTROPIC RECTANGULAR PLATES

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ABSTRACT

The generalized integral transform technique (GITT) is employed to obtain an exact solution for the bending problem of fully clamped orthotropic rectangular plates. The use of the GITT approach in the analysis of the transverse deflection equation leads to a coupled system of fourth order differential equations in the dimensionless longitudinal spatial variable. The resulting transformed ODE system is then numerically solved by making use of the subroutine DBVPFD from IMSL Library. Numerical results with automatic global accuracy control are produced for different values of aspect ratio. Critical comparisons with previously reported numerical results are performed with excellent agreement. Several sets of reference results for clamped orthotropic rectangular plates are also provided for future covalidation purposes.

Key Words: orthotropic rectangular plates, bending, integral transform

1. INTRODUCTION

The bending of orthotropic rectangular plates with clamped edges has been studied over the past century, due to its relevance in engineering [1]. Many different methods have been studied to solve the plate bending problem. Recently, Li et al. [2] proposed a double finite sine integral transform method to obtain exact solutions of bending of plates under arbitrary loadings without any trial functions. Zhong and Li [3] developed a new symplectic method for analysis of a rectangular thin plate with all edges clamped, and also obtained exact solutions. In addition, using Galerkin method, Mbakogu and Pavlovic [4] obtained an approximate solution by means of *Mathematica* [5]. Dalaei and Kerr [6] and Aghdam and Falahatgar [7] utilized the extended Kantorovich method to generate a closed form approximate solution for the deflections of a clamped orthotropic plate subjected to a uniformly distributed load for various aspect ratios. Meleshko [1] presented a detailed review of the superposition methods and provided a simplification of the numerical algorithm. For the purpose of analysing moderately thick plates with

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the transverse shear effects, a spline finite element method has been developed by Shen and He [8]. Adopting the method of modified two-variable and the method of mixing perturbation, the problems of the nonlinear asymmetrical bending for orthotropic rectangular thin plate with variable thickness is studied by Huang [9]. Furthermore, Civalek [10] proposed a discrete singular convolution approach to give a numerical solution of three-dimensional problem of thick rectangular plates.

In this work, we applied the generalized integral transform technique (GITT) to obtain the exact solution of the bending problem of fully clamped orthotropic rectangular thin plates. Unlike the double finite sine integral transform method proposed by Li et al. [2], GITT is mathematically more general, which has been successfully applied in heat and fluid flow problems [11–13]. The algorithm can be constructed quite straightforwardly as follows: i) an appropriate auxiliary eigenvalue problem is solved for the eigenvalues and related normalized eigenfunctions, ii) the original partial differential equation is integral-transformed into an infinite set of coupled ordinary differential equations for the transformed potentials, which is truncated at a finite N-terms to allow computational solution, and iii) the truncated ODE system is then numerically solved by well-established scientific libraries such as IMSL Numerical Library [14] or by symbolic numerical software such as *Mathematica* [5].

The most interesting features of the technique are the automatic and straightforward global error control procedure, which makes it particularly suitable for benchmarking purposes, and the only mild increase in overall computational effort with increasing number of independent variables, both features due to its inherent hybrid nature of numerical-analytical solution methodology [15].

The paper is organized as follows. In the next section, the mathematical formulation of the problem of a uniformly-loaded orthotropic rectangular plate with clamped edges is presented. In Section 3, the exact analytical solution is obtained by carrying out integral transform. Numerical results with automatic global accuracy control are then presented in Section 4 for different values of aspect ratio. A comparison against the previous work [16, 17] is also performed to assess the accuracy and convergence of the present approach. Finally, a fully clamped orthotropic plate is examined, aiming at bringing a set of benchmark results for future comparison.

2. MATHEMATICAL FORMULATION

Consider an orthotropic rectangular plate with constant thickness and all four edges clamped, as illustrated in Fig. 1. The principle axes of orthotropy coincide with the x and y direction. The bending problem is governed by the following equation [16, 18]:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q, \tag{1}$$

where w and q are the transverse deflection of the plate and the applied transverse loading, respectively. D_x and D_y represent the flexural rigidities about the y and x axes, respectively. H, the effective torsional rigidity, is given by $H = D_1 + 2D_{xy}$, where D_{xy} denotes the torsional rigidity and D_1 is defined in terms of the reduced Possion's ratios ν_1 and ν_2, as $D_1 = \nu_2 D_x = \nu_1 D_y$.

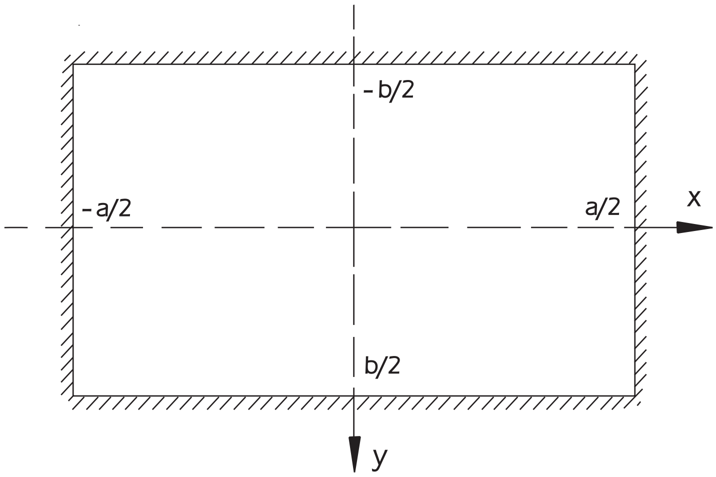

Figure 1: Clamped orthotropic rectangular plate under consideration.

Furthermore, the bending moments M_x and M_y, the torsional moment M_{xy}, the transverse shear forces Q_x and Q_y can be determinated by the following relations:

$$M_x = -\left(D_x \frac{\partial^2 w}{\partial x^2} + D_1 \frac{\partial^2 w}{\partial y^2}\right), \tag{2a}$$

$$M_y = -\left(D_1 \frac{\partial^2 w}{\partial x^2} + D_y \frac{\partial^2 w}{\partial y^2}\right), \tag{2b}$$

$$M_{xy} = -2D_{xy} \frac{\partial^2 w}{\partial x \partial y}, \tag{2c}$$

$$Q_x = -\frac{\partial}{\partial x}\left(D_x \frac{\partial^2 w}{\partial x^2} + H \frac{\partial^2 w}{\partial y^2}\right), \tag{3a}$$

$$Q_y = -\frac{\partial}{\partial y}\left(H \frac{\partial^2 w}{\partial x^2} + D_y \frac{\partial^2 w}{\partial y^2}\right). \tag{3b}$$

In the specific case of isotropy, we have $\nu_1 = \nu_2 = \nu$, $D_x = D_y = H = D$, $D_1 = \nu D$ and $D_{xy} = (1 - \nu)D/2$, where the symbols D and ν represent the flexural rigidity and Poisson's ratio. With these relations, the foregoing equations can be considerably simplified.

The boundary conditions for such problem are given by

$$w = \frac{\partial w}{\partial x} = 0, \quad \text{at} \quad x = \pm\frac{a}{2}, \tag{4a}$$

$$w = \frac{\partial w}{\partial y} = 0, \quad \text{at} \quad x = \pm\frac{b}{2}. \tag{4b}$$

Equation (1) and boundary conditions (4) complete the mathematical formulation of the problem. By introducing the following dimensionless parameters

$$\xi = \frac{x}{a/2}, \quad \eta = \frac{y}{b/2}, \quad c = \frac{b}{a}, \quad \alpha = \frac{H}{D_x}, \quad \beta = \frac{D_y}{D_x}, \quad \tilde{w} = \frac{w}{a^4 q/D_x}, \tag{5}$$

the mathematical formulation in dimensionless form can be written as follows

$$\frac{\partial^4 \tilde{w}}{\partial \xi^4} + \frac{2\alpha}{c^2}\frac{\partial^4 \tilde{w}}{\partial \xi^2 \partial \eta^2} + \frac{\beta}{c^4}\frac{\partial^4 \tilde{w}}{\partial \eta^4} = \frac{1}{16}. \tag{6a}$$

The boundary conditions in dimensionless form are written as

$$\tilde{w}\left(-1,\eta\right) = 0, \quad \frac{\partial \tilde{w}\left(-1,\eta\right)}{\partial \xi} = 0, \tag{6b,c}$$

$$\tilde{w}\left(1,\eta\right) = 0, \quad \frac{\partial \tilde{w}\left(1,\eta\right)}{\partial \xi} = 0, \tag{6d,e}$$

$$\tilde{w}\left(\xi,-1\right) = 0, \quad \frac{\partial \tilde{w}\left(\xi,-1\right)}{\partial \xi} = 0, \tag{6f,g}$$

$$\tilde{w}\left(\xi,1\right) = 0, \quad \frac{\partial \tilde{w}\left(\xi,1\right)}{\partial \xi} = 0. \tag{6h,i}$$

3. INTEGRAL TRANSFORM SOLUTION

Following the ideas in the generalized integral transform technique [15], the auxiliary eigenvalue problems needed for the integral transformation process are chosen from homogeneous versions of the original problems. For the problem (6), the eigenvalue problem is given as

$$\frac{d^4 Y_i\left(\eta\right)}{d\eta^4} = \mu_i^4 Y_i\left(\eta\right), \quad -1 < \eta < 1, \tag{7a}$$

with the following boundary conditions

$$Y_i\left(-1\right) = 0, \quad \frac{dY_i\left(-1\right)}{d\eta} = 0, \tag{7b,c}$$

$$Y_i\left(1\right) = 0, \quad \frac{dY_i\left(1\right)}{d\eta} = 0, \tag{7d,e}$$

where $Y_i\left(\eta\right)$ and μ_i are, respectively, the eigenfunctions and eigenvalues of problem (7), which satisfy the following orthogonality property

$$\int_{-1}^{1} Y_i\left(\eta\right) Y_j\left(\eta\right) d\eta = \delta_{ij} N_i, \tag{8}$$

with $\delta_{ij} = 0$ for $i \neq j$, and $\delta_{ij} = 1$ for $i = j$. The norm, or normalization integral, is written as

$$N_i = \int_{-1}^{1} Y_i^2(\eta)\, d\eta. \tag{9}$$

Problem (7) is readily solved analytically to yield

$$Y_i(\eta) = \begin{cases} \dfrac{\cos(\mu_i \eta)}{\cos(\mu_i)} - \dfrac{\cosh(\mu_i \eta)}{\cosh(\mu_i)}, & \text{for } i \text{ odd}, \\ \dfrac{\sin(\mu_i \eta)}{\sin(\mu_i)} - \dfrac{\sinh(\mu_i \eta)}{\sinh(\mu_i)}, & \text{for } i \text{ even}, \end{cases} \tag{10a,b}$$

and the transcendental equation for the eigenvalues becomes

$$\tanh(\mu_i) = \begin{cases} -\tan(\mu_i), & \text{for } i \text{ odd}, \\ \tan(\mu_i), & \text{for } i \text{ even}, \end{cases} \tag{11a,b}$$

while the norm is evaluated to yield

$$N_i = 2, \quad i = 1, 2, 3, \ldots \tag{12}$$

The eigenvalue problem (7) allows definition of the following integral transform pair

$$\bar{w}_i(\xi) = \int_{-1}^{1} \tilde{Y}_i(\eta)\, \tilde{w}(\xi, \eta)\, d\eta, \quad \text{transform}, \tag{13a}$$

$$\tilde{w}(\xi, \eta) = \sum_{i=1}^{\infty} \tilde{Y}_i(\eta)\, \bar{w}_i(\xi), \quad \text{inversion}, \tag{13b}$$

where $\tilde{Y}_i(\eta)$ is the normalized eigenfunction

$$\tilde{Y}_i(\eta) = \frac{Y_i(\eta)}{N_i^{1/2}} = \frac{Y_i(\eta)}{\sqrt{2}}. \tag{14}$$

Now, to perform the integral transform process, the dimensionless equation (6a) is multiplied by the operator $\int_{-1}^{1} \tilde{Y}_i(\eta)\, d\eta$ and the inverse formula (13b) are applied, yielding, after some mathematical manipulations, the following set of ordinary differential equations

$$\frac{d^4 \bar{w}_i}{d\xi^4} + \frac{2\alpha}{c^2} \sum_{j=1}^{\infty} D_{ij} \frac{d^2 \bar{w}_j}{d\xi^2} + \frac{\beta}{c^4} \mu_i^4 \bar{w}_i = \frac{1}{16} Q_i, \quad i = 1, 2, 3, \ldots, \tag{15a}$$

where the coefficients are analytically determined from the following integrals

$$D_{ij} = \int_{-1}^{1} \tilde{Y}_i \frac{d^2\tilde{Y}_j}{d\eta^2} d\eta, \quad Q_i = \int_{-1}^{1} \tilde{Y}_i d\eta. \tag{15b,c}$$

In a similar manner, the boundary conditions are also integral transformed in the η direction to yield

$$\bar{w}_i(-1) = 0, \quad \frac{d\bar{w}_i(-1)}{d\xi} = 0, \tag{16a,b}$$

$$\bar{w}_i(1) = 0, \quad \frac{d\bar{w}_i(1)}{d\xi} = 0. \tag{16c,d}$$

The integral transformation then eliminates the coordinate η and equations (15) and (16) offer an ODE system in the ξ direction for the transformed potentials $\bar{w}_i(\xi)$, subjected to the corresponding integral transformed boundary conditions.

For computational purposes, the coefficients (15b,c) are computed through the software of symbolic manipulation *Mathematica* [5]. In order to solve the ordinary differential system, infinite summations should be truncated to a sufficiently large order for computational purposes. This truncated system is then solved by a computer program developed in Fortran 90, based on the use of the subroutine DBVPFD from IMSL Library [14]. This subroutine offers an interesting combination of accuracy control, and for this problem the error 10^{-4} is selected.

Once $\bar{w}_i(\xi)$ have been numerically evaluated, the analytical inversion formula (13b) recovers the dimensionless function $\tilde{w}(\xi, \eta)$. Then use the relations (5) to obtain the desired deflection function $w(x, y)$, straightforwardly. The bending moments and the transverse shear forces can be obtained using the expression (2, 3).

4. RESULTS AND DISCUSSION

We now present numerical results for a fully clamped isotropic rectangular plate with different values of the aspect ratio c. In order to allow comparisons with previous results available in the literature [16, 17], we consider two types of load distribution: i) a uniform load of intensity q_0 and ii) linearly varying hydrostatic pressure with the intensity $q_1(x/a + 1/2)$. The solution of the systems (15a) is obtained with $N \leq 128$ to analyze the convergence behavior.

First, the numerical code is validated by making an analysis of the convergence behavior of the transverse deflection w at the center point $(0, 0)$ for an isotropic plate under uniform load with different truncation orders N, as shown in Table I, We can observe that convergence is essentially achieved with a reasonably low truncation orders ($N \leq 8$). For a full convergence to four significant digits, more terms are required. The results are in excellent agreement with the series solutions presented by Timoshenko and Woinowsky-Krieger [16]. The transverse deflection corresponding to the case of infinite aspect ratio ($c = \infty$) is also given in Table I. As can be observed, the convergence rate is slow with the error being 0.54% for $N = 128$.

Table I: Convergence of the transverse deflections $w|_{x=0,y=0}$ $\left(q_0a^4/D\right)$ for an isotropic plate under uniform load with different truncation orders N, and comparison with benchmark series solution (labeled as B. S.) from Timoshenko and Woinowsky-Krieger [16]. ($\nu = 0.3$)

c	$N=4$	$N=8$	$N=16$	$N=32$	$N=64$	$N=128$	B. S.
1.0	0.001261	0.001267	0.001266	0.001266	0.001266	0.001265	0.00126
1.1	0.001501	0.001510	0.001510	0.001508	0.001508	0.001508	0.00150
1.2	0.001715	0.001724	0.001727	0.001727	0.001727	0.001727	0.00172
1.3	0.001897	0.001910	0.001914	0.001914	0.001914	0.001914	0.00191
1.4	0.002049	0.002069	0.002070	0.002071	0.002068	0.002068	0.00207
1.5	0.002172	0.002197	0.002196	0.002197	0.002197	0.002197	0.00220
1.6	0.002269	0.002300	0.002300	0.002300	0.002300	0.002300	0.00230
1.7	0.002344	0.002381	0.002382	0.002382	0.002382	0.002382	0.00238
1.8	0.002401	0.002445	0.002446	0.002446	0.002446	0.002446	0.00245
1.9	0.002442	0.002493	0.002495	0.002496	0.002496	0.002496	0.00249
2.0	0.002471	0.002529	0.002534	0.002533	0.002533	0.002533	0.00254
∞	0.002104	0.002332	0.002463	0.002532	0.002568	0.002586	0.00260

Table II: Convergence of the bending moments $M_x|_{x=a/2,y=0}$ $\left(q_0a^2\right)$ for an isotropic plate under uniform load with different truncation orders N, and comparison with benchmark series solution (labeled as B. S.) from Timoshenko and Woinowsky-Krieger [16]. ($\nu = 0.3$)

c	$N=4$	$N=8$	$N=16$	$N=32$	$N=64$	$N=128$	B. S.
1.0	-0.05106	-0.05142	-0.05137	-0.05134	-0.05134	-0.05133	-0.0513
1.1	-0.05768	-0.05819	-0.05814	-0.05811	-0.05810	-0.05810	-0.0581
1.2	-0.06330	-0.06398	-0.06395	-0.06391	-0.06391	-0.06391	-0.0639
1.3	-0.06789	-0.06878	-0.06877	-0.06873	-0.06872	-0.06872	-0.0687
1.4	-0.07151	-0.07266	-0.07266	-0.07262	-0.07259	-0.07259	-0.0726
1.5	-0.07429	-0.07570	-0.07571	-0.07567	-0.07566	-0.07566	-0.0757
1.6	-0.07636	-0.07804	-0.07809	-0.07805	-0.07804	-0.07804	-0.0780
1.7	-0.07783	-0.07981	-0.07990	-0.07986	-0.07984	-0.07984	-0.0799
1.8	-0.07882	-0.08111	-0.08125	-0.08120	-0.08119	-0.08119	-0.0812
1.9	-0.07943	-0.08204	-0.08223	-0.08219	-0.08217	-0.08217	-0.0822
2.0	-0.07974	-0.08268	-0.08294	-0.08289	-0.08287	-0.08287	-0.0829
∞	-0.06734	-0.07462	-0.07880	-0.08103	-0.08217	-0.08275	-0.0833

Tables II and III present the convergence behavior of the bending moments M_x and the transverse shear forces Q_x at the edge point $(a/2, 0)$, as well as a comparison with those results presented by Timoshenko and Woinowsky-Krieger [16] and Wojtaszak [17], respectively. As would be expected, convergence rates of the bending moments are slower than ones of the transverse deflection, and faster comparing to the transverse shear forces field. This is due to the fact that the bending moments and the transverse shear forces are proportional to the second and third derivatives of the deflection function, respectively. Again, the agreement with references [16, 17] is excellent.

Table III: Convergence of the transverse shear forces $Q_x|_{x=a/2,y=0}\,(q_0 a)$ for an isotropic plate under uniform load with different truncation orders N, and comparison with benchmark series solution (labeled as B. S.) from Wojtaszak [17]. ($\nu = 0.3$)

c	$N = 4$	$N = 8$	$N = 16$	$N = 32$	$N = 64$	$N = 128$	B. S.
1.00	-0.4428	-0.4491	-0.4452	-0.4423	-0.4414	-0.4412	-0.44
1.25	-0.4881	-0.5009	-0.4980	-0.4949	-0.4938	-0.4936	-0.49
1.50	-0.4994	-0.5200	-0.5190	-0.5159	-0.5146	-0.5143	-0.52
1.75	-0.4935	-0.5218	-0.5232	-0.5204	-0.5189	-0.5185	-0.52
2.00	-0.4812	-0.5165	-0.5205	-0.5180	-0.5165	-0.5160	-0.52
∞	-0.4040	-0.4477	-0.4728	-0.4862	-0.4930	-0.4965	-0.50

Table IV: Convergence of the transverse deflections $w|_{x=0,y=0}\,\left(q_1 a^5/D\right)$ for an isotropic plate under linearly varying hydrostatic pressure with different truncation orders N, and comparison with benchmark series solution (labeled as B. S.) from Timoshenko and Woinowsky-Krieger [16]. ($\nu = 0.3$)

c	$N = 4$	$N = 8$	$N = 16$	$N = 32$	$N = 64$	$N = 128$	B. S.
0.5	0.00008000	0.00008009	0.00008009	0.00008009	0.00008009	0.00008009	0.000080
2/3	0.0002166	0.0002170	0.0002171	0.0002171	0.0002171	0.0002171	0.000217
1.0	0.0006304	0.0006334	0.0006336	0.0006336	0.0006336	0.0006336	0.00063
1.5	0.001086	0.001099	0.001099	0.001099	0.001099	0.001098	0.00110
∞	0.001052	0.001166	0.001231	0.001266	0.001284	0.001293	0.00130

Table V: Convergence of the bending moments $M_x|_{x=a/2,y=0}\,\left(q_1 a^3\right)$ for an isotropic plate under linearly varying hydrostatic pressure with different truncation orders N, and comparison with benchmark series solution (labeled as B. S.) from Timoshenko and Woinowsky-Krieger [16]. ($\nu = 0.3$)

c	$N = 4$	$N = 8$	$N = 16$	$N = 32$	$N = 64$	$N = 128$	B. S.
0.5	-0.01140	-0.01148	-0.01147	-0.01146	-0.01146	-0.01146	-0.0115
2/3	-0.01857	-0.01873	-0.01872	-0.01871	-0.01871	-0.01871	-0.0187
1.0	-0.03301	-0.03346	-0.03346	-0.03344	-0.03344	-0.03344	-0.0334
1.5	-0.04483	-0.04605	-0.04616	-0.04615	-0.04614	-0.04613	-0.0462
∞	-0.04040	-0.04477	-0.04728	-0.04861	-0.04930	-0.04964	-0.0500

Next, we perform a similar analysis about the transverse deflections and bending moments for an isotropic plate under linearly varying hydrostatic pressure $q_1\,(x/a + 1/2)$, as presented in Tables IV and V. The same behavior is obtained as the previous cases. The excellent convergence rate of the transverse deflections can be noticed for four significant digits, with values as low as $N \leq 16$ ($c = 0.5$ and 1.5). These results are obtained for a very small CPU time.

Table VI: Deflections, bending moments and transverse shear forces for an orthotropic $(D_y = 4D_x, D_{xy} = 0.85D_x, \nu_1 = 0.075, \nu_2 = 0.3)$ under uniform load with truncation order $N = 200$.

c	$w\|_{x=0,y=0}\ (q_0a^4/D_x)$	$M_x\|_{x=a/2,y=0}\ (q_0a^2)$	$Q_x\|_{x=a/2,y=0}\ (q_0a)$
1.0	0.0005224	-0.02842	-0.3287
1.1	0.0006912	-0.03400	-0.3600
1.2	0.0008722	-0.03969	-0.3891
1.3	0.001057	-0.04528	-0.4154
1.4	0.001240	-0.05061	-0.4383
1.5	0.001415	-0.05555	-0.4577
1.6	0.001579	-0.06002	-0.4736
1.7	0.001731	-0.06402	-0.4864
1.8	0.001866	-0.06751	-0.4965
1.9	0.001983	-0.07051	-0.5041
2.0	0.002088	-0.07307	-0.5097
∞	0.002592(0.00260)*	-0.08296(-0.0833)	-0.4978(-0.50)

* The results in parentheses are from an isotropic plate, when c approaches infinity [16].

Table VII: Deflections, bending moments and transverse shear forces for an orthotropic $(D_y = 4D_x, D_{xy} = 0.85D_x, \nu_1 = 0.075, \nu_2 = 0.3)$ under linearly varying hydrostatic pressure with truncation order $N = 200$.

c	$w\|_{x=0,y=0}\ (q_1a^5/D_x)$	$M_x\|_{x=a/2,y=0}\ (q_1a^3)$	$Q_x\|_{x=a/2,y=0}\ (q_1a^2)$
1.0	0.0002612	-0.02057	-0.2597
1.1	0.0003456	-0.02385	-0.2777
1.2	0.0004362	-0.02707	-0.2938
1.3	0.0005291	-0.03016	-0.3078
1.4	0.0006208	-0.03305	-0.3198
1.5	0.0007087	-0.03569	-0.3298
1.6	0.0007907	-0.03805	-0.3379
1.7	0.0008657	-0.04014	-0.3443
1.8	0.0009331	-0.04195	-0.3493
1.9	0.0009929	-0.04350	-0.3530
2.0	0.001044	-0.04482	-0.3556
∞	0.001296(0.00130)*	-0.04977(-0.0500)	-0.3484

* The results in parentheses are from an isotropic plate, when c approaches infinity [16].

For future comparison, a fully clamped orthotropic plate, the same one proposed by Li et al. [2], is examined. Two sets of results are produced for the plate, also considering two types of load distribution: i) a uniform load of intensity q_0 and ii) linearly varying hydrostatic pressure with the intensity $q_1\,(x/a + 1/2)$, as shown in Tables VI and VII, respectively. In the present analysis, we use a relative high truncation orders ($N = 200$) for a sufficient accuracy. It is interesting to compare the calculated values corresponding to the limiting case ($c = \infty$) to such results obtained from an isotropic plate [16], and the agreement between the dimensionless results can be explicitly observed. Clearly, it follows that as the aspect ratio c approaches infinity, one-way beam bending (along the shorter side) predominates [4].

5. CONCLUSIONS

The generalized integral transform technique (GITT) has shown in this work to be adequate approach for the analysis of bending problem of a fully clamped orthotropic rectangular thin plate, providing an exact numerical-analytical solution for the transverse deflections, bending moments and transverse shear forces. The numerical results obtained are in excellent agreement with the previous results reported in the literatures [16, 17]. This approach can be either employed for benchmarking purposes, yielding sets of reference results with controlled accuracy or, alternatively, as an engineering simulation tool with lower truncation orders and exceptional computational performances. It is important to notice that the methodology developed is applicable to any boundary conditions, and may also to the analysis of bending problems of circular cylindrical shells and multilayer composite plates.

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REFERENCES

1. Meleshko, V. V. Bending of an elastic rectangular clamped plate: Exact versus ‘engineering’ solutions. *Journal of Elasticity.*, **Vol. 48 (1)**, pp. 1–50 (1997).

2. Li, R., Zhong, Y., Tian, B., Liu, Y. M. On the finite integral transform method for exact bending solutions of fully clamped orthotropic rectangular thin plates. *Applied Mathematics Letters.*, **Vol. 22 (12)**, pp. 1821–1827 (2009).

3. Zhong, Y., Li, R. Exact bending analysis of fully clamped rectangular thin plates subjected to arbitrary loads by new symplectic approach. *Mechanics Research Communications.*, **Vol. 36 (6)**, pp. 707–714 (2009).

4. Mbakogu, F. C., Pavlovic, M. N. Bending of clamped orthotropic rectangular plates: a variational symbolic solution. *Computers & Structures.*, **Vol. 77 (2)**, pp. 117–128 (2000).

5. Wolfram, S. *The Mathematica Book, 5th ed.* Wolfram Media/Cambridge University Press, Champaign, Illinois (2003).

6. Dalaei, M., Kerr, A. D. Analysis of clamped rectangular orthotropic plates subjected to a uniform lateral load. *International Journal of Mechanical Sciences.*, **Vol. 37 (5)**, pp. 527–535 (1995).

7. Aghdam, M. M., Falahatgar, S. R. Bending analysis of thick laminated plates using extended kantorovich method. *Composite Structures.*, **Vol. 62 (3-4)**, pp. 279–283 (2003).

8. Shen, P. C., He, P. X. Bending analysis of rectangular moderately thick plates using spline finite-element method. *Computers & Structures.*, **Vol. 54 (6)**, pp. 1023–1029 (1995).

9. Huang, J. Y. Uniformly valid asymptotic solutions of the nonlinear unsymmetrical bending for orthotropic rectangular thin plate of four clamped edges with variable thickness. *Applied Mathematics and Mechanics-English Edition.*, **Vol. 25 (7)**, pp. 817–826 (2004).

10. Civalek, O. Three-dimensional vibration, buckling and bending analyses of thick rectangular plates based on discrete singular convolution method. *International Journal of Mechanical Sciences.*, **Vol. 49 (6)**, pp. 752–765 (2007).

11. Cotta, R. M. *Integral Transforms in Computational Heat and Fluid Flow.* CRC Press, Boca Raton, Florida (1993).

12. Cotta, R. M., Mikhailov, M. D. *Heat Conduction - Lumped Analysis, Integral Transforms, Symbolic Computation.* John Wiley & Sons, Chichester, England (1997).

13. Cotta, R. M. *The Integral Transform Method in Thermal and Fluids Science and Engineering.* Begell House, New York (1998).

14. IMSL. *Fortran Library version 5.0, MATH/LIBRARY Volume 2.* Visual Numerics, Inc., Houston, Texas (2003).

15. Guerrero, J. S. P., Cotta, R. M. Benchmark integral transform results for flow over a backward-facing step. *Computers & Fluids.*, **Vol. 25 (5)**, pp. 527–540 (1996).

16. Timoshenko, S. P., Woinowsky-Krieger, S. W. *Theory of Plates and Shells.* McGraw-Hill, New York (1959).

17. Wojtaszak, I. A. The calculation of maximum deflection, moment, and shear for uniformly loaded rectangular plates with clamped edges. *Journal of Applied Mechanics.*, **Vol. 4**, pp. 173–176 (1937).

18. Szilard, R. *Theory and Analysis of Plates: Classical and Numerical Methods.* Prentice-Hall, New Jersey (1974).