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Evidence of Wigner Rotation Phenomena in the Beam Splitting Experiment at the LCLS

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Abstract

A well-known result from particle tracking states that, after a microbunched electron beam is kicked, its trajectory changes while the orientation of the microbunching wavefront remains as before. Experiments at the LCLS showed the surprising effect that radiation in the kicked direction is produced practically without suppression. This fact could be explained if, in contrast with conventional understanding, the orientation of the microbunching wavefront is readjusted along the new direction of the electron beam. In previous papers we presented a kinematical description of the experiment. We showed that when the evolution of the electron beam modulation is treated according to relativistic kinematics, the orientation of the microbunching wavefront in the ultrarelativistic asymptotic is always perpendicular to the electron beam velocity. In the interest of keeping discussion as simple as possible, in these papers we refrained from using more advanced theoretical concepts to explain or analyze the wavefront rotation more elegantly or concisely. For example, in our previous explanations we only hinted to the relation of this phenomenon with the concept of Wigner rotation. However, this more abstract view of wavefront rotation underlines its elementary nature. To most physicists, the Wigner rotation is known as a fundamental effect in elementary particle physics. As is known, the composition of non collinear boosts does not result in a simple boost but, rather, in a Lorentz transformation involving a boost and a rotation, the Wigner rotation. Here we show that during the LCLS experiments, a Wigner rotation was actually directly recorded for the first time with a ultrarelativistic, macroscopic object. In fact, an ultrarelativistic electron bunch in an XFEL modulated at nm-scale is a macroscopic (in the 10 μm-scale), finite-size object. The purpose of this paper is to point out the very important role that Wigner rotation plays in the analysis and interpretation of experiments with ultrarelativistic, microbunched electron beams in FELs. After the beam splitting experiment at the LCLS it became clear that, in the ultrarelativistic asymptotic, the projection of the microbunching wave vector onto the beam velocity is a Lorentz invariant, similar to the helicity in particle physics.

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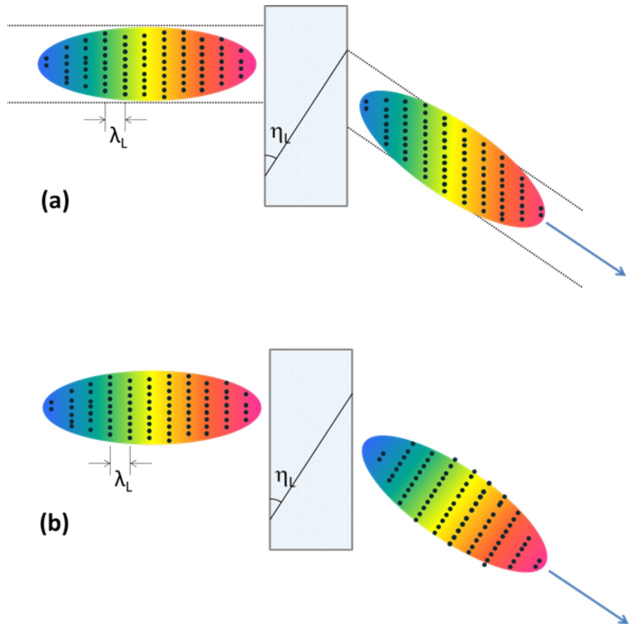

Fig. 1. Illustration of the problem, which arises according to classical particle tracking when a microbunched electron beam is deflected by a dipole magnet by an angle η_L. (a) According to particle tracking results, after passing the dipole the microbunching is preserved, but only along its original direction. (b) Using a different convention the orientation of the microbunching wavefront is always perpendicular to the electron beam velocity.

1 Introduction

A recent "beam splitting" experiment at the LCLS [1] apparently demonstrated that after a microbunched electron beam is kicked on a large angle compared to the divergence of the FEL radiation, the microbunching wavefront is readjusted along the new direction of motion of the kicked beam. Therefore, coherent radiation from the undulator placed after the kicker is emitted along the kicked direction without suppression.

Computers allow for simulations of the "beam splitting" setup, but the emission of coherent undulator radiation in the kicked direction cannot be explained with the help of the existing beam physics codes: the experimental result is in strong qualitative disagreement with simulations. Particle tracking codes predict that, after an electron beam is kicked, there is a change in the trajectory of the electron beam, while the orientation of the microbunching wavefront remains as before. In other words, the kick results in a difference between directions of the electron motion and of the microbunching wave vector, Fig. 1 (a).

In XFEL simulations it is generally accepted that coherent radiation from an undulator placed after the kicker is emitted along the normal to the microbunching wavefront, i.e along the microbunching wave vector. Therefore, when the angular kick exceeds the divergence of the output radiation, emission in the direction of the electron beam motion is strongly suppressed. However, the results of the "beam splitting" experiment at the LCLS, demonstrated that even the direction of emission of coherent undulator radiation is beyond the predictive power of conventional theory.

In our previous papers [2, 3] we explained this puzzle. We showed that the orientation of the microbunching wavefront in an XFEL is related to a conventional choice. The microbunching wavefront can be considered as a plane of simultaneous events, but deciding what events are simultaneous is only a matter of convention. In fact, we cannot give any experimental method by which simultaneity between two events in different places can be ascertained.

Maxwell's theory is usually treated under the standard time order, which is based on the use of clocks in uniform motion with respect to the laboratory frame and synchronized by light-signals. In contrast to this, particle dynamics is usually treated under a different time order, which is based on the use of clocks at rest with respect to the laboratory frame and synchronized by light-signals. This essential point has never received attention in the beam physics community. We showed [3] that a "translation" of particle tracking results to the electromagnetic world picture predicts a surprising effect, in complete contrast to conventional treatment. Namely, in the ultrarelativistic asymptotic, the orientation of the plane of simultaneity (that is the microbunchhing wavefront) is always perpendicular to the beam velocity, Fig. 1 (b). This effect explains the production of coherent undulator radiation from a modulated electron beam in the kicked direction without the strong suppression predicted by conventional theory.

In this paper we present an alternative path to explain the wavefront rotation. In the interest of keeping discussion as simple as possible, in our previous work [2, 3] we have refrained from using advanced theoretical concepts to explain or analyze the wavefront rotation phenomenon more elegantly, or concisely. In particular, the relation between wavefront rotation and Wigner rotation [5, 6, 7] is only hinted to in the previous explanations. To most physicists, the Wigner rotation is known as a fundamental effect in elementary particle physics. As is known, the composition of non-collinear Lorentz boosts does not results in a different boost but in a Lorentz transformation involving a boost and a spatial rotation, the Wigner rotation.

The rotation of the microbunching wave vector after beam kicking is one concrete physical example of Wigner rotation, which can be directly recorded in experiments with ultrarelativistic macroscopic objects. In fact, an ultrarelativistic electron bunch modulated at nanometer-scale in XFELs has a macroscopic finite-size of order of 10 μm. We hold the recent beam-splitting experiment at the LCLS as a direct experimental evidence that the microbunching wave vector actually underwent Wigner rotation after the electron beam was kicked by a large angle with respect to the radiation divergence, limited only by the beamline aperture. Such effect is fundamental and it may be of immediate practical importance in the analysis and interpretation of experiments with ultrarelativistic microbunched electron beams at FELs.

In this paper we show that if the velocity of our modulated electron bunch is close to the velocity of light, Lorentz transformations work out in such a way that the rotation angle of the microbunching wave vector coincides with the angle of rotation of the velocity. In this case, the value of the wave vector along the direction of the bunch motion is a Lorentz invariant. The wave vector of a laser pulse behaves precisely in the same way: during the motion along a curvilinear trajectory, the wave vector of the radiation is always aligned with the direction of motion of the laser pulse. It follows from the previous reasoning that in the large momentum (or zero mass) limit, whatever we know about the kinematics of a laser pulse can immediately be applied to an ultrarelativistic modulated electron bunch. In [4] we accounted for time dependent effects due to finite bunch duration, and demonstrated that it is possible to solve the kinematical problem for a modulated ultrarelativistic electron bunch subject to a kick by direct use of the kinematics of a laser pulse.

In his famous articles [5, 6, 7] Wigner developed a method that enables calculating the change in spin orientation of an elementary particle under a stepwise change in its velocity vector, the so-called Wigner rotation. We actually demonstrated that the description of microbunching wavefront rotation can be reduced to a Wigner rotation, and that the microbunching wave vector behaves in the same way as a particle spin during the motion along a curvilinear trajectory. Wigner demonstrated that, in the large momentum limit, the spin rotation angle coincides with the velocity rotation angle. In this case the projection of the spin along the direction of the particle motion, that is the helicity, is a Lorentz invariant. We note that, as discussed above, the spin of photons behaves precisely in the same way during the motion along a curvilinear trajectory [8].

Due to the Wigner rotation, a spinning mass moving with relativistic velocity exhibits the well known Thomas precession [9]. Wigner rotation and Thomas precession are closely related to each other and have a purely kinematical origin. The relation between them caused several misunderstandings in literature. Mainly two different expressions for the Thomas precession can be found in literature and there is a serious discrepancy between the results of different works. In [10] it is emphasized that in no way can two different expressions for the Thomas precession lead to the same result in the same frame of reference. The aim of review [10] is to analyze the complicated situation related to the Thomas precession and elucidate which of the expressions in literature is correct. According to [10] the correct expression for the evolution of the spin of a particle in the laboratory frame, in the ultrarelativistic asymptotic, is always such that the spin is aligned with the direction of the particle motion, i.e. in the ultrarelativistic asymptotic helicity is a Lorentz invariant [11]. According to the incorrect (in the view of [10]) but standard solution, the spin rotates instead in the opposite direction and, asymptotically, helicity is not a Lorentz invariant [12]. Usually, in defense of the standard expression it is pointed out that precise measurements of the magnetic-moment anomaly of a highly relativistic electron obey the BMT equation [13], of which Thomas precession is part. Therefore, these measurements can be taken as indirect experimental confirmations of the correctness of the standard expression [14]. In contrast with the existing literature, we voice the opinion that both expressions for Thomas precession are correct in the same (for example the laboratory) frame of reference, with the understanding that they are correctly interpreted. Having a spin rotation in the laboratory frame, or not, is similar to the case with a microbunching wavefront rotation: both situations are closely associated with the conventionality of simultaneity.

Whenever we have a theory containing an arbitrary convention, we should examine what parts of the theory depend on the choice of that convention and what parts do not. We may call the former convention-dependent, and the latter convention-invariant part. Clearly, physically meaningful results must be convention-invariant. We state that in the case of Thomas rotation, the Thomas angle of rotation is convention-dependent and has no direct physical meaning: with a suitable clocks synchronization convention, this term can be eliminated. In other words, different expressions for Thomas rotation in a single (e.g. the laboratory) reference system can be different only because they are based on the use of different synchronization conventions. Let us consider a spinning particle under the action of force, which changes its translational motion. The question arises as what happens to the direction of its angular momentum in this case. If there is no torque, then the particle is spinning freely and the direction of its angular momentum should not change. However, Lorentz transformations are not commutative in the general case, and what happens to the direction of the angular momentum depends on the choice of a convention. According to the Wigner convention, there is no rotation of the angular momentum in the non-inertial rest frame connected with the particle. In this case, in the laboratory frame helicity is Lorentz invariant in the ultrarelativistic asymptotic. According to the standard convention used in the description of Thomas precession instead, there is no rotation of the angular momentum in the laboratory frame [15]. Under such convention there is Thomas precession of the angular momentum in the rest frame. In other words, whether the angular momentum rotates or not in the laboratory frame is matter of convention.

It is well known that the Thomas precession is closely associated to the relativity of simultaneity. However, the relativity of simultaneity associated to Thomas precession should not be confused with the conventional nature of distant simultaneity in a single inertial frame, which is related, instead to the choice of convention outlined before: even within a single reference frame the definition of simultaneity of events is, in fact, a matter of convention. In our case of interest the beam radiation problem is simplified in the convention where there is no wavefront rotation in the rest frame, but the wavefront is readjusted in the laboratory frame, whereas, in the standard convention underlying particle tracking, there is no wavefront rotation in the laboratory frame, and the laws of electrodynamics loose their symmetry. This does not mean at all that the standard conventional choice of synchronization underlying particle tracking (or Thomas precession) is a-priory inconvenient. As a matter of fact, the most suitable formalism is usually suggested by the problem itself.

2 The covariant equation for the microbunching wave vector

As discussed in the introduction, in our previous papers [2, 3] we refrained from using advanced theoretical concepts to explain or analyze the puzzling results of the beam splitting experiment at the LCLS [1]. In this section we present a different but equivalent derivation for explaining the apparent wavefront readjusting. In particular, we start with the concept of Fermi-Walker parallel transport [17], applicable to accelerated "non-rotating" vectors in flat space-time to arrive at the wavefront rotation by yet another path.

In order to fix the stage, let us call the metric tensor in Minkowski space-time with g. Components of g in the laboratory frame, and in any other inertial frame are indicated by $g^{\alpha\beta} = g_{\alpha\beta} = \mathrm{Diag}(-1, 1, 1, 1)$, with "Diag" the diagonal matrix.

Consider a gyroscope, which undergoes evolution under the action of forces applied to its center of mass (i.e. without additional torques). According to Newtonian pre-relativistic mechanics we expect the tip of the gyroscope to point in the same direction during the acceleration in the judgement of any inertial observer. This is also the case in the judgement of an accelerated frame, which does not undergo rotations. However, when one deals with relativistic mechanics, the axes of a moving frame are not, in general, parallel to those of a stationary frame, meaning that when we discuss about relativistic motion, the tip of our gyroscope, although ”non-rotating”, can change its direction in the judgement of an inertial observer. The Fermi-Walker transport [17] codifies mathematically the concept of ”non-rotation” for any four-vector K evolving along the proper time τ with four-velocity V under the four-acceleration A:

$$\frac{dK}{d\tau} = \frac{1}{c^2}\left[(A \cdot K)\, V - (V \cdot K) A\right] \tag{1}$$

Consider now a microbunched, ultrarelativistic electron beam, moving with constant velocity along the z axis in the laboratory frame. In first approximation it can be described in terms of a charge density

$$\rho_L(t, \vec{r}) = \rho_0\{1 + a\mathrm{Re}[f(t, \vec{r})]\}\ , \tag{2}$$

where $f(t, \vec{r}) = \exp[i(\omega_L t - \vec{k}_L \cdot \vec{r})]\}$ and a is the modulation level. The function f is just a plane wave propagating along z with velocity v_L and with frequency $\omega_L = 2\pi v_L/\lambda_L$. Here λ_L is the modulation wavelength in the laboratory frame, and the suffix L indicates the laboratory. The wave-vector is easily found as $\vec{k}_L = (0, 0, \omega_L/v_L)$. Our plane wave describes both microbunching orientation and wavelength; in Minkowski space it is characterized by the four wave-vector K, whose components in the laboratory frame are $K_L^\alpha = (\omega_L/c, \vec{k}_L) = (\omega_L/c, 0, 0, \omega_L/v_L)$.

Note that since the modulus of K is, in any inertial frame, $K_L^\alpha K_{L\alpha} = (4\pi^2/\lambda_L^2)/\gamma_L^2 > 0$, with γ_L the relativistic Lorentz factor in the laboratory frame, we have that K is a space-like vector which tends to a null-like vector for $\gamma_L \longrightarrow \infty$. Also note that if this were an electromagnetic wave, $K^\alpha K_\alpha = 0$ and K would always be a null-like vector. In the rest-frame, the components of K become $K_R^\alpha = (0, 0, 0, 2\pi/\lambda_R) = (0, 0, 0, 2\pi/(\lambda_L\gamma_L))$, with R referring to quantities in the rest frame, while those of V become $V_R^\alpha = (c, 0, 0, 0)$. Finally we note that the four-vector K is orthogonal to the four-vector V in any frame, since $K_L^\alpha V_{L\alpha} = \gamma_L(-\omega_L + \vec{k}_L \cdot \vec{v}_L) = 0$. Similarly, in the rest frame of the electron beam $\omega_R = 0$ and $\vec{k}_R = (0, 0, 2\pi/\lambda_R) = (0, 0, 2\pi\gamma_L/\lambda_R)$, so that the components of K in the rest frame are $K_R^\alpha = (0, \vec{k}_R) = (0, 0, 0, 2\pi/\lambda_R)$. By definition of rest frame, the four velocity V has, instead, components $V_R^\alpha = (c, 0, 0, 0)$, so that, consistently with what we found in the laboratory frame, $K \cdot V = 0$.

The process of coherent radiation production can be pictured in the rest frame as in Fig. 2. The period of the microbunching increases of a factor γ_L from λ_L to $\lambda_R = \gamma_L\lambda_R$. Moreover, in the ultrarelativistic limit and for a small undulator parameter K_w [1], the undulator magnetic field is approximatively

[1] In order to understand the physical principles clearly, we take the limit for a

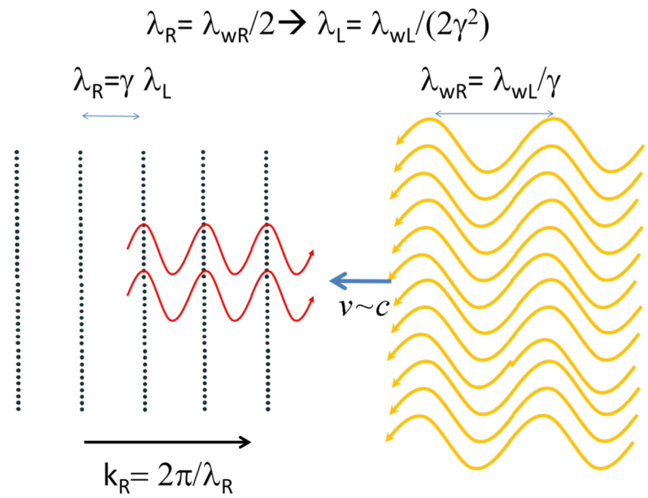

Fig. 2. Viewed from the rest frame, the process of coherent radiation production from a microbunched electron beam in an undulator can be seen as elastic scattering of a plane wave (resulting from the Lorentz-transformation of the magnetic field of the undulator) from a volume grating.

transformed, in the rest frame, into an electromagnetic plane wave with wavelength $\lambda_{wR} = \lambda_{wL}/\gamma_L$, where λ_{wL} is the period of the undulator in the laboratory frame. Therefore, the emission of coherent radiation can be seen as elastic scattering of the normal incident plane wave from a volume grating constituted by the electron charge density. The Bragg condition simply reads $\lambda_R = \lambda_{wR}/2$, which reduces to the usual resonance condition $\lambda_L = \lambda_{wL}/(2\gamma_L^2)$ in the laboratory frame for $K_w \ll 1$.

Here we will be interested in small values of kick angle $\eta_L \ll 1$ in the laboratory frame, and the effect that we want to discuss is related with the rotation of $\vec{k}_L$ of an angle η_L. Therefore, we can safely neglect second order effects in η_L^2 and the complexities related with the acceleration process. Since during the kick there is no torque and $K \cdot V = 0$, we can consider the evolution of K as a function of the proper time τ as that given in Eq. (1), with the simplification $K \cdot V = 0$. The situation is mathematically identical, at least for $\eta_L \ll 1$, to that of a spin four-vector S, which also obeys Eq. (1) under the exact constraint $S \cdot V = 0$ *for any value of* τ. Our microbunched beam is thus formally identical to the spin of an ultrarelativistic particle. In this case, the right hand side of Eq. (1) only includes the term in $K \cdot A$:

$$\frac{dK}{d\tau} = \frac{1}{c^2}(A \cdot K)\, V \tag{3}$$

small undulator parameter. Nevertheless, our considerations remain valid for any value of K_w, if the undulator is helical. In this way we can avoid the complications related with a planar undulator at arbitrary values of K_w.

Using again the fact that $\eta_L \ll 1$, we see that we can approximatively drop the term in $A \cdot K$ in Eq. (3). This can be shown noting that in the laboratory frame the acceleration at the end of the bend is $\vec{a}_{L1} = \vec{a}_L(\cos\eta_L, 0, \sin\eta_L)$ with $A^\alpha_{L1} = (0, \gamma_L^2 \vec{a}_{L1})$. Therefore $K \cdot A \sim \eta_L$. Hence, the change in the laboratory-frame components of K due to a bend of angle η_L is of order η_L^2 and $A \cdot K$ can be dropped as well. We conclude that our evolution equation for K can be approximated with:

$$\frac{dK}{d\tau} = 0 \tag{4}$$

We proceed showing that different conventional choices are compatible with Eq. (4). Similarly to a gauge theory, these choices will not change the final physical result of our investigations. However, one must be consistent in fixing a convention, and in working with it until the end.

2.1 *First convention: $\vec{k}_L$ is constant*

Let us first enforce the results from usual particle tracking. We then have that $\vec{k}_L$, the wave vector in the laboratory frame, remains constant (again, up to second order corrections in η_L). Unsurprisingly, this constraint is consistent with Eq. (4), and viceversa Eq. (4) with the constraint of no evolution of $\vec{k}_L$ confirms the particle tracking results in the laboratory frame:

$$\begin{aligned} &\frac{d\vec{k}_L}{d\tau} = 0 \\ &\frac{dk^0_L}{d\tau} = \frac{1}{c}\frac{d\omega_L}{d\tau} = 0 \end{aligned} \tag{5}$$

If we are interested in the rest frame instead we should use the transformation between laboratory frame L and rest frame R:

$$\begin{aligned} &\vec{k}_L = \vec{k}_R + \frac{\gamma^2}{\gamma+1}(\vec{\beta}\cdot\vec{k}_R)\vec{\beta} \\ &k^0_L = \gamma\vec{\beta}\cdot\vec{k}_R \end{aligned} \tag{6}$$

Here $\vec{\beta} = \vec{v}/c$. Substituting Eq. (6) into Eq. (5) we obtain:

$$
\frac{d\vec{k}_R}{d\tau} + \frac{\gamma^2}{\gamma+1}\frac{d(\vec{\beta}\cdot\vec{k}_R)}{d\tau}\vec{\beta} + \frac{\gamma^2}{\gamma+1}(\vec{\beta}\cdot\vec{k}_R)\frac{d\vec{\beta}}{d\tau} = 0
$$
$$
\gamma\frac{d(\vec{\beta}\cdot\vec{k}_R)}{d\tau} = 0 \tag{7}
$$

that can be combined into the single equation

$$
\frac{d\vec{k}_R}{d\tau} = -\frac{\gamma^2}{\gamma+1}(\vec{\beta}\cdot\vec{k}_R)\frac{d\vec{\beta}}{d\tau} \tag{8}
$$

Here $d\vec{\beta}$ is the change in the relative velocity between the rest and the laboratory frame, $\vec{\beta}$, during the proper-time increment $d\tau$.

Since $A\cdot K = 0$, we have $0 = A\cdot K = -a_R^0 k_R^0 + \vec{a}_R\cdot\vec{k}_R = \vec{a}_R\cdot\vec{k}_R$. Therefore Eq. (8) can also be written as

$$
\frac{d\vec{k}_R}{d\tau} = \frac{\gamma^2}{\gamma+1}\left[\left(\vec{k}_R\cdot\frac{d\vec{\beta}}{d\tau}\right)\vec{\beta} - (\vec{k}_R\cdot\vec{\beta})\frac{d\vec{\beta}}{d\tau}\right] \tag{9}
$$

We now use the relation $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$ to obtain:

$$
\frac{d\vec{k}_R}{d\tau} = \vec{k}_R\times\left(\frac{\gamma^2}{\gamma+1}\vec{\beta}\times\frac{d\vec{\beta}}{d\tau}\right) \tag{10}
$$

Eq. (10) is analogous to the expression for the Thomas precession usually found in literature for the spin in the rest frame, $\vec{S}_R$, and can be cast in the exact expression in literature in terms of laboratory-frame quantities by substituting $\vec{k}_R$ with $\vec{S}_R$ and by using the relation $dt = \gamma_L d\tau$, with t the time in the laboratory frame [12]:

$$
\frac{d\vec{S}_R}{dt} = \vec{S}_R\times\left(\frac{\gamma^2}{\gamma+1}\vec{\beta}\times\frac{d\vec{\beta}}{dt}\right) \tag{11}
$$

It should be noted, however, that after the substitution $dt = \gamma_L d\tau$, all the quantities in Eq. (11) with the exception of $\vec{S}_R$ refer to the laboratory frame, while $\vec{S}_R$ is measured by the observers in the rest frame. Therefore, this equation is in fact a precession equation neither with respect laboratory frame nor with respect to the rest frame. Mathematically, it is nevertheless absolutely correct [16].

We derived Eq. (10) from the Fermi-Walker transport, Eq. (1), for the four-vector K. In our approximation for $\eta_L \ll 1$ it behaves exactly as a particle axial four-spin and is a space-like vector orthogonal to the 4-velocity V. When $V \cdot K = 0$, the Fermi-Walker transport simplifies to the Fermi-transport in Eq. (4). In agreement with [15] we find that the Thomas precession is not due to the remaining kinematical term $(A \cdot K)V$ in Eq. (4). In fact, for our particular motion we approximated $A \cdot K = 0$ as well. As pointed out in [15], the Thomas precession is actually due to the transformation law between $\vec{k}_L$ and $\vec{k}_R$, Eq. (6). Eq. (10) tells us that if we impose the results from particle tracking, $\vec{k}_L$ in the laboratory frame, which is completely analogous to a particle spin in the laboratory frame, does not rotate. This is obvious due to the conditions imposed by particle tracking, see Eq. (5). At the same time, $\vec{k}_R$ undergoes Thomas precession in the rest frame, which is obtained, instant after instant, by a simple boost of the laboratory frame. Such precession, as well known in literature, has opposite direction compared to the rotation of the velocity in the laboratory frame.

2.2 *Second convention: $\vec{k}_R$ is constant*

Let us now set aside particle tracking results for a moment. If we impose that in the rest frame $\vec{k}_R$ remains constant up to the second order in the rotation angle $\eta_L = \eta_R/\gamma_L$, then we still approximatively have $A \cdot K = 0$ for any τ. And then Eq. (4) is still valid. This is a very important point, because it shows that our system can obey Eq. (4) with two apparently very different constraint: $\vec{k}_L$ = constant, which we studied previously, and $\vec{k}_R$ = constant. By this we see that choosing $\vec{k}_L$ = constant or $\vec{k}_R$ = constant is merely a matter of convention, while the evolution equation is Eq. (4) in both cases. Let us explore the consequence of our second choice, that in the rest frame $\vec{k}_R$ remains orthogonal to $\vec{a}_R$. Obviously, Eq. (5) is not anymore valid. This is unsurprising, because we set the evolution of $\vec{k}_R$ automatically with our conventional choice so that

$$\frac{d\vec{k}_R}{d\tau} = 0 \tag{12}$$

while the fourth component of K in the rest-frame is and remains zero by definition of rest frame. If we are interested in the laboratory frame instead we should use the transformation between rest R and laboratory frame L, that is the inverse of Eq. (6). This is given by

$$\vec{k}_R = \vec{k}_L - \frac{\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{k}_L)\vec{\beta} \tag{13}$$

where again the fourth component of K in the rest-frame is simply zero. Substituting Eq. (13) into Eq. (12) we obtain

$$\frac{d\vec{k}_L}{d\tau} - \frac{\gamma}{\gamma+1}\frac{d(\vec{\beta}\cdot\vec{k}_L)}{\mathrm{d}\tau}\vec{\beta} - \frac{\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{k}_L)\frac{d\vec{\beta}}{d\tau} = 0 \tag{14}$$

Since $K_L^\alpha V_{L\alpha} = \gamma(-\omega_L + \vec{k}_L\cdot\vec{v}_L) = 0$, it follows that $d(\vec{\beta}\cdot\vec{k}_L)/d\tau = 0$. Therefore

$$\frac{d\vec{k}_L}{d\tau} = \frac{\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{k}_L)\frac{d\vec{\beta}}{d\tau} \tag{15}$$

Here as before $d\vec{\beta}$ is the change in the relative velocity between the rest and the lab frame, $\vec{\beta}$, in the proper-time increment $d\tau$.

Similarly as before, since $A\cdot K = 0$, we have $0 = A\cdot K = -a_L^0 k_L^0 + \vec{a}_L\cdot\vec{k}_L = \vec{a}_L\cdot\vec{k}_L$. Therefore Eq. (8) can also be written as

$$\frac{d\vec{k}_L}{d\tau} = -\frac{\gamma}{\gamma+1}\left[\left(\vec{k}_L\cdot\frac{d\vec{\beta}}{d\tau}\right)\vec{\beta} - (\vec{k}_L\cdot\vec{\beta})\frac{d\vec{\beta}}{d\tau}\right] \tag{16}$$

Using once more the relation $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$ we obtain:

$$\frac{d\vec{k}_L}{d\tau} = -\vec{k}_L\times\left(\frac{\gamma}{\gamma+1}\vec{\beta}\times\frac{d\vec{\beta}}{d\tau}\right) \tag{17}$$

Eq. (17) tells us that if we impose that in the rest frame $\vec{k}_R$ remains orthogonal to $\vec{a}_R$, then the microbunching wavefront must rotate in the laboratory frame. In particular, Eq. (17) tells us that our vector $\vec{k}_L$ undergoes a Wigner rotation in the laboratory frame as

$$\frac{d\vec{k}_L}{dt} = \omega_W \times \vec{k}_L \tag{18}$$

with

$$\omega_W = \left(1 - \frac{1}{\gamma_L}\right)\Omega_L \tag{19}$$

where Ω_L is the constant angular velocity during the kick. Therefore, after a kick of an angle η we expect a rotation of the $\vec{k}_L$ vector given by

$$\delta\Phi = \left(1 - \frac{1}{\gamma_L}\right)\eta_L \tag{20}$$

From Eq. (17) follows that, in the ultrarelativistic limit $\gamma_L \longrightarrow \infty$, wave vector $\vec{k}_L$ rotates exactly as the velocity vector $\vec{v}_L$. This fact is confirmed by a comparison between Eq. (17) and Eq. (10). They differ from a sign and a γ_L factor. Thus, in the ultra-relativistic limit, the projection of the microbunching wave vector onto the beam velocity is Lorentz invariant similar to the helicity in particle physics.

Finally, it should be noted that, since the vector K should conserve its norm and since $\vec{k}_L$ undergoes only a rotation, we have

$$\frac{dk_L^0}{dt} = 0 \; . \tag{21}$$

We should stress once more that the two constraints studied above only refer to conventional choices, which do not alter the physical reality described by the evolution equation for K in four dimensions. In fact Eq. (4), which is derived from the Fermi-Walker transport including the approximate physical constraints of orthogonality of K with A and V, is satisfied in both cases when $\vec{k}_L$ remains orthogonal to $\vec{a}_L$ or when $\vec{k}_R$ remains orthogonal to $\vec{a}_R$ up to the second order in the rotation angle of the velocity η_L. In other words, the evolution of K does not change. However, the choice of one of the two conventions implies a different evolution of $\vec{k}_L$ and $\vec{k}_R$ compared to the other choice. The physical reason for this apparent contradiction lies in the very reason of existence of Wigner rotation. When we define a "rest frame" of the electrons at any time τ we do have a certain amount of arbitrariness. By definition, a "rest frame" is a frame where the electrons are instantaneously at rest. During the acceleration process such a frame is a function of τ. For a particle with $A = 0$, its evolution in space-time is just a straight line, which coincides with the direction of the time axis. Therefore, for an accelerated particle, we must impose that the time axis must be chosen such that the three-dimensional spatial coordinates lie in the three-dimensional hyperplane orthogonal to V. This, however, fixes the direction of the spatial coordinates system up to a rotation, which is arbitrary and does not include physical information. This explains why the description of the evolution of the spatial part of K in the rest frame can be different, while the four-dimensional evolution of K is, consistently, unvaried. Therefore, Eq. (12) shows no rotation of $\vec{k}_R$ in the rest frame, while Eq. (10) shows a rotation of $\vec{k}_R$ in the rest frame, while in both cases Eq. (4) is satisfied. It should also be noted that different conventions not only pertain the definition of rest frame, in our case, but also the definition of the laboratory frame. Laboratory and rest frames are, in fact, related by Lorentz boosts (depending on τ) so that we have results in Eq. (12) or (10), depending on our choice of convention.

3 Discussion and Conclusions

The microbunching wavefront can be considered as a plane of simultaneous events. Establishing simultaneous events is only a matter of convention and the orientation of the microbunching wave vector of an ultrarelativistic electron beam has no definite objective meaning. In contrast to this, the direction of emission of coherent radiation from a microbunched electron beam obviously has a direct objective meaning. In particle tracking, the only relativistic equation necessary for the description of the electron beam is $d\vec{p}/dt = \vec{F}$, where $\vec{p} = m\gamma\vec{v}$. In other words, in order to describe the dynamical processes in the relativistic electron beam, it is sufficient to take into account the relativistic dependence of the electron momentum on the velocity. According to conventional particle tracking, a kick along the x axis is equivalent to a Galilean coordinate transformation as $x' = x - v_x t$. This transformation is completed with the invariance of simultaneity; in other words, if two electrons arrive simultaneously at a certain position z down the beam, i.e. $\Delta t = 0$, then after the transformation above the same two electrons reach position $z' = z$ once more simultaneously, i.e. $\Delta t' = 0$. The absolute character of temporal simultaneity between two events is a consequence of the identity $t' = t$. As a result of the kick, the transformation of the time and spatial coordinates of any event has the form of a Galilean boost rather than a Lorentz boost. However, this evolution of the electron beam under the Galilean boost poses a problem when dealing with Maxwell's equations. In fact, the d'Alembertian, which enters in basic equations of electromagnetism, is not a Galilean invariant.

There are two possible ways of coupling fields and particles in this situation. The first, Lorentz's way, consists in a "translation" of Maxwell's electrodynamics to the particle tracking world-picture. The second, Einstein's way, consists viceversa in a "translation" of particle tracking results to the electromagnetic world picture using relativistic kinematics for the description of the electron beam evolution. It is necessary to mention that in the case of the beam splitting experiment at the LCLS we deal indeed with an ultra relativistic electron beam ($c - v \simeq 10^{-8}c$), and with transverse velocity after the kick which is very much smaller than speed of light ($v_x/c < 10^{-4}$), so that the first approximation over the parameter v_x/c yields a correct quantitative description. This greatly simplified description allows for both the above-described ways of coupling fields and particles at small angle of velocity rotation to be applied with the same ease.

Let us consider the first "translation" of the d'Alembertian to particle tracking (absolute time) world picture. After properly transforming the d'Alembertian we can see that the inhomogeneous wave equation for the electric field in the laboratory frame after the kick has nearly but not quite the usual, standard form that takes when there is no common, uniform translation of electrons in the transverse direction with velocity v_x. The main difference consists in the "interference" term $\partial^2/\partial t \partial x$ which arises when applying our Galileo boost. The discussion can be simplified by the use of a mathematical trick, without direct solution of the modified wave equation. Lorentz found that the solution of the electrodynamic problem under the absolute-time convention can be obtained with minimal efforts by formally desynchronizing the absolute time (which Lorentz called the "true" time) t to the "local" time $t' = t - xv_x/c^2$ and using t' without changing the d'Alembertian [18]. It is immediately seen by direct calculations that a shift of time is what is needed in order to the eliminate interference term. The effect of this time transformation is just a dislocation in the timing of processes. This transformation has the effect of rotating of the wave front on the angle v_x/c, in accordance with the experimental results of the LCLS beam splitting experiment.

Let us now consider the second "translation". Production of coherent radiation in the kicked direction can in fact be also explained on the basis of relativistic kinematics, when the evolution of our microbunched electron beam is treated under the Einstein's time order. On the one hand, it is well known that the wave equation remains form-invariant with respect to Lorentz transformations. On the other hand, if we make a Lorentz boost in the x direction to describe the kick in the laboratory frame, we automatically introduce the "local" time $t' = t - xv_x/c^2$ and the effect of this transformation is just a wavefront rotation. In other words, in the first order in v_x/c, the Galilean transformation described above, completed by the introduction of the "local" time, is mathematically equivalent to the Lorentz transformation just described here: it does not matter which convention and hence transformation or "translation" is used to describe the same reality.

We note that even in the non-relativistic limit, when we can neglect second order corrections in v_x/c, which are intrinsically relativistic, Lorentz and Galileo transformations are different. The difference is in the term xv_x/c^2 in the Lorentz transformation for time, which is a first order correction. Yet we underline that this term is only conventional and has no direct physical meaning. In other word, differences that arise between Galilean and Lorentz transformations in the non-relativistic limit are only to be ascribed to the use of different synchronization conventions.

Since the formulation of special relativity, most researchers assume that Lorentz transformations immediately follow from the postulates of the theory of relativity. However these postulates alone are not sufficient to obtain Lorentz transformations: one additionally needs to synchronize spatially separated moving clocks with the help of light signals. If this done using the Einstein's synchronization convention, Lorentz transformation follow. However, if the same clocks are synchronized following a different synchronization convention, other transformations are obtained. In order to get a Galilean transformation, we synchronize clocks in the rest system with the usual Einstein procedure involving light signals. Then, since we want to perform measurements in an inertial frame moving e.g. with velocity v_x with respect to the rest system, it is necessary to synchronize the moving clocks. This can be done with the help of the previously synchronized clocks at rest without involving light signals, by adjusting the moving clocks to zero whenever they fly past a clock at rest that shows zero as well [20].

We emphasize that many debates around the expression for the rotation of a particle spin [10, 19] are strictly related to the problem of wavefront rotation of a microbunched electron beam, and are based on the deep-rooted, subjective belief in the existence of an absolute wavefront or spin orientation in the laboratory frame. In the absolute time convention underlying particle tracking or spin tracking in accelerators, there is no wavefront or spin rotation in the laboratory frame after a kick. This absence of spin or wavefront rotation is orthodoxly accepted as physical reality because this is a result of the dynamical evolution (tracking) in laboratory frame. This is no mistake and this approach keeps being consistent. However, using this convention, the laws of field theory loose their symmetry. As a result, first, one needs to properly transform the field equations before coupling them with particle sources in order to obtain correct results and, second, given the complicated equations resulting from this process, its usefulness may be questioned. Very adequate fields-particles coupling can be made, instead, within the Wigner approach with the help of Lorentz boosts from the rest frame where there is no wavefront, nor spin rotation. Following this approach, it is not difficult to derive that in the large momentum limit the microbunching wave vector or spin is always aligned with the direction of the particle motion. This convention has a fundamental advantage: within its framework, field theory equations can be used in their usual standard form.

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