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SIMULATION OF LOCAL INSTABILITIES WITH THE USE OF REDUCED ORDER MODELS

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ABSTRACT

The development of an advanced reduced order model (ROM) with four heated channels, taking into account local, regional and core-wide oscillations, is described. The ROM contains three sub-models: a neutron-kinetic model (describing neutron transport), a thermal-hydraulic model (describing the coolant flow) and a heat transfer model (describing heat transfer between the fuel and the coolant). All these three models are coupled to each other, using two feedback mechanisms: void feedback and doppler feedback. Each of the sub-models is described by a set of reduced ordinary differential equations, derived from the corresponding time space-dependent partial differential equations by using different types of approximations and mathematical techniques. All three models were developed from past ROMs and, subsequently, were modified in order to fit the purpose of our investigations. One of the novelties of the present ROM is that it takes into account the effect of the first three neutronic modes, namely the fundamental, the first and the second azimuthal modes, as well as the effect of local oscillations on these modes.
In order to have a proper representation of both azimuthal modes, a four heated channel ROM was developed. Another modification, compared to earlier work, is the determination of the coupling reactivity coefficients for both void fraction and fuel temperature, which were calculated explicitly by evaluating cross-section perturbations with the help of the SIMULATE-3 and the CORESIM codes.
The ROM was thereafter applied to a channel instability event that occurred at the Swedish Forsmark-1 BWR in 1996/1997. The time signals for each of the modes were generated from the ROM and compared with the measurements, performed at the plant. Some qualitative comparison between the ROM and the measurements was made. The results could bear some significance in understanding the instability event and its coupling mechanism to core-wide oscillations.

Key Words: BWR instability, local perturbation, reduced order model, density wave oscillations, decay ratio.

1. INTRODUCTION

In conceptual considerations of Boiling Water Reactors (BWRs), it is often plausible and sufficient to use linear methods, namely to investigate oscillations of the system parameters around the equilibrium point to get an idea of the stability properties of the core. Such a type of analysis is valid as long as the operational point is a hyperbolic fixed point. This methodology has become more and more popular in reactor safety analysis over the last years. Despite the coarse simplicity of the model, it gives quite accurate and significant results and does not demand any complicated numerical calculations. Moreover, in most cases it provides a good intuition and understanding of the physical phenomena which drive the instability in BWRs. The crucial point of such approaches is the assumption of the linearity of the system which dramatically simplifies the whole model. Thus, in such models, any feedback effect which inevitably leads to the non-linear character of the system, is automatically excluded.

However, as it was shown in many papers, there are many cases when the assumption of the linear behavior of the system cannot be applied and leads to completely erroneous results (non-hyperbolic fixed points). In such specific cases, one needs to take into account the fact that the dynamics of the real BWR system is non-linear due to the strong coupling between different phenomena. One such example is the case of large amplitude oscillations when one cannot describe the system response only with linear terms and thus, higher order terms should be included as well. Another argument for the non-linear description of the system comes from the fact that it yields a better understanding of the dependence of the stability properties of the system on different operational parameters as well as the various mechanisms, which induce even more pathological instabilities.

Moreover, at this point, it is worth mentioning that there are mainly two difficulties in the study of non-linear systems which make the application of the linear approach impossible. The first difficulty is a conceptual one, namely it is not possible to split the induced neutron noise as the product between the system transfer function, which depends on the equilibrium parameters, and the deviation of the system parameters from the steady-state (unperturbed) values. This concept is considered to be the base of the linear stability analysis. The second difficulty is a practical one and related to the fact that it is not always possible to get full analytical solution of the complex strongly coupled and inhomogeneous systems. In such cases, the use of system codes could be an option. But in reality, the structure of these system codes is quite complicated and involves thousands of variables which makes it impossible to get any insight into the physical mechanisms of the BWR stability behavior. An alternative way is then to simplify the description of the system both from the neutron-kinetic and thermal-hydraulic point of view in such a way that it still reproduces the main features of the BWR system.

Such kind of models are usually referred to as Reduced Order Models. The main idea of the ROM modeling is to reduce the general partial differential equations, which describes the reactor behavior, to simplified ordinary differential equations, eliminating the complexity of the system and providing deeper understanding of the physical phenomena which drive instabilities in BWRs.

Over the past few years, a number of various ROMs were developed and applied to investigation of different instability events. Some of these models are very simple, others are quite advanced. The simplest ROM represents a one-dimensional heated channel with constant heat generation.

Such a model is quite useful in modeling so-called self-sustained Density Wave Oscillations (DWOs). Another more advanced prototype of ROM includes a point-kinetic module to simulate non-constant power generation as well as feedback effects [1]. It is obvious that such a model is able to reconstruct only the so-called global instabilities, which are related to the fundamental mode. Since quite some time ago, it was demonstrated that in addition to the global instability phenomena, the so-called regional instability can occur, which is usually associated with one of the azimuthal (radial) modes. As a consequence, another prototype of ROM, containing two thermal-hydraulic channels with one of the azimuthal modes included, was developed [2, 3]. While this model can indeed study two modes (fundamental + one of the azimuthal modes) at the same time, in real BWRs both azimuthal modes exist simultaneously and their joint behavior carries also some important information about the system and its stability properties. Thus, one of the goals of the present ROM is to include the effect of both azimuthal modes, as well as the fundamental one, to simulate and study different interesting phenomena, in particular the time-dependent rotation of the symmetry line between two radial modes (which was observed in many measurements). It is obvious that in order to get a proper description of both the first and the second azimuthal modes, one needs to construct a ROM with at least four heated channels and such a development work was undertaken and is reported hereafter [4].

Another important case from the BWR stability point of view is when two spacial oscillation modes with different stability properties but with close frequencies coexist, for instance, a local and a global oscillation modes or two local ones. In particular, it was shown that the DR as a single global indicator fails [5]. It is thus desirable to investigate such local instabilities with some simplified models like a ROM in order to get deeper insight into the physical mechanisms which induce and govern such instability phenomena. Hence, another particular goal of the current ROM development was to take into account the effect of local perturbations combined with the core-wide perturbation, thus giving an opportunity to study and investigate more realistic instability events.

As an event of interest for ROM analysis, we chose the Forsmark-1 channel instability event that took place at $P = 63.3\%$ of the nominal power and $F = 39.1\%$ of the nominal flow in 1996/1997 [6]. This instability event was presumably created by one or several unseated fuel bundles, causing self-sustained Density Wave Oscillations (DWOs) and, hence, induced local power oscillations. One of the reasons to choose this event as an example comes from the interesting phenomena which were observed in this event, namely the strong space-dependence of the decay ratio and the oscillating symmetry line between the azimuthal modes. Another reason is related to the nature of the instability itself in this event, namely local oscillations which were triggered by self-sustained density wave oscillations.

To this end, a four heated channel ROM was developed in order to introduce the effect of both azimuthal modes. Furthermore, the local oscillations were introduced into the ROM (in a simple manner) in order to simulate the effect of self-sustained DWOs. Finally, some characteristic results of the simulation are discussed and some possible continuation for this investigation is suggested.

2. DEVELOPMENT OF THE FOUR CHANNEL ROM

The brief description of three sub-models which were implemented in our ROM development will be given. Firstly, the reduction procedure which allows one to transform the 3D space time-dependent two-group diffusion equations into the point kinetic time-dependent equations is presented. Only the first three modes, namely the fundamental, the first and the second azimuthal modes, are taken into consideration. Secondly, we reduce the general energy balance equations for the fuel rod heat conduction to ordinary differential equations, assuming two piecewise quadratic spatial approximations for the fuel pellet temperature and applying the variational principle approach [1]. Finally, we derive the reduced Ordinary Differential Equations (ODEs) for the two coolant phase regions (single- and two-phase regions). We start from the general flow cross-sectional averaged balance equations, assuming proper spatial quadratic distribution for the enthalpy and quality, respectively, and then, apply the variational method [1]. In the following, only the main features of the different models are presented. The full derivation of all models can be found in [4].

2.1 Neutron kinetic model

As is customary in reactor physics, we start with the three-dimensional spacial time-dependent two-energy group diffusion equations, written in the operator (matrix) form as:

$$\bar{\bar{v}}^{-1} \cdot \frac{\partial \bar{\Psi}(\bar{r},t)}{\partial t} = [(1-\beta)\cdot \bar{\bar{F}}(\bar{r},t) - \bar{\bar{L}}(\bar{r},t)] \cdot \bar{\Psi}(\bar{r},t) + \sum_{l=1}^{6} \lambda_l \cdot C_l(\bar{r},t)\cdot \bar{X}_l, \tag{1}$$

$$\frac{\partial C_l(\bar{r},t)}{\partial t}\bar{X}_l = \beta_l \bar{\bar{F}}(\bar{r},t)\cdot \bar{\Psi}(\bar{r},t) - \lambda_l \cdot C_l(\bar{r},t)\bar{X}_l, \tag{2}$$

where

- $\bar{X}_l$ is a unit vector, i.e. $\bar{X}_l = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.
- $\bar{\Psi}(\bar{r},t)$ is the neutron flux vector which consists of two components: the fast flux $\bar{\Psi}_1(\bar{r},t)$ and the thermal flux $\bar{\Psi}_2(\bar{r},t)$.
- $\bar{\bar{L}}(\bar{r},t)$ is the net loss matrix operator, which represents neutron leakage through diffusion, scattering and absorption.
- $\bar{\bar{F}}(\bar{r},t)$ is the fission production matrix operator, which represents the neutron production through fission reactions.
- $\bar{\bar{v}}^{-1}$ is the inverse velocity matrix operator, i.e. $\bar{\bar{v}}^{-1} = \begin{pmatrix} \frac{1}{v_1} & 0 \\ 0 & \frac{1}{v_2} \end{pmatrix}$.
- $C_l(\bar{r},t)$ is the concentration of the delayed neutron precursors for the lth group.

Other notations are standard. The boundary conditions are defined as $\bar{\Psi}(\bar{r}_B,t) = 0$ where $\bar{r}_B$ is a point at the extrapolated boundary. Next, we apply first order perturbation theory, namely

considering the small fluctuations around the critical state for the neutron flux and the precursor density i.e.

$$\bar{\Psi}(\bar{r},t) = \bar{\varphi}_0(\bar{r}) + \delta\bar{\Psi}(\bar{r},t), \tag{3}$$

$$C_l(\bar{r},t)\cdot\bar{X}_l = (C_{0,l}(\bar{r}) + \delta C_l(\bar{r},t))\cdot\bar{X}_l, \tag{4}$$

are given rise by the fluctuations of the fission $\bar{\bar{F}}(\bar{r},t)$ and loss $\bar{\bar{L}}(\bar{r},t)$ cross section operators around their static values:

$$\bar{\bar{F}}(\bar{r},t) = \bar{\bar{F}}_0(\bar{r}) + \delta\bar{\bar{F}}(\bar{r},t), \tag{5}$$

$$\bar{\bar{L}}(\bar{r},t) = \bar{\bar{L}}_0(\bar{r}) + \delta\bar{\bar{L}}(\bar{r},t). \tag{6}$$

Further, we expand both the space time-dependent oscillating part of the neutron flux vector $\delta\bar{\Psi}(\bar{r},t)$, as well as the space time-dependent oscillating part of the concentration of the delayed neutron precursors $\delta C_l(\bar{r},t)\cdot\bar{X}_l$ in terms of lambda (reactivity) modes as:

$$\delta\bar{\Psi}(\bar{r},t) = \sum_{n=0}^{\infty}\bar{\bar{P}}_n(t)\bar{\Phi}_n(\bar{r}), \tag{7}$$

$$\delta C_l(\bar{r},t)\cdot\bar{X}_l = \sum_{n=0}^{\infty}C_{nl}(t)\bar{\bar{F}}_0\bar{\Phi}_n(\bar{r})\Lambda_n. \tag{8}$$

In the above, Λ_n is the prompt neutron generation time, $\bar{\Phi}_0(\bar{r}) = \bar{\varphi}_0(\bar{r})$ is the static flux and $\bar{\Phi}_n(\bar{r})$ is the eigenvector, satisfying the following λ eigenvalue problem:

$$\frac{1}{k_n}\bar{\bar{F}}_0(\bar{r})\bar{\Phi}_n(\bar{r}) = \bar{\bar{L}}_0(\bar{r})\bar{\Phi}_n(\bar{r}), \tag{9}$$

where $n = 1, ..., \infty$ and $\lambda_n = 1/k_n$ is the corresponding eigenvalue.

Substituting Eqs. (3)-(8) into Eqs. (1) and (2), multiplying the resulting equation by the adjoint eigenmode $\bar{\Phi}^{\dagger}_m(\bar{r})$, utilizing equation (9), assuming one group of delayed neutrons and taking into account only the first three modes, after integration and some rearrangements, one gets the following point-kinetic equations:

$$\frac{dP_0(t)}{dt} = \frac{1}{\Lambda_0}(\rho^s_0 - \beta)P_0(t) + \frac{1}{\Lambda_0}\sum_{n=1}^{3}\rho^F_{0n}P_n(t) + \lambda C_0(t), \tag{10}$$

$$\frac{dC_0(t)}{dt} = \frac{\beta}{\Lambda_0}P_0(t) - \lambda C_0(t), \tag{11}$$

$$\frac{dP_1(t)}{dt} = \frac{1}{\Lambda_1}(\rho^s_1 - \beta)P_1(t) + \frac{1}{\Lambda_1}\sum_{n=1}^{3}\rho^F_{1n}P_n(t) + \lambda C_1(t), \tag{12}$$

$$\frac{dC_1(t)}{dt} = \frac{\beta}{\Lambda_1}P_2(t) - \lambda C_1(t), \tag{13}$$

$$\frac{dP_2(t)}{dt} = \frac{1}{\Lambda_2}(\rho^s_2 - \beta)P_2(t) + \frac{1}{\Lambda_2}\sum_{n=1}^{3}\rho^F_{2n}P_n(t) + \lambda C_2(t), \tag{14}$$

$$\frac{dC_2(t)}{dt} = \frac{\beta}{\Lambda_2} P_1(t) - \lambda C_2(t), \tag{15}$$

where ρ_s is the static reactivity, ρ^F_{mn} are the dynamic feedback reactivities. In the last equations, we utilized the fact that $\Lambda_m = \Lambda_{mm} >> \Lambda_{mn}$ for $n \neq m$ and neglected the contribution of delayed feedback reactivities.

The dynamic feedback reactivities ρ^F_{mn} reflect the feedback mechanism between the neutron kinetics and thermal-hydraulics in terms of void fraction/fuel temperature. In the linear approximation, the feedback reactivities for both void fraction and fuel temperature can be expressed as:

$$\rho^F_{mn}(t) = \rho^V_{mn}(t)|_{T_f(t)=const}| + \rho^D_{mn}(t)|_{\alpha(t)=const}| = \sum_{l=1}^{4} C^V_{mn,l}(\alpha_l(t) - \alpha_0) + \sum_{l=1}^{4} C^D_{mn,l}(T_{f,l}(t) - T_{f0}), \tag{16}$$

where l stands for the channel number and α_0 and T_{f0} are the steady state void fraction and fuel temperature, correspondingly.

2.2 Heat transfer model

For the heat conduction model, we start with the general three-dimensional time space-dependent energy balance equation, written for a single fuel rod as:

$$\rho c_p \frac{\partial}{\partial t} T(\bar{r}, t) = q'''(\bar{r}, t) - \bar{\nabla} \cdot \bar{q}''(\bar{r}, t), \tag{17}$$

where ρ is the density of the fuel rod, c_p is the specific heat of the fuel rod under constant pressure, $q'''(\bar{r}, t)$ is the volumetric heat production per unit time and per unit fuel rod volume, and $\bar{q}''(\bar{r}, t)$ is the heat flux.

Furthermore, neglecting the axial heat conduction, assuming azimuthal symmetry in the radial direction and taking into account three radial regions in the fuel rod (namely: fuel pellet, gap and cladding), the one-dimensional heat conduction equation for the fuel rod in dimensionless form is written as:

- for $0 < r < r_p$

$$\frac{1}{\alpha_p} \frac{\partial \Theta_p(r,t)}{\partial t} = \frac{\partial^2 \Theta_p(r,t)}{\partial r}) + \frac{1}{r} \frac{\partial \Theta_{p,}(r,t)}{\partial r} + q'''(t), \tag{18}$$

- for $r_g < r < r_c$

$$\frac{1}{\alpha_c} \frac{\partial \Theta_c(r,t)}{\partial t} = \frac{\partial^2 \Theta_c(r,t)}{\partial r} + \frac{1}{r} \frac{\partial \Theta_{p,j\phi}(r,t)}{\partial r}, \tag{19}$$

where r_p, r_g and r_c are the outer pellet, gap and clad radii, respectively; $\Theta_p(r,t)$ and $\Theta_c(r,t)$ are the dimensionless temperature deviations from the steady-state values in the fuel pellet and cladding, respectively; $q'''(t)$ is the dimensionless power density, produced in the fuel, and α_p, α_c are the pellet and cladding diffusivities, correspondingly. Here, it should be pointed out that the heat balance equations (18)-(19) are given only for two fuel rod radial regions, namely the

pellet and the cladding while the gap is eliminated via properly adjusted boundary conditions. In addition, both equations should be written separately for the single-phase and two-phase regions. For the sake of simplicity, the equations will be kept generic for any of the two regions.

Next, we define the temperature profile for the cladding without solving the clad equation (19) directly. Utilizing the fact that the clad thermal diffusivity α_c is ten times larger than that of the fuel pellet α_p, one can assume that the dynamical clad temperature profile does not deviate much from its steady-state distribution. Thus, using the logarithmic spatial steady-state temperature distribution in the cladding, $\Theta_c(r,t)$ can be approximated in time and space as:

$$\Theta_c(r,t) = b_2(t)\log(r) + b_3(t), \tag{20}$$

where b_i, $i = 2,3$ are complicated coefficients which depend on the design, operational parameters and fuel pellet surface temperature $\Theta_p(r_p,t)$.

Then, we derive the ODEs for the fuel pellet temperature by reducing the corresponding PDEs via the variational method. Here, we assume that the fuel pellet temperature distribution can be approximated through two piece-wise quadratic spatial functions with time-dependent expansion coefficients, written as:

$$\Theta_p(r,t) = \begin{cases} T_1(t) + \eta_1(t)r + \eta_2(t)r^2 & \text{if} \quad 0 < r < r_d. \\ \\ T_2(t) + \sigma_1(t)r + \sigma_2(t)r^2 & \text{if} \quad r_d < r < r_p \end{cases} \tag{21}$$

where r_d stands for the point of discontinuity between $r = 0$ and $r = r_p$. The time-dependent expansion coefficients $\eta_i(t)$ and $\sigma_i(t)$, $i = 1,2$, can be expressed through $T_i(t)$, $i = 1,2$, and the design parameters, utilizing discontinuity and boundary conditions.

Finally, we apply the variational principle to derive the ODEs for the expansion coefficients $T_i(t)$. The basic idea behind the variational approach is to find the proper approximation for the function profile, which has a minimum error, compared with the exact solution. Thus, we get the following reduced differential equations, describing the fuel rod conduction dynamics:

$$\frac{dT_{1,l,j\phi}(t)}{dt} = p_{11,j\phi}T_{1,l,j\phi}(t) + p_{21,j\phi}T_{2,l,j\phi}(t) + p_{31,j\phi}[c_q(P_0(t) - \widetilde{P}_0(t)) + c_q\xi_1 P_1(t) + c_q\xi_2 P_2(t)], \tag{22}$$

$$\frac{dT_{2,l,j\phi}(t)}{dt} = p_{12,j\phi}T_{1,l,j\phi}(t) + p_{22,j\phi}T_{2,l,j\phi}(t) + p_{32,j\phi}[c_q(P_0(t) - \widetilde{P}_0(t)) + c_q\xi_1 P_1(t) + c_q\xi_2 P_2(t)], \tag{23}$$

where p_{ij} are complicated coefficients which depend on the design and operational parameters, $j\phi$ stands for the single (1ϕ) or two phase (2ϕ) regions, l is the channel number (varying between 1 and 4), and $\widetilde{P}_0$ is the steady-state value of the fundamental mode. It should be mentioned that in the last two equations we replace the power density $q'''(t)$ with the contribution from each of the three modes. This concludes our derivation of the ODEs for the heat conduction in one fuel rod and the results are given by equations (22)-(23), written for the time-dependent expansion coefficients of the fuel pellet temperature profile.

2.3 Thermal-hydraulic model

In this section, the description of the thermal-hydraulic model for our ROM is given. Since we assume two axial coolant regions, namely single-phase and two-phase regions, with a constant flow cross-sectional area, the description is performed for each of the two regions separately. We then demonstrate how to transform the PDEs, describing the thermal-hydraulic processes, into simplified ODEs, applying the variational method.

2.3.1 Single-phase region

We start with the three local conservation equations written for mass, momentum and enthalpy, respectively, as:

$$\frac{\partial \rho(\bar{r},t)}{\partial t} + \bar{\nabla}\cdot(\rho\bar{v})(\bar{r},t) = 0, \tag{24}$$

$$\frac{\partial(\rho\bar{v})}{\partial t}(\bar{r},t) + \bar{\nabla}\cdot(\rho\bar{v}\otimes\bar{v})(\bar{r},t) = \bar{\nabla}\cdot\bar{\bar{\tau}}(\bar{r},t) - \bar{\nabla}\cdot(P(\bar{r},t)\bar{\bar{I}}) + \rho(\bar{r},t)\bar{g}, \tag{25}$$

$$\rho(\bar{r},t)\frac{\partial h(\bar{r},t)}{\partial t} + (\rho\bar{v})(\bar{r},t)\cdot\bar{\nabla}h(\bar{r},t) =$$

$$-\bar{\nabla}\cdot\bar{q}^{''}(\bar{r},t) + q^{'''}(\bar{r},t) + \bar{\bar{\tau}}(\bar{r},t) : [\bar{\nabla}\otimes\bar{v}(\bar{r},t)] + \frac{\partial P(\bar{r},t)}{\partial t} + \bar{v}(\bar{r},t)\cdot\bar{\nabla}P(\bar{r},t) \tag{26}$$

where $\otimes$ stands for the tensor multiplication and $\bar{\bar{I}}$ is the unit tensor. Then, assuming the coolant flow mainly in the axial direction and neglecting the radial flow, after cross-section averaging, one gets the following simplified thermal-hydraulic balance equations, written in the dimensionless form as:

$$\frac{\partial v(z,t)}{\partial z} = 0, \tag{27}$$

leading to

$$v(z,t) = v(t) = v_{inlet}(t) \tag{28}$$

and

$$\frac{\partial h(z,t)}{\partial t} + v_{inlet}(t)\frac{\partial h(z,t)}{\partial z} = N_\rho N_r N_{pch,1\phi}, \tag{29}$$

$$\frac{\partial P_{1\phi}(z,t)}{\partial z} = -\frac{dv_{inlet}(t)}{dt} - \frac{1}{Fr} - N_{f,1\phi}v_{inlet}^2. \tag{30}$$

Here, $N_{f,\phi}$,N_r and N_ρ are dimensionless numbers, Fr is the Froude number and $N_{pch,1\phi}(t)$ is the time-dependent phase change number in the single phase region which is proportional to the wall heat flux $q_{1\phi}^{''}$. Assuming the following space time-dependent quadratic approximation for the single phase enthalpy profile $h(z,t)$:

$$h(z,t) \approx h_N(z,t) = h(0,t) + p_1(t)z + p_2(t)z^2, \tag{31}$$

replacing $h(z,t)$ in (29) via (31) and applying the variational method to the resulting equations, one gets the following ODEs for the time-dependent expansion coefficients for the single-phase enthalpy:

$$\frac{dp_{1,l}(t)}{dt} = \frac{6}{\mu_l(t)}[N_\rho N_r N_{pch,1\ phi,l}(t) - v_{inlet,l}(t)p_{1,n}(t)] - 2v_{inlet,l}(t)p_{2,l}(t), \tag{32}$$

$$\frac{dp_{2,l}(t)}{dt} = -\frac{6}{\mu_l^2(t)}[N_\rho N_r N_{pch,1\ phi,l}(t) - v_{inlet,l}(t)p_{1,l}(t)], \tag{33}$$

where $\mu(t)$ is the boiling boundary (the axial point in the reactor core where the boiling starts) for each heated channel.

2.3.2 Two-phase region

Similar to the single-phase region, we start with the three local conservation equations written for mass, momentum and enthalpy, for each phase, respectively, as:

$$\frac{\partial \rho_k(\bar{r},t)}{\partial t} + \bar{\nabla}\cdot(\rho_k \bar{v}_k)(\bar{r},t) = 0, \tag{34}$$

$$\frac{\partial(\rho_k \bar{v}_k)}{\partial t}(\bar{r},t) + \bar{\nabla}\cdot(\rho_k \bar{v}_k \otimes \bar{v}_k)(\bar{r},t) = \bar{\nabla}\cdot\bar{\bar{\tau}}(\bar{r},t) - \bar{\nabla}\cdot(P_k(\bar{r},t)\bar{\bar{I}}) + \rho_k(\bar{r},t)\bar{g}, \tag{35}$$

$$\rho_k(\bar{r},t)\frac{\partial h_k(\bar{r},t)}{\partial t} + (\rho_k \bar{v}_k)(\bar{r},t)\cdot\bar{\nabla}h_k(\bar{r},t) =$$

$$-\bar{\nabla}\cdot\bar{q}_k''(\bar{r},t) + q_k'''(\bar{,}t) + \bar{\bar{\tau}}_k(\bar{r},t) : [\bar{\nabla}\otimes\bar{v}_k(\bar{r},t)] + \frac{\partial P_k(\bar{r},t)}{\partial t} + \bar{v}_k(\bar{r},t)\cdot\bar{\nabla}P_k(\bar{r},t), \tag{36}$$

where $k = l, v$ stands for the coolant phase (l for the liquid phase and v for the vapor phase). Basically, the above equations should be solved for each phase separately. However, in order to reduce the number of equations, we use mixture quantities, namely we replace the single- and two-phase quantities with the two-phase averaged (mixture) quantities, performing cross-section averaging. Further, using the homogeneous equilibrium model (namely, both the liquid and vapor phases are assumed to be in equilibrium, and $v_l = v_v$), equations (34)-(36) can be rewritten in dimensionless form as:

$$\frac{\partial}{\partial t}\rho_m(z,t) + v_m(z,t)\frac{\partial \rho_m(z,t)}{\partial z} = -\rho_m(z,t)N_{pch,2\phi}(t), \tag{37}$$

$$\frac{\partial P_{2\phi}(z,t)}{\partial z} = -\rho_m(z,t)\frac{\partial v_m(z,t)}{\partial t}$$

$$-\rho_m(z,t)v_m(z,t)\frac{\partial v_m(z,t)}{\partial z} - \frac{1}{Fr} - N_{f,2\phi} * \rho_m(z,t)v_m^2(z,t), \tag{38}$$

$$\frac{\partial v_m(z,t)}{\partial z} = N_{pch,2\phi}(t) \tag{39}$$

where the index m stands for the mixture quantities, $P_{2\phi}(z,t)$ is the total pressure drop in the two-phase region, induced by acceleration (first two terms on the right hand side of second equation), gravity (third term) and friction (fourth term), $N_{pch,2\phi}(t)$ is the time-dependent phase change number in the two-phase region which is proportional to the wall heat flux $q_{2\phi}''$.

Next, assuming the following quadratic approximation for the space time-dependent profile of the quality:

$$x(z,t) \approx x_2(z,t) = N_\rho N_r(u_1(t)(z-\mu(t) + u_2(t)(z-\mu(t)^2), \tag{40}$$

and expressing the two-phase coolant density via quality as:

$$\rho_m(z,t) = \frac{N_\rho N_r}{N_\rho N_r + x(z,t)}, \tag{41}$$

after substituting Eqs. (40) and (41) into equation (37) and applying the variational method to the resulting equation, one gets the following ODE for the time-dependent expansion coefficients for the quality:

$$\frac{du_{1,l}(t)}{dt} = \frac{1}{f_{2,l}(t)}(f_{3,l}(t)f_{1,l}(t) + f_{4,l}(t)), \tag{42}$$

$$\frac{du_{2,l}(t)}{dt} = \frac{1}{f_{5,l}(t)}(f_{3,l}(t)f_{1,l}(t) + f_{6,l}(t)), \tag{43}$$

where $f_i(t)$, $i = 1...6$, and $M_j(t)$, $j = 2,3$ are complicated functions of time, depending on the design and operational parameters, as well as on phase variables, i.e. inlet velocity $v_{inlet}(t)$, pellet temperature time-dependent coefficients $T_i(t)$, $i = 1,2$, phase change number $N_{pch,2\phi}$, mixture density $\rho_m(t)$ and boiling boundary $\mu(t)$.

3. INTRODUCTION OF A LOCAL NOISE SOURCE INTO THE ROM MODEL

In the previous sections, we briefly described the four-channel ROM which was developed to study the combined effect of the first three modes. From a neutron-kinetic point-of-view, it becomes obvious that such a model can only reconstruct the system response induced by the global ("in-phase") or regional ("out-phase") or combined oscillations, but it is not capable to simulate any effect due to local perturbations, for instance, self-sustained DWOs. As many experiments show, this phenomenon, which is the main concern of our paper, should be modeled so that one can correctly estimate the stability properties and the margins to instability of the system, as well as gain physical insight. It was thus decided to introduce the effect of a self-sustained DWO into the ROM. We shall now give a short overview of the methodology which we applied to reconstruct local oscillations in our ROM.

Prior to present the modeling of the effect of a self-sustained DWO into the ROM, it is necessary to highlight the purpose of our investigations. In this work, we focus our attention to a self-sustained DWO, and its effect on the rest of the core. Namely, a self-sustained DWO will induce fluctuations of the neutron flux throughout the entire core. Such fluctuations will in turn induce changes of the macroscopic cross-sections throughout the entire core due to void fraction and fuel temperature fluctuations, which themselves result from the effect of the self-sustained DWO on the neutron flux. These cross-section fluctuations will themselves excite core-wide instabilities (global and regional). The model presented hereafter studies the effect of such a self-sustained DWO on the stability of the core.

In reality, the situation is much more involved. The core-wide response due to a self-sustained DWO also has some effect on the self-sustained DWO itself and it is not granted at all that the self-sustained DWO can be maintained (i.e. self-sustained).

In the investigations reported hereafter, this effect is neglected. This is equivalent to saying that the self-sustained DWO is artificially self-sustained. Not making this assumption would require

a system code modeling approach, which is exactly the type of the approach a ROM tries to avoid.

Combining Eqs. (1), (2) and (3)-(6) from the previous section and assuming one group of delayed neutrons, the equations for the fluctuating parts can be written as:

$$\bar{\bar{v}}^{-1} \cdot \frac{\partial \delta\bar{\Psi}(\bar{r},t)}{\partial t} = [(1-\beta)\cdot\delta\bar{\bar{F}}(\bar{r},t) - \delta\bar{\bar{L}}(\bar{r},t)]\cdot(\bar{\Phi}_0(\bar{r},t)+\delta\bar{\Psi}(\bar{r},t)) + \lambda\cdot(C_0(\bar{r})+\delta C(\bar{r},t))\cdot\bar{X}$$

$$+[(1-\beta)\cdot\bar{\bar{F}}_0(\bar{r},t) - \bar{\bar{L}}_0(\bar{r},t)]\cdot(\bar{\Phi}_0(\bar{r})+\delta\bar{\Psi}(\bar{r},t)), \tag{44}$$

$$\frac{\partial \delta C(\bar{r},t)}{\partial t}\bar{X} = \beta\delta\bar{\bar{F}}(\bar{r},t)\cdot(\bar{\Phi}_0(\bar{r})+\delta\bar{\Psi}(\bar{r},t)) - \lambda\cdot(C_0(\bar{r})+\delta C(\bar{r},t))\bar{X}$$

$$+\beta\bar{\bar{F}}_0(\bar{r})\cdot(\bar{\Phi}_0(\bar{r})+\delta\bar{\Psi}(\bar{r},t)). \tag{45}$$

Equations (44)-(45) describe the system response, induced by both a local and a global (core-wide) perturbations. Moreover, one can study the effect of each perturbation separately, using the same approach. That is achieved by introducing the corresponding noise sources into the cross-section perturbations $\delta\bar{\bar{F}}$ and $\delta\bar{\bar{L}}$. Thus, the right-hand side of Eq. (44) contains two types of sources: the global one (we call it "homogeneous") which excite the whole core and the local one (we call it "inhomogeneous" source) which only affects the channel where the self-sustained DWO exists.

The solution of this kind of problem can be found by first estimating the response of the system to a pure local oscillation (solving the inhomogeneous problem). Thereafter, one removes the effect of the local perturbation from equations (44)-(45) and solve the remaining equations, using the projection on different eigenmodes. As will be shown later on, such equations are identical with the ones we used in the previous section (neutron-kinetic model) and will provide the solution to core-wide perturbations. Nevertheless, in the present case, the effect of the local perturbation on the cross-sections throughout the entire core is accounted for. Thus, the general solution can be given as the superposition of the system responses to both a local and a core-wide perturbation.

Accordingly, we first split the perturbations in the material data, as well as the corresponding system response (perturbations in the neutron flux), into a "homogeneous" (i.e. core-wide oscillations) and "inhomogeneous" part (i.e. local oscillations) i.e.

$$\delta\bar{\bar{X}S}(\bar{r},t) = \delta\bar{\bar{X}S}^h(\bar{r},t) + \delta\bar{\bar{X}S}^i(\bar{r},t), \tag{46}$$

$$\delta\bar{\Psi}(\bar{r},t) = \delta\bar{\Psi}^h(\bar{r},t) + \delta\bar{\Psi}^i(\bar{r},t), \tag{47}$$

where i stands for inhomogeneous, h for homogeneous and $\bar{\bar{XS}} = \bar{\bar{F}}, \bar{\bar{L}}$. Thus, using the linearized version of Eqs. (44)-(45) for the local perturbations (i.e. neglecting second-order terms), one gets:

$$\bar{\bar{v}}^{-1} \cdot \frac{\partial \delta\bar{\Psi}^i(\bar{r},t)}{\partial t} = [(1-\beta)\cdot\delta\bar{\bar{F}}^i(\bar{r},t) - \delta\bar{\bar{L}}^i(\bar{r},t)]\cdot\bar{\Phi}_0(\bar{r},t) + \lambda\cdot\delta C^i(\bar{r},t)\cdot\bar{X} +$$

$$[(1-\beta)\cdot\bar{\bar{F}}_0(\bar{r},t) - \bar{\bar{L}}_0(\bar{r},t)]\cdot\delta\bar{\Psi}^i(\bar{r},t), \tag{48}$$

$$\frac{\partial \delta C^i(\bar{r},t)}{\partial t}\cdot\bar{X} = \beta\delta\bar{\bar{F}}^i(\bar{r},t)\cdot\bar{\Phi}_0(\bar{r},t) - \lambda\cdot\delta C^i(\bar{r},t)\bar{X} + \beta\bar{\bar{F}}_0(\bar{r})\cdot\delta\bar{\Psi}^i(\bar{r},t). \tag{49}$$

Assuming the following delta-function form of the local perturbations in the absorbtion cross section:

$$\delta\bar{\bar{X}}S^i(\bar{r},t) = \bar{\bar{A}}\exp^{\iota\omega t}\,\delta(\bar{r}-\bar{r}_0), \tag{50}$$

the general solution of Eqs.(48)-(49) can be written as:

$$\delta\bar{\Psi}^i(\bar{r},t) = \bar{\bar{B}}\exp^{\iota\omega t}\,\bar{\varphi}^i(\bar{r},\bar{r}_0,\omega), \tag{51}$$

$$\delta C^i(\bar{r},t) = \bar{\bar{C}}\exp^{\iota\omega t}\,\bar{\varphi}^i(\bar{r},\bar{r}_0,\omega), \tag{52}$$

where $\bar{r}_0$ stands for the location of the local cross section perturbation and $\bar{\bar{A}}$, $\bar{\bar{B}}$ and $\bar{\bar{C}}$ are matrix coefficients. Substituting Eqs. (51)-(52) into Eqs. (48)-(49) and eliminating the coefficient $\bar{\bar{C}}$ leads to:

$$\bar{\bar{v}}^{-1}\iota\omega\bar{\bar{B}}\bar{\varphi}^i(\bar{r}) - [(1-\frac{\iota\omega\beta}{\lambda+\iota\omega})\bar{\bar{F}}_0(\bar{r}) - \bar{\bar{L}}_0(\bar{r})]\bar{\bar{B}}\bar{\varphi}^i(\bar{r}) = [(1-\frac{\iota\omega\beta}{\lambda+\iota\omega})\delta\bar{\bar{F}}^i(\bar{r},t) - \delta\bar{\bar{L}}(\bar{r},t)]\bar{\Phi}_0(\bar{r}). \tag{53}$$

Solving this last equation for $\bar{\bar{B}}\bar{\varphi}^i(\bar{r})$ and combining this result with Eqs. (51) and (52), one gets the explicit solutions for the neutron flux $\delta\bar{\Psi}^i(\bar{r},t)$ and the concentration of the delayed neutron precursors $\delta C^i(\bar{r},t)$ of the inhomogeneous problem (48)-(49).

Next, we derive the equation for the homogeneous problem. Subtracting Eqs. (48)-(49) from Eqs. (44)-(45), respectively, taking into account Eqs. (46)-(47) and neglecting the cross-terms between the homogeneous and inhomogeneous problems due to the smallness, as well as the cross-products between the inhomogeneous perturbation and the corresponding inhomogeneous solution, one has:

$$\begin{aligned}\bar{\bar{v}}^{-1}\cdot\frac{\partial\delta\bar{\Psi}^h(\bar{r},t)}{\partial t} &= [(1-\beta)\cdot\delta\bar{\bar{F}}^h(\bar{r},t) - \delta\bar{\bar{L}}^h(\bar{r},t)]\cdot\delta\bar{\Psi}^h(\bar{r},t) + \lambda\cdot\delta C^h(\bar{r},t)\cdot\bar{X} \\ &\quad +[(1-\beta)\cdot\bar{\bar{F}}_0(\bar{r},t) - \bar{\bar{L}}_0(\bar{r},t)]\cdot\delta\bar{\Psi}^h(\bar{r},t),\end{aligned} \tag{54}$$

$$\frac{\partial\delta C^h(\bar{r},t)}{\partial t}\bar{X} = \beta\delta\bar{\bar{F}}^h(\bar{r},t)\cdot\delta\bar{\Psi}^h(\bar{r},t) - \lambda\cdot\delta C^h(\bar{r},t)\bar{X} + \beta\bar{\bar{F}}_0(\bar{r})\cdot\delta\bar{\Psi}^h(\bar{r},t), \tag{55}$$

where $\delta\bar{\Psi}^h(\bar{r},t)$ and $\delta C^h(\bar{r},t)$ are defined as :

$$\delta\bar{\Psi}^h(\bar{r},t) = \delta\bar{\Psi}(\bar{r},t) - \delta\bar{\Psi}^i(\bar{r},t),$$

$$\delta C^h(\bar{r},t) = \delta C(\bar{r},t) - \delta C^i(\bar{r},t),$$

which are exactly the same equations as in the previous section (neutron-kinetic model). It is important to emphasize here that the derivation of the homogeneous problem is only consistent with the derivation of the ROM neutron-kinetic model in a linear sense, since the above derivation neglects the cross-products between the homogeneous and inhomogeneous problems, as well as the cross-terms between the inhomogeneous perturbation and the corresponding inhomogeneous solution. As could be earlier noticed, the main strength of the presented methodology is its relative simplicity if linear theory is used. Further investigations are nevertheless necessary in order to find out the effect of the non-linear terms related to the inhomogeneous problem in the ROM.

Eqs. (54)-(55) are then solved by expanding the neutron flux $\delta\bar{\Psi}^h(\bar{r},t)$ and concentration of the delayed neutron precursors $\delta C^h(\bar{r},t)$ in terms of eigenfunctions in the same manner as was done

before. Thus, taking into account only the first three eigenmodes of the flux expansion, the full solution of Eqs. (44)-(45) is given as:

$$\delta\bar{\Psi}(\bar{r},t) = \sum_{n=0}^{2} P_n(t)\bar{\Phi}_n(\bar{r}) + \bar{\bar{B}}\exp^{\iota\omega t}\,\bar{\varphi}(\bar{r},\bar{r}_0,\omega) \tag{56}$$

Equations (54)-(55) clearly indicate that the introduction of the local oscillations into the model does not directly affect the form of the neutron-kinetic model. This means that one does not need to introduce any extra term directly into the neutron-kinetic part of the ROM. On the other hand, from the full solution (56) one can clearly see one extra term in the power oscillations which is due to the change of the core-wide cross-sections resulting from the neutron noise induced by a self-sustained DWO. Since the power oscillations strongly influence the fuel temperature, one needs to modify the heat-transfer model to get the correct representation of the local perturbations in the ROM. At this point, it is worth to point out that the local oscillations will induce a change of the power produced in each of four heated channels. These changes in the fuel temperature and of the resulting void production perturb the cross-sections (feedback effects) in $\delta\bar{\bar{F}}^h$ and $\delta\bar{\bar{L}}^h$.

4. ANALYSIS OF THE NUMERICAL RESULTS AND COMPARISON WITH THE PLANT MEASUREMENTS

The time-dependent amplitude factors for each of the three modes, namely the fundamental, the first and the second azimuthal modes, were calculated by performing the numerical integration of 42 ODEs. The cross-section data were extracted directly from the output file of the SIMULATE-3 code with input data corresponding to the Forsmark-1 instability event of 1996/1997. As for the design parameters, data typical for a BWR system were used. A large number of calculations were performed, out of which only some results will be given below. In the following, we will focus our attention to the qualitative comparison between the ROM simulated signals and the real measurements.

First, we consider the case without the introduction of any self-sustained DWO, namely the case when we have only a global instability. This is equivalent to the introduction of a global constant perturbation into the ROM at the beginning of the transient. The excitation of the system is done via inlet flow by adding small fluctuations to the inlet velocity for each of the four channels. Obviously, the system can be perturbed in any other way, but the asymptotic system response will be qualitatively the same. The results of the numerical integration for all three modes are given in Fig. 1 (left figure). Due to the smallness of the amplitudes of the azimuthal modes, they are demonstrated separately in Fig. 1 (right figure).

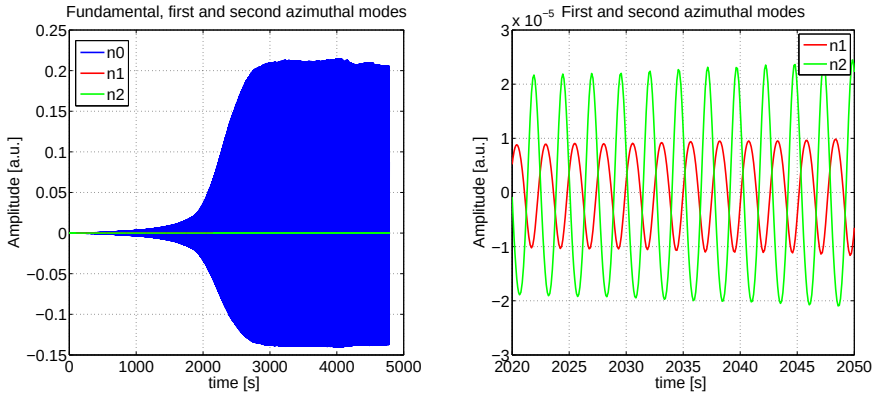

Figure 1: Time evolutions of the fundamental, first and second azimuthal modes as computed by the ROM.

As Fig. 1 (left figure) shows, for this specific instability event, the ROM gives an unstable fundamental mode. Thus, the stability behavior in the present ROM description is mainly driven by the fundamental mode reaching limit cycle conditions. Compared to the measurements, the "out-of-phase" oscillations should have much larger amplitudes. Moreover, from Fig. 1 (right figure), it is seen that the time-dependent rotation of the symmetry line between the two azimuthal modes, as was noticed in the measurements, is not reconstructed by the present ROM. In order to be consistent with earlier results provided by system code calculations [6] indicating the completely stable core behavior, we artificially stabilized the fundamental mode (by adjusting the ratio between the downcomer cross section and the heated channel cross section). From now on, this adjusted ROM is used in the analysis.

Next, we consider the case where the local noise source is included in the ROM, following the methodology described in the previous section. As earlier stated, one does not need to make any significant modification of the ROM, but rather to modify the heat-transfer model by adding some additional power source accounting for the local neutron source. The space-dependence of the neutron noise induced by the local perturbations was calculated by using the CORE SIM tool [7]. The real stability measurements, corresponding to this instability event, indicate that there are at least two local noise sources (two unseated fuel assemblies) present in the reactor core. Taking into account this information, we introduced two artificial noise sources with slightly different stability properties, but quite close frequencies into our ROM.

Some results of the ROM simulation for the case of two coexisting local perturbations are given in Figs. 2-4. In Fig. 2 (right figure), we show the time-dependent behavior of the total system response (including both the system and the source contributions) which were decomposed into the fundamental, the first and the second azimuthal modes. It has to be noted that there are several interesting features which can be observed in this case, compared to the previous one. One of them is that all three modes are oscillating and driven by the external local neutron sources. As a result, all three modes exhibit an unstable limit cycle behavior. From a physical point of view, it is more interesting to compare the relative weight between the noise source and its corresponding response. For this reason, in Fig. 2 (left figure), we demonstrate the time

evolution of the maximum amplitude of neutron noise.

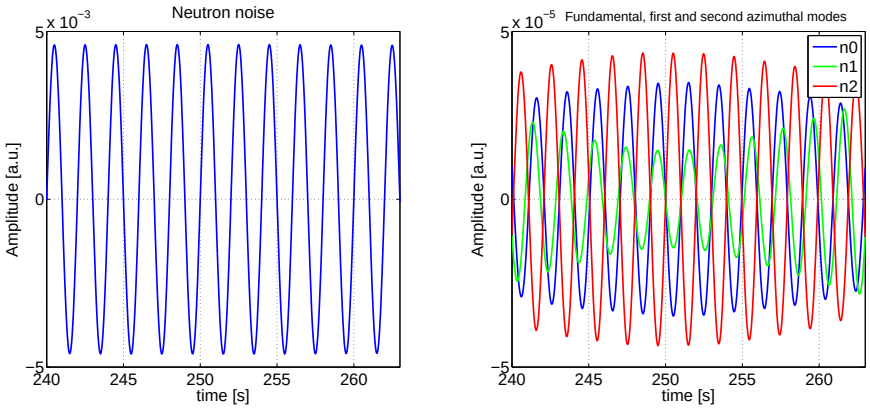

Figure 2: Time evolution of the local neutron noise (left figure) and the corresponding response, decomposed into the fundamental, the first and second azimuthal modes (right figure), as computed by the ROM.

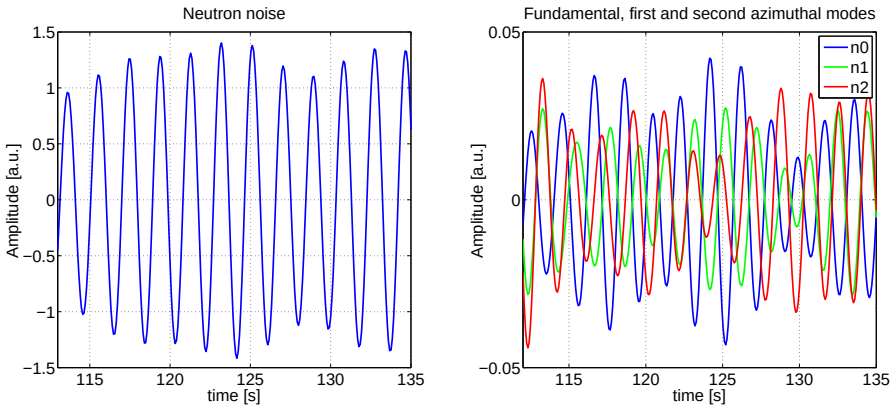

Figure 3: Time evolution of the local neutron noise (left figure) and the corresponding response (right figure), decomposed into the fundamental, the first and the second azimuthal modes, as computed from the measurements.

For the purpose of comparison, in Fig. 3, we demonstrate the time evolution of the first three modes, extracted from the real measurements by using a modal decomposition (right figure) together with neutron noise, measured in one radial location of the core (the location was chosen as the one corresponding to the largest noise amplitude) (left figure). It should be noted that all signals were pass-band filtered between 0.4 and 0.6 Hz, which corresponds to the frequency range of self-sustained DWOs. From Figs. 2-3, it is seen that both cases exhibit qualitatively a

similar behavior. Namely, all three modes are excited and oscillate with comparable amplitudes. Furthermore, the mode amplitudes have approximately the same relative strength compare to the total neutron noise. Another important feature, which is observed in the measurements and now is also reproduced by ROM simulation, is the oscillating symmetry line. Thus, the last figures give some indication that the ROM results are qualitatively similar to the ones provided by the measurements and thus can give some insight into the understanding of the local instabilities.

Fig. 4 further shows the pure system response (the source being excluded, right figure) and the pure source contribution (left figure). Both were decomposed into the first 3 modes. Comparing Figs. 2 and 4 , it is seen that the main contribution into the total system response comes from the local neutron source itself, rather than from the excited system modes (pure system response), driven by this local neutron source. This is explained by the fact that the system is inherently stable. Thus, one can conclude that in the case of Forsmark-1 local instability event, the main contribution into the total system response, induced by the local neutron source, comes primarily from the source itself and the system does not give any significant contribution.

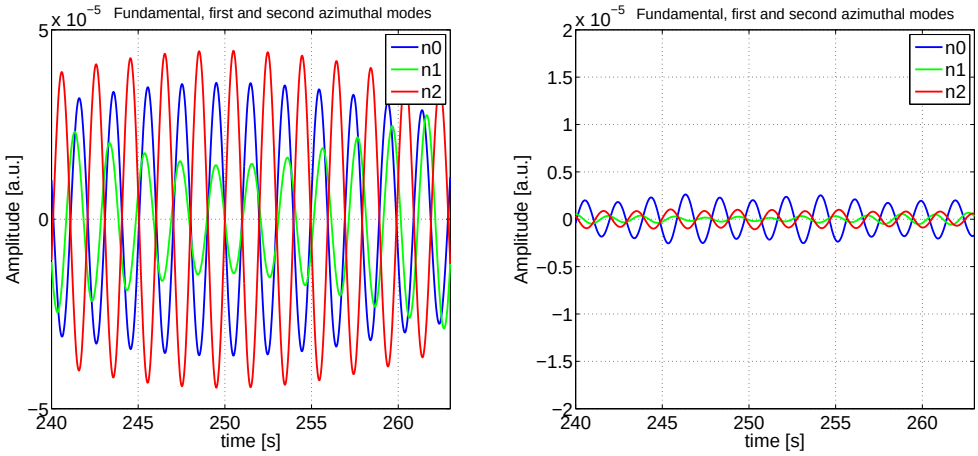

Figure 4: Time evolution of the neutron noise source (without any contribution from the system response, left figure) and the corresponding system response (local source being excluded, right figure), decomposed into the fundamental, the first and the second azimuthal modes, as computed by the ROM.

5. CONCLUSIONS

A four heated channel ROM, taking into account the effect of the fundamental, the first and the second azimuthal modes, was developed and applied to the stability analysis of the Forsmark-1 instability event of 1996/1997. The feedback reactivity coefficients were directly calculated from the cross-section data without any approximation. Furthermore, two local sources were introduced into the present ROM, to study the effect of local perturbations on the reactor core stability within the scope of ROM modeling. The results of the ROM simulations were compared with the decomposed measurement signals.

The main conclusion of this paper is that at least with the ROM chosen, in the case of the local instability event analyzed, the main contribution into the total system response comes from the local neutron source itself while the pure system contribution is quite small. This statement agrees with the fact that in all system code analysis, stable behavior of the reactor core was indicated. Another conclusion from the present ROM analysis is that the Forsmark-1 instability event of 1996/1997 is driven by an external local neutron noise source, induced by a self-sustained density wave oscillation.

This paper also demonstrates that a ROM can simulate not only core-wide instabilities, but also the effect of local ones in a simplified manner. Another interesting finding of this paper is that, with two external local sources, one can reconstruct some of the features observed in actual measurements. This gives some insight into the nature of local instabilities and their effect on core wide oscillations. However, some differences between the ROM results and the modal decomposition of the measurements data remain and thus require further investigations.

Further investigations are also necessary to identify the effects of the non-linear terms coming from the local noise source and that appear in the ROM, which is itself a non-linear model. Such terms have been neglected in the present study. The computational burden that the inclusion of such non-linear terms would imply would then have to be weighted against the simplicity of the model presented in the present paper. The modeling of the non-linear terms will be the subject of another paper.

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