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DESIGNING SHIELDS FOR KeV PHOTONS WITH GENETIC ALGORITHMS

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ABSTRACT

Shielding of x-ray sources and low energy gamma rays is often accomplished with lead aprons, comprising a thin layer (0.5 mm to 1 mm) of lead or similar high-Z material. In previous work the authors used Genetic Algorithms to explore the design of a shadow shield for space applications. Now those techniques have been applied to the problem of shielding humans from low energy gamma radiation. This paper uses a simple geometry to explore layering various materials as a method to reduce mass and dose for thin gamma shields. The genetic algorithms discover layers of materials with various Z is in fact more effective than an equivalent mass of Pb alone for lower energy gammas, but as the incident radiation energy increases the efficacy of such layering diminishes. The utility of varying Z for lower energy gammas is in part due to their complementary K-edges, where one material compensates for the transmission that would occur just below the K-edge in another material.

Key Words: Algorithms, Shielding, Radiation, Evolution.

1. INTRODUCTION

The design of a wearable shield is really a multi-objective function, trading off mass with dose reduction effectiveness. Radiation workers are generally willing to trade dose for mobility, but only up to a point. Our goal in this paper is to look for non-obvious ways that a more effective shield can be built without an increase in mass. Some recent work on optimizing non-Pb shields for x-rays by McCaffrey, et.al.[1] point in an interesting direction. McCaffrey and colleagues investigated a number of bi-layer shielding configurations for x-ray sources with energies under 100 kVp. Through both simulation and measurement they found that using a low-Z/high-Z ordering produced a shield that was superior in dose reduction compared to an equivalent mass of lead (of 0.5 mm thickness).

Our goal in this paper is two fold. First, to see if Genetic Algorithms, GA, can match the results from McCaffrey's physical experiments [1] and ad-hoc search for bi-layers. Second to see if GA can provide even better shielding at these low photon energies. Hopefully, because of the non-biased search that Genetic Algorithms provide, we can find un-expected shield designs.

2. THE PROBLEM SPACE

Our low level gamma shielding investigations are all performed using MCNP on a simple slab geometry, as shown in Fig. 1. At $z = -T - 0.1$ there is a plane that contains a circular source of

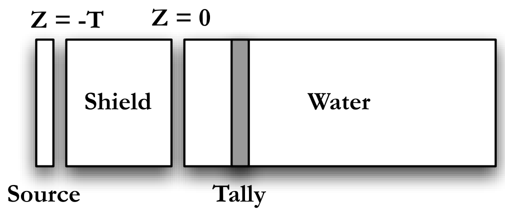

Figure 1: The geometry for the MCNP simulations.

Table I: Low Energy Gamma Shielding Materials

Material	Density (gm/cm^3)	Z
tungsten	19.25	74
Lead	11.342	82
Bismuth	9.78	83
Tin	7.365	50
antimony	6.697	51
Aluminum	2.7	13

radius 5 cm centered at $(5, 5)$. Starting at $z = 0$ and continuing to the full shield thickness, T, in the negative z direction there are a series of layers. The number of layers will depend on the GA parameters. A dose tally (F6) is computed averaged over a cell starting 0.95 cm past the shield, and continuing for 0.1 cm. This tally is used to calculate the deep dose equivalent, DDE. Before and after the tally, up to 25.05 cm the space is filled with water. This water phantom provides a model for the body of a person. This water may reflect or scatter radiation back into our primary tally. The x and y boundaries at 0 and 10 cm are reflective, creating a slab geometry. The z boundaries are both vacuum boundaries.

The shielding materials we choose are listed in Table I. These materials were chosen to include the materials from McCaffrey's physical experiments [1] and to then provide a reasonable spread from low-Z to high-Z values. All of the materials are metals, solid at room temperature, and relatively easy to fabricate.

2.1 Investigation of high-Z to low-Z ordering

To validate this geometry we tried to replicate the results from the McCaffrey paper. For this exercise we performed multiple runs of MCNP, building a 16 layer shield of $T = 0.5\,\text{mm}$. Each shield has a number of layers, from 0–15 of tungsten, with the remainder of the shield built from antimony. So, for example, one run would have 0 layers of tungsten and 16 layers of antimony, another would have 5 layers of tungsten and 11 layers of antimony. Moreover, each of these realizations was reproduced in two orientations: one set of results put tungsten, a high-Z material, closest to the source while the other put antimony, a low-Z material, closest to the source. In all cases the DDE was computed.

The results for a 50 keV gammas source are shown in Figure 2. These results match McCaffrey's in that the low-Z/high-Z ordering produced a lower dose than the high-Z/low-Z order at this x-ray energy. Starting from the left, with 0 layers of antimony, the two geometries match. Moving to the right, we add layers of antimony by removing layers of tungsten. The bottom line shows the DDE for Sb-W, while the upper line shows the DDE for W-Sb. As we add antimony, both values for the DDE increase. But the value for W-Sb, where tungsten is near the source, grows faster. This high-Z/low-Z configuration doesn't reduce the dose as well as the Sb-W configuration.

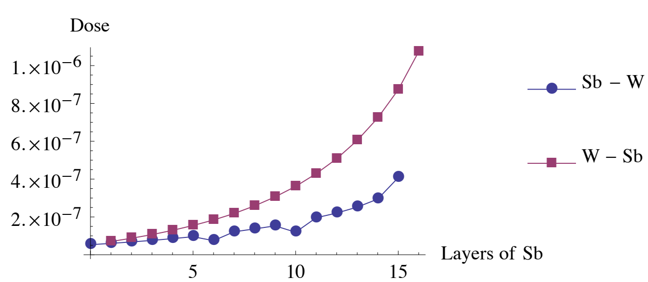

Figure 2: 50kev with antimony and tungsten

Performing this same experiment with a 100 keV gamma source energy demonstrates another expected result, shown in Figure 3, which is that higher energy gammas are less effected by the high-Z / low-Z order. This effect was also shown in McCaffrey's paper.

3. DESIGNING A SHIELD WITH GA

Having confirmed McCaffrey's finding that low-Z/high-Z ordering is preferable in our geometry, at lower energies, the next step is to define our genetic algorithm and apply it to the shielding problem.

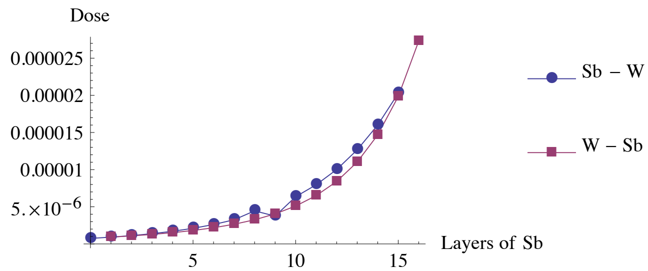

Figure 3: 100kev with antimony and tungsten

3.1 Defining a fitness function

We define a fitness function that will try to meet a mass goal, and once that mass goal is met it will then seek to reduce the dose. The mass goal will be the mass of a fixed thickness of lead. This is a standard shield material, often used in the past for shielding aprons, so we are essentially trying to weigh less than or equal a standard shield, and then try to beat it on dose. The thickness of lead will generally be fixed to a value smaller than the full allowed shield thickness. This allows lighter materials to fill in space to reduce dose. For example, we might allow the GA to build a shield that is 1 mm thick, but compare its mass to 0.5 mm of lead.

The fitness of each shield s will be determined as follows:

1. Calculate the mass of a prototypical shield made of lead, $m(S)$
2. Calculate the dose with the shield, $D(s)$ using the deep dose tally
3. Compare the mass with the shield to the mass of the lead shield
   - If the shield is heavier than the lead shield, calculate the mass of the shield, $m(s)$, and the fitness is $-(m(s)/m(S))$
   - Otherwise the fitness is $1 - 1000 \times D(s)$

In other words, the fitness of shield s is

$$F(s) = \begin{cases} 1 - 1000 \times D(s) & \text{if } m(s) < m(S) \\ 0 - m(s)/m(S) & \text{otherwise.} \end{cases} \quad (1)$$

The arbitrary factor of 1000 scales the fitness scores into a more human readable range, but does not effect the ordering of shields.

3.2 Defining the genetic algorithm

We will use two different genetic algorithms. One, called simply GA, is a standard generational genetic algorithm [2], described further below. The other algorithm is called multi-grid genetic algorithm (MGGA)[3], and is described in more detail below.

Using the fitness function from Eq. 1 we will run several experiments for GA and MGGA. The final geometry will have a total of 16 layers, with seven possible materials in each layer, the 6 materials from Table I, plus vacuum. The thickness of the layers will be $T/16$, where T is the maximum allowed thickness of the shield. With 7 materials in 16 layers there are $7^{16} \approx 3.3 \times 10^{13}$ possible shield configurations. The challenge is to efficiently search this large space to find the best shields.

The encoding for each shield is a string of integers, one for each shield layer. The integers are the numbers 0-6 indicating an index into the list of available materials: vacuum, tungsten, lead, bismuth, tin, antimony, and aluminum.

All GA and MGGA will be performed using tournament selection with a tournament size of 5. There will be a 25% chance that the worst individual in a tournament will win to help preserve diversity. Reproduction will rely on uniform crossover, mutation, and direct copy, with probabilities 70%, 20% and 10% respectively. Uniform selection will allow each child to use the materials from any layer from either parent, however, we will create two children for each crossover reproduction, these two children will be the result of each parent being the primary and the secondary. For example, if the first layer for parent A is lead, and the first layer for parent B is aluminum, we will create 2 children with cross over, one with start with lead and the other with aluminum. Each subsequent layer of the first child will come randomly from A or B, but the same layer in the second child will always come from the other parent. We ran each genetic algorithm three times with both GA and MGGA, unless otherwise stated, to look for the best shield.

One of the tools we are studying as a way of making genetic algorithms more efficient is Multi-Grid Genetic Algorithms (MGGA). MGGA is a meta-algorithm that runs a series of generational genetic algorithms in order. The last generation of each GA phase is used to produce the first generation of the next phase, which runs on a more refined spatial mesh than the previous GA, so the MGGA runs using 4, then 8, then 16 layers in the shield. In previous work, [3], we have found that MGGA can produce a better solution than GA with the same resources. To move between phases each layer will simply be cut into two pieces for the next phase. From a shielding standpoint the shields will be identical as they move between phases. For example, a shield in phase one that has the chromosome 0123 will turn into 00112233 if moved to the second phase.

The GA runs are given 1000 individuals in each population for 20 generations. The Multi-Grid Genetic Algorithm MGGA will run with the following generational parameters:

- 4 layers - 250 individuals per generation with 10 generations
- 8 layers - 500 individuals per generation with 15 generations
- 16 layers - 500 individuals per generation with 15 generations

4. RESULTS

The results of these genetic algorithm runs are summarized in Table II. Each entry in the table represents the“best” shield from the three runs performed at that energy and thickness. There

Table II: Summary of Results

Energy	Shield Thickness	Dose Reduction	Lead Thickness	Algorithm
50 keV	1 mm	0.796	0.5 mm	GA
50 keV	1 mm	0.80	0.5 mm	MGGA
100 keV	1 mm	0.314	0.5 mm	GA
100 keV	1 mm	0.249	0.5 mm	MGGA
200 keV	1 mm	0.013	0.5 mm	GA
200 keV	1 mm	0.002	0.5 mm	MGGA
200 keV	4 mm	0.043	2 mm	GA
200 keV	4 mm	0	2 mm	MGGA
300 kev	4 mm	0.021	2 mm	GA
300 kev	4 mm	0	2 mm	MGGA

are five values in each row of the table: the energy of the gammas produced at the source, the total possible thickness of the shield, the thickness of lead being compared against in the fitness function, Eq. 1, the algorithm used, GA or MGGA, and the reduction in dose of the genetic algorithm design shield over a the lead shield of equivalent mass. This dose reduction is defined as $1 - D(s)/D(S)$, where s is the shield being evaluated, and S is the lead sheild. So a value of 80% means that the dose is 80 percent less than the lead shield being compared to.

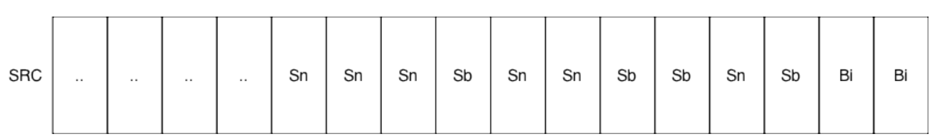

Figure 4: 50kev Best MGGA 1mm shield

Reviewing these results, notice first that the genetic algorithms are able to do a great job of beating a lead shield at 50 kev. The MGGA design shield allows only 20% of the dose of an all lead shield. This shield uses tin, antimony and then bismuth to protect the water phantom, as shown in Fig. 4; it also displays the expected low-Z/high-Z configuration that McCaffrey et al.

[1] saw, but while they restricted themselves to bi-layers, we see here that in fact 3 materials is an even better choice. While this optimal shield can potentially be twice as thick as the lead shield, it has the same mass. The fitness function is designed in such a way that when shields are competing on dose, there is no selective pressure to further reduce mass. Therefore when shields are competing on dose there is no selective pressure to have a mass lower than the mass of the lead shield, $m(S)$. Only when a shield has a mass greater than $m(S)$ is there a selective pressure on mass. Therefore the best shields from the genetic algorithms will tend to have the same mass as the lead sheild, and a lower or equal dose as the lead sheild.

A second important item to notice in these results is that dose reduction achieved with the optimal shields identified by the genetic algorithms decreases as the source energy is increased. At 200 keV the GA only beat the 1 mm lead shield by 1.3%. Allowing the shield to be thicker, 4 mm, and comparing to a thicker lead shield, 2 mm, GA was able to move this value up to 4.3% percent, but this is still little improvement over an all lead shield. As the source energy increases more of the efficacy of the shield comes from Compton scatter, and the positive effect of differing atomic structure is lost in favor of higher electron density. The final shield produced by GA, pictured in Fig. 5 is basically all Bismuth, and does not represent any smart layering.

Figure 5: 200kev Best GA 4mm shield

Finally, it is worth noting that MGGA was unable to beat GA for the higher energy shields. In all cases MGGA produced a pure lead shield. This change in MGGA's success rate will be investigated further in later work. Most likely it represents a degenerate problem where the MGGA produces mostly lead shields in the first phase and is unable to sufficiently alter the composition at later phases sufficiently to beat lead. If this is the case, the solution will be to begin the MGGA with a larger number of layers so that it cannot stagnate too easily.

All of the shields listed in Table II are pictured in APPENDIX A. From the graphics we can see how MGGA stagnates on lead, and how GA is moving to Bismuth, a high-Z, but less dense material than lead.

5. CONCLUSIONS

The genetic algorithms were able to show a clear success in discovering an excellent layering to lower the dose due to 50 keV gammas. We were able to show an 80% reduction in dose using a low-Z/high-Z layering scheme, compared to an identical mass of lead. This would represent a significant lowering of dose to x-ray technicians, and this layered shield could be deployed in either wearable aprons or in the wall shielding of an x-ray facility. In addition, we expect that we could achieve a dose similar to 0.5 mm of lead, but at a reduced weight; this would be useful for either x-ray technicians or for patients, who must often be covered by lead drapes. At higher energies the genetic algorithms were still able to improve the shield, compared to the same mass as lead, but only marginally, and with decreasing success as the energy increased.

Unlike in our earlier work, on this problem MGGA is not showing significantly better results than GA. In the future we will look into why this may be happening, including the possibility that MGGA is being allowed to stagnate in the first phase. If this is the case, then an insufficiently diverse population is moved from one phase to the next, and this can be addressed through the injection of additional mutation.

ACKNOWLEDGEMENTS

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APPENDIX A

Shields from Table II, GA and MGGA are pictured together, with the best GA shield on top.

SRC	Sb	Sb	Sn	Sn	Sb	..	Sb	..	..	Sn	..	Sb	Sb	Sb	Pb	Bi
SRC	..	..	..	..	Sn	Sn	Sn	Sb	Sn	Sn	Sb	Sb	Sn	Sb	Bi	Bi

Figure 6: 50kev Best GA and MGGA shield

SRC	Pb	Pb	Bi	Bi	W	..	..	..	W	Bi	..	..	..	..	..	..
SRC	Pb	Pb	Pb	Pb	W	..	..	..	..	..	..	..	Sb	W	..	..

Figure 7: 100kev Best GA and MGGA shield

SRC	Bi	Bi	Bi	Bi	..	Bi	..	..	..	Bi	Bi	..	Pb	..	Bi	..
SRC	Pb	Pb	Pb	Pb	..	..	Pb	Pb	Pb	Pb	..	..	..	..	..	..

Figure 8: 200kev Best GA and MGGA shield at 1mm

SRC	Bi	Bi	..	..	Bi	..	Bi	..	..	Bi	Bi	..	Bi	Pb	..	Bi
SRC	Pb	Pb	Pb	Pb	Pb	Pb	Pb	Pb	..	..	..	..	..	..	..	..

Figure 9: 200kev Best GA and MGGA shield at 4mm

SRC	..	Bi	..	..	Bi	..	Bi	Bi	Bi	Bi	Bi	..	..	..	Pb	Bi
SRC	..	..	..	..	..	..	..	..	Pb	Pb	Pb	Pb	Pb	Pb	Pb	Pb

Figure 10: 300kev Best GA and MGGA shield