DIFFRACTION LIMIT OF REFRACTIVE COMPOUND LENS

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A compound X-ray and neutron lenses is an array of lenses with a common axis. The resolution limited by aberration and by diffraction. Diffraction limit comes from theory based on absorption aperture of the compound refractive lenses. Beam passing through transparent lenses form Airy pattern. Results of calculation of diffraction resolution limit for non-transparent X-ray and neutron lenses are discussed.

Introduction

Up to now using refractive optics for X-rays and neutrons were unreasonable due to small value of refraction index and large value of absorption and scattering. Real part of refraction index n is negative both for X-rays and neutrons:

$$n=1-\delta+i\,(\lambda/4\pi)\mu \qquad (1)$$

The focal length R of the individual refractive neutron or X-ray concave lens is more than 100 m and it is inappropriate for practical using:

$$F=R/2\delta \qquad (2)$$

The focal distance of stack of N refractive lenses is N times smaller than the focal distance of individual lens due to the geometrical optics. So the stack of the biconcave refractive lenses can be suitable for X-rays and neutron experiments (Fig.1).

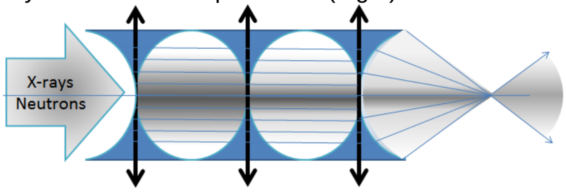

Fig. 1. Schematic view of compound refractive lens.

Refractive optics are used for 5-100 keV X-rays and for a neutrons with wavelength 0.2-10 nm [1-6]. Typical refractive neutron lens consists of 30-100 individual lenses, made from low absorbing material MgF_2, with radius 25 mm. Used in experiments refractive neutron lenses have focal distance 8.7 m and transmission 50% for 1.32 nm neutron beam, and measured gain was 9.7. The main parameters of the X-ray refractive lenses are aperture 5-500 microns, the number of refracting lenses 5-500, focal length 0.1-2 m, transmission 0.1-90%. Absorption and scattering in the material define optical properties of the X-ray or neutron lens.

Spot size formed by refractive lens is equal to size of source image. Gain or concentration of the flux is equal to ratio areas of the lens aperture to the source image multiply to the lens transmission. The diffraction limit spot size defined by aberration and by diffraction depends on size of lens aperture. Absorption diaphragm R_D for the parabolic refractive lenses the radius of the diaphragm defined as:

$$R_D = \sqrt{\frac{R}{2\mu N}} \leq R\ . \qquad (3)$$

Absorption diaphragm is used for transmission and flux calculations. In classical optics absorption diaphragm define resolution of the lens R_{diff} [1]:

$$R_{diff} = 0{,}61 \cdot \lambda \frac{F}{2R_D} \geq \lambda\ . \qquad (4)$$

R_{diff} is the radius of an Airy disk is the central bright circular region of the pattern produced by light diffracted when passing through a small circular aperture (Fig. 2). The central disk is surrounded by less intense concentric rings, so light intensity takes local maximum and minimum while it decreases away from the center [7]:

$$A(r) = A_0 \frac{2J_1(kRr/\lambda)}{kRr/\lambda} \qquad (5)$$

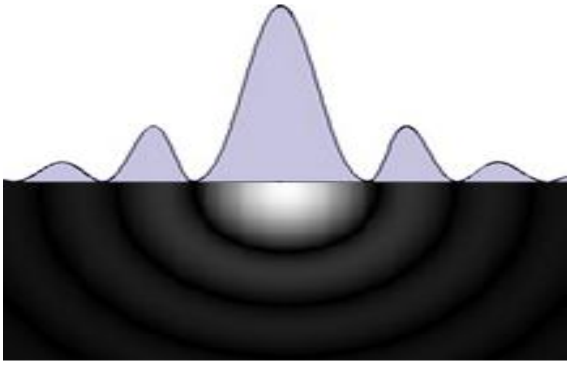

Fig. 2. The diffraction pattern resulting from a uniformly illuminated circular aperture.

The distance between the central maximum and the first minimum is the size of an Airy disk. In classical optics, the Airy disk is the smallest focused spot of light that a lens with a circular aperture can make. The width of the Airy disk is used to define the theoretical maximum resolution for an optical system. Using dimensionless coordinates, the radius of an Airy disk depends on numerical aperture *NA and* equals 3.83 in optical units *λ/(2πNA)*. About 84% of the total light intensity is in the central Airy disk, the remainder 16 % is distributed in the concentric rings.

The Rayleigh criterion for the diffraction limit to resolution states that two points are resolvable if the center of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other. The first minimum is at an angle of θ, so that two point objects are just resolvable if they are separated by the angle:

$$\theta=1.22\lambda/D. \qquad (6)$$

Diffraction limit

Here we discuss diffraction limit of the compound refractive biconcave lenses. Distribution of x-ray radiation in a refracting x-ray lens can be described by theory of monochromatic electromagnetic plane waves. Light in the form of a plane wave in space is said to be linearly polarized. it allows to describe a light wave in scalar approach. The monochromatic electromagnetic plane waves is described as follows:

$$E=A_0\cos(wt-kr) \qquad (7)$$

where w – frequency, r – radius vector, k – wave vector.

A wave front is the locus of points having the same phase *kr=const*. The plane waves change amplitude and a phase in refractive x-ray lens. The amplitude may be described by law *P(x)* taking into account attenuation due to thickness of the lens material. Changes of amplitude and phase in lens material can be described by complex function of a transmission *T*:

$$T(x) = P(x)\exp(i\,\varphi(x)) \qquad (8)$$

A wave front at the exit of lens doesn't plane, it is described by law φ (x):

$$\varphi(x) = k\delta L(x), \qquad (9)$$

Intensity distribution for circular hole is described as:

$$I = \left(\int_0^R \int_0^{2\pi} U\, e^{-\mu L(x)} e^{-i k r \cos\varphi} r\, d\varphi\, dr\right)^2 \qquad (10)$$

where μ - linear coefficient of attenuation of a material, q-defines change of a wave vector at diffraction. Condition $I=0$ defines the size of Airy disk and diffraction limit of the refractive X-ray lens.

Results of calculations intensity distribution for ideal infinite planar parabolic lens show that condition I=0 should be changed:

$$I = \left(\int_{-\infty}^{\infty} e^{-\mu x^2} e^{-ikx'\sin\varphi} dx\right)^2 = \frac{\pi}{\mu} e^{-\frac{k^2 \sin^2\varphi}{2\mu}} \qquad (11)$$

Distribution of intensity consists of one central peak. Size of the central peak is depends on linear coefficient of attenuation of a material and define diffraction limit of the compound refractive lenses:

$$R_{dif} = \frac{\lambda F\sqrt{\mu}}{\pi} \qquad (12)$$

At great values of absorption diffraction picture is characterized by monotonously decreasing function. Intensity of b the size of an effective diaphragm is proportional to $\mu^{-1/2}$.The central disk is not surrounded by less intense concentric rings.

Intensity distribution for ideal circular infinite parabolic lens show that intensity does not depends on direction:

$$I = \left(\int_0^{\infty}\int_0^{2\pi} e^{-\mu x^2} e^{-ikx'\sin\varphi} d\varphi\, dx\right)^2 = \frac{\pi^2}{\mu^2} e^{-\frac{k^2}{2\mu}} \qquad (13)$$

Intensity distribution from circular parabolic refractive lens similar to the point source.

So structure of the central maximum varied from Airy disk for transparent optical systems to monotonously decreasing function for system with absorption and scattering.

Results of calculations diffraction radius Q for refractive X-ray lens (100 microlenses, radius equal 100 microns) for different absorption coefficient are made. Dependence of the diffraction radius in optical units ($2\pi/\lambda$) is shown in Fig.3.

The size of Airy disk depends on absorption in the material of the compound X-ray lens. Calculations show that for small values of linear coefficient of attenuation of the material μ the diffraction results are correspond to classical optics and diffraction resolution does not depends on attenuation up to $\mu = 10^{-4}\ cm^{-1}$. Absorption leads to reduction of intensity of diffraction peaks and insignificant increasing of the Airy disk size. The radius of an effective diaphragm of R_d is equal to the radius of X-ray lens *R.*

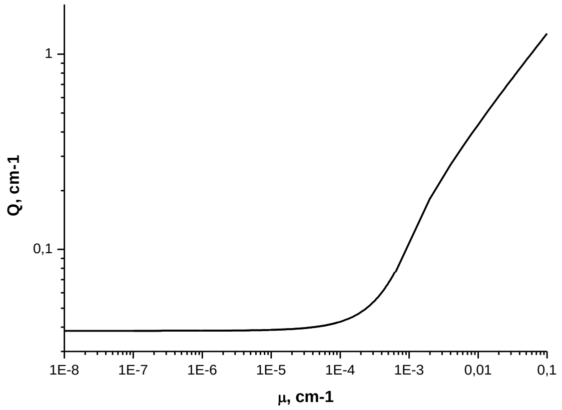

Fig. 3. Dependence of the diffraction radius on attenuation coefficient.

At values of attenuation 10^{-4}-10^{-1}cm^{-1} diffraction resolution proportional to μ. Attenuation of the beam in material blurs of the diffraction picture that corresponds to reduction of radius of an effective diaphragm R_d. Distribution of intensity is similar to classical diffraction picture: decreasing intensity runs a number of maximums and minimums. Values of the size of an effective diaphragm R_D can be determined by a diffraction picture.

At great values of attenuation μ>1 diffraction picture is characterized by monotonously decreasing function. The size of central peak is proportional to $\mu^{-1/2}$.Refractive system with significant attenuation is similar to the point source.

Conclusion

A compound X-ray lenses is an array of microlenses with a common axis. The resolution limited by aberration and by diffraction. Diffraction limit comes from theory based on absorption aperture of the compound refractive lenses. So structure of the central maximum varied from Airy disk for transparent optical systems to monotonously decreasing function for system with absorption and scattering and to isotropic distribution for point source. Results of calculation of diffraction resolution limit for X-ray lenses are discussed.

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