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MEASUREMENT OF REACTIVITY IN ADS REACTORS CONSIDERING AN EXPONENTIAL DECAY AFTER AN INTERRUPTION IN THE EXTERNAL PROTON SOURCE

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ABSTRACT

The online monitoring of reactivity in ADS reactors is of paramount importance for the operations of such systems. This work is dedicated to the prediction of reactivity from a decay of the neutron population after a pulse from the external source. For that, a pulse from an external source in an ADS reactor was simulated with Serpent Reactor Physics code. From the data obtained, it was possible to make an adjustment, based on a combination of exponentials. The coefficient of the exponential for the dominating term of the sum of exponentials is compared to the simplified solution of the neutron diffusion equation, thus obtaining the reactivity. The method used for the adjustment has the advantage of not requiring data that is equally spaced, and of being easily programmable, waiving the use of specific software for linear adjustments. The preliminary results of the research showed a 750 pcm deviation in relation to the value found of -3,630 pcm obtained through point kinetics, and as a result should be the object of further study.

1. INTRODUCTION

Reactivity is one of the most important properties in a nuclear reactor to be monitored and is directly related to the effective multiplication factor of the neutrons (k_{eff}) [1]. For ADS reactors, considered to be 4th generation ones, the choice of geometry and composition are the main factors that define the value of the k_{eff}. For the sake of operating safety, and to ensure that the reactors can be easily shutdown in the case of an accident, most o the proposals take a value for the k_{eff} between 0.95 and 0.98 into account. On the other hand, the smaller the k_{eff} is, the higher the intensity of the neutron source should be.

As an example of the relevance in choosing such factors, a proposal for a reactor that uses thorium as fuel, driven by an accelerator (Accelerator-Driven Thorium Reactor - ADTR), is undergoing a feasibility study. This project started as a result of an agreement between the UK and Switzerland and consists of a lead-cooled rapid reactor [2]. This reactor entails a self-supported thorium fuel cycle of ten years, using plutonium as fission trigger. The lead works as a spallation target and as a coolant at the same time. This reactor is capable of generating 600MWe at a 0.995 k_{eff} , needing a 3-4 MW accelerator. As a result of the k_{eff} being so close

to 1, it is necessary to use quick-action control rods to guarantee its shutdown. This proposal deviates large part from the ADS reactors as regards the multiplication factor, such as, for example, the MYRRHA. Such a reactor is designed to generate 50-100MWe with a k_{eff} of 0.95 and a 3.2MW accelerator [3]. This points to the need for a more powerful accelerator, if compared to the electrical power of the MYRRHA with that of the reactor discussed before. In this comparison, it is possible to see that it is possible to attain a power around six times higher with an external source of the same order of magnitude, by varying the k_{eff}. Therefore, a continuous monitoring of the reactivity in these systems is fundamental.

This article is dedicated to the development of a method capable of monitoring reactivity online in ADS reactors. For the measuring of reactivity one should consider the length of time after a pulse from the external proton source in which the neutron flow produced by the protons, as measured on the inner detectors of the core, decays in the shape of an exponential. For variations as swift as this, the contribution of the delayed neutrons could be ignored, simplifying the neutron diffusion equation, having the exponential function as the solution of the partial differential equation.

Some studies indicate that the data gathered with internal detectors may be adjusted in the shape of a combination of exponentials, and that the dominating coefficient in this summation can be compared to the exponential obtained from the solution of the differential equation for the simplified neutron diffusion equation.

This proposal for online monitoring is feasible when taking the MYRRHA reactor into account, where the external proton source is interrupted for 200ms at every second of normal operation for the appropriate measurements. The volume of data obtained in this time frame is sufficient to yield the data needed to make the adjustment of the exponentials.

For the production of this proposal, an ADS reactor based on the MYRRHA reactor [4] was simulated and, in order to simplify the data then collected, the flow of neutrons in the internal detector was obtained after one single pulse from the external proton source. The data on the neutron flow was adjusted as a combination of exponentials, based on the method of adjustment of exponentials, as presented in [4]. The calculation routines were implemented in the Matlab software. The coefficients obtained in the adjustment were compared with the exponential that represents the solution of the partial simplified differential equation for the neutron diffusion equation to obtain the reactivity.

This work is divided as follows: Section 2 presents the methodology proposed, with its mathematical formulation, and with the simulation of the reactor under study. Section 3 provides the results and section 4 presents the conclusions.

2. METHODOLOGY

This Section presents the methodology used, including a brief explanation of the neutron kinetic equations, of reactor simulation, and of the method used for the adjustment of the exponentials.

2.1. Neutron Diffusion Equation

Proposals based on the calculation of reactivity in an exponential adjustment having been gaining prominence in the study of ADS reactors. This method has become quite attractive as it can be used online and because of its need of few kinetic parameters. In order to obtain it, one has to consider the neutron diffusion equations [1], taking the external neutron source, as follows:

$$\frac{\partial}{\partial t}n(t)=\left[\frac{\rho-\beta}{\Lambda}\right]n(t)+\sum_{i=1}^{n}\lambda_i c_i(t)+Q(t), \tag{1}$$

$$\frac{dc_i(t)}{dt}=\frac{\beta}{\Lambda}n-\lambda_i c_i(t)\ , \tag{2}$$

where $n(t)$ is the neutron density at a given moment, ρ is the reactivity of the system, $Q(t)$ is a term to refer the external source, $c(t)$ is a term to refer the concentration of precursors, β is the total delayed neutron fraction, λ is the effective decay constant and Λ is the mean generation time between the appearance and subsequent absorption of neutrons inducing fission.

Delayed neutrons can be disregarded for the case of a single pulse from the external source [5]. With it, equations (1) and (2) can be simplified, to obtain:

$$\frac{\partial}{\partial t}n(t)=\left[\frac{\rho-\beta}{\Lambda}\right]n(t). \tag{3}$$

Equation (3) has a simple solution, given by:

$$n(t)=n(t_0)\exp(\alpha.t), \tag{4}$$

where

$$\frac{\rho-\beta}{\Lambda}=\alpha\ . \tag{5}$$

According to [6], it is possible to make an exponential adjustment with the data obtained from the in-core internal detectors of sub-critical reactors. Such an adjustment is compared to Equation (4) and can be done as a linear combination of exponentials such as:

$$y(t)=\sum_{i=1}^{n}a_i e^{-b_i t}\ , \tag{6}$$

with $y(t)$ being the quantity of neutrons in time t.

Commercial computer software such as Origin and Matlab provide an exponential adjustment with 2 terms at most, that is, $n=2$ as in Equation (6). For an exponential adjustment with 3 or

more terms, software developed in specialist research laboratories is frequently used, such as MINUIT, developed at CERN [7].

In order to make an adjustment with 3 or more terms, in the computational point of view more easy and accessible, it is necessary to use some mathematical method. It is possible to find an adjustment with 3 exponentials in [4]. In order to evaluate the reactivity with a higher degree of precision, the method was expanded to an exponential adjustment with 4 terms, as described in the next section. The method used in this work is based on the method presented in [4] and, according to [8] the only method that is adequate for non-equally spaced data.

2.2. Exponential adjustment

To illustrate the process used, one can consider a function written in terms of 4 exponentials, that is, doing $n = 4$ in Equation (6), such as:

$$y(t) = a_1 e^{-b_1 t} + a_2 e^{-b_2 t} + a_3 e^{-b_3 t} + a_4 e^{-b_4 t}. \tag{7}$$

Through simplification, term $y(t)$ will be noted as y. Equation (7) is then derived four times, that is, the number of terms used in the summation. With it, one gets y' y'', y''', y''''.

Term y'''' is then written as a result of the smaller derivatives, to produce:

$$\begin{aligned} &y'''' - (b_1 + b_2 + b_3 + b_4)\, y''' - \left(-b_1 b_2 - b_1 b_3 - b_1 b_4 - b_2 b_3 - b_2 b_4 - b_3 b_4\right) y'' \\ &- \left(-b_1 b_2 b_3 - b_1 b_2 b_4 - b_2 b_3 b_4\right) y' - \left(b_1 b_2 b_3 b_4\right) y = 0. \end{aligned} \tag{8}$$

The initial conditions for Equation (8) are given by:

$$y_0 = a_1 + a_2 + a_3 + a_4, \tag{9}$$

$$y_0' = -a_1 b_1 - a_2 b_2 - a_3 b_3 - a_4 b_4, \tag{10}$$

$$y_0'' = a_1 b_1^2 + a_2 b_2^2 + a_3 b_3^2 + a_4 b_4^2, \tag{11}$$

$$y_0''' = -a_1 b_1^3 + a_2 b_2^3 + a_3 b_3^3 + a_4 b_4^3. \tag{12}$$

With it, Equation (8) is integrated four times to produce:

$$y = a + bt + c\frac{t^2}{2} + d\frac{t^3}{3} e\int_0^t y\,dt + f\int_0^t\int_0^t y\,dtdt + g\int_0^t\int_0^t\int_0^t y\,dtdtdt + h\int_0^t\int_0^t\int_0^t\int_0^t y\,dtdtdtdt, \tag{13}$$

where:

$$a = a_1 + a_2 + a_3 + a_4 \,, \tag{14}$$

$$b = (-a_1 b_1 - a_2 b_2 - a_3 b_3) - e\left(a_1 + a_2 + a_3 + a_4\right) \,, \tag{15}$$

$$c = \left(a_1 b_1^2 + a_2 b_2^2 + a_3 b_3^2 + a_4 b_4^2\right) - e(-a_1 b_1 - a_2 b_2 - a_3 b_3) - f\left(a_1 + a_2 + a_3 + a_4\right) \,, \tag{16}$$

$$\begin{aligned} d = &\left(-a_1 b_1^3 - a_2 b_2^3 + a_3 b_3^3 + a_4 b_4^3\right) - e\left(a_1 b_1^2 + a_2 b_2^2 + a_3 b_3^2 + a_4 b_4^2\right) \\ &- f(-a_1 b_1 - a_2 b_2 - a_3 b_3) - g\left(a_1 + a_2 + a_3 + a_4\right), \end{aligned} \tag{17}$$

$$e = \left(-b_1 - b_2 - b_3 - b_4\right) \,, \tag{18}$$

$$f = \left(-b_1 b_2 - b_1 b_3 - b_1 b_4 - b_2 b_3 - b_2 b_4 - b_3 b_4\right) \,, \tag{19}$$

$$g = \left(-b_1 b_2 b_3 - b_1 b_2 b_3 - b_1 b_3 b_4\right) \,, \tag{20}$$

$$h = (-b_1 b_2 b_3 b_4) \,. \tag{21}$$

The integrals of Equation (13) can be solved numerically, as variables y and t are known. With it, and in order to make the notion simpler, each integral is denoted as:

$$F = \int_0^t y dt \,, \tag{22}$$

$$G = \int_0^t \int_0^t y dt dt \,, \tag{23}$$

$$H = \int_0^t \int_0^t \int_0^t y dt dt dt \,, \tag{24}$$

$$I = \int_0^t \int_0^t \int_0^t \int_0^t y dt dt dt dt \,. \tag{25}$$

Equation (13) can be re-written in a matrix way as:

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 1\, t_1\, t_1^2\, t_1^3\; F_1\; G_1\; H_1\; I_1 \\ 1\, t_2\, t_2^2\, t_2^3\; F_2\; G_2\; H_2\; I_2 \\ \vdots\;\vdots\;\vdots\;\vdots\;\;\vdots\;\;\vdots\;\;\vdots\;\;\vdots \\ 1\, t_n\, t_n^2\, t_n^3\; F_n\; G_n\; H_n\; I_n \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{pmatrix}. \tag{26}$$

By simplifying the notation, is is possible to re-write:

$$Y = X.\beta\,, \tag{27}$$

where β can be estimated by making:

$$Y = X.\hat{\beta} + \varepsilon. \tag{28}$$

The errors related to Equation (28) will not be dealt with in this work. With it, $\hat{\beta}$ can be obtained by doing:

$$\hat{\beta} = \left(X'.\mathrm{X}\right)^{-1}.X'\mathrm{Y}\,. \tag{29}$$

By having vector $\hat{\beta}$, it is possible to build a system made by Equations (14) to (21) with the aim of determining terms a_i and b_i of Equation (7).

Due to the size of the system of equations, we resorted to the Maple software. However, the expressions related to coefficients a_i for 4 terms are excessively large, but for the calculation of reactivity only the coefficients b_i have relevance, and they can be obtained with the roots of Equation

$$x^4 + e.x^3 - f.x^2 + g.x - h = 0. \tag{30}$$

As a result of the difficulty to calculate the terms a_i for an exponential adjustment with 4 terms, the mathematical induction to select the appropriate term in the summation of exponentials to be used to calculate the reactivity will take into account an exponential adjustment with 3 terms, as described in [4]. However, for such analyses, one needs to obtain the data from a sub-critical reactor.

The method to obtain reactivity from an exponential adjustment has already been used in some experimental facilities such as Yalina [9] and MUSE [10], with the use of the MINUIT software for the exponential adjustment. In order to obtain the data, a reactor was simulated in the

Serpent reactor Physics code, using the MYRRHA reactor [11] as the basis, as shown in the next sub-section.

2.3. Simulation of the Myrrha Reactor

The Serpent reactor physics code was chosen as it is one of the few codes that allow simulating a reactor controlled by an external source. Fig. 1 shows a cross-section of the reactor simulated, as produced by the code.

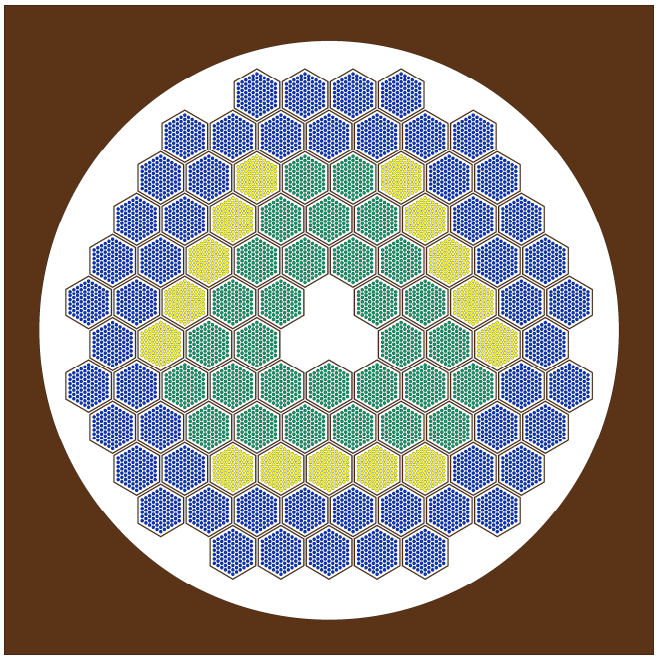

Figure 1: Cross-section of the reactor simulated.

The reactor simulated consists of 99 fuel elements and each element is filled with 91 rods. The fuel rod parameters are shown in Table 1. The composition of the materials used in the simulation is shown in Tables 2, 3, and 4. The kinetic parameters obtained from the Serpent code are shown in Table 5.

Table 1: General fuel rod parameters.

Parameters	Size (cm)
Fuel radius	0.3075
Inner fuel rod radius	0.3175
Outer fuel rod radius	0.3275
Rod pitch	0.855
Fuel element pitch	0.962
Active height	120

Table 2: Composition of the core fuel (density = 11 g/cm³).

Element	Fuel (mass fraction)
O	0.1185
^{235}U	0.0521
^{238}U	0.7253
^{238}Pu	0.0010
^{239}Pu	0.0343
^{240}Pu	0.0108
^{241}Pu	0.0301
^{242}Pu	0.0030

Table 3: Composition of the coolant (density = 10 g/cm³).

Element	Mass fraction
^{82}Pb	0.45
^{83}Bi	0.55

Table 4: Composition of the reflector (density = 5,3908 g/cm³).

Element	Atomic Density (atoms/barn. cm)
^{23}Na	7.15E+01
^{54}Fe	2.77E+01
^{56}Fe	4.35E+02
^{57}Fe	1.01E+01
^{58}Fe	1.34E+00
^{58}Ni	1.99E+00
^{60}Ni	7.67E-01
^{61}Ni	3.33E-02
^{62}Ni	1.06E-01
^{64}Ni	2.71E-02
^{50}Cr	3.06E+00
^{52}Cr	5.91E+01
^{53}Cr	6.70E+00
^{54}Cr	167E+00
^{55}Mn	3.12E+00
^{91}Zr	3.33E+00

Table 5: The Kinetic parameters as obtained from the Serpent code.

β	$\lambda(s^{-1})$	$\Lambda(s)$
0.0118	0.784	4.02E-7

In order to obtain data on the decay of the neutron population, the reactor simulated was subjected to a single neutron pulse. In order to collect the data, counters were allocated to measure the quantity of neutrons per time unit that crossed each one of the regions of the reactor, i.e., green region (fuel-in), yellow (fuel-middle), and blue (fuel-out).

Having grasped the methodology and the data needed for its implementation, the results obtained are provided in the next section.

3 RESULTS

With the coefficients obtained in Section 2.2 applied to the reactor simulated in Section 2.3, it was possible to make the calculations. With it, we list some typical values, obtained for coefficients b_i for an adjustment containing 3 exponentials:

$$b_1 = 4.5 \cdot 10^6 \tag{31}$$

$$b_2 = 3.9 \cdot 10^5 \tag{32}$$

$$b_3 = 2.0 \cdot 10^5 \tag{33}$$

Fig. 2 shows the adjustment with 3 exponentials, compared with the data obtained from the counters and each one of the terms of the exponential, plotted independently.

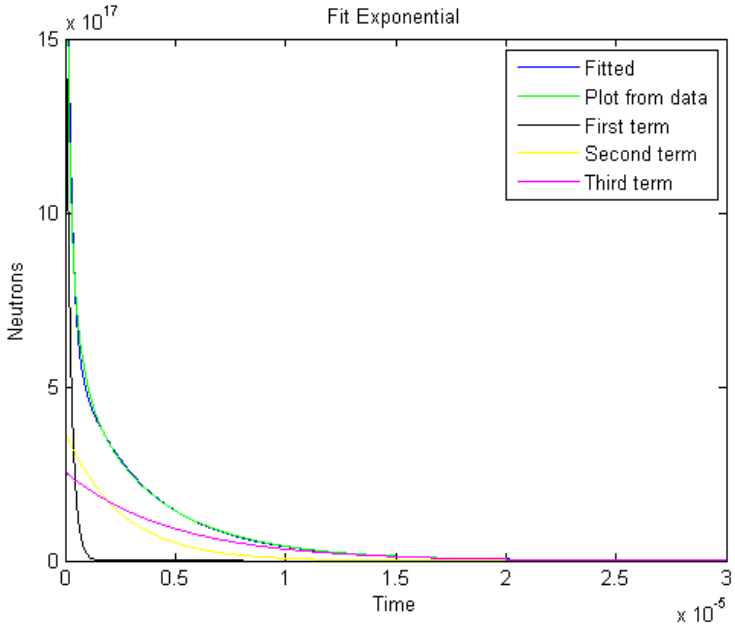

Figure 2: Plot with the adjusted function, compared with the data and with each one of the terms of the exponential.

By plotting Fig. 2 again, with the **y** axis in the log scale, it is possible to see the dominating term, as follows:

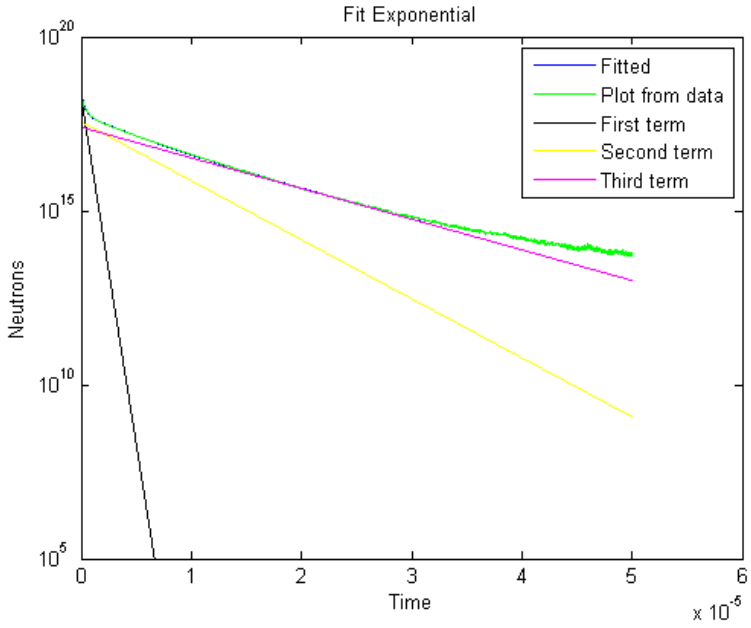

Figure 3: 'y' axis in log scale.

With it, it is possible to see in Fig. 3 that the third term is the dominating one in this process. It is possible to arrive at this same conclusion by looking directly at the coefficients, as the exponential is written in the form of $e^{-b_n t}$, leading the smaller b_n to represent the term that decays in a slower way. Therefore, the smaller coefficient b_n found was used in Equation (5).

With the kinetic parameters obtained in Section 2.3, it is possible to make the calculation of the reactivity for 4 exponentials, for each one of the regions in the reactor, and compare it with inverse kinetics, as shown in Table 6:

Table 6: Calculation of reactivity for keff=0.9574 with a percentage deviation in relation to inverse kinetics.

	Reactivity (pcm)	
Calculated by inverse kinetics	-3630	Percentage deviation
Calculated at 'fuel in'	-4830	33
Calculated at 'fuel middle'	-5150	41
Calculated at 'fuel out'	-4380	20

With it, it is possible to observe that the smallest percentage deviation in reactivity as calculated based on the detectors in relation to the value for reactivity obtained through inverse kinetics was found in the neutron counter at the fuel-out point, that is, in the regions closer to the peripheral area of the active reactor core.

4. CONCLUSIONS

This work presented a methodology to calculate reactivity that can be used in an online manner in ADS reactors, using the analysis of the exponential decay of the neutron population as a basis, after a pulse from the external source. The data obtained during the decay of the neutron population after the external source pulse are adjusted through a sum of exponentials. This adjustment is then compared to the simplified solution of the point kinetics equations, from which it is possible to obtain the reactivity. For this goal, it was shown an exponential adjustment method in Section 2.2 that is recommended by the authors. In spite of the fact that the smaller percentage deviation found in the calculation of the reactivity was 20%, this method is promising, as it does not require many kinetic parameters. The authors recommend more studies related to this method in order to reduce the errors found.

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