297/82

IC/81/226

INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS

A HIGGSLESS SU(6) MODEL WITH $SU(4)_{EC} \times SU(2)_L \times U(1)_Y$ AS INTERMEDIATE SYMMETRY

Cao Chang-qi

and

Ding Xing-fu

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1981 MIRAMARE-TRIESTE

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

A HIGGSLESS SU(6) MODEL WITH $SU(4)_{EC} \times SU(2)_L \times U(1)_Y$ AS INTERMEDIATE SYMMETRY *

Cao Chang-qi** and Ding Xing-fu
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

No elementary Higgs field is introduced in accordance with the DSB idea. Instead of technicolour formulation, an extended colour $SU(4)_{EC}$ is considered, which is supposed to cause vacuum condensation to break $SU(2)_L$ at the suggested mass scale of 1 TeV and then break itself to $SU(3)_C$. The intermediate symmetry group $SU(4)_{EC} \times SU(2)_L \times U(1)_Y$ is further unified to a simple group SU(6). This model shows some interesting features, especially that it fits the existing neutral current parameters quite well, yet with expressions different from those in the W-S model. Some severe problems are also mentioned.

MIRAMARE - TRIESTE
November 1981

* To be submitted for publication.

** On leave of absence from Department of Physics, Peking University, Beijing, People's Republic of China.

I. INTRODUCTION

The grand unified theory is attractive in that it accounts for the quantization of electric charge and embeds strong, weak and electromagnetic interactions into a unified theoretical scheme with a single coupling constant. But the introduction of Higgs scalars will damage much of its esthetic advantages by the appearance of other forms of interactions such as Yukawa interactions and interactions among the scalars themselves. As a result, the number of free parameters in the theory also increase enormously. This actually undermines the basic idea of grand unification. Furthermore, the vast difference of order of magnitude between the grand unification mass scale and that of the W bosons. which is usually referred to as gauge hierarchy, can only be achieved in this case through fine adjustment of parameters in the Higgs potentials to 24 decimal places, which makes the theory very unnatural.

The gauge hierarchy suggests that the breakdown of $SU(2)_L \times U(1)_Y$ is most likely of a pure dynamical nature. Making a further step in this direction leads to the idea that all symmetry breaking may be dynamical and there are no elementary Higgs scalars at all. In this case the only interaction in the Lagrangian is a gauge interaction and the analogues of the usual scalar fields are no more than combined systems of fermion pairs. Susskind [1] pointed out that for the asymptotic free non-Abelian gauge theory, owing to the logarithmic dependence of coupling constant with q^2, the vast difference in the order of mass scales only corresponds to a relatively not large change of coupling constant, which suggests that dynamical symmetry breaking (DSB) may be able to offer an explanation for gauge hierarchy.

Usually the symmetry at intermediate energies is taken to be $SU(3)_C \times SU(2)_L \times U(1)_Y$. For DSB a natural idea is that the breaking of $SU(2)_L \times U(1)_Y$ is caused by the interaction of $SU(3)_C$, which makes quark and antiquark to form bound pairs and to condense in vacuum. But a slight examination shows that this is impossible, because the Goldstone bosons thus developed must be the pions, while in reality the pions are not eaten by W_μ and Z_μ bosons. In addition, if pions were Goldstone bosons, the mass acquired by W_μ and Z_μ, as estimated from the pion decay constant, would be three orders smaller than that in the W-S model. This situation made some authors [1),2)] propose a new interaction called technicolour which is stronger than the colour interaction and responsible for the desired condensation at a scale of 1 TeV (rather than a fraction of 1 GeV, the scale of QCD). This means that the symmetry at intermediate energies will be enlarged to $G_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y$, with G_{TC} representing the technicolour gauge group. Nevertheless, there is another possibility which will be considered in this paper in some detail, that is, instead of direct multiplication of $SU(3)_C$ with some G_{TC}, the $SU(3)_C$ is embedded in a larger simple group G_{EC} called extended colour gauge group. The reasoning is that, despite the fact that G_{EC} has the same coupling constant as its subgroup $SU(3)_C$, it is possible for some reaction channels in G_{EC} to be more attractive *) than those in $SU(3)_C$, because for G_{EC} there are more gauge bosons and in certain cases their interactions may add constructively.

If we limit ourselves to consider only SU(n) group, the smallest extended colour group will be SU(4). In this case the symmetry at intermediate energies is $SU(4)_{EC} \times SU(2)_L \times U(1)_Y$, and $SU(2)_L \times U(1)_Y$ is broken by a presumed condensation at 1 TeV induced by $SU(4)_{EC}$ interaction, which subsequently breaks to $SU(3)_C$ by itself, leading to the low energy symmetry $SU(3)_C \times U(1)_e$.

The SU(4) symmetry was considered by Pati and Salam [3] and some others several years ago. The main distinction between this work and the earlier works lies in that the SU(4) here breaks at an intermediate energy, say 10^2 GeV, rather than at some high energy level, which makes the masses of gauge bosons associated with generators of $SU(4)_{EC}/SU(3)_C$ be of the same order as M_W and M_Z. In this scheme, not only the structure of neutral currents is different from the standard model but also some new charged currents will appear. Whether these results are consistent with the known facts of particle physics will be the severe test of this tentative proposal.

For the sake of reducing the number of parameters and also carrying the idea of unification through, we try to unify the intermediate symmetry group $SU(4)_{EC} \times SU(2)_L \times U(1)_Y$ further to a simple group SU(6). The breaking of SU(6) down to $SU(4)_{EC} \times SU(2)_L \times U(1)_Y$ is also assumed to be caused by the condensation of fermion pairs like a tumbling theory proposed by Raby et al. [4] In order to get three generations of ordinary fermions and to meet the requirements of asymptotic freedom, of absence of anomalies and of no bizarre colour fermions, we take the whole left-handed fermions in the representation $[3]_6 + 3[2]_6 + 6[5]_6$, with the symbol $[k]_n$ denoting the totally antisymmetric representations formed by a product of k fundamental representations of SU(n). The coefficients here for the three irreducible representations meet the criterion proposed by Frampton [5] that they have no common factors. Nevertheless,

*) For instance, the one boson exchange potential for reaction channel $\psi^1_L + \psi^2_L \to \phi$ is proportional to $\frac{1}{2} g^2[C_2(R_\phi) - C_2(R_1) - C_2(R_2)]$, in which C_2's are Casimir operators and R_ϕ, R_1, R_2 stand for the representations related to ϕ, ψ^1_L, ψ^2_L. Hence its strength not only depends on coupling constant g^2 but also on relevant Casimir operators [3]. When $C_2(R\ ) - C_2(R_1) - C_2(R_2) < 0$, the potential is attractive. It deserves to be mentioned that the effect of symmetry breaking has not been taken into account.

they are still unsatisfactory, because for DSB the fact that not all the coefficients are equal to unity implies unwanted Goldstone bosons, which turns out to be one of the main problems for this model.

At present there is no reliable theory for DSB. Does DSB really take place? If so, in which channel and at which mass scale will it occur? Which direction, if there are several inequivalent directions for the condensate, will the vacuum condensate take? All these questions are still not clear yet. The purpose of this paper is not to try to derive the pattern of DSB according to some basic rules, but is instead to see whether it is possible to choose phenomenologically some channels of condensation to realize the required pattern of symmetry breaking mentioned above, namely $SU(6) \to SU(4)_{EC} \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_e$, and to see, in the case that this pattern of breaking is possible, what consequences will result. In determining which directions the vacuum condensate may take, some effective Higgs formulation will often be utilized.

We shall show that for the fermion assignment [3] + 3[2] + 6[5] three generations of low masses will result, with some heavy particles and also some additional massless neutral particles, but the t-quark possesses abnormal properties: both t_L and t_R have value $\frac{1}{2}$ for I_3, and belong to two SU(2) doublets $\begin{pmatrix} t \\ b \end{pmatrix}_L$ and $\begin{pmatrix} t \\ \beta \end{pmatrix}_R$, respectively. At low energies the right-handed current of t-quark will not manifest itself, for the mass of β is presumably of order 1 TeV. In this scheme the ordinary leptons are $SU(4)_{EC}$ singlets, the particles of "fourth" colour as proposed by Pati and Salam [3] (here they are labelled index zero, so hereinafter they will be called of zeroth colour) are referred to as exotic leptons which, except for three neutral massless ones, presumably have much larger masses and distinct "weak" interactions.

The charged currents at low energies now contain a new part related to the off-diagonal generators of $SU(4)_{EC}/SU(3)_C$. The associated bosons are denoted by V_μ's. Under the condition that the mixing among certain neutral exotic leptons is extremely small, the new charge currents will not cause any obvious trouble, because in this case the V_μ bosons connect the ordinary quarks only to presumably heavy exotic leptons.

The structure of neutral currents also shows some new features. Now there are two neutral currents which can be specified with two mixing angles θ_1 and θ_2. But the known eight neutral current parameters for ν-e, ν-quark and e-quark scattering are only related to θ_1, with expressions different from those in the Weinberg-Salam model. Nevertheless, their theoretical values determined by use of renormalization group equations, fit the experimental data quite well.

The model is also interesting in that it presents many of predictions about new particles and new weak interactions at the TeV energy region, to which new experiments will be sensitive. On the other hand, there are also some severe problems which seem to make this SU(6) model unlikely to be a realistic one. Nevertheless, at least the group $SU(4)_{EC} \times SU(2)_L \times U(1)_Y$ as an intermediate symmetry, according to our view, deserves further investigation.

II. PATTERNS OF SYMMETRY BREAKING

The gauge group SU(6) has 35 generators, the associated gauge bosons are denoted generally by A_i^j $(i,j = 0,1,2,3,4,5)$ with (0,1,2,3) labelling $SU(4)_{EC}$ indices and (5,6) labelling SU(2) indices. It is convenient for later reference to specify the special symbols for those off-diagonal bosons which are to acquire masses in result of symmetry breakings. Their charges are also listed:

$$\left(\begin{array}{cccc|cc} & V_1^+ & V_2^+ & V_3^+ & X_0^+ & Y_0^+ \\ \hline V_1 & & & & X_1^+ & Y_1^+ \\ V_2 & & & & X_2^+ & Y_2^+ \\ V_3 & & & & X_3^+ & Y_3^+ \\ \hline X_0 & X_1 & X_2 & X_3 & & W^+ \\ Y_0 & Y_1 & Y_2 & Y_3 & W^- & \end{array}\right) \left(\begin{array}{cccc|cc} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -1 & 0 \\ \hline -\frac{1}{3} & & & & -\frac{4}{3} & -\frac{1}{3} \\ -\frac{1}{3} & & & & -\frac{4}{3} & -\frac{1}{3} \\ -\frac{1}{3} & & & & -\frac{4}{3} & -\frac{1}{3} \\ \hline 1 & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & 0 & 1 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -1 & 0 \end{array}\right)$$

in which (1,2,3) stand for $SU(3)_C$ indices.

The whole left-handed fermions are assigned to the representation $[3]_6 + 3[2]_6 + 6[5]_6$, among which $[3]_6$ is real. Nevertheless, the fermions in $[3]_6$ cannot have bare mass, because such SU(6) invariant mass term are forbidden by Fermi statistics. We shall see hereinafter that three components in $[3]_6$, actually the t_i^c $(i = 1,2,3)$, are supposed to survive to ordinary mass scale, which accounts for the abnormality of the t quark.

Since no elementary Higgs scalar is introduced, the Lagrangian density has a simple form

$$\mathcal{L} = -\frac{1}{4} \operatorname{tr} \boldsymbol{F}_{\mu\nu} \boldsymbol{F}_{\mu\nu} - \bar{\psi} \gamma_\mu (\partial_\mu - ig\boldsymbol{A}_\mu) \psi \qquad (2.1)$$

with ψ denoting the whole left-handed fermions (in a reducible representation) Those fermions transforming according to irreducible representations $[3]_6$, $[2]_6$ and $[5]_6$ are denoted by ψ_{ijk}, ψ_{ij} and ψ^i, respectively.

The representation matrices L_α for ψ_{ijk}, ψ_{ij} and ψ^i can be constructed directly from the fundamental representation matrices, so that whenever we specify a generator, we only need to write out explicitly the corresponding 6×6 matrix for fundamental representation, which will be denoted by $\hat{F}_\alpha$ and normalized according to

$$\operatorname{tr} \hat{F}_\alpha \hat{F}_\beta = \frac{1}{2} \delta_{\alpha\beta} . \qquad (2.2)$$

Also for convenience of later reference, we specify in advance the matching of fermions into conjugate pairs in result of subsequent symmetry breakings, together with their charge. The boldface types are used to denote those interaction bases which are or can be chosen as mass eigenstates.

Symbols and matching

$$[3]_6: \quad \underset{\psi_{0ij}}{\begin{pmatrix} 0 & \mathbf{q}_3^c & -\mathbf{q}_2^c & \mathbf{T}_1 & B_1 \\ & 0 & \mathbf{q}_1^c & \mathbf{T}_2 & B_2 \\ & & 0 & \mathbf{T}_3 & B_3 \\ & & & 0 & \mathbf{L}^+ \\ & & & & 0 \end{pmatrix}_L} \quad \underset{\psi_{1ij}}{\begin{pmatrix} 0 & \mathbf{L}^- & \mathbf{b}_3^c & t_3^c \\ & 0 & -\mathbf{b}_2^c & -t_2^c \\ & & 0 & \mathbf{q}_1 \\ & & & 0 \end{pmatrix}_L} \quad \underset{\psi_{2ij}}{\begin{pmatrix} 0 & \mathbf{b}_1^c & t_1^c \\ & 0 & \mathbf{q}_2 \\ & & 0 \end{pmatrix}_L} \quad \underset{\psi_{3ij}}{\begin{pmatrix} 0 & \mathbf{q}_3 \\ & 0 \end{pmatrix}_L}$$

$$3[2]_6: \begin{pmatrix} 0 & \beta_1 & \beta_2 & \beta_3 & \ell_\tau^+ & n_\tau^c \\ & 0 & T_3^c & -T_2^c & t_1 & b_1 \\ & & 0 & T_1^c & t_2 & b_2 \\ & & & 0 & t_3 & b_3 \\ & & & & 0 & \tau^+ \\ & & & & & 0 \end{pmatrix}_L, \quad \begin{pmatrix} 0 & S_1 & S_2 & S_3 & \ell_\mu^+ & n_\mu^c \\ & 0 & c_3^c & -c_2^c & c_1 & s_1 \\ & & 0 & c_1^c & c_2 & s_2 \\ & & & 0 & c_3 & s_3 \\ & & & & 0 & \mu^+ \\ & & & & & 0 \end{pmatrix}_L, \quad \begin{pmatrix} 0 & D_1 & D_2 & D_3 & \ell_e^+ & n_e^c \\ & 0 & u_3^c & -u_2^c & u_1 & d_1 \\ & & 0 & u_1^c & u_2 & d_2 \\ & & & 0 & u_3 & d_3 \\ & & & & 0 & e^+ \\ & & & & & 0 \end{pmatrix}_L.$$

$$6\,[5]_6: \quad (\lambda_B\ B_1^c\ B_2^c\ B_3^c\ \ell_\tau^-\ \lambda_\tau)_L,\ (\lambda_S\ S_1^c\ S_2^c\ S_3^c\ \ell_\mu^-\ \lambda_\mu)_L,\ (\lambda_D\ D_1^c\ D_2^c\ D_3^c\ \ell_e^-\ \lambda_e)_L.$$

$$(n_\tau\ b_1^c\ b_2^c\ b_3^c\ \tau^-\ \nu_\tau)_L,\ (n_\mu\ s_1^c\ s_2^c\ s_3^c\ \mu^-\ \nu_\mu)_L,\ (n_e\ d_1^c\ d_2^c\ d_3^c\ e^-\ \nu_e)_L.$$

Charges

$$[3]_6: \begin{pmatrix} 0 & -\frac{2}{3} & -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ & 0 & -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ & & 0 & \frac{2}{3} & -\frac{1}{3} \\ & & & 0 & 1 \\ & & & & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & -1 & \frac{1}{3} & -\frac{2}{3} \\ & 0 & \frac{1}{3} & -\frac{2}{3} \\ & & 0 & \frac{2}{3} \\ & & & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & \frac{1}{3} & -\frac{2}{3} \\ & 0 & \frac{2}{3} \\ & & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & \frac{2}{3} \\ & 0 \end{pmatrix}.$$

$$[2]_6: \begin{pmatrix} 0 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 1 & 0 \\ & 0 & -\frac{2}{3} & -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ & & 0 & -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ & & & 0 & \frac{2}{3} & -\frac{1}{3} \\ & & & & 0 & 1 \\ & & & & & 0 \end{pmatrix}$$

$$[5]_6: \quad \left(0\ \ \tfrac{1}{3}\ \ \tfrac{1}{3}\ \ \tfrac{1}{3}\ \ -1\ \ 0\right).$$

The evolution equation for coupling constant g is of the form

$$M \frac{\partial g(M)}{\partial M} = \beta\, g^3(M) + O(g^5) \quad , \tag{2.3}$$

where

$$\beta = -\frac{1}{3(4\pi)^2}\,[11\, C_2(A) - 2T_f] \tag{2.4}$$

with $C_2(A)$ denoting the quadratic Casimir operator for the adjoint representation and T_f is given by

$$T_f\, \delta_{ij} = \mathrm{Tr}\,(\hat{L}_i\, \hat{L}_j) \tag{2.5}$$

in which $\hat{L}_i$ are representation matrices for the whole left-handed fermions. For SU(n) group, $C_2(A) = n$ and

$$T_f(n,k) = \frac{1}{2}\begin{pmatrix} n-2 \\ k-1 \end{pmatrix} = \frac{1}{2}\,\frac{(n-2)!}{(k-1)!\,(n-k-1)!} \tag{2.6}$$

where (n.k) refers to totally antisymmetric representation $[k]_n$.

Above the grand unification mass scale, the value of $2T_f$ for the whole left-handed fermions is equal to 24, so that $\beta < 0$ and the theory is asymptotically free. At a certain mass scale M_1 where the coupling constant reaches an appropriate value, a condensation of fermion pairs presumably results. It is assumed to take place in the channel

$$[3]_6 \times [3]_6 \to \underline{35} \quad .$$

with $\underline{35}$ denoting the adjoint representation. Recall that the channel $[3]_6 \times [3]_6 \to [0]_6$ is forbidden by Fermi statistics. The SU(6) one-boson exchange potential for this channel is attractive, since

$$C_2(A) - 2\, C_2(6,3) = -\frac{9}{2} \quad .$$

In view of HSB (symmetry breaking via Higgs particles), this condensation corresponds to assuming a 35-dimensional Higgs scalar.

In order to examine whether it is possible to break SU(6) into $SU(4) \times SU(2) \times U(1)$, an effective Higgs formulation is employed, with $\chi_i^{\,j}$ standing for the 35-dimensional condensate. Obviously, if the VEV (vacuum expectation value) of $\chi_i^{\,j}$ takes the form

$$\langle \chi_i{}^j \rangle_0 = w \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix} \qquad (2.7)$$

the desired symmetry breaking will result. The most general self-interaction potential among $\chi_i{}^j$ is

$$V(\chi) = -\frac{1}{2} a \operatorname{tr} \chi^2 - \frac{1}{4} b (\operatorname{tr} \chi^2)^2 + \frac{1}{2} c \operatorname{tr} \chi^4 + d \operatorname{tr} \chi^3. \quad (2.8)$$

According to Ruegg's analysis [6], (2.7) is likely to be the case when coefficients in (2.8) meet a certain condition.

It is convenient to take the generator of group $U(1)_Y$, the hypercharge Y, to have integral quantum numbers, namely

$$\hat{Y} = - \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix} . \qquad (2.9)$$

Such $\hat{Y}$ is not normalized, the normalized generator which satisfies (2.3) is related to $\hat{Y}$ by

$$\hat{F}_Y = \frac{1}{\sqrt{24}} \hat{Y} \quad . \qquad (2.10)$$

The coupling constants associated with $\hat{F}_Y$ and $\hat{Y}$ are denoted by g_Y and G_Y, respectively, with $g_Y = \sqrt{24}\ G_Y$.

We decompose $[3]_6$ with respect to irreducible representations of $SU(4)_{EC} \times SU(2)_L$, which are denoted by $([\ell]_4, [m]_2)$, to find out which components will acquire masses through this condensation. The result is as follows, with hypercharge Y also specified explicitly

$$[3]_6 \rightarrow ([3]_4, [0]_2, -3) + ([2]_4, [1]_2, 0) + ([1]_4, [2]_2, 3)$$

in which the values of Y for $([\ell]_4\ [m]_2)$ are determined by the formula

$$Y = -\ell + 2m \quad . \qquad (2.11)$$

$([1]_4, [2]_2, 3)$, namely $([1]_4, [0]_2, 3)$ can form an $SU(4)_{EC} \times SU(2)_L \times U(1)_Y$ invariant with $([3]_4, [0]_2, -3)$ and consequently they are to become conjugates of each other and acquire a superheavy mass of order M_1. $([2]_4, [1]_2, 0)$ remains massless because $([2]_4, [1]_2, 0) \times ([2]_4, [1]_2, 0) \rightarrow ([0]_4, [0]_2, 0)$ is forbidden by Fermi statistics. The massive quartet $([1]_4, [0]_2, 3)$ will be denoted by (L^+, Q_1, Q_2, Q_3), with L^- referred to as an exotic lepton of zeroth colour and Q_i as exotic quarks, since both of their left-handed and right-handed components are $SU(2)_L$ singlets.

The sixteen gauge bosons associated with generators of $SU(6)/SU(4)_{EC} \times SU(2)_L \times U(1)_Y$ also acquire masses of order M_1, which are called $X_{i\mu}$ and $Y_{i\mu}$ bosons.

In mass range much below M_1, the coupling constants will evolve in accordance with renormalization group equation for group $SU(4)_{EC} \times SU(2)_L \times U(1)_Y$ with (L^+, Q_1, Q_2, Q_3) decoupled. Decomposing 3[2] + 6[5] in the same way as before, we get the total $SU(4)_{EC} \times SU(2)_L \times U(1)_Y$ content of the remaining massless fermions as

$$([2]_4, [1]_2, 0) + 3([2]_4, [0]_2, -2) + 3([1]_4, [1]_2, 1) + 3([0]_4, [2]_2, 4)$$
$$+ 6([4]_4, [1]_2, -2) + 6([3]_4, [2]_2, 1)$$

among which only $([2]_4, [1]_2, 0)$ comes from $[3]_6$.

The values of $2T_f$ for subgroups $SU(4)_{EC}$, $SU(2)_L$ and $U(1)_Y$ below the mass scale M_1 are 22, 24 and 18, respectively. Hence the extended colour group $SU(4)_{EC}$ is also asymptotic free and is supposed to induce a second condensation at mass scale M_2 of the desired order of 1 TeV. The condensation channel is taken to be

$$([2]_4, [1]_2, 0) \times ([2]_4, [0]_2, -2) \rightarrow ([0]_4, [1]_2, -2)$$

for which the one-boson exchange potential is also attractive, since $C_2(4,0) - 2C_2(4,2) = -5$.

The content of fermion multiplets listed above contains three $([2]_4, [0]_2, -2)$ multiplets, coming from three $[2]_6$, respectively, but only one $([2]_4, [1]_2, 0)$ multiplet. So in general it is some linear combination of the three $([2]_4, [0]_2, -2)$ that will condense with $([2]_4, [1]_2, 0)$. Nevertheless, one can always perform a transformation among the three $[2]_6$ to make the condensate only relate to the first one.

The bound pair formed by this condensation is an $SU(4)_{EC}$ singlet, so that it does not break $SU(4)_{EC}$. However, it is a doublet with respect to $SU(2)_L$ and of non-zero hypercharge, like the Higgs field introduced in the W-S model, and consequently breaks $SU(2)_L \times U(1)_Y$ to an $U(1)'$, leading to a symmetry $SU(4)_{EC} \times U(1)'$. As before one may use an effective doublet $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ to represent the condensate $([0]_4, [1]_2, -2)$ and write down the effective potential as

$$V(\phi) = -\frac{1}{2}\mu^2\phi^+\phi + \frac{1}{2}\lambda(\phi^+\phi)^2 \quad . \tag{2.12}$$

The VEV, which minimizes $V(\phi)$, can always be taken as

$$\langle\phi\rangle = v\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad , \quad v^2 = \frac{\mu^2}{\lambda} \tag{2.13}$$

so that $W_\mu^\pm$ and $Z_{1\mu}$($Z_{1\mu} = \cos\theta_1 W_{3\mu} - \sin\theta_1 B_\mu$, with B_μ standing for the gauge boson associated with the hypercharge) will acquire mass through this condensation, while $A'_\mu = \sin\theta_1 W_{3\mu} + \cos\theta_1 B_\mu$ remains massless. The values of θ_1 as well as M_W, M_{Z1} are given by

$$tg^2\theta_1 = \frac{2}{3}\frac{g_Y^2}{g_L^2} \tag{2.14}$$

$$M_W^2 = \frac{1}{2}g_L^2 v^2 \quad , \quad M_{Z1}^2 = \frac{M_W^2}{\cos^2\theta_1} \quad . \tag{2.15}$$

The expressions for $\hat{K}'$ and $\hat{K}_1$, the generator associated with A'_μ and $Z_{1\mu}$, respectively, as well as for the corresponding coupling constants are:

$$\hat{K}' = \hat{I}_3 + \frac{1}{4}\hat{Y} \quad , \quad G' = g_L \sin\theta_1 \quad , \tag{2.16}$$

$$\hat{K}_1 = \hat{I}_3 - \frac{1}{4}tg^2\theta_1 \hat{Y}, \ G_1 = g_L\cos\theta_1 \quad . \tag{2.17}$$

Hereinafter the current coupled to $Z_{1\mu}$ is referred to as the first neutral current.

Under the $SU(4)_{EC} \times U(1)'$, $([2]_4, [1]_2, 0)$ and $([2]_4, [0]_2, -2)$ which form the condensate, decompose as

$$([2]_4, [1]_2, 0) \longrightarrow \begin{Bmatrix} [2]_4, \frac{1}{2} \\ [2]_4, -\frac{1}{2} \end{Bmatrix} \quad , \tag{2.18}$$

$$([2]_4, [0]_2, -2) \longrightarrow ([2]_4, -\frac{1}{2}) \quad . \tag{2.19}$$

Hence the fermions which acquire masses through the second condensation are sextet $([2]_4 \frac{1}{2})$ in (2.18) and $([2]_4, -\frac{1}{2})$ in (2.19). They become conjugates of each other with masses of order M_2 ($\sim$1 TeV) and will be denoted by

$$\begin{pmatrix} 0 & \beta_1 & \beta_2 & \beta_3 \\ & 0 & T_3^c & -T_2^c \\ & & 0 & T_1^c \\ & & & \end{pmatrix}_L \quad .$$

The left-handed fields β_L and right-handed fields T_R are SU(2) singlets, while the right-handed fields β_R and left-handed fields T_L belong to two SU(2) doublets, respectively. As regards $SU(3)_C$, T_i and β_i each make up a triplet. Hereinafter they are also referred to as exotic quarks (T_i differ from the ordinary u_i quarks with regard to neutral currents).

The fermions which still remain massless are

$$3([2]_4,-\tfrac{1}{2}) + 3([1]_4,\tfrac{3}{4}) + 3([1]_4,-\tfrac{1}{4}) + 3([0]_4,1) + 6([0]_4,0) + 6([0]_4,-1) + 6([3]_4,\tfrac{1}{4}) \cdot$$

Below mass scale M_2, T and β decouple from the interaction, hence the value of $2T_f$ for $SU(4)_{EC}$ reduces to 18 *). The $SU(4)_{EC}$ interaction of course remains asymptotically free, so a third condensation is supposed to occur at mass scale M_3 in the channel

$$([2]_4, -\frac{1}{2}) \times ([3]_4, \frac{1}{4}) \longrightarrow ([1]_4, -\frac{1}{4})$$

with $C_2(4,1) - C_2(4,2) - C_2(4,3) = -\frac{5}{2}$. Denoting the $SU(4)_{EC}$ quartet effective scalar by φ, one has

$$V(\varphi) = -\frac{1}{2}\mu'^2\varphi^+\varphi + \frac{1}{4}\lambda'(\varphi^+\varphi)^2 \quad . \tag{2.20}$$

*) At the range from M_2 to M_W, the $2T_f$ for $SU(2)_L$ and U(1) also reduce to 21 and 16, respectively.

The VEV, which minimizes $V(\varphi)$, can always be taken as

$$\langle\varphi_0\rangle_0 = v' = \sqrt{\frac{\mu'^2}{\lambda'}} \quad , \quad \langle\varphi_1\rangle_0 = \langle\varphi_2\rangle_0 = \langle\varphi_3\rangle_0 = 0 \tag{2.21}$$

$SU(4)_{EC} \times U(1)'$ is therefore broken to $SU(3)_C \times U(1)_e$, yielding mass

$$M_V^2 = \frac{1}{2} g_C^2 v'^2 \tag{2.22}$$

for the charged boson V_μ. The neutral boson B'_μ among $SU(4)_{EC}/SU(3)_C$, which is associated with the generator

$$\hat{F}_{Y'} = \frac{1}{\sqrt{24}} \hat{Y}' \quad , \quad \hat{Y}' = \begin{pmatrix} 3 &&&&& \\ & -1 &&&& \\ && -1 &&& \\ &&& -1 && \\ &&&& 0 & \\ &&&&& 0 \end{pmatrix} \tag{2.23}$$

will mix with A'_μ to give a massive $Z_{2\mu}$ and a massless A_μ which is referred to as a photon :

$$\begin{aligned} Z_{2\mu} &= \cos\theta_2 B'_\mu - \sin\theta_2 A'_\mu \\ A_\mu &= \cos\theta_2 A'_\mu + \sin\theta_2 B'_\mu \quad . \end{aligned} \tag{2.24}$$

It is easy to find out the generators $\hat{K}_2$ and $\hat{Q}$ which are associated with $Z_{2\mu}$ and A_μ respectively, as well as corresponding coupling constants G_2 and e, with the result that

$$\begin{aligned} \hat{K}_2 &= \hat{Q} - \frac{1}{12 \sin^2\theta_2} \hat{Y}' \quad , \quad \hat{Q} = \hat{I}_3 + \frac{1}{4}\hat{Y} + \frac{1}{12}\hat{Y}' \\ \sin^2\theta_2 &= \frac{1}{1 + 6g_C^2\left(\frac{1}{g_L^2} + \frac{3}{2}\frac{1}{g_Y^2}\right)} \\ \frac{1}{e^2} &= \frac{1}{g_L^2} + \frac{3}{2}\frac{1}{g_Y^2} + \frac{1}{6}\frac{1}{g_C^2} = \frac{1}{6g_C^2 \sin^2\theta_2} \\ G_2^2 &= e^2 \operatorname{tg}^2\theta_2 = g_L^2 \sin^2\theta_1 \sin^2\theta_2 \quad . \end{aligned} \tag{2.25}$$

The mass of charged V_μ when expressed in terms of θ_2 and M_W is of the form

$$M_V^2 = \frac{1}{6} k^2 \sin^2\theta_1 \operatorname{ctg}^2\theta_2 M_W^2 \ , \quad k^2 = \frac{v'^2}{v^2} \quad , \tag{2.26}$$

while the mass of $Z_{2\mu}$ relates to M_V through

$$M_{Z2}^2 = \frac{3}{2} \frac{M_V^2}{\cos^2\theta_2} \tag{2.27}$$

The $([2]_4, -\frac{1}{2})$ and $([3]_4, \frac{1}{4})$ which participate in the condensate are decomposed with respect to $SU(3)_C \times U(1)_e$ to yield

$$([2]_4, -\tfrac{1}{2}) \to ([2]_3, -\tfrac{2}{3}) + ([1]_3, -\tfrac{1}{3})$$

$$([3]_4, \tfrac{1}{4}) \to ([3]_3, 0) + ([2]_3, \tfrac{1}{3})$$

which implies the fermions that gain mass this time must belong to $([1]_3, -\frac{1}{3})$ and $([2]_3, \frac{1}{3})$. Note that there are three $([2]_4, -\frac{1}{2})$ multiplets, among which one comes from $[3]_6$ and other two from the second and third $[2]_6$, and that six $([3]_4, \frac{1}{4})$ multiplets come from six $[5]_6$ respectively. So the mass eigenstates for the three colour triplets $([1]_3, -\frac{1}{3})$, which are denoted by B_{iL}, S_{iL} and D_{iL}, are in general mixtures of the interaction bases owing to the fact that they come from three unsymmetric $([2]_4, -\frac{1}{2})$, while the six $([2]_3, \frac{1}{3})$ which are completely symmetric can always be re-defined to make the three mass eigenstates B^c_{iL}, S^c_{iL} and D^c_{iL} be just the first three ones.

These particles of mass M_2 are $SU(2)_L$ singlets except for B_{iL}. B,S,D are all referred to as exotic quarks, since B_{iL} also differ from the ordinary b_{iL} with regard to neutral currents.

The remaining massless fermions decompose with respect to $SU(3)_C \times U(1)_e$ as

$$3([1]_3,\tfrac{2}{3}) + 3([2]_3,-\tfrac{2}{3}) + 3([1]_3,-\tfrac{1}{3}) + 3([2]_3,\tfrac{1}{3}) + 6([0]_3,-1) + 6([0]_3,1) + 15([0]_3,0)$$

so that they form real representation(reducible) of $SU(3)_C \times U(1)_e$, which is essential for any theory to be reasonable.

The value of $2T_f$ for $SU(3)_C$ is 12, therefore SU(3) interaction is also asymptotically free, which is presumed to induce further condensations in channel $([1]_3, \frac{2}{3}) \times ([2]_3, -\frac{2}{3}) \to ([0]_3, 0)$ as well as $([1]_3, -\frac{1}{3}) \times ([2]_3, \frac{1}{3}) \to ([0]_3, 0)$ with $C_3(3,0) - C_2(3,1) - C_2(3,2) = -\frac{8}{3}$. All the $SU(3)_C$ triplets match up to form three generations of ordinary quarks except that t_R has a different I_3 value.

The mixing also occurs in this condensation. For the right-handed fields d_R, s_R and b_R we are free to pick the interaction bases to coincide with the mass eigenstates, but the other mass eigenstates are mixtures of interaction bases.

As for the masses of leptons (which refer to SU(3) singlets), apart from the superheavy exotic ones, all the others do not acquire masses through condensation. They become massive only by means of some radiation correction mechanism. For example, e, μ, τ can acquire masses via ordinary quarks as intermediate states (Fig.1) and the qualitative relation $m_e < m_\mu < m_\tau$ follows naturally from $m_d \ll m_s \ll m_b$, but quantitatively there are some problems.

The charged exotic leptons ℓ acquire masses through diagrams in Fig.2 which implies ℓ_e, ℓ_μ and ℓ_τ will in general mix with each other.

Comparing Fig.2 with Fig.1 and taking into account that D, S, B are presumably heavier than b, one may suppose ℓ_e, ℓ_μ, ℓ_τ are also heavier than τ, which accounts for why they are not observed yet.

Similarly the neutral exotic leptons become massive through diagrams like Fig.3(a). It is reasonable to assume that the amplitudes of diagrams like Fig.3(b) are far smaller than those of Fig.3(a), implying n's are almost unmixed with λ's. The exotic leptons λ's are therefore substantially massless.

We note that no substantial mixing of n_e with λ_e (and neither n_μ with λ_μ) is essential for this model, otherwise there will be severe trouble as will be seen in the next section.

III. EVALUATION OF PARAMETERS

We shall examine what consequence will result in the case that the above-mentioned pattern of symmetry breaking is realized in reality. In this section it is shown that by renormalization group equations we can basically determine various parameters for the high energy as well as for the intermediate energy scale, only one parameter namely $k^2 \equiv (\frac{v'}{v})^2$ remains undetermined.

The pattern of symmetry breaking can be described as

$$SU(6) \xrightarrow{M_1} SU(4)_{EC} \times SU(2)_L \times U(1)_Y \xrightarrow{M_3} SU(3)_C \times U(1)_e$$

with the second and the third breakings at the same scale M_3. This is due to the fact that W_μ and $Z_{1\mu}$ acquire masses only of order 10^2 GeV, presumably nearer to the scale M_3, other than of order 1 TeV as the fermions participating in the second condensation do (the reduction of M_W and M_{Z1} comes from their dependence on g_L^2). In the treatment of evolution equations, we can simply take the pattern of symmetry breaking as above without introducing appreciable errors.

For the sake of simplicity only leading terms are kept in the evolution equations and all thresholds are treated as step functions. The masses of various exotic particles other than the superheavy L and Q_1 are taken as follows: 10^3 GeV, namely M_2, for T, β; M_3 for B, S, D, ℓ, n; zero for λ. The mass of the unobserved t-quark is also taken as M_3. It deserves to be mentioned that the results are insensitive to the precise value of M_3. For definite, M_3 will be taken in the numerical calculation to be 10^2 GeV.

Among the parameters to be determined are:

1) M_1, α_6 at high energy scale

2) α_C, α_L, α_Y, M_W, M_{Z1}, M_{Z2}, M_V at intermediate energy scale.

The expressions for α_e as well as mixing angles θ_1, θ_2 in terms of α_C, α_L and α_Y *) are

$$\frac{1}{\alpha_e} = \frac{1}{\alpha_L} + \frac{3}{2}\frac{1}{\alpha_Y} + \frac{1}{6}\frac{1}{\alpha_C} \tag{3.1}$$

$$\sin^2\theta_1 = \frac{1}{1 + \frac{3}{2}\frac{\alpha_L}{\alpha_Y}} \quad , \quad \sin^2\theta_2 = \frac{\alpha_e}{6\alpha_C} \tag{3.2}$$

or inversely

$$\alpha_C = \frac{\alpha_e}{6\sin^2\theta_2} \quad , \quad \alpha_L = \frac{\alpha_e}{\sin^2\theta_1\cos^2\theta_2} \quad , \quad \alpha_Y = \frac{3\alpha_e}{2\cos^2\theta_1\cos^2\theta_2} \tag{3.3}$$

The boson masses when expressed in terms of G_F, α_e, θ_1, θ_2 and k^2 are of the form

$$M_W^2 = \frac{e^2}{4\sqrt{2}\, G_F \sin^2\theta_1\cos^2\theta_2} \quad , \quad M_{Z1}^2 = \frac{M_W^2}{\cos^2\theta_1} \quad ,$$
$$M_V^2 = \frac{k^2 e^2}{24\sqrt{2}\, G_F \sin^2\theta_2} \quad , \quad M_{Z2}^2 = \frac{3}{2}\frac{M_V^2}{\cos^2\theta_2} \tag{3.4}$$

*) Hereinafter α_C, α_L, α_Y, θ_1, θ_2 all refer to values at M_3, unless otherwise noted.

From the evolution equations together with the $2T_f$ values in the region M_1 to M_2 and M_2 to M_3, one gets

$$\frac{1}{\alpha_Y} = \frac{1}{\alpha_6(M_1)} + \frac{3}{\pi}\ln\frac{M_1}{M_2} + \frac{8}{3\pi}\ln\frac{M_2}{M_3},$$

$$\frac{1}{\alpha_L} = \frac{1}{\alpha_6(M_1)} + \frac{1}{3\pi}\ln\frac{M_1}{M_2} - \frac{1}{6\pi}\ln\frac{M_2}{M_3}, \tag{3.5}$$

$$\frac{1}{\alpha_C} = \frac{1}{\alpha_6(M_1)} - \frac{11}{3\pi}\ln\frac{M_1}{M_2} - \frac{13}{\pi}\ln\frac{M_2}{M_3},$$

so that

$$\frac{1}{\alpha_e} = \frac{8}{3}\frac{1}{\alpha_C} + \frac{14}{\pi}\ln\frac{M_1}{M_3} + \frac{2}{3\pi}\ln\frac{M_2}{M_3}.$$

Therefore one has

$$M_1 = M_3\, e^{\frac{\pi}{14}\left(\frac{1}{\alpha_e} - \frac{8}{3\alpha_C}\right)} \left(\frac{M_3}{M_2}\right)^{\frac{1}{21}} \cong M_3\, e^{\frac{\pi}{14}\left(\frac{1}{\alpha_e} - \frac{8}{3\alpha_C}\right)}, \tag{3.6}$$

together with

$$\sin^2\theta_1 = \frac{2}{7}\,\frac{1 + 5\sin^2\theta_2 - \frac{\alpha_e}{12\pi}\ln\frac{M_2}{M_3}}{\cos^2\theta_2} \cong \frac{2}{7}\,\frac{1+5\sin^2\theta_2}{\cos^2\theta_2}. \tag{3.7}$$

Note that (3.2) already shows how $\sin^2\theta_2$ is determined directly by α_e and α_C, now (3.7) further gives $\sin^2\theta_1$ in terms of $\sin^2\theta_2$. It still remains to determine α_e and α_C. The evolution equation for $\alpha_C(M)$ is of the form:

$$\frac{1}{\alpha_e(M)} = \frac{1}{\alpha_e(0)} - \frac{1}{3\pi}\sum_f q_f^2 \ln\frac{M^2}{m_f^2}, \tag{3.8}$$

where the sum extends over all fermions of charge q_f with mass $m_f^2 < M^2$ taking $\alpha(0) = \frac{1}{137}$ and evaluating the contributions of e, μ, u, d, s, c, τ, b individually according to their own masses, one gets

$$\alpha_e^{-1}(100\text{ GeV}) = 128.4 \tag{3.9}$$

$\alpha_C(M)$ at the b threshold m_b is evaluated by the formula

$$\frac{1}{\alpha_C(m_b)} = \frac{25}{6\pi}\ln\frac{m_b}{\Lambda} \tag{3.10}$$

so that

$$\frac{1}{\alpha_C(100\text{ GeV})} = \frac{1}{\alpha_C(m_b)} + \frac{23}{6\pi}\ln\frac{100\text{ GeV}}{m_b}$$
$$= \frac{25}{6\pi}\ln\frac{m_b}{\Lambda} + \frac{23}{6\pi}\ln\frac{100\text{ GeV}}{m_b} \tag{3.11}$$

With $m_b = 4.5$ GeV, $M_3 = 100$ GeV and Λ assuming the values 0.3 GeV, 0.4 GeV and 0.5 GeV one gets the results as follows:

	$1/\alpha_C$	$1/\alpha_L$	$1/\alpha_6$	M_1 (GeV)	$\sin^2\theta_2$	$\sin^2\theta_1$	M_W	M_{Z1}
$\Lambda = 0.3$ (GeV)	7.4	38.4	36.2	3.5×10^{12}	9.6×10^{-3}	0.30	70	84
$\Lambda = 0.4$	7.0	38.3	36.1	4.4×10^{12}	9.1×10^{-3}	0.30	70	84
$\Lambda = 0.5$	6.7	38.3	36.0	5.3×10^{12}	8.7×10^{-3}	0.30	70	84

We see that the grand unification mass scale M_1, which stands for M_X and M_Y and is crucial to the proton lifetime, turns out to be too small, and that the value of $\sin^2\theta_1$, which determines the existing eight Fermi type parameters for neutral current interaction (as will be seen in the next section), is almost fixed at the value 0.30. In fact even when α_e/α_C increases by a factor of 2 or reduce to zero, the $\sin^2\theta_1$ only changes to 0.32 or to $\frac{2}{7}$, respectively. As a result, the values of M_W and M_{Z1} are also stable.

IV. NEUTRAL CURRENTS, NEW CHARGED CURRENTS AND PROTON DECAY

$SU(4)_{EC} \times SU(2)_L \times U(1)_Y$ differs from $SU(3)_C \times U(2)_L \times U(1)_Y$ in that it has two neutral currents together with three new charged currents associated with $V_{i\mu}$. The two neutral current interactions are of the form:

$$G_1 J_{1\mu} Z_{1\mu} = i G_1 \bar{\Psi} \gamma_\mu (\hat{I}_3 - \tfrac{1}{4} tg^2\theta_1 \hat{Y}) \Psi Z_{1\mu}$$

$$= i \frac{G_1}{\cos^2\theta_1} \bar{\Psi} \gamma_\mu (\hat{I}_3 - \sin^2\theta_1 \hat{Q} + \tfrac{1}{12} \sin^2\theta_1 \hat{Y}') \Psi Z_{1\mu}, \tag{4.1}$$

$$G_2 J_{2\mu} Z_{2\mu} = i G_2 \bar{\Psi} \gamma_\mu (\hat{Q} - \frac{1}{12 \sin^2\theta_2} \hat{Y}') \Psi Z_{2\mu}$$

The Q and I_3 quantum numbers for ordinary quarks and leptons are as usual except for $I_3(t_R)$, which now assumes a value of $\frac{1}{2}$. The Y and Y' quantum numbers for ordinary quarks and leptons are

	left-handed quarks	d_R, s_R, b_R	u_R c_R t_R	left-handed leptons	right-handed leptons
Y	1	-1	2 2 0	-2	-4
Y'	-1	-1	2 2 2	0	0

It deserves to be mentioned that as far as leptons are concerned the second neutral boson $Z_{2\mu}$ behaves like a heavy photon with a different charge unit, so that it does not couple to the neutrino. In addition, one sees that t_R differs from u_R and c_R only in their first neutral current interaction.

The effective Hamiltonian of four-fermion neutral current interaction is of the form

$$\mathcal{H}_{eff} = -\frac{G_1^2}{2M_{Z1}^2} J_{1\mu} J_{1\mu} - \frac{G_2^2}{2M_{Z2}^2} J_{2\mu} J_{2\mu} \tag{4.2}$$

Among the experimentally investigated processes mediated via neutral current interactions are

(I) $\nu + e \to \nu + e$

(II) $\nu + u \to \nu + u$, $\nu + d \to \nu + d$

(III) $e + u \to e + u$, $e + d \to e + d$ (parity violation part).

Usually the $\mathcal{H}_{eff}$'s for these processes are parametrized with ten phenomenological parameters:

$$\mathcal{H}_{eff}^{I} = \frac{G_F}{\sqrt{2}} \bar{\nu} \gamma_\mu (1+\gamma_5) \nu \, [\bar{e} \gamma_\mu (g_V + g_A \gamma_5) e],$$

$$\mathcal{H}_{eff}^{II} = \frac{G_F}{\sqrt{2}} \bar{\nu} \gamma_\mu (1+\gamma_5) \nu \Big[\bar{u} \gamma_\mu \{U_L (1+\gamma_5) + U_R (1-\gamma_5)\} u + \bar{d} \gamma_\mu \{D_L (1+\gamma_5) + D_R (1-\gamma_5)\} d \Big], \tag{4.3}$$

$$\mathcal{H}_{eff}^{III} = \frac{G_F}{\sqrt{2}} \bar{e} \gamma_\mu \gamma_5 e \Big[\tfrac{1}{2} \tilde{\alpha} (\bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d) + \tfrac{1}{2} \tilde{\gamma} (\bar{u} \gamma_\mu u + \bar{d} \gamma_\mu d) \Big] + \frac{G_F}{\sqrt{2}} \bar{e} \gamma_\mu e \Big[\tfrac{1}{2} \tilde{\beta} (\bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d) + \tfrac{1}{2} \tilde{\delta} (\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d) \Big]$$

Comparing these with (4.1) and (4.2) one gets the expressions for these ten parameters in terms of θ_1, θ_2 and k. The results are tabulated as follows with corresponding expressions in W-S model also listed for comparison:

	g_V	g_A	U_L	U_R	D_L	D_R	$\tilde{\alpha}$	$\tilde{\gamma}$	$\tilde{\beta}$	$\tilde{\delta}$
SU(6)	$2\sin^2\theta_1 - \frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2} - \frac{3}{4}\sin^2\theta_1$	$-\frac{1}{2}\sin^2\theta_1$	$\frac{1}{4}\sin^2\theta_1 - \frac{1}{2}$	$\frac{1}{4}\sin^2\theta_1$	$\frac{7}{4}\sin^2\theta_1 - 1$	$\frac{3}{4}\sin^2\theta_1$	$\tilde{\beta}(6)$	$\tilde{\delta}(6)$
W-S	$2\sin^2\theta_W - \frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2} - \frac{2}{3}\sin^2\theta_W$	$-\frac{2}{3}\sin^2\theta_W$	$\frac{1}{3}\sin^2\theta_W - \frac{1}{2}$	$\frac{1}{3}\sin^2\theta_W$	$2\sin^2\theta_W - 1$	$\frac{2}{3}\sin^2\theta_W$	$-1 + 4\sin^2\theta_W$	0

with

$$\tilde{\beta}(6) = (-1 + 4\sin^2\theta_1)(1 - \tfrac{1}{4}\sin^2\theta_1) - \tfrac{4}{k^2}\sin^2\theta_2.$$

$$\tilde{\delta}(6) = \tfrac{1}{4}\sin^2\theta_1 (1 - 4\sin^2\theta_1) - \tfrac{4}{k^2}\sin^2\theta_2.$$

Among the ten parameters the first eight as well as the difference $\tilde{\beta}(6) - \tilde{\delta}(6)$ (= $-1 + 4 \sin^2\theta_1$) are independent of θ_2 and k. At present only the empirical values for the first eight parameters are available [7], which will be compared in the following with the theoretical values. Those for $SU(2)_L \times U(1)_Y$ (in which $\sin^2\theta_W$ is determined phenomenologically) and for SU(5) are also listed

	g_V	g_A	U_L	U_R	D_L	D_R	$\tilde{\alpha}$	$\tilde{\gamma}$
Emperical	0.06 ± 0.08	-0.52 ± 0.06	0.32 ± 0.03	-0.17 ± 0.02	-0.43 ± 0.03	-0.01 ± 0.05	-0.72 ± 0.25	0.36 ± 0.28
SU(6) ($\sin^2\theta_1 = 0.30$)	0.10	$-\frac{1}{2}$	0.27*	−0.15	−0.42	0.075*	−0.47	0.23
SU(5) ($\sin^2\theta_W = 0.20$)	−0.10*	$-\frac{1}{2}$	0.37*	−0.13*	−0.43	0.067*	−0.60	0.13
$SU(2)_L \times U(1)_Y$ ($\sin^2\theta_W = 0.235$)	−0.030*	$-\frac{1}{2}$	0.343	−0.157	−0.422	0.0783*	−0.530	0.157

The superscript * means that the value indicated lies outside the empirical range. One sees the fit of SU(6) predicted values with empirical data quite well, even better than those of SU(5).

The effect of mixing of fermions on the neutral current structure will be briefly discussed. The quarks d,s,b have the similar quantum numbers of I_3, Q and Y', so that their mixing does not effect the structure of neutral currents. The same applies for u_L, C_L and t_L. The only modification comes from the mixing among u_R, c_R and t_R. However the second neutral current remains unchanged, only an additional term $\Delta J_{1\mu}$ is added to $J_{1\mu}$ with

$$\Delta J_\mu = \frac{i}{2\cos^2\theta_1}\left(U_{13}^* \bar{u}_R + U_{23}^* \bar{c}_R + U_{33}^* \bar{t}_R\right)\gamma_\mu \left(U_{13} u_R + U_{23} c_R + U_{33} t_R\right) , \tag{4.4}$$

where U_{ij} stand for the elements of mixing matrix so that

$$(u_R \; c_R \; t_R) = (\mathbf{u}_R \; \mathbf{c}_R \; \mathbf{t}_R)\, U \quad . \tag{4.5}$$

(4.4) implies appearance of flavour change neutral currents among $\mathbf{u}_R$, $\mathbf{c}_R$, $\mathbf{t}_R$, together with a modification of their neutral current strength. This mixing must be small enough to avoid confliction with the existing data.

As regards to the new charged current interaction, only those parts related to u, d, c, s are needed to write out explicitly

$$\sqrt{2}\, i g_c \left[\epsilon_{ijk} \bar{D}_j \gamma_\nu u_k^c + \bar{l}_e^+ \gamma_\nu u_i + \bar{n}_e^c \gamma_\nu d_i + \epsilon_{ijk} \bar{S}_j \gamma_\nu c_k^c + \bar{l}_\mu^+ \gamma_\nu c_i + \bar{n}_\mu^c \gamma_\nu s_i\right]_L V_\nu^{i\dagger}$$
$$+ \frac{i}{\sqrt{2}} g_c \left[\bar{n}_e^c \gamma_\nu d_i + \bar{n}_\mu^c \gamma_\nu s_i\right]_R V_\nu^{i\dagger} \quad . \tag{4.6}$$

On account of

$$\frac{g_c^2}{M_V^2} = \frac{1}{k^2}\,\frac{g_L^2}{M_W^2} \tag{4.7}$$

together with k presumably smaller than unity, the new charged current interactions will be even stronger than the ordinary weak interactions. Its empirical absence is attributed to its association with heavy particles. It will manifest itself in the heavy particle decay if both of them exist.

It remains to consider the effect of this new charged current interaction on the decay of the proton. The $V_{1\mu}$ bosons alone cannot mediate proton decay. But with additional W_μ interaction, diagrams of higher order which may lead to proton decay are possible, such as Fig.4.

Nevertheless, these processes are unimportant because the phase space is extremely suppressed by the occurrance of so many final state particles. In case n_e^c is considerably mixed with λ_e^c, however, the proton lifetime may be greatly reduced by the process shown in Fig.5. So this mixing is required to be small (say $\lesssim 10^{-5}$). In addition, n-λ mixing leads to other unobserved processes such as in Fig.6. But the upper limit set by these processes is not so stringent as that by proton decay.

As far as n-λ mixing can be ignored, the proton decay is mainly mediated via superheavy bosons $X_{i\mu}$ and $Y_{i\mu}$ (i = 1,2,3) with diagrams similar to that in SU(5) theory. The interaction of $X_{0\mu}$ and $Y_{0\mu}$ with u,d,e is of the form:

$$-i\sqrt{2}\, g_6 \left[(\bar{D}_i \gamma_\mu u_i + \bar{n}_e^c \gamma_\mu e^+)_L X_{0\mu}^\dagger + (\bar{D}_i \gamma_\mu d_i + \bar{l}_e^+ \gamma_\mu e^+)_L Y_{0\mu}^\dagger\right] \tag{4.8}$$

so that it is unimportant to the proton decay.

The values of M_X and M_Y obtained in the previous section is of order 10^{12}-10^{13} GeV, which is too small. This is one of the main difficulties for this SU(6) model. It may be that the values of M_X and M_Y responsible for the proton lifetime differs considerably from those at the grand unification scale because the magnitude q^2 related to proton decay is only of order 1 GeV^2 and in the case of DSB M_X and M_Y is presumably soft. Nevertheless, we do not think this effect can account for such a large difference as two order of magnitude.

V. BRIEF SUMMARY

The possibility of a Higgsless grand unified model SU(6) with intermediate symmetry $SU(4)_{EC} \times SU(2)_1 \times U(1)_Y$ is discussed with the following results. The neutral currents fit the empirical data quite well. The new charged current also does not lead to difficulty as far as n-λ mixing is very small. The three generations of ordinary quarks are not precisely similar: the t quark is abnormal. Among the main difficulties are: appearance of unwanted Goldstone bosons associated with the breaking of additional global symmetries, a too small grand unification scale leading to an unacceptably short lifetime for proton decay, absence of current masses for ordinary quarks.

These problems may imply thatthe unification of $SU(4)_{EC} \times SU(2)_L \times U(1)_Y$ to SU(6) is unsuitable. Nevertheless the $SU(4)_{EC} \times SU(2)_L \times U(1)_Y$ itself as a possible intermediate symmetry may merit attention, because it is compatible with the present data and, in addition, predicts a lot of new particles as well as new interactions so that beyond the 100 GeV we still meet a luxuriant oasis.

ACKNOWLEDGMENTS

The authors would like to thank Professor Abdus Salam, the International Atomic Eenrgy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

REFERENCES

1) L. Susskind, Phys. Rev. D20, 2619 (1979).

2) S. Weinberg, Phys. Rev. D13, 974 (1976).

3) J.C. Pati and Abdus Salam, Phys. Rev. D8, 1240 (1973); D10, 275 (1974).

4) S. Raby, S. Dimopoulos and L. Susskind, Nucl. Phys. B169, 373 (1980).

5) P.H. Frampton, Phys. Letters 88B, 299 (1979).

6) H. Ruegg, SLAC-PUB-2518 (1980).

7) F. Dydak, Internaional Conference on High-Energy Physics, Proceedings, Geneva (1979), Vol.I, Sec.1.

FIGURE CAPTIONS

Fig.1 Diagrams generating masses for leptons e, μ, τ.

Fig.2 Diagrams generating masses for charged exotic leptons ℓ's.

Fig.3 Diagrams generating masses for neutral exotic leptons.

Fig.4 Diagram for proton decay via interactions mediated by V_μ and W_μ bosons.

Fig.5 Proton decay via V_μ and W_μ bosons together with n-λ mixing.

Fig.6 Diagrams leading to the processes $\pi^0 \to$ neutral particles and $K^- \to \pi^-$ + neutral particles in case of n-λ mixing.

FIG.1

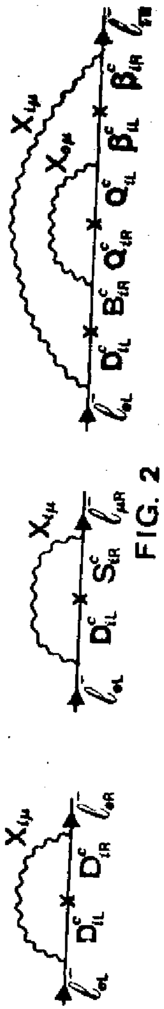

FIG. 2

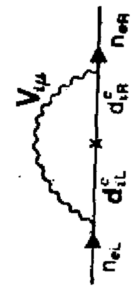

FIG.3(1)

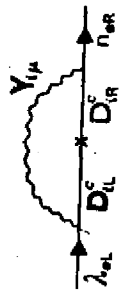

FIG.3(2)

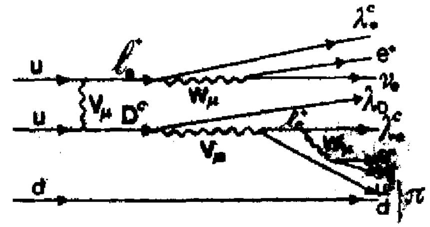

FIG.4

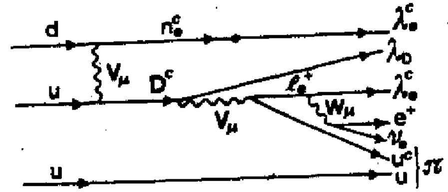

FIG.5

FIG.6

IC/81/154 J. CHELA-FLORES - Tests for complete breakdown of discrete symmetry in $D^0 - \bar{D}^0$

IC/81/155 J. CHELA-FLORES - Pairing in hadron structure.

IC/81/156 C.C. HAZRA and M.K. PARIDA - Factorization hypothesis and estimation of parity violating parameters in nuclear force.

IC/81/157 INT.REP.* A.H. NAYYAR and G. MURTAZA - Motion of a magnetic soliton about a lattice soliton in a Heisenberg chain.

IC/81/158 INT.REP.* J.A. MAGPANTAY - Higgs phenomenon via gauge invariants.

IC/81/159 INT.REP.* R.V. SHARMA - A note on the hard sphere model of surface entropy of liquid metals.

IC/81/160 INT.REP.* A. BREZINI and G. OLIVIER - Self-consistent theory of localization in binary alloys.

IC/81/161 INT.REP.* S. CHATTERJEE and S.V. SUBRAMANYAN - Radiation induced depinning of a charge density wave system.

IC/81/162 INT.REP.* S.I. HUSAIN and Z. AHSAN - A differentiable structure on space-time manifold corresponding to total pure radiation.

IC/81/163 INT.REP.* ZHANG LI-YUAN - An approximate method for calculating electron-phonon matrix element of a disordered transition metal and relevant comments on superconductivity.

IC/81/164 INT.REP.* ZHANG LI-YUAN and YIN DAO-LE - Energy band theory of heterometal superposed film and relevant comments on superconductivity in heterometal systems.

IC/81/165 INT.REP.* D. KUMAR - Proposal for a non-linear reactor.

IC/81/166 INT.REP. S. NARISON - On the Laplace transform of the Weinberg sum rules.

IC/81/167 INT.REP.* F. CLARO - Local fields in ionic crystals.

IC/81/168 ZHANG LI-YUAN - Peierls instability and superconductivity in substitutionally disordered pseudo one-dimensional conductors.

IC/81/169 INT.REP.* S.G. KAMATH - Penguin effects in $\Delta S \neq 0$ B decays in a five flavour $SU(2) \times U(1)$ model.

IC/81/170 INT.REP.* G. SENATORE and M.P. TOSI - Optimized random phase approximation for the structure of liquid alkali metals as electron-ion plasmas.

IC/81/171 INT.REP.* Z. BREZINI and G. OLIVIER - Study of the mobility edge in a Cayley tree.

IC/81/172 L. MASPERI and C. OMERO - Role of gauge invariance in a variational and mean-field calculation.

IC/71/173 A. KUNDU - Energy-spectrum against topological charge in a non-linear model in three space dimensions.

* Internal Reports: Limited distribution.

THESE PREPRINTS ARE AVAILABLE FROM THE PUBLICATIONS OFFICE, ICTP, PO BOX 586, I-34100 TRIESTE, ITALY.

IC/81/174 INT.REP.* A. HOLAS and K.S. SINGWI - Comment on the frequency dependence of the local field factor of an electron liquid.

IC/81/175 INT.REP.* G. KALMAN and K.I. GOLDEN - Correlation correction to plasmon dispersion: Rebuttal of a reply by Ichimaru, Totsuji, Tange and Pines.

IC/81/176 AHMED OSMAN - Elastic and rearrangement scattering between two interacting deuterons as a four-body problem.

IC/81/177 INT.REP.* S.P. de ALWIS and J.E. KIM - Supersymmetric SU(11), the invisible axion and proton decay.

IC/81/178 P. CARINI, G. KALMAN and K.I. GOLDEN - Exact dynamical polarizability for one-component classical plasmas.

IC/81/179 INT.REP.* LEAH MIZRACHI - Comments on "dyons of charge $e\theta/2\pi$".

IC/81/180 V. PAAR and S. BRANT - Signature effect in SU(6) model for quadrupole phonons.

IC/81/181 INT.REP.* Z.G. KOINOV and I.Y. YANCHEV - Path-integral calculation of the density of states in heavily doped strongly compensated semiconductors in a magnetic field.

IC/81/182 INT.REP.* C. PANAGIOTAKOPOULOS - Regularization of the quantum field theory of charges and monopoles.

IC/81/183 INT.REP.* A. SADIQ and M. AHSAN KHAN - Entropy of percolating and random infinite clusters.

IC/81/184 A.H. BASSYOUNI - Stark broadening of isolated U.V.-lines of LiI, AlI and SiI by a D.C.-ARC plasma without and with Cs.

IC/81/185 A.H. BASSYOUNI - Imprisonment of resonance radiation in atomic absorption spectroscopy.

IC/81/186 B. BUTI - Interaction of turbulent solar wind with cometary plasma tails.

IC/81/187 M.K. PARIDA and A. RAYCHAUDHURI - Low mass parity restoration, weak interaction phenomenology and grand unification.

IC/81/188 J. PRZYSTAWA - Symmetry and phase transitions.

IC/81/189 P. BUDINICH - On "conformal spinor geometry": An attempt to "understand" internal symmetry.

IC/81/190 Y. FUJIMOTO - The axion, fermion mass in SU(9) G.U.Th.

IC/81/191 U. KASPER - Remarks on the non-uniqueness of the canonical energy-momentum tensor of classical field theories.

IC/81/192 J. TARSKI - Path integral over phase space, their definition and simple properties.

IC/81/193 H.P. BERG - Classical mechanics of a breathing top.

IC/81/194 INT.REP.* K.G. AKDENIZ - On classical solutions of Gürsey's conformal-invariant spinor model.

IC/81/195 INT.REP.* ZHANG LI-NING - Graded de-Sitter space-time, group and algebra.

IC/81/196 INT.REP.* P. NIKOLOV - On the correspondence between infinitesimal and integral description of connections.

IC/81/197 A.R. CHOUDHARY - On an improved effective range formula.

IC/81/198 INT.REP.* K.R. PAINTER, P.J. GROUT, N.H. MARCH and M.P. TOSI - A model of the ice-d-electron metal interface.

IC/81/199 TEJ SRIVASTAVA - On the representation of generalized Dirac (Clifford) algebras.

IC/81/200 R. JENGO, L. MASPERI and C. OMERO - Variational discussion of the Hamiltonian Z(N) spin model in 1+1 and 2+1 dimensions.

IC/81/201 INT.REP.* D.A. AKYEAMPONG - Supersymmetry breaking in a magnetic field.

IC/81/202 INT.REP.* D.K. SRIVASTAVA - Integrals involving functions of the type $(WS)^q$.

IC/81/203 A. BULGAC, F. CARSTOIU, O. DUMITRESCU and S. HOLAN - Fermi liquid model of alpha decay.

IC/81/204 ANDRZEJ GOZDZ - Equivalence of the L-diminishing operator and the Hill-Wheeler projection technique and its application to the five-dimensional harmonic oscillator.

IC/81/205 M. CIAFALONI - Soft gluon contributions to hard processes.

IC/81/206 INT.REP.* Summer Workshop on High-Energy Physics (July-August 1981) (Collection of lecture notes).

IC/81/207 INTREP.* E. BOBULA and Z. KALICKA - Sufficient condition for generation of multiple solidification front in one-dimensional solidification of binary alloys.

IC/81/209 INT.REP.* A. SADIQ - Cultural isolation of third-World scientists.

IC/81/210 E.W. MIELKE - Reduction of the Poincaré gauge field equations by means of a duality rotation.

IC/81/211 ABDUS SALAM and J. STRATHDEE - On Kaluza-Klein theory.

IC/81/212 INT.REP.* N.H. MARCH and M.P. TOSI - Saturation of Debye screening length and the free energy of superionic PbF_2 .

IC/81/213 H.R. KARADAYI - Anatomy of grand unifying groups.

IC/81/214 A.R. CHOUDHARY - A study of S-wave n-p scattering using a new effective range formula.

IC/81/215 S. OLSZEWSKI - Hartree-Fock approximation for the one-dimensional electron gas.

IC/81/216 T. SRIVASTAVA - A two-component wave-equation for particles of spin-$\frac{1}{2}$ and non-zero rest mass (Part I).

IC/81/217 R.V. SHARMA, G. SENATORE and M.P. TOSI - Short-range ionic correlations in gold-cesium melts.