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Abstract

The moment of inertia of the yrast superdeformed band in ^{149}Gd exhibits an unexpected bifurcation at high rotational frequency. States differing by four units of angular momentum show an energy shift of about 60 eV. This indicates the remnant of a new quantum number associated with the fourfold

rotational symmetry.

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The identification of rotational bands in atomic nuclei about forty years ago led to the conclusion that nuclei may possess a deformed shape [1]. In most cases spin sequences with ΔI=2, connected with E2-transition matrix elements, form the rotational bands. This implies R-invariance (C_2 symmetry), i.e., the intrinsic Hamiltonian is invariant under a rotation of 180° around an axis perpendicular to the symmetry axis and the signature, $r = \exp(\pi i I) = \exp(\pi i \alpha)$, is a good quantum number [2]. In some nuclei there are indications of a further reduction of symmetry as the rotational spectra are consistent with reflection asymmetric shapes which in the extreme limit means ΔI=1 rotational band sequences with alternating parity [3].

In this Letter we report on experimental evidence for the presence of a *higher* symmetry in the rotational spectra. The yrast superdeformed band in ^{149}Gd [5,6] shows an unexpected staggering. At high rotational frequencies the ΔI=2 rotational band is perturbed and two ΔI=4 rotational sequences emerge with an energy splitting of about 120 eV. This feature suggests the remnant of a quantum number associated with an invariance of the Hamiltonian under a rotation of 90° around the rotation axis (C_4-symmetry). A classical analogy of this effect is a C_4-bifurcation in nonlinear dynamics [4].

The yrast superdeformed band of ^{149}Gd has been studied using the Eurogam [7] multidetector array. In its phase I configuration, the Franco-British γ-ray spectrometer sited at the Daresbury Nuclear Structure Facility consisted of 44 large-volume HPGe detectors. Each individual Ge crystal was surrounded by a bismuth germanate (BGO) Compton-suppression shield. In the present experiment, the ^{149}Gd residual nucleus was populated via the ^{124}Sn(^{30}Si,5n)^{149}Gd reaction at a bombarding energy of 158 MeV. The target consisted of a stack of two tin foils of 0.5 mg cm^{-2} thickness enriched to 98% in ^{124}Sn. The event trigger required at least six unsuppressed Ge detectors to fire. With this experimental setup, approximately 1×10^9 events were acquired during a three-day time period. After unpacking the higher-fold events, there were a total of $\sim 3 \times 10^9$ four-fold Compton-suppressed γ-ray coincidences in the data set.

The high-energy part of the γ-ray spectrum corresponding to the yrast superdeformed band in ^{149}Gd is presented in Fig. 1 and the γ-ray energies are listed in Table I. The analysis of four- and five-fold coincidence events [8] produces extremely clean spectra which require little background subtraction. This was essential for a very precise determination of γ-ray energies. The background spectra corresponding to the n-fold data were generated by selecting the same energy windows from (n-1)-fold γ-ray coincidences. The experimental uncertainties on the transition energies quoted in Table I take into account both the statistical errors from the coincidence spectrum and its associated background spectrum. Furthermore, strongly contaminated transitions were avoided in the gating procedure. For example, the band member at 1167 keV has not been selected since this γ ray is very close in energy with a transition belonging to the normal-deformed level scheme [9] in the spin region where the superdeformed band decays. It is therefore natural to observe, in the final one-dimensional spectrum of Fig. 1, an apparently larger intensity for this transition compared with the intensities of other band members. The new γ-ray transition at 1730 keV represents the highest rotational frequency ($\hbar\omega = E_\gamma/2$) observed so far in a discrete superdeformed band in the A~150 mass region. Assuming the theoretical spin assignments of Ragnarsson [10,11], this γ ray is emitted by a state having an angular momentum of $I =$ 139/2 $\hbar$.

The deduced $\Im^{(2)}$ dynamical moment of inertia of the yrast superdeformed band in ^{149}Gd is plotted in Fig. 2 as a function of rotational frequency. This quantity, related to the curvature of the excitation energy as a function of spin, can be extracted from the energy difference between two consecutive quadrupole transitions in the band by $\Im^{(2)}(I) = 4\hbar^2/\Delta E_\gamma(I)$ where $\Delta E_\gamma(I) = E_\gamma(I+2) - E_\gamma(I)$. The dynamical moment of inertia therefore does not depend on the knowledge of the spin I but only on the measured γ-ray energies. The $\Im^{(2)}$ and ΔE_γ values are also listed in Table I. In the rare-earth region, the gross behavior of $\Im^{(2)}$ associated with the occupancy of high-N intruder orbitals is rather well understood [12,13]. For example, the configuration $\pi 6^2 \nu 7^1$ assigned to this band not only reproduces the absolute magnitude but also the slope of $\Im^{(2)}$ as a function of $\hbar\omega$. However, as seen in Fig. 2, the moment of inertia exhibits an anomalous staggering at frequencies greater than $\hbar\omega = 0.49$ MeV. The amplitude of the oscillations is weak but definitely outside the experimental error bars so that the staggering effect certainly originates from a perturbation of the superdeformed energy levels. At the highest observed frequencies ($\hbar\omega \geq 0.79$ MeV) the oscillation pattern becomes irregular and it may invert but the irregularity could also be due to an accidental degeneracy with another band having the same spin and parity. It should be pointed out that the observed $\Im^{(2)}$ oscillations cannot be an artefact related to the non-linearity of the detector electronics because (i) the yrast superdeformed bands in ^{151}Tb and ^{152}Dy studied with the same experimental device do not show the effect and (ii) there were some hints of $\Im^{(2)}$ staggering in a study of ^{149}Gd performed [6] with a different γ-ray spectrometer.

The effects of the perturbation on ΔE_γ can be experimentally determined by comparing the $\Delta E_\gamma(I)$ values with a smooth reference calculated with the help of the expression $\Delta E_\gamma^{ref}(I) = [\Delta E_\gamma(I+2) + 2\Delta E_\gamma(I) + \Delta E_\gamma(I-2)]/4$. As shown in Fig. 3, the absolute value of $\Delta E_\gamma(I) - \Delta E_\gamma^{ref}(I)$ in the region between $\hbar\omega \simeq 0.50$ and 0.75 MeV corresponds on average to 230 $\pm$ 43 eV, with alternating signs. This pattern can be produced by a similar alternating effect in the γ-ray transition energies, the E_γ values being alternately shifted up and down by 115 $\pm$ 22 eV. The superdeformed energy levels are consequently separated into two sequences with the spin values I, $I+4$, $I+8$, ... and $I+2$, $I+6$, $I+10$, ..., respectively. The ΔI=2 spin states are however still connected with strong E2 transition matrix elements, the levels being, for example, alternately pushed up and down by 58 $\pm$ 11 eV relative to their unperturbed positions. Considering the fact that these superdeformed states are at an excitation energy of $\sim$20 MeV above the ground state, the observed shifts correspond to a 10^{-6} perturbation on the energy levels.

Due to the regularity of the perturbation over many transitions the phenomenon causing it is unlikely to be of chaotic origin. Thus explanations based on the interaction with the states in the first well, or the coupling to the fission channel [14] are less probable. The presence of the two regular ΔI=4 families in a rotational band suggests an explanation based on a fourfold rotational symmetry. For the quantal system which is invariant under a rotation of $\pi/2$ about the x-axis (i.e., invariant with respect to the point group C_4), a good quantum number appears, $r_4 \equiv \exp(\frac{1}{2}\pi i I)$. By introducing the r_4-exponent quantum number, α_4, defined by means of $r_4 = \exp(\frac{1}{2}\pi i \alpha_4)$, the following relation holds:

$$I = \alpha_4 \bmod 4, \tag{1}$$

where α_4 can take the values {0,1,2,3} (even-A) and $\{\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}\}$ (odd-A). Excitations with the same α_4 are expected to form one family of states. In the presence of the C_4-type perturbation the rotational energy can be written as

$$E(I) = \tilde{E}(I) + (-1)^{\frac{1}{2}[\alpha_4 - \alpha]} C_0 \tag{2}$$

where $\tilde{E}(I)$ is a smooth function of I and C_0 is the magnitude of the perturbation. Examples of molecular rotations in the presence of C_4 symmetry can be found in Ref. [15].

In atomic nuclei there have been a few examples indicating an approximate fourfold rotational symmetry. For instance, the 4p-4h band in ^{16}O built upon the $I^\pi = 0^+_2$ level at 6.0494 MeV [16] can be understood in terms of four alpha particles in a plane forming the kite shape [17], i.e., close to the (C_4-invariant) square configuration. For this band, the parameter C_0 =194 keV. Examples of $\Im^{(2)}$ oscillations have been pointed out by Peker *et al.* [18] in the octupole $K^\pi = 0^-$ bands of ^{236}U (C_0 =361 eV) and ^{238}U (C_0 =46 eV). However, in all these cases the $\Im^{(2)}$ oscillations are not as regular as in ^{149}Gd and they take place over much shorter spin sequences. The C_0-values for ^{149}Gd, ^{236}U, and ^{238}U are three orders of magnitude smaller than that for the kite-shaped ^{16}O suggesting that the former cases are associated with dynamical effects.

Hamamoto using the particle plus-rotor-model description has pointed out [19] that a strongly triaxial system with irrotational moments of inertia ($\gamma = -30°$), for which the two principal moments of inertia, $\Im_2$ and $\Im_3$, are equal and smaller than $\Im_1$, is invariant with respect to C_4. However, according to calculations [12] the superdeformed bands in the A$\sim$150 mass region are fairly rigid with respect to γ and the moments of inertia differ from irrotational ones because of weak pairing correlations at superdeformed shapes.

A possible origin of the fourfold symmetry manifestation could be the coupling to the hexadecapole field, suggested in Ref. [18]. The yrast superdeformed band in ^{149}Gd is expected [12] to have a large β_4 deformation of $\sim$0.08 which changes with rotational frequency, indicating softness to the hexadecapole distortion. In the limit of large angular momentum, the hexadecapole phonon becomes aligned along the axis of rotation (x-axis) producing a perturbation invariant with respect to $R_1(\pi/2)$. While two components of this field, namely $Q^4_{0_x}$ and $Q^4_{2_x}$, lead to a small renormalization of the quadrupole fields with M_x=0 and 2, the $Q^4_{4_x}$ interaction is the first non-trivial one [20]. Such a perturbation can cause a small (second order) splitting between states with different α_4 values. Unfortunately, due to the smallness of the effect it is difficult to confirm this scenario by microscopic calculations.

Several observations regarding the ΔI=4 effect in ^{149}Gd can be drawn. Firstly, there is evidence that the staggering effect in $\Im^{(2)}$ may also be present in other superdeformed bands [6,21] based on configurations involving one N=7 neutron but is definitely absent in bands based on two N=7 neutrons (e.g., yrast superdeformed bands in ^{151}Tb and ^{152}Dy). Secondly, the staggering sets in at a rotational frequency slightly above $\hbar\omega$=0.4 MeV at which the pair of N=6 protons becomes aligned [12]. This suggests that the effect could either be associated with the polarization of the superdeformed ^{152}Dy core by three aligned high-N holes, or by the mutual proton-neutron interaction between the N=6 and N=7 valence holes, or both. Thirdly, it is interesting to note that the K^π=0$^-$ bands in ^{236,238}U discussed by Peker *et al.* also involve the N=6 protons and N=7 neutrons. These observations suggest that the microscopic origin of the bifurcation is linked with these high-N orbitals.

In summary, we have observed bifurcation of the dynamical moment of inertia at high spin in the yrast superdeformed band of ^{149}Gd. It is related to states differing by four units of angular momentum having the same perturbation and indicates evidence of a quantum number associated with the fourfold rotational symmetry. The observed perturbation is extremely small and only the analysis of four- and five-fold coincidences has allowed the determination of the γ-ray transition energies with the precision required to reveal a systematic effect.

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TABLE I. Measured γ-ray transition energies, energy differences between two consecutive transitions, and dynamical moment of inertia of the yrast superdeformed band in ^{149}Gd.

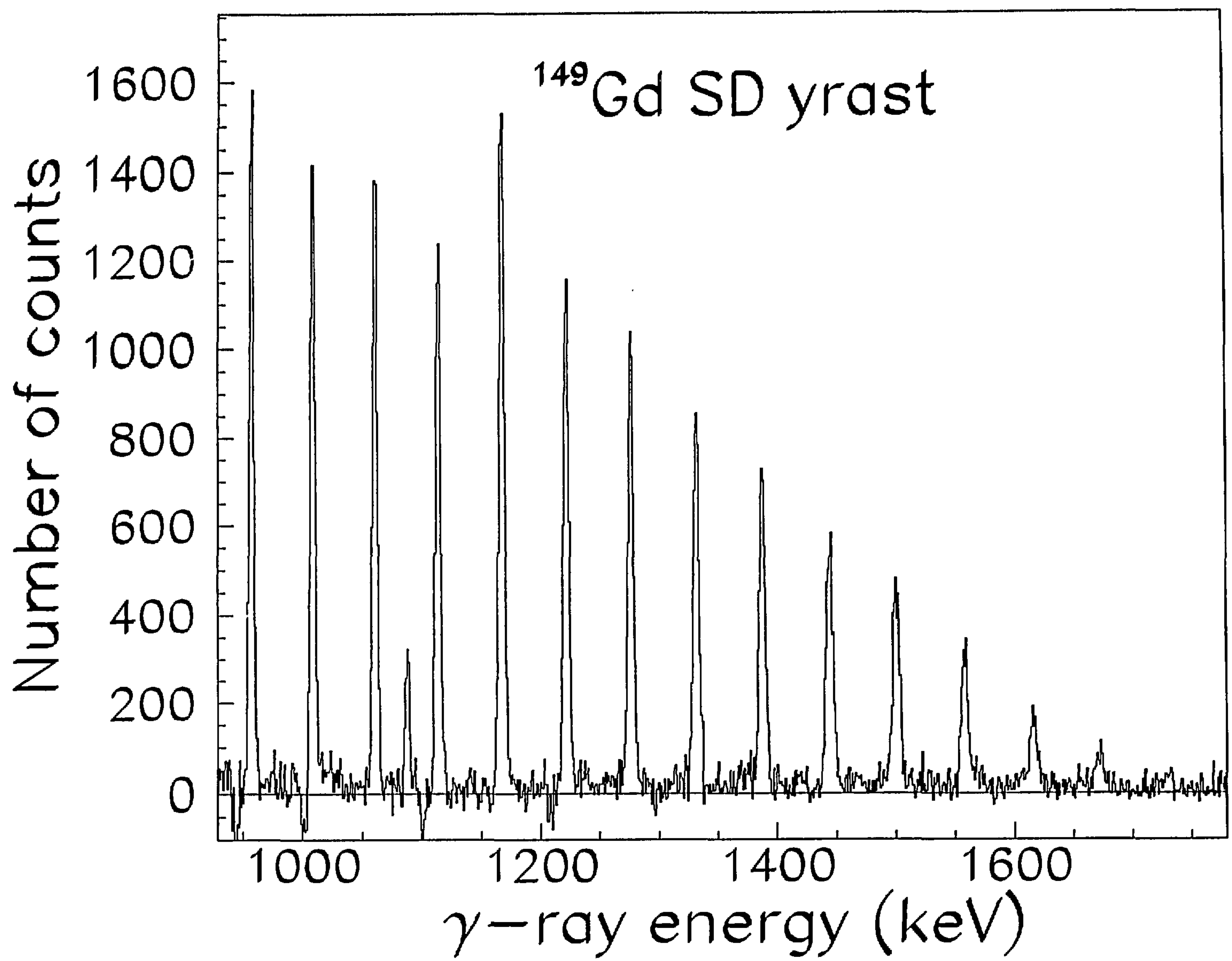
E_γ (keV)	ΔE_γ (keV)	$\Im^{(2)}$ ($\hbar^2$ MeV^{-1})
617.8(1)	46.4(1)	86.2(2)
664.2(1)	47.6(1)	84.0(2)
711.8(1)	47.9(1)	83.5(2)
759.7(1)	48.4(1)	82.6(2)
808.1(1)	49.0(1)	81.6(2)
857.1(1)	49.6(1)	80.6(2)
906.7(1)	50.4(1)	79.4(2)
957.1(1)	51.6(1)	77.5(2)
1008.7(1)	52.0(1)	76.9(2)
1060.7(1)	53.1(1)	75.3(2)
1113.8(1)	53.4(2)	74.9(3)
1167.2(2)	54.6(2)	73.3(3)
1221.8(1)	54.7(1)	73.1(2)
1276.5(1)	55.5(1)	72.1(2)
1332.0(1)	55.6(1)	71.9(2)
1387.6(1)	56.6(1)	70.7(2)
1444.2(1)	56.3(2)	71.0(3)
1500.5(2)	57.3(3)	69.8(3)
1557.8(2)	57.9(4)	69.1(4)
1615.7(3)	56.4(5)	70.9(6)
1672.1(4)	57.8(9)	69.2(11)
1729.9(8)		

FIGURES

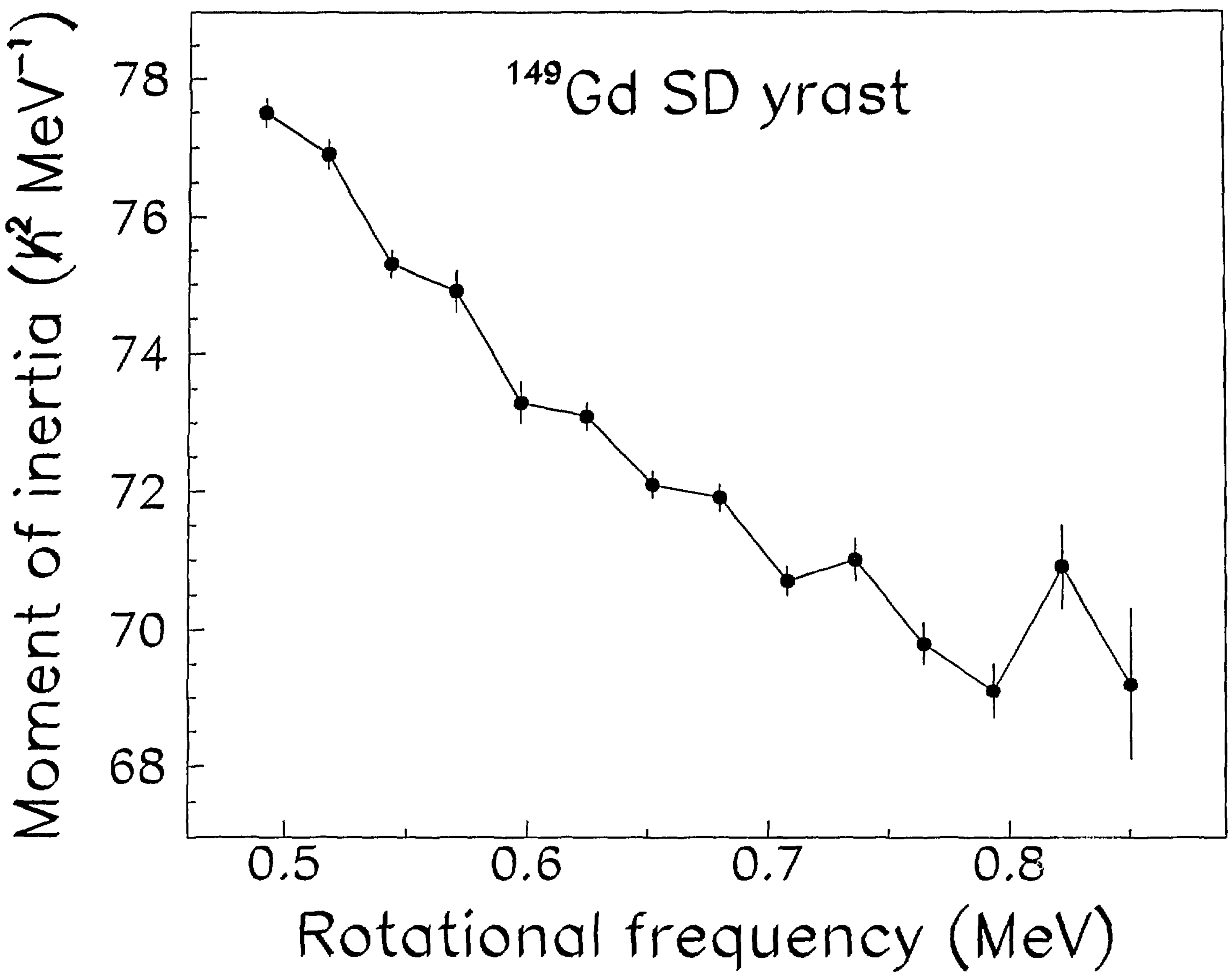
FIG. 1. Partial γ-ray spectrum corresponding to the yrast superdeformed band in ^{149}Gd obtained by summing a number of combinations of 3-dimensional energy windows set on four-fold coincidence events. A small fraction of the corresponding double-gated spectrum has been subtracted.

FIG. 2. Dynamical moment of inertia of the yrast superdeformed band in ^{149}Gd corresponding to the γ-ray energy region displayed in Fig. 1. The solid line joining the points is intended to guide the eye.

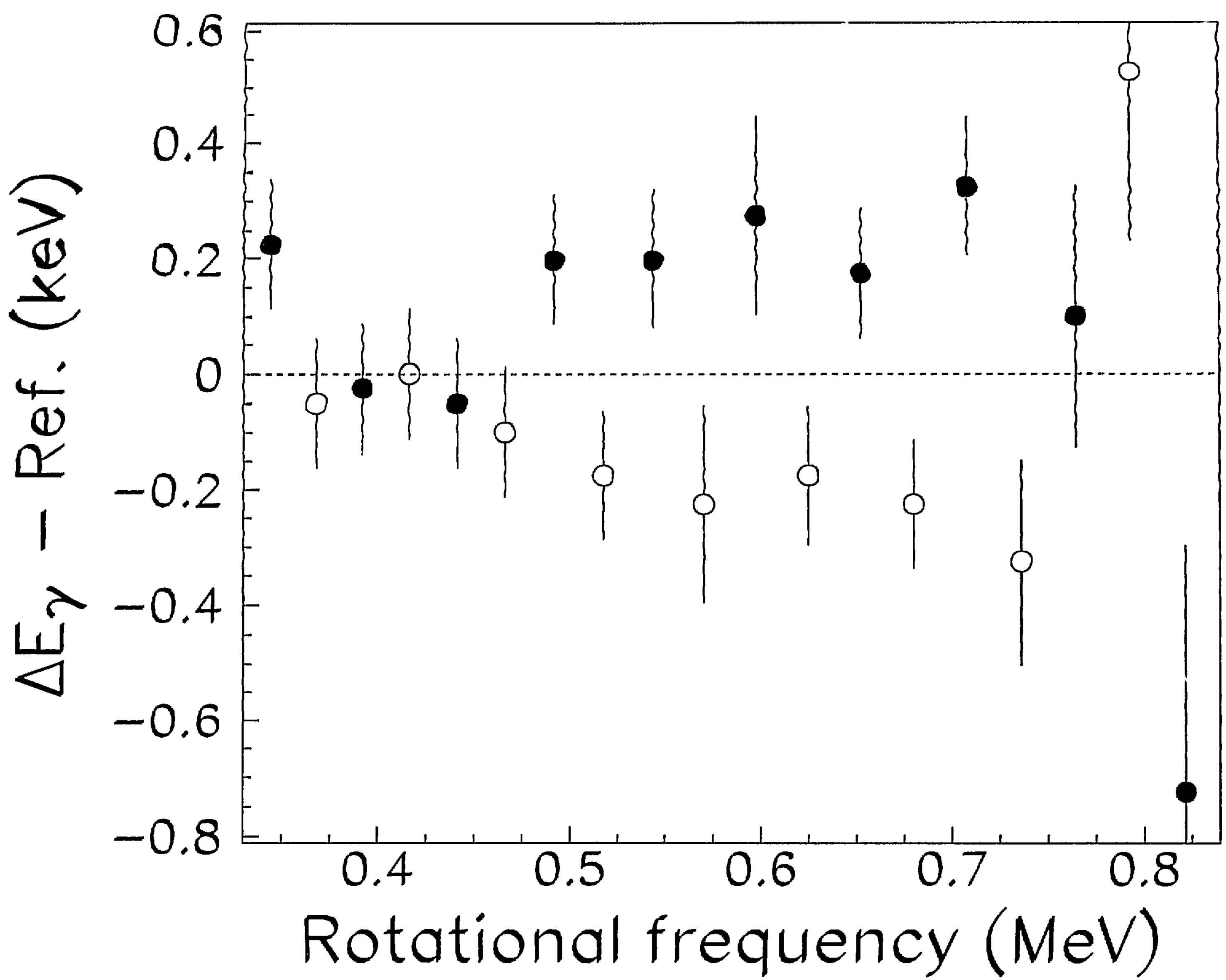
FIG. 3. Energy differences ΔE_γ between two consecutive γ-ray transitions of the superdeformed band in ^{149}Gd as a function of rotational frequency after subtraction of a smooth reference given by $\Delta E_\gamma^{ref}(I) = [\Delta E_\gamma(I+2)+2\Delta E_\gamma(I)+\Delta E_\gamma(I-2)]/4$. Filled and empty symbols refer to different values of α_4. The staggering effect sets in just above $\hbar\omega$=0.4 MeV, i.e., just after alignment of the N=6 proton pair.



^{149}Gd SD yrast
Number of counts
0
200
400
600
800
1000
1200
1400
1600
1000
1200
1400
1600
γ−ray energy (keV)



^{149}Gd SD yrast
Moment of inertia ($\hbar^2$ MeV^{-1})
78
76
74
72
70
68
0.5
0.6
0.7
0.8
Rotational frequency (MeV)



0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
ΔEγ − Ref. (keV)
0.4
0.5
0.6
0.7
0.8
Rotational frequency (MeV)